Theoretical Introduction to LHC Physics

1. Bosons and Partons

M. E. Peskin, SLAC
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Lecture 1: Bosons and Partons

Ingredients of the Standard Model
Properties of the W and Z
Parton Model
the Drell-Yan Process

Lecture 2. Higgs Boson

Precision tests of the electroweak model
Oblique corrections and constraints
Properties of the SM Higgs boson

Lecture 3. Supersymmetry

Models of electroweak symmetry breaking
Principles of supersymmetry
Supersymmetry spectroscopy
Supersymmetry and electroweak symmetry breaking

The goal of this school is to prepare you to think about physics results from the LHC.

At the LHC, we hope to discover laws of physics that are now unknown.

However, any new physics must stand on the foundation of what is already understood. In these lectures, I will present some essential features of the ‘Standard Model’ of elementary particle physics.

The Standard Model is a theory of vector bosons, fermions, and one scalar boson that gives an internally self-consistent model of strong, weak, and electromagnetic interactions.   The model is well tested and, plausibly, explains all aspects of elementary particle behavior seen in accelerator experiments.

The predictions of the model divide into two classes. First, there are predictions that are derived in weak-coupling perturbation theory. Second, there are predictions of the theory that require analysis of strongly coupled quantum field theory. For LHC physics, we need to understand both aspects. We collide protons, strong interaction bound states, but the reaction we are most interested in involve weak interactions of the proton components.

In this lecture, I will give a general introduction to both aspects of the model.

We first discuss weakly coupled theories of vector bosons and fermions. The most general renormalizable Lagrangian for such a system is very simple. It is a Yang-Mills theory.

Let G be a simple Lie group with structure constants f^{abc}

$$[t^a, t^b] = i f^{abc} t^c$$

Then the unique renormalizable Lagrangian is

$$\mathcal{L} = -\frac{1}{4g^2}(F^a_{\mu\nu})^2$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu$$

The field strength $F^a_{\mu\nu}$ is associated with a covariant derivative

$$D_\mu = \partial_\mu - i A^a_\mu t^a$$

by the relation

$$[D_\mu, D_\nu] = -i F^a_{\mu\nu} t^a$$

This theory has G as a local ('gauge') symmetry, a property essential for forming a consistent renormalizable quantum theory.

Fermion and boson matter fields couple to the vector fields by using the covariant derivative in their Lagrangian. For example, for a massless fermion in the representation R of G

$$\mathcal{L} = \psi^\dagger (i\overline{\sigma}^\mu D_\mu)\psi$$

with

$$\overline{\sigma}^\mu = (1, -\vec{\sigma})^\mu \qquad D_\mu = \partial_\mu - iA_\mu^a t_R^a$$

I have written the fermion as a 2-component fermion field. That is the minimal spin-1/2 representation of the Lorentz algebra.

We can also add mass terms for the fermions. In the Standard Model, it turns out, all fermion mass terms are forbidden by symmetry, unless we add a scalar or some other ingredient to the model. In this lecture, I will say the very minimum about masses, reserving that discussion for later.

That is the whole structure.

We can have several commuting local symmetry groups and several representations of fermions. Then the most general Lagrangian is

$$\mathcal{L} = -\frac{1}{4}\sum_i \frac{1}{g_i^2}(F^a_{i\mu\nu})^2 + \sum_j \psi_j^\dagger (i\overline{\sigma}\cdot D_\mu)\psi_j$$

We need to pick a set of gauge groups G representations R, and coupling constants g that describe the real world. Then all of the consequences of the Standard Model must come out of this formula (and assumptions about how masses are generated).

The correct choices are that G should be a product of three groups

$$G = U(1) \times SU(2) \times SU(3)$$

describing the electroweak and strong interactions, and that the fermions should give three copies of the representations

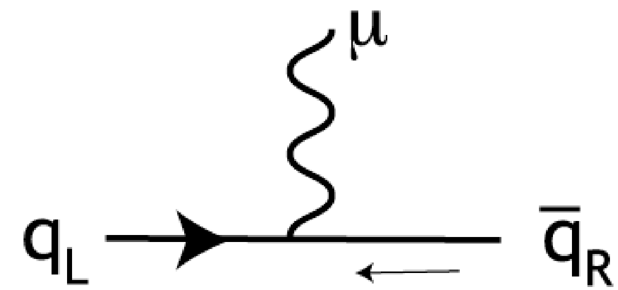
$$\begin{array}{ll}
L = \begin{pmatrix} \nu \\ e \end{pmatrix} & (-\frac{1}{2}, \frac{1}{2}, 1) \\
\overline{e} & (+1, 0, 1) \\
Q = \begin{pmatrix} u \\ d \end{pmatrix} & (+\frac{1}{6}, \frac{1}{2}, 3) \\
\overline{u} & (-\frac{2}{3}, 0, \overline{3}) \\
\overline{d} & (+\frac{1}{3}, 0, \overline{3})
\end{array}$$

indicated by the charge under the U(1), the spin under the SU(2), and the representation under the SU(3).

The fermions shown are left-handed; their antiparticles, with the opposite quantum numbers, are right-handed.

Here is a simple example of the dynamics in the gauge theory Lagrangian: the coupling of a massive vector boson to a massless fermion and antifermion:

$$i\mathcal{M} = iG\ \epsilon^*_\mu(W)\ u^\dagger(q^\dagger)\overline{\sigma}^\mu u(q)$$



Then, with the spinors

$$u^\dagger(q^\dagger) = \sqrt{2E}\begin{pmatrix}-1\\0\end{pmatrix} \qquad u(q) = \sqrt{2E}\begin{pmatrix}0\\1\end{pmatrix}$$

We find

$$i\mathcal{M} = iG\, m_W\ \epsilon^*_\mu(W)\ (0,1,-i,0)^\mu$$

The vector is proportional to the $J=1,\ J^3=-1$ polarization vector, as should be expected.

Then, for example, the process $d\overline{u} \rightarrow W^- \rightarrow \ell^- \overline{\nu}$ yields a W boson with $J^3 = -1$.

A complete process $d\overline{u} \rightarrow \ell^- \overline{\nu}$ has an angular distribution

$$\begin{aligned} \frac{d\sigma}{d\cos\theta}(d\overline{u} \rightarrow \ell^- \overline{\nu}) &\sim |(0,1,-i,0)\cdot(0,\cos\theta,i,-\sin\theta)|^2 \\ &\sim |1+\cos\theta|^2 \end{aligned}$$

Similarly,

$$\frac{d\sigma}{d\cos\theta}(u\overline{d} \rightarrow \ell^+ \nu) \sim |1-\cos\theta|^2$$

Compare the QED formula for $e^+e^- \rightarrow \mu^+\mu^-$

$$\begin{aligned} \frac{d\sigma}{d\cos\theta} &= \frac{2}{4}\cdot\frac{\pi\alpha^2}{2s}[(1+\cos\theta)^2 + (1-\cos\theta)^2] \\ &= \frac{\pi\alpha^2}{2s}(1+\cos^2\theta) \\ &= \frac{\pi\alpha^2}{s}[\frac{u^2}{s^2} + \frac{t^2}{s^2}] \end{aligned}$$

We can now compute the width and partial widths of the W boson. To be a little more careful about the normalization, write

$$W^{\pm} = \frac{1}{\sqrt{2} g_2}(A^1 \mp i A^2) \qquad\qquad \sigma^{\pm} = \frac{1}{2}(\sigma^1 \pm i\sigma^2)$$

The kinetic term of SU(2) bosons written earlier is normalized to

$$\mathcal{L} = -\frac{1}{2g_2^2}(A^1 \Box A^1 + A^2 \Box A^2)$$

so with the definition of the W field above

$$\mathcal{L} = -W^+ \Box W^-$$

Then the W vertex becomes

$$\psi^\dagger \overline{\sigma}^\mu A^a_\mu (\frac{\sigma}{2})^a \psi = \frac{g_2}{\sqrt{2}} \psi^\dagger \overline{\sigma}^\mu \left[W^+_\mu \sigma^+ + W^-_\mu \sigma^- \right] \psi$$

or

$$i\mathcal{M}(u_L \overline{d}_R \to W^+_L) = i \frac{g_2}{\sqrt{2}} m_W \cdot \sqrt{2}$$

Now we can work out all of the properties of the W boson.

We need the experimental values:

$$m_W = 80.4 \text{ GeV} \qquad \frac{g_2}{4\pi} = \alpha_w = 1/29.6$$

Then

$$\Gamma(W^+ \to \ell^+ \nu) = \frac{1}{3} \frac{1}{2m_W} \frac{1}{8\pi} \left| g_2 m_W \right|^2 = \frac{\alpha_w}{12} m_W = 226 \text{ MeV}$$

Similarly, including a factor 3 for color, and a QCD correction,

$$\Gamma(W^+ \to u\bar{d}) = (\alpha_w m_W/12) \cdot 3 \cdot (1 + \frac{\alpha_s(m_W)}{\pi}) = 702 \text{ MeV}$$

This gives

$$\Gamma_W = 3 \cdot 0.226 + 2 \cdot 0.702 = 2.08 \text{ GeV}$$

and the branching ratios:

$$BR(W^+ \to e^+ \nu) = 11\% \qquad BR(W^+ \to u\bar{d}) = 34\%$$

To discuss the Z boson and the weak neutral current, I need to say more about the vector boson mass matrix. I will start by postulating the following mass matrix of SU(2)XU(1) bosons:

$$m^2 \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ B \end{pmatrix} = \frac{v^2}{4} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & -1 \\ & & -1 & 1 \end{pmatrix} \begin{pmatrix} A^1 \\ A^2 \\ A^3 \\ B \end{pmatrix}$$

In the Lagrangian, this reads

$$\mathcal{L} = \frac{v^2}{8}[(A^1)^2 + (A^2)^2 + (A^3 - B)^2]$$

This mass matrix has the following properties:

It couples SU(2) (I^3) and U(1) (Y) gauge bosons.
It contains a zero eigenvector (which will give the photon).
The pure SU(2) part is isospin-symmetric (custodial symmetry).

The mass eigenstates are given by the W fields above and by rewriting

$$\frac{A^3_\mu}{g_2} = \frac{g_2 Z_\mu + g_1 A_\mu}{\sqrt{g_1^2 + g_2^2}} \qquad \frac{B_\mu}{g_1} = \frac{-g_1 Z_\mu + g_2 A_\mu}{\sqrt{g_1^2 + g_2^2}}$$

The following notation is useful:

$$\frac{g_1}{g_2} \equiv \tan\theta_w \qquad \frac{g_1^2}{g_1^2 + g_2^2} = s_w^2$$

Then

$$-\frac{1}{2g_2^2} A^3 \Box A^3 - \frac{1}{2g_1^2} B \Box B = -\frac{1}{2}(A \Box A + Z \Box Z)$$

and

$$\frac{v^2}{8}(A^3 - B)^2 = \frac{1}{2}\frac{v^2}{4}(g_1^2 + g_2^2) Z^2$$

The mass eigenstate vector bosons are thus:

$$\begin{aligned} A &: \qquad m^2 = 0 \\ W^\pm &: \qquad m^2 = g_2^2 v^2/4 \\ Z &: \qquad m^2 = (g_1^2 + g_2^2) v^2/4 \end{aligned}$$

From these formulae, $m_W^2/m_Z^2 = c_w^2 = 1 - s_w^2$

Experimentally, $m_W^2/m_Z^2 = 0.777 \quad s_w^2 = 0.23$ (not so bad)

The couplings of A and Z are derived from the basic form

$$A^3_\mu I^3 + B_\mu Y$$

by the substitutions on the previous slide

$$\begin{aligned} &= \frac{g_1 g_2 (I^3 + Y)}{\sqrt{g_1^2 + g_2^2}} A_\mu + \frac{g_2^2 I^3 - g_1^2 Y}{\sqrt{g_1^2 + g_2^2}} Z_\mu \\ &= eQA_\mu + \frac{e}{s_w c_w} Q_Z Z_\mu \end{aligned}$$

In these formulae, I identify $e^2 = \frac{g_1^2 g_2^2}{g_1^2 + g_2^2}$ $\quad g_1^2 + g_2^2 = \frac{e^2}{s_w^2 c_w^2}$

and $\quad Q = (I^3 + Y) \qquad Q_Z = (I^3 - s_w^2 Q)$

From these formulae, we can compute the properties of the Z:

For any fermion species

$$\Gamma(Z^0 \to f\overline{f}) = \frac{1}{3}\frac{1}{2m_Z}\frac{1}{8\pi}\left|\sqrt{2}m_Z\frac{g_2}{c_w^2}Q_Z\right|^2 = \frac{\alpha_w m_Z}{6c_w^2}Q_Z^2$$

The values of Q_Z are:

	ν	e	u	d	
$Q_Z^2 =$	0.250	0.073	0.120	0.179	L,Q
	-	0.053	0.024	0.006	$\overline{e},\overline{u},\overline{d}$
sum $=$	0.250	0.126	0.144	0.185	

Then we find for the total width:

$$\begin{aligned}\Gamma_Z &= (667\ \mathrm{MeV})\cdot[3\cdot 0.250 + 3\cdot(0.126) + (3.1)\cdot(3\cdot 0.185 + 2\cdot 0.144)] \\ &= 2.49\ \mathrm{GeV}\end{aligned}$$

and for the branching ratios:

	3ν	e	u	d
$BR(Z^0 \to f\overline{f})$	20.%	3.4%	11.9%	15.3%

As along as we only deal with leptons, we can easily compute the cross sections for W and Z production:

$$\begin{aligned}\sigma(e^+e^- \to Z^0) &= \frac{1}{2s}\int \frac{d^3p}{(2\pi)^3 2E_Z}(2\pi)^4\delta^{(4)}(p_1+p_2-p_Z)\ \frac{1}{4}\Big|\frac{\sqrt{2}g_2 m_Z}{c_w}\Big|^2(Q_{ZL}^2+Q_{ZR}^2)\\ &= \frac{1}{4}\frac{4\pi\alpha_w}{c_w^2}(Q_{ZL}^2+Q_{ZR}^2)\cdot 2\pi\delta(s-m_Z^2)\end{aligned}$$

or finally

$$\sigma(e^+e^- \to Z^0) = \frac{2\pi^2\alpha_w}{c_w^2}(Q_{ZL}^2+Q_{ZR}^2)\delta(s-m_Z^2)$$

To include a finite boson width

$$\delta(s-m_Z^2) \to \frac{m_Z\Gamma_Z/\pi}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}$$

However, at the LHC, we do collider hadrons, so we must include some results from the SU(3) part of the Standard Model, QCD.

Frank Petriello will give a comprehensive introduction to QCD in his lectures at this school. But I would like to introduce the most basic concepts now.

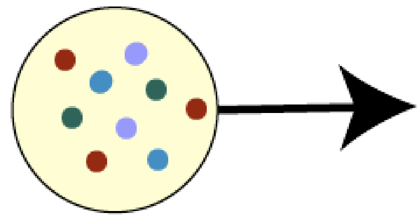
The coupling constant of QCD exhibits asymptotic freedom

$$\frac{g_3^2}{4\pi} \equiv \alpha_s(Q) = \frac{\alpha_s(Q_0)}{1 + (b_0/2\pi)\log Q/Q_0} = \begin{cases} 0.18 & \text{at } Q = 10 \text{ GeV} \\ 0.12 & \text{at } Q = 100 \text{ GeV} \end{cases}$$

At high Q, QCD is a weak interaction and perturbation theory applies.
at low Q, QCD is a strong interaction. We find quark confinement and hadronic bound states.

To compute the rates of reactions at the LHC, it would be good to deal as little as possible with the nonperturbative aspects of QCD. However, we are colliding protons, so we need to know something about the proton bound state. How do we encode that information ?

Consider a proton at high energy.



Each constituent has most of its momentum in the direction of the proton's momentum. In order for a constituent to have high momentum transverse to the proton direction, it must have exchanged large momentum with another constituent. This is a weak effect and can be added later using QCD perturbation theory.

So, model the proton as a collection of collinear massless constituents (partons). Let

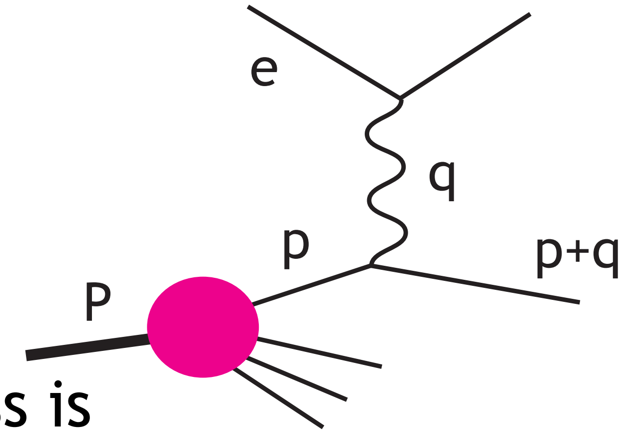
$$d\xi \ f_p(\xi)$$

be the probability of find the parton p (e.g. a u quark or a gluon) at the fraction ξ $(0 < \xi < 1)$ of the proton's momentum.

This is the parton model, as first introduced by Feynman.

The parton model had its first success in application to deep inelastic scattering: $e^- p \to e^- + X$ at energies $E_e \gg m_p$.

In the parton model, this is described as electron-quark scattering:



The cross section for the underlying process is

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{\hat{s}} Q_f^2 \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right)$$

obtained by crossing our earlier result for $e^+e^- \to \mu^+\mu^-$. The hats denote the invariants in the reaction of partons. Then the parton model cross section for ep scattering should then be

$$\sigma = \int d\xi \sum_f Q_f^2 f_f(\xi) \int d\cos\theta \; \frac{\pi\alpha^2}{\hat{s}} \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$

In the deep inelastic experinents, we measures the recoil momentum of the electron. Then it is possible to infer the momentum transfer q. From this, all of the invariants can be computed.

$$\hat{t} = q^2 \equiv -Q^2 \qquad \hat{s} = 2k \cdot p \qquad \hat{u} = 2(k-q) \cdot p$$

The initial quark momentum is $p = \xi P$. Now comes Feynman's remarkable observation: the final quark must be on shell, so

$$(p+q)^2 = (\xi P + q)^2 = 2\xi P \cdot q - Q^2$$

(ignoring $P^2 = m_p^2 \ll Q^2$). Then

$$\xi = \frac{Q^2}{2P \cdot q} \equiv x$$

In each event, we know the momentum fraction ξ of the struck quark ! Further,

$$\frac{\hat{u}}{\hat{s}} = \frac{2(k-q) \cdot p}{2k \cdot p} = 1 - y \text{ , with } \; y \equiv \frac{2q \cdot P}{2k \cdot P}$$

The value of y is observable, equal to the fraction of the e- energy that is tranfered to the hadronic system in the lab frame. Note also

$$\int d\cos\theta = \int 2d\hat{u}/\hat{s} = \int 2dy$$

Finally, we obtain

$$\frac{d\sigma}{dxdy} = \left[\sum_f Q_f^2 x f_f(x)\right] \cdot \frac{2\pi\alpha^2 s}{Q^4}[1 + (1-y)^2]$$

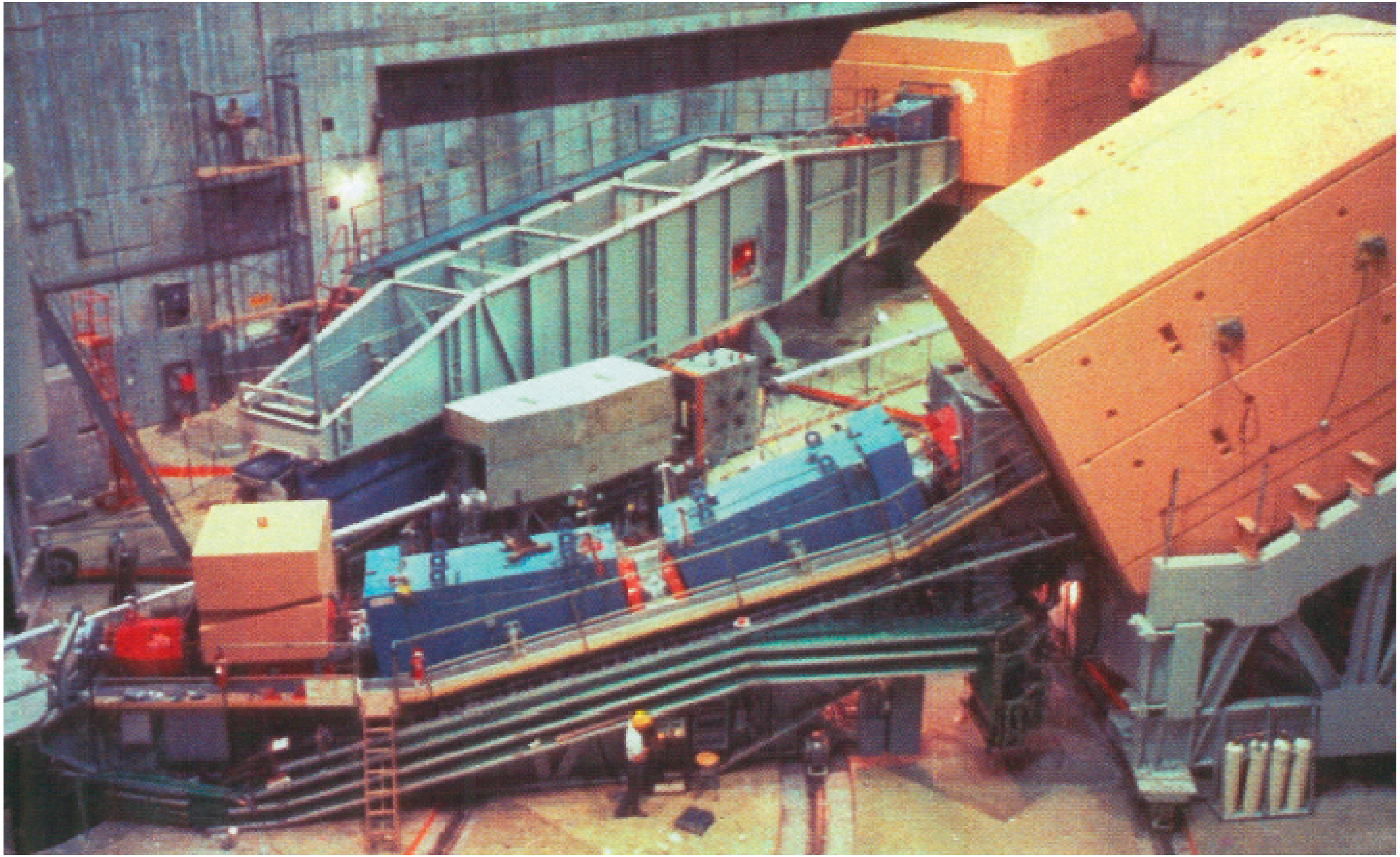
where f is summed over all species of quarks and antiquarks.

The factorization of this formula is remarkable. The quantity

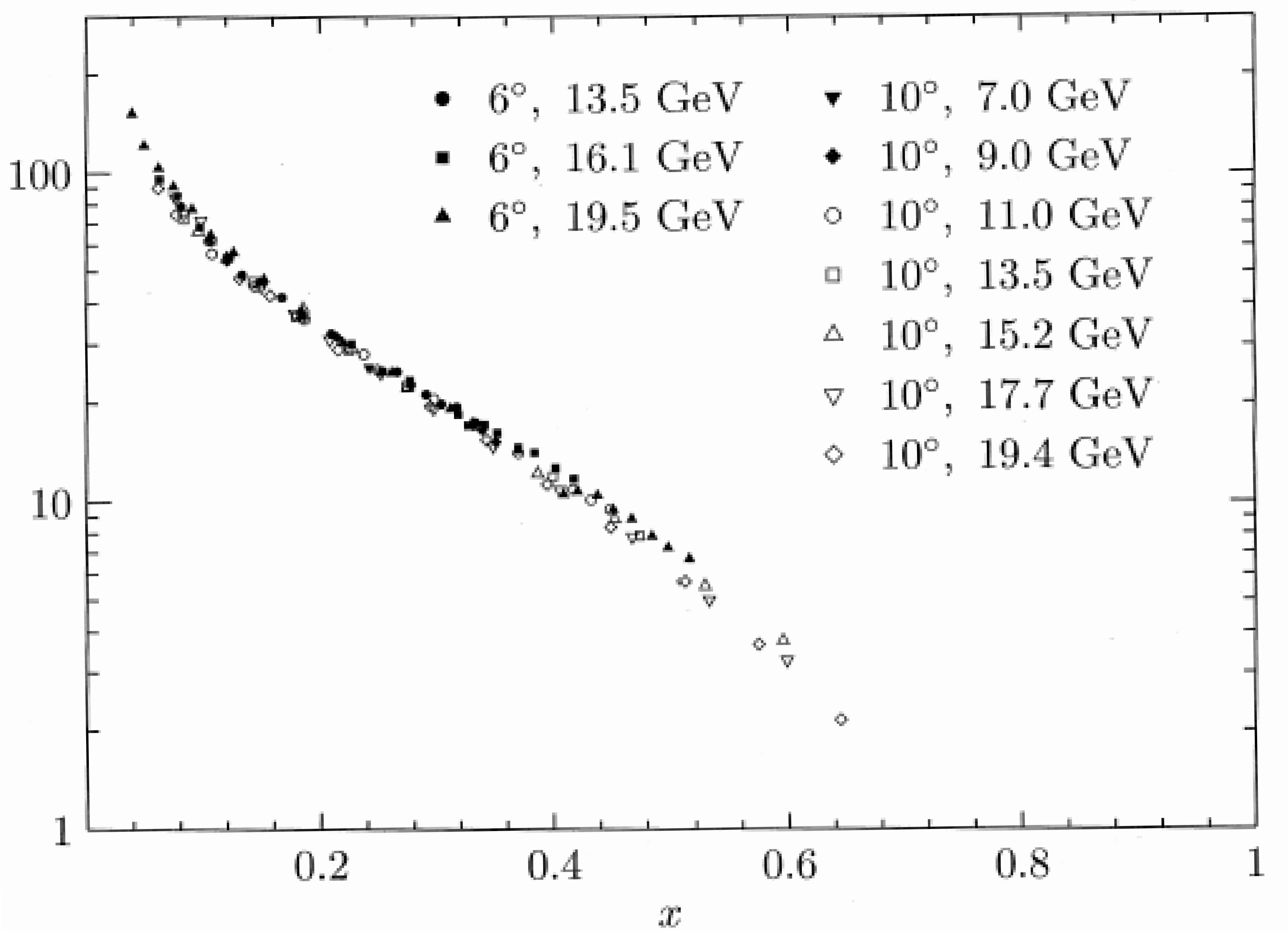
$$F_2(x) = \sum_f Q_f^2 x f_f(x)$$

is predicted to depend only on x and to be independent of Q^2 . This property is called Bjorken scaling.

The original data from SLAC supported this scaling strongly. Today, with data over a larger range of Q^2, we know that also has a slow dependence on Q^2 . As Frank Petriello will explain, this Q^2 dependence of the parton distributions is predicted by QCD, and the observed variation is in good agreement with the predictions.

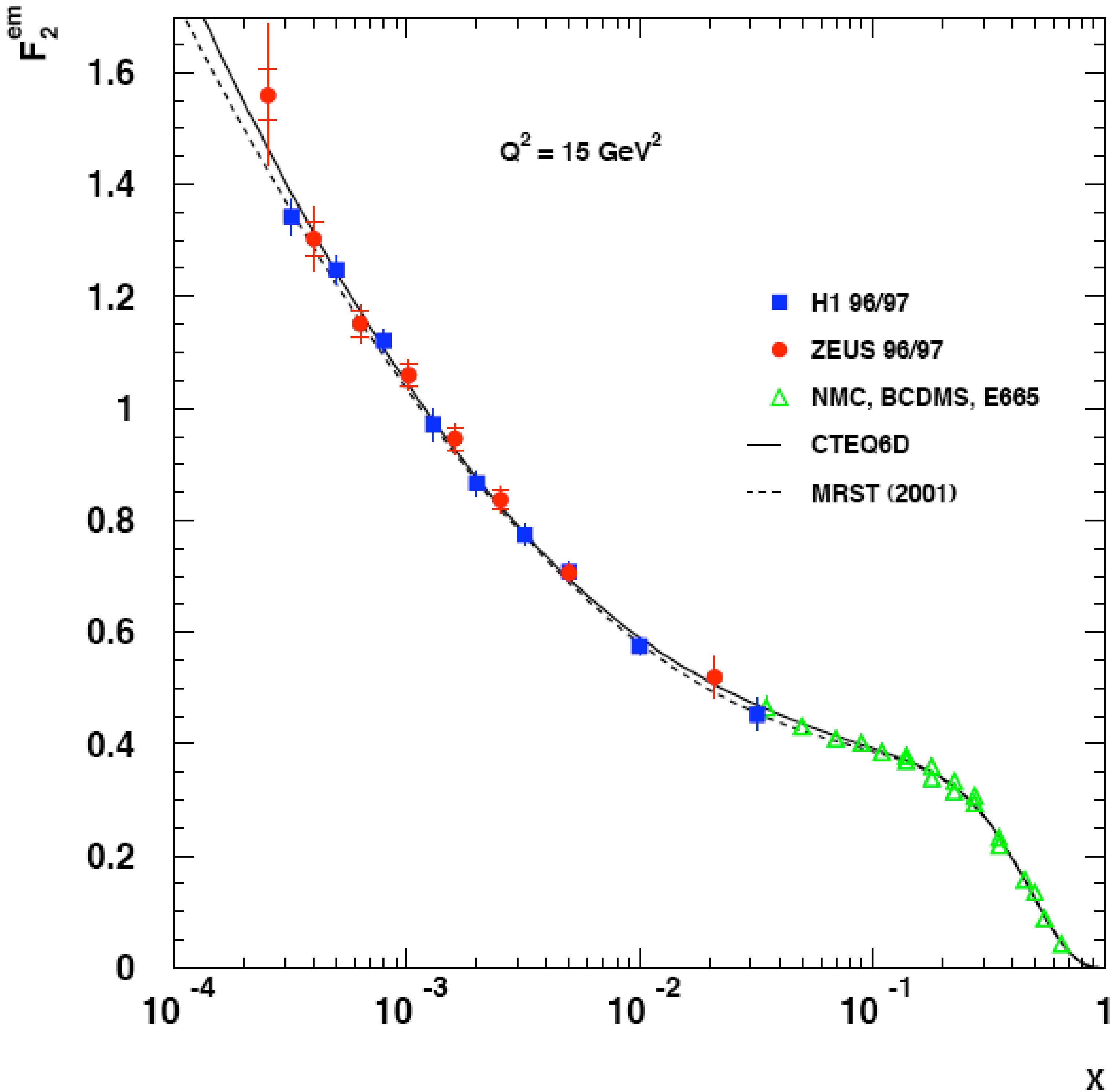


the SLAC-MIT deep inelastic scattering experiment
1967

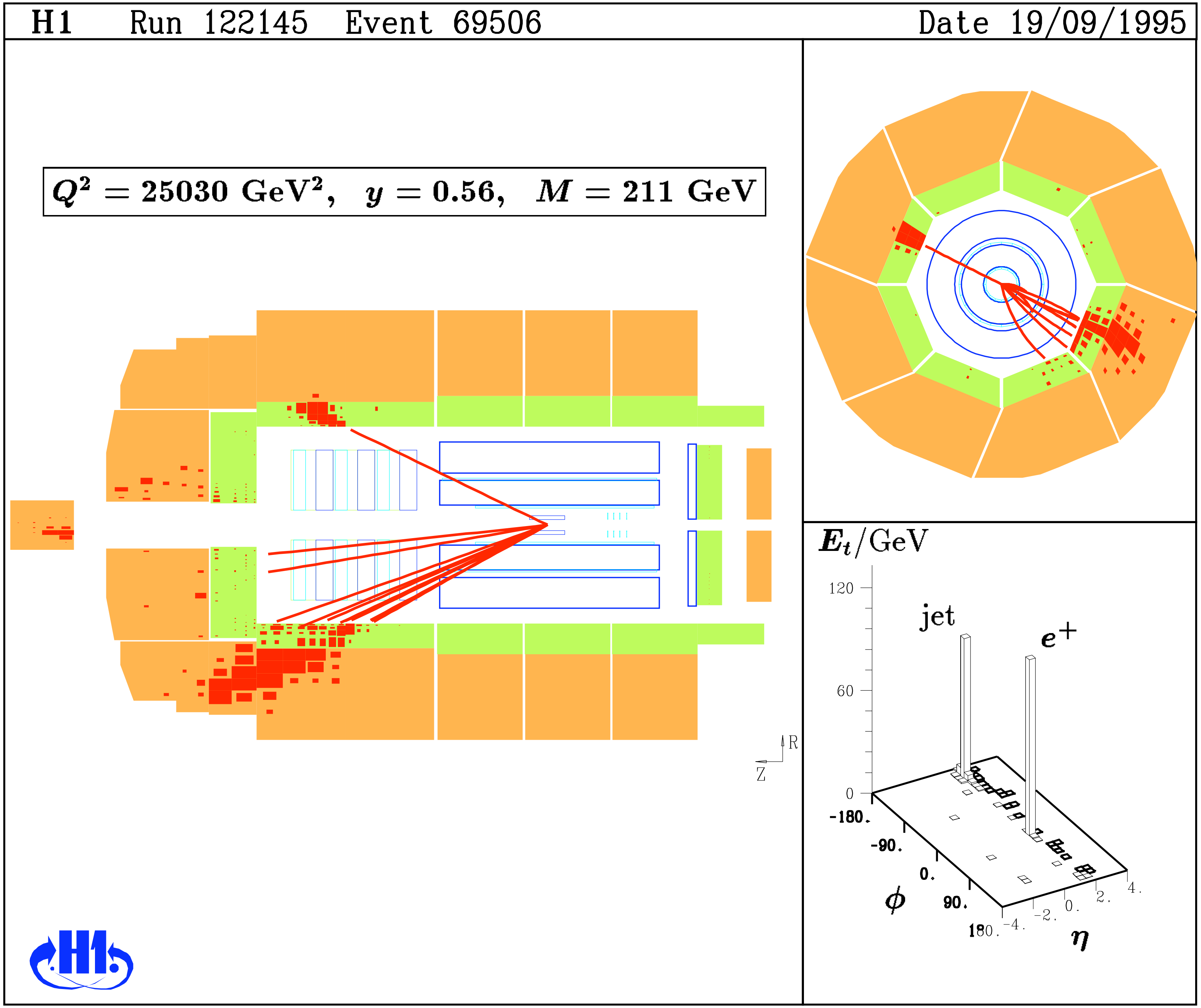


6°, 13.5 GeV
6°, 16.1 GeV
6°, 19.5 GeV
10°, 7.0 GeV
10°, 9.0 GeV
10°, 11.0 GeV
10°, 13.5 GeV
10°, 15.2 GeV
10°, 17.7 GeV
10°, 19.4 GeV
100
10
1
0.2
0.4
0.6
0.8
1
x

SLAC-MIT

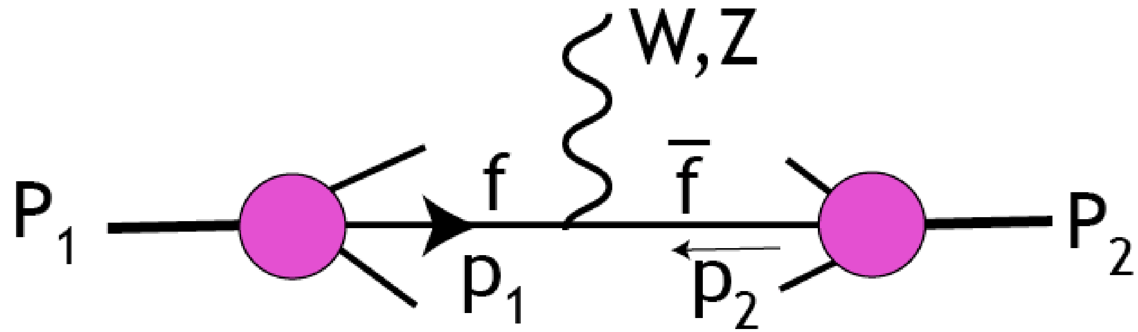


F_2^{em}
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
$Q^2 = 15\ GeV^2$
H1 96/97
ZEUS 96/97
NMC, BCDMS, E665
CTEQ6D
MRST (2001)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x



H1 - 920 GeV p x 27.6 GeV e-

Now apply this logic to the cross section for W and Z production -- the Drell-Yan process. The picture is



The quark level cross section formulae are

$$\sigma(q\bar{q} \to Z^0) = \frac{2\pi^2\alpha_w}{3c_w^2}(Q_{ZL}^2 + Q_{ZR}^2)\delta(s - m_Z^2)$$

$$\sigma(u\bar{d} \to W^+) = \frac{\pi^2\alpha_w}{3c_w^2}\delta(s - m_W^2)$$

The factor 1/3 appears because the quark and antiquark must be of the same color to annihilate. Then we find

$$\sigma(pp \to Z^0) = \int dx_1 dx_2 \sum_f f_f(x_1) f_{\overline{f}}(x_2)[Q_{ZL}^2 + Q_{ZR}^2] \frac{2\pi^2\alpha_w}{3c_w^2}\delta(s - m_Z^2)$$

$$\sigma(pp \to W^+) = \int dx_1 dx_2 \, [f_u(x_1) f_{\overline{d}}(x_2) + \cdots] \frac{\pi^2\alpha_w}{3c_w^2}\delta(s - m_W^2)$$

In this case, there are two partons with unknown momentum fractions. But it is possible to identify both fractions for each event.

$$m_W^2 = \hat{s} = (p_1 + p_2)^2 = 2p_1 \cdot p_2 = 2x_1 x_2 P_1 \cdot P_2 = x_1 x_2 s$$

The rapidity of the W or Z is given by

$$Y = \frac{1}{2} \log \frac{q^0 + q^3}{q^0 - q^3} = \frac{1}{2} \log \frac{q^+}{q^-} = \frac{1}{2} \log \frac{x_1}{x_2}$$

Then

$$x_1 = (m_W/s) e^{Y} \qquad x_2 = (m_W/s) e^{-Y}$$

From these formulae

$$dx_1 dx_2 = \frac{d\hat{s}}{\hat{s}} dY$$

Then the Drell-Yan cross section can be written

$$\frac{d\sigma}{dY}(pp \to W^+) = \left[x_1 f_u(x_1) x_2 f_{\overline{d}}(x_2) + \cdots \right] \frac{\pi^2 \alpha_w}{3 m_W^2}$$

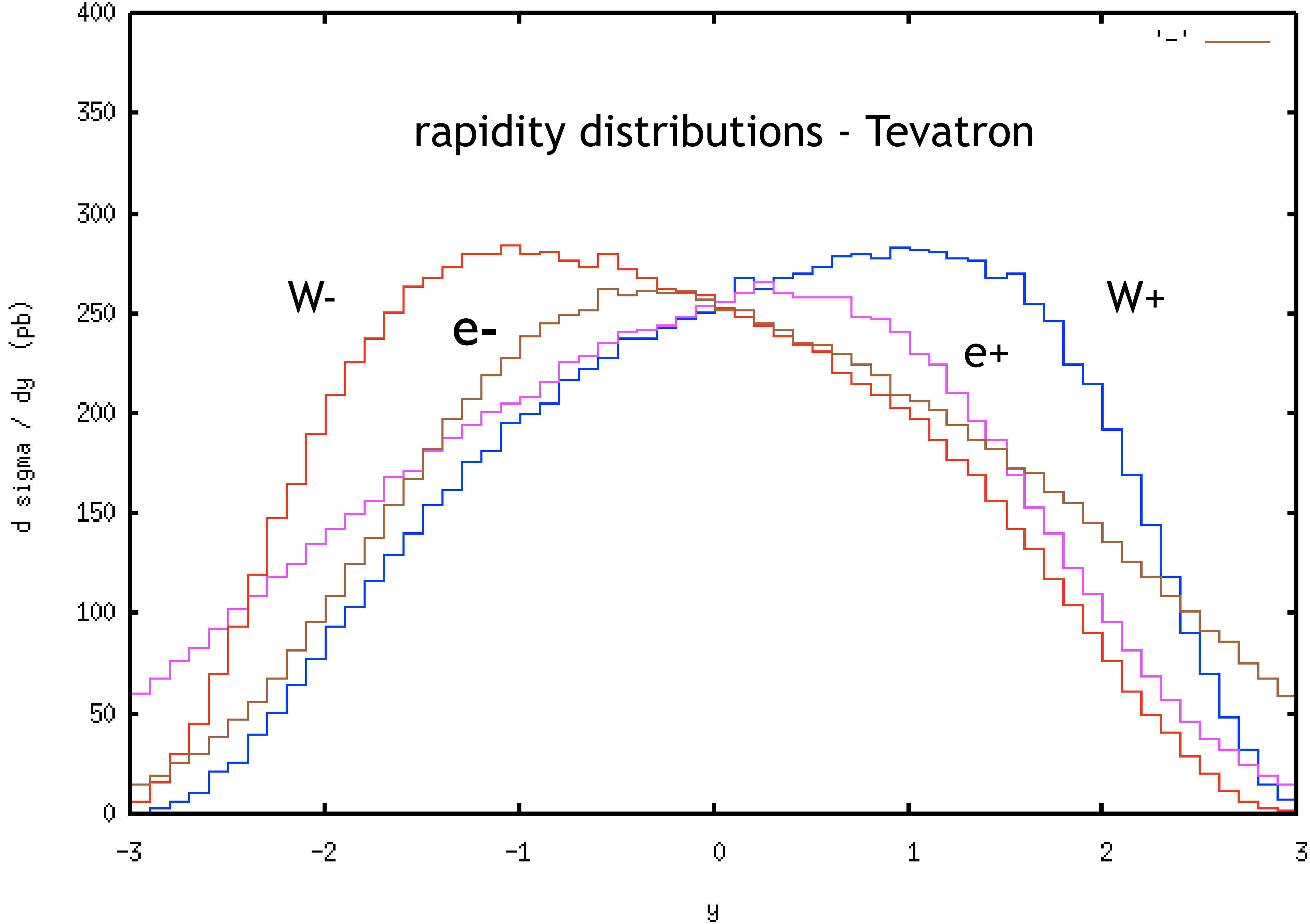
This factorization is called Drell-Yan scaling.

This formula has an interesting physical consequence.

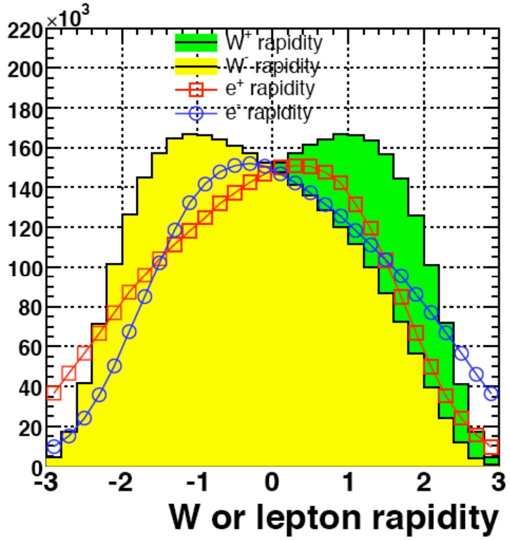
Consider first proton-antiproton scattering at the Tevatron.

The proton has hard quarks and the antiproton has hard antiquarks. The u quarks are more likely to carry a large fraction of the momentum than the d quarks. Then most W+ will be made from $u\overline{d}$ and go forward, while most W- will be made from $d\overline{u}$ and go backward.

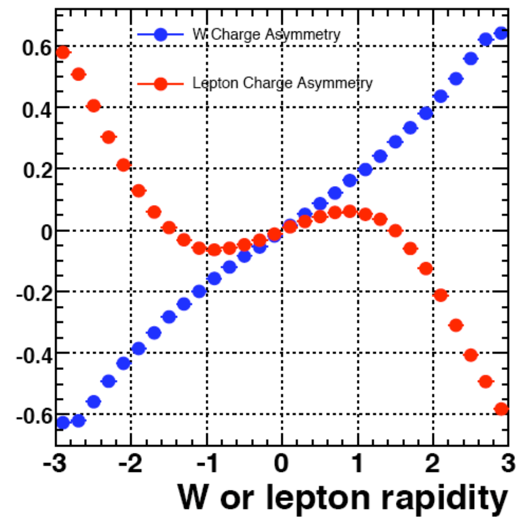
Next, recall that the leptons from W+ go backward with respect to the quark direction, while the leptons from W- go forward with respect to the quark direct. This gives a symmetric pattern in which the leptons are more central than the W's.



rapidity distributions - Tevatron
'-'
W-
e-
e+
W+
d sigma / dy (pb)
y
400
350
300
250
200
150
100
50
0
-3
-2
-1
0
1
2
3

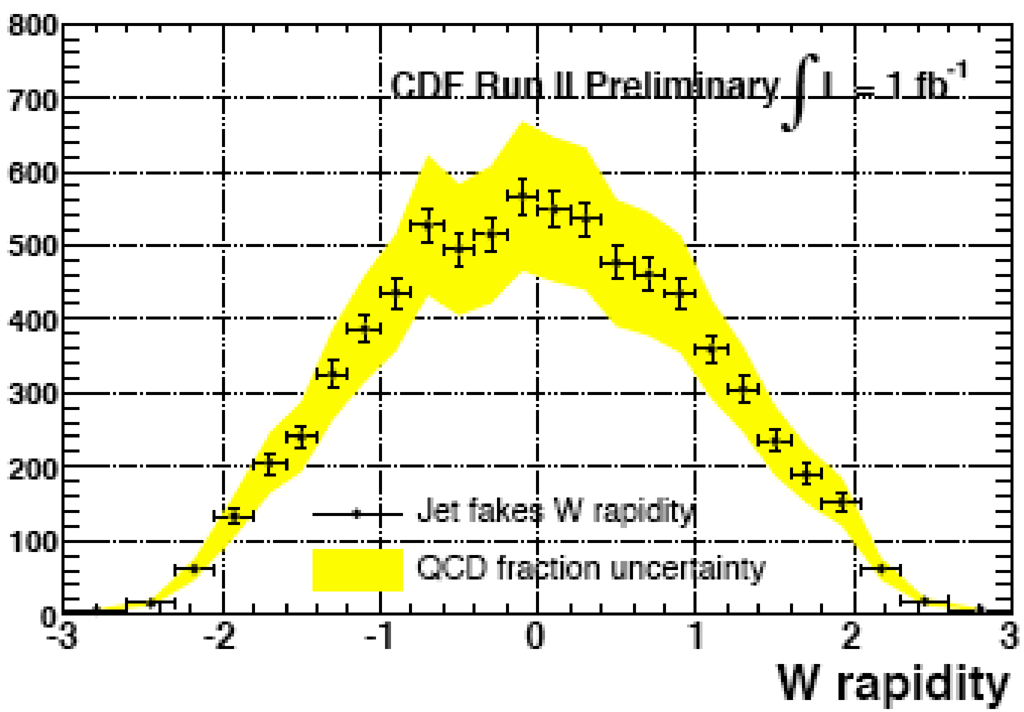


W+ rapidity
W- rapidity
e+ rapidity
e- rapidity
W or lepton rapidity

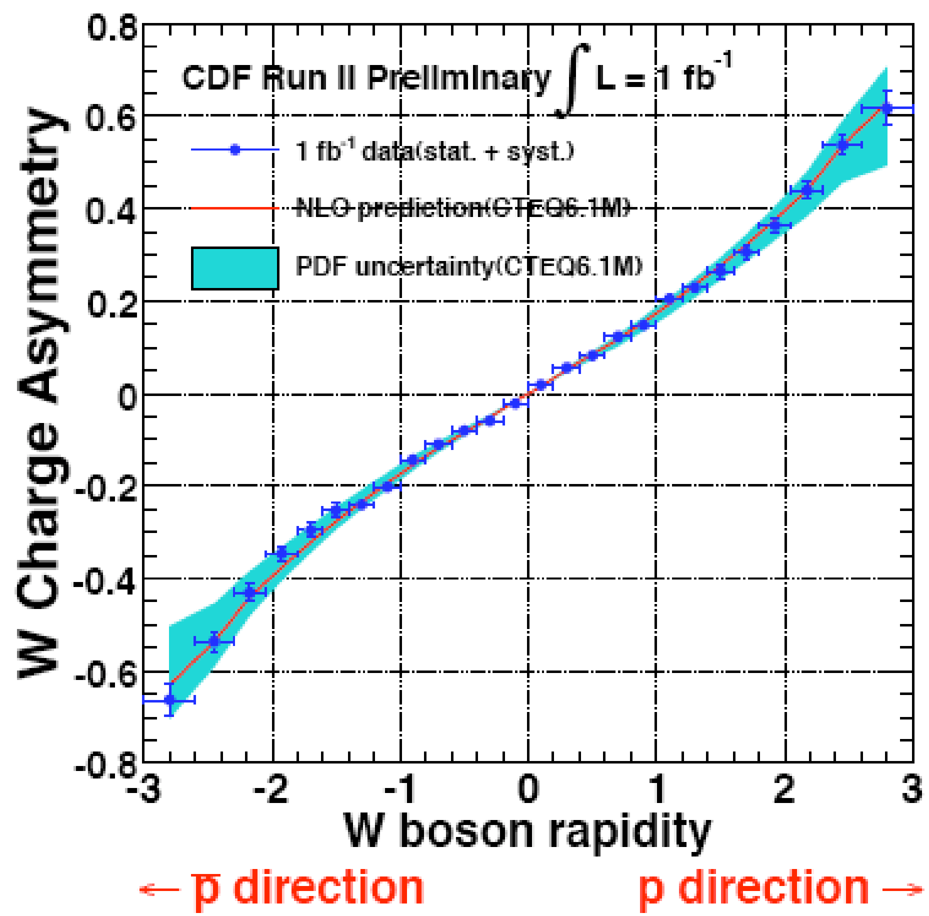


W Charge Asymmetry
Lepton Charge Asymmetry
W or lepton rapidity

CDF

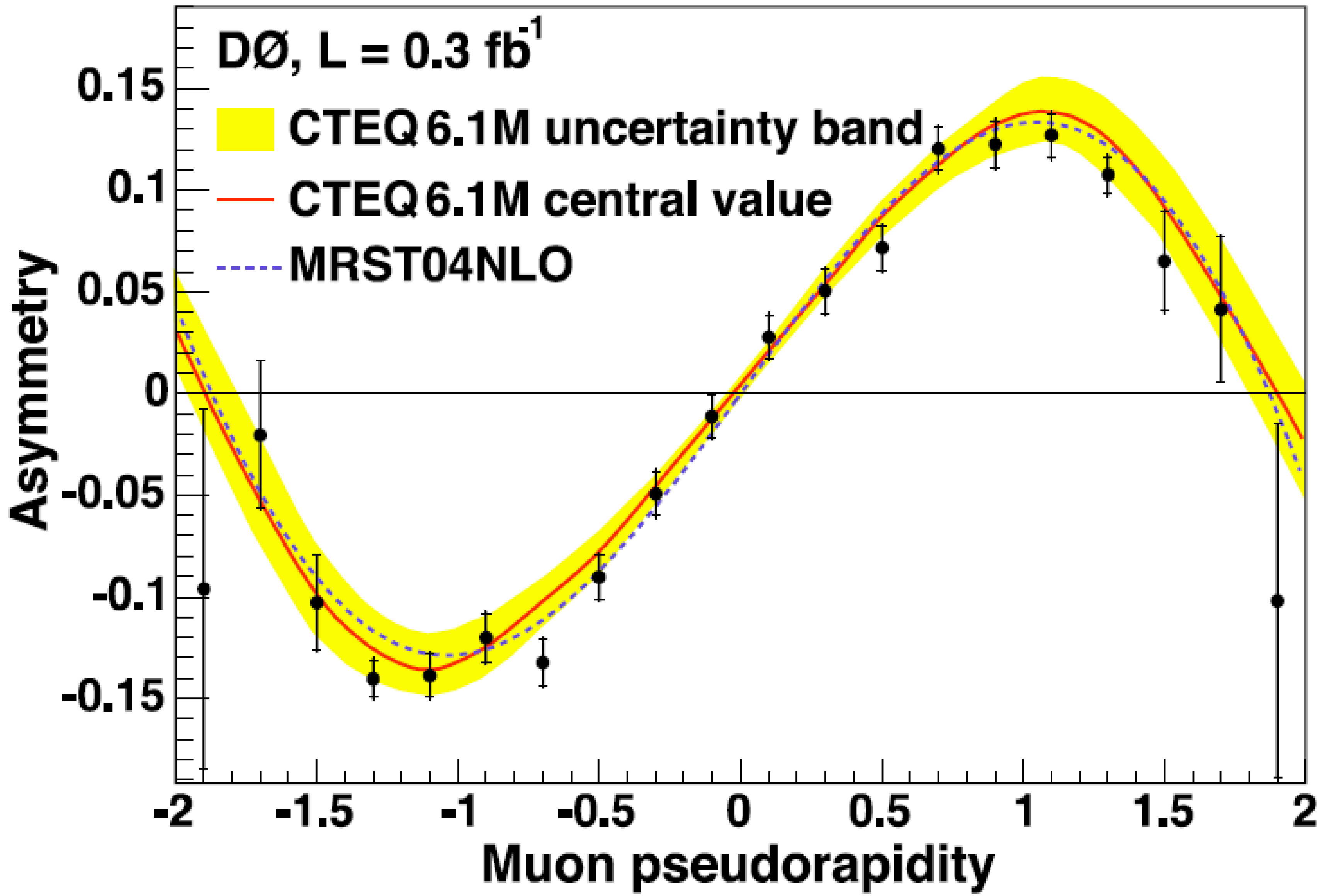


CDF Run II Preliminary ∫L = 1 fb-1
Jet fakes W rapidity
QCD fraction uncertainty
W rapidity



CDF Run II Preliminary ∫L = 1 fb-1
1 fb-1 data(stat. + syst.)
NLO prediction(CTEQ6.1M)
PDF uncertainty(CTEQ6.1M)
W Charge Asymmetry
W boson rapidity
← p̄ direction
p direction →

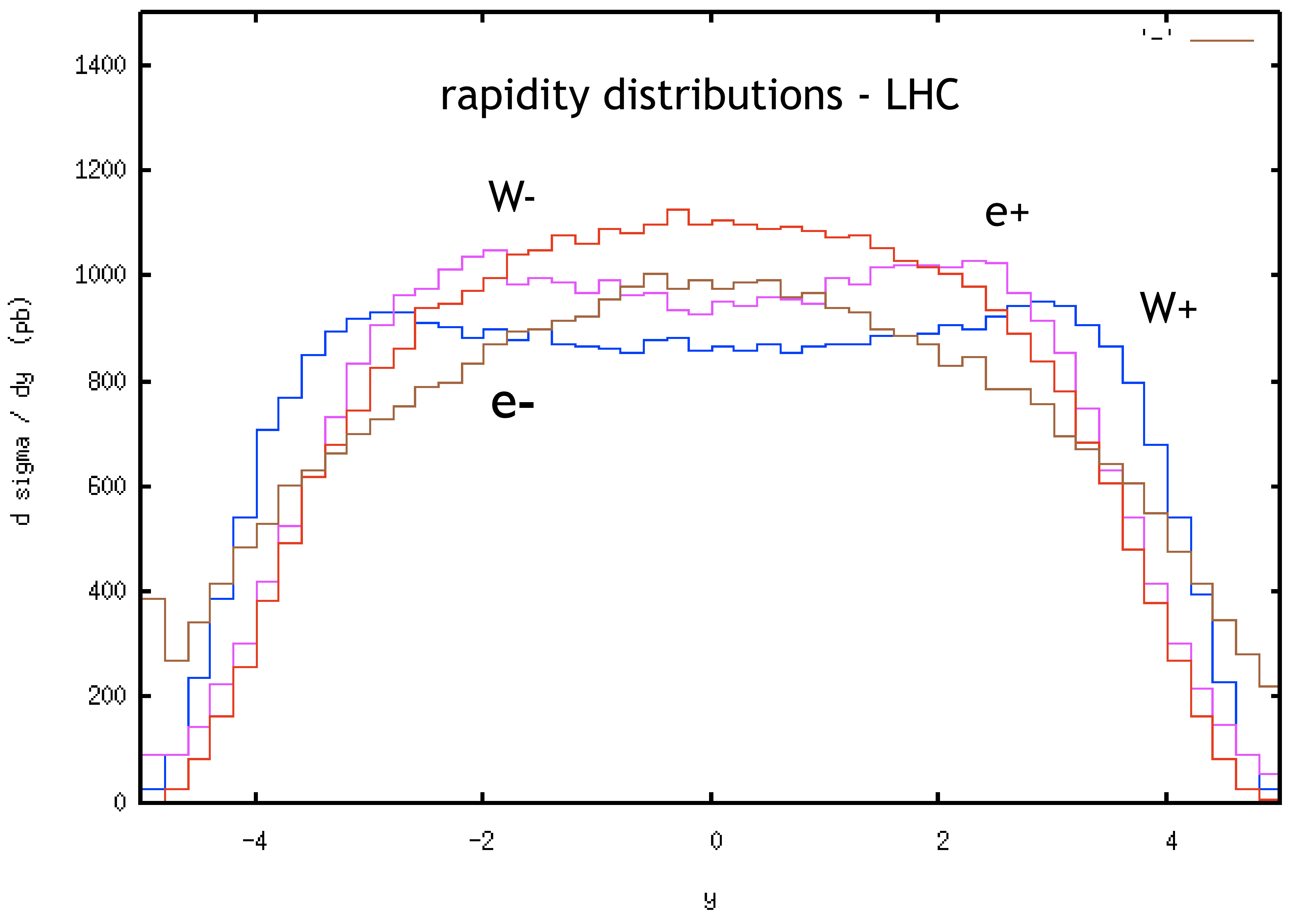
Muon charge asymmetry vs. muon rapidity, from DO



What about at the LHC ? Here W's are made by annihilation of valence quarks on low-momentum antiquarks.

Both the W+ and W- distributions must be symmetric about Y=0. However, the W+ distribution will be broader in Y, reflecting the higher momentum of the u quarks.

The W+'s then decay to a backward lepton, giving a narrow distribution for the l+'s. The W-'s decay to a forward lepton, broadening the distribution of the l-'s.



'_'
rapidity distributions - LHC
W-
e+
W+
e-
1400
1200
1000
800
600
400
200
0
d sigma / dy (pb)
-4
-2
0
2
4
y

The parton model is a very general tool for computing cross sections at the LHC. It applies both the Standard Model and Beyond the Standard Model physics. As the school progresses, you will see many applications of this model to LHC processes.