Theoretical Introduction to LHC Physics

2. Higgs Boson

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Zuoz Summer School
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In the previous lecture, I described the structure of the Standard Model couplings to massless quarks and leptons. In the 1990's, this structure was tested by precision experiments on the Z resonance in e+e- annihilation at SLC and LEP. These tests continue with the precision measurement of the W boson mass at the Tevatron. I will now discuss these tests and some of their implications.

The first question we must address is: What exactly is the definition of the Standard Model that we will test ? The Standard Model has many parameters. How do we fix them ?

Actually, the model of electroweak interactions in the previous lecture has only 3 parameters that affect its tree-level predictions: g_1, g_2, v Other parameters, such as α_s, m_t appear at the 1-loop level. To set up precision tests, we need at least to fix the tree-level parameters.

To fix these three parameters, we need 3 precision measurements. A standard set is:

$$\begin{aligned} \alpha^{-1}(m_Z) &= 128.95\,(5) \\ G_F &= 1.16637\,(1) \times 10^{-5}\ \mathrm{GeV}^{-2} \\ m_Z &= 91.1876\,(21)\ \mathrm{GeV} \end{aligned}$$

Note that I take the running QED coupling at the Z mass. The primary source of error is the value of the hadronic contribution to the vacuum polarization, which must be determined from data on the e+e- hadronic cross section.

If g_1, g_2, v are extracted from these three measurements using tree-level formulae, the corresponding value of s_w^2 is given by

$$\sin^2 2\theta_0 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2}$$

The value is: $s_0^2 = 0.23107\,(4)$

We can compare this value to other values of s_w^2 obtained, for example from the value of a Z decay rate or asymmetry or from m_W/m_Z .

The accuracy of these numbers is such that 1-loop corrections must be applied systematically to make this comparison.

How many such tests can we obtain ?

In yesterday's lecture, I displayed a table of values of the couplings Q_Z^2 . Here is that table again, augmented by the values of

$$S_f = Q^2_{ZLf} + Q^2_{ZRf} \qquad A_f = \frac{Q^2_{ZLf} - Q^2_{ZRf}}{Q^2_{ZLf} + Q^2_{ZRf}}$$

The S_f determine the Z partial widths and branching ratios. The A_f determine the parity asymmetry of Z decay in the various channels. Most notably, the A_f take very different values for different quark and lepton species.

	ν	e	u	d	
$Q_Z^2 =$	0.250	0.073	0.120	0.179	L,Q
	-	0.053	0.024	0.006	$\overline{e}, \overline{u}, \overline{d}$
$S_f =$	0.250	0.126	0.144	0.185	
$A_f =$	1.00	0.16	0.67	0.94	

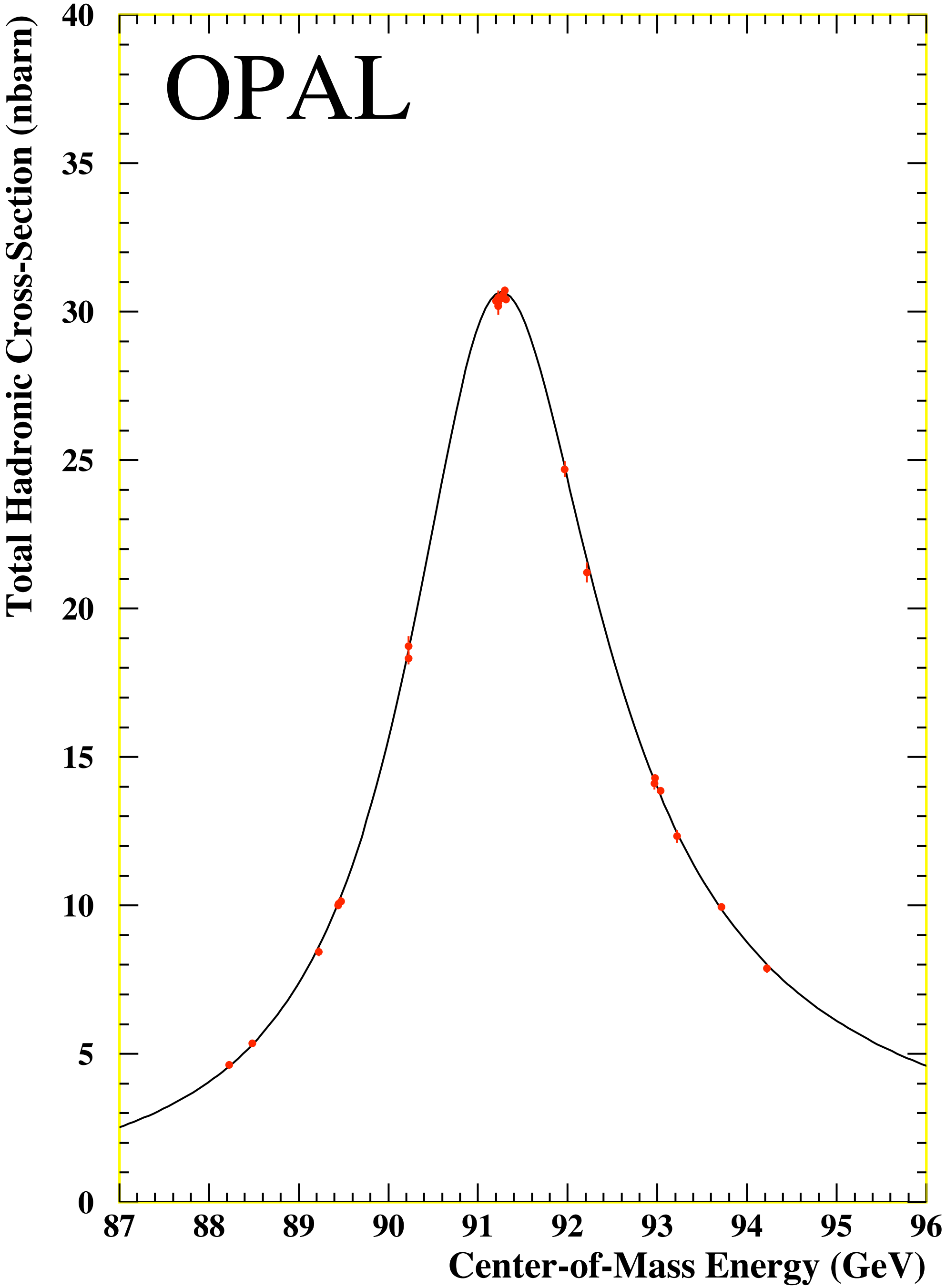
The most stringent test of the S_f comes from the measurement of the total width of the Z. This can be extracted from measurement of the Z resonance line shape. That study actually brings in radiative corrections from all three of the fundamental interactions -- electroweak corrections to the e+e-Z vertex, QCD corrections to the decay amplitude to quarks, and QED corrections (up to NNNLL) which distort the resonance shape through initial state radiation.

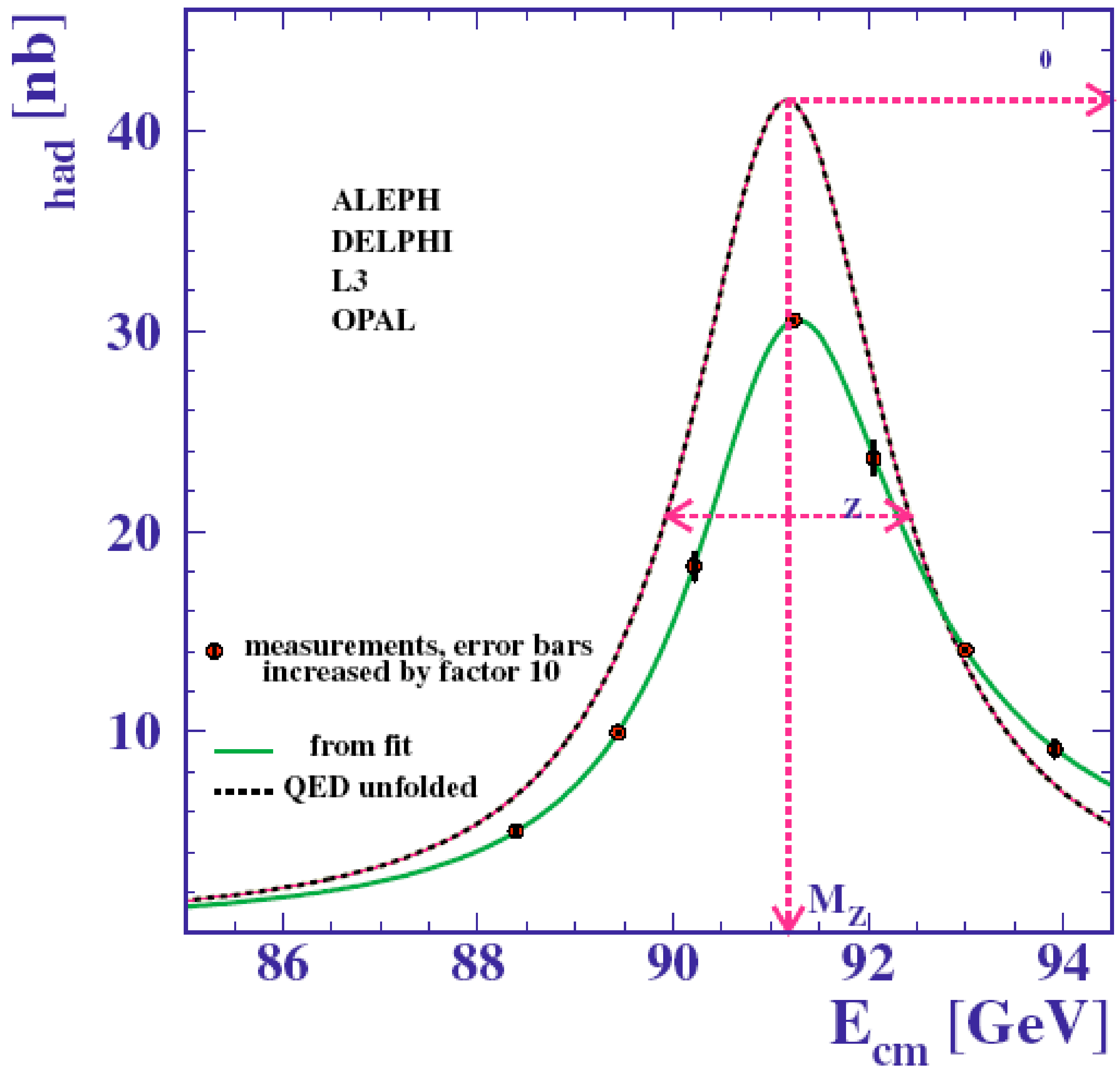
The final results is the very impressive value

$$\Gamma_Z = 2.4952\,(23)$$

in good agreement with the Standard Model predictions. A measure of the quality of the result is the constraint on the width of the Z to invisible final states, quoted as a number of neutrinos:

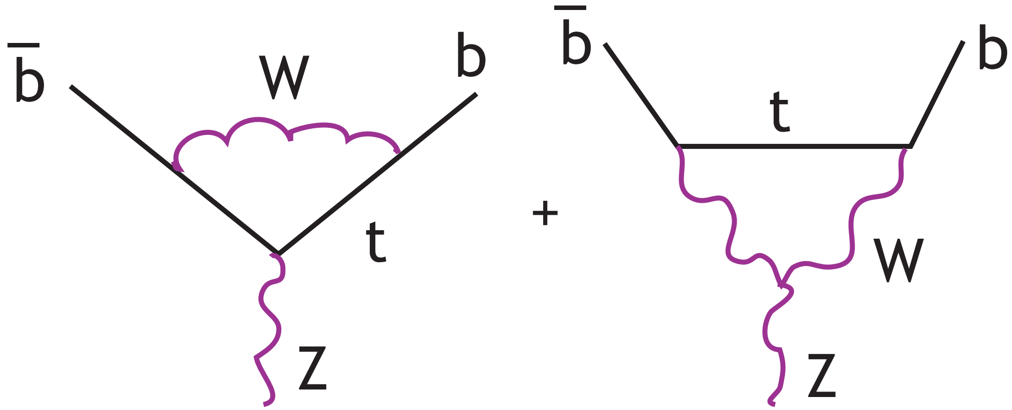
$$n_\nu = 2.9840\,(82)$$


OPAL
Total Hadronic Cross-Section (nbarn)
0
5
10
15
20
25
30
35
40
87
88
89
90
91
92
93
94
95
96
Center-of-Mass Energy (GeV)



composite of the four LEP experiments, showing the effect of ISR

A special interesting partial width is $\Gamma(Z \to b\bar{b})$. The diagrams



contribute a correction to the b_L Z charge,

$$Q_{ZbL} = -(\frac{1}{2} - \frac{1}{3}s_w^2 - \frac{\alpha}{16\pi s_w^2}\frac{m_t^2}{m_W^2})$$

This is a -2% correction to the partial width. It is easier to measure the quantity

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{ hadrons})}$$

which is almost independent of s_w^2 with the SM charge assignments for b and $\bar{b}$.

The final result is:

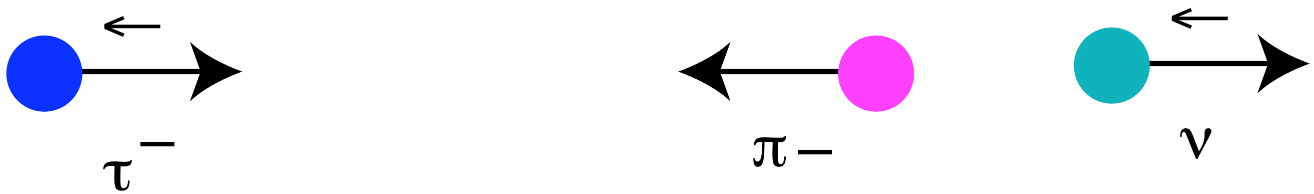
$$R_b = 0.21629 \pm 0.00066 \qquad (\pm 0.3\%)$$

to be compared to 0.21586 expected in the Standard Model, confirming the -2% shift due to the t-W diagrams.

Measuring the A_f is trickier:

There are three methods to measure the A_f for leptons:

1. use the fact that the τ lepton decays through weak interactions in a way that analyzes its polarization



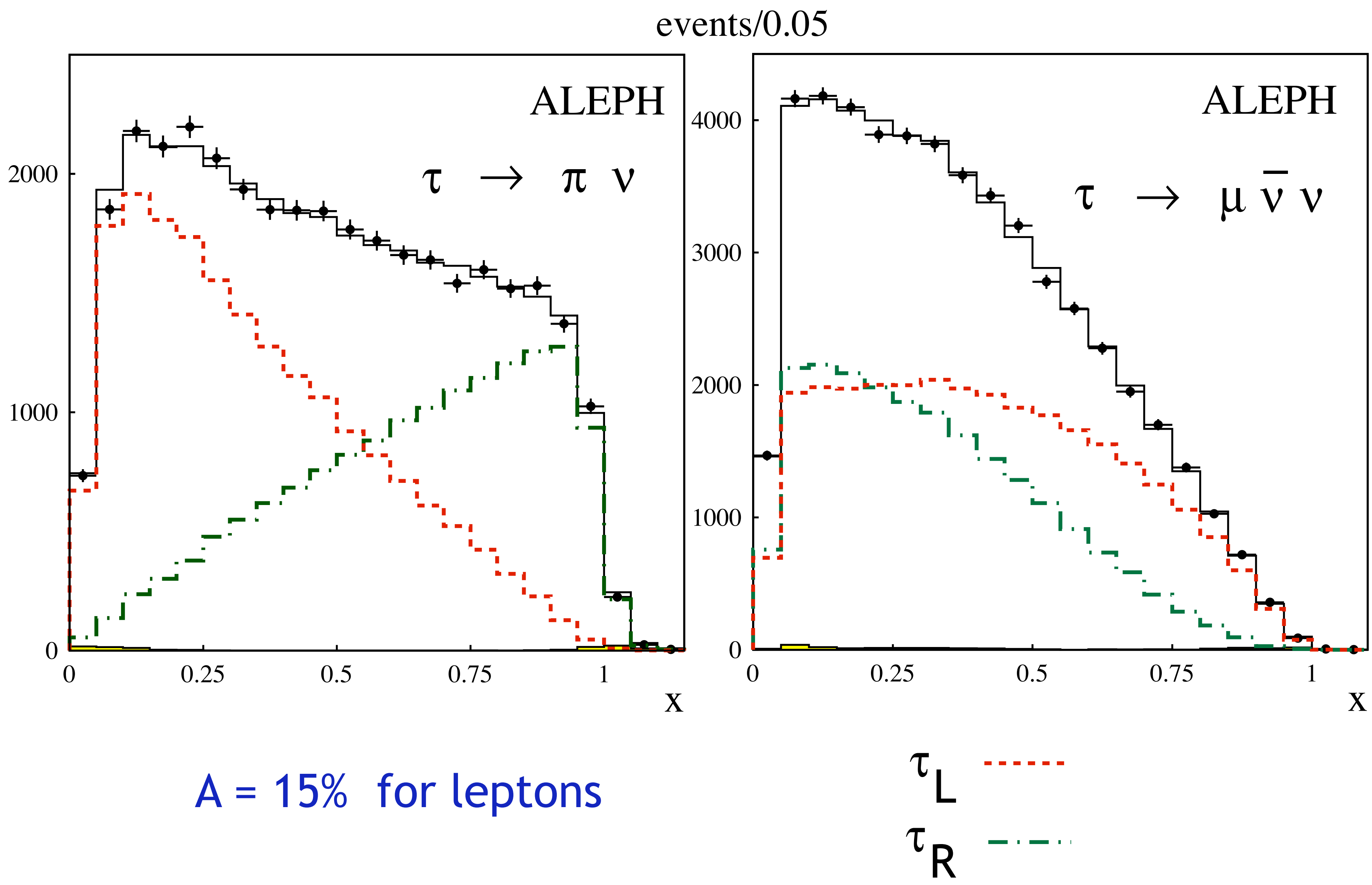
2. create a polarized e^- beam, and measure the relative ability of e^-_L and e^-_R beams to produce the Z resonance

3. measure the forward-backward asymmetry in unpolarized

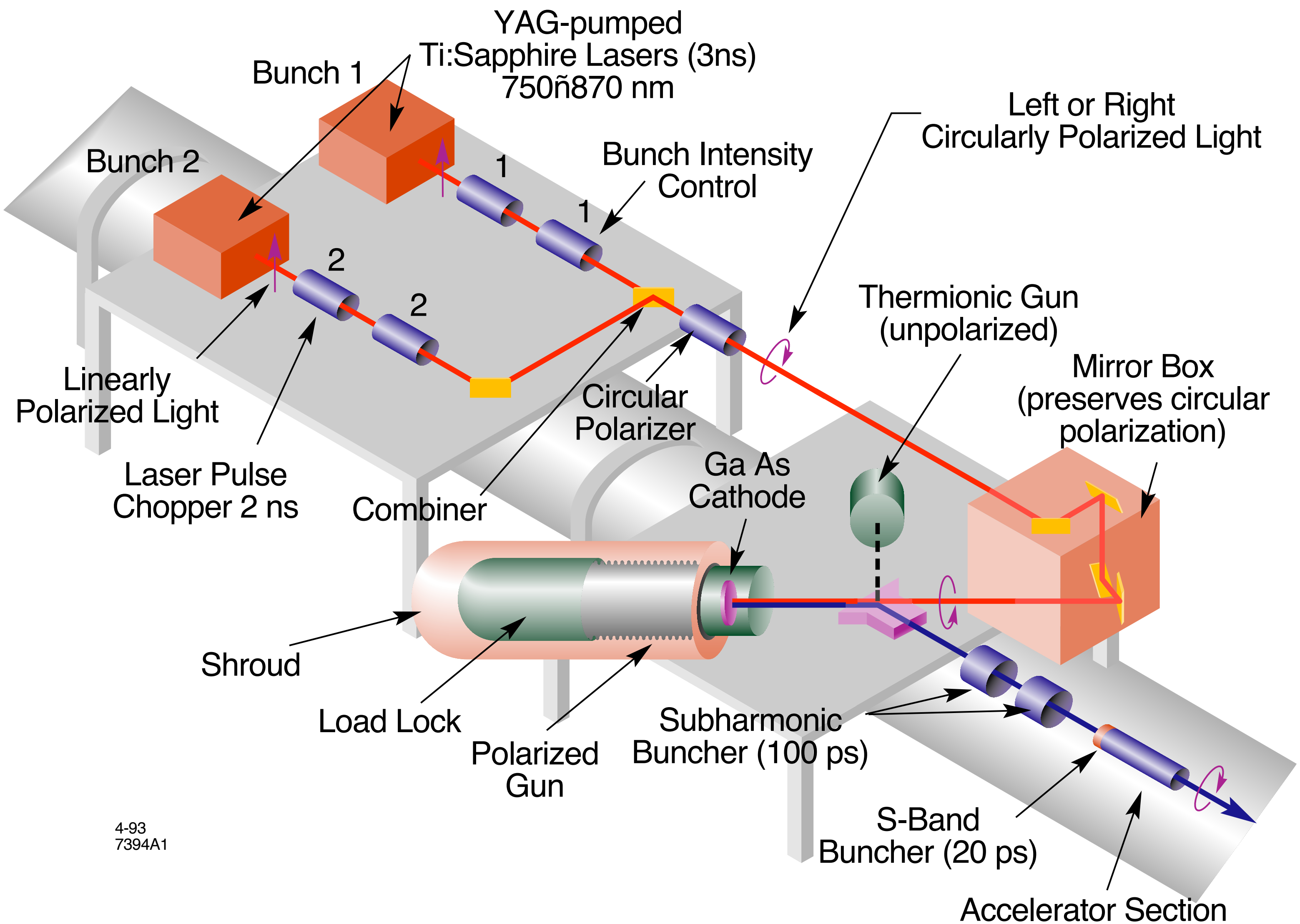
$$e^+e^- \to b\bar{b}: \quad A_{FB} = (3/4)A_e A_b$$

Because A_b is almost maximal, this measures A_e .

Energy distribution of tau decay products



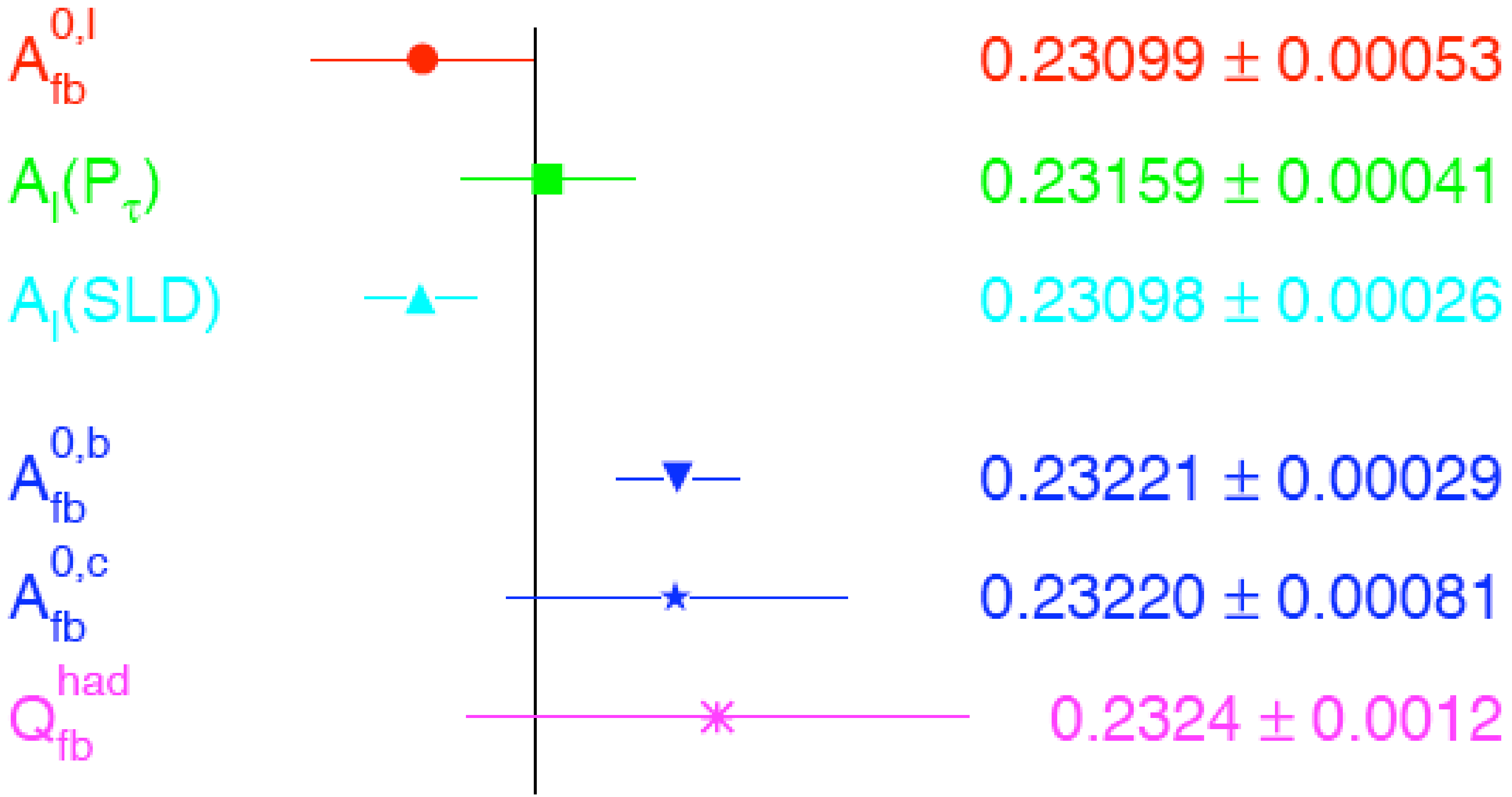
A = 15% for leptons


YAG-pumped
Ti:Sapphire Lasers (3ns)
750ñ870 nm
Bunch 1
Bunch 2
1
1
2
2
Bunch Intensity
Control
Left or Right
Circularly Polarized Light
Thermionic Gun
(unpolarized)
Mirror Box
(preserves circular
polarization)
Linearly
Polarized Light
Laser Pulse
Chopper 2 ns
Combiner
Circular
Polarizer
Ga As
Cathode
Shroud
Load Lock
Polarized
Gun
Subharmonic
Buncher (100 ps)
S-Band
Buncher (20 ps)
Accelerator Section
4-93
7394A1

A_ℓ is very sensitive to s_w^2 and so gives the most precise tests for this parameter:

$$A_\ell = 8(\frac{1}{4} - s_w^2)$$

Here are the results (LEP EW Working Group):

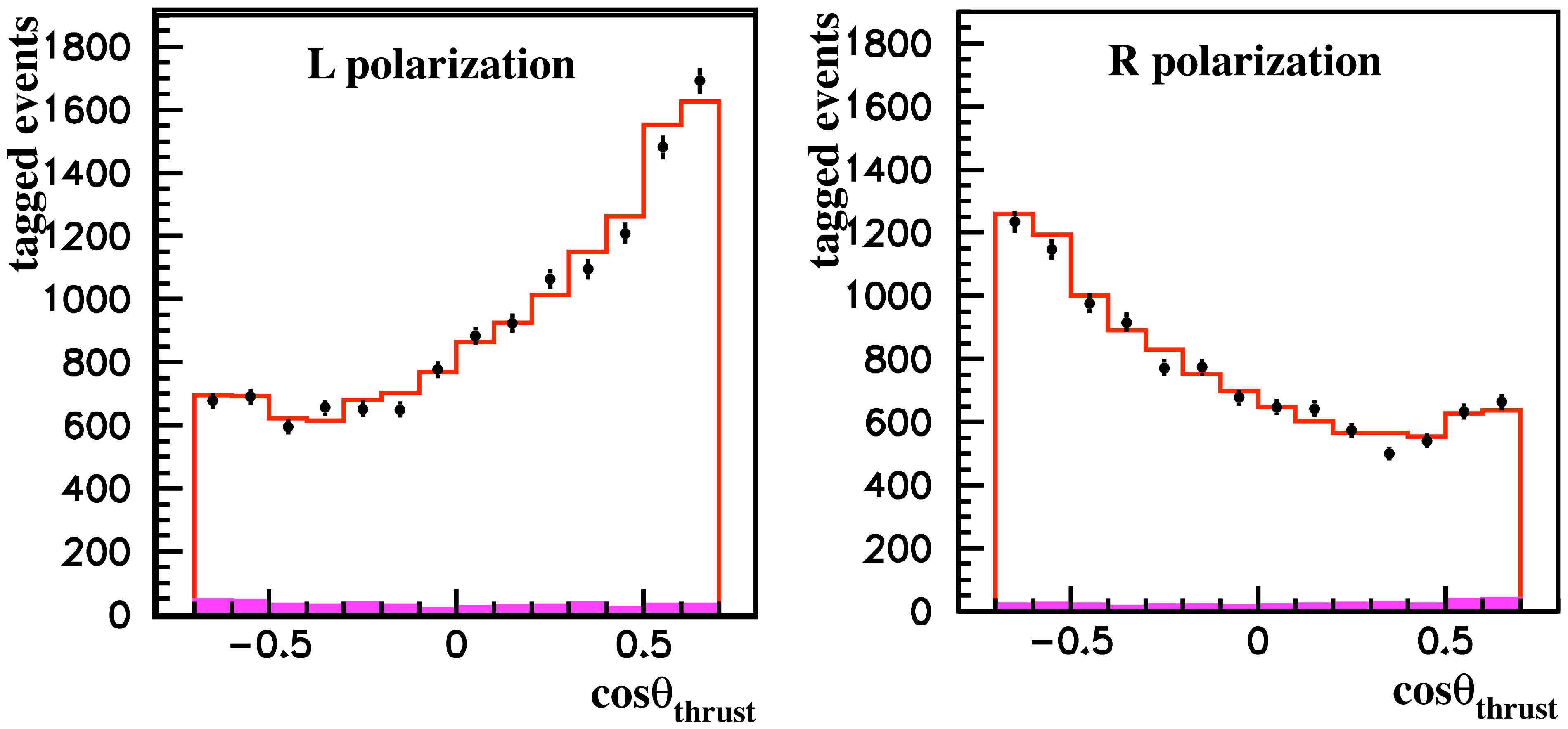


With polarized electron beams, the asymmetries A_b, A_c can be measured from the formulae

$$\frac{d\sigma}{d\cos\theta}(e^-_L e^+ \to b\bar{b}) \quad \sim \quad Q^2_{ZLb}(1+\cos\theta)^2 + Q^2_{ZRb}(1-\cos\theta)^2$$
$$\frac{d\sigma}{d\cos\theta}(e^-_R e^+ \to b\bar{b}) \quad \sim \quad Q^2_{ZLb}(1-\cos\theta)^2 + Q^2_{ZRb}(1+\cos\theta)^2$$

The SLD experiment made this measurement using the polarized beams available at the SLC.

Angular distribution of b jets using polarized electrons



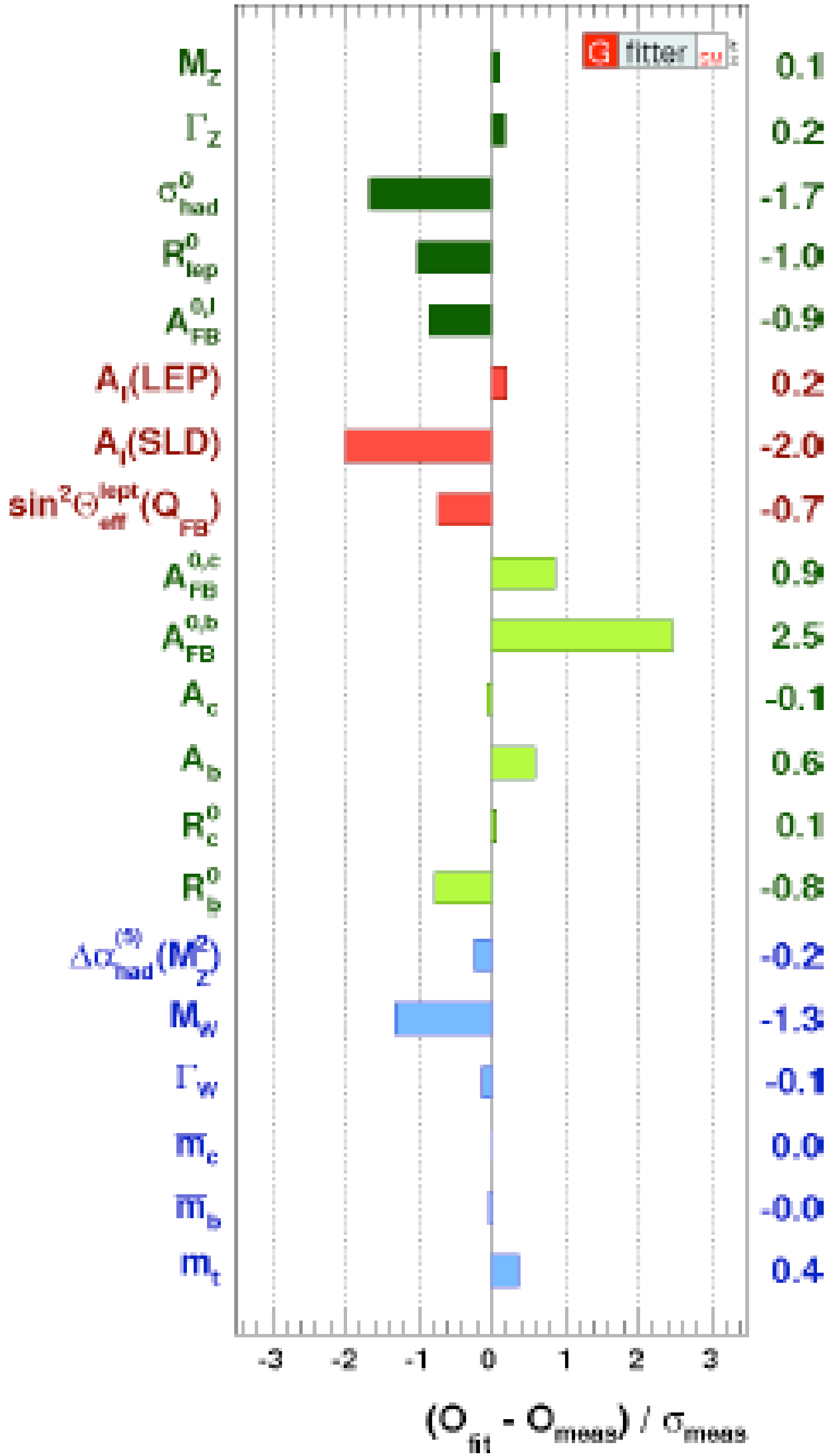
in agreement with Ab = 94% for b quarks SLD

We can interpret these results in several ways.

The most straightforward way is to compare the observables to a reference Standard Model, with the best-fit values of

$$g_1, g_2, v, \alpha_s, m_t, m_h$$

The result from the Gfitter group presented at ICHEP 2010 is shown on the next slide.



Standard Model fit pull distribution:

Gfitter group
ICHEP 2010

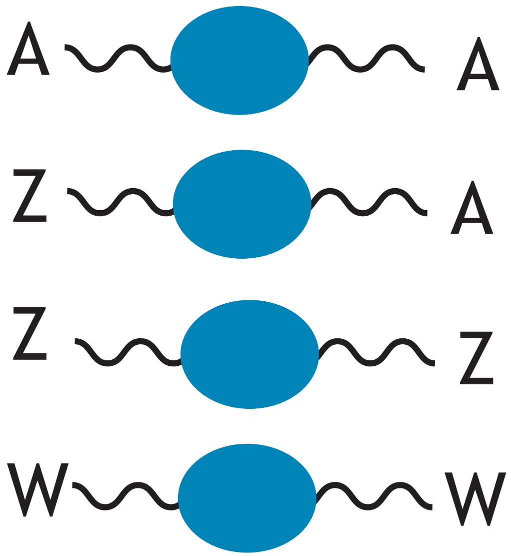
Another way to make this comparison is to consider the bounds on models of new physics. Takeuchi and I proposed a very general framework for doing this.

We assumed that new particles do not couple directly to light quarks and leptons. Then they affect the precision electroweak observables only through vacuum polarization corrections. These can be analyzed in a very general way. Corrections of this type are called oblique.

In most models of electroweak symmetry breaking, the dominant source of new physics corrections is through vacuum polarization diagrams. Also, the contributions to precision electroweak observables from the top quark and the Higgs boson are of this form.

I will now present this method of analysis.

To begin the analysis, define



$$= ie^2\Pi_{QQ}g^{\mu\nu}$$

$$= i\frac{e^2}{s_w c_w}(\Pi_{3Q} - s_w^2\Pi_{QQ})g^{\mu\nu}$$

$$= i\frac{e^2}{s_w^2 c_w^2}(\Pi_{33} - 2s_w^2\Pi_{3Q} + s_w^2\Pi_{QQ})g^{\mu\nu}$$

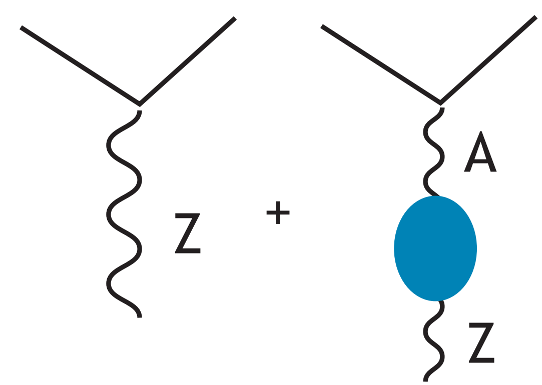
$$= i\frac{e^2}{s_w^2}\Pi_{11}g^{\mu\nu}$$

These amplitudes gives corrections to precision electroweak observables, for example

W mass $$m_W^2 = \frac{e^2v^2}{4s_w^2} + \frac{e^2}{s_W^2}\Pi_{11}(m_W^2)$$

value of s_w^2 obtained from Z asymmetries:

$$s_*^2 = s_w^2 - e^2(\Pi_{3Q} - s_w^2\Pi_{QQ})/m_Z^2$$



These formulae are not yet adequate, because the vacuum polarization amplitudes on the previous slide are UV divergent and need renormalization. A convenient way to do this is to define the parameters used there

$$e^2, \; s_w^2, \; v^2$$

by reference to the three accurately known electroweak observables. Then, for example, we would compute the differences

$$s_*^2 - s_0^2 \; , \qquad m_W/m_Z - c_0$$

which must be finite, by the renormalizability of the electroweak theory. To do this, we must take into account that the observables α, G_F, m_Z are also shifted by vacuum polarization corrections, for example,

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8m_W^2}\left(1 + \frac{e^2}{s_w^2 m_W^2}\Pi_{11}(0)\right)$$

At the end, we find a simple formalism. The shifts depend on two combinations of vacuum polarization amplitudes

$$S = \frac{16\pi}{m_Z^2}(\Pi_{33}(m_Z^2) - \Pi_{33}(0) - \Pi_{3Q}(m_Z^2))$$

$$T = \frac{4\pi}{s_w^2 m_W^2}(\Pi_{11}(0) - \Pi_{33}(0))$$

(and a third, U, which is typically very small).

In terms of these quantities, the shifts take simple forms.

$$\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left(-\frac{1}{2}S + c^2 T\right)$$

$$s_*^2 - s_0^2 = \frac{\alpha}{c^2 - s^2}\left(\frac{1}{4}S - s^2 c^2 T\right)$$

The heavy particles of the Standard Model, top and Higgs, fit into the framework. We find contributions to S and T.

top: $$S = \frac{1}{6\pi} \log \frac{m_t^2}{m_Z^2} \qquad T = \frac{3}{16\pi s^2 c^2} \frac{m_t^2}{m_Z^2}$$

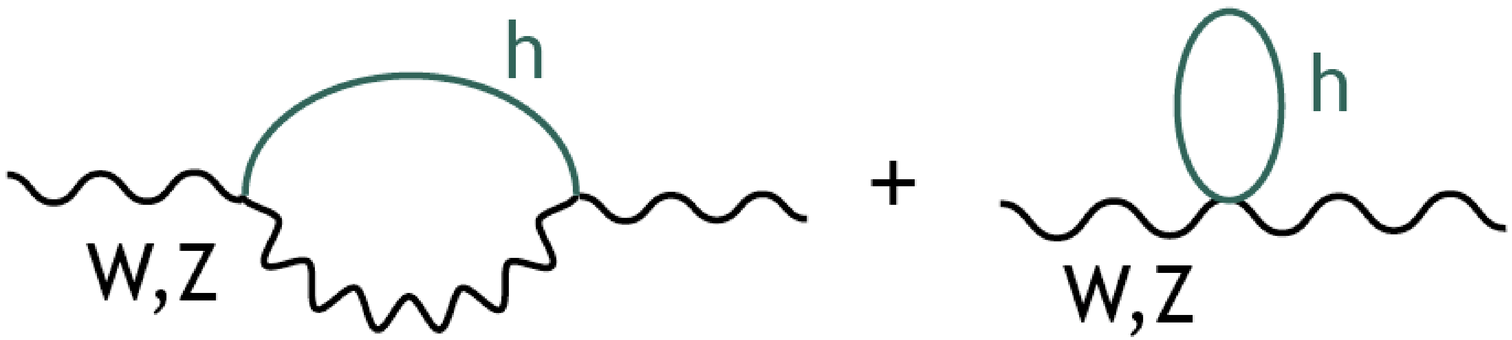
Higgs: $$S = \frac{1}{12\pi} \log \frac{m_h^2}{m_Z^2} \qquad T = -\frac{3}{16\pi c^2} \log \frac{m_h^2}{m_Z^2}$$

A new doublet of heavy fermions gives $S = \dfrac{1}{6\pi}$

and a (potentially enormous) contribution to T proportional to

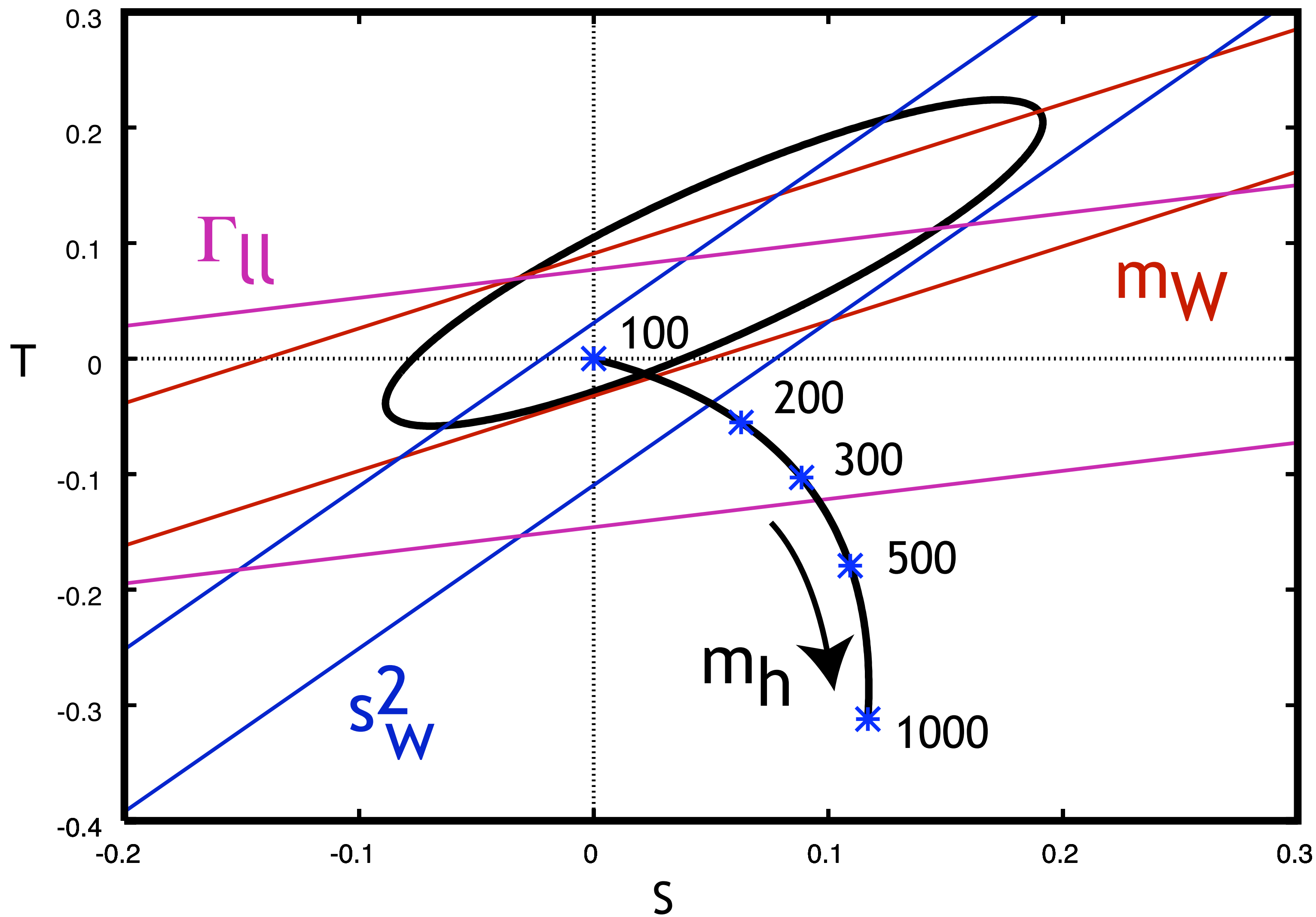
$$\frac{|m_U^2 - m_D^2|}{m_Z^2}$$

The contributions from the Higgs boson come from the diagrams

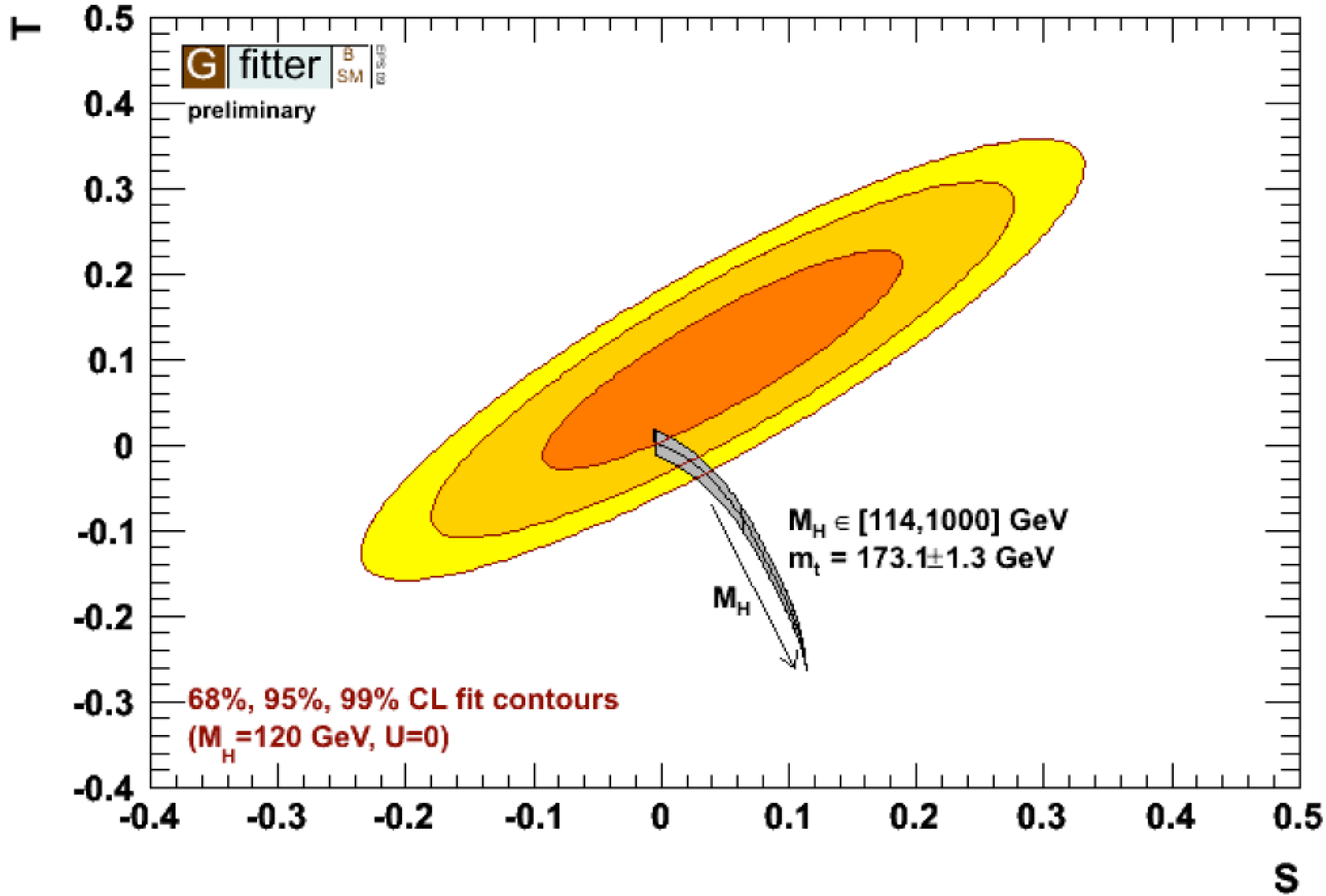


which are part of the cancellation of UV divergences in the W and Z vacuum polarizations. We will see later that the vertices are those by which the Higgs gives mass to the W and Z bosons. If you do not believe in the Higgs boson, you need to explain how to treat these terms. In particular, sending $m_h \longrightarrow \infty$ gives large corrections to the electroweak observables.

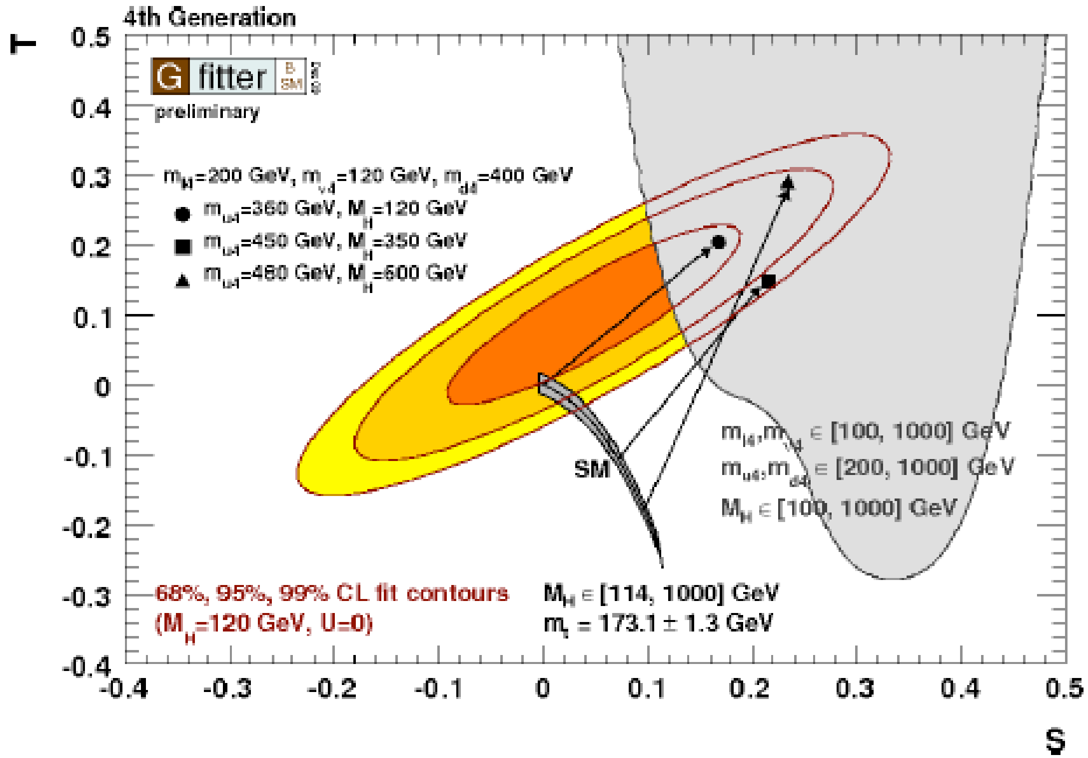
Here is a sample determination of S,T from three additional precision measurements. The Standard Model line has a fixed $m_t = 171\ \mathrm{GeV}$ and varying m_h .

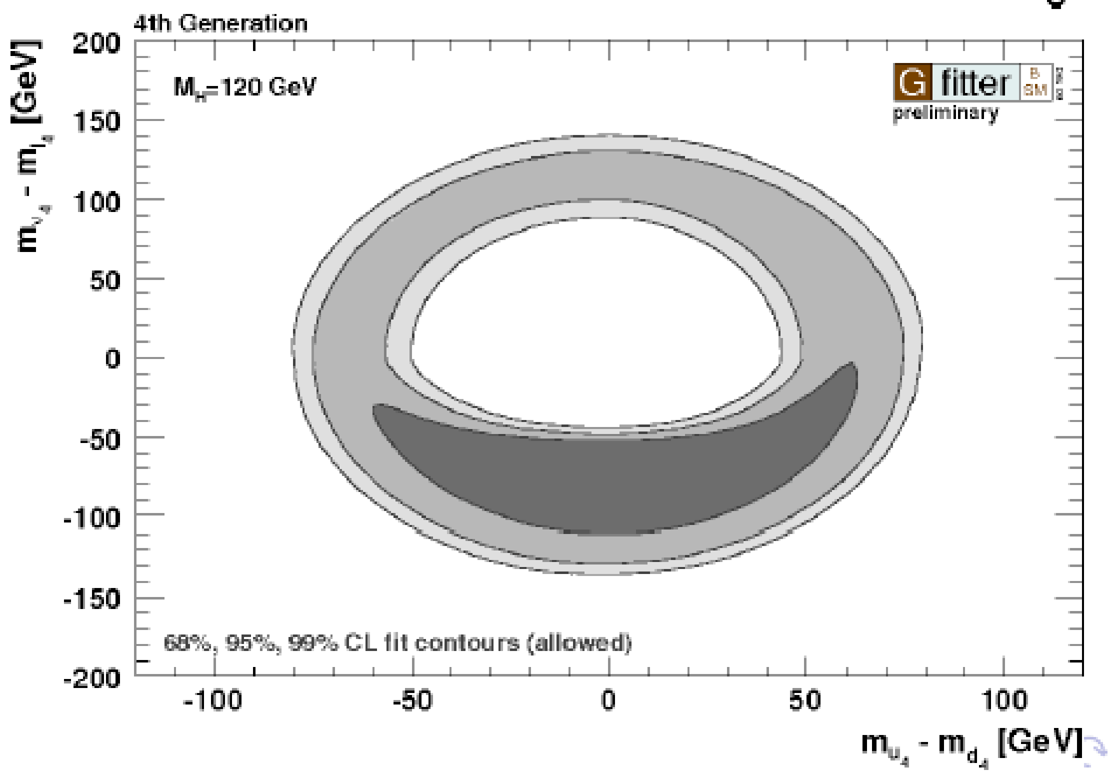


LEP EWWG: within the MSM $m_h < 144\ (182)\ \mathrm{GeV} \quad (95\%\ \mathrm{CL})$



Here is the allowed (S,T) region from Gfitter / ICHEP 2010



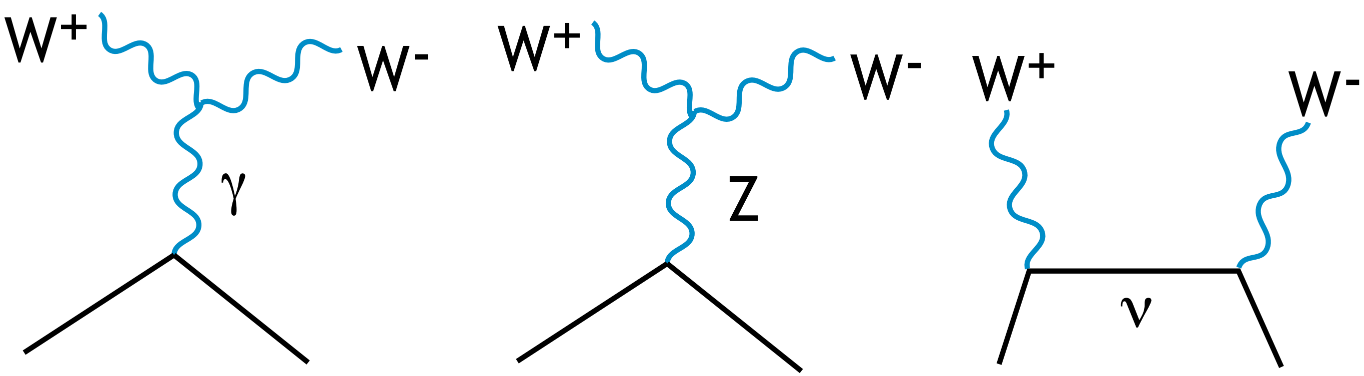


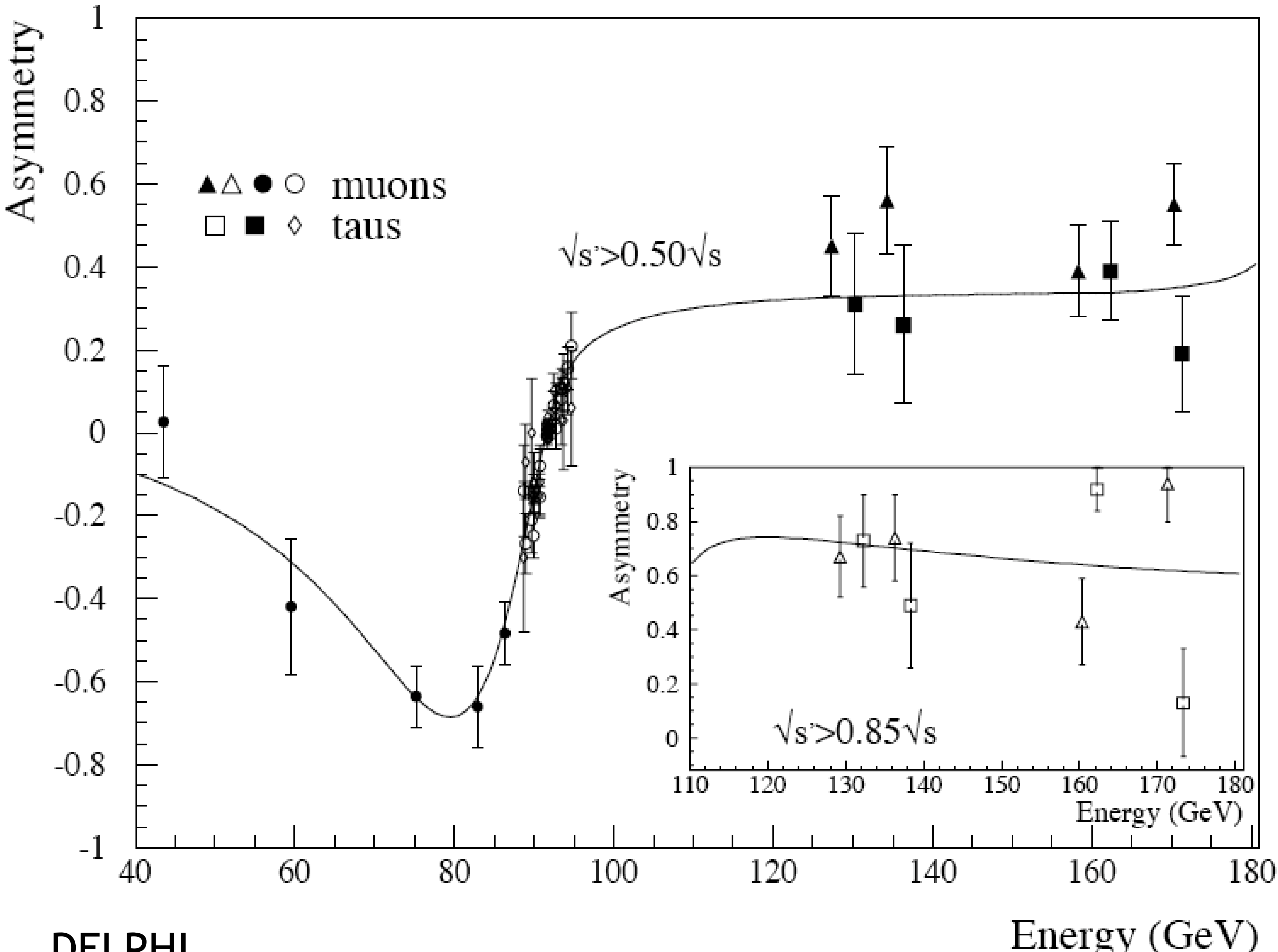
Here is an analysis of a 4th generation model

shown by the Gfitter group at ICHEP 2010

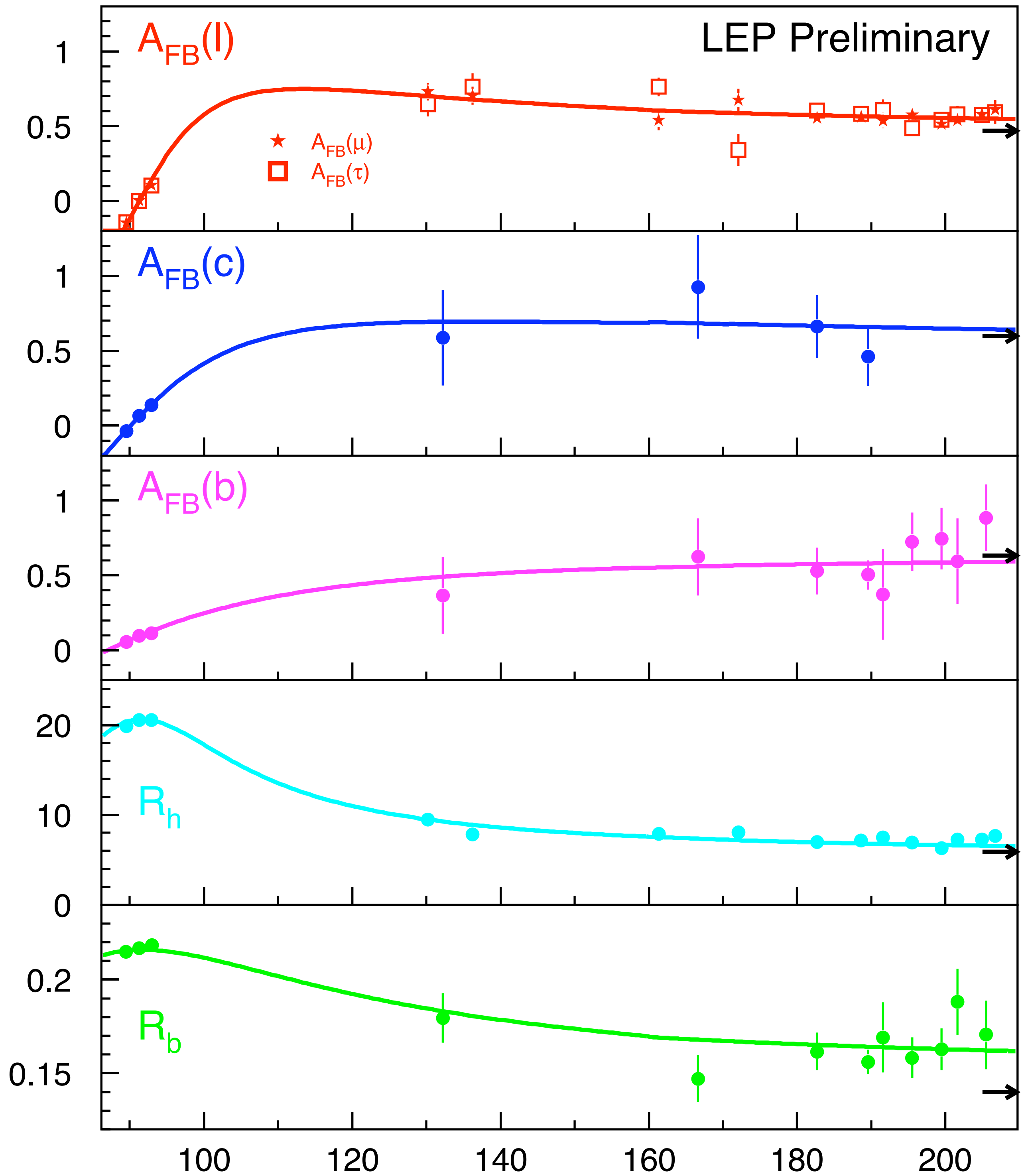
Further evidence for the SU(2)XU(1) gauge structure is given by measurements at LEP2 of $e^+e^- \longrightarrow f\overline{f}$ and $e^+e^- \longrightarrow W^+W^-$.

The latter process involves the nonlinear 3-gauge boson coupling and tests that this interaction of the Yang-Mills form.

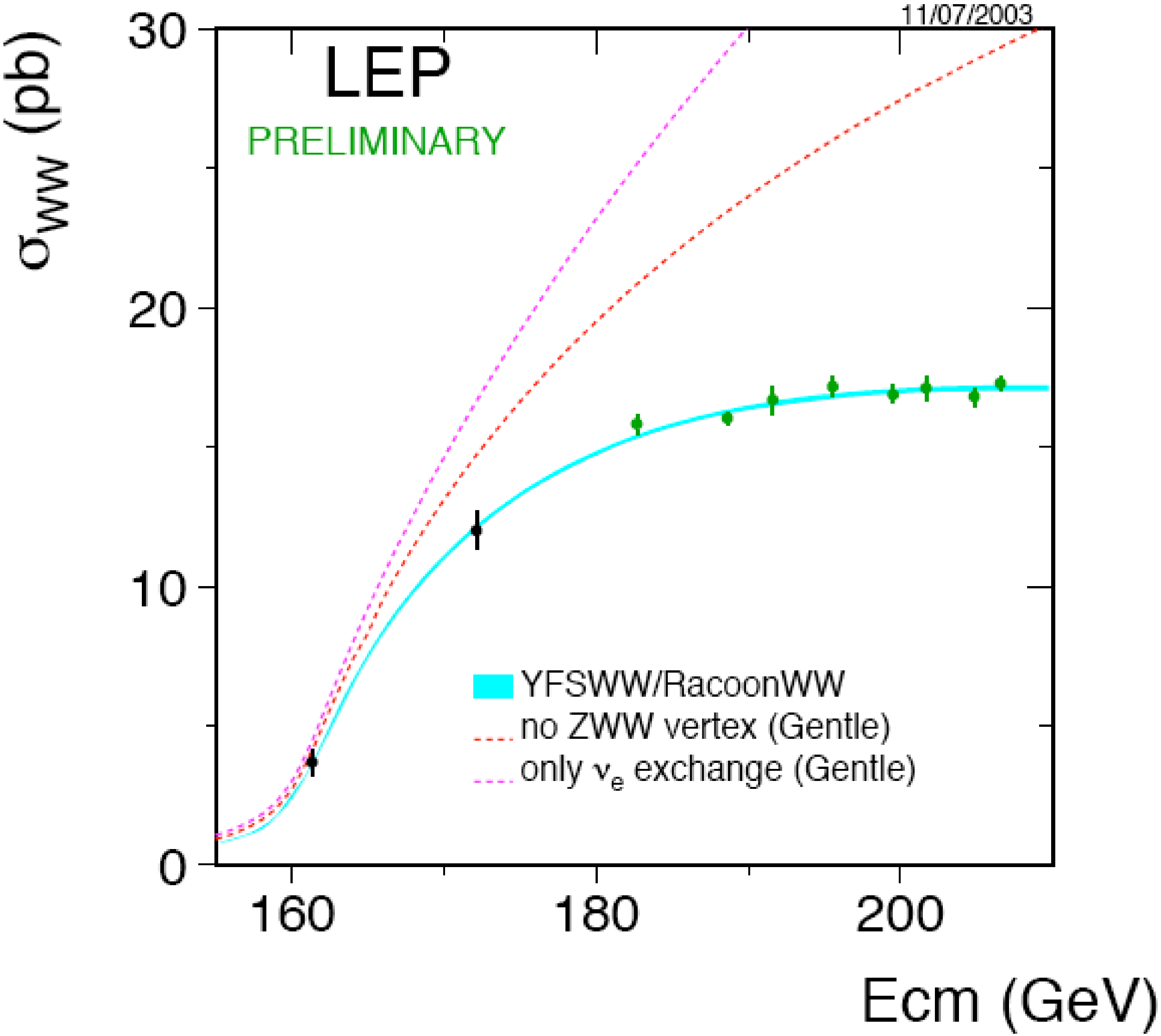



Asymmetry
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
muons
taus
√s'>0.50√s
40
60
80
100
120
140
160
180
Energy (GeV)
Asymmetry
√s'>0.85√s
110
120
130
140
150
160
170
180
Energy (GeV)

DELPHI

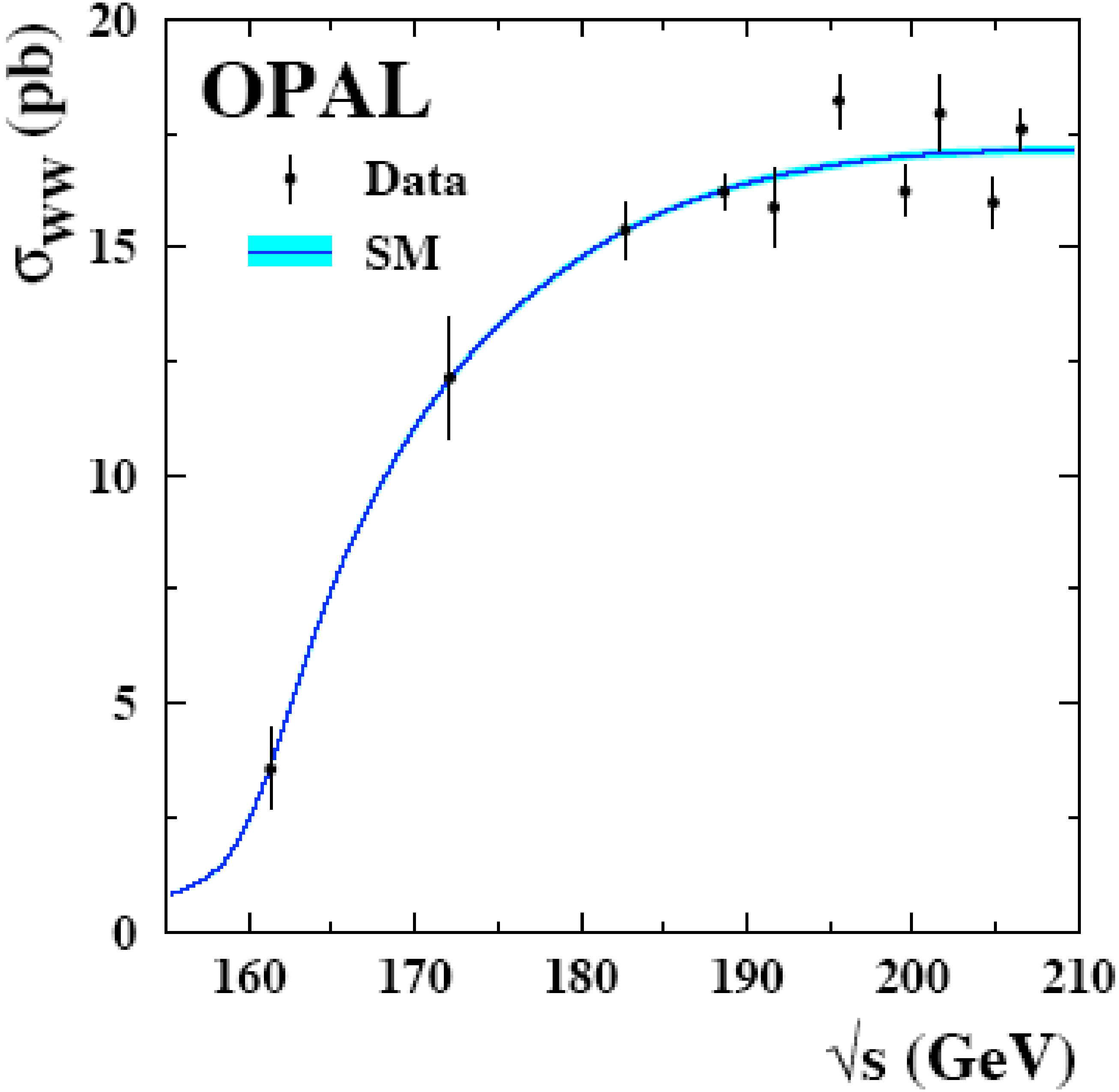


data compilation
by Hildreth



11/07/2003
LEP
PRELIMINARY
σ_{WW} (pb)
30
20
10
0
160
180
200
Ecm (GeV)
YFSWW/RacoonWW
no ZWW vertex (Gentle)
only ν_e exchange (Gentle)

OPAL 2007



OPAL
Data
SM
σ_{WW} (pb)
$\sqrt{s}$ (GeV)
0
5
10
15
20
160
170
180
190
200
210

Now we are ready to discuss mass generation in the Standard Model. First, we are ready to appreciate that this is a major problem:

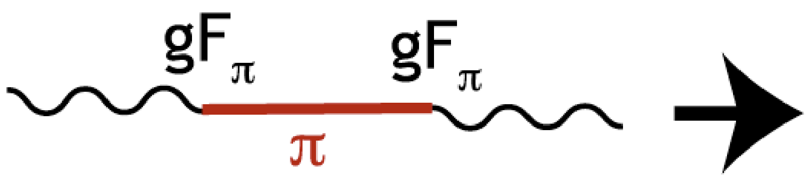
The vector bosons obey the Yang-Mills equations, essentially, generalized Maxwell equations. These equations forbid any masses for the W and Z bosons.

The fermions coupled to W and Z in a way that displays different quantum numbers for the L and R species. We cannot mix these states by a mass term without violating the SU(2)XU(1) symmetry.

Adding explicit SU(2)XU(1) symmetry breaking will ruin the model, making it nonrenomalizable and spoiling the sharp and correct predictions.

The only option is to add fields that break the SU(2)XU(1) symmetry spontaneously. That is, the Lagrangian is SU(2)XU(1) invariant, but the ground state is not.

The coupling of W and Z to these fields will cause these bosons to become massive. By a general argument given in my textbook, the coupling of a gauge boson to a Goldstone boson resulting from spontaneously broken symmetry leads to a mass for the gauge boson. This method of mass generation is called the Higgs mechanism.   This idea does not yet require or make reference to a Higgs particle.



$$\Pi^{\mu\nu}(q) = g^2 F_\pi^2 (g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2})$$

Nevertheless, the Higgs particle gives the simplest explicit realization of a symmetry-breaking mechanism.

Begin with the symmetry-breaking sector alone. Introduce a scalar field

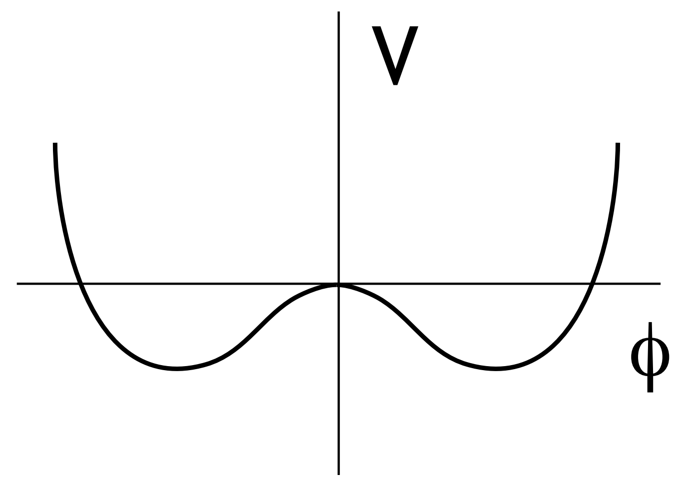
$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad (\frac{1}{2}, \frac{1}{2}, 1)$$

Let the Lagrangian be

$$\mathcal{L} = D^\mu \varphi^\dagger D_\mu \varphi - V(\varphi) \qquad V = -\mu^2 |\varphi|^2 + \lambda (|\varphi|^2)^2$$

Then a minimum of the potential is

$$\varphi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad v = \mu / \sqrt{\lambda}$$



A general parametrization of $\varphi(x)$ is

$$\varphi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi^1(x) + i\pi^2(x) \\ v + h(x) + i\pi^3(x) \end{pmatrix}$$

But $\pi^1(x), \pi^2(x), \pi^3(x)$ can be removed by SU(2) gauge transformations. The field $h(x)$ cannot be removed; this is a physical scalar field.

I claimed that any broken symmetry state would give masses to vector bosons. These masses are found in

$$
\begin{aligned}
D^\mu \varphi^\dagger D_\mu \varphi &= -\varphi^\dagger(-iA^a\frac{\sigma^a}{2} - iBY)^2\varphi \\
&= \frac{1}{2}v^2 \begin{pmatrix}0 & 1\end{pmatrix} (A^a\frac{\sigma^a}{2} + \frac{1}{2}B)^2 \begin{pmatrix}0 \\ 1\end{pmatrix} \\
&= \frac{1}{2}\frac{v^2}{4}\left[(A^1)^2 + (A^2)^2 + (-A^3 + B)^2\right]
\end{aligned}
$$

It is quite nontrivial that we find the exact form that I claimed yesterday was needed to give the Standard Model mass structure.

Quark and lepton masses come from new terms that we must add to the Lagrangian

$$\mathcal{L} = y_e \varphi^\dagger L \overline{e} + y_d \varphi^\dagger Q \overline{d} + y_u \epsilon_{ab} \varphi_a Q_b \overline{u} + h.c.$$

You can check that each term is SU(2) invariant and has total Y = 0. Without having the object φ available, it would not be possible to write symmetric terms coupling the L and R fermions. Substituting the ground state value of φ, we find quark and lepton mass terms with masses

$$m_f = \frac{y_f v}{\sqrt{2}}$$

The theory of a simple Higgs boson has one more nontrivial feature with respect to fermion mass generation. In a theory with 3 generations, the most general fermion coupling to Higgs is more complicated:

$$\mathcal{L} = Y_e^{ij}\varphi^\dagger L^i \overline{e}^j + Y_d^{ij}\varphi^\dagger Q^i \overline{d}^j + Y_u^{ij}\epsilon_{ab}\varphi_a Q_b^i \overline{u}^j + h.c.$$

However, it is possible to diagonalize the coupling matrices

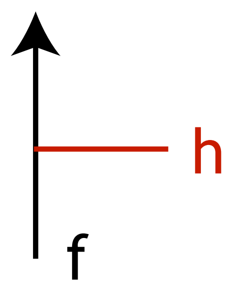
$$Y_e = U_e^\dagger y_e V_e \quad Y_d = U_d^\dagger y_d V_d \quad Y_u = U_u^\dagger y_u V_u$$

and absorb the tranformations U,V into redefinitions of the quark and lepton fields. Then these factors cancel and disappear from the Standard Model Lagrangian except that the W couplings aquire a generation mixing: $V_{CKM} = U_u U_d^\dagger$.

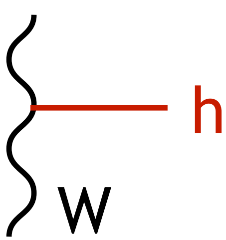
Among alternative theories of electroweak symmetry breaking, only a few have such a simple cancellation of dangerous flavor-changing terms.

If we assume that electroweak symmetry breaking is due to a single Higgs field, we expect to see one additional particle, the scalar h^0. The mass of the h^0 is not determined by any current observation. However, given that mass, the couplings of the h^0 are fixed, and we can compute its cross sections and branching ratios.

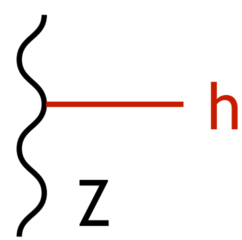
We can find the Feynman rules for the h^0 by noting that this field appears in the combination $(v + h^0)$. Then from the Higgs-induced mass terms, we find



$$= -i\frac{m_f}{v}$$



$$= 2i\frac{m_W^2}{v}g^{\mu\nu}$$



$$= 2i\frac{m_Z^2}{v}g^{\mu\nu}$$

Using these Feynman rules, it is straightforward to work out the partial widths of the Higgs boson.   These calculations have some theoretically interesting features (which we can discuss offline).

For the Higgs decays to fermions

$$\Gamma(h^0 \to f\overline{f}) = \frac{1}{2m_h}\frac{1}{8\pi} \cdot 2m_h^2 \frac{m_f^2}{v^2} = \frac{\alpha_w m_h}{8}\frac{m_f^2}{m_W^2}$$

times the color factor 3 for quarks.

For the Higgs decays to vector bosons

$$\Gamma(h^0 \to W^+W^-) = \frac{1}{2m_h}\frac{1}{8\pi}\frac{4m_W^2}{v^2}(g^{\mu\nu} - \frac{q_+^\mu q_+^\nu}{m_W^2})(g_{\mu\nu} - \frac{q_{-\mu}q_{-\nu}}{m_W^2})$$

This works out to

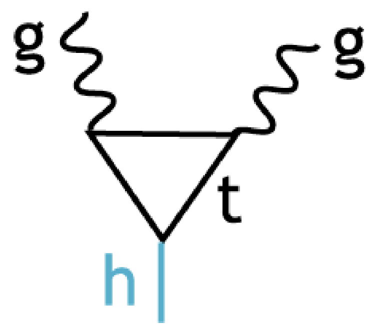
$$\Gamma(h^0 \to W^+W^-) = \frac{\alpha_w}{16}\frac{m_h^3}{m_W^2}(1 - 4\frac{m_W^2}{m_h^2} + 12\frac{m_W^4}{m_h^4})(1 - 4\frac{m_W^2}{m_h^2})^{1/2}$$

$$\Gamma(h^0 \to Z^0Z^0) = \frac{\alpha_w}{32}\frac{m_h^3}{m_W^2}(1 - 4\frac{m_Z^2}{m_h^2} + 12\frac{m_Z^4}{m_h^4})(1 - 4\frac{m_Z^2}{m_h^2})^{1/2}$$

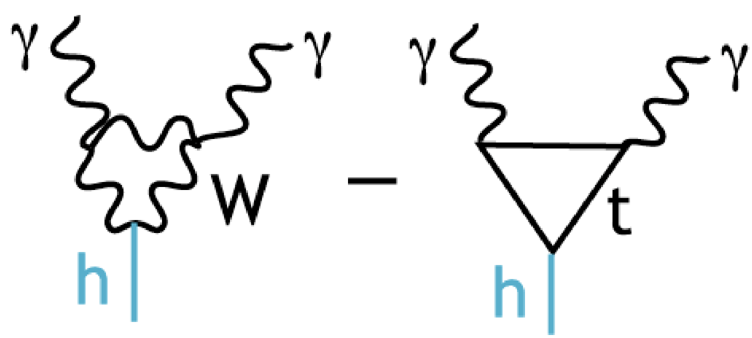
The dependence m_h^3/m_W^2 is strange but correct and causes these modes to dominate all others when kinematically allowed.

Indeed, even below the threshold for $h^0 \to W^+W^-$, decays to one W or Z on-shell and one far off-shell can compete with the dominant on-shell decay $h^0 \to b\bar{b}$.

Two addition sets of decays arise only a the 1-loop level but can still be relevant for a light Higgs boson. I give the formulae in the limit $m_h \ll m_W$



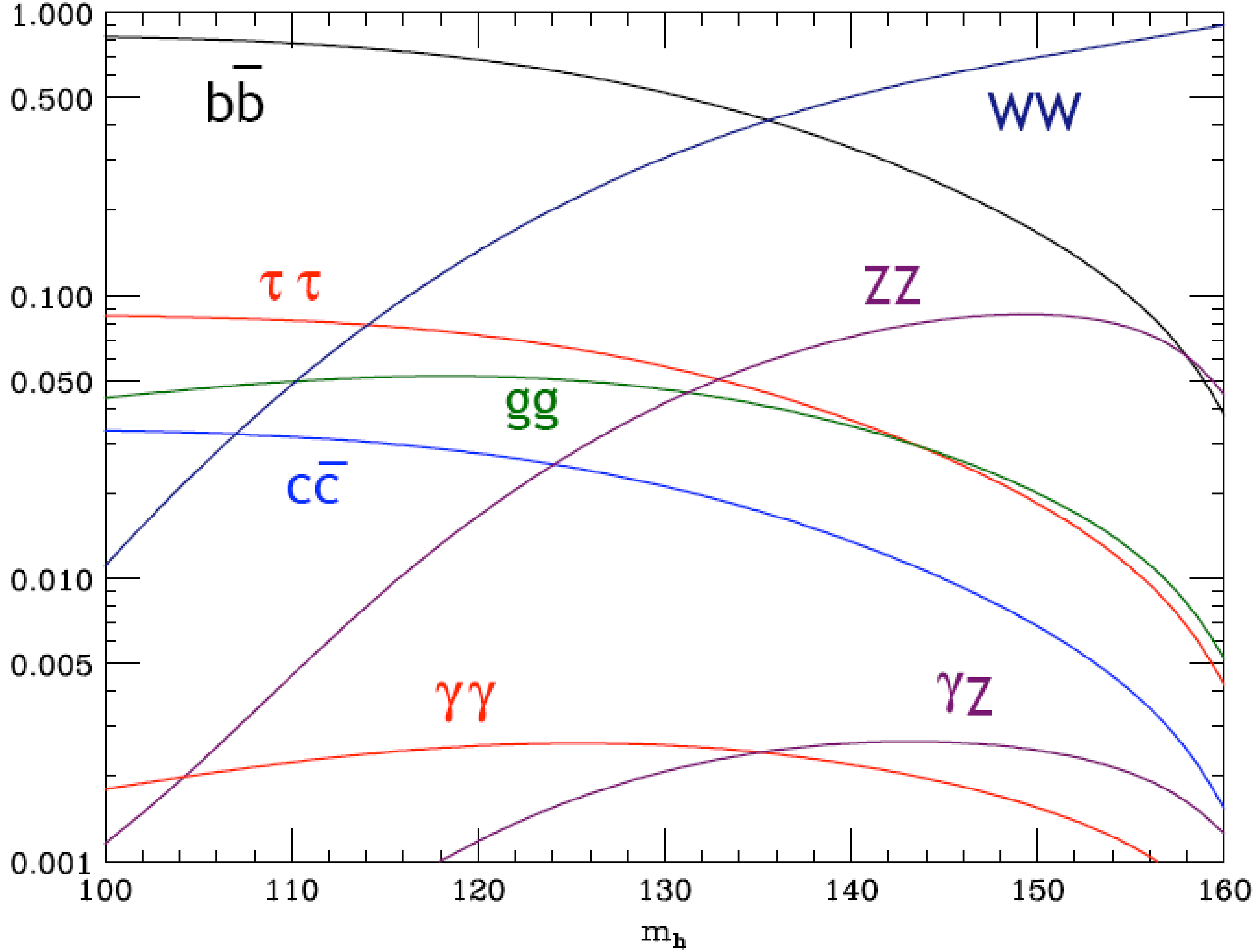
$$\Gamma(h^0 \to gg) = \frac{\alpha_w \alpha_s^2}{288\pi^2} \frac{m_h^3}{m_W^2}$$



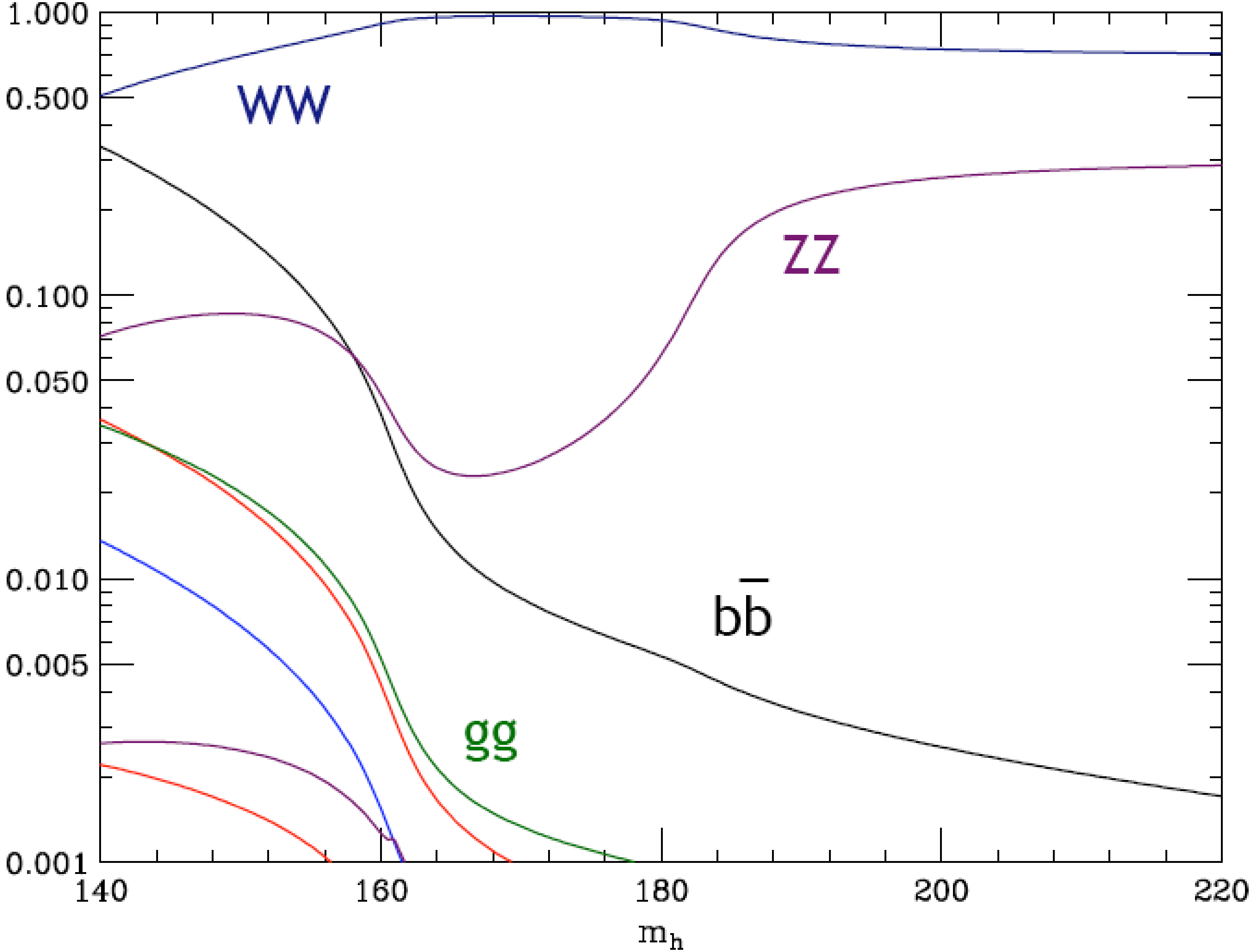
$$\Gamma(h^0 \to \gamma\gamma) = \frac{\alpha_w \alpha^2}{576\pi^2} \frac{m_h^3}{m_W^2} \left| \frac{21}{4} - \frac{4}{3} \right|^2$$

Putting all of the pieces together:

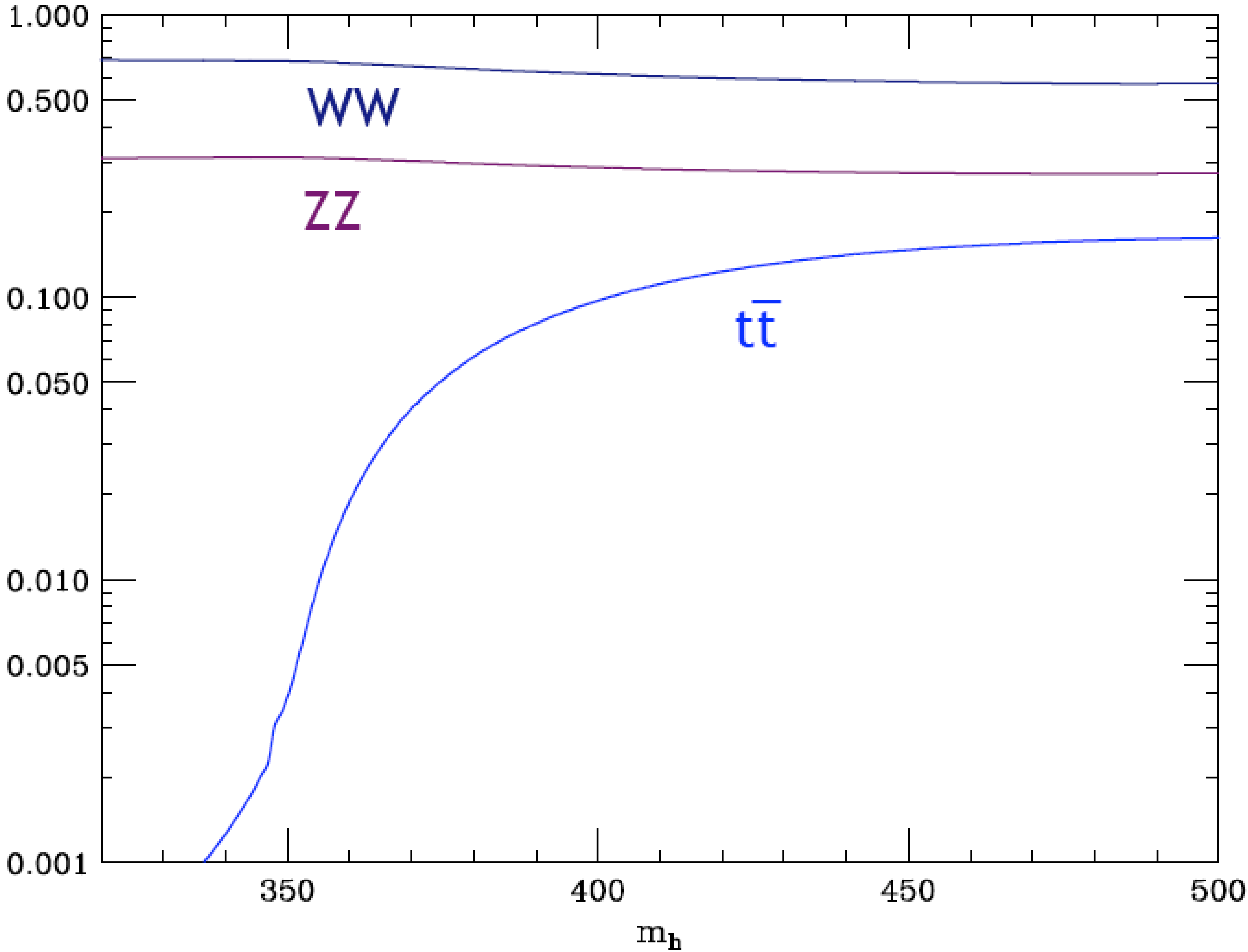
h branching ratios -- light Higgs



h branching ratios -- Higgs near W threshold



h branching ratios -- heavy Higgs



Through the formula

$$\sigma(A \to h^0 \to B) \sim \Gamma(h^0 \to A) \cdot BR(h^0 \to B)$$

these results for the partial widths also provide information about the Higgs production cross sections at the Tevatron and the LHC.

For example, the amplitude for $h^0 \to gg$ induces a production channel at hadron colliders $gg \to h^0$. However, I will leave that discussion to the other lecturers.