Time evolutions of quantum systems and quantum Zeno effect

Saverio Pascazio
Dipartimento di Fisica and INFN
Bari, Italy

Paul Scherrer Institut, 19 April 2024

Zeno of Elea

Zeno was an Eleatic philosopher, a native of Elea in Italy, son of Teleutagoras, and the favorite disciple of Parmenides. He was born about 488 BC, and at the age of forty accompanied Parmenides to Athens

The flying arrow is at rest.

At any given moment it is in a space equal to its own length, and therefore is at rest at that moment. So, it is at rest at all moments. The sum of an infinite number of these positions of rest is not a motion.

Zeno was born here

Archimedes

Quantum Zeno effect

quantum system	ψ
Hamiltonian	H
Schrödinger equation	$\psi_t = e^{-iHt}\psi_0$

$$\mathcal{A}(t) = \langle \psi_0 | \psi(t) \rangle = \langle \psi_0 | e^{-iHt} | \psi_0 \rangle$$

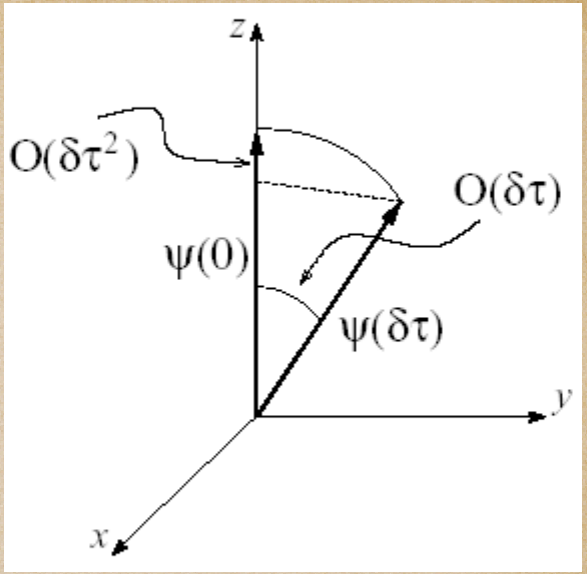
$$p(t) = |\mathcal{A}(t)|^2 = |\langle \psi_0 | e^{-iHt} | \psi_0 \rangle|^2$$

survival amplitude and probability

(always valid; both in QM and QFT)

$$p(t) = |\langle \psi_t | \psi_0 \rangle|^2 = 1 - t^2/\tau_Z^2$$

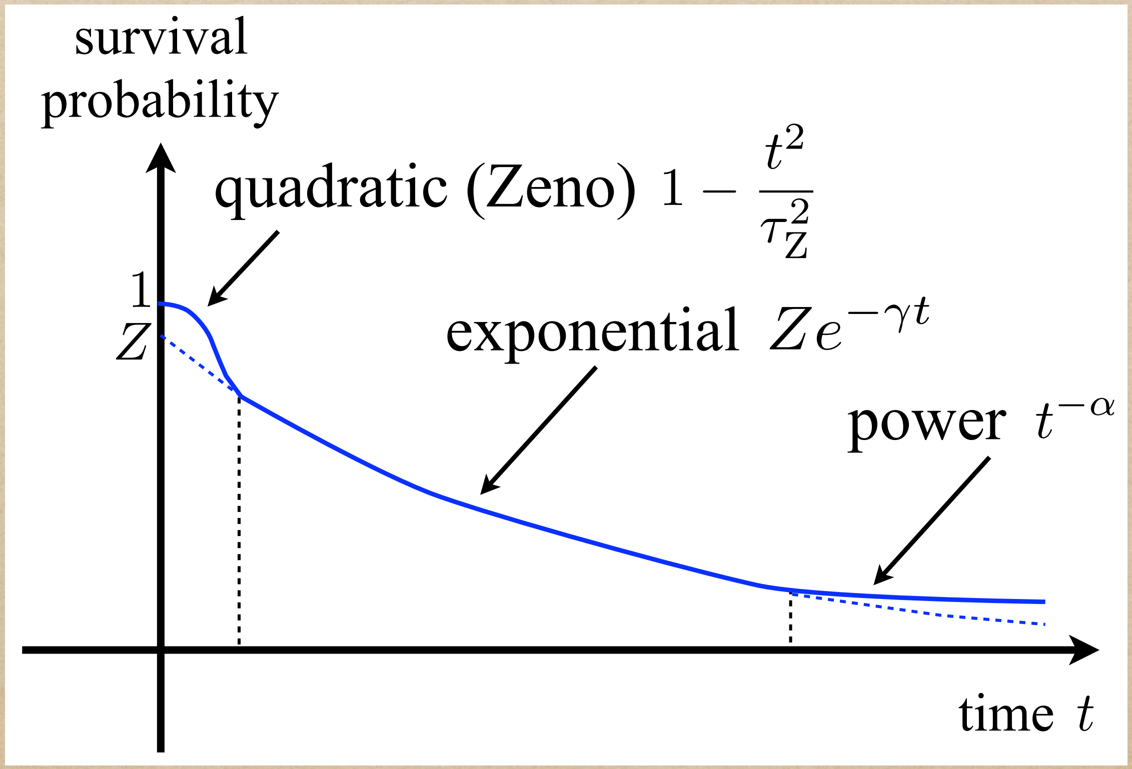
$$\tau_Z^{-2} \equiv \langle \psi_0 | H^2 | \psi_0 \rangle - \langle \psi_0 | H | \psi_0 \rangle^2$$



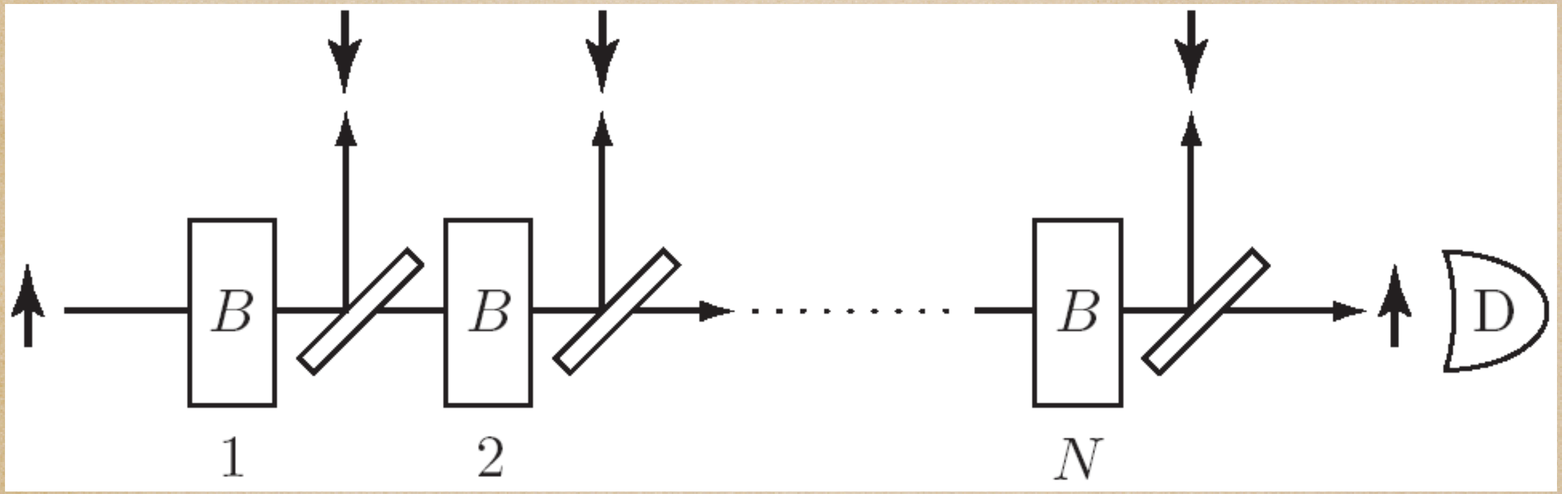
$$[p\,(t/N)]^N \simeq \left(1 - \frac{t^2}{\tau_Z^2 N^2}\right)^N \xrightarrow{N\to\infty} 1$$

Misra and Sudarshan 1977

always valid, also for “unstable” systems



Experimental proposal: neutron spin



$$p(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \cos\left(\frac{t}{\tau_Z}\right)^2$$

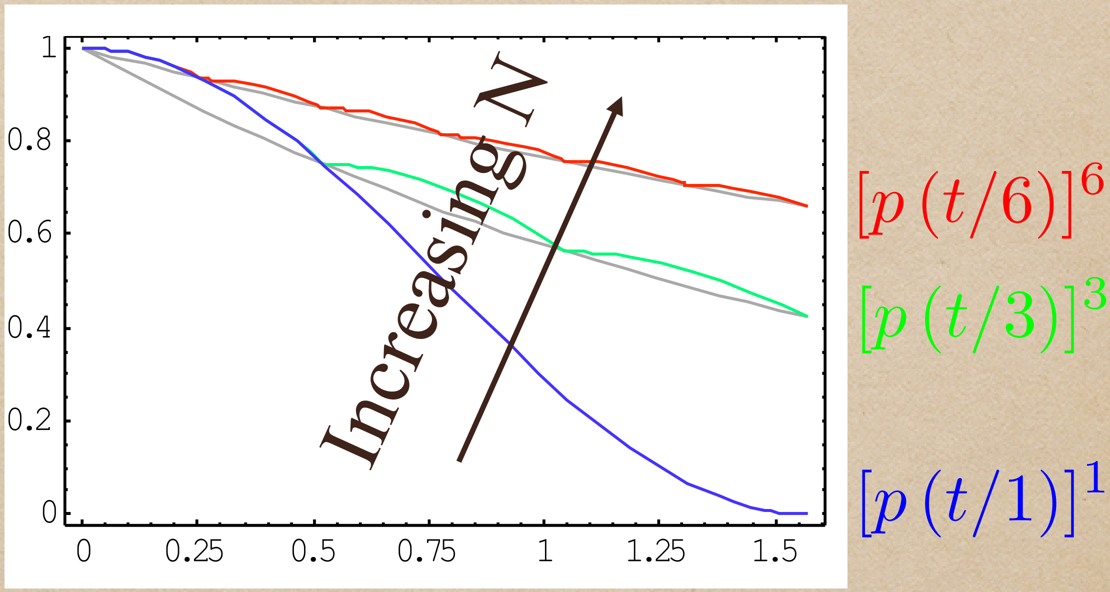
$$p^{(N)}(t) = p\left(\frac{t}{N}\right)^N = \cos\left(\frac{t}{N\tau_Z}\right)^{2N}$$

$$p^{(N)}(t) \xrightarrow{N\to\infty} 1$$

P., Namiki, Badurek, Rauch, Phys. Lett. A **169**, 155 (1993)

$$[p\,(t/N)]^N \simeq \left(1-\frac{t^2}{\tau_Z^2 N^2}\right)^N \xrightarrow{N\to\infty} 1$$

Quantum Zeno effect



many experiments on many physical systems
(and applications: quantum control)

Quantum Zeno effect

- A consequence of general principles of quantum physics
- A quantum measurement perturbs the system under observation
- And entails a projection (“collapse”) of the wave function
- Projection onto the state that is the outcome of the measurement

$$\mathcal{A}(t) = \langle\psi_0|\psi(t)\rangle = \langle\psi_0|e^{-iHt}|\psi_0\rangle$$

$$p(t) = |\mathcal{A}(t)|^2 = |\langle\psi_0|e^{-iHt}|\psi_0\rangle|^2$$

Fourier/Laplace transform

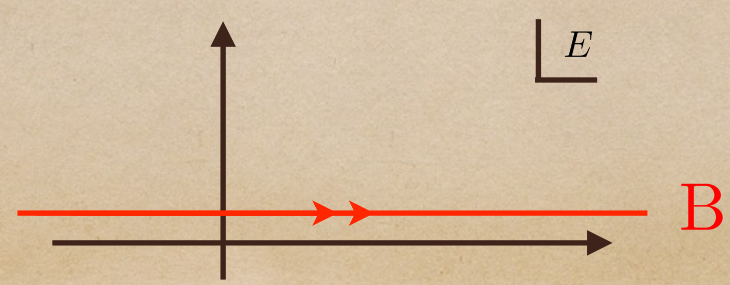
$$\mathcal{A}(E) = \int_0^\infty dt\, e^{iEt}\mathcal{A}(t) = \langle\psi_0|\frac{i}{E-H}|\psi_0\rangle$$

$$= \frac{i}{E-\omega_0-\Sigma(E)}$$

self-energy function

$$\mathcal{A}(t) = \int_{\mathrm{B}} \frac{dE}{2\pi}\, e^{-iEt}\mathcal{A}(E) = \frac{i}{2\pi}\int_{\mathrm{B}} dE\, \frac{e^{-iEt}}{E-\omega_0-\Sigma(E)}$$

Bromwich path



$$\mathcal{A}(t) = \frac{i}{2\pi}\int_B dE \, \frac{e^{-iEt}}{E-\omega_0-\Sigma(E)} = \mathcal{A}_{\text{pole}}(t) + \mathcal{A}_{\text{cut}}(t)$$

"small"

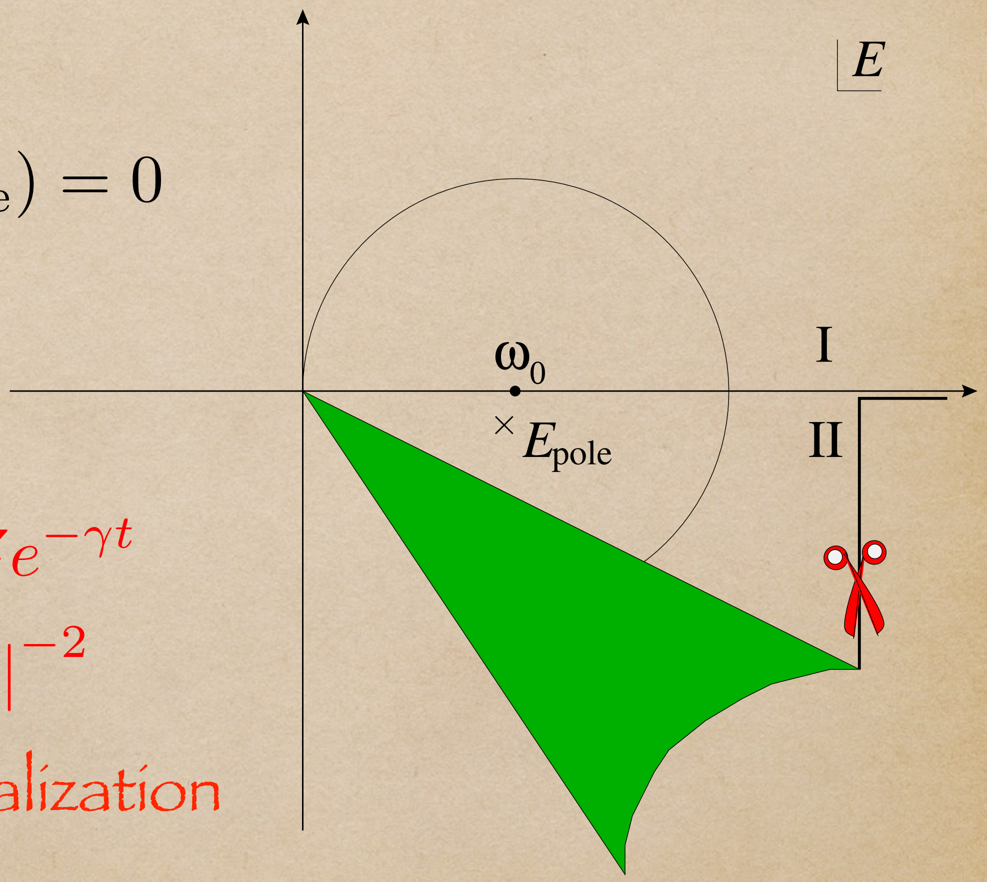
$$E_{\text{pole}} - \omega_0 - \Sigma_{II}(E_{\text{pole}}) = 0$$

Weisskopf-Wigner

$$P(t) \simeq |\mathcal{A}_{\text{pole}}(t)|^2 = Ze^{-\gamma t}$$

$$Z = |1-\Sigma'_{II}(E_{\text{pole}})|^{-2}$$

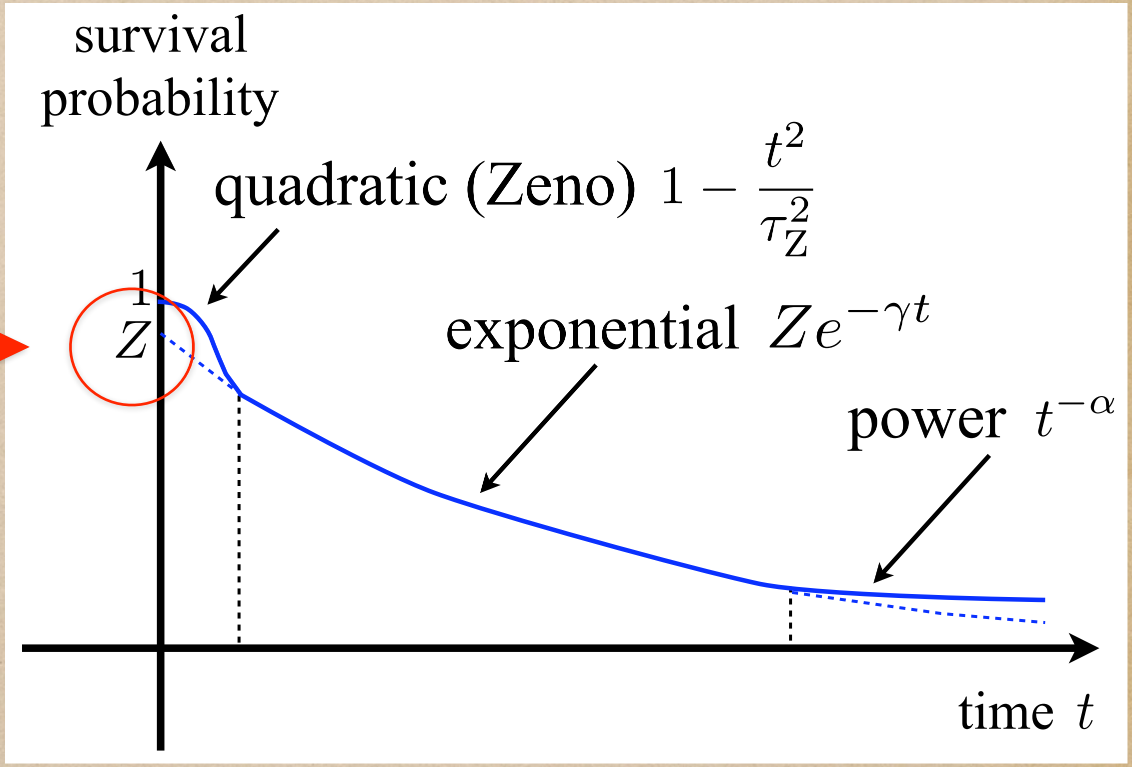
wave-function renormalization



Araki etal (1957), Schwinger 1960

notice

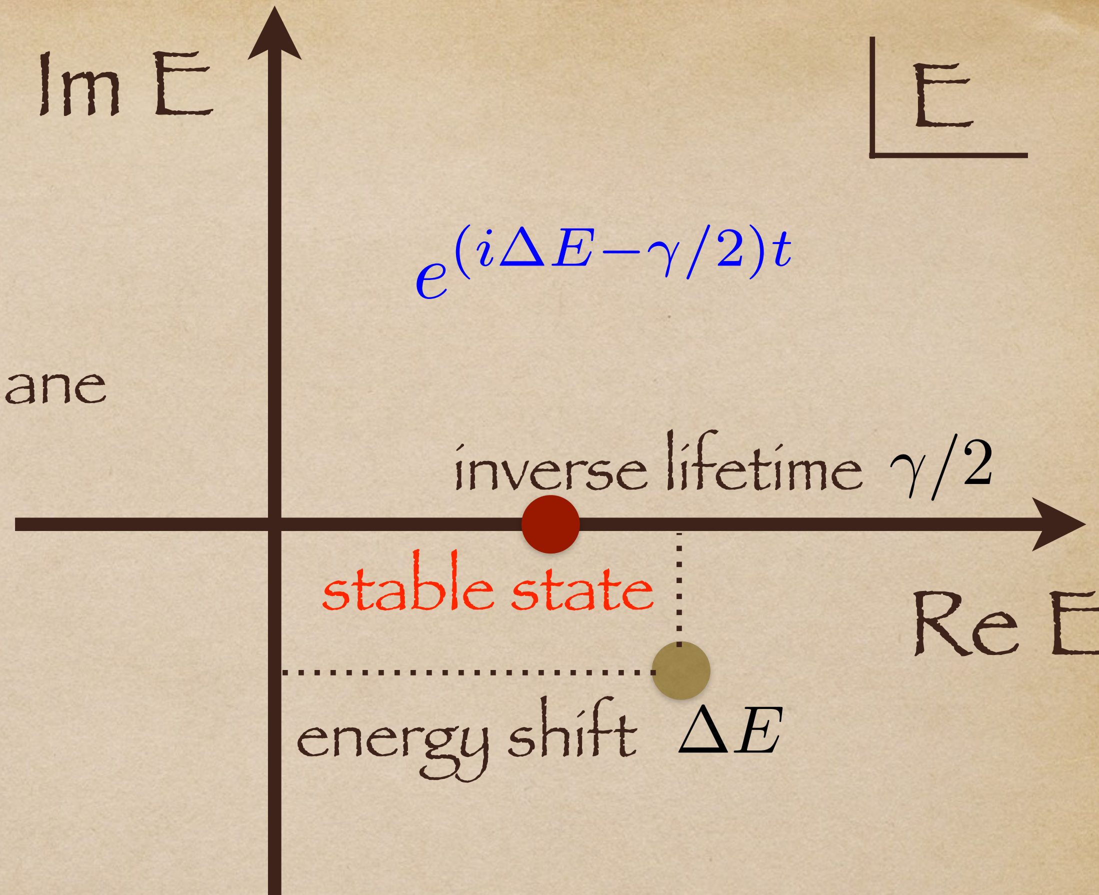
wave-function renormalization



fast tutorial

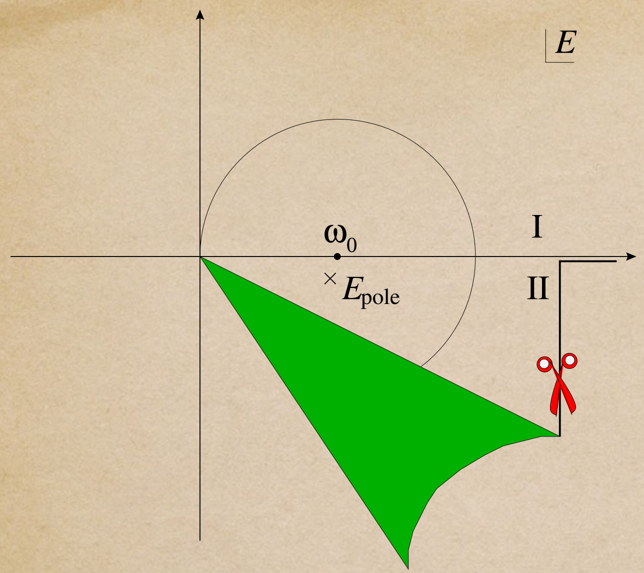
$\boxed{E}$

complex energy plane



Schwinger (simple poles);

Araki et al (proof of Fermi "Golden rule") in general



$$\mathcal{A}(t) = \mathcal{A}_{\rm pole}(t) + \mathcal{A}_{\rm cut}(t)$$

$$P(t) \simeq |\mathcal{A}(t)|^2$$

interference!

therefore

- Zeno
- Z (wf renormalisation)
- pole + cut
- what else?

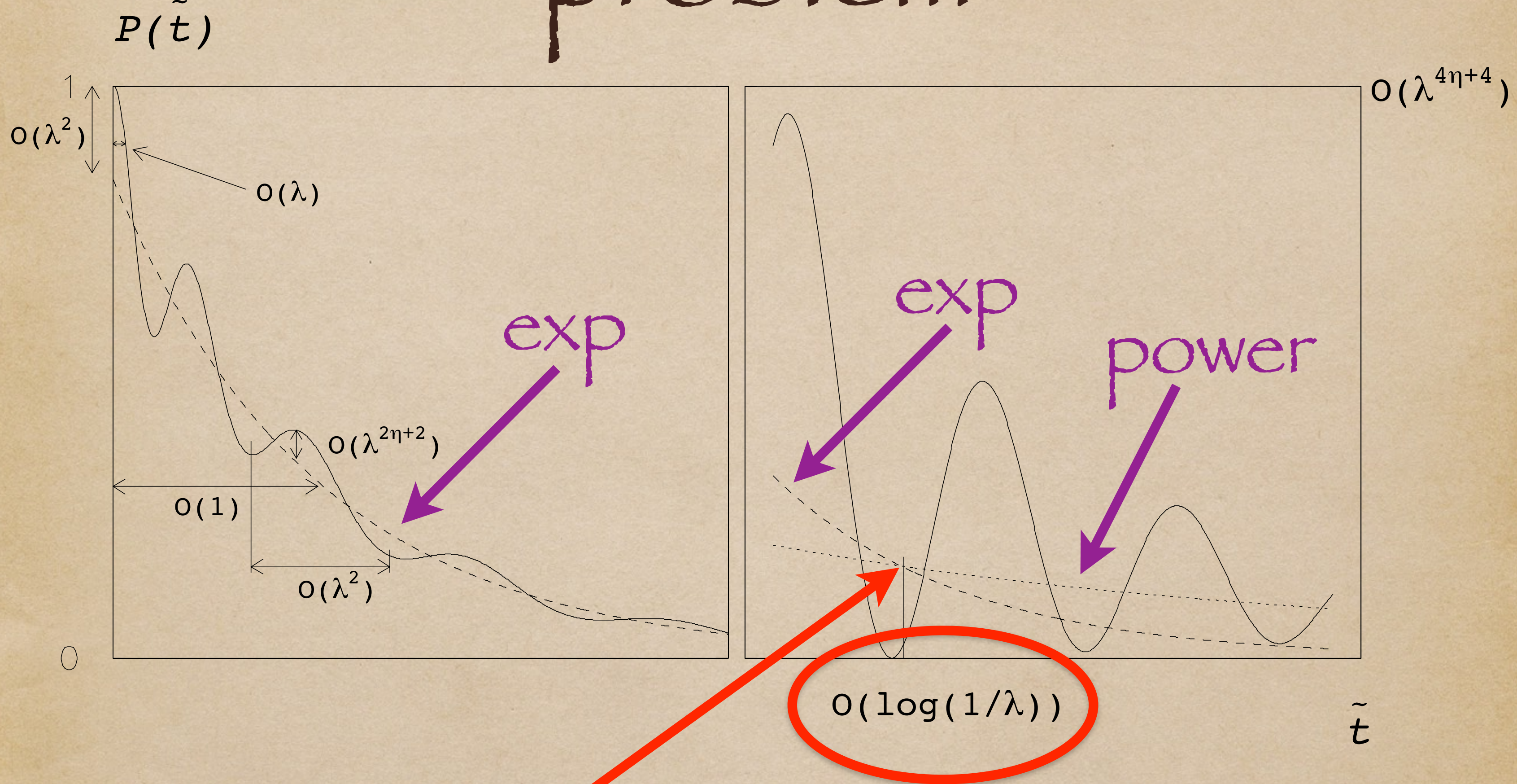
Paley and Wiener

$$\int_{-\infty}^{+\infty} \frac{|\log p(t)|}{1+t^2} \mathrm{d}t < +\infty$$

if spectrum of H bounded from below

incompatible with $p(t) \sim e^{-\alpha t^\beta}$

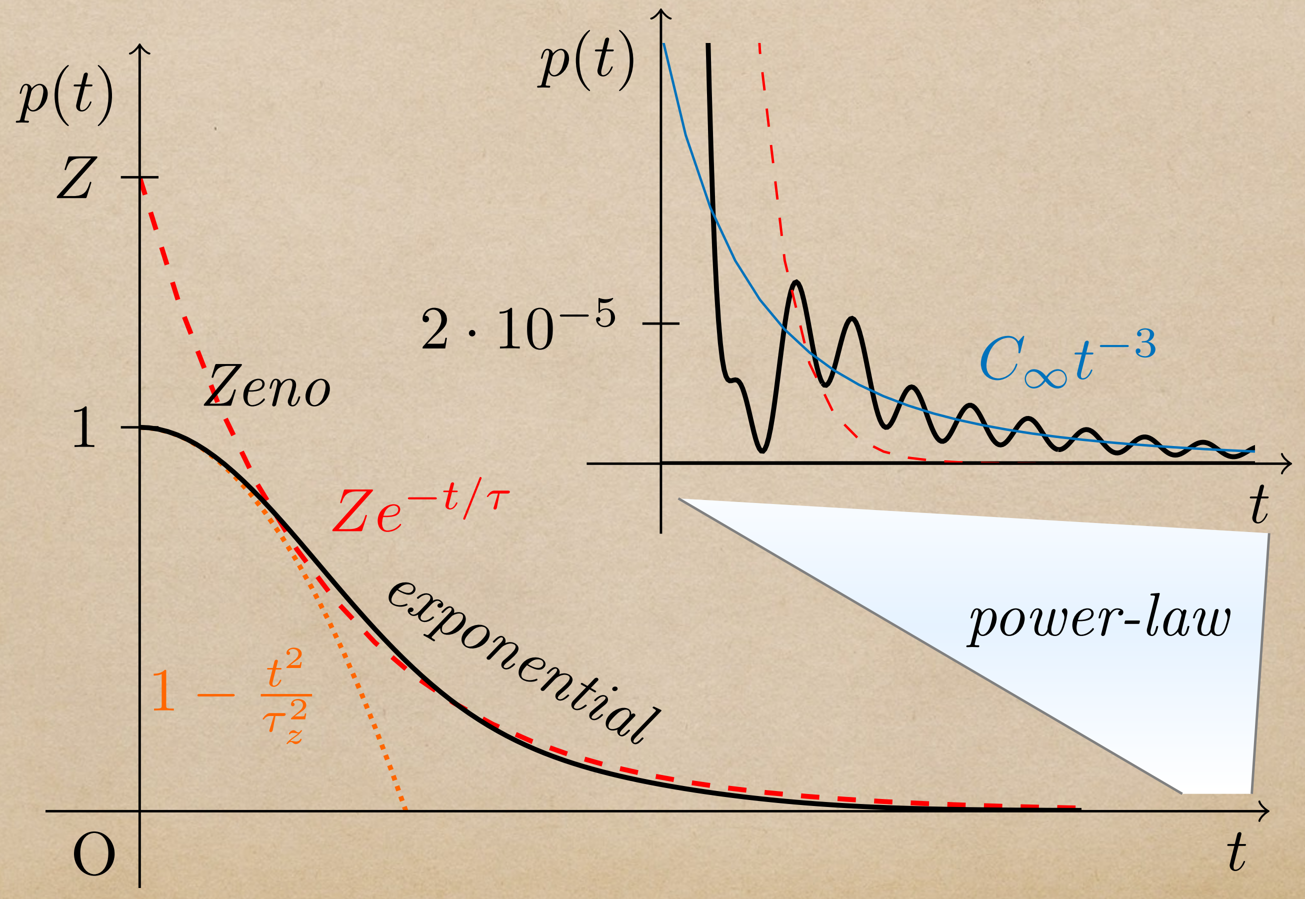
problem



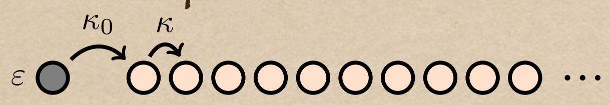
at transition, initial state heavily depleted!

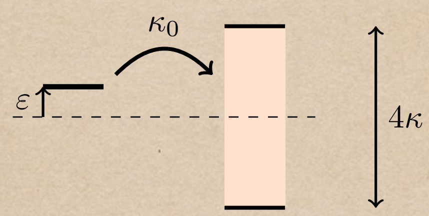
(say, 100 lifetimes!)

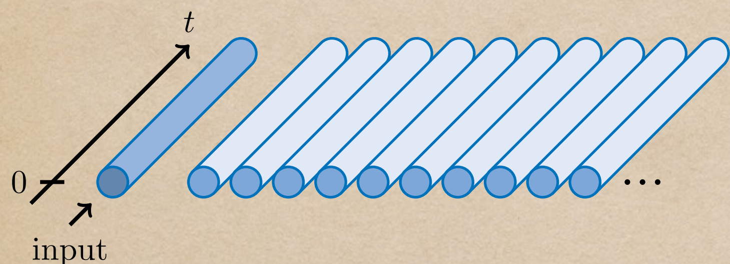
in a real system

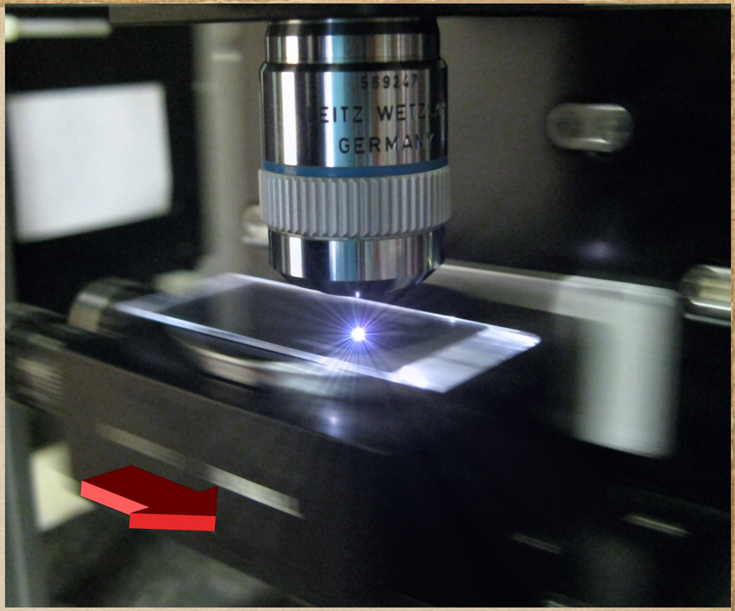


experiment (Sciarrino + Crespi and Osellame)

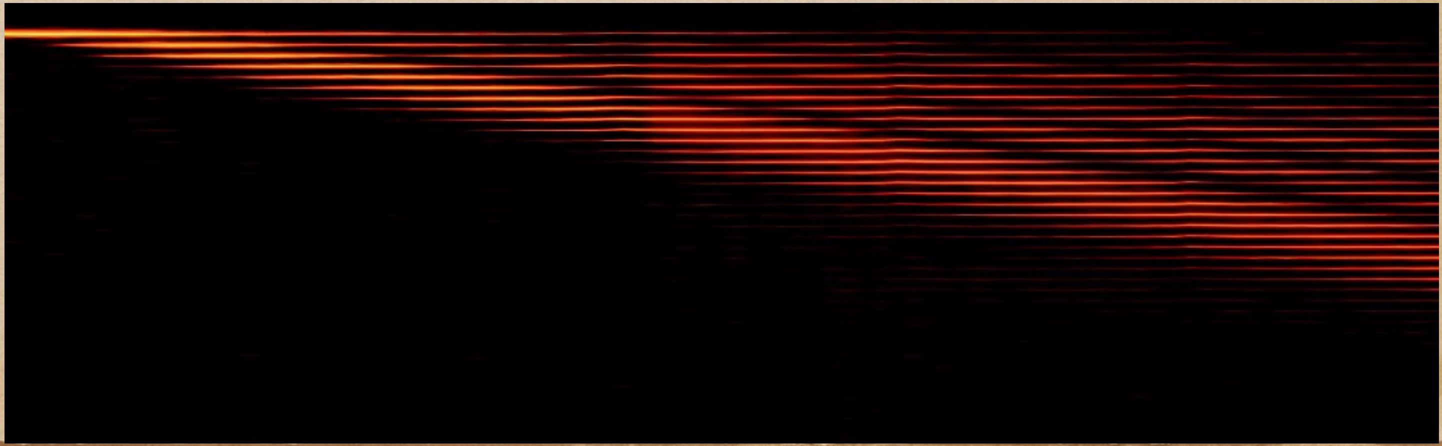


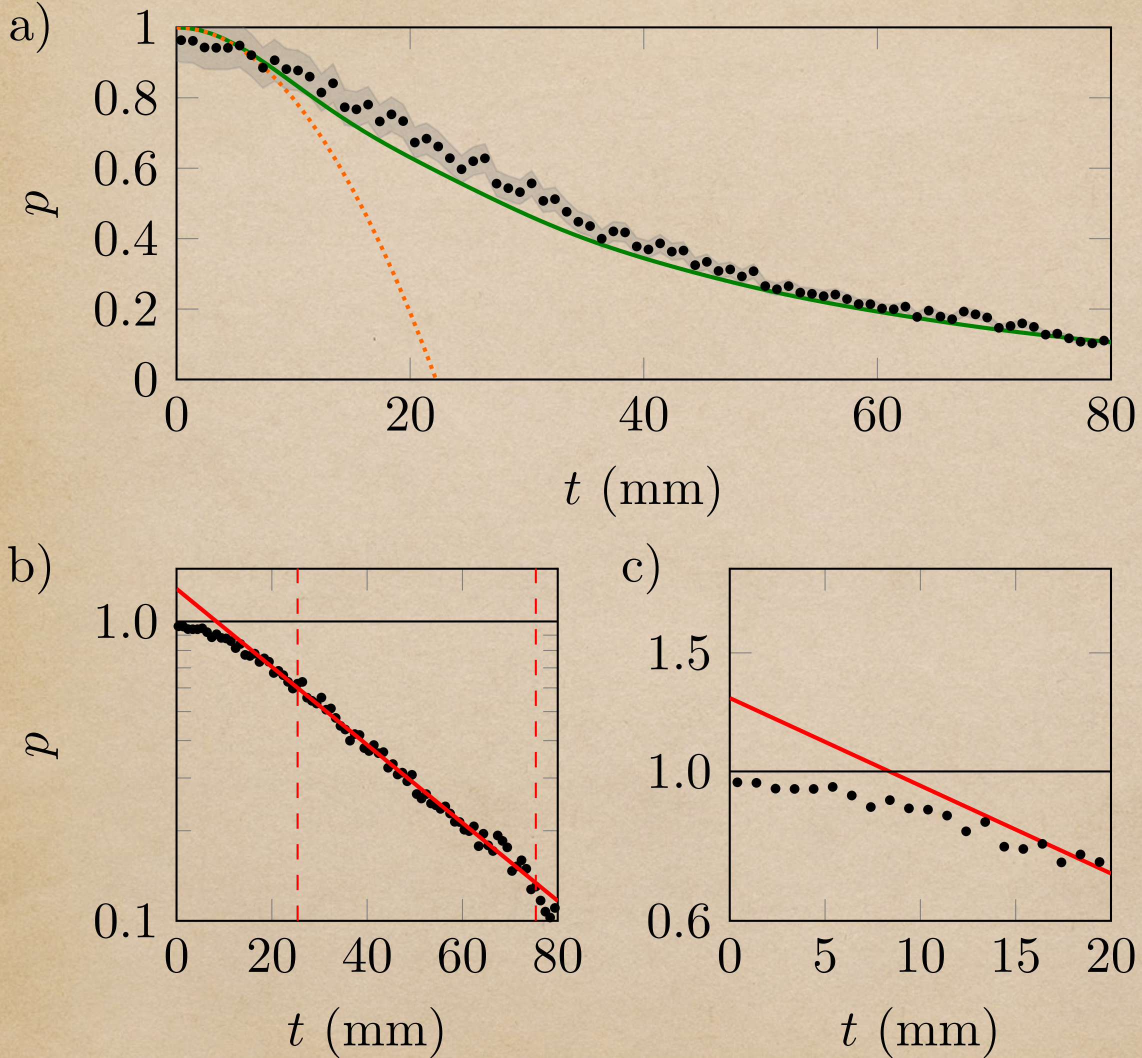




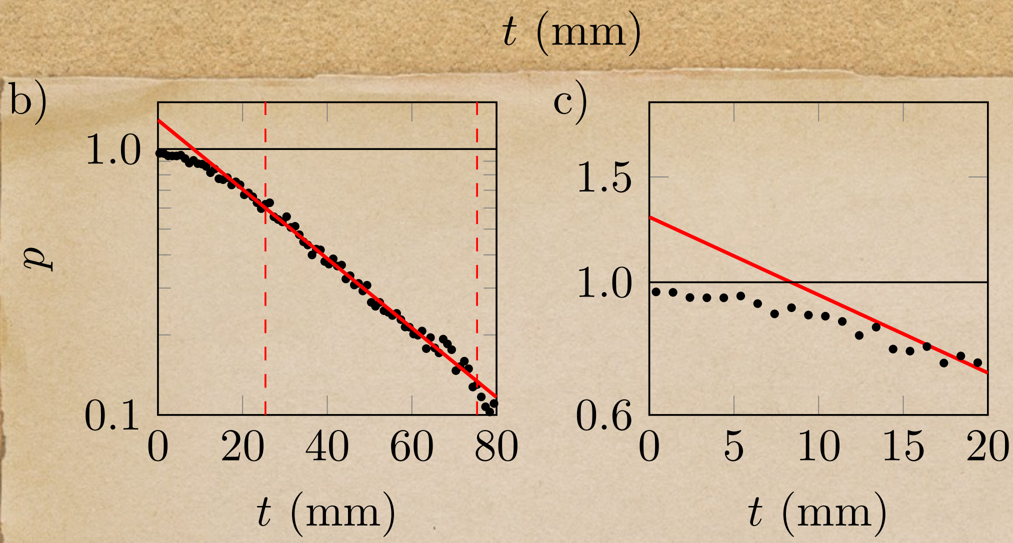


$|\mathrm{IN}\rangle \rightarrow$




a)
1
0.8
0.6
0.4
0.2
0
p
0
20
40
60
80
t (mm)
b)
1.0
0.1
p
0
20
40
60
80
t (mm)
c)
1.5
1.0
0.6
0
5
10
15
20
t (mm)

$$Z \simeq 1.23$$



$Z \simeq 1.23$

PHYSICAL REVIEW LETTERS

VOLUME 86 26 MARCH 2001 NUMBER 13

From the Quantum Zeno to the Inverse Quantum Zeno Effect

P. Facchi,[1,3] H. Nakazato,[2] and S. Pascazio[3]

[1]*Atominstitut der Österreichischen Universitäten, Stadionallee 2, A-1020, Wien, Austria*
[2]*Department of Physics, Waseda University, Tokyo 169-8555, Japan*
[3]*Dipartimento di Fisica, Università di Bari and Istituto Nazionale di Fisica Nucleare, Sezione di Bari, I-70126 Bari, Italy*
(Received 21 June 2000)

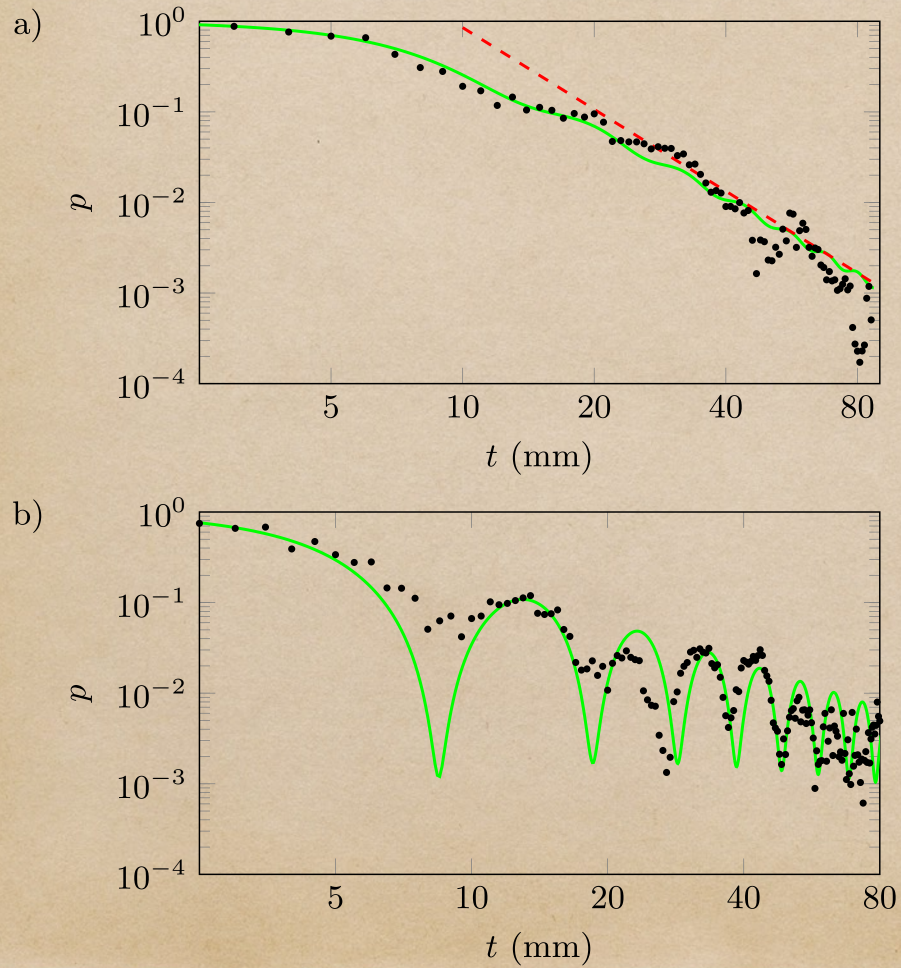
The temporal evolution of an unstable quantum mechanical system undergoing repeated measurements is investigated. In general, by changing the time interval between successive measurements, the decay can be accelerated (inverse quantum Zeno effect) or slowed down (quantum Zeno effect), depending on the features of the interaction Hamiltonian. A geometric criterion is proposed for a transition to occur between these two regimes.

DOI: 10.1103/PhysRevLett.86.2699 PACS numbers: 03.65.Xp

$$x(t) = \sqrt{Z}\, e^{-\gamma_0 t/2 - i\alpha(t)} + x_{\rm cut}(t),$$
$$Z = |1 - \Sigma'(E_{\rm pole})|^{-2}, \qquad (9)$$

where the exponential term (first term) is due to the contribution of a simple pole $E_{\rm pole}$ on the second Riemannian sheet in the complex energy plane, while the second term is the result of a contour integration [1]. The lifetime γ_0^{-1} is given by the Fermi golden rule, computed according to the Weisskopf-Wigner approximation. The quantity Z is the square of the residue of pole of the propagator (yielding wave function renormalization in quantum field the ... t, alth ugh for a stable state $Z < 1$ (due to probability c n-ser tion in the Källén-Lehmann representation), fo an uns ... is of order (coupling constant)^2 and modifies the exponential law both at short and long times, yielding the characteristic quadratic and power-law behaviors. The survival probabil-

power law



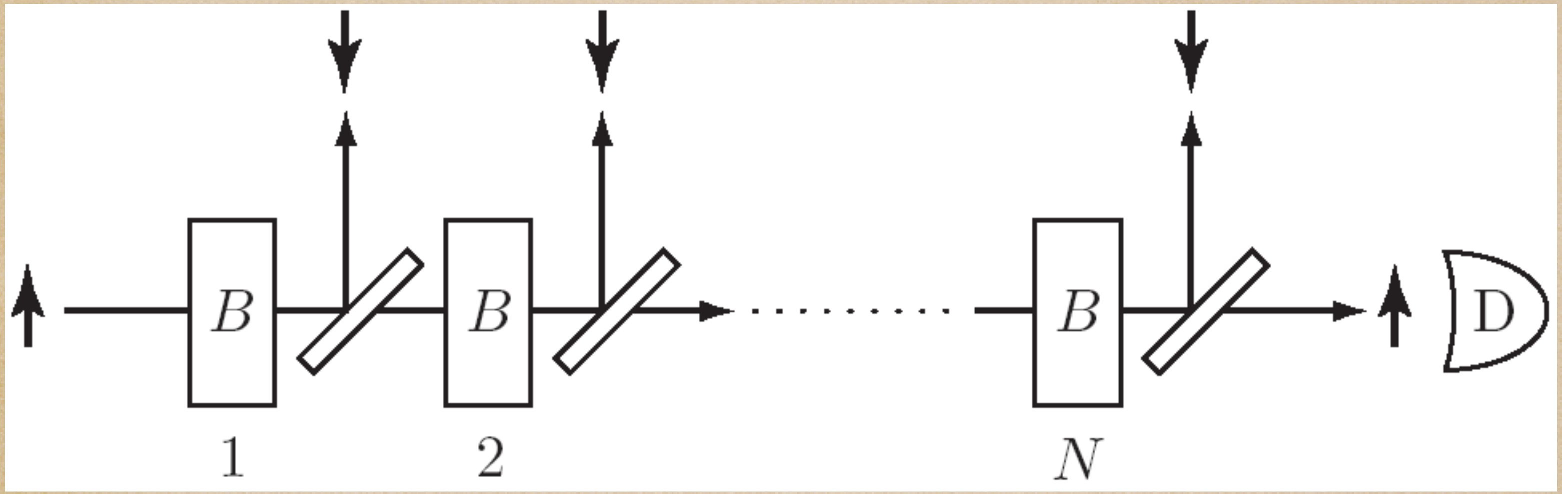
$$p(t) \sim \frac{1}{t^3}$$

notice interference

back to short times

- Seminal ideas
- Recent experiments

Reminder: neutron spin



$$p(t) = \left|\langle \Psi(0) | \Psi(t) \rangle\right|^2 = \cos\left(\frac{t}{\tau_Z}\right)^2$$

$$p^{(N)}(t) = p\left(\frac{t}{N}\right)^N = \cos\left(\frac{t}{N\tau_Z}\right)^{2N}$$

$$p^{(N)}(t) \xrightarrow{N \to \infty} 1$$

P., Namiki, Badurek, Rauch, Phys. Lett. A **169**, 155 (1993)

History

von Neumann,1932

Beskow and Nilsson,1967

Khalfin 1968

Friedman 1972

Misra and Sudarshan, 1977

photon polarization, nuclear spin isomers

ions (individual), neutron spin

Landau-Zener tunneling Bose-Einstein condensates

Cavity QED

Applications: decoherence in quantum computing

efficient preservation of spin polarized gases

Zeno tomography

(main) Experiments (Cook 1988)

Itano, Heinzen, Bollinger, and Wineland 1990

Nagels, Hermans, and Chapovsky 1997

Kwiat, White, Mitchell, Nairz, Weihs, Weinfurter and Zeilinger 1999

Wunderlich, Balzer, and Toschek, 2001

Fischer, Gutierrez-Medina, Raizen, 2001

Streed, Mun, Boyd, Campbell, Medley, Ketterle, Pritchard, 2006

Bernu, Sayrin, Kuhr, Dotsenko, Brune, Raimond, Haroche 2008

ALL: one dimensional Zeno effect!

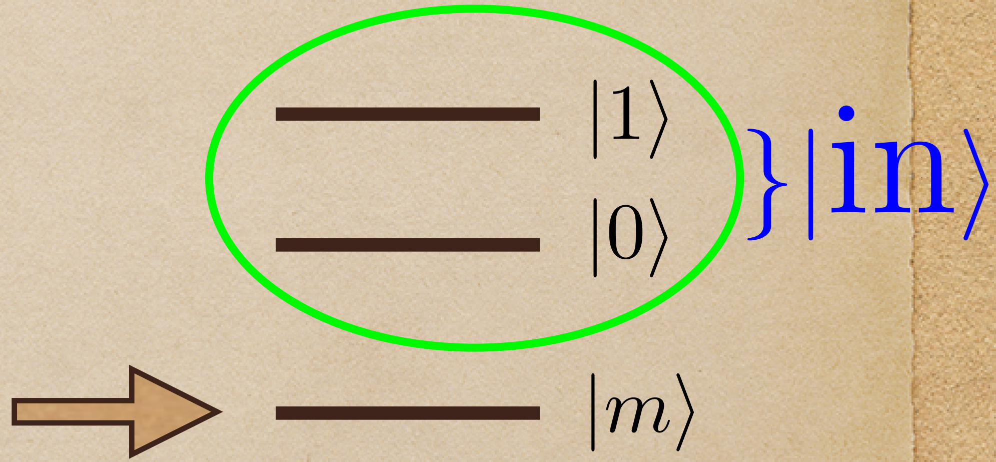
but system need not be 1D...

example:

measure

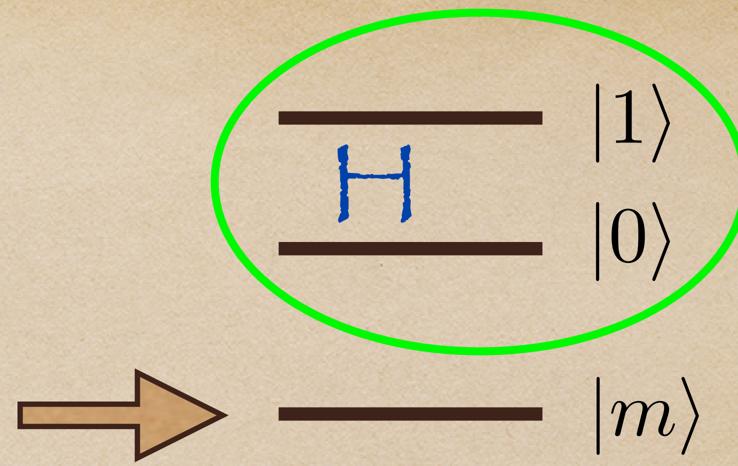
system remains in initial state

more interesting example:



measure

system remains in subspace |in> defined by (negative result, nonselective) measurement



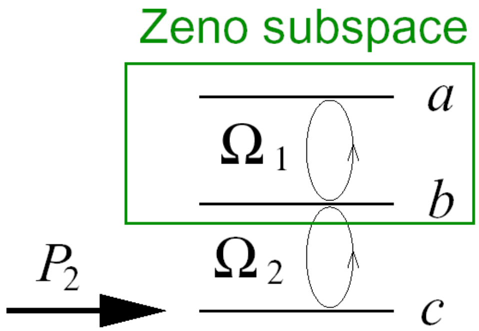
system remains in subspace defined by
(negative result, nonselective) measurement

what if there is a Hamiltonian?

Quantum Zeno DYNAMICS

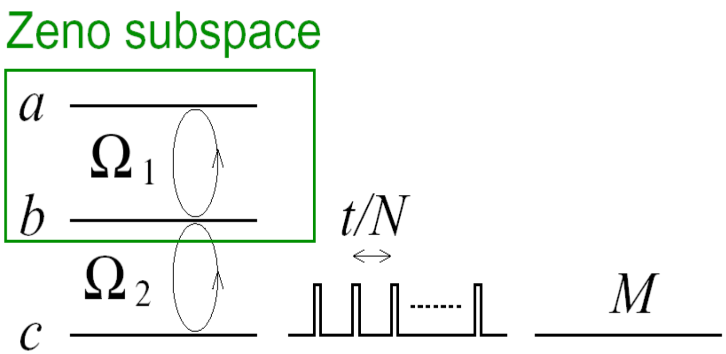
in Quantum Zeno SUBSPACE

3 STRATEGIES to obtain Zeno subspaces



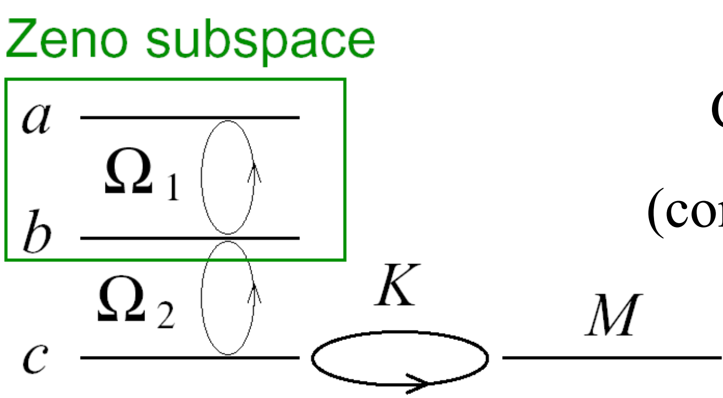
Measurements

(projections)



Unitary kicks

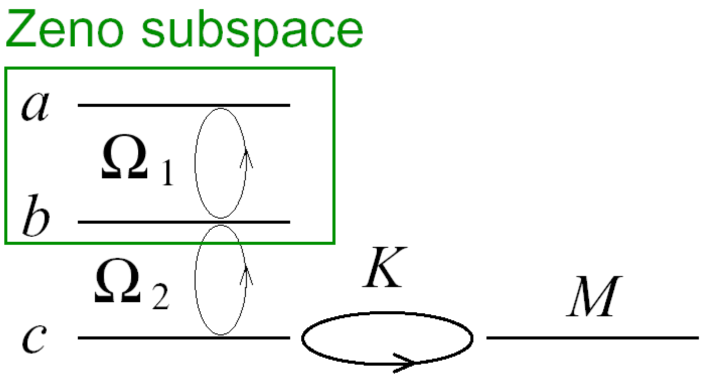
("bang bang")



Continuous coupling

(continuous measurement)

e.g.: let us look at continuous coupling



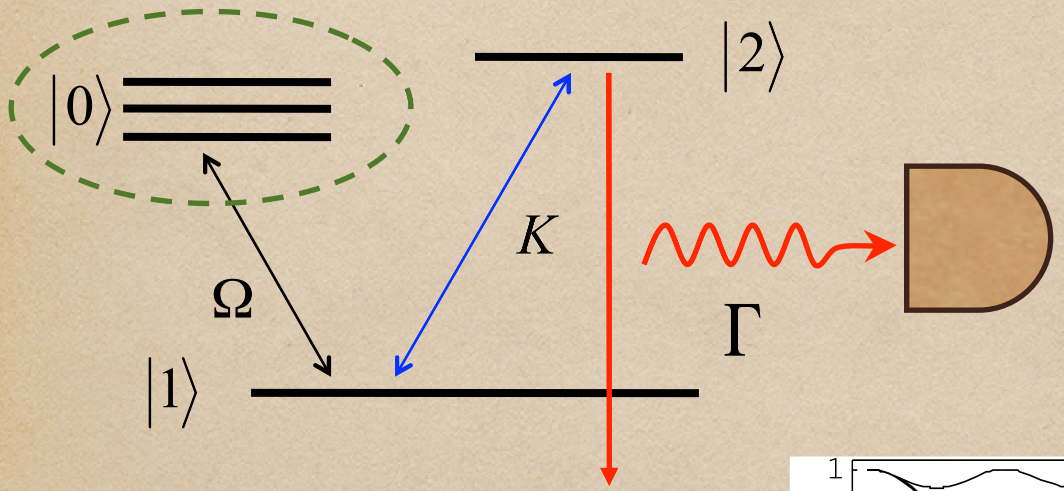
$$KH_c = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K \\ 0 & 0 & K & 0 \end{pmatrix}$$

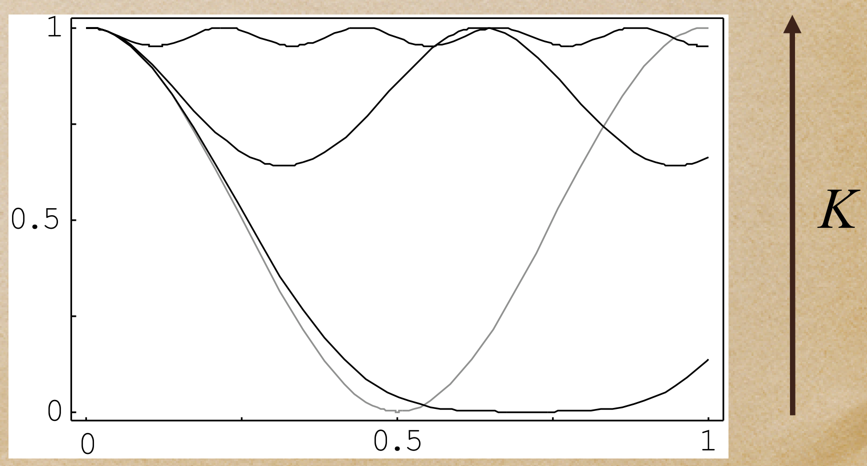
$$H = \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & \Omega_2 & 0 \\ 0 & \Omega_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = H_Z$$

Zeno limit $(K \to \infty)$

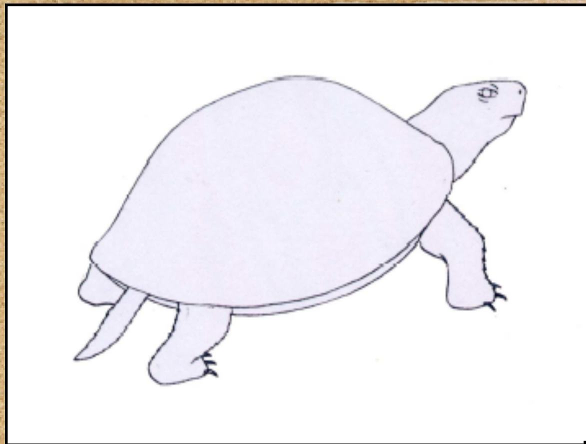
What causes the Zeno effect

Zeno subspace

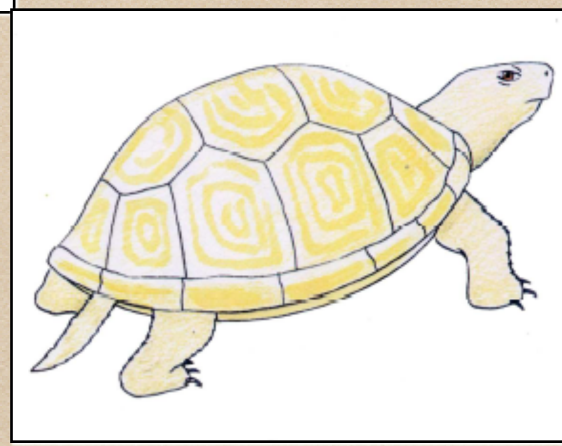




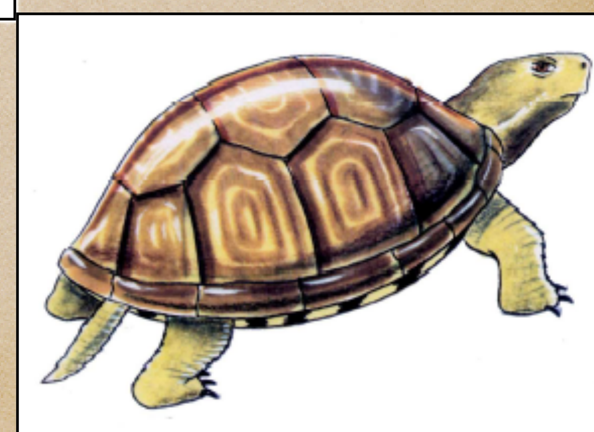
Quantum Zeno subspaces



Dynamical superselection sectors



Coupling or N



Experiments

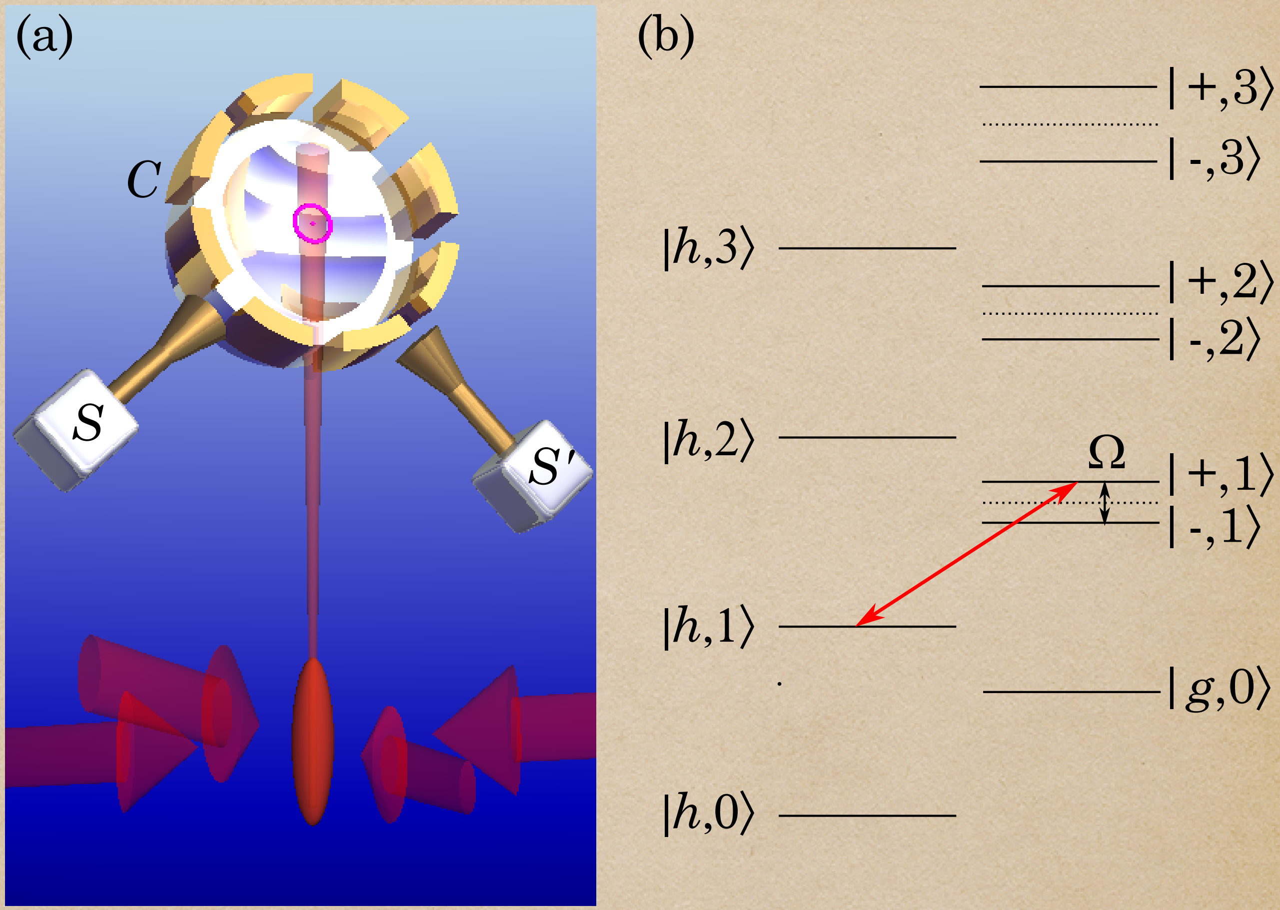
Experimental realization of quantum zeno dynamics
F. Schäfer, I. Herrera, S. Cherukattil, C. Lovecchio, F.S. Cataliotti, F. Caruso & A. Smerzi
Nature Communications 5, 3194 (2014)

Confined quantum Zeno dynamics of a watched atomic arrow
Adrien Signoles, Adrien Facon, Dorian Grosso, Igor Dotsenko, Serge Haroche, Jean-Michel Raimond, Michel Brune & Sébastien Gleyzes
Nature Physics 10, 715–719 (2014)

Quantum dynamics of an electromagnetic mode that cannot have N photons
L. Bretheau, P. Campagne-Ibarcq, E. Flurin, F. Mallet, B. Huard
Science, 348, 776-779 (2015)

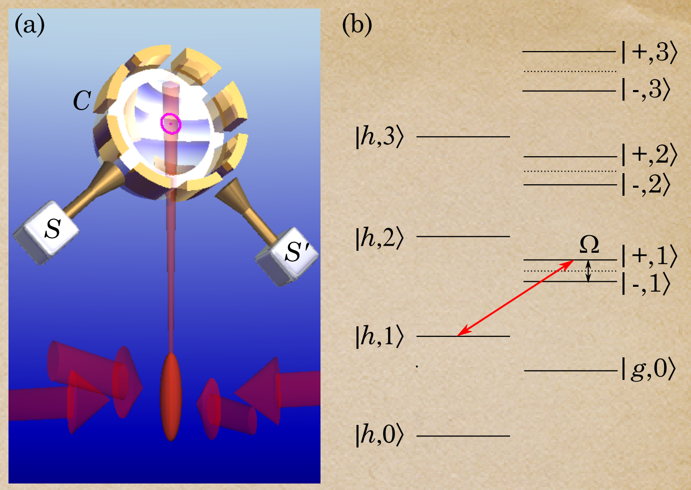
QZD in Cavity QED

atomic fountain



$$|\pm, n\rangle = \frac{1}{\sqrt{2}}(|e, n-1\rangle \pm |g, n\rangle), \quad n \geq 1$$

Hamiltonians



laser

$$H = \alpha a^\dagger + \alpha^* a$$

cavity

$$V = \frac{\hbar\Omega}{2}\left(|e\rangle\langle g|a + |g\rangle\langle e|a^\dagger\right)$$

implement measurement of s

$$P = 1 - |s\rangle\langle s|$$

Zeno Hamiltonian

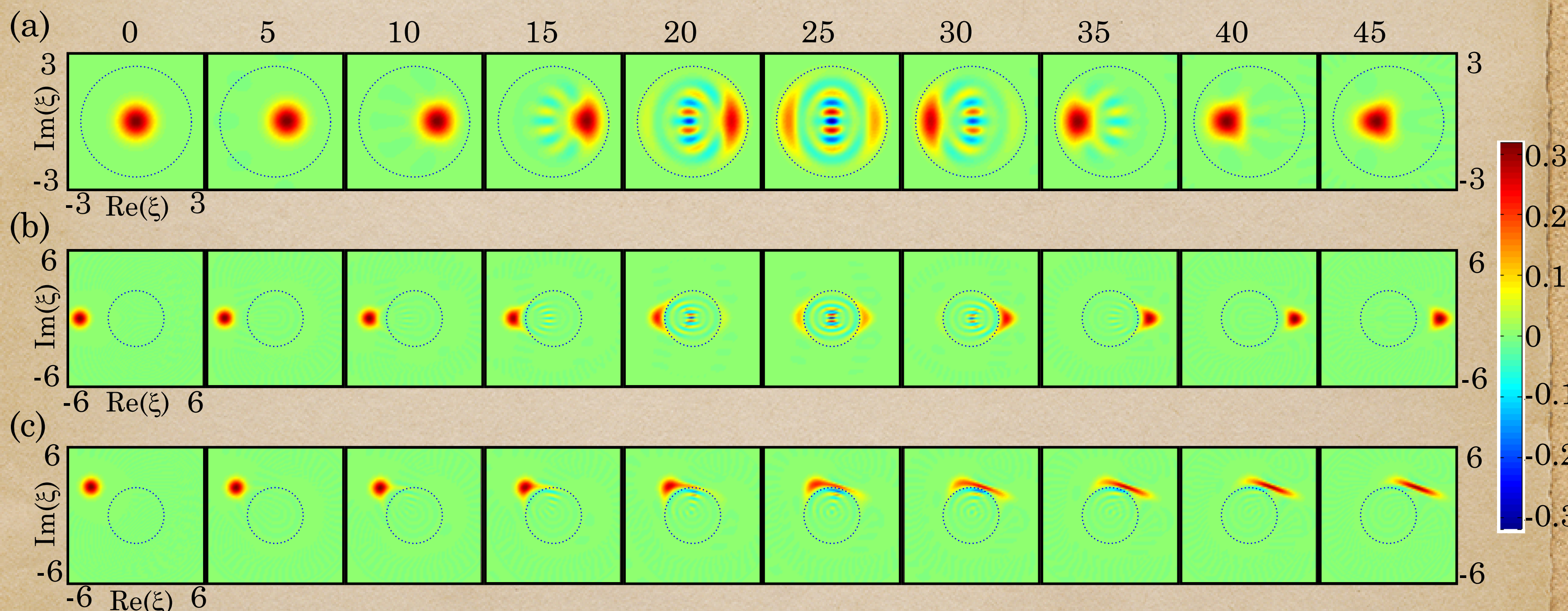
$$H_Z = P_{<s} H P_{<s} + P_{>s} H P_{>s} = H_{<s} + H_{>s}$$

$$H = \begin{pmatrix} 0 & \alpha & 0 & 0 & 0 & 0 & 0 & \dots \\ \alpha^* & 0 & \sqrt{2}\alpha & 0 & 0 & 0 & 0 & \\ 0 & \sqrt{2}\alpha^* & 0 & \sqrt{3}\alpha & 0 & 0 & 0 & \\ 0 & 0 & \sqrt{3}\alpha^* & 0 & \sqrt{4}\alpha & 0 & 0 & \\ 0 & 0 & 0 & \sqrt{4}\alpha^* & 0 & \sqrt{5}\alpha & 0 & \\ 0 & 0 & 0 & 0 & \sqrt{5}\alpha^* & 0 & \sqrt{6}\alpha & \\ 0 & 0 & 0 & 0 & 0 & \sqrt{6}\alpha^* & 0 & \\ \vdots & & & & & & & \ddots \end{pmatrix}$$

s=4

$$\xrightarrow{\text{Zeno}} \begin{pmatrix} 0 & \alpha & 0 & 0 & 0 & 0 & 0 & \dots \\ \alpha^* & 0 & \sqrt{2}\alpha & 0 & 0 & 0 & 0 & \\ 0 & \sqrt{2}\alpha^* & 0 & \sqrt{3}\alpha & 0 & 0 & 0 & \\ 0 & 0 & \sqrt{3}\alpha^* & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{6}\alpha & \\ 0 & 0 & 0 & 0 & 0 & \sqrt{6}\alpha^* & 0 & \\ \vdots & & & & & & & \ddots \end{pmatrix} = H_Z$$

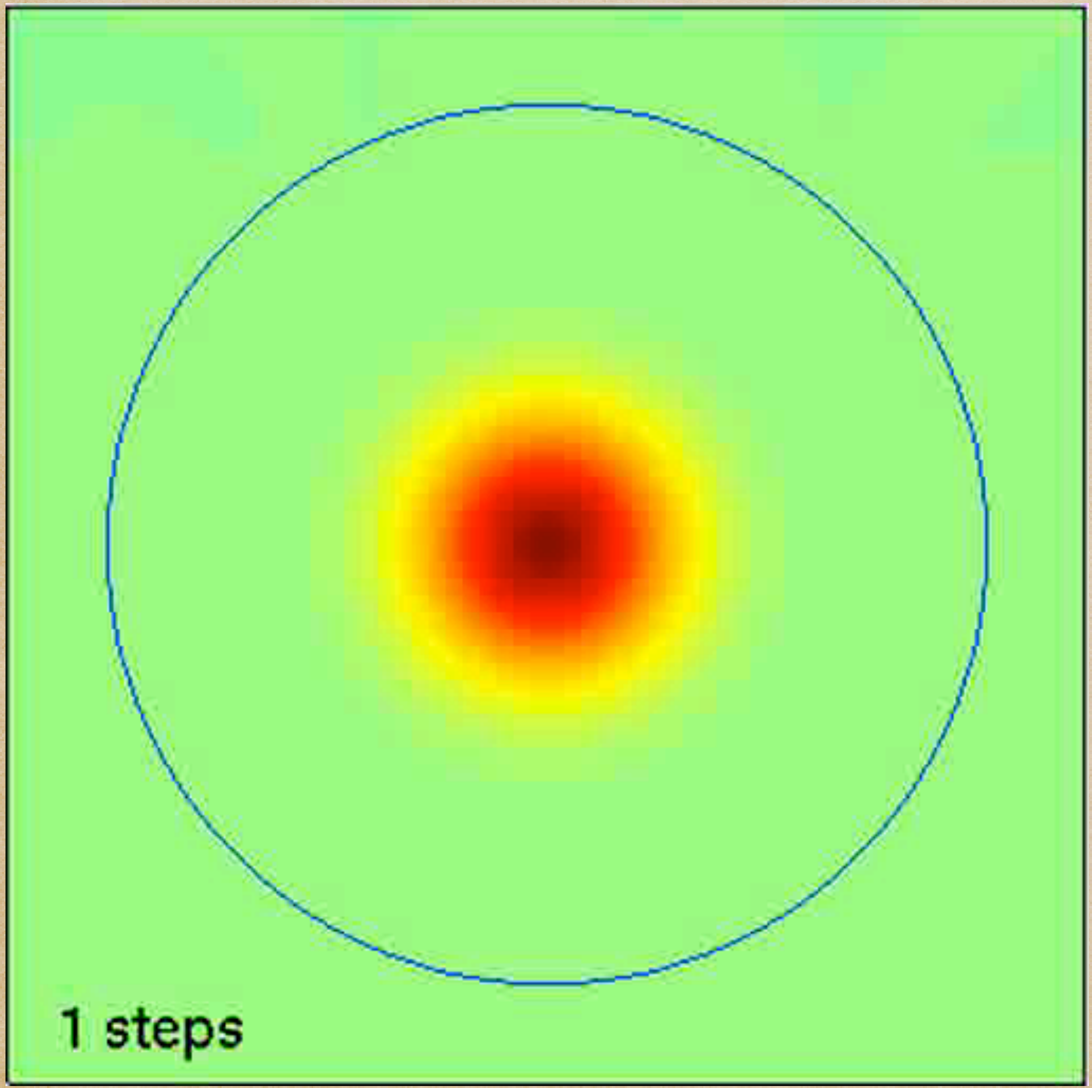
s=6: phase space view



“exclusion circle”

“tailoring” non-classical fields

Quantum Zeno dynamics



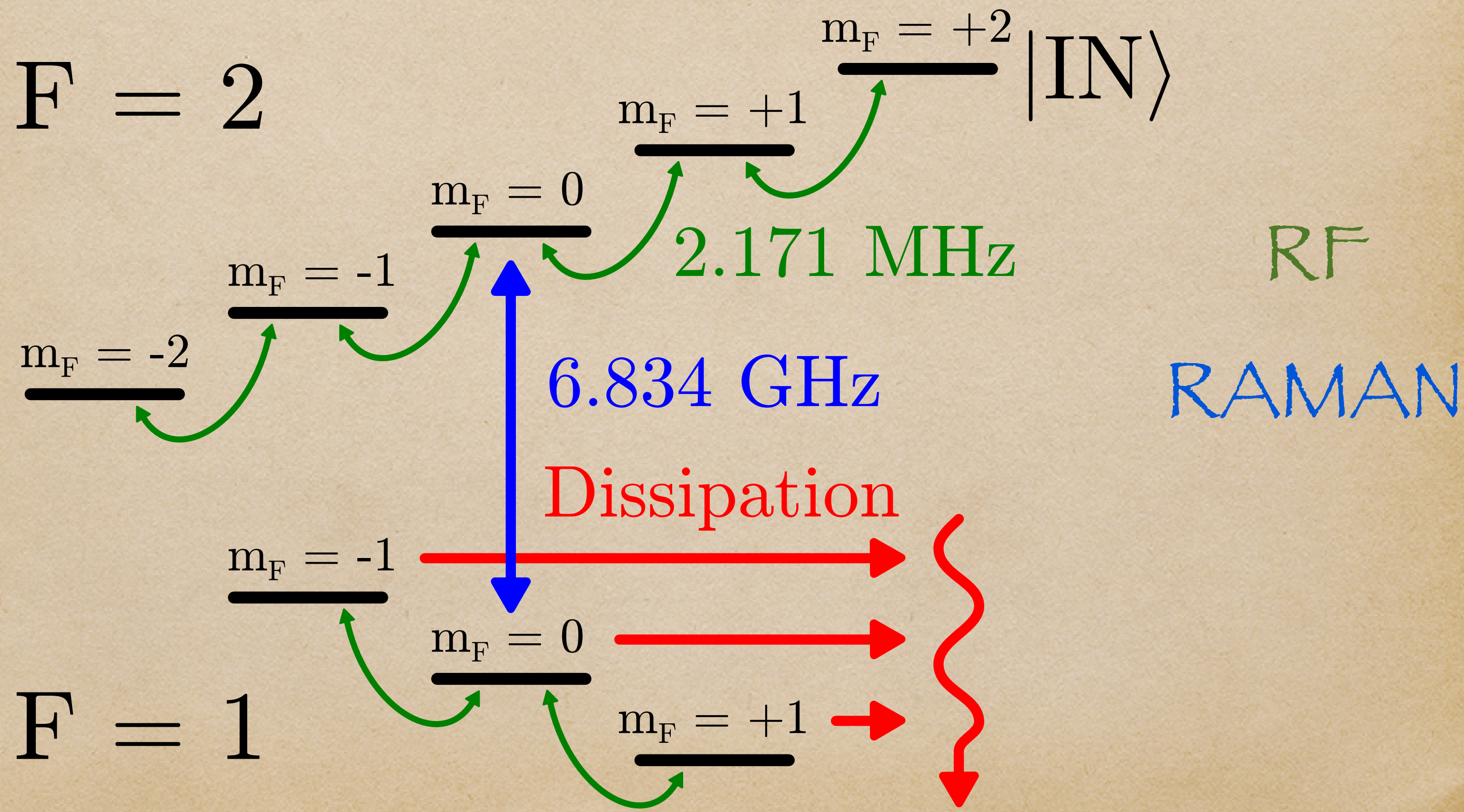
exclusion circle
in phase space

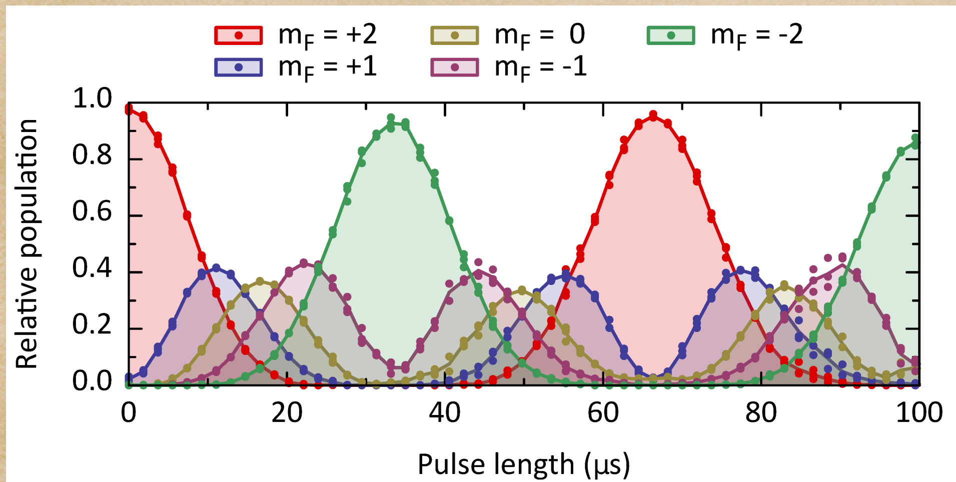
think of the way
the measurement
is performed!

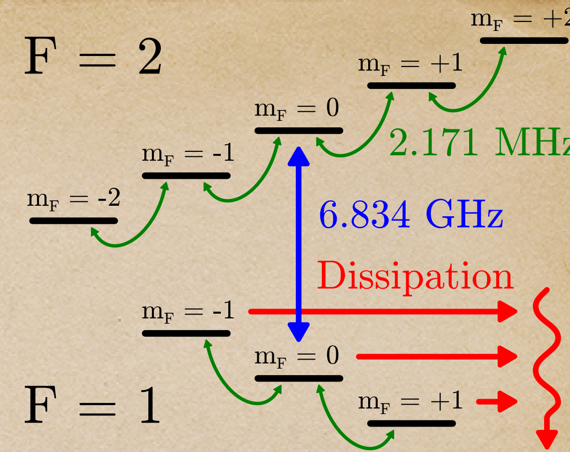
FIRST EXPERIMENT

Schaefer, Herrera, Cherukattil, Lovecchio, Cataliotti, Caruso, Smerzi

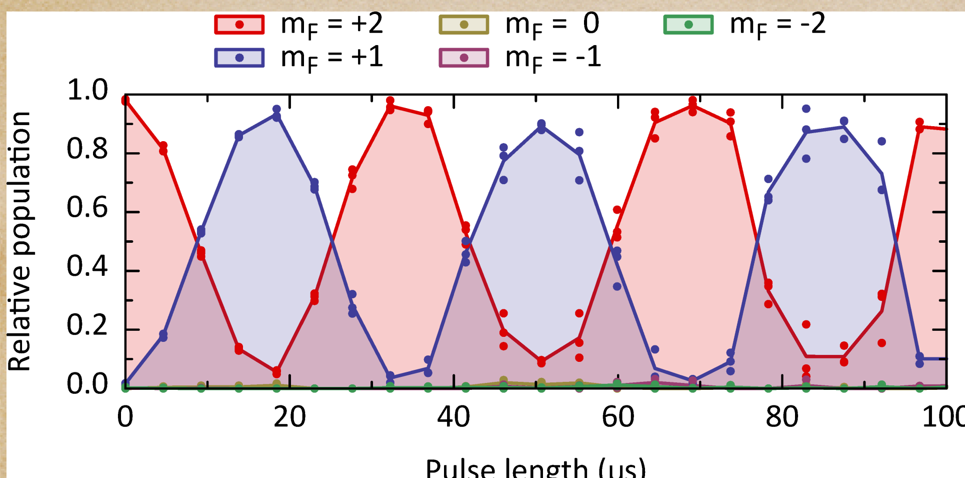
Florence 2014







only RF on



RF and RAMAN on !

Free particle in n dimensions

$$H = \frac{p^2}{2M} = -\frac{\hbar^2 \Delta}{2M}, \quad U(t) = \exp(-iHt/\hbar) \quad \text{in } L^2(\mathbb{R}^n)$$

$\Omega \subset \mathbb{R}^n$ compact domain, $P = \chi_\Omega(x)$ spatial projection

Zeno dynamics $V_Z(t) = \lim_{N\to\infty} \left[V\left(\frac{t}{N}\right)\right]^N, \quad V(s) = PU(s)P$

How does the particle move inside Ω? Does it leak out?

The weak limit $V_Z(t)$ exists and yields

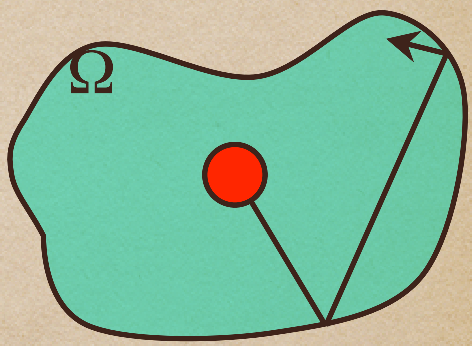
$V_Z(t) = \exp(-iH_\Omega t/\hbar)P,$

a unitary group in $L^2(\Omega)$

with Zeno Hamiltonian

$$H_\Omega = -\frac{\hbar^2 \Delta}{2M}, \quad D(H_\Omega) = H^2(\Omega) \cap H_0^1(\Omega)$$

Dirichlet boundary conditions



Free particle in a box with perfectly reflecting hard walls

Facchi, Marmo, Pascazio, Scardicchio, Sudarshan 2003
Exner and Ichinose, 2005

…although there is NO wall!

a final comment

$$U^{(N)}(t) = \underbrace{\left(e^{-iTt/N}e^{-iVt/N}\right)\left(e^{-iTt/N}e^{-iVt/N}\right)\cdots\left(e^{-iTt/N}e^{-iVt/N}\right)}_{N\text{ times}}$$

$$= \left(e^{-iTt/N}e^{-iVt/N}\right)^N$$

Feynman/Trotter

$$\sim e^{-i(T+V)t} = e^{-iHt} \qquad H = T + V$$

$$V^{(N)}(t) = \underbrace{\left(Pe^{-iHt/N}\right)\left(Pe^{-iHt/N}\right)\cdots\left(Pe^{-iHt/N}\right)}_{N\text{ times}}$$

$$= \left(Pe^{-iHt/N}\right)^N$$

Zeno (Faddeev)

$$\sim e^{-iPHPt}P$$

$$W^{(N)}(t) = \underbrace{\left(e^{-iH_1t/N}e^{-iH_2t/N}\right)\left(e^{-iH_1t/N}e^{-iH_2t/N}\right)\cdots\left(e^{-iH_1t/N}e^{-iH_2t/N}\right)}_{N\text{ times}}$$

$$= \left(e^{-iH_1t/N}e^{-iH_2t/N}\right)^N$$

$$\sim e^{-i(\overline{H}_1+\overline{H}_2)t} \quad ?$$

q. chaos/control

Casati-Chirikov et al

+ Berry et al

School of Athens
(Raphael's Room of the Signature, Vatican Museums)