PSI Particle Theory Seminar
March 13, 2008

Sudakov Logarithms for Neutral-Current Drell–Yan Processes at the LHC

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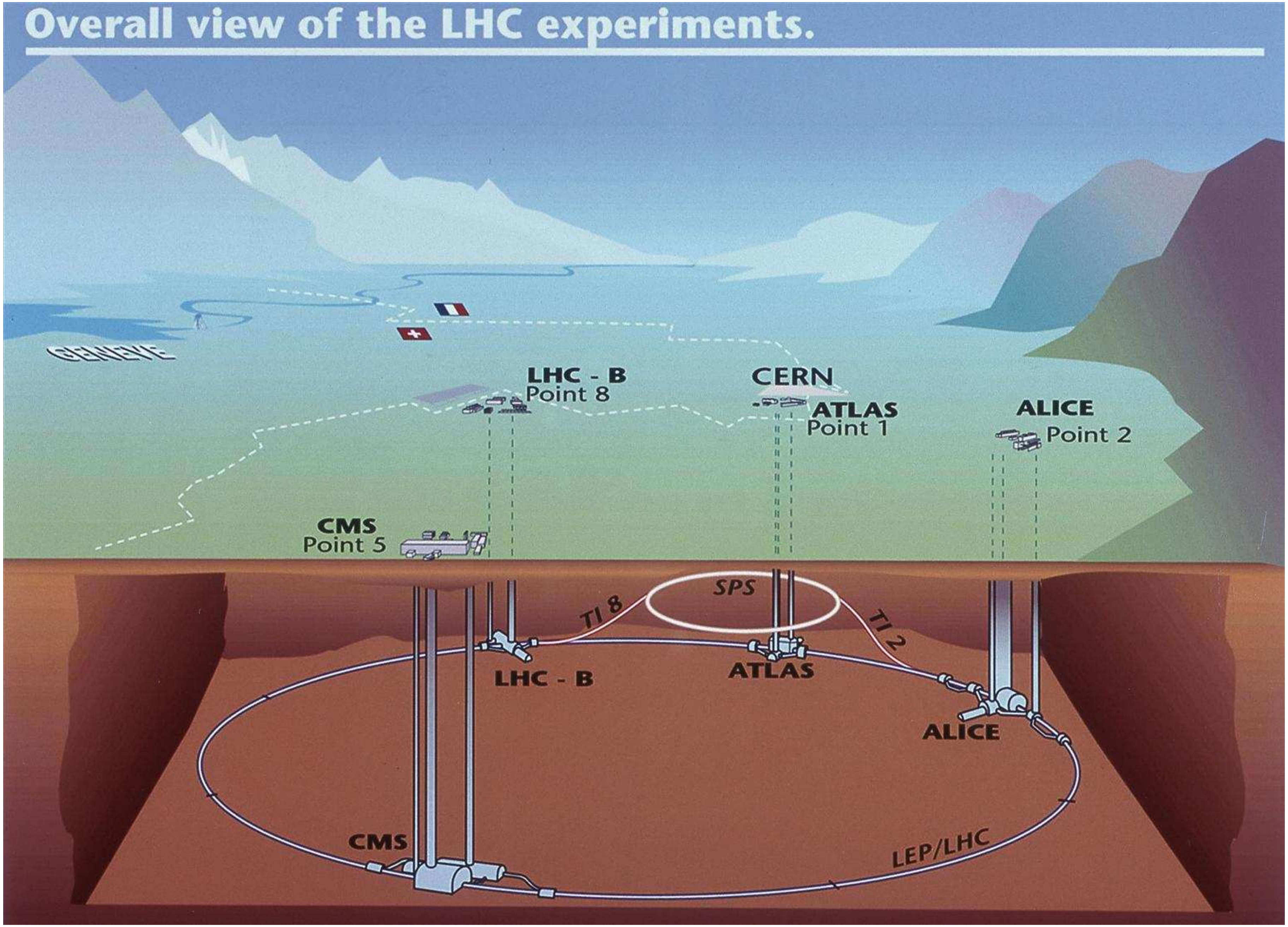
Overall view of the LHC experiments.
GENEVE
LHC - B
Point 8
CERN
ATLAS
Point 1
ALICE
Point 2
CMS
Point 5
SPS
TI 8
TI 2
LHC - B
ATLAS
ALICE
CMS
LEP/LHC

Overview

I Drell–Yan processes at the LHC

- precision physics with Drell–Yan processes
- physics motivations
- status of loop calculations & tools for neutral-current processes
- Les Houches 2005/2007 & TeV4LHC

II Electroweak Sudakov logarithms

- large logarithms in electroweak corrections
- one- & two-loop effects
- existing Sudakov results at two loops

III Four-fermion scattering: two-loop calculation

- factorization into QED contributions, form factor & reduced amplitude
- SU(2) form factor: calculation & results
- electroweak results

IV Implementation of Sudakov corrections for LHC

- running couplings & input parameter schemes
- partonic cross sections
- hadronic distributions

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I Drell–Yan processes at the LHC

Precision collider physics

Precision measurements and electroweak (EW) + QCD corrections for observables at **LEP**, **SLC** and **Tevatron** enabled us to

- probe Standard Model (SM) of EW & strong interactions as a quantum field theory,
- test SM consistency by comparing direct with indirect measurements of model parameters $(m_t, M_W, \sin^2\theta_{\text{eff}}, \ldots)$,
- constrain SM Higgs boson mass,
- search for small deviations from SM predictions ⇒ indirect signals of new physics
- exclude or constrain new physics models.

⇒ Continue precision physics at **LHC** (and **ILC**/**CLIC**):

- possibly discover Higgs and/or new physics particles
- measure new particles' masses, couplings, spin
- test consistency, search for deviations, constrain model parameters, . . .

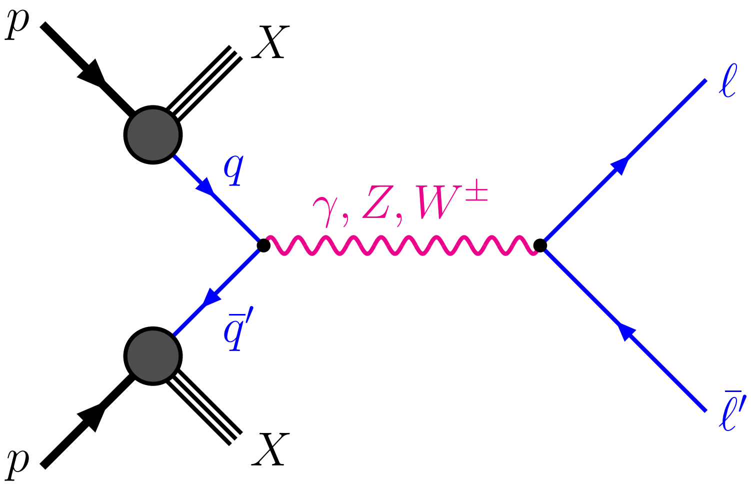
Theoretical input needed for precision physics

- precise predictions for signal and background processes (SM and beyond)
- include all relevant higher-order corrections
- partonic and hadronic processes → PDF's, fragmentation
- estimate of theoretical uncertainties
- total cross sections and kinematic distributions with experimental/realistic cuts
- implementation in event generators

Drell–Yan (DY) processes

↪ production of a lepton–antilepton pair with high invariant mass $M_{\ell\ell}$ (or high p_T^ℓ's) in a hadron–hadron collision: $H_1\, H_2 \to \ell\, \bar{\ell}' + X$ $(H_{1,2} = p, \bar{p})$ Drell, Yan '70, '71

LHC $(H_1 = H_2 = p)$:



Partonic reaction

- neutral-current DY: $q\,\bar{q} \to \gamma/Z \to \ell^-\,\ell^+$
- charged-current DY: $q\,\bar{q}' \to W^\pm \to \ell^\pm\,\nu/\bar{\nu}$

Factorization

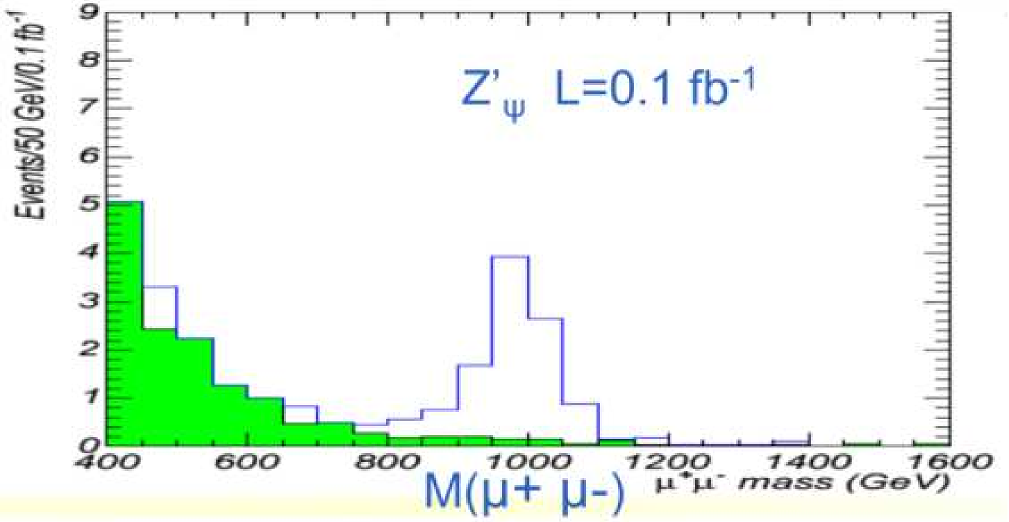
$$\sigma_{\text{tot}} = \sum_{i,j} \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2\, f_i^{H_1}(x_1,\mu^2)\, f_j^{H_2}(x_2,\mu^2) \int \mathrm{d}\sigma\Big(q_i(x_1 p_1) + \bar{q}_j(x_2 p_2) \to \ell + \bar{\ell}', \mu^2\Big)$$

Physics motivations

Why consider Drell–Yan at LHC?

- easy detection: high-p_T lepton pair or lepton + missing p_T (typically $p_T > 20\,\text{GeV}$ in central detector region)
- large cross sections: $\sigma(p\,p \to e^- e^+,\, \mu^- \mu^+) \sim 1.8\,\text{nb}$, $\sigma(p\,p \to e\,\nu_e,\, \mu\,\nu_\mu) \sim 17\,\text{nb}$
  $\hookrightarrow \mathcal{O}(10^8, 10^9)$ events for $\mathcal{L} = 100\,\text{fb}^{-1}$
- detector calibration and luminosity monitoring (∼ Bhabha at LEP)
- validating and constraining PDF's
- precision measurement of $M_{W,Z}$ and $\Gamma_{W,Z}$ at the W, Z peak
- background to new physics searches at high $M_{\ell\ell}$ $\Longrightarrow$

  $\hookrightarrow$ need precise theoretical prediction for high-energy tail



talk by Menici at IFAE 2006

Status of loop calculations & tools for neutral-current processes

QCD

- total cross sections up to $\mathcal{O}(\alpha_s^2)$ and resummation (RESBOS)
  Altarelli et al. '84; Hamberg et al. '91; van Neerven et al. '92; Giele et al. '93; Balazs et al. '97
- fully differential distributions up to $\mathcal{O}(\alpha_s^2)$ (MCFM, FEWZ)
  Campbell et al. '02; Anastasiou et al. '04; Dixon et al. '03; Melnikov et al. '06
- merged/matched with parton shower (HERWIG, MC@NLO, ALPGEN, SHERPA, ...)
  Frixione et al. '02; Mangano et al. '03; Kraus et al. '05

Electroweak

- complete EW $\mathcal{O}(\alpha)$ corrections (ZGRAD2, HORACE, SANC)
  Baur et al. '02; Carloni Calame et al. '07; Bardin et al. '07
- multi-photon radiation (HORACE, ZINHAC)
  Carloni Calame et al. '05, '07; Płaczek et al.

⇒ Need combined tools for QCD & EW!

Strong enhancement by EW Sudakov logs at high energies
⇒ might need dominant EW 2-loop corrections

Les Houches 2005 Workshop "Physics at TeV Colliders" & Tevatron-for-LHC (TeV4LHC) Electroweak Working Groups

hep-ph/0604120
arXiv:0705.3251 [hep-ph]

Tuned comparison of available Monte Carlo (MC) programs for **charged-current DY**:

- common setup for comparison:
  cuts ($p_T^\ell > 20\,\text{GeV}$, $|\eta_\ell| < 2.5$), input parameter scheme, which distributions, . . .
- estimate remaining theoretical uncertainties
  (missing higher-order corrections, PDF uncertainties, . . .)
- identify necessary improvements to match experimental precision

Les Houches 2007

arXiv:0803.0678 [hep-ph]

Ongoing comparisons for **neutral-current DY**

Mini-Workshop "Physics with LHC early data"

in Glasgow, November 26–30, 2007 `http://ppewww.physics.gla.ac.uk/~samir/MiniWorkshop`

- continue comparison for neutral-current DY (QCD & EW)
- implement 2-loop EW Sudakov logs in MC programs

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Large logarithms in electroweak corrections

EW corrections at high energies $\sqrt{s} \sim$ TeV $\gg M_{W,Z}$

- dominated by Sudakov logarithms $\alpha^n \ln^j(s/M_W^2)$, $j = 2n$,
- subleading logs $(0 \leq j < 2n)$ also important (cancellations!)

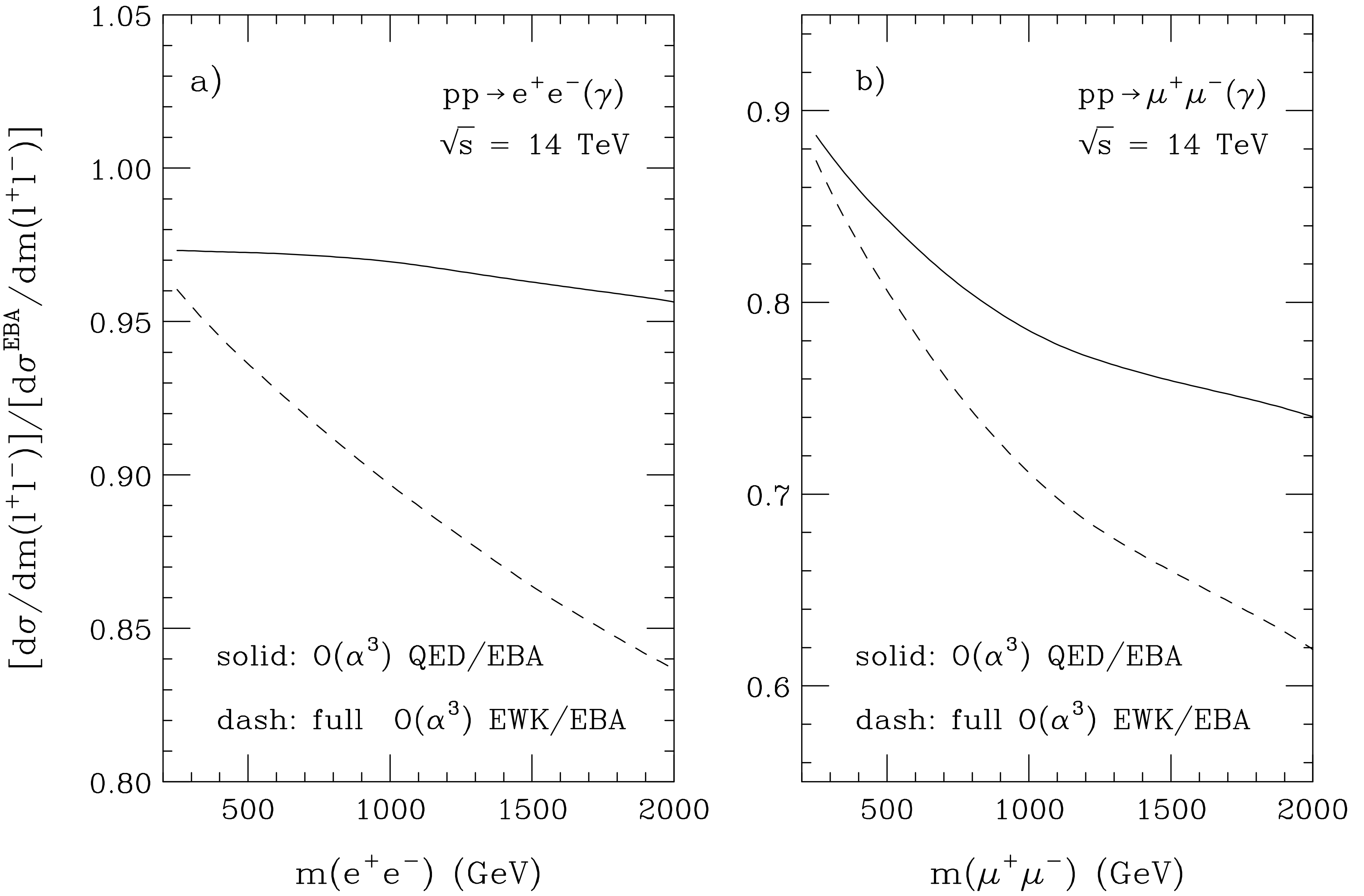
Origin of large logs

- mass singularities: emission of soft/collinear gauge bosons from external particles
- remnants from UV singularities: $\ln(s/\mu^2)$

Massless gauge bosons: real emission of soft/collinear photons/gluons cannot be detected ⇒ mass singularities cancel (KLN theorem)

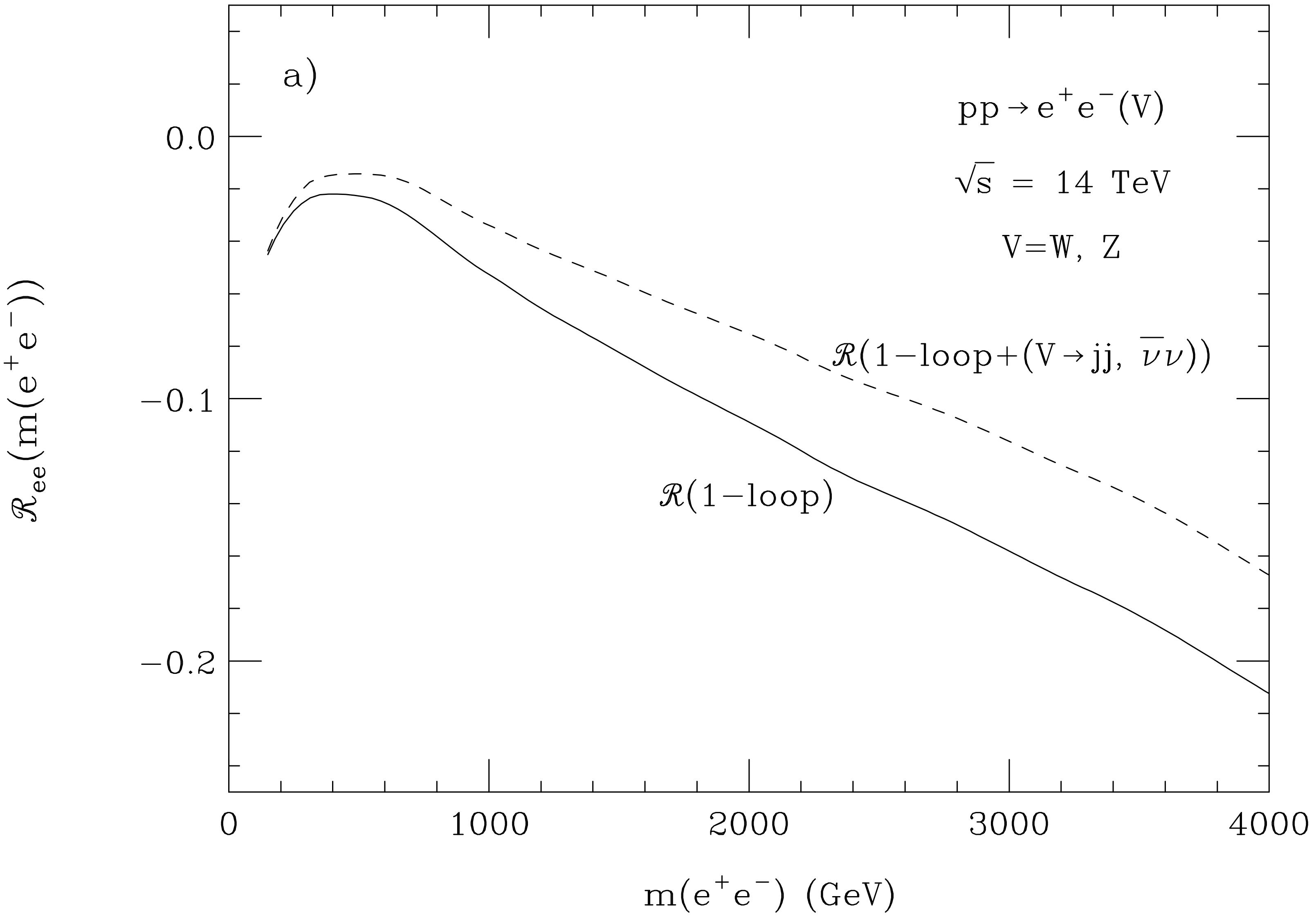
Massive gauge bosons: real emission of W's, Z's can be detected separately
↪ large logs remain present in exclusive observables,
↪ even in inclusive observables (Block–Nordsieck violations)

One-loop effects of EW Sudakov logarithms



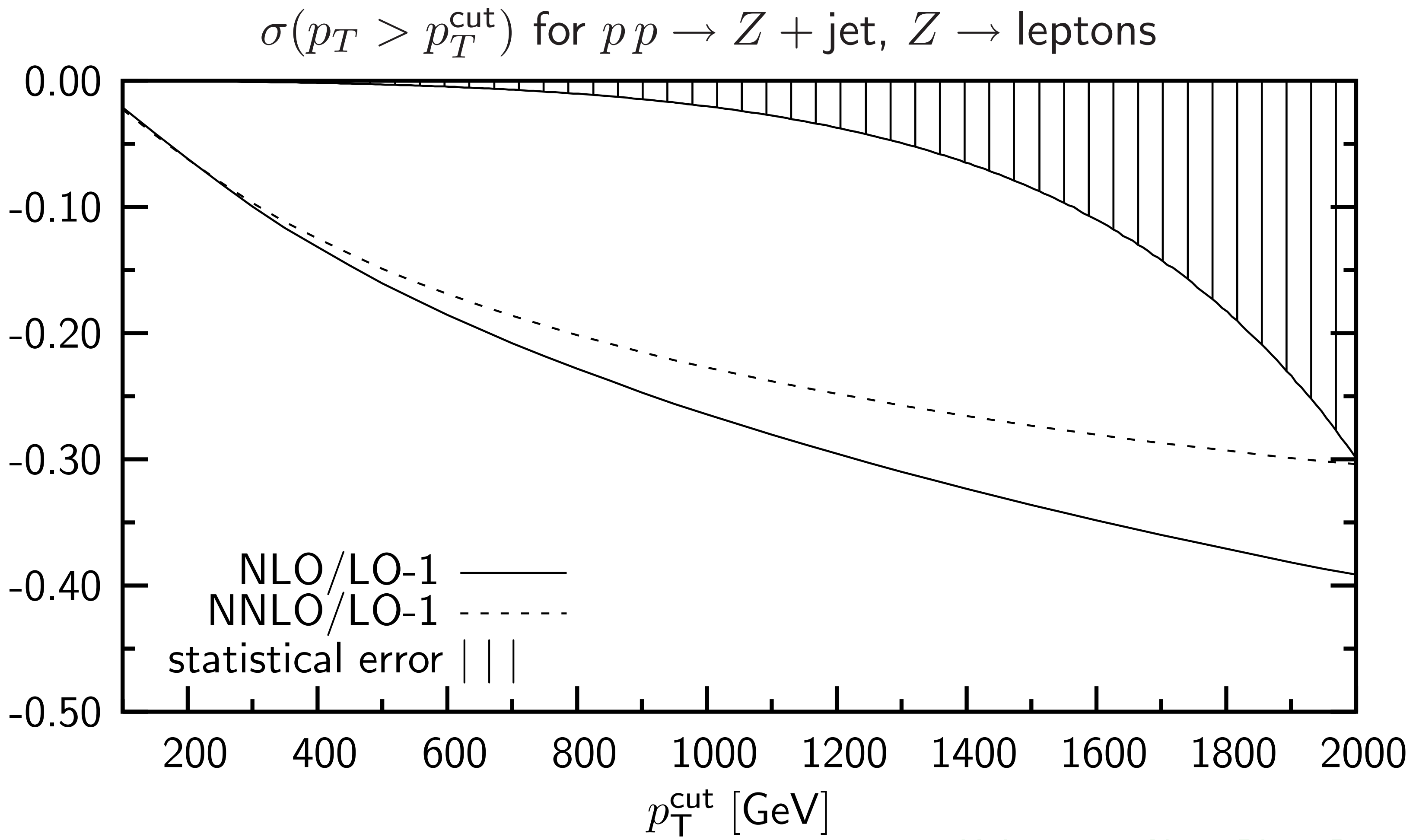
Baur et al., PRD65 (2002) 033007

Real W, Z emission: only partial cancellation



Baur, Phys. Rev. D75 (2007) 013005

Two-loop effects of EW Sudakov logarithms



Kühn et al., Nucl. Phys. B727 (2005) 368

Size of corrections at $p_T^{\text{cut}} \sim 1\,\text{TeV}$ vs. statistical error $\sigma_{\text{stat}} \simeq 2\%$ ($\mathcal{L} = 300\,\text{fb}^{-1}$):

- 1-loop: $-26\% \simeq -13\,\sigma_{\text{stat}}$
- dominant 2-loop terms: $+4\% \simeq +2\,\sigma_{\text{stat}}$

Existing Sudakov results at two loops

Universal structure of mass singularities ⇒ use **evolution equations**

valid for unbroken SU(2)×U(1) ($M_\gamma = M_Z = M_W$) and for pure QED:

$$\alpha^2\Bigg[\underbrace{\underbrace{C_{\text{LL}}\ln^4\left(\frac{s}{M_W^2}\right)}_{\substack{\text{Fadin, Lipatov,}\\ \text{Martin, Melles '99}}} + \underbrace{C_{\text{NLL}}\ln^3\left(\frac{s}{M_W^2}\right)}_{\text{Melles '00, '01}}}_{\text{arbitrary processes}} + \underbrace{\underbrace{C_{\text{NNLL}}\ln^2\left(\frac{s}{M_W^2}\right)}_{\substack{\text{Kühn, Moch, Penin,}\\ \text{Smirnov '99–'01}}} + \underbrace{C_{\text{N}^3\text{LL}}\ln\left(\frac{s}{M_W^2}\right)}_{\substack{\text{B.J., Kühn, Penin,}\\ \text{Smirnov '03–'05}}}}_{\text{massless } f\,\bar f \to f'\,\bar f' \text{ processes}}\Bigg]$$

Recently: $e^-\,e^+ \to W^+\,W^-$ [Kühn, Metzler, Penin '07], SCET approach [Chiu, Golf, Kelley, Manohar '07]

Explicit 2-loop calculations based on EW Lagrangian:

$$\alpha^2\Bigg[\underbrace{\underbrace{C_{\text{LL}}\ln^4\left(\frac{s}{M_W^2}\right)}_{\substack{\text{Melles '00;}\\ \text{Hori, Kawamura, Kodaira '00;}\\ \text{Beenakker, Werthenbach '00, '01}}} + \underbrace{C_{\text{NLL}}^{\text{ang}}\ln\left(\frac{t}{s}\right)\ln^3\left(\frac{s}{M_W^2}\right)}_{\text{Denner, Melles, Pozzorini '03}}}_{\text{arbitrary processes}} + \underbrace{\underbrace{C_{\text{NLL}}^{\text{rem}}\ln^3\left(\frac{s}{M_W^2}\right)}_{\substack{\text{Pozzorini '04;}\\ \text{Denner, B.J., Pozzorini '06(–'08)}}}}_{n\text{-fermion processes}}\Bigg]$$

⇒ agree with evolution equations

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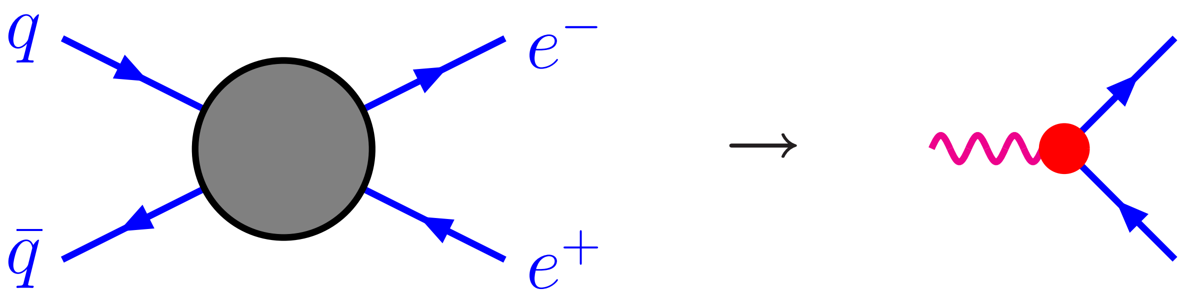
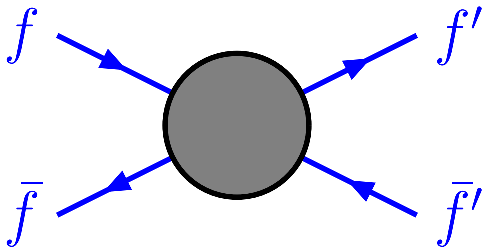


J.H. Kühn, A.A. Penin, *hep-ph/9906545*
J.H. Kühn, A.A. Penin, V.A. Smirnov, *Eur. Phys. J. C 17 (2000) 97*
J.H. Kühn, S. Moch, A.A. Penin, V.A. Smirnov, *Nucl. Phys. B 616 (2001) 286*
B. Feucht (Jantzen), J.H. Kühn, S. Moch, *Phys. Lett. B 561 (2003) 111*
B. Feucht (Jantzen), J.H. Kühn, A.A. Penin, V.A. Smirnov, *Phys. Rev. Lett. 93 (2004) 101802*
B. Jantzen, J.H. Kühn, A.A. Penin, V.A. Smirnov, *Phys. Rev. D 72 (2005) 051301(R)*
B. Jantzen, J.H. Kühn, A.A. Penin, V.A. Smirnov, *Nucl. Phys. B 731 (2005) 188*
B. Jantzen, V.A. Smirnov, *Eur. Phys. J. C 47 (2006) 671*

Four-fermion scattering: $f\bar{f} \to f'\bar{f}'$, important class of processes (→ Drell–Yan)

Factorization of QED contributions:



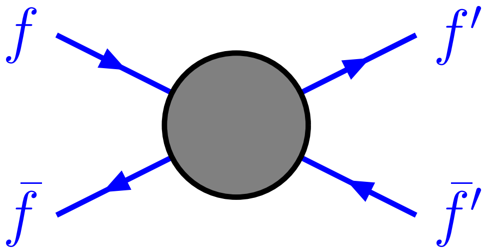
$$A = U_{\text{QED}} \cdot A_{\text{EW}}$$

- QED factor U_{QED} → IR singularities from virtual massless photons (regularized dimensionally or by small photon mass, compensated by real corrections)
- amplitude A_{EW} → remaining electroweak contributions, IR-safe
- calculate A_{EW} by evaluating A/U_{QED} with $M_\gamma = M_W$
  ↪ works at NNLL accuracy ✓

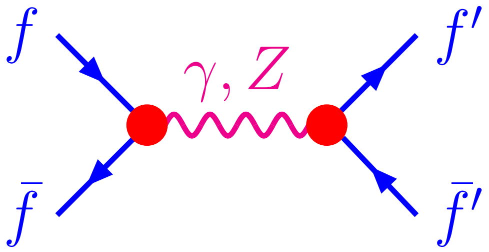
Problem at N^{3}LL (2-loop linear log):
mixing of gauge groups SU(2)×U(1) through Higgs mechanism
⇒ use simplified model without mixing: B.J., Kühn, Penin, Smirnov '04, '05

- factorization of QED contributions works at N^{3}LL accuracy
- single mass parameter: $M = M_W = M_{Z=W^3} = M_{\gamma=B}$
- include mass difference $(M_Z - M_W)$ by expansion around $M_Z \approx M_W$
- remaining error $\sim \mathcal{O}(\sin^2\theta_W) \sim 20\%$ in coefficient of linear 2-loop log

Factorization into form factor and reduced amplitude:

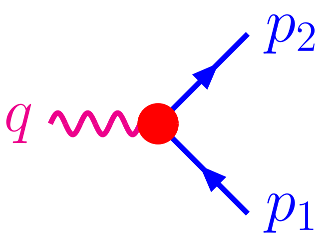


$$A_{\text{EW}} = \frac{ig^2}{s} F^2 \tilde{A}$$



$$\times \tilde{A}$$

Form factor F of vector current:



$$= F \cdot \bar{u}(p_2)\, \gamma^\mu\, u(p_1) + \mathcal{O}(\text{fermion masses})$$

High-energy behaviour $s \sim |t| \sim |u| \gg M_{W,Z}^2$

references: see Kühn et al. '01

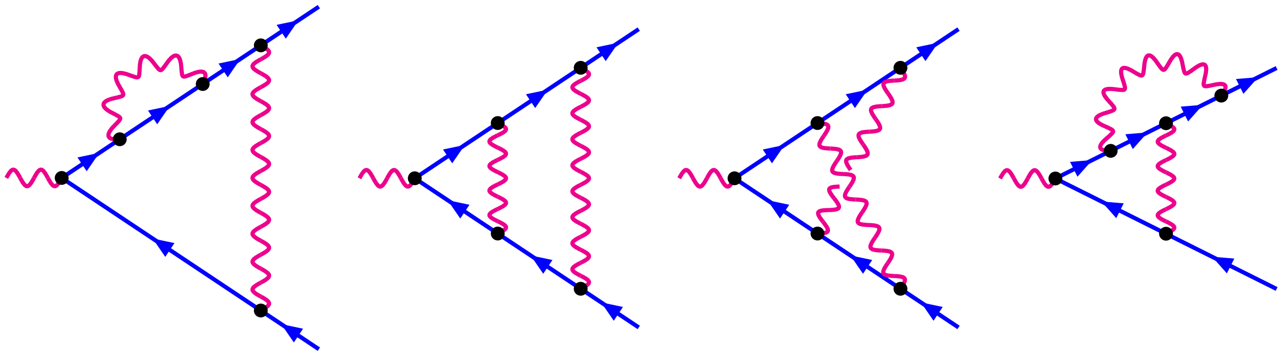
- all double logs $\alpha^n \ln^{2n}$ ⇝ form factors F^2
- reduced amplitude $\tilde{A}$ → only single logs $\alpha^n \ln^n$
- $\tilde{A}$ obtained from 1-loop and massless 2-loop calculations via an evolution equation:

$$\frac{\partial \tilde{A}}{\partial \ln s} = \chi\big(\alpha(s)\big)\, \tilde{A}\,, \qquad \chi = \text{matrix of soft anomalous dimensions}$$

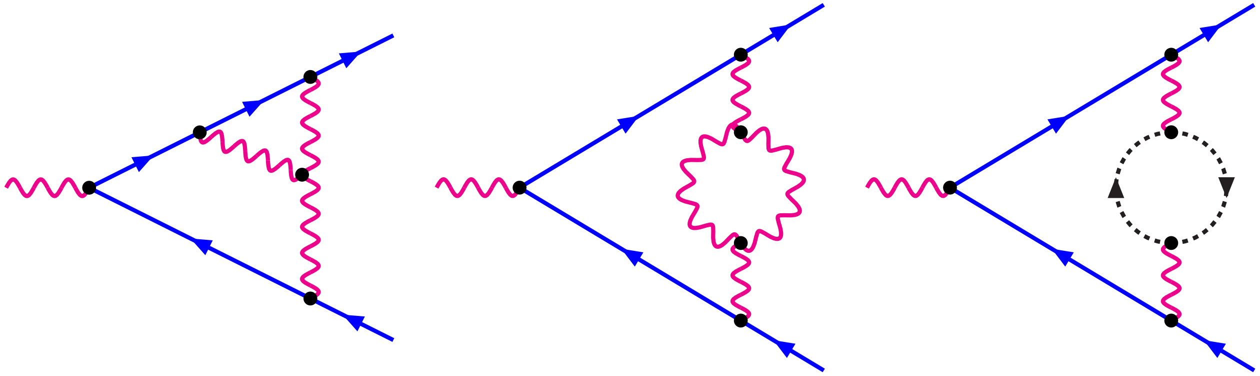
⇒ For full logarithmic (N^{3}LL) 2-loop amplitude: need form factor F

SU(2) form factor in two loops: vertex diagrams

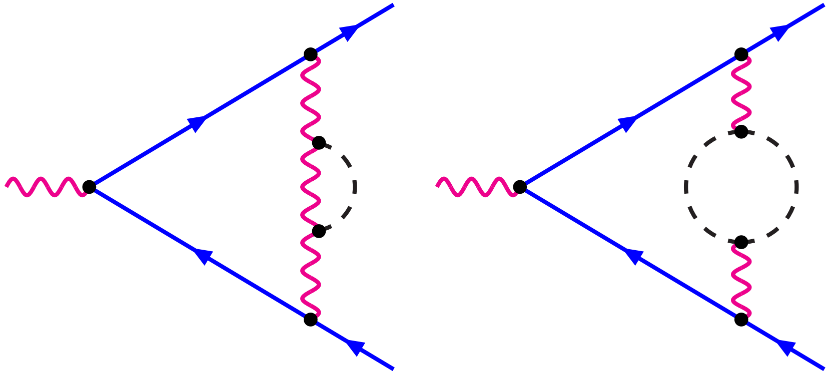
Abelian:
↪ U(1)



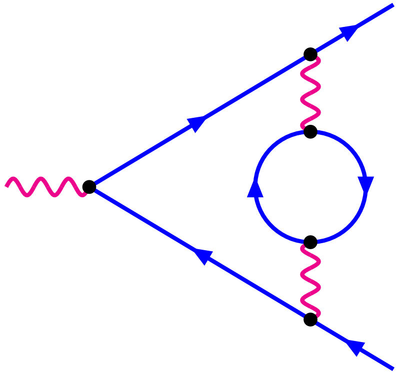
non-Abelian:



Higgs:



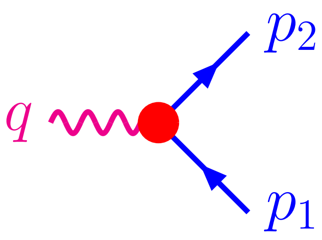
fermion:



+ 1-loop×1-loop corrections + renormalization

High-energy behaviour of the form factor

↪ Sudakov limit:



$$= F(Q^2) \cdot \bar{u}(p_2)\, \gamma^\mu\, u(p_1)$$

- momentum transfer $-q^2 \equiv Q^2 \gg M^2 \equiv M^2_{W,Z}$

  $\Big[$Euclidean $Q^2 > 0,$ real $F \xrightarrow[\text{continuation}]{\text{analytic}}$ Minkowskian $Q^2 = -s - i0 < 0\Big]$
- neglect fermion masses ⇒ external on-shell fermions: $p_1^2 = p_2^2 = 0$
- logarithmic approximation: neglect terms suppressed by a factor of M^2/Q^2

  ↪ works well for 2-loop n_f contribution where the exact result in M^2/Q^2 is known

  B.F., Kühn, Moch '03

  ⇒ contains powers of the large log $\ln(Q^2/M^2)$

  ⇒ leading order of asymptotic expansion in M^2/Q^2
- only 2-loop logs $\ln^{4,3,2,1}$, non-logarithmic constant more difficult
- choose $M_{\text{Higgs}} = M_W$ ⇒ calculation easier, affects only N^{3}LL, small error
- methods: expansion by regions & Mellin–Barnes representations B.J., Smirnov '06

SU(2) form factor in two loops: result

$$\left(\frac{\alpha}{4\pi}\right)^2 \Bigg[\frac{9}{32}\ln^4\left(\frac{Q^2}{M^2}\right) - \frac{43}{48}\ln^3\left(\frac{Q^2}{M^2}\right) + \left(\frac{7}{8}\pi^2 - \frac{235}{48}\right)\ln^2\left(\frac{Q^2}{M^2}\right)$$

confirmed ✓

$$+ \left(\frac{13}{2}\sqrt{3}\,\mathrm{Cl}_2\left(\frac{\pi}{3}\right) + \frac{15}{4}\sqrt{3}\pi - \frac{61}{2}\zeta_3 - \frac{11}{24}\pi^2 + \frac{65}{4}\right)\ln\left(\frac{Q^2}{M^2}\right) \Bigg]$$

new!

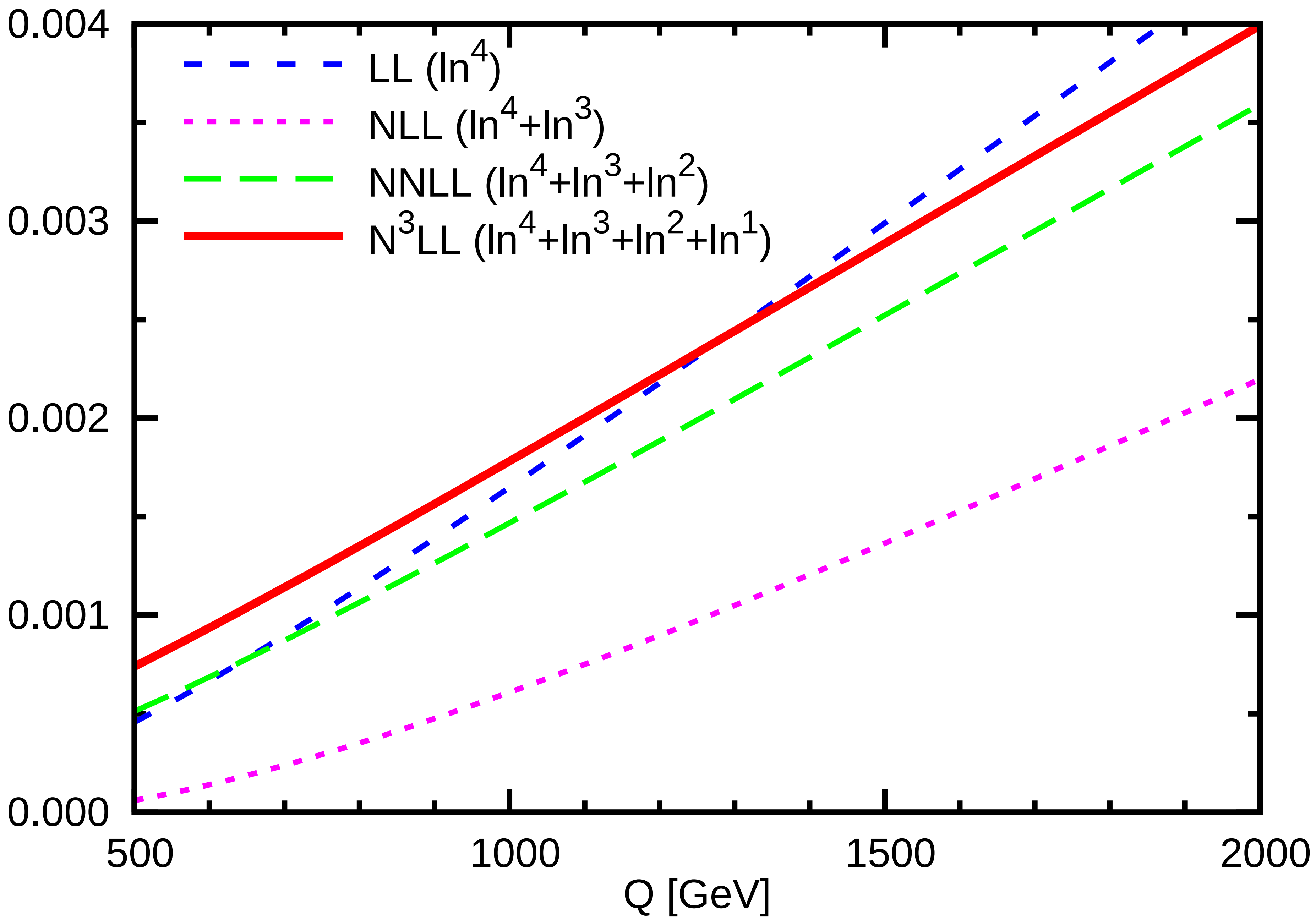
B.J., Kühn, Moch '03; B.J., Kühn, Penin, Smirnov '04, '05; B.J., Smirnov '06

$\ln^{4,3,2}$: Kühn, Moch, Penin, Smirnov '01

Sizes of logarithmic contributions (at $Q = 1$ TeV in per mil):

Abelian:	$+\,0.3\ln^4$	$-\,1.7\ln^3$	$+\,8.2\ln^2$	$-\,11\ln$	$+\,15$
	$+1.6$	-2.0	$+1.9$	-0.5	$+0.1$
fermionic:		$-\,1.0\ln^3$	$+\,9.5\ln^2$	$-\,26\ln$	$+\,42$
		-1.2	$+2.2$	-1.2	$+0.4$
non-Abelian + Higgs:		$+\,1.8\ln^3$	$-\,14\ln^2$	$+\,43\ln$	$-\,\ldots$
		$+2.1$	-3.2	$+2.0$	
total:	$+\,0.3\ln^4$	$-\,0.9\ln^3$	$+\,3.7\ln^2$	$+\,6.9\ln$	
	$+1.6$	-1.0	$+0.9$	$+0.3$	

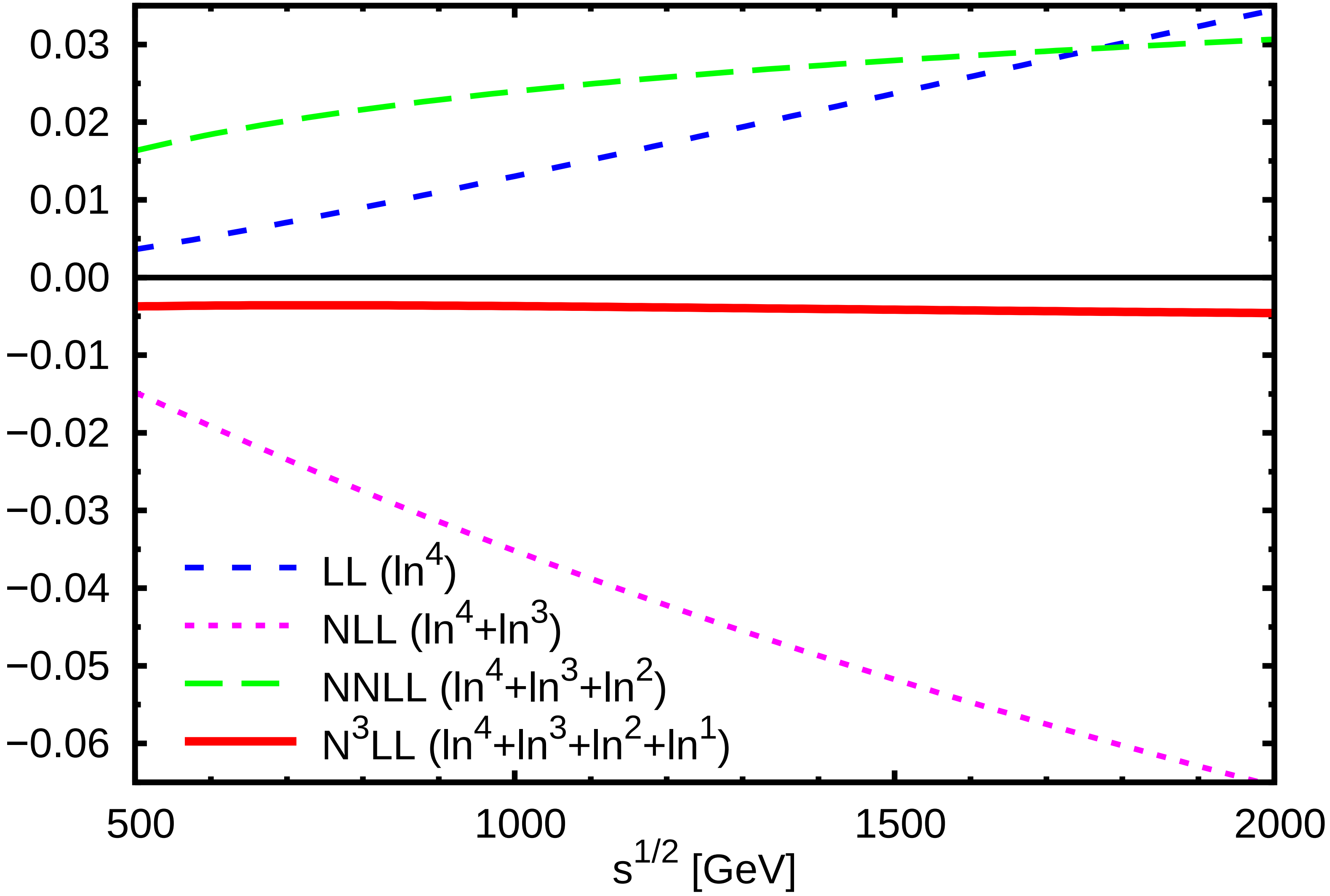
SU(2) form factor in two loops: result (2)



Electroweak results: example $\sigma(q\,\bar{q} \to e^-\,e^+)$ $(q = d, s)$

numerical 2-loop result:

$$\left(\frac{\alpha_{\text{ew}}}{4\pi}\right)^2 \left[+2.79\ln^4\left(\frac{s}{M_W^2}\right) - 51.98\ln^3\left(\frac{s}{M_W^2}\right) + 321.34\ln^2\left(\frac{s}{M_W^2}\right) - 757.35\ln\left(\frac{s}{M_W^2}\right) \right]$$



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Running couplings & input parameter schemes

Mathematica file with all contributions up to 2 loops A. Penin

Rewriting Mathematica code with all features needed for implementation: B.J.

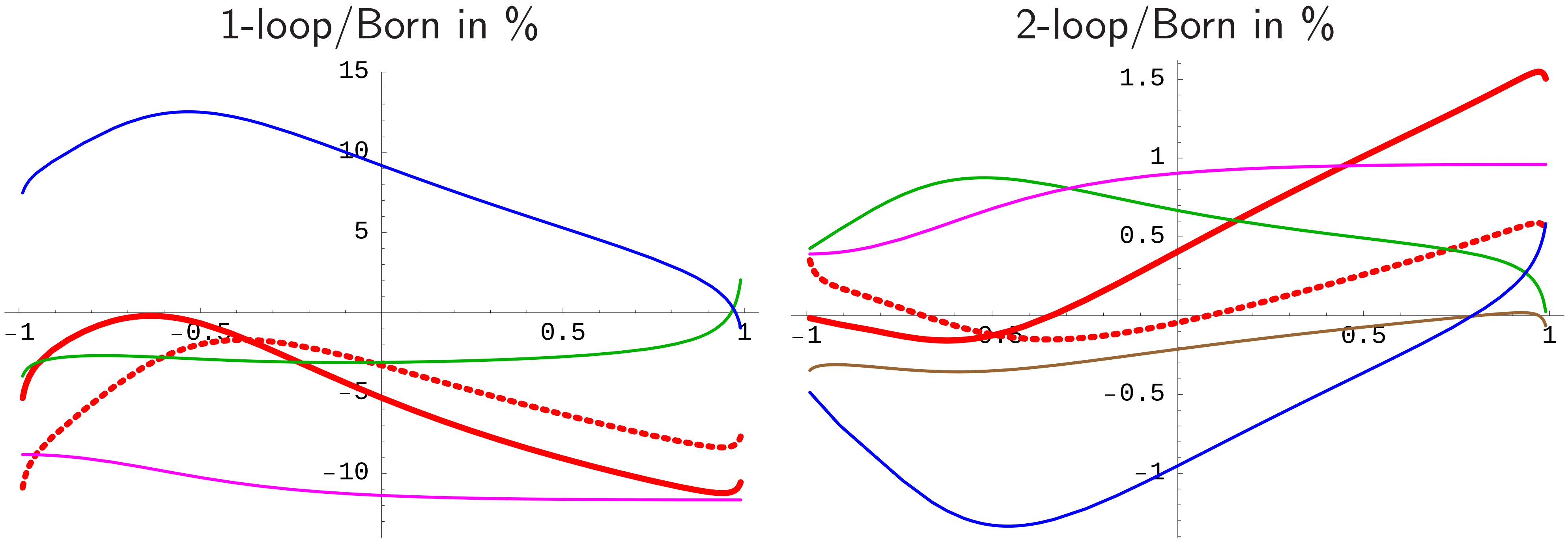
- couplings and mixing angles from Born amplitude are renormalized at scale s
  $\hookrightarrow$ implement 2-loop running $s \to M_W^2$:
  EW (incl. Yukawa) & optionally QCD contributions Jones '81; Arason et al. '91
- fully differential cross section (unpolarized or polarized)

Fortran code implementing Mathematica results: S. Pozzorini

- optional conversion from $\overline{\text{MS}}$ to G_μ scheme $\Rightarrow$ closer to setup of MC programs
- provides ratios 1-loop/Born & 2-loop/Born
- switches & variables allow flexible settings

Implementation in MC programs **ZGRAD2** & **HORACE** D. Wackeroth, C.M. Carloni Calame

Partonic cross sections: angular dependence for $u\,\bar{u} \to e^-\,e^+$



as a function of $\cos\theta$ for $s = 1\,\text{TeV}^2$

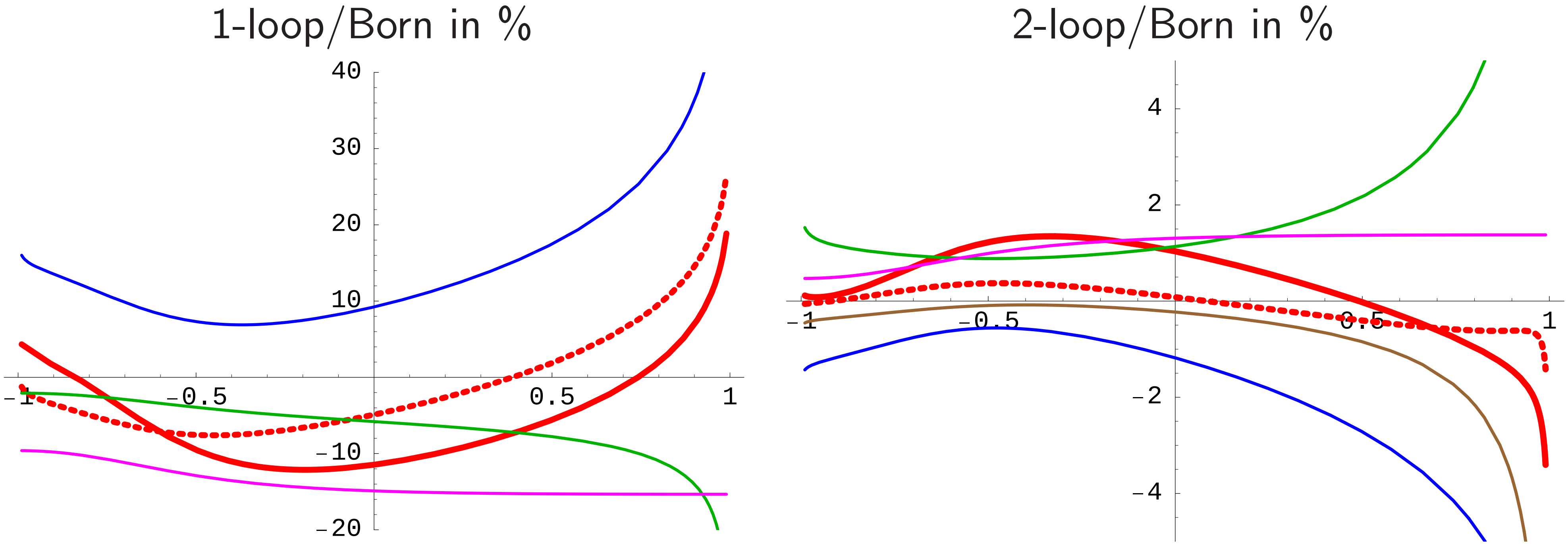
colors: full result; individual logs LL, NLL, NNLL, N^{3}LL

dashed: without running of Born couplings

Strong angular dependence!

(even in LL from normalization with unpolarized cross section)

Partonic cross sections: angular dependence for $d\,\bar{d} \rightarrow e^-\,e^+$

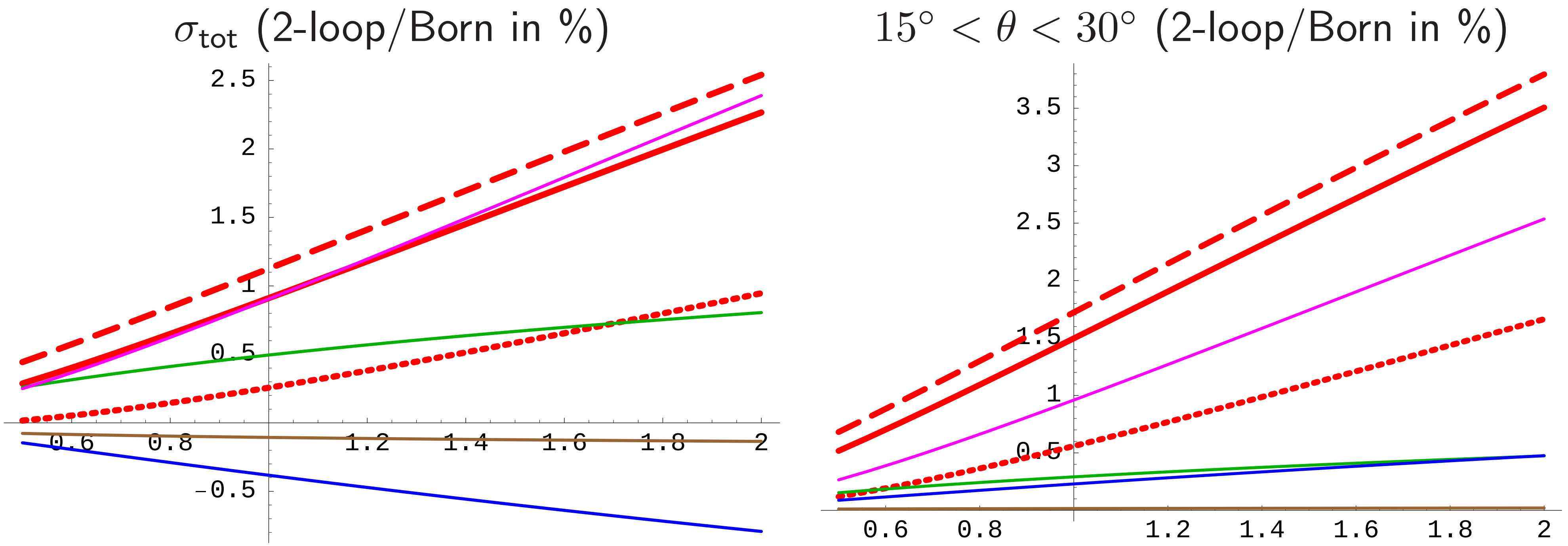


as a function of $\cos\theta$ for $s = 1\,\text{TeV}^2$

colors: **full result**; individual logs LL, NLL, NNLL, N^{3}LL

dashed: without running of Born couplings

Partonic cross sections: $u\,\bar{u} \to e^-\,e^+$



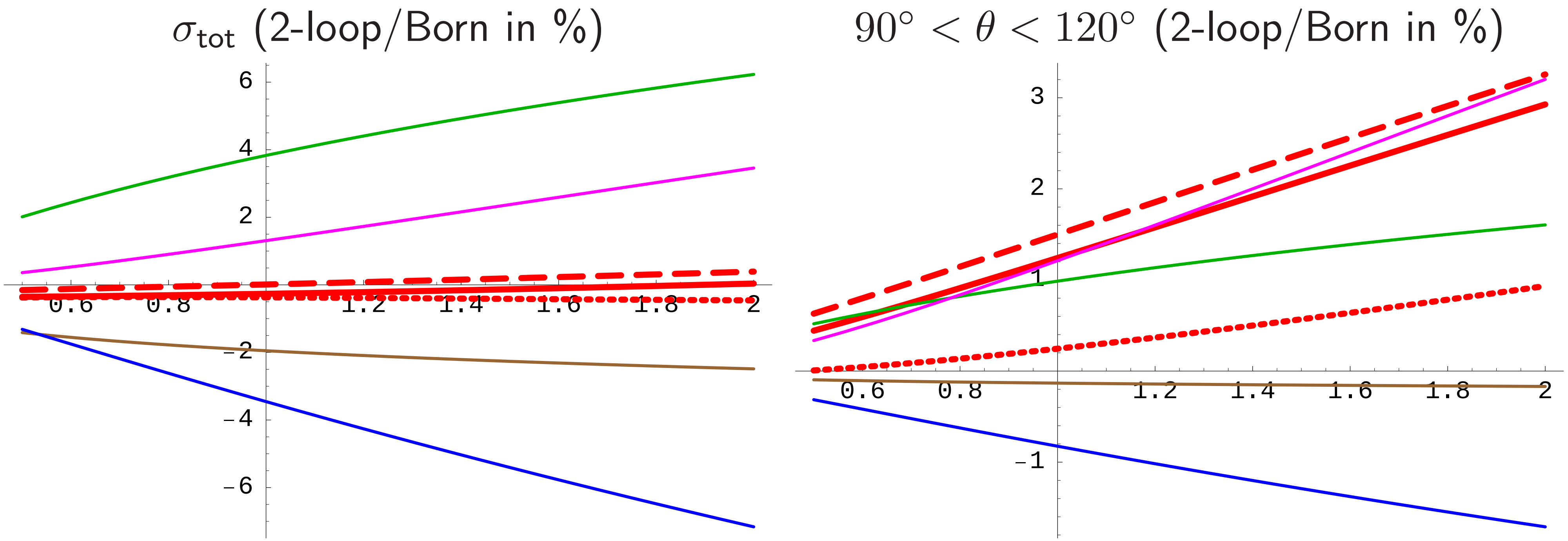
as a function of $\sqrt{s}\,[\text{TeV}]$

colors: **full result**; individual logs LL, NLL, NNLL, N^{3}LL

long-dashed: with additional QCD contributions to the running

short-dashed: without running of Born couplings

Partonic cross sections: $d\,\bar{d} \to e^-\,e^+$


σ_{tot} (2-loop/Born in %)

$90^\circ < \theta < 120^\circ$ (2-loop/Born in %)

as a function of $\sqrt{s}\,[\text{TeV}]$

colors: full result; individual logs LL, NLL, NNLL, N^{3}LL

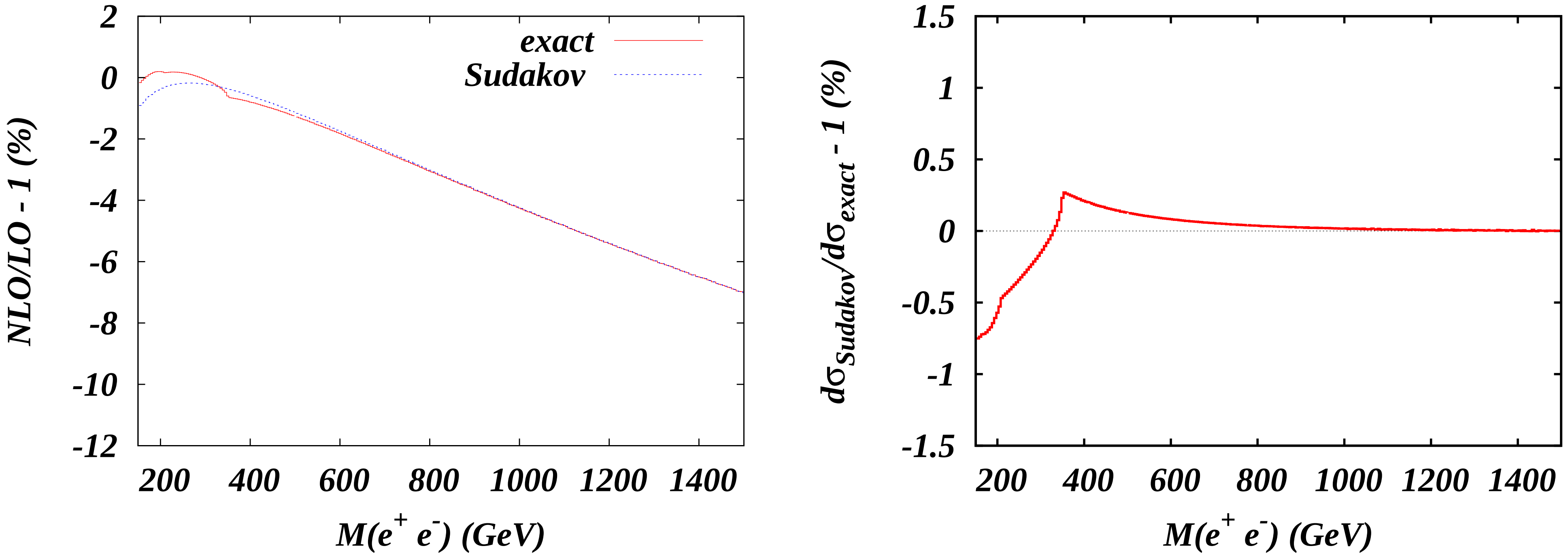
long-dashed: with additional QCD contributions to the running

short-dashed: without running of Born couplings

Hadronic distributions: 1-loop comparison of Sudakov corrections with ZGRAD2

1-loop/Born in % (ZGRAD2, Sudakov)

(Sudakov/ZGRAD2 -1) in %

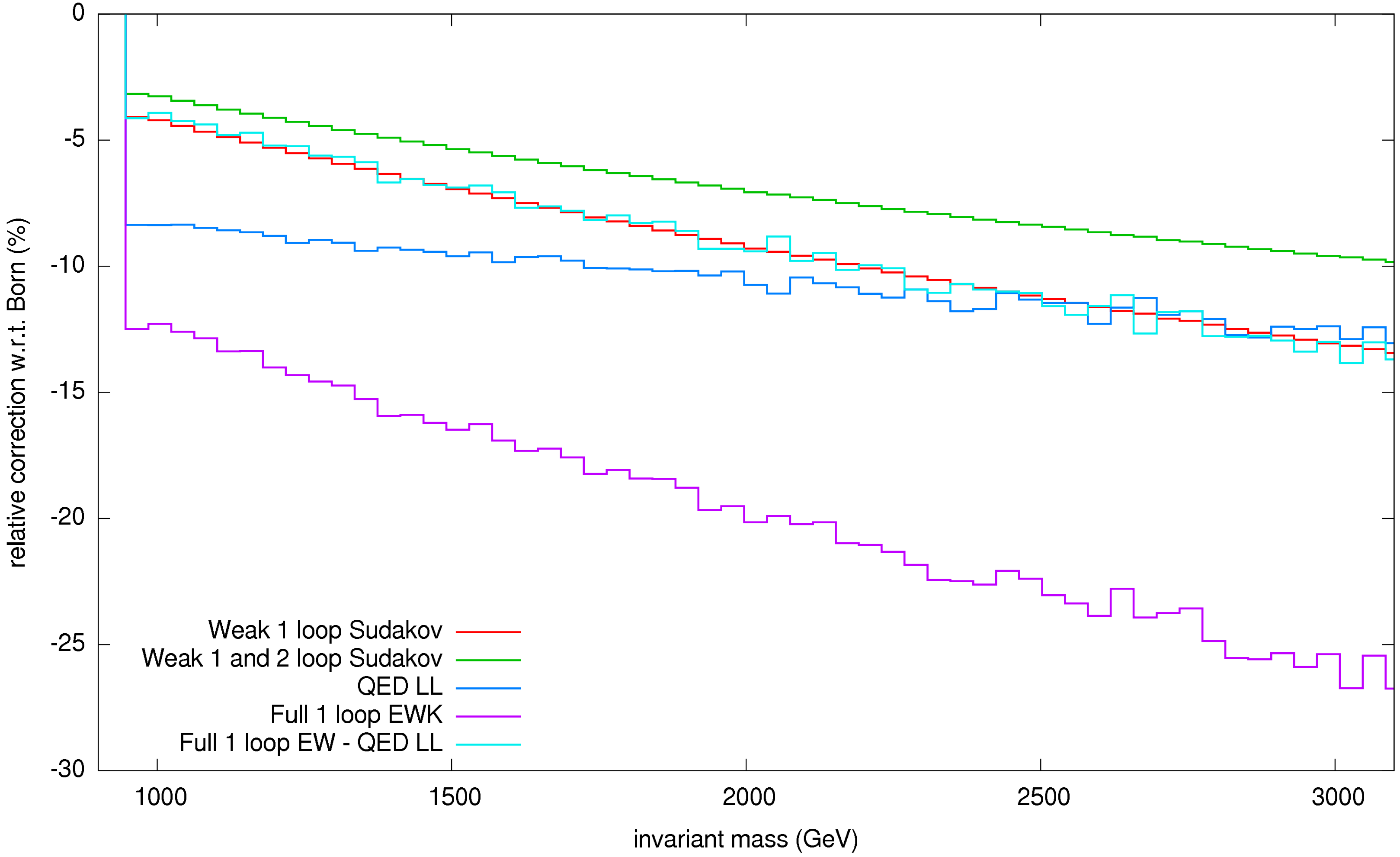


$$p\,p \longrightarrow e^-\,e^+$$

D. Wackeroth

↪ Same setup for separation of 1-loop QED contribution from EW corrections

Hadronic distributions: 1- & 2-loop Sudakov corrections with HORACE



$$p\,p \to \mu^-\,\mu^+$$

C.M. Carloni Calame

⇒ good 1-loop agreement after subtraction of QED leading logs

⇒ 2-loop Sudakov corrections of several %

Overview

I Drell–Yan processes at the LHC

- precision physics with Drell–Yan processes
- physics motivations
- status of loop calculations & tools for neutral-current processes
- Les Houches 2005/2007 & TeV4LHC

II Electroweak Sudakov logarithms

- large logarithms in electroweak corrections
- one- & two-loop effects
- existing Sudakov results at two loops

III Four-fermion scattering: two-loop calculation

- factorization into QED contributions, form factor & reduced amplitude
- SU(2) form factor: calculation & results
- electroweak results

IV Implementation of Sudakov corrections for LHC

- running couplings & input parameter schemes
- partonic cross sections
- hadronic distributions

V Summary

V Summary

Drell–Yan processes

- important for precision physics at LHC
- implement best available theoretical predictions into MC program(s)

Electroweak Sudakov logarithms

- large impact at high energies, also 2-loop corrections important

Four-fermion scattering

- 2-loop calculation of amplitude and form factor presented
- large logarithmic contributions, but also large cancellations

Implementation of Sudakov corrections for LHC

- Mathematica & Fortran code → coupling & scheme conversions
- first results for distributions of hadronic cross sections
- good agreement at 1 loop, preliminary 2-loop distributions

Included in **Les Houches 2007 Summary Report** of the “Standard Model Handles and Candles” Working Group C. Buttar et al., arXiv:0803.0678 [hep-ph]

↪ separate publication from Glasgow workshop with more detailed studies will follow