Non Standard Interaction in Neutrino Oscillation

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"Closing in on the Standard Model, Zuoz Summer School"

Outline

- Introduction to neutrino oscillations
  - Brief history on neutrino oscillations.
- Theory of neutrino oscillations
- Introduction to non standard interaction
  - Motivation.
  - Formalism.
- Non standard interaction in neutrino physics
  - Mathematical formulation by considering large θ_{13}.
  - NSI during propagation (ϵ^m) of the neutrino.
  - NSI at the source and detector(ϵ^s and ϵ^d).
- Bounds on the NSI parameters
  - Bounds on the NSI parameters during propagation (ϵ^m).
- Summary and conclusion

Introduction to neutrino oscillation

Brief history on neutrino oscillations

- In 1998, SK collaboration reported strong evidence for neutrino oscillations in their atmospheric neutrino data.
- In 2001, SNO provided compelling evidence for neutrino oscillation from their solar neutrino data.
- Solar neutrino experiments combined with KamLAND have measured the solar neutrino parameters Δm^2_{21} and θ_{12}.
- Atmospheric neutrino experiments such as Super-Kamiokande together with K2K and MINOS have determined the atmospheric neutrino parameters $|\Delta m^2_{31}|$ and θ_{23}.
- Very recently reactor based neutrino experiments such as Daya Bay and Reno have discovered θ_{13}.

Theory of neutrino oscillation

- In the Shrödinger picture,

$$i\frac{d|\nu_\alpha(t)\rangle}{dt} = \mathcal{H}_0|\nu_\alpha(t)\rangle; \qquad |\nu_\alpha(0)\rangle = |\nu_\alpha\rangle,$$

- where,

$$\mathcal{H}_0 = \frac{1}{2E}\left[U\begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31}\end{pmatrix}U^\dagger + A\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0\end{pmatrix}\right].$$

- Standard matter interaction

$$A = 2\sqrt{2}EG_F N_e, \qquad (0)$$

PMNS matrix

- Furthermore, the standard parametrisation of the U (PMNS) matrix is

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- $s \Rightarrow \sin\theta$, $c \Rightarrow \cos\theta$, $\delta \Rightarrow$ CP violating dirac phase,
- $\rho, \sigma \Rightarrow$ CP violating majorana phase.

B. Pontecorvo, Sov. Phys. JETP (1958)
Z. Maki, M.Nakagawa and S. Sakata (1962)

Probability of neutrino oscillation

- The general oscillation probability

$$P_{\nu_\alpha \to \nu_\beta} = |\langle \nu_\beta | e^{-i\mathcal{H}L} | \nu_\alpha \rangle|^2,$$

- In terms of the PMNS matrices

$$\begin{aligned} P(\nu_{e_l} \to \nu_{e_m}) &= \delta_{lm} - 4 \sum_{i>j} \mathrm{Re}\left(U_{li} U_{lj}^* U_{mi}^* U_{mj}\right) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E}\right) \\ &- 2 \sum_{i>j} \mathrm{Im}\left(U_{li} U_{lj}^* U_{mi}^* U_{mj}\right) \sin\left(\frac{\Delta m_{ij}^2 L}{2E}\right) \end{aligned}$$

- 3-flavour oscillation probability in the limit $\Delta m_{21}^2 \to 0$

$$P(\nu_\mu \to \nu_e) = \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

Introduction to non standard interaction

Motivation

- It is convenient to represent the impact of heavy fields at high energies, by adding an infinite tower of non-renormalizable oparators of $d > 4$.

$$\mathcal{L}^{eff} = \mathcal{L}_{SM} + \frac{\delta \mathcal{L}^{d=5}}{\Lambda} + \frac{\delta \mathcal{L}^{d=6}}{\Lambda^2} + \dots$$

- Particularly the angle θ_{13} can be mimicked by the NSI parameters in a neutrino experiment.
- But with the discovery of the angle θ_{13}, we would like to deduce the bounds on these NSI parameters using both the superbeam experiments and the reactor based experiments.

Formalism of NSI

- Energies much below the W boson mass, the charged current interaction can be written with a 6-dimensional operators.

$$\mathcal{L}_{CC}^{eff} = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_\alpha \gamma^\rho (1-\gamma_5) l_\alpha\right] \left[\bar{f} \gamma_\rho (1-\gamma_5) f'\right] + h.c.$$

$$\mathcal{L}_{NC}^{eff} = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_\alpha \gamma^\rho (1-\gamma_5) \nu_\alpha\right] \left[\bar{f} \gamma_\rho (C_V + C_A \gamma_5) f\right] + h.c,$$

- With NSI the Lagrangian gets modified as,

$$\mathcal{L}_{NSI} = \frac{G_F}{\sqrt{2}} \sum_{f,f'} \epsilon_{\alpha\beta}^{CC,f,f'} \left[\bar{\nu}_\alpha \gamma^\rho (1-\gamma_5) l_\beta\right] \left[\bar{f} \gamma_\rho (1-\gamma_5) f'\right]$$

$$+ \frac{G_F}{\sqrt{2}} \sum_{f,f'} \epsilon_{\alpha\beta}^{NC,f} \left[\bar{\nu}_\alpha \gamma^\rho (1-\gamma_5) \nu_\beta\right] \left[\bar{f} \gamma_\rho (1-\gamma_5) f\right] + h.c.$$

Mathematical formulation for large θ_{13} perturbation theory

- Standard matter interaction $\hat{A} = \frac{A}{\Delta m^2_{31}} = 0.06 \sim \epsilon$.
- $\alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sim 0.03 \sim \epsilon$
- For large $\theta_{13} \sim 8.83^\circ$, $\sin\theta_{13} \sim \sqrt{\epsilon}$.
- **Convenient to work in the tilde basis**, $\tilde{\nu}_\alpha = (U^\dagger_{23})_{\alpha\beta}\nu_\beta$.

H. Minakata et al. (2009, 2011)
R. Adhikary, Sabyasachi Chakraborty, A. Dasgupta, S. Roy(2012).

Different orders of the Hamiltonian with respect to perturbation parameter $\sqrt{\epsilon}$

- Zeroth order Hamiltonian in the tilde basis

$$\tilde{H}_0 = \frac{\Delta m^2_{31}}{2E}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

- Perturbative part $(\sqrt{\epsilon})$ of the Hamiltonian in the tilde basis

$$\tilde{H}_1(\sqrt{\epsilon}) = \frac{\Delta m^2_{31}}{2E}\begin{bmatrix} 0 & 0 & s_{13}e^{-i\delta} \\ 0 & 0 & 0 \\ s_{13}e^{i\delta} & 0 & 0 \end{bmatrix} \ldots\ldots$$

Incorporating NSI during propagation

- The Hamiltonian consisting of the NSI parameters during propagation is

$$\mathcal{H}_{NSI} = \frac{\Delta m^2_{31}}{2E}\hat{A}\begin{bmatrix} \epsilon^m_{ee} & \epsilon^m_{e\mu} & \epsilon^m_{e\tau} \\ \epsilon^m_{e\mu} & \epsilon^m_{\mu\mu} & \epsilon^m_{\mu\tau} \\ \epsilon^m_{e\tau} & \epsilon^m_{\mu\tau} & \epsilon^m_{\tau\tau} \end{bmatrix}.$$

- In the tilde basis, this Hamiltonian becomes

$$\tilde{\mathcal{H}}_{NSI} = U^\dagger_{23}\mathcal{H}_{NSI}U_{23}.$$

- Perturbative Hamiltonian is now redefined in the form

$$\tilde{H}_1 \rightarrow \tilde{H}_1 + \tilde{\mathcal{H}}_{NSI}$$

Evaluating the S-matrix

- Definition of the S matrix

$$\tilde{S}(L) = T\exp\left[-i\int_0^L dx\tilde{\mathcal{H}}(x)\right]$$

- S matrix in the flavor basis is

$$S = U_{23}\tilde{S}U_{23}^{\dagger}$$

- Since the S matrix changes the flavor of a neutrino state after traversing a length L,

$$\nu_\alpha = S_{\alpha\beta}\nu_\beta(0)$$

- The oscillation probability of the neutrino, changing the flavor from $\alpha \to \beta$, considering standard matter interaction and NSI during propagation is

$$P(\nu_{\beta\to\alpha}; L) = |S_{\alpha\beta}|^2.$$

Incorporating NSI at the source and detector

- In presence of the NSI at the source and at the detector,

$$|\nu^s_\alpha\rangle = \left(|\nu_\alpha\rangle + \sum_{\beta=e,\mu,\tau} \epsilon^s_{\alpha\beta}|\nu_\beta\rangle\right)$$

$$,\langle\nu^d_\beta| = \left(\langle\nu_\beta| + \sum_{\alpha=e,\mu,\tau} \epsilon^d_{\alpha\beta}\langle\nu_\alpha|\right)$$

- The total expression for the probability becomes,

$$P_{\nu^s_\alpha \to \nu^d_\beta} = |\langle\nu^d_\beta|S(L)|\nu^s_\alpha\rangle|^2$$
$$= |[(1+\epsilon^d)^T \; e^{-i\mathcal{H}L} \; (1+\epsilon^s)^T]_{\beta\alpha}|^2,$$

- At $L = 0$,

$$P^{ND}_{\nu_\mu \to \nu_e} = |\epsilon^d_{\mu e}|^2 + |\epsilon^s_{\mu e}|^2 + 2|\epsilon^d_{\mu e}||\epsilon^s_{\mu e}|\cos(\phi^d_{\mu e} \sim \phi^s_{\mu e}).$$

T. Ohlsson, H. Zhang (2009).

Bounds on the NSI parameters

- NSI parameters at the source and detector are highly constrained $\mathcal{O}(10^{-3})$.
- Bounds on the NSI parameters during propagation are not very stringent.

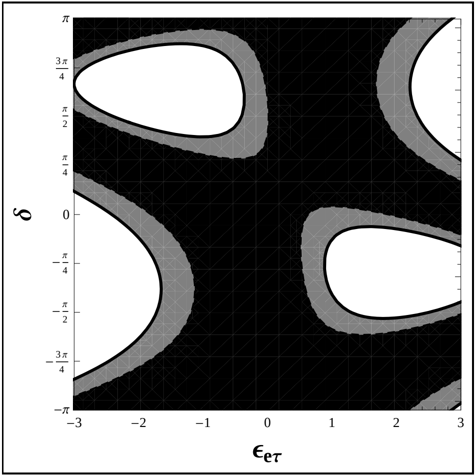
$$|\epsilon^m_{\alpha\beta}| \leq \begin{bmatrix} 4.2 & 0.33 & 3.0 \\ 0.33 & 0.068 & 0.33 \\ 3.0 & 0.33 & 21 \end{bmatrix}$$

- Our aim is to bound these NSI parameters using the results from T2K and Daya Bay,

Carla Biggio, Mattias Blennow, Enrique Fernandez-Martinez (2009)

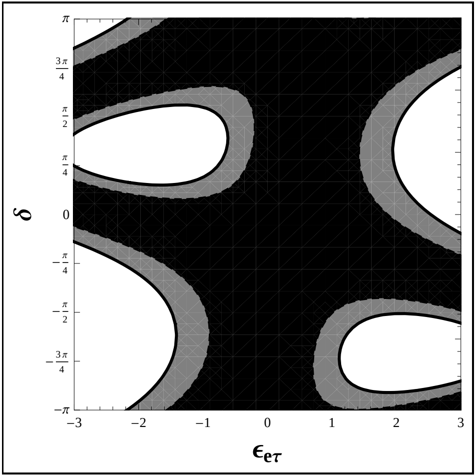
Variations of $\epsilon_{e\tau} - \delta$

- Excluded regions are, white at 90% confidence level, and grey+white at 66% confidence level, for inverted hierarchy.



Variations of $\epsilon_{e\tau} - \delta$

- Excluded regions are, white at 90% confidence level, and grey+white at 66% confidence level, for normal hierarchy.



Summary and Conclusion

- Using large θ_{13} we have presented $P_{\nu_\mu \to \nu_e}$ perturbatively, upto second order in ϵ.
- In the probability expression the NSI parameters $|\epsilon^s_{\mu\mu}|$, $|\epsilon^s_{\mu\tau}|$, and $|\epsilon^d_{ee}|$ are much more suppressed.
- Most prominent terms in the expression of the probability are $|\epsilon^s_{\mu e}|$, $|\epsilon^d_{\mu e}|$ and $|\epsilon^d_{\tau e}|$.
- We were able to constrain the NSI parameters during propagation using the recent results from the T2K and Daya Bay experiment.
- Significant constraint on $\epsilon^m_{e\tau}$ and $\epsilon^m_{\tau\tau}$ were given from our work.
- The knowledge of delta from future baseline experiments could provide better understanding about the possible strength of NSI.

Thank you

Back-up

Solar neutrino oscillation

- In 2001, SNO provided compelling evidence for neutrino oscillation from their solar neutrino data.
- SNO, SAGE, Kamiokande provided answer to the long standing solar neutrino problem.

Introduction to neutrino oscillation

- Solar neutrino experiments combined with KamLAND have measured the solar neutrino parameters Δm^2_{21} and θ_{12}.
- Atmospheric neutrino experiments such as Super-Kamiokande together with K2K and MINOS have determined the atmospheric neutrino parameters $|\Delta m_{31}|^2$ and θ_{23}.
- Very recently reactor based neutrino experiments such as Daya Bay and Reno have discovered θ_{13}.
- The CP violating dirac phase δ_{CP} is still unknown.

Oscillation parameters

- Mass squared differences,

$$\Delta m^2_{21} = (7.6 \pm 0.20) \times 10^{-5} eV^2, \quad |\Delta m_{31}|^2 = (2.4^{+0.12}_{-0.08}) \times 10^{-3} eV^2,$$

- Mixing angles,

$$\sin^2 2\theta_{12} = 0.8704^{+0.022}_{-0.016}, \qquad \sin^2 2\theta_{23} = 1^{+0.07}_{-0.06},$$

- Recent reactor based experiments e.g. Reno and Daya-Bay, provide compelling evidence for a large θ_{13},

$$\theta_{13} = 8.83^\circ$$

Types of neutrino experiments

Reactor neutrino experiments

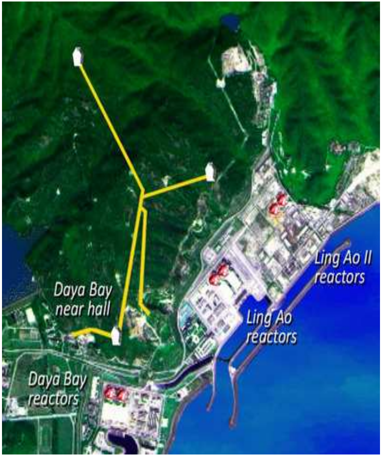
e.g. Daya Bay and Double Chooz.

Superbeam experiments

e.g. T2K and NoνA.

Reactor neutrino experiment

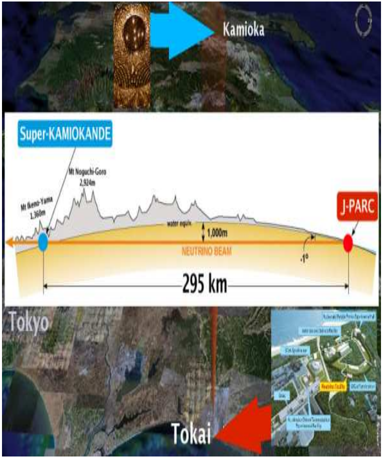
- Nuclear power plant emitting $\bar{\nu}_e$.
- Baseline length $\sim$ 1 Km.
- Energy of the neutrino beam is around a few MeV.
- Searches for the signal for non zero θ_{13}.
- e.g Daya bay, discovered θ_{13} at 5.2σ



F. P. An et al. (2012)

Superbeam experiment

- Muons are produced via pion decay.
- Baseline length is usually several hundred kilometres.
- Energy of the neutrino beam is a few GeV.
- Apart from θ_{13}, additional information is provided for δ_{CP} and the mass hierarchy.
- e.g T2K experiment.



T2K Collaboration: K.Abe et al.(2011)

- In the Shrödinger picture,

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- where,

$$\mathcal{H}_0 = \frac{1}{2E}\left[U\begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix}U^\dagger + A\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right].$$

- Standard matter interaction

$$A = 2\sqrt{2}EG_F N_e, \qquad \text{(-21)}$$

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- Furthermore, the standard parametrisation of the U (PMNS) matrix is

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Probability of neutrino oscillation

- The general oscillation probability

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- 3-flavour oscillation probability in the limit $\Delta m_{21}^2 \to 0$

$$P(\nu_\mu \to \nu_e) = \sin^2\theta_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

Motivation

- It is convenient to represent the impact of heavy fields at high energies, by adding an infinite tower of non-renormalizable oparators of $d > 4$.

$$\mathcal{L}^{eff} = \mathcal{L}_{SM} + \frac{\delta\mathcal{L}^{d=5}}{\Lambda} + \frac{\delta\mathcal{L}^{d=6}}{\Lambda^2} + ...$$

- These non-renormalizable operators must be invariant under the standard model gauge group.
- In our work we are considering such operators of dimension six, known as the non standard interaction operators.

Motivation

- NSI parameters can cloud the sesitivity of the oscillation parameters.
- Particularly the angle θ_{13} can be mimicked by the NSI parameters in a neutrino experiment.
- But with the discovery of the angle θ_{13}, we would like to deduce the bounds on these NSI parameters using both the superbeam experiments and the reactor based experiments.

Formalism of NSI

- Standard model charged current interaction

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \left[\bar{l}_\alpha \gamma^\mu \frac{1-\gamma_5}{2} \nu_\alpha W_\mu^+ + h.c. \right]$$

- Standard model neutral current interaction

$$\mathcal{L}_{NC} = \frac{g}{2\cos\theta_W} \bar{\nu}_\alpha \gamma_\mu \frac{1-\gamma_5}{2} \nu_\alpha Z_\mu^0$$

Formalism of NSI

- Energies much below the W boson mass, the charged current interaction can be written with a 6-dimensional operators.

$$\mathcal{L}_{CC}^{eff} = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_\alpha \gamma^\rho (1-\gamma_5) l_\alpha\right] \left[\bar{f} \gamma_\rho (1-\gamma_5) f'\right] + h.c.$$

$$\mathcal{L}_{NC}^{eff} = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_\alpha \gamma^\rho (1-\gamma_5) \nu_\alpha\right] \left[\bar{f} \gamma_\rho (C_V + C_A \gamma_5) f\right] + h.c,$$

- With NSI the Lagrangian gets modified as,

$$\mathcal{L}_{NSI} = \frac{G_F}{\sqrt{2}} \sum_{f,f'} \epsilon_{\alpha\beta}^{CC,f,f'} \left[\bar{\nu}_\alpha \gamma^\rho (1-\gamma_5) l_\beta\right] \left[\bar{f} \gamma_\rho (1-\gamma_5) f'\right]$$

$$+ \frac{G_F}{\sqrt{2}} \sum_{f,f'} \epsilon_{\alpha\beta}^{NC,f} \left[\bar{\nu}_\alpha \gamma^\rho (1-\gamma_5) \nu_\beta\right] \left[\bar{f} \gamma_\rho (1-\gamma_5) f\right] + h.c.$$

Formalism of NSI

- $\epsilon^{CC,f,f'}$ $\Rightarrow$associated with charged current interaction $\Rightarrow$ NSI at the source and at the detector.
- $\epsilon^{CC,f,f'}$ $\Rightarrow$ can have any complex structure.
- $\epsilon^{NC,f}$ $\Rightarrow$associated with the neutral current interaction $\Rightarrow$ NSI during propagation of the neutrino.
- $\epsilon^{NC,f}$ $\Rightarrow$ is hermitian.
- $|\epsilon| \sim \frac{M_W^2}{M_{NSI}^2} \sim 10^{-2}$

J. Kopp, M. Lindner, T. Ota and J. Sato (2008)

Mathematical formulation for large θ_{13} perturbation theory

- Standard matter interaction $\hat{A} = \frac{A}{\Delta m^2_{31}} = 0.06 \sim \epsilon$.
- $\alpha = \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sim 0.03 \sim \epsilon$
- For large $\theta_{13} \sim 8.83^\circ$, $\sin\theta_{13} \sim \sqrt{\epsilon}$.
- **Convenient to work in the tilde basis**, $\underline{\tilde{\nu}_\alpha = (U^\dagger_{23})_{\alpha\beta}\nu_\beta}$.

H. Minakata et al. (2009, 2011)
R. Adhikary, Sabyasachi Chakraborty, A. Dasgupta, S. Roy(2012).

Different orders of the Hamiltonian with respect to perturbation parameter $\sqrt{\epsilon}$

- Zeroth order Hamiltonian in the tilde basis

$$\tilde{H}_0 = \frac{\Delta m^2_{31}}{2E} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

- Perturbative part ($\sqrt{\epsilon}$) of the Hamiltonian in the tilde basis

$$\tilde{H}_1(\sqrt{\epsilon}) = \frac{\Delta m^2_{31}}{2E} \begin{bmatrix} 0 & 0 & s_{13}e^{-i\delta} \\ 0 & 0 & 0 \\ s_{13}e^{i\delta} & 0 & 0 \end{bmatrix}$$

- Perturbative part (ϵ) of the Hamiltonian in the tilde basis

$$\tilde{H}_1(\epsilon) = \frac{\Delta m^2_{31}}{2E}\begin{bmatrix} \hat{A} + \alpha s^2_{12} + s^2_{13} & \alpha c_{12}s_{12} & 0 \\ \alpha c_{12}s_{12} & \alpha c^2_{12} & 0 \\ 0 & 0 & -s^2_{13} \end{bmatrix}$$

- Perturbative part ($\epsilon^{\frac{3}{2}}$) of the Hamiltonian in the tilde basis,

$$\tilde{H}_1(\epsilon^{\frac{3}{2}}) =$$

$$-\frac{\Delta m^2_{31}}{2E}\begin{bmatrix} 0 & 0 & \left(\alpha s^2_{12} + \frac{1}{2}s^2_{13}\right) s_{13}e^{-i\delta} \\ 0 & 0 & \alpha c_{12}s_{12}s_{13}e^{-i\delta} \\ \left(\alpha s^2_{12} + \frac{1}{2}s^2_{13}\right) s_{13}e^{i\delta} & \alpha c_{12}s_{12}s_{13}e^{i\delta} & 0 \end{bmatrix}$$

- Perturbative part (ϵ^2) of the Hamiltonian in the tilde basis,

$$\tilde{H}_1(\epsilon^2) = \frac{\Delta m^2_{31}}{2E}\alpha \begin{bmatrix} s^2_{12}s^2_{13} & \frac{1}{2}c_{12}s_{12}s^2_{13} & 0 \\ \frac{1}{2}c_{12}s_{12}s^2_{13} & 0 & 0 \\ 0 & 0 & -s^2_{12}s^2_{13} \end{bmatrix}.$$

Incorporating NSI during propagation

- The Hamiltonian consisting of the NSI parameters during propagation is

$$\mathcal{H}_{NSI} = \frac{\Delta m^2_{31}}{2E}\hat{A}\begin{bmatrix} \epsilon^m_{ee} & \epsilon^m_{e\mu} & \epsilon^m_{e\tau} \\ \epsilon^m_{e\mu} & \epsilon^m_{\mu\mu} & \epsilon^m_{\mu\tau} \\ \epsilon^m_{e\tau} & \epsilon^m_{\mu\tau} & \epsilon^m_{\tau\tau} \end{bmatrix}.$$

- In the tilde basis, this Hamiltonian becomes

$$\tilde{\mathcal{H}}_{NSI} = U^{\dagger}_{23}\mathcal{H}_{NSI}U_{23}.$$

- Perturbative Hamiltonian is now redefined in the form

$$\tilde{H}_1 \rightarrow \tilde{H}_1 + \tilde{\mathcal{H}}_{NSI}$$

Evaluating the S-matrix

- S matrix in the tilde basis is related to the S matrix in the flavor basis

$$S(L) = U_{23}\tilde{S}(L)U_{23}^{\dagger}$$

- Definition of the S matrix

$$\tilde{S}(L) = T\exp\left[-i\int_0^L dx\tilde{\mathcal{H}}(x)\right]$$

- To evaluate $\tilde{S}(L)$ perturbatively, we choose

$$\Omega(x) = e^{i\tilde{H}_0 x}\tilde{S}(x)$$

- Where $\Omega(x)$ follows the evolution equation

$$i\frac{d}{dx}\Omega(x) = H_1\Omega(x).$$

- And H_1 is written in the form

$$H_1 \equiv e^{i\tilde{H}_0 x}\tilde{H}_1 e^{-i\tilde{H}_0 x}.$$

- Perturbative solution of $\Omega(x)$ can be written as

$$\begin{aligned}\Omega(x) &= 1+(-i)\int_0^x dx' H_1(x') + (-i)^2\int_0^x dx' H_1(x')\int_0^{x'} dx'' H_1(x'') \\ &+ (-i)^3\int_0^x dx' H_1(x')\int_0^{x'} dx'' H_1(x'')\int_0^{x''} dx''' H_1(x''') + \mathcal{O}(\epsilon^4).\end{aligned}$$

- After the deduction of $\Omega(x)$ we write the S matrix as

$$\tilde{S}(x) = e^{-i\tilde{H}_0 x}\Omega(x).$$

- S matrix in the flavor basis is

$$S = U_{23}\tilde{S}U_{23}^{\dagger}$$

- Since the S matrix changes the flavor of a neutrino state after traversing a length L,

$$\nu_{\alpha} = S_{\alpha\beta}\nu_{\beta}(0)$$

- The oscillation probability of the neutrino, changing the flavor from $\alpha \rightarrow \beta$, considering standard matter interaction and NSI during propagation is

$$P(\nu_{\beta\rightarrow\alpha}; L) = |S_{\alpha\beta}|^2.$$

Incorporating NSI at the source and detector

- In presence of the NSI at the source and at the detector, the neutrino states become a superposition of pure orthonormal flavor states,

$$|\nu_\alpha^s\rangle = \left(|\nu_\alpha\rangle + \sum_{\beta=e,\mu,\tau} \epsilon^s_{\alpha\beta}|\nu_\beta\rangle\right),$$

$$\langle\nu_\beta^d| = \left(\langle\nu_\beta| + \sum_{\alpha=e,\mu,\tau} \epsilon^d_{\alpha\beta}\langle\nu_\alpha|\right)$$

- Due to the non unitary of the neutrino states, they follow the following properties,

$$\sum_{\alpha=e,\mu,\tau} |\nu_\alpha^s\rangle\langle\nu_\alpha^s| \neq \mathbf{1}, \sum_{\beta=e,\mu,\tau} |\nu_\beta^d\rangle\langle\nu_\beta^d| \neq \mathbf{1},$$

$$\langle\nu_\alpha^s|\nu_\beta^s\rangle \neq \delta_{\alpha\beta}, \langle\nu_\alpha^d|\nu_\beta^d\rangle \neq \delta_{\alpha\beta}.$$

T. Ohlsson, H. Zhang (2009).

Expression for the probability of oscillation

- By taking into account all these effects, the expression for the probability becomes,

$$\begin{aligned} P_{\nu_\alpha^s \to \nu_\beta^d} &= |\langle \nu_\beta^d | S(L) | \nu_\alpha^s \rangle|^2 \\ &= |[(1+\epsilon^d)^T \; e^{-i\mathcal{H}L} \; (1+\epsilon^s)^T]_{\beta\alpha}|^2, \end{aligned}$$

- The total oscillation probability would have the form

$$P(\nu_\alpha \to \nu_\beta) = P^{(0)}_{\alpha\beta} + P^{(1)}_{\alpha\beta} + P^{(3/2)}_{\alpha\beta} + P^{(2)}_{\alpha\beta}$$

Specific choices of the NSI during propagation

- Upto this point we have not mentioned about the order of magnitude of the NSI parameters during propagation.
- We will consider two different regimes of the NSI parameters during propagation. e.g. $\epsilon^m \sim \sqrt{\epsilon}$ and $\epsilon^m \sim \epsilon$.

Large NSI parameters during propagation, $\epsilon^m \sim \sqrt{\epsilon}$

$$\tilde{H}_1(\epsilon^{3/2}) = -\frac{\Delta m^2_{31}}{2E} s_{13}$$

$$\begin{bmatrix} 0 & 0 & (\alpha s^2_{12} + \frac{1}{2} s^2_{13}) e^{-i\delta} \\ 0 & 0 & \alpha c_{12} s_{12} e^{-i\delta} \\ (\alpha s^2_{12} + \frac{1}{2} s^2_{13}) e^{i\delta} & \alpha c_{12} s_{12} e^{i\delta} & 0 \end{bmatrix}$$

$$+ \frac{\Delta m^2_{31}}{2E} \hat{A} U^\dagger_{23} \begin{bmatrix} \epsilon^m_{ee} & \epsilon^m_{e\mu} & \epsilon^m_{e\tau} \\ \epsilon^m_{e\mu} & \epsilon^m_{\mu\mu} & \epsilon^m_{\mu\tau} \\ \epsilon^m_{e\tau} & \epsilon^m_{\mu\tau} & \epsilon^m_{\tau\tau} \end{bmatrix} U_{23}.$$

- We obtain the oscillation probability for the $\nu_\mu \to \nu_e$ channel.

Non standard interaction in neutrino oscillation

Salient features about the probability expression

- Assuming $\epsilon^s, \epsilon^d, \epsilon^m = 0$

$$P^{Vacuum}_{\nu_\mu \to \nu_e} = s^2_{2\times 13} s^2_{23} \sin^2 \left[\frac{\Delta m^2_{31} L}{4E} \right].$$

- At $L = 0$,

$$P^{ND}_{\nu_\mu \to \nu_e} = |\epsilon^d_{\mu e}|^2 + |\epsilon^s_{\mu e}|^2 + 2|\epsilon^d_{\mu e}||\epsilon^s_{\mu e}| \cos(\phi^d_{\mu e} \sim \phi^s_{\mu e}).$$

  This term is coined as the near detector effect.

- The probability expression contains terms proportional to $\sin\delta$ combined with $\sin\left(\frac{\Delta m^2_{21} L}{2E}\right)$.
- Similarly the expression contains terms proportional to $\cos\delta$ combined with $\sin\left(\frac{\Delta m^2_{31} L}{4E}\right)^2$.

Salient features about the probability expression

- The NSI terms which play crucial role in $\nu_\mu \to \nu_e$ oscillations are $\epsilon^s_{\mu e}, \epsilon^s_{\mu\mu}, \epsilon^s_{\mu\tau}$, and $\epsilon^d_{ee}, \epsilon^d_{\mu e}, \epsilon^d_{\tau e}$.
- In the expression for the probability $|\epsilon^s_{\mu\mu}|$, $|\epsilon^s_{\mu\tau}|$ and $|\epsilon^d_{ee}|$ are coupled with a factor of $\sin^2\theta_{13}$.
- $|\epsilon^s_{\mu e}|$, $|\epsilon^d_{\mu e}|$, and $|\epsilon^d_{\tau e}|$ are coupled with a term $\sin\theta_{13}$.

Non standard interaction in neutrino oscillation

Small NSI parameters during propagation, $\epsilon^m \sim \epsilon$

$$\tilde{H}_1(\epsilon^2) = -\frac{\Delta m^2_{31}}{2E}\alpha \begin{bmatrix} s^2_{12}s^2_{13} & \frac{1}{2}c_{12}s_{12}s^2_{13} & 0 \\ \frac{1}{2}c_{12}s_{12}s^2_{13} & 0 & 0 \\ 0 & 0 & -s^2_{12}s^2_{13} \end{bmatrix} + \frac{\Delta m^2_{31}}{2E}\hat{A}U^\dagger_{23} \begin{bmatrix} \epsilon^m_{ee} & \epsilon^m_{e\mu} & \epsilon^m_{e\tau} \\ \epsilon^m_{e\mu} & \epsilon^m_{\mu\mu} & \epsilon^m_{\mu\tau} \\ \epsilon^m_{e\tau} & \epsilon^m_{\mu\tau} & \epsilon^m_{\tau\tau} \end{bmatrix} U_{23}$$

- Again by following the same procedure, we obtain the $\nu_\mu \to \nu_e$ oscillation probability for $\epsilon^m \sim \epsilon$.
- As $\epsilon^m \sim \epsilon$, therefore in the $\nu_\mu \to \nu_e$ oscillation channel NSI during propagation terms are very much suppressed.

Bounds on the NSI parameters

- NSI parameters at the source and detector are highly constrained $\mathcal{O}(10^{-3})$.
- Bounds on the NSI parameters during propagation are not very stringent.

$$|\epsilon^{m}_{\alpha\beta}| \leq \begin{bmatrix} 4.2 & 0.33 & 3.0 \\ 0.33 & 0.068 & 0.33 \\ 3.0 & 0.33 & 21 \end{bmatrix}$$

- Our aim is to bound these NSI parameters using the results from T2K and Daya Bay,

Carla Biggio, Mattias Blennow, Enrique Fernandez-Martinez (2009)

Numerical analysis

- We have solved the evolution equation

$$i\hbar c\frac{d}{dx}S_{\beta\alpha}(x)=\sum_{\eta}\mathcal{H}_{\beta\eta}S_{\eta\alpha},$$

- After including the source and detector effects of NSI

$$\mathcal{A}_{\beta\alpha}=\frac{1}{N^{s}_{\alpha}N^{d}_{\beta}}[(1+\epsilon^{d})^{T}\ S\ (1+\epsilon^{s})^{T}]_{\beta\alpha}.$$

- The oscillation probability

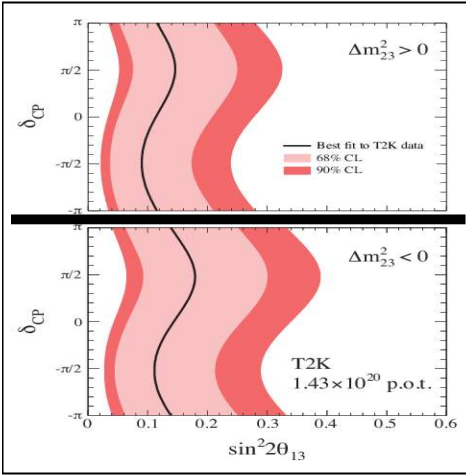
$$P_{\nu^{s}_{\alpha}\rightarrow\nu^{d}_{\beta}}=|\mathcal{A}_{\beta\alpha}|^{2}.$$

- To reduce the number of NSI parameters,

$$\epsilon^{s}_{\alpha\beta}=\epsilon^{d*}_{\beta\alpha}.$$

Algorithm of the numerical analysis

- Our goal is to tune with the experimental results of T2K and Daya Bay.
- T2K ν_e appearance plot

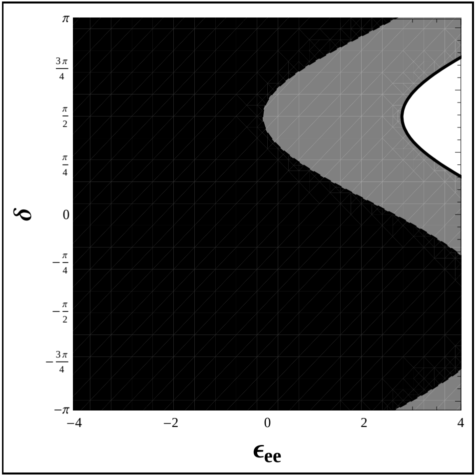


Numerical analysis

- We have used the T2K obtained range of probability of oscillation using the constraints on $\delta - \sin^2 2\theta_{13}$.
- We have fixed the value of $\theta_{13} = 8.8^\circ$ discovered by the Daya Bay experiment.
- After that we have found out the variations of $\theta_{13} - \delta$.
- We have considered one NSI parameter at a time to deduce these variations.

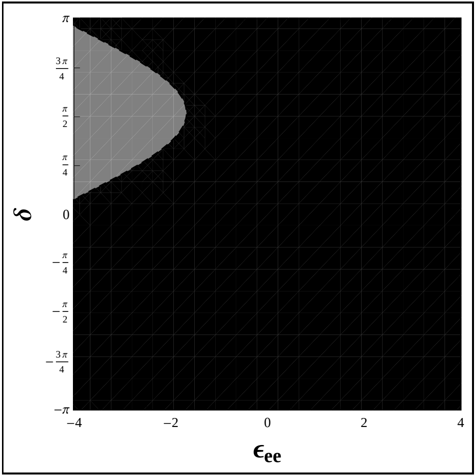
Variations of $\epsilon_{ee} - \delta$

- Excluded regions are, white at 90% confidence level, and grey+white at 66% confidence level, for inverted hierarchy.



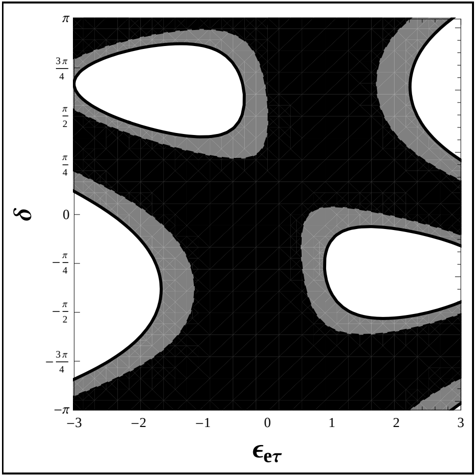
Variations of $\epsilon_{ee} - \delta$

- Excluded regions are, white at 90% confidence level, and grey+white at 66% confidence level, for normal hierarchy.



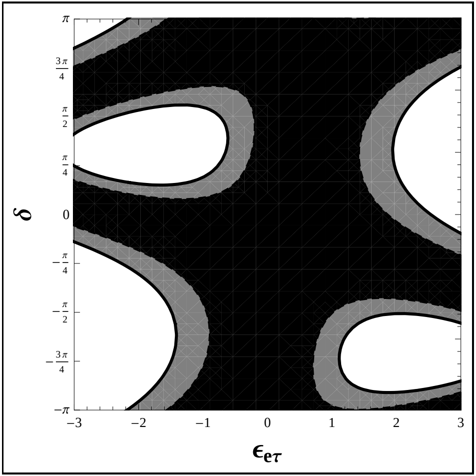
Variations of $\epsilon_{e\tau} - \delta$

- Excluded regions are, white at 90% confidence level, and grey+white at 66% confidence level, for inverted hierarchy.



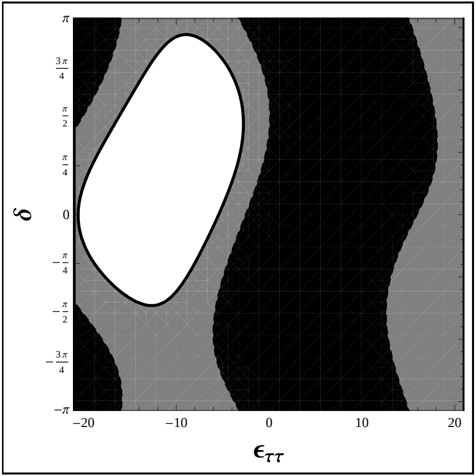
Variations of $\epsilon_{e\tau} - \delta$

- Excluded regions are, white at 90% confidence level, and grey+white at 66% confidence level, for normal hierarchy.



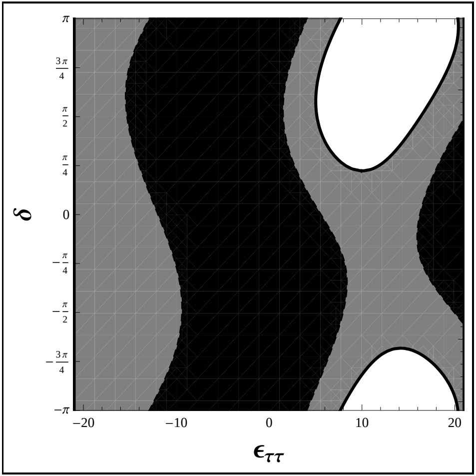
Variations of $\epsilon_{\tau\tau} - \delta$

- Excluded regions are, white at 90% confidence level, and grey+white at 66% confidence level, for inverted hierarchy.



Variations of $\epsilon_{\tau\tau} - \delta$

- Excluded regions are, white at 90% confidence level, and grey+white at 66% confidence level, for normal hierarchy.



Summary and Conclusion

- Using large θ_{13} we have presented $P_{\nu_\mu \to \nu_e}$ perturbatively, upto second order in ϵ.
- In the probability expression the NSI parameters $|\epsilon^s_{\mu\mu}|$, $|\epsilon^s_{\mu\tau}|$, and $|\epsilon^d_{ee}|$ are much more suppressed.
- Most prominent terms in the expression of the probability are $|\epsilon^s_{\mu e}|$, $|\epsilon^d_{\mu e}|$ and $|\epsilon^d_{\tau e}|$.
- We were able to constrain the NSI parameters during propagation using the recent results from the T2K and Daya Bay experiment.
- Significant constraint on $\epsilon^m_{e\tau}$ and $\epsilon^m_{\tau\tau}$ were given from our work.
- The knowledge of delta from future baseline experiments could provide better understanding about the possible strength of NSI.

Thank You !

Beta beams

- Accelerate unstable ions and circulate them in a storage ring.
- Unstable ions would decay and produce highly boosted ν_e beam.
- Baseline is several hundred kilometres.
- Apart from θ_{13}, additional information is provided for δ_{CP} and the mass hierarchy.
- e.g Memphys experiment.