Physics of Monopoles in QCD

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Abstract

We review the physics of monopoles in non-Abelian gauge theories. The format of the review is determined by the fact that there exists a lot of data on the monopoles accumulated via numerical lattice simulations. Therefore, the emphasize is made on the continuum-limit formulation of the theory which would allow to interpret the data in field theoretical terms. In particular, we discuss the monopole-antimonopole potential at short and larger distances as well as a dual formulation of the gluodynamics, relevant to the physics of the confinement. This review is an adaptation for the purposes of the Zuoz School of earlier reviews and original papers written in collaboration with M.N. Chernodub, F.V. Gubarev, and M.I. Polikarpov.

1 Basic notions

1.1 Introduction

This review is devoted to the physics of monopoles in non-Abelian theories. The magnetic monopoles is an important and fascinating subject. It is fascinating because the magnetic monopoles [1] is a beautiful and challenging

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theoretical construction which occupies the minds of many theorists for about 70 years. The first 50 years of development of the theory of the magnetic monopoles were summarized in an illuminating review by Coleman [2]. However, the construction might look too exotic to be relevant to the real world. Especially the famous Dirac string which is transporting magnetic flux to the monopole from the infinity. Therefore the relevance to "everyday physics" was first dramatically revealed by the non-Abelian gauge theories where the monopoles are invoked to explain the the color confinement. The basic features of the mechanism were confirmed through the lattice simulations, for review and further references see, e.g., [3].

The accumulation of the lattice data constitutes the main development since the Coleman's review. For a theorist, the excitement is that the theory can now be confronted with the data. True, the monopoles observed are not exactly those introduced by Dirac, but rather their close akins, that is monopoles of non-Abelian gauge theories (moreover, for the sake of definiteness we concentrate on the simplest gauge group, that is $SU(2)$). Also true, the data are numerical mostly and obtained on the lattice so that their interpretation in terms of the continuum theory may not be so straightforward. But nevertheless the monopoles are basicly the same Dirac monopoles and one can test the theoretical ideas discussed for years.

At a closer look, the issue is not so much to compare the data with the theory as to develop the theory itself since in many respect it is in its infancy. This makes the topic of the magnetic monopoles to fit the purposes of the present school very well: the magnetic monopoles will be with us for the years to come and ask for fresh ideas and understanding of very non-trivial theoretical issues.

Coming closer to the substance of the lectures, we should notice first of all that the field of the so to say lattice monopoles is very rich and varied by itself. More specifically, let us mention here three topics:

(a) The numerical data refer mostly to the monopoles with a double magnetic charge, $|Q_m| = 2$ where the units are fixed by the Dirac quantization condition for the gluons. Classically, there are no stable solutions with $|Q_m| = 2$ [4] and, therefore, quantum effects seem to be absolutely crucial even to introduce such monopoles. As a result, the theory of these monopoles is practically nonexistent.

(b) There are recent measurements on the lattice [5, 6] of the interaction potential between the fundamental monopoles with $|Q_m| = 1$. Unlike the case of the $|Q_m| = 2$ monopoles, the interaction of the fundamental monopoles is

in fact quite well understood. The fact, which might be not well appreciated by the community.

(c) There exist surprisingly simple phenomenological descriptions of the $|Q_m| = 2$ monopoles which are so poorly understood on the purely theoretical side. We mean here models like the Abelian Higgs model which provide quantitative support to the old idea of the dual-superconductor mechanism [7] and work surprisingly well at least in some cases, for review and further references see [3, 8].

In this review we will emphasize some new points related to each of the items (a)-(c) listed above and which are based mostly on the original papers [9, 10, 11]. The new points, although they refer to various topics, are unified by a common approach. The starting point is that we consider monopoles within the fundamental gluodynamics while the more traditional path is to introduce monopoles within an effective theory intended to mimic QCD in the infrared region [3, 8]. Also, we understand the continuum gluodynamics rather as the limiting case of the lattice formulation. As a result, one allows for certain singular gauge transformations which are not included in more traditional frameworks.

1.2 Dirac monopoles, Dirac strings, Dirac veto

The Dirac monopole of charge Q_m, by definition, produces a radial magnetic field similar to the electric field of a point-like charge,

$$\mathbf{H} = (4\pi)^{-1} Q_m \, (\mathbf{r}/r^3)$$

. One can easily construct a corresponding vector potential:

$$A_r \;=\; A_\theta \;= 0, \quad A_\phi \;=\; \frac{Q_m}{4\pi} \frac{(1+\cos\theta)}{r\sin\theta}. \tag{1}$$

The analogy between the electric and magnetic charges is somewhat formal, however. Namely, because of the conservation of the magnetic flux, the radial magnetic field of the monopole should be supplemented by the magnetic field of a string which brings in the flux spread out uniformly by the radial component of the field. Thus, we actually have

$$\mathbf{H} \;=\; \mathbf{H}_{rad} + \mathbf{H}_{string} \, . \tag{2}$$

The presence of the string is exhibited, in particular, by the explicit expression for the potential $\mathbf{A}$ above.

The Dirac string is unphysical and there is a number of constraints imposed on the theory to ensure that the string does not produce any physical effect. First, there is the *Dirac veto* which forbids any direct interaction with the string. The condition is not trivial at all, in fact. Indeed, let us consider perturbation theory and the scattering of a charged particle off the string. In perturbative approach one uses the basis o plane wave functions. Which implies that the in-going particle can be found at any space-point and overlaps, therefore, with the string. Which is a violation of the Dirac veto. Thus, a simple minded Born approximation can be misleading. Of course, this consideration alone does not allow us yet to conclude whether this overlap of the plane waves with the string is significant. But a little bit more involved estimates do indicate that the overlap is important numerically and the perturbation theory can bring wrong conclusions. We shall examine these issues in much more detail in the chapter devoted to the radiative corrections and here just state that a consistent treatment of the Dirac string is always non-perturbative in nature because of the Dirac veto. This simple observation allows to resolve many theoretical puzzles.

The best known constraint on the Dirac string seems to be *the Dirac quantization condition* which ensures absence of the Aharonov-Bohm effect for the electrons scattered off the string:

$$Q_e \oint_{string} \mathbf{A} \mathrm{d}\mathbf{x} \;=\; Q_e\, Q_m \;=\; 2\pi k\,, \tag{3}$$

where Q_e is the electric charge of the electron and k is an integer number. Note that as a result of the quantization condition the interaction of the monopoles is described by a large coupling constant if the electrons interact weakly and vice versa. Also, the interaction of an electron with the monopole is always governed by a coupling of order unit and is never weak. This is another reason for the use of non-perturbative methods in the theory of monopoles, which is of course always a challenge.

1.3 Energy of the Dirac string

Unlike the quantization condition (3), the energy of the Dirac string is rarely discussed. It is worth to emphasize, therefore, that naively the energy of the string is infinite in the ultraviolet.

Indeed, let us estimate the energy of the string:

$$\epsilon_{string} \sim \int (\mathbf{H}_{string})^2 \, \mathrm{d}^3 r \;\sim\; \frac{(Length)}{Q_e^2 (Area)} \;\sim\; Q_e^{-2} \Lambda_{UV}^2 (Length) \quad . \tag{4}$$

We accounted here for the fact that the magnetic flux is quantized (see above),

$$(flux) \;\equiv\; \int \mathbf{H}\cdot d\mathbf{s} \;=\; \frac{2\pi k}{Q_e}$$

and tended the cross section of the string denoted by $(Area)$ to zero at the end of the calculation. Thus, we substituted $(Area)^{-1}$ by Λ_{UV}^2.

The radial part of the magnetic field is also associated with an infinite energy:

$$\epsilon_{rad} \;\sim \int (\mathbf{H}_{rad})^2 \, \mathrm{d}^3 r \;\sim\; \frac{1}{r_0} \;\sim\; \Lambda_{UV}\,. \tag{5}$$

Note that this ultraviolet divergence is linear, i.e. weaker than the divergence due to the string, see Eq. (4).

The infinite magnetic field of the string may have more subtle manifestations as well. Consider interaction of two magnetic monopoles with magnetic charge $\pm Q_m$ placed at distance R from each other. Then, by the analogy with the the case of two electric charges, we would like to have the following expression for the interaction energy:

$$\epsilon_{int} \;=\; \int \mathbf{H}_{1,rad}\mathbf{H}_{2,rad} \, \mathrm{d}^3 r \;=\; -\,\frac{Q_m^2}{4\pi}\,\frac{1}{R}\,. \tag{6}$$

Note, however, that if we substitute the sum of the radial and string fields for $\mathbf{H}_{1,2}$, then we have an extra term in the interaction energy:

$$\tilde{\epsilon}_{int} \;=\; \int (\mathbf{H}_{1,rad}\mathbf{H}_{2,string} + \mathbf{H}_{1,string}\mathbf{H}_{2,rad}) \, \mathrm{d}^3 r \;=\; +\,2\,\frac{Q_m^2}{4\pi}\,\frac{1}{R}\,. \tag{7}$$

In other words, the account of the string field would flip the sign of the interaction energy! This contribution, although looks absolutely finite, is of course a manifestation of the singular nature of the string magnetic field, $|\mathbf{H}_{string}| \sim (Flux)/(Area)$. Note that the integral in (7) does not depend on the shape of the string.

To maintain the unphysical nature of the Dirac string we should use a regularization scheme which would allow to get rid of these singularities.

1.4 Lattice regularization.

Since the monopoles are naively having divergent energy (or action) in the ultraviolet, the regularization is a crucial issue. Moreover, we would like to follow the lattice formulation since the monopoles are observed on the lattice. And for a good reason, as we shall immediately see.

Consider first the $U(1)$ case. As is emphasized in Ref. [12], the lattice formulation implies that Dirac string which produces no Aharonov-Bohm scattering costs no action as well. The reason is very simple. The lattice action is written originally in terms of the contour integrals like (3) rather than field strength $F_{\mu\nu}$:

$$S \;=\; \sum_p \mathrm{Re}\;\exp\{iQ_e \oint_{\partial p} A_\mu dx^\mu\}\,, \tag{8}$$

where the sum is taken over all the plaquettes p. Thus, the condition (3) means absence, in the lattice formulation, of both the Aharonov-Bohm effect and the quadratic divergence (4). Moreover, it is straightforward to see that the interference term (7) also vanishes.

Later, we will also discuss Dirac strings which correspond to negative plaquettes in the lattice formulation. Such strings are associated with the fundamental monopoles with $Q_m = 1$ in terms of the monopole classification discussed above. The energy of such Dirac strings is infinite in the continuum limit, in agreement with the naive estimate (4). The interference term (7), however, disappears in the lattice formulation in this case as well. later we shall see that this observation is crucial to understand the heavy monopole potential in the continuum limit.

The radial field, $\mathbf{H}_{rad}$ may also cause problems with infinite energy, see (5). The lattice regularization is not much specific in that case, however. The role of r_0 is simply played by the lattice spacing a. Thus, the probability to find a monopole on the lattice is suppressed by the action as:

$$e^{-S} \;\sim\; \exp(-const\cdot Q_e^{-2}\,L/a)\,, \tag{9}$$

where L is the length of the monopole trajectory, and the Q_e^{-2} factor appears because of the Dirac quantization condition (3) which relates the magnetic charge Q_m to the inverse electric charge.

Although the Eq. (9), at first sight, rules out monopoles as physically significant excitations, the fate of the monopoles in the $U(1)$ case depends in fact on the value of the charge Q_e. The point is that the entropy factor grows also exponentially with the length of the monopole trajectory:

$$(Entropy) \;\sim\; \exp(+const'\cdot L/a)\,, \tag{10}$$

where the $const'$ is a pure geometric factor, not related to any coupling constant like Q_e. As a result for $Q_e \sim 1$ there is a phase transition corresponding to the condensation of the monopoles. This phase transition, which is well

studied on the lattice, is the first and striking example of importance of the UV regularization in the non-perturbative sector. Indeed, once the UV divergence (4) is removed by the lattice regularization the monopoles can modify the physics completely (for further comments see [13]).

1.5 Classification of monopoles in non-Abelian theories.

From now on, we will discuss monopoles in unbroken non-Abelian gauge theories, having in mind primarily gluodynamics, i.e. quantum chromodynamics without dynamical quarks. Moreover, for the sake of simplicity we will consider only the $SU(2)$ gauge group.

A natural starting point to consider monopoles in non-Abelian theories is their classification. There are actually a few approaches to the monopole classification and it is important to realize both similarities and differences between them.

The dynamical, or $U(1)$ classification. Within this approach [14], one looks for monopole-like solutions of the classical Yang-Mills equations. Where by the "monopole-like" solutions one understands potentials which fall off as $1/r$ at large r, see Eq. (1). The basic finding is that there are no specific non-Abelian solutions and all the monopoles can be viewed as Abelian-like embedded into the $SU(2)$ group. Moreover, using the gauge invariance one can always choose the corresponding $U(1)$ group as, say, the rotation group around the third direction in the color space. According to this classification, the monopoles are characterized by their charge with respect to a $U(1)$ group and may have, therefore, charges,

$$|Q_m| \;=\; 0,\; 1,\; 2,\; ... \quad . \tag{11}$$

The topological, or Z_2 classification. The Z_2 classification [15] is based entirely on topological arguments. Namely, independent types of monopoles are enumerated by considering the first homotopy group of the gauge group. The SU(2) gauge group is trivial since $\pi_1(SU(2)) = 0$, while in the case of the $SO(3)$,

$$\pi_1(SU(2)/Z_2) \;=\; Z_2, \tag{12}$$

and there exists a single non-trivial topological monopole. We will denote the magnetic charge of such monopoles as $|Q_m| = 1$. Note, however, that the charges $Q_m = \pm 1$ are indistinguishable in fact. As for the charges $Q_m = 2$ they are equivalent, from this point of view, to no magnetic charge at all.

The topological classification (12) is readily understood if one tries to enumerate various types of the Dirac strings whose end points represent monopoles under consideration. Then there is only one non-trivial string, that is the one for which Eq. (3) is satisfied for gluons but not for quarks. Namely, because the $U(1)$ charge associated with gluons is twice as big as that of the particles in the fundamental representation (quarks) we may have

$$\exp\{ ig \oint A_\mu dx^\mu \} \;=\; -1 \tag{13}$$

and such a string is not visible for the isospin-one particles. On the other hand, the standard plaquette action is based on the phase factor evaluated for particles in the fundamental representation. Which means, in turn, that the Dirac string is piercing the negative plaquettes. This observation is the basis for introducing the $|Q_m| = 1$ monopoles via the 't Hooft loop: one changes the sign of β ($\beta \equiv 4/g^2$) on a world sheet. The boundary of this sheet corresponds to the end points of the Dirac string, or the monopole trajectory.

1.6 Z_2 monopoles.

In principle, the $U(1)$ and Z_2 classifications are different. Indeed, while the $U(1)$ classification allows for any integer charge, the Z_2 classification leaves space only for a single non-trivial charge:

$$Q_m \;=\; 0, 1. \tag{14}$$

The reconciliation of the two classifications is that the $U(1)$ solutions with $|Q_m| \geq 2$ are in fact unstable because of the presence massless charged vector particles (gluons) [4]. The instability of the solutions implies that even if the external sources with $|Q_m| \geq 2$ were introduced into the vacuum state of the gluodynamics, charged gluons would fall onto the center because of the strong magnetic interactions. Moreover, one can imagine that as result of this instability the charged fields $A^\pm$ are build up as well.

In a somewhat related way, one can demonstrate the apparent irrelevance of the $|Q_m| = 2$ monopoles by producing an explicit non-Abelian field configuration which looks as a $|Q_m| = 2$ monopole in its Abelian part but has *no* $SU(2)$ *action* at all [9]. This field configuration is a Dirac string with open ends, which correspond to the monopole-anti-monopole pair separated by the distance R. Note that the Abelian flux is still transported along the Dirac string and is still conserved for the radial field. What is lost, however,

is the relation between the Abelian flux and action. In the Abelian case non-vanishing flux means non-vanishing magnetic field and non-vanishing action since the action density is simply $\mathbf{H}^2$. Now the action is $(F^a_{\mu\nu})^2$ and the Abelian part of the $F^3_{\mu\nu}$ can be canceled by the commutator term.

The simplest example illustrating the cancellation between the Abelian component and the commutator term in $F^a_{\mu\nu}$ is produced by the potential:

$$A^a_\mu \;=\; -\frac{2}{r^2}\epsilon^{abc}r^b\frac{\sigma^a}{2} \quad , \tag{15}$$

where σ^2 are the Pauli matrices and r^a is the radius vector. One can readily check by a direct calculation that the corresponding non-Abelian field-strength tensor vanishes identically (some care should be exercised to analyze the singularity at $r = 0$). On the other hand, the Abelian part of the field-strength tensor looks as a monopole of charge $Q_m = 2$. Another way to convince oneself that the field strength corresponding to (15) vanishes is to observe that

$$A^a_\mu \;=\; i(\Omega^0)^\dagger\partial_\mu\Omega^0, \tag{16}$$

where

$$\Omega^0 \;=\; i\sigma^a n^a,$$

and n^a is the unit vector looking from the center of the "monopole" to the observation point.

We pause here to note that the potential (15) does not exhibit any Dirac string. The reason is that we did not gauge rotate it to the form suitable for the $U(1)$ classification of the monopoles. The absence of the Dirac string may look appealing at first sight. The shortcoming of using an explicitly non-Abelian field, like (15) is that one cannot add up in this case the monopole fields if the monopoles are situated at different space points. Which rules out in fact any possibility of an explicit calculation almost in any interesting in applications case. Thus, we do not use such forms in this review although one cannot rule out that in future some progress can be made to overcome the difficulty with adding up the monopoles written in the form similar to (15). Also, writing the potential in the form like (15) produces a new and simple argument to disregard the energy of the Dirac strings connecting monopoles with $|Q_m| = 2$.

Coming back to the problem of writing a filed configuration which in its Abelian part looks as a monopole-antimonopole pair ($|Q_m| = 2$) we will use formulation with a Dirac string. Algebraically the problem becomes more complicated and we reproduce only the final answer [9]. Namely, such a

configuration is generated from the vacuum by the following gauge rotation matrix:

$$\Omega \;=\; \begin{pmatrix} e^{i\varphi}\sqrt{A_D} & \sqrt{1-A_D} \\ -\sqrt{1-A_D} & e^{-i\varphi}\sqrt{A_D} \end{pmatrix}, \tag{17}$$

where φ is the angle of rotation around the axis connecting the monopoles and A_D is the $U(1)$ potential representing pure Abelian monopole pair:

$$A_\mu dx_\mu \;=\; \frac{1}{2}\left(\frac{z_+}{r_+} - \frac{z_-}{r_-}\right) d\varphi \;\equiv\; A_D(z,\rho)d\varphi\,, \tag{18}$$

where $z_\pm = z \pm R/2$, $\rho^2 = x^2 + y^2$, $r_\pm^2 = z_\pm^2 + \rho^2$. Note that the action associated with the Dirac string is considered in this case zero, in accordance with the lattice version of the theory (for details see [9]).

In this example, the monopoles with $|Q_m| = 2$ are a kind of a pure gauge field configurations carrying no action.

It is somewhat more difficult to visualize dynamically the equivalence of the $Q_m = \pm 1$ monopoles, also implied by the Z_2 classification. The mechanism mixing the $Q_m = \pm 1$ solutions seems to be the following. Imagine that we start with, say, $Q_m = +1$ solution. Then a Dirac string carrying the flux corresponding to the $Q_m = -2$ can be superimposed on this solution. It is important at this point that such a Dirac string costs no action (or energy). Then the radial magnetic field can also change its direction since this does not contradict the flux conservation any longer. In a related language, one could say that the $|Q_m| = 2$ monopoles are condensed in the vacuum and as a result the magnetic charge can be changed freely by two units.

As far as interaction of two $|Q_m| = 1$ monopoles is concerned, one might expect that they would interact as an monopole-antimonopole pair. Indeed monopole and antimonopole would attract each other and thus represent the lowest energy state of the system.

1.7 Conclusions # 1

Thus, the physics of the monopoles in non-Abelian theories in the first approximation turns very simple.

Namely, there exist only monopoles with $|Q_m| = 1 \equiv 2\pi/g$ where g is the coupling constant of the non-Abelian $SU(2)$ theory. The monopoles are infinitely heavy and their interaction is Abelian like:

$$V_{m\bar{m}} \;=\; -\,\frac{Q_m^2}{4\pi R} \;=\; -\,\frac{\pi}{g^2 R}\,, \tag{19}$$

where R is the separation between the monopoles.

Clearly enough, this first, or classical approximation falls far beyond an adequate description of the empirical data on the monopoles, see the Introduction. Thus, we are invited to go into more advanced approaches which we would try to introduce step by step.

2 The Zwanziger Lagrangian

2.1 Photons, direct and dual

There is a long standing interest in constructing the dual gluodynamics, for review and further references see [8]. The dual gluon, by definition, interacts with monopoles. The motivation is to realize in the field theoretical language the dual superconductor model of the quark confinement [7] according to which the quarks are connected at large distances by an Abrikosov-type vortex [16]. The key element is the construction of the non-Abelian monopoles, which are usually modeled after the 't Hooft–Polyakov solution. Namely, one introduces first non-Abelian dual gluons interacting with Higgs fields and then assumes condensation of the Higgs fields which mimics the condensation of the monopoles. In the realistic case of the $SU(3)$ gauge group one needs an octet of dual gluons and three octets of the Higgs fields, all of them understood in terms of effective field theory valid in the infrared region.

While such a construction might be viable as an effective theory, we need in fact tools to describe interaction of non-Abelian monopoles at arbitrary short distances as well [9]. Indeed, in the lattice version of the theory external monopoles can be introduced via the 't Hooft loop operator [17] and in the continuum limit these monopoles are point like. Thus, we are encouraged to consider the dual gluodynamics at short distances, or at the *fundamental* level.

It is natural to try a Lagrangian approach to the dual gluodynamics. Indeed, in case of the same 't Hooft loop operator it is known that its expectation value depends only on the boundary, or the monopole trajectory, and not on the shape of the Dirac string. Thus, it seems natural to introduce a dual gluon which would interact directly with point-like monopoles. In the context of electrodynamics, the idea is very old and goes back to papers in Ref. [18]. There are successes and problems inherent to this approach, for a review see [19].

Consider first electrodynamics. A well known example of Lagrangian which describes interaction of a U(1) gauge fields with Abelian point-like monopoles is due to Zwanziger [18]:

$$L_{Zw}(A,B) \;=\; \frac{1}{2}(m\cdot[\partial\wedge A])^2 \;+\; \frac{1}{2}(m\cdot[\partial\wedge B])^2 \;+ \tag{20}$$

$$+\,\frac{i}{2}(m\cdot[\partial\wedge A])(m\cdot{}^*[\partial\wedge B]) - \frac{i}{2}(m\cdot[\partial\wedge B])(m\cdot{}^*[\partial\wedge A]) + i\,j_e\cdot A + i\,j_m\cdot B\,,$$

where j_e, j_m are electric and magnetic currents, respectively, m_μ is a constant vector, $m^2=1$ and

$$[A\wedge B]_{\mu\nu} = A_\mu B_\nu - A_\nu B_\mu\,, \qquad (m\cdot[A\wedge B])_\mu = m_\nu[A\wedge B]_{\mu\nu}\,,$$

$$^*[A\wedge B]_{\mu\nu} \;=\; \frac{1}{2}\,\varepsilon_{\mu\nu\lambda\rho}\,[A\wedge B]_{\lambda\rho}\,.$$

At first sight, we have introduced two different vector fields, A and B, to describe interaction with electric and magnetic charges, respectively. If it were so, however, we would have solved a wrong problem because we need to have a single photon interacting both with electric and magnetic charges. And this is what is achieved by the construct (20). Indeed, the action (20) is not diagonal in the A, B fields and one can convince oneself that the form of the bilinear in A, B interference terms in (20) is such that the field strength tensors constructed on the potentials A and B are in fact related to each other:

$$F_{\mu\nu}(A) \;=\; {}^*F_{\mu\nu}(B)\,, \tag{21}$$

where ${}^*F$ denotes the dual tensor. Eq. (21) implies in turn that there are only two physical degrees of freedom corresponding to the transverse photons which can be described either in terms of the potential A or B. Topological excitations, however, can be different in terms of A and B.

The physical content of (20) is revealed by the propagators for the fields A, B. In the α-gauge one can derive:

$$\begin{aligned} \langle A_\mu A_\nu\rangle \;&=\; \langle B_\mu B_\nu\rangle \;=\; \tfrac{1}{k^2}\,(\delta_{\mu\nu} \;-\; (1-\alpha)\,\tfrac{k_\mu k_\nu}{k^2})\,, \\ \langle A_\mu B_\nu\rangle \;&=\; -\,\langle B_\mu A_\nu\rangle \;=\; \tfrac{i}{k^2(km)}\;{}^*[m\wedge k]_{\mu\nu}\,. \end{aligned} \tag{22}$$

The propagators should reproduce, as usual, the classical solutions. And indeed, the $\langle AA\rangle$, $\langle BB\rangle$ propagators describe the Coulomb-like interaction of two electric charges and magnetic monopoles, respectively. While the $\langle AB\rangle$ propagator reproduces interaction of the magnetic field of a monopole

with a moving electric charge. The appearance of the poles in $(k \cdot m)$ is a manifestation of the Dirac strings.

To summarize, the Zwanziger Lagrangian in electrodynamics [18] reproduces the classical interaction of monopoles and charges. Upon the quantization, it describes the correct number of the degrees of freedom associated with the photon.

2.2 Dual gluon as an Abelian vector field.

Now, if we would approach the problem of constructing a Zwanziger-type Lagrangian for the dual gluodynamics, we immediately come to a paradoxical conclusion that the dual field, if any, is Abelian. Indeed, monopoles associated with, say, $SU(N)$ gauge group are classified according to $U(1)^{N-1}$ subgroups [14] and might be realized as a pure Abelian objects. Thus, there is no place for a non-Abelian dual gluon because the monopoles do not constitute representations of the non-Abelian group.

The function of the classical Lagrangian is, first of all, to reproduce the classical interactions of the monopoles and charges. It is rather obvious that the potential (19) can be derived in the classical approximation from the Lagrangian:

$$L_{dual}(A^a, B) \;=\; \frac{1}{4}(F^a_{\mu\nu})^2 \;+\; \frac{1}{2}(m \cdot [\,\partial \wedge B - i\,{}^*G\,])^2 \;+\; i\,j_m B \;+\; i\,j^a_e A^a\,, \quad (23)$$

where $a = 1, 2, 3$ is the color index, j_m is the magnetic current and $F^a_{\mu\nu}$ is the non-Abelian field strength tensor. The Lagrangian (23) also contains vector field n^a, $n^2 = 1$ in the adjoint representation and antisymmetric tensor $G_{\mu\nu}$ is the 't Hooft tensor [21]:

$$G_{\mu\nu} \;=\; n^a F^a_{\mu\nu} \;-\; \varepsilon^{abc}\, n^a\, (D_\mu n)^b\, (D_\nu n)^c\,. \quad (24)$$

Let us add a few comments on the meaning and rules of using the Lagrangian (23).

(a) First, if the magnetic current is vanishing, $j_m = 0$ then the integration over the field B reproduces the standard Lagrangian of the gluodynamics.

(b) As far as the quantization is concerned, the Lagrangian (23) reproduces the correct degrees of freedom of the free gluons. Indeed, in the limit $g \to 0$ and for $n^a = \delta^{a,3}$ the Lagrangian (23) becomes:

$$L_{dual}(A^a, B) \;=\; \frac{1}{4}(\partial \wedge A^1)^2 + \frac{1}{4}(\partial \wedge A^2)^2 + i\,j_m B + i\,j^a_e A^a +$$

$$+\frac{1}{2}[m\cdot(\partial\wedge A^3)]^2+\frac{1}{2}[m\cdot(\partial\wedge B)]^2+ \tag{25}$$

$$+\frac{i}{2}[m\cdot(\partial\wedge A^3)][m\cdot{}^*(\partial\wedge B)]-\frac{i}{2}[m\cdot{}^*(\partial\wedge A^3)][m\cdot(\partial\wedge B)]\,,$$

which is essentially the Zwanziger Lagrangian (20). Quantization at this point is the same as in the case of a single photon.

(c) Already in the Zwanziger example (20) we have seen that the fields that are mixed up in the Lagrangian have a common source. Namely, in case of the electrodynamics $\partial^*F(A)=\partial F(B)=j_m$. Since it is known [21] that the monopoles in non-Abelian theories serve as sources for the 't Hooft tensor (24) one expect from the very beginning that in case of the gluodynamics the (dual) field strength tensor build up on the dual gluon field B is mixed up with the 't Hooft tensor constructed in terms of the gluon filed A. And, indeed, this is true for (23).

(d) The emergence of the vector n^a in the Lagrangian (23) is of crucial importance. The point is that the origin of the vector n^a goes back to choosing the color orientation of the monopoles. As is emphasized above the monopole solutions are Abelian in nature which means, in particular, that they can be rotated to any direction in the color space by gauge transformations. Thus, picking up a particular n^a is nothing else but using the gauge fixing freedom. Therefore, we can either average over the directions of n^a or fix n^a but evaluate only gauge invariant quantities, like the Wilson loop (note somewhat similar remarks in Ref. [20]).

(e) The Z_2 nature of the monopoles is manifested in the freedom of changing $n^a\to -n^a$, $B_\mu\to -B_\mu$. Indeed, under such transformation the monopole with the charge $Q_m=+1$ is transformed into a monopole with $Q_m=-1$ and vice versa. In the language we used above such a transformation corresponds to adding a Dirac string with a double magnetic flux. We see that the averaging over $\pm n^a$ is a part of the overall averaging over all possible embedding of the $U(1)$ into the $SU(2)$ gauge group.

An apparent application of (23) would be evaluating the running of the coupling g in the expression (19). And, indeed, exploiting the Lagrangian (23) one can approach the problem of the running of the coupling in a way similar to the case of pure electrodynamics, for a review and further references see [19]. We comment on this approach below.

2.3 Conclusions # 2

The use of the Zwanziger Lagrangian is a convenient means to describe interaction of photon with both magnetic and electric charges. Formally,

one introduces two potentials, A_μ, B_μ. However, because of the condition $F_{\mu\nu}(A) = {}^*F_{\mu\nu}(B)$ there is a single photon in fact. One can get rid of one of the potentials. But then the Dirac string emerges explicitly and the formulation becomes non-local.

Another point, very important from the point of view of the phenomenological applications, is that in case of the gluodynamics the dual gluon (that is the field B) is still Abelian like although the "direct" gluons (that is the fields A^a) are in an adjoint representation of the non-Abelian group.

3 More on the monopoles with Q_m=1 and Q_m=2

3.1 Dirac strings in the non-Abelian case

The Dirac strings remain essentially Abelian objects in case of non-Abelian theories as well. A novel feature is that the field transported along the string can be arbitrarily oriented in the color space. Which is a kind of a new gauge freedom.

In more detail, the general one-plaquette action of $SU(2)$ lattice gauge theory (LGT) can be represented as:

$$S_{lat}(U) = \frac{4}{g^2} \sum_p S_p \left(1 - \frac{1}{2} Tr U[\partial p]\right), \tag{26}$$

where g is the bare coupling, ∂p is the boundary of an elementary plaquette p, the sum is taken over all p, $U[\partial p]$ is the ordered product of link variables U_l along ∂p. In particular, if $S_P(x) = x$ then (26) is the standard Wilson action. The exponent of the lattice field strength tensor F_p is defined in terms of $U[\partial p]$:

$$U[\partial p] = e^{i\hat{F}_p} = \cos[\frac{1}{2}|F_p|] + i\tau^a n^a \sin[\frac{1}{2}|F_p|], \tag{27}$$

where $\hat{F} = F^a \cdot \tau^a/2, |F| = \sqrt{F^a F^a}$ and we define $n_p^a = F_p^a/|F_p|$ for $|F_p| \neq 0$, while n_p^a is an arbitrary unit vector for $|F_p| = 0$.

The lattice action (26) depends only on $\cos[\frac{1}{2}|F_p|]$. Therefore the action of the $SU(2)$ LGT possesses not only the usual gauge symmetry, but allows

also for the gauge transformations which shift the field strength tensor by $4\pi k$, $|F_p| \to |F_p| + 4\pi k$, $k \in Z$:

$$e^{i\hat{F}_p} \;=\; \exp\{i|F_p|\hat{n}_p\} \;=\; \exp\{i(|F_p| + 4\pi)\hat{n}_p\} \;=\; \exp\{i(F_p^a + 4\pi n_p^a)\tau^a/2\}. \tag{28}$$

Thus, the symmetry inherent to the lattice formulation can be represented as:

$$F_p^a \;\to\; F_p^a + 4\pi n_p^a, \quad \vec{F}_p \times \vec{n}_p \;=\; 0, \quad n_p^2 = 1. \tag{29}$$

The symmetry (29) is absent in the conventional continuum limit, $\int (F_{\mu\nu}^a)^2 d^4x$. Note that in the continuum limit n_p^a becomes a singular two-dimensional structure ${}^*\Sigma_{\mu\nu}^a$ which is representing the Dirac string world sheet.

3.2 The 't Hooft loop

So far we discussed invisible Dirac strings. As we know, the Dirac strings corresponding to the fundamental monopoles with $Q_m = 1$ are having an infinite action, or energy. In the lattice formulation they correspond to the plaquettes with the phase factor $exp(i\pi)$ rather than $exp(2i\pi)$, as for the invisible Dirac strings. This infinite energy precludes the fundamental monopoles from being dynamical objects present in the QCD vacuum. However, they can be introduced as external objects. The infinite energy of the corresponding Dirac string does not matter than, the same as with the infinite self-energy of heavy quarks introduced via the Wilson loop. The fundamental monopoles can be introduced on the lattice via the 't Hooft loop [17]. In a simplified way, the monopoles are visualized as the end-point of the corresponding Dirac strings which in turn are defined as piercing negative plaquettes. The trick to introduce the negative plaquettes on the lattice is to formally change the sign of the coupling g^2 on a manifold of plaquettes. Then these plaquettes become negative in the limit $|g^2| \to 0$.

Proceeding to more detailed definitions, the 't Hooft loop is formulated then (see [5] and references therein) in terms of the action

$$S(\beta, -\beta) = \beta \sum_{p \notin M} \mathrm{Tr}\, U_p - \beta \sum_{p \in M} \mathrm{Tr}\, U_p \,, \tag{30}$$

where M is a manifold which is dual to the surface spanned on the monopole world-line j. Introducing the corresponding partition function, $Z(\beta, -\beta)$ and considering a planar rectangular $T \times R$, $T \gg R$ contour j one can define

$$V_{m\bar{m}}(R) \;\equiv\; -\frac{1}{T} \ln \frac{Z(\beta, -\beta)}{Z(\beta, \beta)} \,. \tag{31}$$

By the analogy with expectation value of the Wilson loop the quantity $V_{m\bar{m}}(R)$ is referred to as monopole-antimonopole, or heavy monopole potential. The potential $V_{m\bar{m}}(R)$, in a way, is the same fundamental quantity as the heavy-quark potential $V_{Q\bar{Q}}$ and its understanding within the fundamental QCD would be of great importance.

After the experience with working out the continuum limit of the Dirac string world sheet in the preceding subsection, there is no difficulty to figure out that a continuum analog of the 't Hooft loop looks as [11]:

$$H(\Sigma_{\mathcal{C}}) = \exp\left\{\frac{1}{4g^2}\int d^4x\ \left[\left(F^a_{\mu\nu}\right)^2 - \left(F^a_{\mu\nu} + 2\pi^*\Sigma^a_{\mathcal{C}\ \mu\nu}\right)^2\right]\right\}, \qquad (32)$$

$$\Sigma^a_{\mathcal{C}\ \mu\nu} = \int d^2\sigma_{\mu\nu}\ n^a(\sigma)\ \delta^{(4)}(x - \tilde{x}(\sigma))\,, \qquad (33)$$

where the surface $\Sigma^a_{\mathcal{C}}$ spanned on the contour $\mathcal{C}$ is assumed to be non-intersecting. The unit three-dimensional vector field $n^a(\sigma)$, $\vec{n}^2 = 1$ is defined on the world-sheet:

$$n^a(\sigma) \ = \ (t\cdot{}^*F^a)\ \left[(t\cdot{}^*F^b)^2\right]^{-1/2}\,, \qquad (t\cdot F^a) = t_{\mu\nu}(\sigma)\ F^a_{\mu\nu}(\tilde{x})\,, \qquad (34)$$

$$t_{\mu\nu}(\sigma) = \frac{1}{\sqrt{g}}\ \varepsilon^{\alpha\beta}\ \partial_\alpha\tilde{x}_\mu\ \partial_\beta\tilde{x}_\nu\,, \qquad t^2_{\mu\nu} = 2\,, \qquad g(\sigma) = \mathrm{Det}[\ \partial_\alpha\tilde{x}_\mu\ \partial_\beta\tilde{x}_\mu\ ]\,. \qquad (35)$$

Therefore $n^a(\sigma)$ is not an independent variable, it is completely determined by the components of the field strength tensor $F^a_{\mu\nu}$. On the set of points where $(t\cdot{}^*F^a) = 0$ the direction of $n^a(\sigma)$ is arbitrary. It can be shown [10] that the Eq. (32)-(35) define the correct 't Hooft loop operator the expectation value of which depends only on the contour $\mathcal{C}$, not on a particular position of the surface $\Sigma_{\mathcal{C}}$.

3.3 The heavy monopole potential

Having the continuum analog of the 't Hooft loop in hand, we can derive now the potential energy of two fundamental monopoles in the classical approximation.

Indeed, consider the equations of motion in presence of the 't Hooft loop operator:

$$D_\nu\left(F_{\mu\nu}(A) + 2\pi^*\Sigma_{\mu\nu}\right) \ = \ 0, \qquad (36)$$

which should be supplemented by the Bianchi identities:

$$D_\nu{}^*F_{\mu\nu} \ = \ 0. \qquad (37)$$

To appreciate the meaning of the equation of motion (36) let us choose the gauge such that $\Sigma^a_{\mu\nu}$ has a constant color orientation characterized by the vector n^a_0. A particular solution of (36) may be found within the anzatz $A^a_\mu = n^a_0 A_\mu$, for which Eq. (36) reduces to:

$$\partial_\nu \left(\partial_{[\mu} A_{\nu]}\right) \;=\; -\,2\pi \partial_\nu {}^*\Sigma_{\mu\nu}. \tag{38}$$

The solution of this equation in the Landau gauge,

$$A^a_\mu \;=\; -\,n^a_0 \cdot 2\pi \frac{1}{\Delta} \partial_\nu {}^*\Sigma_{\mu\nu}, \tag{39}$$

corresponds to the gauge potential of an Abelian monopoles current $\partial\Sigma$ embedded into the $SU(2)$ group. Thus, $\Sigma_{\mu\nu}$ is the Dirac world sheet.

Derivation of the classical equations of motion (38) is the first step in deriving the interaction of the fundamental monopoles outside any Lagrangian framework.

3.4 The $Q_m = 2$ monopoles as quantum objects

So far we discussed the fundamental monopoles $|Q_m| = 1$ which can be visualized as classical infinitely heavy objects. Because of the infinite energy, they can be used only as probes of the QCD vacuum but play no dynamical role by themselves. The monopoles with the double charge $|Q_m| = 2$ are very different. As discussed above, they do not exist on the classical level. On the other hand, one can argue in a simple way that the $Q_m = 2$ monopoles play a dynamical role on the quantum level. Indeed, as far as the fundamental Lagrangian is concerned, the only role of the quantum corrections is the running of the non-Abelian coupling. In particular, if we consider a lattice coarse enough then $g_{SU(2)}$ becomes of order unity. Obviously, the same coupling governs the physics associated with any $U(1)$ subgroup of the $SU(2)$. However, if the coupling $g_{U(1)}$ becomes of order unity, then there is a phase transition associated with the monopole condensation [12], see discussion in Chapter 1.

Thus, it is very natural to assume that the monopole condensation occurs also in QCD since the running of the coupling allows so to say to scan the physics at all the values of $g_{SU(2)}$ until one runs into a phase transition.

However, even if one accepts such speculations, there remains a very important unresolved question. Namely, it is not clear which $U(1)$ subgroup of the full non-Abelian group is to be selected as the classification group for

the monopoles. The most common approach here is to rely on the empirical data. In a way, it is forced on us since the phase transition is expected to happen at $g^2_{SU(2)} \sim 1$ where analytical approaches are hardly possible. From the lattice simulations it is known that the monopoles in the Maximal Abelian projection appear to be most relevant, see [3] for review and further references.

Instead of reviewing this material once more – which would take us far beyond the scope of the present review – we will highlight some features of a new kind of monopoles introduced in Ref. [10]. The basic idea behind this construction is to make monopoles as much geometrical objects as possible.

3.5 Geometrical monopoles

The construction of the new kind monopoles is in few steps which we will briefly outline now.

(*i*) The usual starting point to introduce monopoles is to fix some $U(1)$ for the whole lattice and then look for the Dirac strings and monopoles with respect to this $U(1)$. The starting point of [10] is somewhat different. Namely, it is the observation that each Wilson loop defines in a natural way its own $U(1)$. Indeed, turn back to the expression (27) for the plaquette action which is actually true for any Wilson loop. Then, it is clear that each Wilson loop defines the vector $\hat{F}_p$ and the "natural" $U(1)$ is the group of rotation around this vector (in the color space). In this way, one can define a $U(1)$ group for each plaquette. The definition of the $U(1)$ subgroup varies from one plaquette to another, emphasizing the non-Abelian nature of the underlying theory.

(*ii*) The plaquette action is $1/2 \cos\phi$ and is invariant under $\phi_p \to \phi_p + 2\pi k$, as is emphasized in sect. 1.3. Now to detect the Dirac strings we should be able to somehow define the integer k. For a plaquette, the natural decomposition is

$$\phi_p \;=\; \phi_1 + \phi_2 + \phi_3 + \phi_4, \tag{40}$$

where the phases ϕ_i $(i = 1, ..., 4)$ are associated with the corresponding links. The decomposition (40) comes about naturally in the basis of the *coherent states*. Indeed, for a particular coherent state the whole evolution may be reduced to a phase factor:

$$|\psi(t)\rangle \;=\; e^{i\phi(t)}|\psi(0)\rangle. \tag{41}$$

Moreover, the coherent states can be explicitly constructed in terms of the link matrices, for details and further references see [10]. As a result, for any

given lattice fields configuration, one can determine k and detect the Dirac strings in this way. The monopoles are defined then as the end points of the strings.

(*iii*) The phase $\phi(t)$ can in fact be decomposed into the dynamical and Berry phase. It is useful for this purpose to introduce a single-valued state vector $|\tilde{\psi}\rangle$ defined as

$$|\tilde{\psi}(T)\rangle \;=\; |\tilde{\psi}(0)\rangle\,, \tag{42}$$

where T is the period of the motion so that at $t = T$ the system comes back to the same point in the parameter space as at the moment $t = 0$. Then

$$\phi(T) \;=\; \delta + \gamma \;=\; -\int_0^T \langle\tilde{\psi}|H\,|\tilde{\psi}\rangle + i\int_C \langle\tilde{\psi}|\frac{\partial}{\partial\lambda_i}|\tilde{\psi}\rangle\, d\lambda^i\,, \tag{43}$$

where λ_i are parameters, $\lambda_i(T) = \lambda_i(0)$ and C is a closed contour in the parameter space.

3.6 Choice of the gauge and the numerical results

The steps (*i*) – (*iii*) described above fully determine monopoles as geometrical objects. As a mathematical construct, it certainly appears very appealing. However, from the physical point of view the crucial observation is the gauge dependence of the monopoles constructed in this way. As a result, the monopoles are devoid, generally speaking, of any physical meaning. It is amusing that one can actually specify the conditions for the monopoles to be physical objects. Namely, the monopole density ρ should satisfy the renormgroup equation:

$$\rho \;=\; \text{const}\;\cdot\;\beta^{153/121}\;\exp\left(-\frac{9\pi^2}{11}\beta\right), \tag{44}$$

where $\beta \equiv 4/g^2$. The condition (44) is a very strong constraint and there is no much surprise that the monopoles defined according to the procedure outlined in the preceding subsection, generally speaking, do not satisfy (44).

To continue with the physics, we need a physically motivated choice of the gauge. At first sight, such a choice is impossible. However, one can argue [22, 10] that the Landau gauge is a proper gauge. The Landau gauge on the lattice is defined by the requirement that the functional

$$R \;=\; \sum_l \left(1 - \frac{1}{2}\mathrm{Tr}U_l\right) \tag{45}$$

is minimal on the gauge orbit (U_l denote the link matrices). In the naive continuum limit (45) reduces to $R = 1/4 \int (A^a_\mu)^2$.

The logic behind the choice (45) is as follows. In the continuum limit both the Dirac strings and monopoles correspond to singular gauge potentials A^a_μ. It is easy to imagine, therefore, that one can generate an arbitrary number of spurious strings and monopoles by going to arbitrary large potentials A, so to say inflated by the gauge transformations. On the other hand, by minimizing potentials one may hope to squeeze the number of the topological defects to its minimum and these topological defects may be physically significant.

And, indeed, the numerical simulations indicate that the geometrical monopoles defined in the Lorenz gauge are physical objects, i.e. their density satisfies the condition (44). There are also other indications that the geometrical monopoles are physical [10]. For example, there is an excess of the non-Abelian action associated with them.

3.7 Conclusions # 3

The fundamental monopoles with $Q_m = 1$ are essentially classical objects interacting as Abelian monopoles. On the lattice, their interaction can be measured via the 't Hooft loop allowing for a full account of the quantum corrections as well. We have worked out a continuum analog of the 't Hooft loop but have not considered yet the radiative corrections in the continuum limit.

The monopoles with $Q_m = 2$ are very different. They unify properties of field-theoretical and statistical objects. Namely, on one hand the monopoles are defined locally in terms of the link matrices. However, the link matrices are gauge dependent and the existence or non-existence of a monopole at a particular point is devoid of physical meaning for this reason. On the other hand, if one introduces gauge fixing in a physically reasonable way, the statistical properties of the monopoles, such as their density, satisfy very non-trivial renormgroup constraints and are physical.

4 Radiative corrections

4.1 Perturbative treatment and the role of the Dirac string

In this chapter, we will consider a more advanced theoretical topic, that is the radiative corrections to the Coulomb-like interaction (19) at short

distances. The reader more interested in the phenomenological applications could proceed directly to the next chapter.

Obviously enough, one would expect that the radiative corrections result in the standard, non-Abelian running of the coupling g^2. Which is indeed our main conclusion. Moreover, since for a constant vector n^a the non-Abelian monopole essentially coincides with the Dirac monopole, there is no much specific about the derivation of the running of the coupling. And, indeed, our considerations overlap to a great extent with those given in the original papers [23, 24, 25] and in the reviews [2, 19]. Still, we feel that it is useful to present the arguments, may be in a new sequence, to emphasize the points crucial for our purposes.

Let us emphasize from the very beginning that the evaluation of the radiative corrections addresses in fact two different, although closely related problems. That is, running of the coupling and stability of the classical solutions. Both aspects are unified, of course, into evaluation of a single loop in the classical background. However, the running of the coupling can be clarified by keeping track of the ultraviolet logs, $\ln \Lambda_{UV}$ alone. These logs are universal since in the ultraviolet all the external fields can be neglected. Therefore, the coefficient in front of $\ln \Lambda_{UV}$ can be found by evaluating the loop graph with two external legs, i.e. the graph corresponding to the standard polarization operator in perturbation theory. This is true despite of the fact that the monopole field is strong (i.e. the product of the magnetic and electric coupling is of order unit). On the other hand, the stability of the classical solution is decided by the physics in the infrared. Here one needs to consider the particular dynamical system, monopoles in our case, and the fact that the magnetic charge is of order $1/g$ can be crucial.

Consider first the running of the coupling. Moreover, for the sake of definiteness we concentrate on the Dirac monopole with the minimal magnetic charge interacting with electrons and in one-loop approximation [19, 23, 24, 25]. As we have just mentioned, the crucial point here is that only loops with insertion of two external (i.e., monopole) fields can be considered despite of the fact that there is no perturbative expansion at all.

Then, the evaluation of, say, first radiative correction to the propagator $\langle B_\mu B_\nu \rangle$ in the Zwanziger formalism (22) seems very straightforward and reduces to taking a product of two $\langle AB \rangle$ propagators and inserting in between the standard polarization operator of two electromagnetic currents. The result is [23]:

$$\langle B_\mu B_\nu \rangle(k) \;=\; \frac{\delta_{\mu\nu}}{k^2}(1-L) + \frac{1}{(k\cdot m)^2}(\delta_{\mu\nu} - m_\mu m_\nu)L, \qquad (46)$$

$$L \;=\; \frac{\alpha_{el}}{6}\ln\Lambda^2_{UV}/k^2$$

where we neglected the electron masses so that the infrared cut-off is provided, in the logarithmic approximation, by the momentum k.

At first sight, there is nothing disturbing about the result (46). Indeed, we have a renormalization of the original propagator which is to be absorbed into the running coupling, and a new structure with the factor $(k\cdot m)^{-2}$ which is non-vanishing, however, only on the Dirac string. The latter term would correspond to the renormalization of the Dirac-string self-energy which we do not follow in any case since it is included into self-energy of the external monopoles. What is, actually, disturbing is that according (46) the magnetic coupling runs exactly the same as the electric charge,

$$\langle A_\mu A_\nu\rangle(k) \;=\; (1-L)\frac{\delta_{\mu\nu}}{k^2}\,,$$

violating the Dirac quantization condition.

The origin of the trouble is not difficult to figure out. Indeed, using the propagator $\langle AB\rangle$ while evaluating the radiative corrections is equivalent, of course, to using the full potential corresponding to the Dirac monopole A^{cl}_D. Then, switching on the interaction with electrons would bring terms like $A^{cl}_D\bar\psi\gamma\psi$. Since A_D includes the potential of the string electrons do interact with the Dirac string and we are violating the Dirac "veto" which forbids any direct interaction with the string.

Let us demonstrate that, indeed, the incorrect treatment of the Dirac string changes the sign of the radiative correction. This can be done in fact in an amusingly simple way. First, let us note that it is much simpler to remove the string if one works in terms of the field strength tensor, not the potential. Indeed, we have $\mathbf{H} \;=\; \mathbf{H}_{rad} + \mathbf{H}_{string}$ while in terms of the potential $\mathbf{A}$ any separation of the string would be ambiguous (see Eq. (1)).

Thus, we start with relating the potential, or energy to the interference term in the $\mathbf{H}^2$ field:

$$V_{m\bar m} \;=\; \int \mathbf{H}_1\cdot\mathbf{H}_2\,\mathrm{d}^3r\,. \tag{47}$$

Now, it is not absolutely trivial, how we should understand the product $\mathbf{H}_1\cdot\mathbf{H}_2$. Indeed, we emphasized in section 1.2 that the string field is to be removed from this interference term, see Eq (7). Thus, in the zero, or classical approximation we have:

$$\mathbf{H}_1\cdot\mathbf{H}_2 \;\equiv\; \mathbf{H}_{1,rad}\cdot\mathbf{H}_{2,rad}\,. \tag{48}$$

However, if we use the standard technique of an external field:

$$A_\mu \;=\; A_\mu^{class} + a_\mu \tag{49}$$

and substitute (1) as the classical background then the first radiative correction would bring the product of the total $\mathbf{H}_1 \cdot \mathbf{H}_2$ which includes also the string contribution[1]. Indeed, the result in the log approximation would be as follows:

$$\delta(\mathbf{H}_1 \cdot \mathbf{H}_2) \;=\; L(\mathbf{H}_{1,string} + \mathbf{H}_{1,rad}) \cdot (\mathbf{H}_{2,string} + \mathbf{H}_{2,rad}) \;=\; -L\mathbf{H}_{1,rad} \cdot \mathbf{H}_{2,rad}, \tag{50}$$

where at the last step we have used the observation (7).

Now, it is clear how we could ameliorate the situation. Namely, to keep the Dirac string unphysical we should remove the string field from the expression (50) which arises automatically if we use the propagators (22) following from the Zwanziger Lagrangian. Thus, we introduce:

$$(\mathbf{H}_1 \cdot \mathbf{H}_2)' \;\equiv\; \mathbf{H}_{1,rad} \cdot \mathbf{H}_{2,rad} \tag{51}$$

and change $\mathbf{H}_1 \cdot \mathbf{H}_2$ in the expression (50) into (51) so to say by hand. The justification is that we should remove the effect of the string field from any observable.

Then we reverse the sign of the radiative correction and the final result is

$$(V_{m\bar{m}})_{class} \;\equiv\; -\frac{\pi}{g_0^2}\frac{1}{R} \;\to\; -\frac{\pi}{g^2(R)}\frac{1}{R}. \tag{52}$$

One might wonder, how it happens that the couplings in the electric and magnetic potential run in opposite ways. Indeed, now we reduced the product $\mathbf{H}_1 \cdot \mathbf{H}_2$ to exactly the same form as the product $\mathbf{E}_1 \cdot \mathbf{E}_2$ in case of two electric charges (since the radial magnetic and electric fields are the same, up to a change of the overall constants). The resolution of the paradox is that the renormalization of the electric and magnetic fields are indeed similar in the language of the Lagrangian. However, the small corrections to the Lagrangian and Hamiltonian are related as:

$$\delta L \;=\; -\,\delta H. \tag{53}$$

Since $\mathbf{E}^2$ and $\mathbf{H}^2$ enter with the same sign into the expression for the Hamiltonian and with the opposite signs into the Lagrangian, Eq (53) implies that

[1] At this point we assume in fact that Λ_{UV} is smaller than the inverse size of the string, which is convenient for our purposes here. Other limiting procedures could be considered as well, however.

the running of the couplings in the electric V_e and magnetic V_m potentials are opposite in sign. Which is, of course, in full agreement with expectations since $V_e \sim g^2$ and $V_m \sim g^{-2}$.

Thus, it is not difficult to derive the running of the magnetic coupling following only the ultraviolet log, $\ln \Lambda_{UV}$. Note, however, that the same arguments would go through without change if we started with, say, monopoles with $Q_m = 2$. But such monopoles are unstable [4] and this is a much more drastic effect than the would-be running of the coupling.

Thus, to investigate the stability of the classical solution one has, generally speaking, to consider all orders in $g_e\, g_m \sim 1$. It is known that single monopoles with $Q_m = 1$ are stable. The stability of the monopole-antimonopole system, which we are interested in, has never been investigated analytically in detail because of the complexity of the problem. However, there is no known mechanism which could cause instability of the classical monopole-antimonopole solution. Moreover, we checked numerically that the classical solution is indeed stable [9].

4.2 Why the "right way" is correct?

Thus, our exercise with evaluating the running of the magnetic coupling has brought mixed results. On one hand, we were able to derive that the product of electric and magnetic coupling constants is not renormalized, as one would expect. On the other hand, to derive this we had to go actually beyond the Lagrangian approach and remove the effect of the magnetic field of the string. Now we ask the next question, why this removing was the correct procedure.

Let us reexamine the grounds for the Lagrangian approach, in their generality. Any monopole involves also a Dirac string and, as a result, a world sheet, not just particle trajectories. If we stop here, then the conclusion would be that there is no Lagrangian approach to the problem. However, we are aware that the 't Hooft loop operator depends only on its boundary, which is the monopoles trajectory, j_m. And this is the real basis for the hopes for the Lagrangian formulation. Now, we see that the Dirac veto is not respected by the Lagrangian formulation and, therefore, the possibility arise that the world sheet swept by the Dirac string is still somehow important. Thus, we will outline in this subsection an approach [9] which is based on derivation of a continuum analog of the lattice 't Hooft loop operator and avoids any direct use of Lagrangians.

One could consider along these lines also the radiative corrections [11]. We will not go into details here but let us mention, how it comes about

that the "Dirac veto" is observed and virtual particles do not interact with the Dirac string. We will substantiate this point here on the example of the spin interaction. Since the Yang-Mills quanta possess spin, there exists interaction which is generalization of the non-relativistic expression $\sigma \cdot \mathbf{H}$. In particular, if there exists classical field directed in third direction in the color space, $(F^3_{\mu\nu})_{cl}$ then its interaction with the quantum charged fields $a^{\pm}_{\mu}$ contains the term

$$g\,(F^3_{\mu\nu})_{cl}\,a^+_\mu a^-_\nu\ . \tag{54}$$

In the Zwanziger formalism, the $(F^3_{\mu\nu})_{cl}$ means the whole magnetic field, the field of the string including. Then the interaction (54) brings the term $\mathbf{H}_{1,string}\cdot\mathbf{H}_{2,radial}$ on the level of the quantum corrections. As we emphasized in sect. 1.2, this term actually proportional to $\mathbf{H}_{1,radial}\cdot\mathbf{H}_{2,radial}$ which is responsible for the coupling running. In this way the term $\mathbf{H}_{1,string}\cdot\mathbf{H}_{2,radial}$, if it arises, brings in a "wrong" sign of the radiative correction.

On the other hand, in our formulation of the continuum analog of the 't Hooft loop operator, see Eq. (32), there is no spin interaction of the virtual particles with the string magnetic field. This is crucial to prove [11] that the coupling governing the monopole-antimonopole interaction indeed runs as g^{-2}.

4.3 Conclusions # 4

Second, we discussed how far one can go with a Lagrangian formulation of the dual gluodynamics a la Zwanziger. To test the Lagrangian approach we evaluated the running of the coupling in the monopole-antimonopole potential. The conclusion is that one can get the correct running of the coupling by imposing the Dirac veto which forbids the interaction of virtual particles with the Dirac string. This requirement is not inherent to the Lagrangian approach (the same is true for the Zwanziger Lagrangian in the $U(1)$ case), however. It can be derived by studying the continuum analog of the 't Hooft loop operator.

5 Phenomenological applications

5.1 The effective Lagrangian and the Casimir scaling

The standard way to develop a phenomenology is to assume that the monopoles condense. We will follow the suit and modify the Zwanziger Lagrangian (23)

by adding an effective Higgs interaction where the role of the Higgs field is played by the monopole field ϕ_m:

$$S_{eff} \;=\; S_{dual}(A^a, B) + S_{Higgs}(B, \phi_m)\,, \tag{55}$$

where S_{Higgs} is the standard action of the Abelian Higgs model. The vacuum expectation value of the Higgs, or monopole field is, of course, of order Λ_{QCD}.

Despite its apparent simplicity, Eq. (55) is highly speculative. Namely, it unifies so to say fundamental gluons, A^a, their dual counterpart B which is an Abelian gauge boson, and ϕ_m which is presumably an effective scalar field. One may justify the use of (55) by assuming that the effective size of the monopoles with $Q_m = 2$ is in fact numerically small, although generically it is of order Λ_{QCD}. While in our presentation here we follow mostly the lines of Refs. [9, 11, 26], let us note that similar consequences arise within the models [27, 28] also introducing a new mass scale.

What is also specific about the Lagrangian (55) is that the dual gluon is a $U(1)$ gauge boson. The color symmetry is maintained by averaging over all possible embeddings of the (dual) $U(1)$ into $SU(2)$, see the discussion in section 2.2. The confinement mechanism inherent to (55) is the formation of the Abrikosov-Nielsen-Olesen string which can be considered already on the classical level. More generally, the Lagrangian (55) exhibits the *Abelian dominance* in the confining region which is the dominance of Abelian-like field configurations in the full non-Abelian theory. This dominance is common to all the realizations of the dual-superconductor model of confinement [7] and is strongly supported by the lattice data [3]. What we avoid, however, is the *breaking of* $SU(2)$ *to* $U(1)$ which is inherent to the models [8] which start with the dual gluons in the adjoint representation and then add effective isospin one Higgs fields. Such models have well-known principal difficulties with, say, describing interaction of the adjoint sources, see, e.g. [29].

To the contrary, the model (55) can be applied to consider the static interaction of the sources belonging to various representations of $SU(2)$. One of the basic facts here, established through the numerical simulations on the lattice [30], is the so called Casimir scaling. The phenomenon of the Casimir scaling is that the static potential is described by a sum of the Coulomb-like and linear terms:

$$V_j(r) \;\approx\; -\,j(j+1)\,\frac{\alpha_s}{\pi\; r} \;+\; j(j+1)\,\sigma\, r\,, \tag{56}$$

where j labels the representation (we consider the $SU(2)$ case) and σ is independent of j. Note that at large distances one expects qualitatively

different behavior of the potential for integer and half-integer spins j because of the string breaking in case of the integer representations. However, at presently measured distances Eq. (56) turns to be a very good approximation to the potential.

The potential of the type (56) does arise in the classical approximation in the model (55) because there are classical string solutions. However, the tension of the string is now a dynamical quantity which can be found as a function of the parameters of the model, that is vector and Higgs masses:

$$\sigma \;=\; \sigma_j(\, m_H/m_V\,)\,. \tag{57}$$

In particular, the Casimir scaling holds in the London limit,

$$\frac{\sigma_{j_1}}{\sigma_{j_2}} \;=\; \frac{j_1(j_1+1)}{j_2(j_2+1)}\,, \quad \text{if} \quad m_H \gg m_V\,. \tag{58}$$

Thus, the model (55) can incorporate the Casimir scaling.

However, the description of the profile of the confining string is the best if $m_H \approx m_V$ [31]. Thus, there is mismatch with (58). Since the functions of m_H/m_V involved in the fits are rather smooth, it is possible to get a compromise description which is valid in both cases, say, with 20% accuracy. Which is not bad at all keeping in mind that we are using a classical approximation. Nevertheless, the fact that the Casimir scaling works at a per cent level [30] remains a kind of an unexplained mystery in the classical approximation. Further analysis of this point might be needed.

5.2 Unconventional power corrections

We introduced (55) as an effective Lagrangian. Now we will describe a rather paradoxical situation that this Lagrangian seem to provide with better phenomenology of the power corrections to the parton approximation than the conventional QCD approach. Namely, there are novel $1/Q^2$ corrections inherent to Higgs models [26, 27] which are absent in the standard considerations[2]. Moreover, these corrections seem to fit the data at all the distances measured so far, that is $r \geq 0.1$fm. Example of this type was found in Refs. [26, 32]. Further support came from the instanton physics [27]. We have already reviewed the unconventional power corrections in [13] and will be brief here.

[2]Literally, the model (55) which introduces averaging over all the embedding of the Abelian dual gluon into $SU(2)$ has not been ever discussed so far. However, as far as the power corrections are concerned there is no difference from the cases considered in [27, 26, 32].

In the standard approach, the power corrections are given by matrix elements of various operators constructed on the quark and gluon fields [33]. For our set up, the central point is that for the vacuum state the simplest matrix element had dimension $d = 4$,

$$\langle 0|\alpha_s(G^a_{\mu\nu})^2|0\rangle \;\sim\; \Lambda^4_{QCD}\,, \tag{59}$$

and, as a result, there are no Λ^2_{QCD}/Q^2 corrections [33]. On the other hand, the Higgs model has a mass parameter built it. This mass parameter can thought of as the mass of the vector particle, m^2_V. Let us list a few examples where the two approaches lead to different predictions:

(1) First, within the Higgs model the dual gluon gets mass and the monopole potential becomes Yukawa type:

$$V_{m\bar{m}} \;\to\; \frac{\pi}{g^2 r} e^{-m_V r}. \tag{60}$$

The prediction has already been confirmed by the data [5]. In the conventional approach, one should have to remove at least the term $\sim m^2 r$ at short distances. The quality of the data might be not so good as to rule this out, but the possibility looks quite bizarre.

(2) Since the dual and "ordinary" gluon are in fact the same particles (see discussion in section 2) one would assume that the massiveness of the dual gluon implies the massiveness of the gluon interacting with the color. But this is not true [26]! There is no analyticity in this sense. And the reason is again problems with the Dirac veto which we had already chance to discuss in connection with the radiative corrections (see section 2.3). Namely, as far as we discuss only the "dual world", one can forget about the Dirac strings. However, if we introduce color sources $Q\bar{Q}$ into the vacuum with $\langle\phi_M\rangle \neq 0$ we should respect the Dirac veto. The ordinary operator product expansion, or perturbative expansion in m^2_V/Q^2 do not respect this veto – like ordinary perturbation theory does not do this either (see section 2.1). The correct treatment demonstrates that their is a linear correction to the quark potential at short distances:

$$\delta V_{Q\bar{Q}} \;=\; \sigma_0 r\,, \tag{61}$$

where σ_0 is calculable function of m_H, m_V. Within the standard approach there is no such term, for explanations and further references see [13]. The data do support the presence of the linear term. Amusingly enough, the data refer exclusively to the the non-perturbative potential, and there is no need

for painful separation of (small) power corrections against the perturbative "background".

(3) The linear term (61) can be rephrased as the statement that the gluon has a tachyonic mass. Indeed, the ordinary mass would give a negative σ_0, as seen from the expansion of the Yukawa potential at short distances. The introduction of a tachyonic gluon mass in the framework of the QCD sum rules allows to resolve in an absolutely natural way long standing problems with the phenomenology based on the QCD sum rules [34].

(4) The last but not the least point in our discussion concerns the instanton density [27]. The conventional approach predicts that the deviations from the 't Hooft instanton density due to non-trivial background vacuum fields is of the fourth order in the instanton size ρ:

$$dn(\rho) \;=\; dn_{pert.}(\rho)\,(1 \;+\; \frac{\pi^4\rho^4}{2g^4}\,\langle 0|g^2(G^a_{\mu\nu})^2|0\rangle \;+\; ...) \qquad (62)$$

The data, on the other hand are beautifully fitted by a quadratic correction, inherent to (55). Note however, that the coefficient in front of the quadratic term has been fitted rather than calculated from (55) so far.

5.3 Conclusions # 5

We have proposed in this section a phenomenological Lagrangian (55) which unifies the Higgs mechanism for the Abelian dual gluon with full $SU(2)$ symmetry of the ordinary gluodynamics. The full study of the consequences from this formulation is still awaiting its time to come.

However, it seems promising that the color SU(2) is not broken at any step despite the Higgs mechanism. This allows to broaden applications of the effective Lagrangian and incorporate, to certain accuracy, the Casimir scaling.

Also, emergence of the mass of the dual gluon in the effective Lagrangian approach provides with a natural framework to introduce the novel $1/Q^2$ corrections. Phenomenologically, these corrections bring crucial improvements to the existing phenomenology. Moreover, generically the corrections are of the same type as those associated with ultraviolet renormalons (see, e.g., [35]). However, within the effective Lagrangian approach these corrections should disappear in the limit of infinite Q^2 which is not true for the ultraviolet renormalons and has not been supported by any data so far.

6 Conclusions

In this review we considered various effects related to the monopoles in unbroken non-Abelian gauge theories. In conclusion, let us reiterate the main points (see also conclusions to the Sections 1–5):

(i) Fundamental monopoles with the magnetic charge $|Q_m| = 1$ are introduced as external objects via the 't Hooft loop. The corresponding inter-monopole potential $V_{m\bar{m}}(r)$ can be evaluated at short distances from first principles, see Sections 2,4. In the Lagrangian approach similar to that of Zwanziger, the dual gluon interacting with point-like external monopoles appears as an *Abelian gauge field* (see Section 2 and [9, 11] for details).

(ii) Monopoles with the magnetic charge $|Q_m| = 2$ are pure quantum objects which can be studied so far only numerically. We discussed briefly the newly introduced [10] geometrical monopoles which appear to be physical objects.

(iii) Effective Lagrangian which assumes condensation of the monopoles incorporates the Abelian dominance at distances, where the effects of confinement are crucial, without breaking $SU(2)$ to $U(1)$. In the London limit, it reproduces the the Casimir scaling phenomenon. There are further phenomenological consequences, in particular, the evaluation of the potential $V_{m\bar{m}}(r)$ at larger distances (see Section 5 and [9, 11] for details).

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