Theoretical Introduction to LHC Physics

3. Supersymmetry

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In the previous lecture, I discussed the minimal Higgs scalar as an explicit model of SU(2)XU(1) symmetry breaking. Now you might ask:

Do I believe that this model is correct ?

No.

If not, what should replace it ?

That is the subject of this lecture.

There is nothing observationally wrong with the minimal Higgs model.

The problem with it is that it does not explain why electroweak symmetry is broken.

Normally, in physics, we have an explanation for major phenomena. For example, we have superconductivity because of Cooper pairing, magnetism because of Hund's rule.

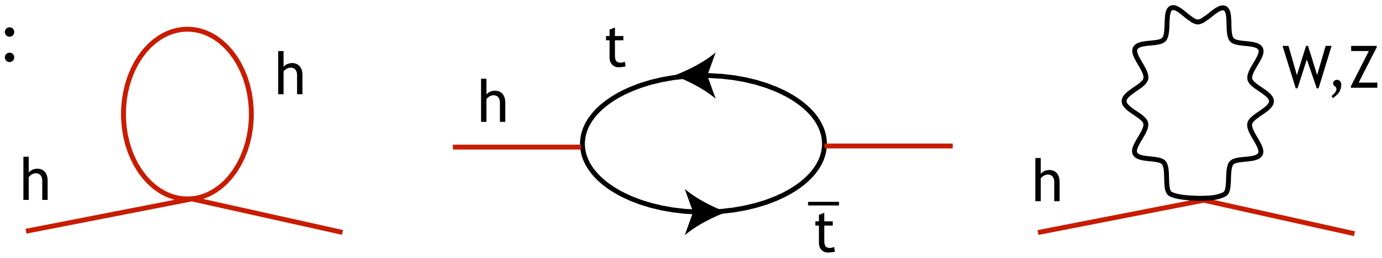
In the minimal Higgs model we postulate the potential:

$$V = \mu^2 |\varphi|^2 + \lambda(|\varphi|^2)^2$$

with $\mu^2 < 0$. Why the negative sign ? No response.

More sophisticated theorists might give more sophisticated explanations. One example is the gauge hierarchy problem.

Assume that the potential above is correct in the fundamental Lagrangian of Nature. Consider the 1-loop corrections to the parameter μ^2:



$$\mu^2 - \mu^2_{\text{bare}} = +\frac{\lambda}{8\pi^2}\Lambda^2 - \frac{3y_t^2}{8\pi^2}\Lambda^2 + \frac{3(3g^2+g'^2)}{16\pi^2}\Lambda^2$$

These diagrams are ultraviolet-divergent. We need to take this seriously if we believe that there is no new physics up to a mass scale $\Lambda \gg 1\ \text{TeV}$. These corrections correct μ^2_{bare} additively. If the corrections are much greater than $(100\ \text{GeV})^2$, a strange cancellation is required so that μ^2 will have the value needed to explain the observations. For $\Lambda \sim M_{\text{Planck}}$, the first 33 decimal places must cancel.

Such arguments are just a symptom of the fact that we postulate the specific form of the Higgs field potential without providing any explanation of where it comes from.

Can we, then, calculate the potential responsible for spontaneous SU(2)XU(1) symmetry breaking ?   There are two strategies:

Strong-coupling electroweak symmetry breaking:

Introduce new strong interactions. The Higgs will be composite, or perhaps there will not be a Higgs particle at all. (This follows the analogy between electroweak symmetry breaking and superconductivity.)

Weak-coupling electroweak symmetry breaking:

There is a Higgs field with a potential with the qualitative form of that in the minimal theory. We introduce new particles and forces so that this potential is calculable.

Christophe Grojean will discuss the strong-coupling route in his lecture tomorrow.

The simplest strong-coupling theory is technicolor:

Introduce a new copy of QCD with (U,D) quarks at a mass scale near 1 TeV.   This naturally give the correct pattern of W,Z masses.

Technicolor has very serious problems:

It is very difficult to give masses to quarks and leptons without drastically complicating the theory. The changes produce new flavor-changing processes and give the top quark new strong interactions.

The mixing of technicolor resonances with W,Z gives large corrections to S and T that are excluded by the electroweak data.

As Christophe will explain, modern approaches may (or may not) ameliorate these problems.

The weak-coupling route is not so easy either.

We must cancel the quadratic divergence of the Higgs scalar field mass term. The quadratic divergence of a scalar field mass is generic in quantum field theory. Very special structures are needed to remove this divergence.

After this cancellation occurs, the computation of the Higgs potential must give a negative mass term. Hopefully, this should be a prediction of the theory, not a result of a parameter choice.

Very often, it turns out, loop corrections that involve the top quark Yukawa coupling give contributions to the Higgs potential that have the desired sign.

Three different symmetries have been used in the literature to cancel the divergence in the Higgs mass term:

Shift symmetry: $\delta\varphi = \epsilon$

This is the symmetry that keeps a Goldstone boson massless. In Little Higgs models, the four Higgs fields in φ are Goldstone bosons resulting from a symmetry breaking at 10 TeV.

Mixing with a Gauge Boson: $\delta\varphi = \epsilon^\mu A_\mu$

In an extra-dimensional theory, φ may be the 5th component of a gauge field. Then gauge symmetry forbids the mass term.

Mixing with a Fermion: $\delta\varphi = \epsilon \cdot \psi$

If the Higgs boson mass in related to a fermion mass, and that mass is forbidden by a flavor symmetry, we forbid the Higgs mass also.

All three of these mechanisms require a large superstructure.

For Goldstone bosons and for extra dimensions, this is clear.

For fermions, it is not so clear, until we do some analysis.

So, consider the problem of making boson-fermion mixing an exact symmetry of the Hamiltonian. Let Q_α be the charge that generates this symmetry. Q_α is a 2-component fermion and commutes with the Hamiltonian.

From Q_α, we can construct the operator $\{Q_\alpha, Q^\dagger_\beta\}$. Here are the formal properties of this operator:

1. It is nonzero.
2. It is a Lorentz vector: $\{Q_\alpha, Q^\dagger_\beta\} = 2\sigma^\mu_{\alpha\beta}\ R_\mu$
3. It commutes with H.

Here is a proof of item 1:

$$\begin{aligned} \langle A| \{Q_\alpha, Q_\alpha^\dagger\} |A\rangle &= \langle A| Q_\alpha Q_\alpha^\dagger |A\rangle + \langle A| Q_\alpha^\dagger Q_\alpha |A\rangle \\ &= |Q_\alpha |A\rangle |^2 + |Q_\alpha^\dagger |A\rangle |^2 \end{aligned}$$

There is a problem with this structure, pointed out by Coleman and Mandula. It is very difficulty to have a vector charge that commutes with the Hamiltonian. Energy-momentum and Lorentz invariance already restrict 2-particle scattering amplitudes to a function of one variable, the CM scatting angle. Add another conserved vector charge, and the restrictions become inconsistent: scattering is forbidden.

The only way out is to insist that $R_\mu = P_\mu$. This leads to a very restrictive structure:

$$\{Q_\alpha, Q_\beta^\dagger\} = 2\sigma^\mu_{\alpha\beta}\ P_\mu$$

Q_α is the square root of energy-momentum.

A fermionic symmetry with this property is a supersymmetry.

The fact that the square of a supersymmetry charge is energy-momentum has a striking implication:

Every particle in the theory must be acted on by the supersymmetry charge. This action creates a new particle with the same SM quantum numbers and spin different by 1/2.

We started with a small fix for the Higgs boson, but the result is a complete doubling of the spectrum of particles.

Supersymmetry also places strong restrictions on the particle Lagrangians. Let me present some examples.

Supersymmetry relates a scalar to a Weyl fermion. The basic symmetry multiplet is called a chiral multiplet. It contains one complex-valued scalar field, one Weyl fermion, and one 'auxiliary' field with no associated particles:

$$(\phi, \psi, F)$$

The particle content is (2 spin 0 and 2 spin 1/2 states):

$$\phi, \phi^*, \psi_L, \psi_R^\dagger$$

The most general supersymmetric Lagrangian is

$$\mathcal{L} = (\partial^\mu \phi)(\partial_\mu \phi) + \psi^\dagger i\sigma \cdot \partial \psi + F^\dagger F - V(\phi, \psi, F)$$

where the nonlinear interactions of the fields are built from a function called the superpotential W, which is an analytic function of the complex variable ϕ .

$$V = -F \frac{\partial W}{\partial \phi} + \frac{1}{2} \psi \cdot \psi \frac{\partial^2 W}{\partial \phi^2} + h.c.$$

For a theory with several chiral multiplets, make the obvious generalizatons.

Here are two simple examples of this formalism:

$$W = \frac{1}{2} m \phi^2 \; :$$

$$\begin{aligned} F^\dagger F - V &= F^\dagger F + (mF\phi - \frac{1}{2} m \psi \cdot \psi) + h.c. \\ &= (F^\dagger + m\phi)(F + m^* \phi^\dagger) \quad - |m|^2 \phi^\dagger \phi - (\frac{1}{2} m \psi \cdot \psi + h.c.) \end{aligned}$$

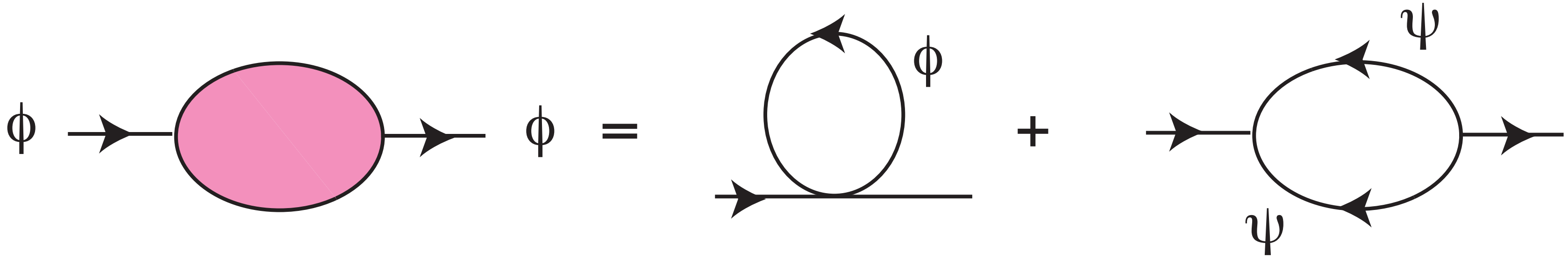
F is a Lagrange multiplier. Solving for F and eliminating it removes the first term. The result is a model with a mass m for the scalar field and an equal Majorana mass for the fermion.

$$W = \frac{1}{3} \lambda \phi^3 \; :$$

$$\begin{aligned} F^\dagger F - V &= F^\dagger F + (F\lambda\phi^2 - \lambda\psi \cdot \psi \, \phi) + h.c. \\ &= -|\lambda|^2 |\phi|^4 - \lambda\psi \cdot \psi \, \phi - h.c. - \text{ eliminated} \end{aligned}$$

This is a merger of two scale-invariant field theories: ϕ^4 theory and Yukawa theory, with massless bosons and fermions.

In the second of these theories, radiative corrections could potentially generate a mass for the ϕ. However, when we compute this explicitly, there is a delicious cancellation:



$$
\begin{aligned}
&= (-4i\lambda^2)\int \frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2} \\
&\qquad -\frac{1}{2}(-2i\lambda)(2i\lambda)\int \frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\Big[\Big(\frac{i\sigma\cdot p}{p^2}\Big)_{\alpha\beta}\epsilon_{\alpha\gamma}\epsilon_{\beta\delta}\Big(\frac{i\sigma\cdot(-p)}{p^2}\Big)_{\gamma\delta}\Big] \\
&= 0
\end{aligned}
$$

There is a general theorem that the superpotential is not renormalized to any order in perturbation theory. This prevents the appearance of quadratic divergences and, ultimately, protects the Higgs mass term in supersymmetric versions of the SM.

Yang-Mills gauge bosons are connected by supersymmery to Weyl fermions, called gauginos. The basic structure is a vector supermultiplet, which contains the fields

$$(A_\mu^a, \lambda^a, D^a)$$

including a real-valued Lagrange multiplier. The particle content is (2 spin 1 and 2 spin 1/2 states for each gauge generator):

$$(A_+^a, A_-^a, \lambda_L^a, \lambda_R^{a\dagger})$$

The Lagrangian is

$$\mathcal{L} = \frac{1}{4}(F_{\mu\nu}^a)^2 + \lambda^{a\dagger} i \not{D} \lambda^a + \frac{1}{2}(D^a)^2$$

The coupling to a chiral multiplet generalizes the SM gauge interaction with couplings to gauginos and D

$$\begin{aligned} \mathcal{L} &= (D^\mu \phi)^* D_\mu \phi + \psi^\dagger i \not{D} \psi + F^\dagger F \\ &\quad - \sqrt{2} g (\phi^\dagger \lambda^a t^a \cdot \psi + h.c.) + g \phi^\dagger D^a t^a \phi \end{aligned}$$

(t^a is the gauge generator, g is the Yang-Mills coupling)

Eliminating D gives a new ϕ^4 term: $-\frac{g^2}{2}|\phi^\dagger t^a \phi|^2$

The gauge couplings respect the nonrenormalization of the superpotential.

If ϕ obtains a vacuum expectation value, the $g\phi^\dagger\lambda\psi$ gaugino interaction becomes a Dirac mass term combining the gaugino and and fermion. In the supersymmetric version of the Higgs mechanism

the vector boson eats the Goldstone boson
the real part of the scalar field gets mass from the $g^2\phi^4$ term
the gaugino and fermion combine to a Dirac fermion

In all, there are 4 boson and 4 fermion states, all with the same mass

$$m^2 = g^2 \langle\phi\rangle^2$$

Now we can apply this formalism to create a supersymmetric (SUSY) generalization of the SM.

Each gauge field of the SM should be assigned to a vector supermultiplet. Each matter field should be assigned to a chiral supermultiplet. In each case, the whole supermultiplet is assigned the U(1)XSU(2)XSU(3) quantum numbers of the SM particle it contains.

The vector multiplets are

$$\begin{aligned} U(1) &: \quad B_m \to B_m, \widetilde{b} \\ SU(2) &: \quad W_m^a \to W_m^a, \widetilde{w}^a \\ SU(3) &: \quad A_m^a \to A_m^a, \widetilde{g} \end{aligned}$$

vector bosons
\+ gauginos

(In the following, I will drop ~'s when they are not needed for clarity.)

Each left-handed fermion obtains a partner that is a complex spin 0 field:

SU(3) x SU(2) x U(1) quantum nos.

$(1, 2, -\frac{1}{2})$	$\begin{pmatrix} \widetilde{\nu} \\ \widetilde{e} \end{pmatrix} , \begin{pmatrix} \nu \\ e \end{pmatrix}$	leptons + sleptons
$(1, 1, +1)$	$\widetilde{\overline{e}} , \overline{e}$	
$(3, 2, +\frac{1}{6})$	$\begin{pmatrix} \widetilde{u} \\ \widetilde{d} \end{pmatrix} , \begin{pmatrix} u \\ d \end{pmatrix}$	quarks + squarks
$(\overline{3}, 1, -\frac{2}{3})$	$\widetilde{\overline{u}} , \overline{u}$	
$(\overline{3}, 1, +\frac{1}{3})$	$\widetilde{\overline{d}} , \overline{d}$	

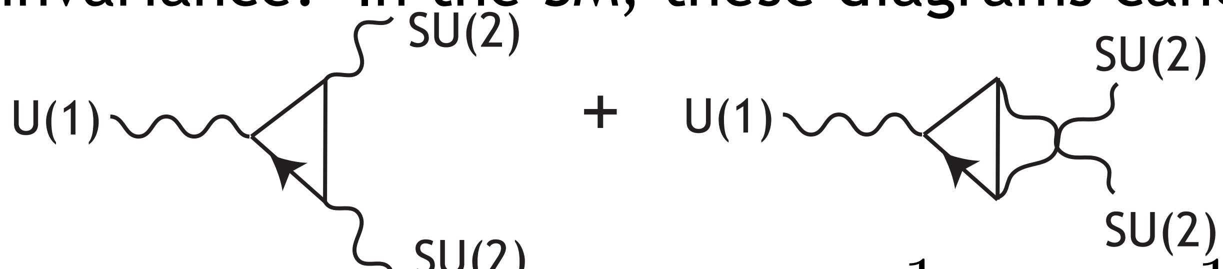
Notice that each Dirac fermion in the SM has two partner fields, one with the left-handed SM couplings, one with the right-handed SM couplings.

What about the Higgs field ? In the MSM, φ has the quantum numbers (1,2, 1/2) or (1,2, -1/2). I claim that, in SUSY, we need both:

$$(1,2,+\frac{1}{2}) \qquad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \; , \; \widetilde{h}_u = \begin{pmatrix} \widetilde{h}_u^+ \\ \widetilde{h}_u^0 \end{pmatrix}$$

$$(1,2,-\frac{1}{2}) \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \; , \; \widetilde{h}_d = \begin{pmatrix} \widetilde{h}_d^+ \\ \widetilde{h}_d^0 \end{pmatrix}$$

There are 2 reasons. The first comes from the cancellation of gauge anomalies. In gauge theories without P and C, the triangle diagram of fermions can give a radiative correction that violates gauge invariance. In the SM, these diagrams cancel, e.g.



$$= \mathrm{tr}[Y\{T^a, T^b\}] = \frac{1}{2}\delta^{ab} \cdot (-\frac{1}{2} + 3 \cdot \frac{1}{6}) = 0$$

To keep this cancellation in SUSY, we need Higgsinos with both Y = +1/2 and Y = -1/2

We also need a superpotential to generate the Yukawa interactions that give mass to the quarks and leptons. This is:

$$W = y_e \epsilon_{ab} H_{da} L_b \overline{e} + y_d \epsilon_{ab} H_{da} Q_b \overline{d} + y_u \epsilon_{ab} H_{ua} Q_b \overline{u}$$

Since W must be an analytic function of fields, $H_u^\dagger$ cannot appear. This is the second reason: We need two Higgs multiplets to give masses to both u and d quarks.

An argument similar to the one I gave for the SM proves that this set of couplings naturally preserves flavor, up to the appearance of the CKM matrix. It is necessary, in this argument, that only one Higgs give mass to each type of quark or lepton.

For later purposes, we will need one more superpotential term:

$$+\mu H_u \cdot H_d$$

which gives mass to Higgses and Higgsinos in a supersymmetric way.

The model we have built is called the

Minimal Supersymmetric Standard Model (MSSM).

At this point, the MSSM has very few parameters. In addition to the gauge and Yukawa couplings that we had already in the SM, there are only 2 parameters:

$$\mu \qquad \tan\beta = \langle H_u \rangle / \langle H_d \rangle$$

the second coming from the fact that we have a 2-Higgs doublet model.

However, in a few slides, I will add many more parameters.

I note in passing that the particle content of the MSSM has a very interesting relation to the idea of Grand Unification of the fundamental interactions. This is the idea that the SM gauge group U(1)XSU(2)XSU(3) results from spontaneous breaking of a very large gauge group (SU(5) or SO(10)) at a very high mass scale.

In Grand Unified Theories, the coupling constants

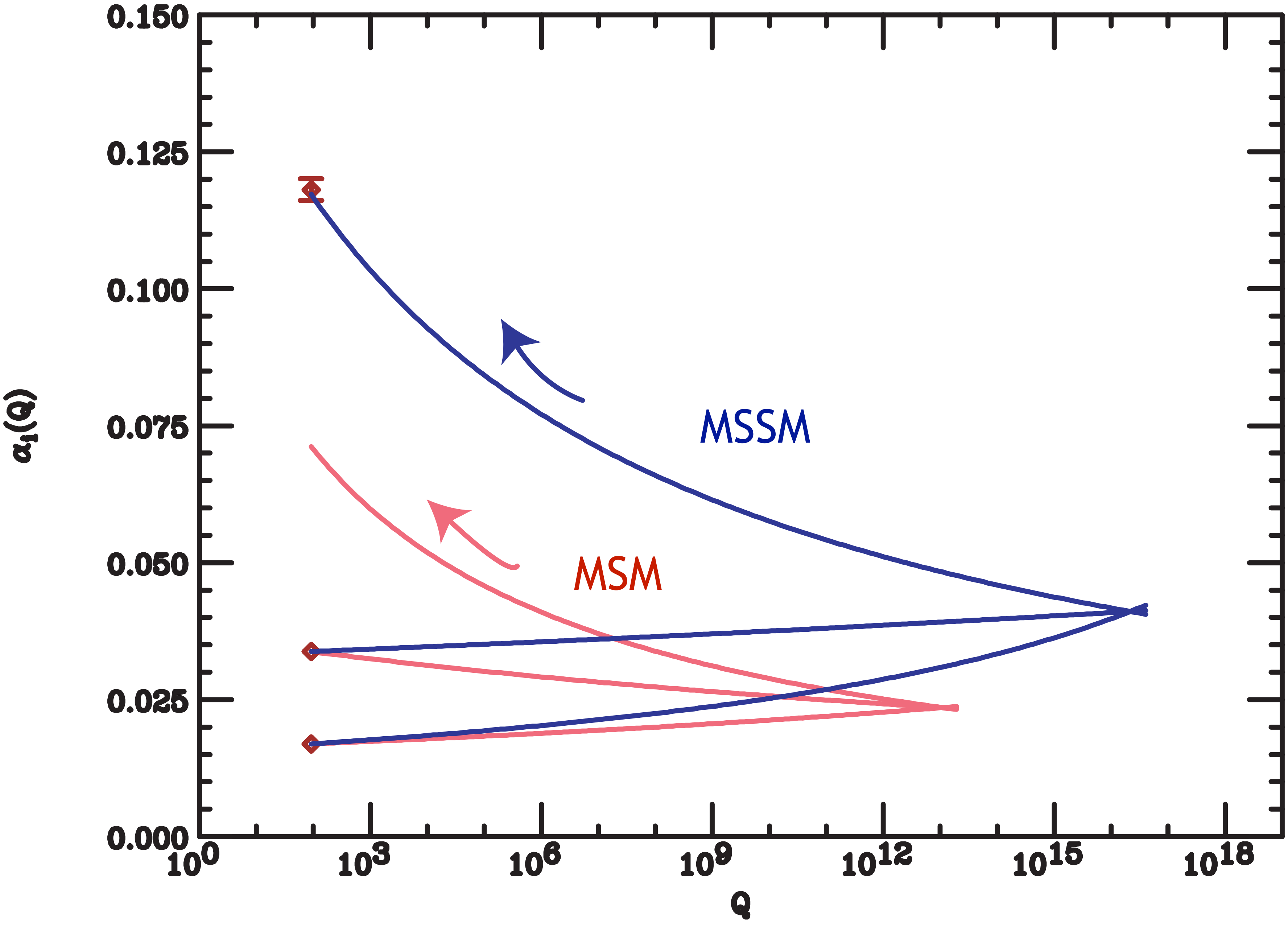
$$\sqrt{\frac{5}{3}}g_1,\ g_2,\ g_3$$

become equal at the unification scale. By evolving these couplings to $Q = m_Z$, we find a relation among the three SM couplings.

In the SM, this calculation does not quite work. From the values of g_1, g_2 measured at the Z, we predict $\alpha_s(m_Z) \sim 0.07$.

In the MSSM, the Higgsinos and gluinos slow down the rise of g_2, g_3 due to asymptotic freedom. Now we predict $\alpha_s(m_Z) \sim 0.12$.

Here is a pictorial view of the grand unification relation:



One important effect is still missing from our model. In the real world, supersymmetry is manifestly broken. There is no scalar electron at 0.51 MeV, there is no fermion at the W mass. So we need to include a mechanism to break supersymmetry.

One's first guess would be to add a simple field like the MSM Higgs field that accomplishes spontaneous SUSY breaking. That does not work. Because of the supersymmetry relation of couplings, it can be shown that any simple weak-coupling model of supersymmetry breaking leads to tau sleptons and b squarks at rougly the same mass as b and tau.

To solve this problem, we need a different approach to the spontaneous breaking of supersymmetry.

I argued that the reason that quarks, leptons, and gauge bosons are at an accessible mass scale is that their masses are protected by SU(2) x U(1). Similarly, the squarks, sleptons, and gauginos will be accessible if their masses are protected by supersymmetry. But supersymmetry involves every possible field in Nature.

As a consequence, supersymmetry breaking anywhere in physics eventually destroys supersymmetry everywhere in physics. By default, supersymmetry breaking in some new, very high-energy interaction will couple to (super)gravity and, through gravitational couplings, induce supersymmetry-breaking terms in the effective Lagrangian for the MSSM. Gauge interactions, appearing as radiative corrections to the supersymmetry breaking physics, can also have this effect.

This leads to the following picture of supersymmetry breaking in Nature:

Supersymmetry is broken in a 'hidden sector' with no direct coupling to the quarks and leptons.

A weak coupling of this sector to the observable sector described by the MSSM induces SUSY-breaking effective interactions in the MSSM. The induced mass terms are of the order of

$$m \sim \frac{\langle F \rangle}{M} \sim \frac{\Lambda^2}{M}$$

where Λ is the scale of new physics in the hidden sector and M is called the messenger scale.

By default, the messenger is gravity. Then $\Lambda \sim 10^{11}\ \mathrm{GeV}$. However, the connection can also be made by gauge interactions, new scalars (moduli), or other mechanisms.

Supersymmetry requires a supersymmetric generalization of gravity, supergravity. Then there is another new particle, the superpartner of the graviton, called the gravitino. Spontaneous supersymmetry breaking gives a mass to the gravitino. This is called the super-Higgs mechanism. The value is

$$m_{\tilde{G}} = \frac{8\pi}{3} \frac{F}{m_{\rm Pl}} \sim \frac{\Lambda^2}{m_{\rm Pl}}$$

If the messenger of supersymmetry breaking is gravity and supergravity, this mass is larger than that mass scale of SM superpartners. If the messenger scale is lower, the gravitino is light, and SM superpartners will decay to the gravitino. The couplings responsible for these decays need not be of gravitational strength.

In this picture, the weak interaction scale is not a fundamental scale in Nature, but rather is derived from the parameters of the hidden sector and the messenger interactions.

Actually, what is derived is the mass scale of squarks, gluinos, and Higgsinos. However, once we break supersymmetry, we will generate a mass for the Higgs boson.

Hopefully, the Higgs mass parameter will be negative and electroweak symmetry will be broken. This in turn will produce the masses of quarks, leptons, and gauge bosons.

To implement this program, we represent supersymmetry breaking phenomenologically by adding to the Lagrangian soft operators that explicitly break supersymmetry:

$$\delta L = -M_f^2 |\widetilde{f}|^2 - \frac{1}{2} m_i \lambda_i \cdot \lambda_i - A W_{\text{Higgs}} - B\mu H_u \cdot H_d - h.c.$$

Eventually, if supersymmetry is correct, we will determine the coefficients experimentally and use these to infer the underlying model of supersymmetry breaking.

A problem with this idea is that we have no control over flavor violation by the soft terms. We can impose flavor conservation by hand, or we can try to invent underlying models that lead to flavor conservation. A common assumption is that the squark and slepton masses are degenerate for the three generations. Then we can rotate away flavor violation in the mass terms.

The soft supersymmetry breaking terms give masses individually to the squarks, sleptons, and gauge bosons. Terms involving the Higgs vevs mix the partners of f and $\overline{f}$, and the partners of gauge and Higgs bosons.

Consider, for example, the partners of $W^+, \ W^-, \ H_u^+, \ H_d^-$ These obtain diagonal masses from the soft gaugino masses and the μ term, and Dirac masses from the Higgs vevs. The mass matrix, written in full, is

$$\begin{pmatrix} \widetilde{w}^- & \widetilde{h}_d^- \end{pmatrix} m_C \begin{pmatrix} \widetilde{w}^+ \\ \widetilde{h}_u^+ \end{pmatrix}$$

with

$$m_C = \begin{pmatrix} m_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}$$

The mass eigenstates are called charginos: $\widetilde{C}^{\pm}_{1,2}$ or $\widetilde{\chi}^{\pm}_{1,2}$

Similarly, the partners of the neutral gauge bosons and neutral Higgs bosons $B_\mu, \; W^0_\mu, \; H^0_u, \; H^0_d$ obtain both Majorana and Dirac mass terms. The mass matrix, acting on

$$(\widetilde{b}, \widetilde{w}^0, \widetilde{h}^0_d, \widetilde{h}^0_u)$$

is

$$m_N = \begin{pmatrix} m_1 & 0 & -m_Z c_\beta s_w & m_Z s_\beta s_w \\ 0 & m_2 & m_Z c_\beta c_w & -m_Z s_\beta c_w \\ -m_Z c_\beta s_w & m_Z c_\beta c_w & 0 & -\mu \\ m_Z s_\beta s_w & -m_Z s_\beta c_w & -\mu & 0 \end{pmatrix}$$

The mass eigenstates are called neutralinos: $\widetilde{N}^0_{1\ldots4}$ or $\widetilde{\chi}^0_{1\ldots4}$

Note that the μ term is needed to prevent the appearance of a massless neutralino and a chargino lighter than the W.

The off-diagonal elements linking the gauginos and Higgsinos are proportional to m_W and m_Z. If the scale of SUSY masses is high,

$$m_1, m_2, |\mu| \gg m_W$$

the mixing problems simplify. Still, there are two distinct cases:

gaugino region: $m_1, m_2 < |\mu|$

here N_1^0, N_2^0, C_1^+ are mainly gaugino, with masses m_1, m_2 . while the heavier states are mainly Higgsino

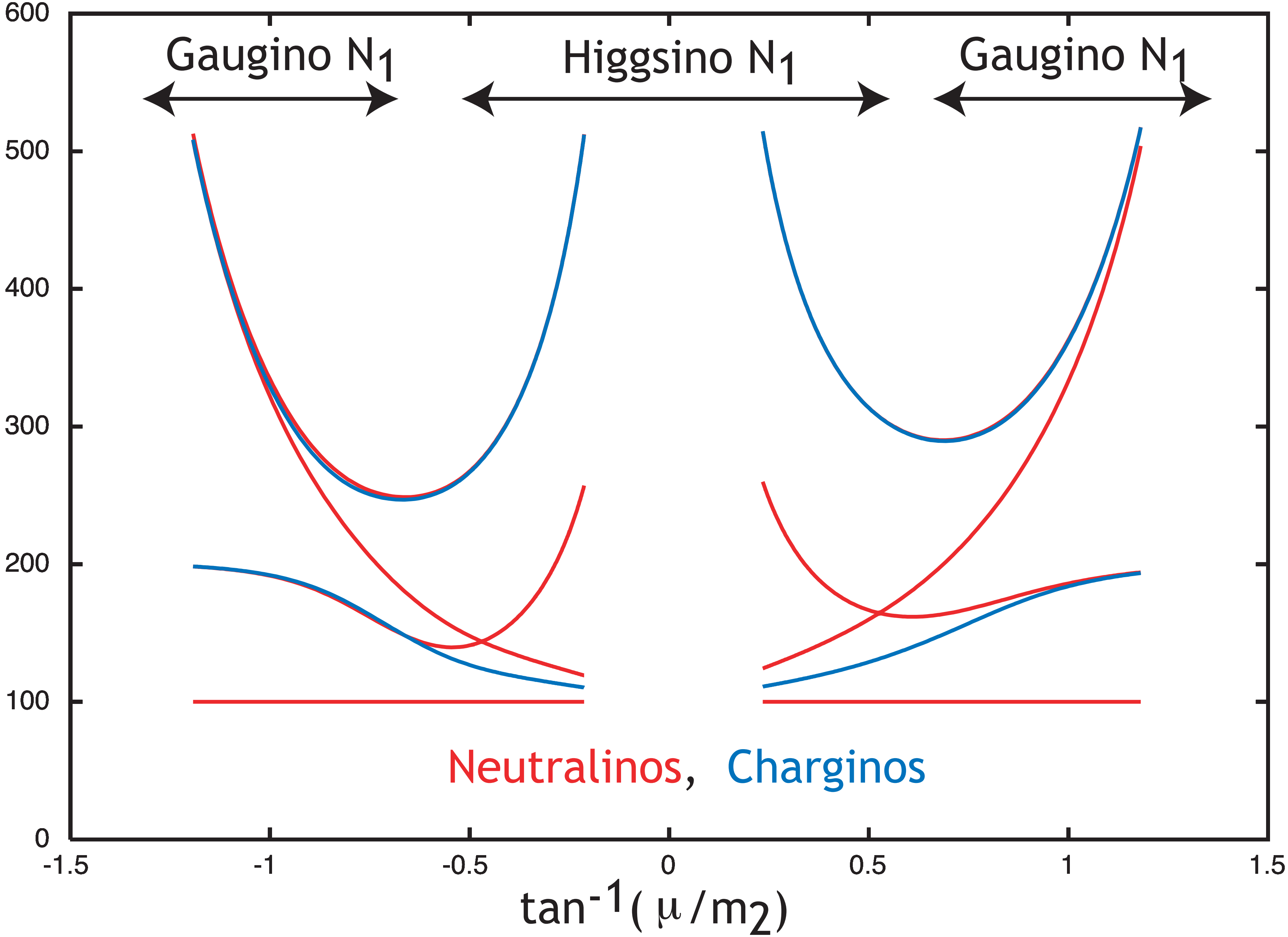
Higgsino region: $m_1, m_2 > |\mu|$

here N_1^0, N_2^0, C_1^+ are mainly Higgsino, with degenerate masses $|\mu|$ while the heavier states are mainly gaugino

It is important for some purposes that the lightest neutralino has both gaugino and Higgsino components.

In my numerical examples, I will put $m_1 \sim 0.5 m_2$ for reasons to be explained later.

along a line with fixed $m(N_1^0)$, varying μ/m_2



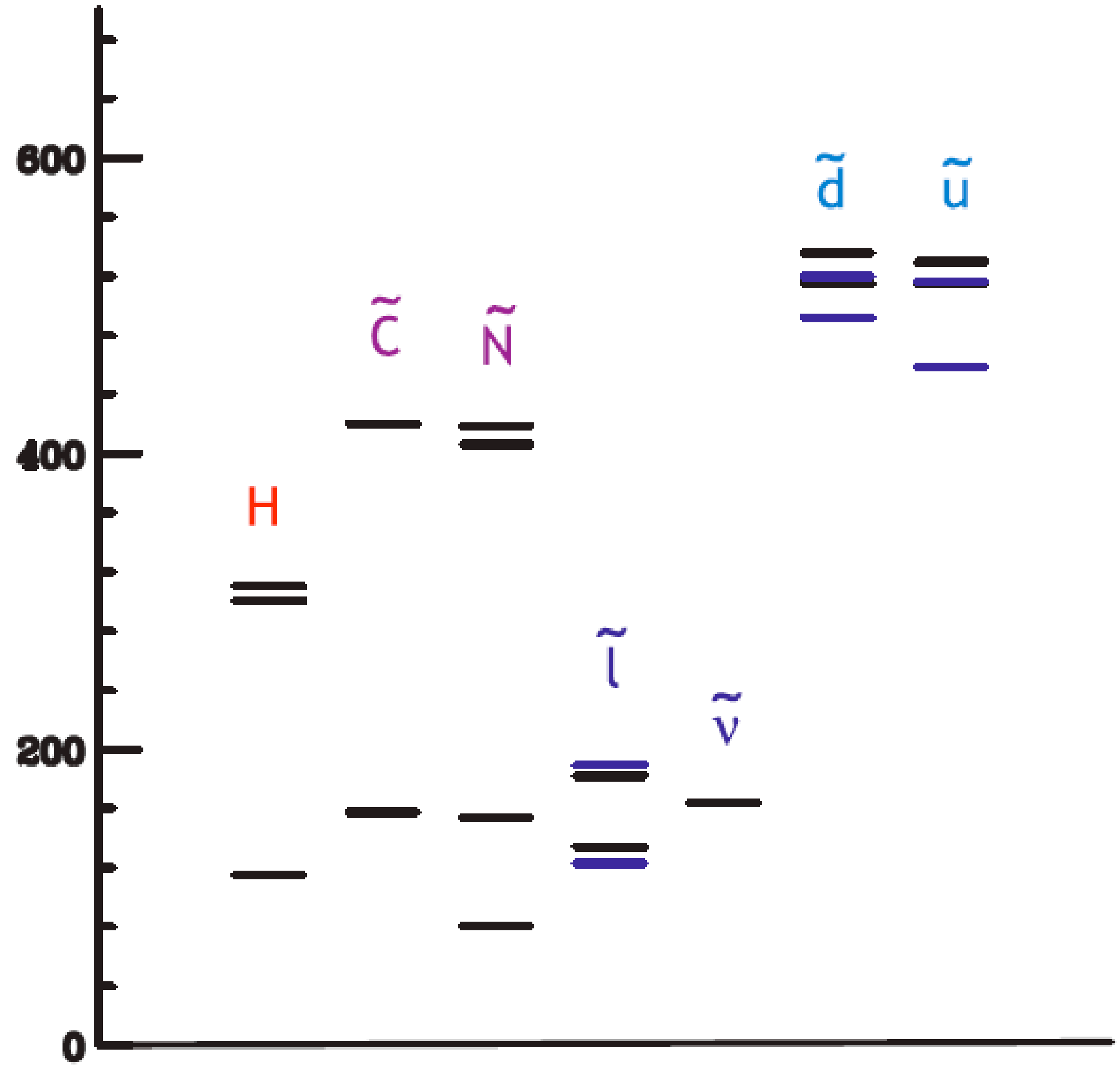
Here is a sample spectrum of SUSY particles.

Note that we are in the gaugino region of N and C, and that the 3rd generation sfermions (in blue) are noticeably split off from the others.

The Higgs spectrum has a light state h^0 and four heavy Higgs bosons

$$(H^0, H^{\pm}, A^0)$$

This is typical of 2-Higgs doublet models.



In principle, I could have constructed this spectrum by giving random values to the soft SUSY-breaking masses. However, what I actually did was to give universal masses to the sfermions and (separately) to the gauginos at the Grand Unification scale and let the mass differences develop by renomalization group running of the soft parameters.

For the gauginos, this story is very simple; the soft masses run with the gauge couplings: $m_i(Q)/\alpha_i(Q) = m_i(M_U)/\alpha_U$

If the three gaugino masses are equal at the Grand Unification scale, then at the weak scale,

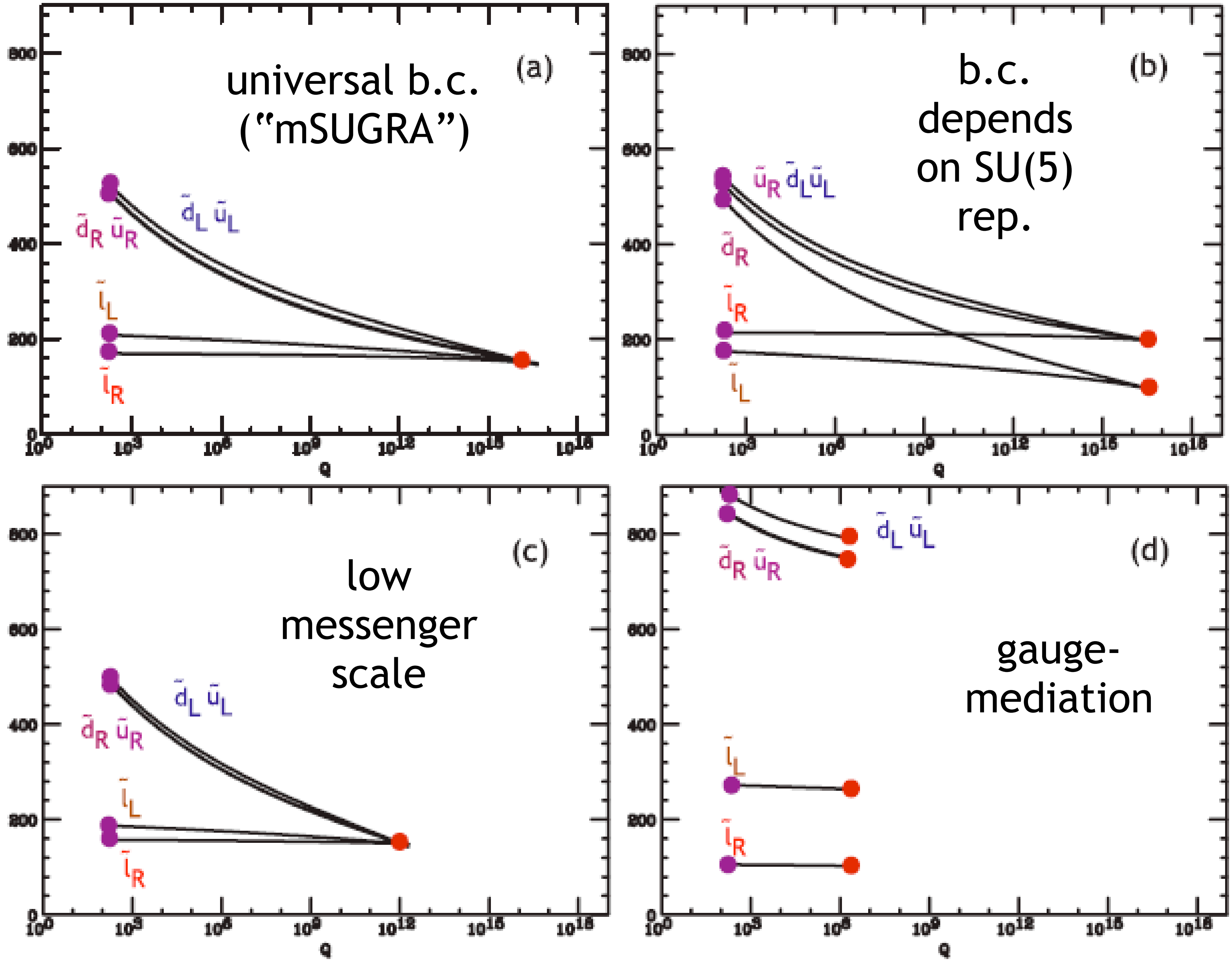
$$m_1 : m_2 : m_3 = \alpha_1 : \alpha_2 : \alpha_3 = 0.5 : 1 : 3.5$$

This relation is called "gaugino unification".

Sfermions obtain positive mass contributions from their gauge and gaugino interactions. These are much larger for squarks than for sleptons.

The next slide shows various different hypotheses for the underlying values of the soft sfermion masses and their modification by renormalization group running. Obviously, this is a small sampling of the possibilities.

What we can measure are the values of the physical masses (and mixings) at the TeV scale. From this, we need to infer the original pattern.



(a)
universal b.c.
("mSUGRA")
$\tilde{d}_L$ $\tilde{u}_L$
$\tilde{d}_R$ $\tilde{u}_R$
$\tilde{l}_L$
$\tilde{l}_R$
Q
(b)
b.c.
depends
on SU(5)
rep.
$\tilde{u}_R$ $\tilde{d}_L$ $\tilde{u}_L$
$\tilde{d}_R$
$\tilde{l}_R$
$\tilde{l}_L$
Q
(c)
low
messenger
scale
$\tilde{d}_L$ $\tilde{u}_L$
$\tilde{d}_R$ $\tilde{u}_R$
$\tilde{l}_L$
$\tilde{l}_R$
Q
(d)
gauge-
mediation
$\tilde{d}_L$ $\tilde{u}_L$
$\tilde{d}_R$ $\tilde{u}_R$
$\tilde{l}_L$
$\tilde{l}_R$
Q

Some qualitative features are apparent:

Typically, squarks are much heavier than sleptons.

Typically, squarks are rather degenerate.

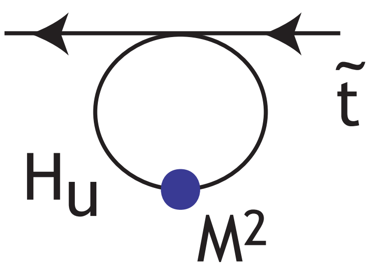
For sleptons, $m(\widetilde{e})/m(\widetilde{\overline{e}})$ can be large. This is an interesting diagnostic of the underlying theory.

It is important that a program of SUSY spectroscopy should

- test gaugino universality
- measure $m(\widetilde{e})/m(\widetilde{\overline{e}})$ and $m(\widetilde{e})/m(\widetilde{q})$
- test generation-independence of sfermion masses

There is one more very important renormalization group effect.

SInce the top quark is heavy, the Yukawa coupling y_t will renormalize the soft masses of the three states that are affected by this coupling: $t, \bar{t}, H_u$. Using the 4-scalar interaction proportional to y_t^2 , we obtain, for example



$$= -iy_t^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2} (-iM_{H_u}^2) \frac{i}{k^2}$$

$$= y_t^2 M_{H_u}^2 \cdot \frac{i}{(4\pi)^2} \log \Lambda^2$$

Comparing to the canonical form of a mass renormalization $-i\delta m^2$, this is a negative contribution to the mass parameter.

All three sfermions obtain a similar negative correction, with a counting factor for the number of colors or isospin states in the loop.

Taking all of these effects into account, we find for the RG equations for the soft masses:

$$\frac{dM_t^2}{d\log Q} = \frac{2}{(4\pi)^2}\cdot 1 \cdot y_t^2[M_t^2 + M_{\bar{t}}^2 + M_{Hu}^2 + A_t^2] - \frac{8}{3\pi}\alpha_3 m_3^2 + \cdots$$

$$\frac{dM_{\bar{t}}^2}{d\log Q} = \frac{2}{(4\pi)^2}\cdot 2 \cdot y_t^2[M_t^2 + M_{\bar{t}}^2 + M_{Hu}^2 + A_t^2] - \frac{8}{3\pi}\alpha_3 m_3^2 + \cdots$$

$$\frac{dM_{Hu}^2}{d\log Q} = \frac{2}{(4\pi)^2}\cdot 3 \cdot y_t^2[M_t^2 + M_{\bar{t}}^2 + M_{Hu}^2 + A_t^2] + \cdots$$

The structure is intriguing: The three fields $(\widetilde{t}, \widetilde{\bar{t}}, H_u)$ all receive negative contributions to their mass parameters. In some sense, they race to see which will first have $M^2 < 0$ as Q is decreased.

H_u wins the race, and we find an instability to the correct pattern of electroweak symmetry breaking. This effect on the Higgs mass is the dominant one as a result of the heaviness of the top quark.

Finally, a little on the phenomenology of supersymmetry at colliders.

Because we know so little about the soft supersymmetry breaking parameters, many different patterns are allowed for the supersymmetry spectrum. An important branch point comes from the relation between the mass of the gravitino and the mass of the lightest SM superpartner (LSP).

If $m_{\tilde{G}} > m_{LSP}$, SM superpartners will decay to quarks and leptons plus the LSP. It is easy to arrange that the LSP is absolutely stable. If the LSP is neutral, it is a stable, neutral weakly interacting particle of mass about 100 GeV. That is, it is a perfect candidate for the particle of cosmic dark matter. Most models realize this by identifying the LSP = N_1 .

If $m_{\tilde{G}} < m_{LSP}$, SM superpartners will still decay to the LSP, but then that particle will decay to the gravitino with a long lifetime, fsec to yr, depending on the messenger scale. In any event, this lifetime is much shorter than cosmological times. Now it is allowed that the LSP can be charged. An attractive choice is LSP = $\widetilde{\tau}$.

Consider first the case LSP = N_1, with the N_1 an absolutely stable particle.

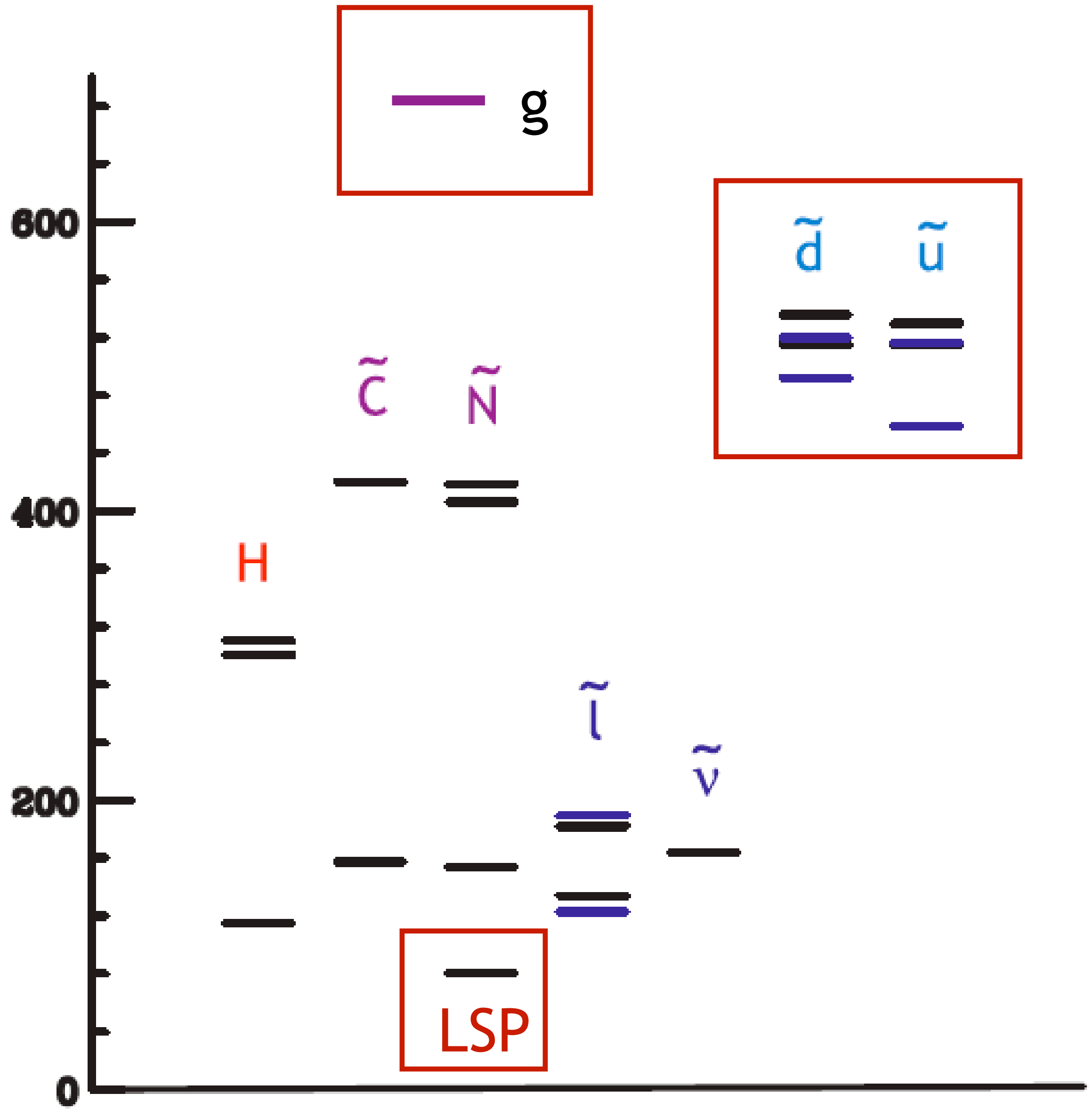
The direct pair production of the N_1 has a small cross section and is almost impossible to observe.

However, we can take advantage of the fact that squarks and gluinos have relatively large QCD pair production cross sections. This idea, that there are colored new particles with large QCD cross sections that decay to the dark matter particle, is generic in models of electroweak symmetry breaking. In particular, it is an essential feature of most models that there is a partner of the top quark in the new particle sector.

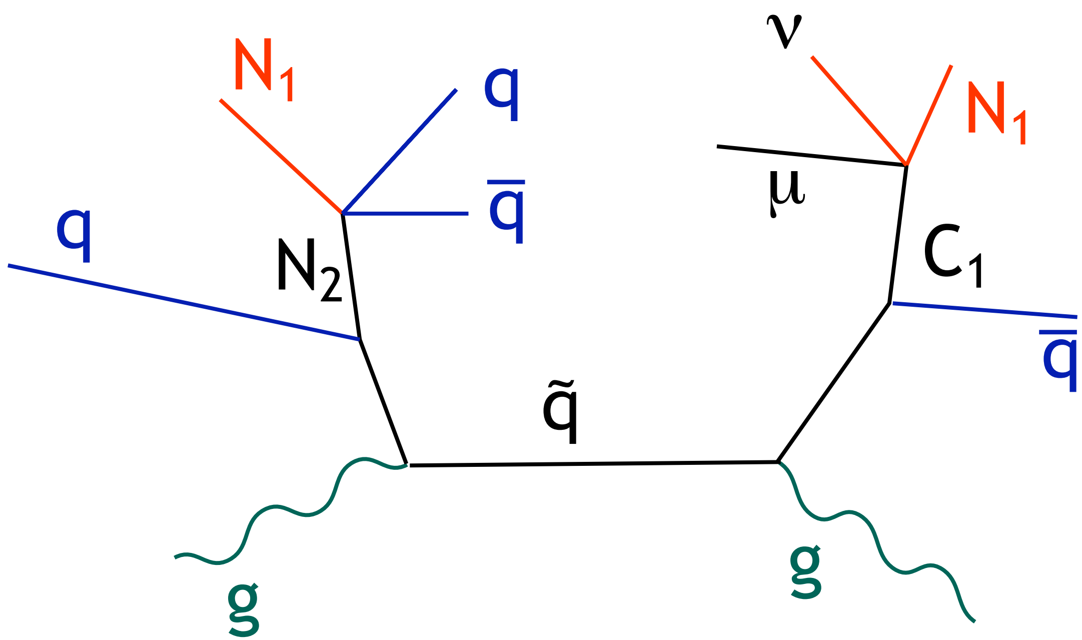
I remind you of the sample supersymmetry spectrum that I showed in Lecture 3.

At the LHC, the largest cross sections will be those for gluino and squark pair production.

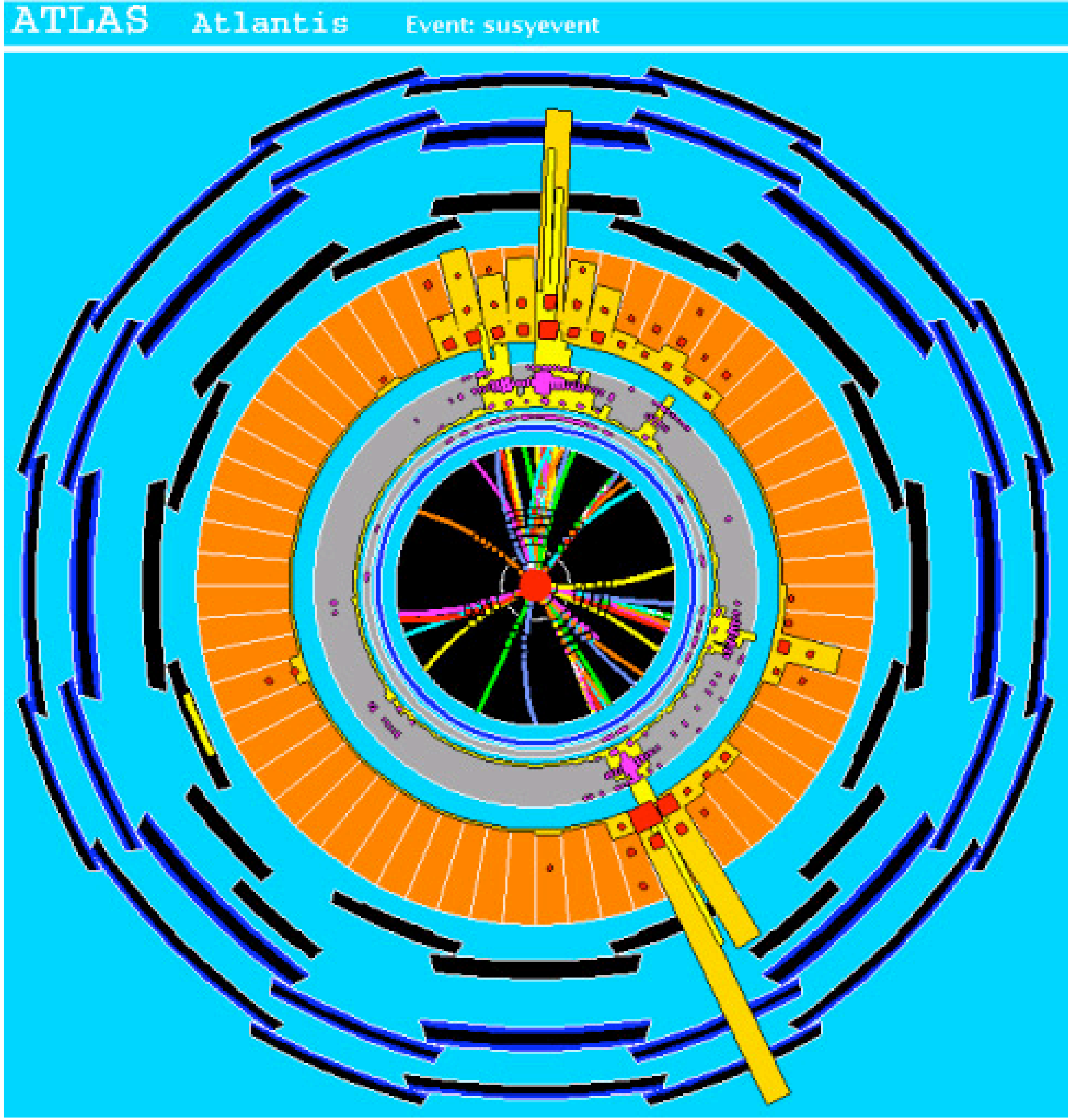
The gluinos and squarks will rapidly decay through a sequence that ends in the lightest neutralino.



The new physics events can be characteristic in having multiple jet production and unbalanced visible momentum. A typical event would have the following form: (Particle labels are for supersymmetry.)



It is expected that events of this kind will appear as a very significant signal above background.



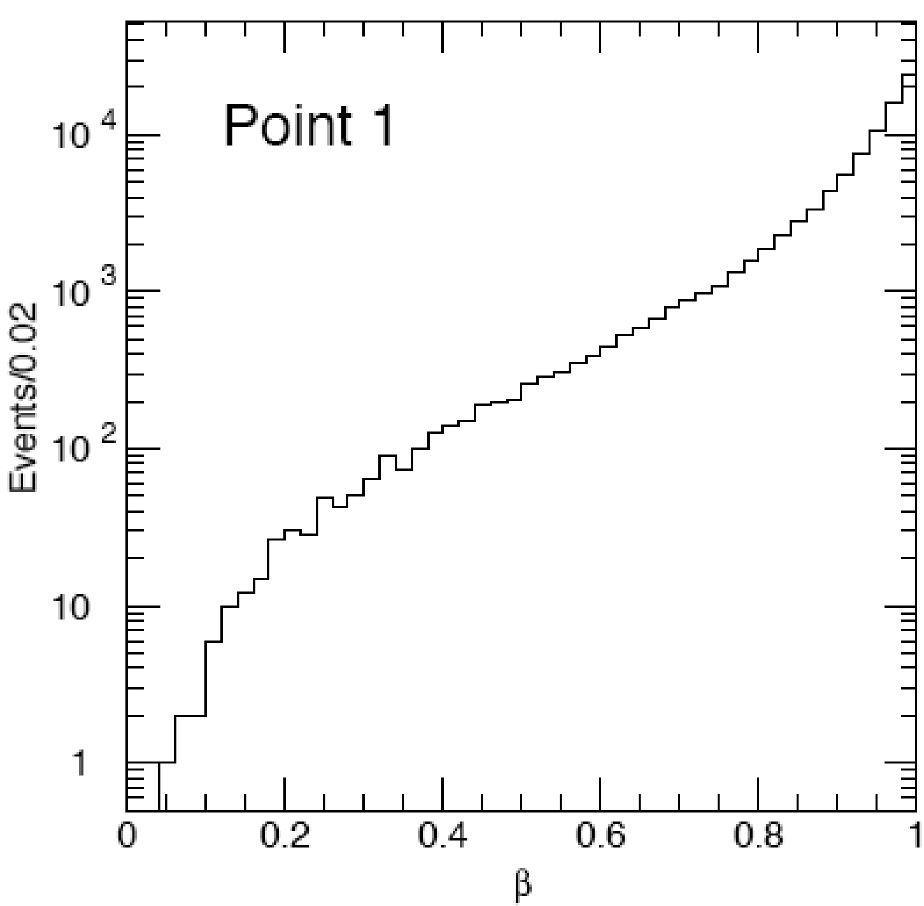
ATLAS
simulation

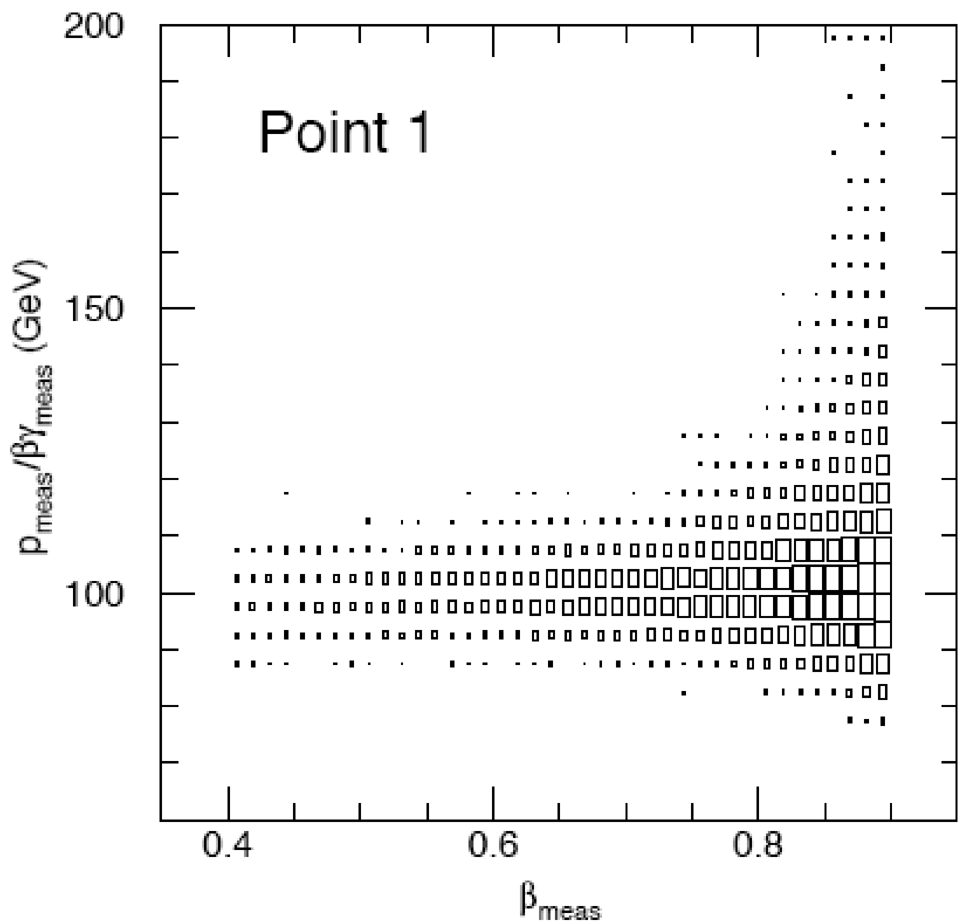
In models with a $\widetilde{\tau}$ LSP, the supersymmetry events have a similar character. The dominant production mechanism is pair production of squarks and gluinos. These particles then decay to quarks, leptons, and quasi-stable $\widetilde{\tau}$ s.

If we are lucky, the $\widetilde{\tau}$ lifetime will be longer than the time of 100 nsec needed for the $\widetilde{\tau}$ to traverse an LHC detector. Then the $\widetilde{\tau}$ will appear as a heavy charged stable particle.

Stable $\widetilde{\tau}$ s then appear as muons which are slow but can still be within the time bucket of the muon system. This is a very easy signature of SUSY compared to the usual ones.

Using $\beta\ vs.\ p$, it is possible to measure the mass to 0.1%.



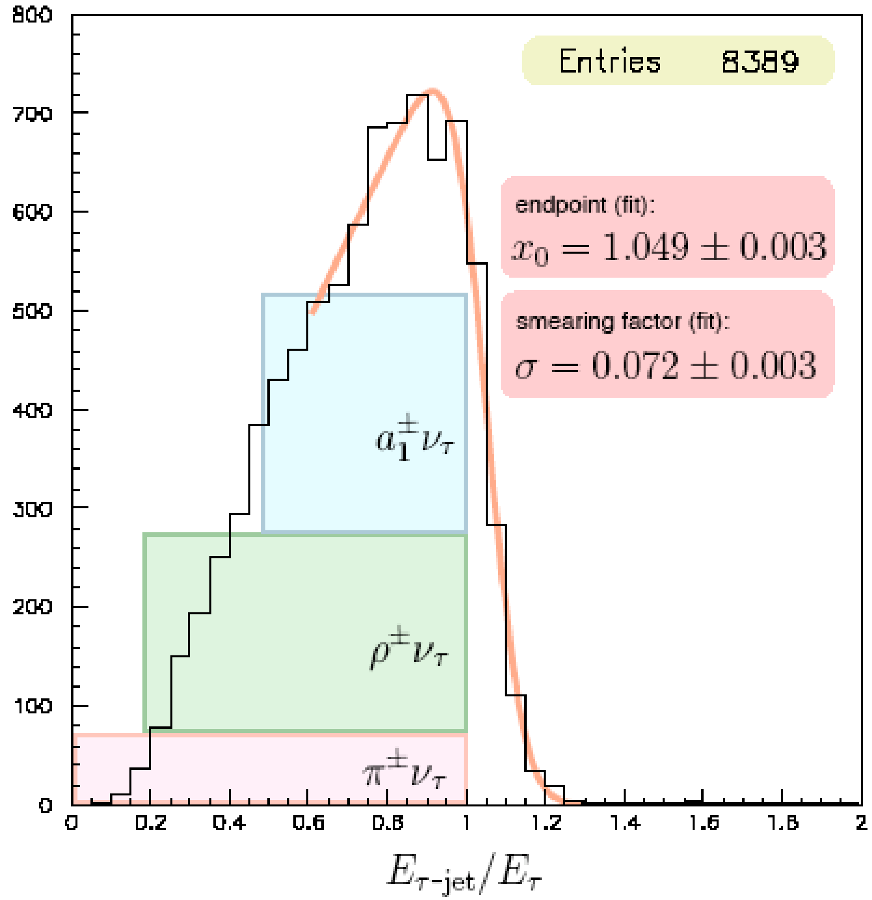


Ambrosanio, Mele, Petrarca, Polesello, Rimoldi

Gauginos decay to the slepton by

$$\widetilde{\chi}_i^0 \rightarrow \tau^+ \widetilde{\tau}^-$$

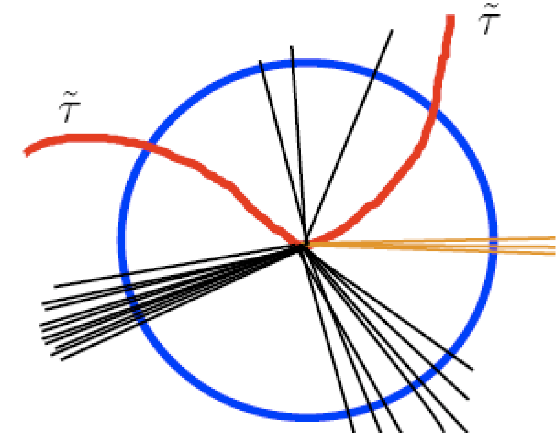
so we can measure the spectrum of gauginos by associating τ jets with staus. Of course, the τ is not observed completely. But in hadronic τ decays, the LHC detectors can see most of the energy.

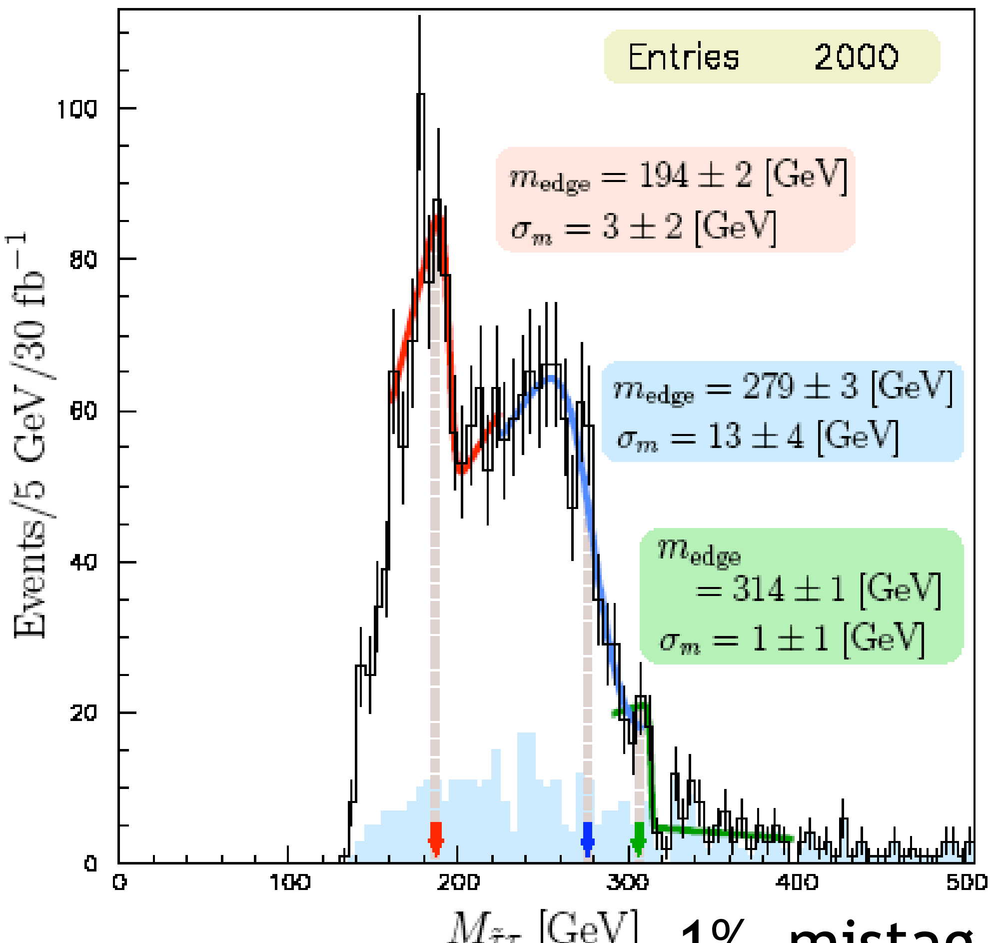


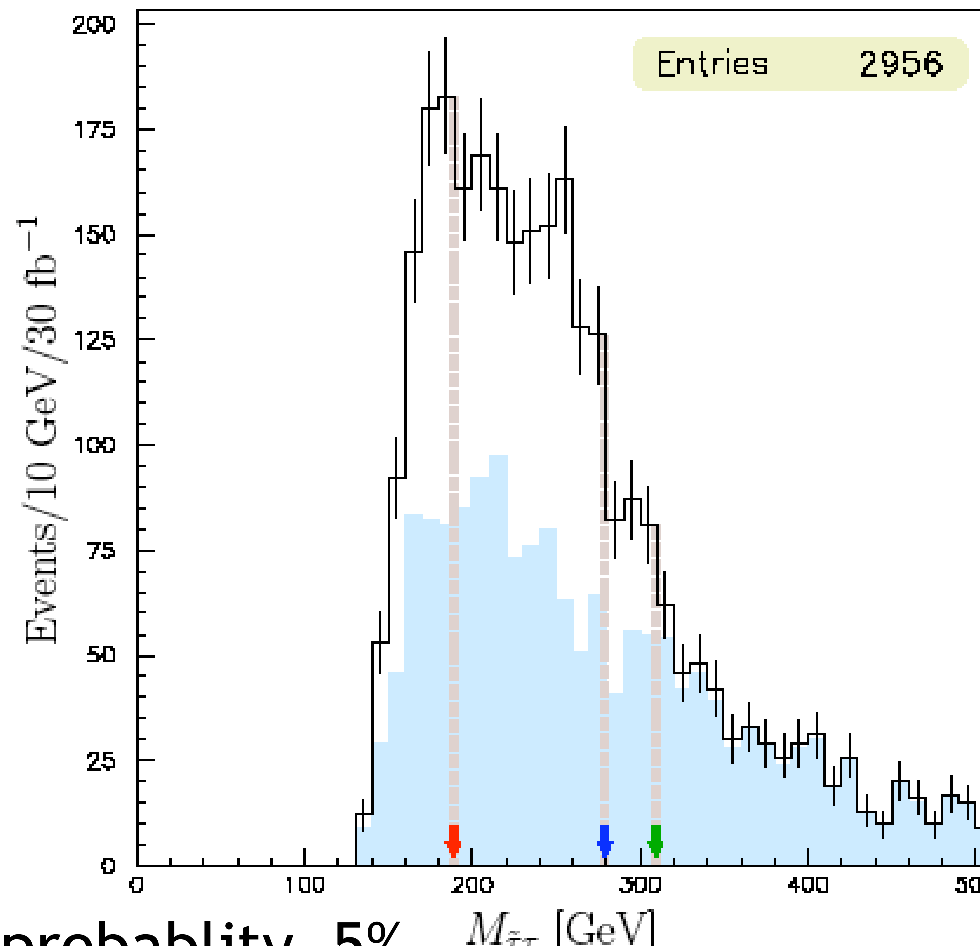
Kitano and Ibe

HERWIG + TAOULA + AcerDET

So we can combine stable $\tilde{\tau}$s with jets and look for resonances. Including detector effects, these appear as kinematic edges. Then, many of the superpartner masses can be obtained directly.







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If these ideas about electroweak symmetry breaking are realized, in any of their possible forms, we are on the brink of an exciting era in physics.

The LHC will reveal a whole spectrum of new particles, with many observables that will allow us to understand their structure and systematics. This will be the first step in unveiling a new set of fundamental interactions of Nature.

Today, the LHC is the place to be ! I wish you the best of luck in this era of discovery.