QCD and MC's for the LHC

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Claims and Aims

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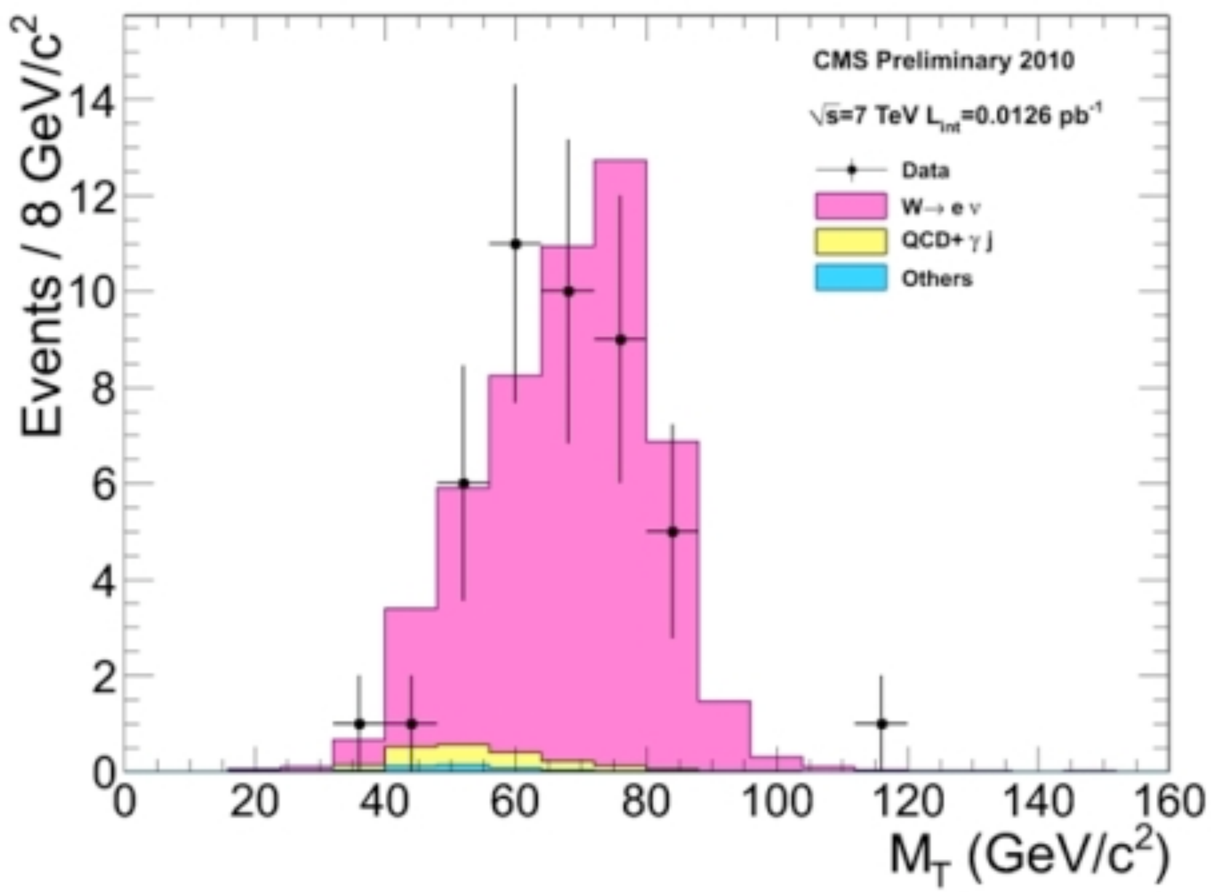
LHC data is there!!!!



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In fact, new dimensions have been added to
Theory ⇔ Experiment interactions

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- physics issues: QCD from high- to low-Q^2, Parton showers, Angular ordering, jet algos
- recent progress: NLO computations, merging Monte Carlo with FO.
- key applications at the LHC: Drell-Yan, Top, Higgs, Jets, BSM,...

Claims and (your) Aims

A mathematica notebook on a simple NLO calculation and other exercises on LHC phenomenology available on the MadGraph Wiki.

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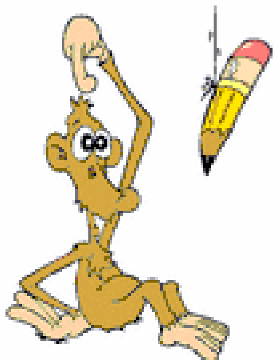

Think

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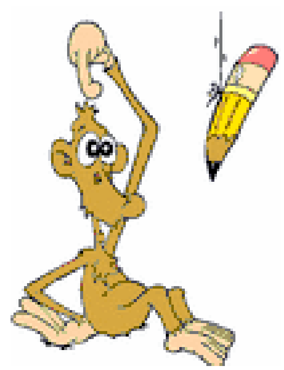

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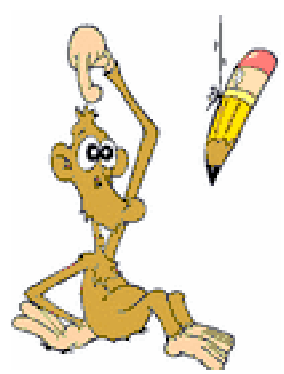

Think

Ask

Work

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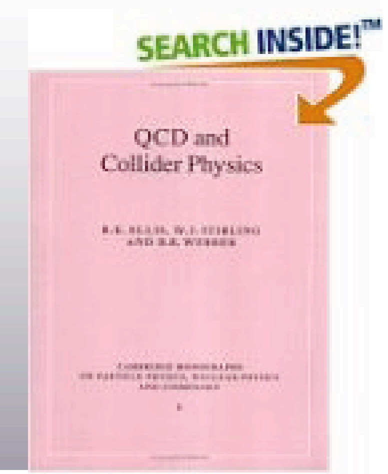
Minimal references and write-ups

Ellis, Stirling, Webber: The pink book

Subtitle:
"All you want to know about perturbative QCD
and never dared to ask..."



*Very useful recent talks/lectures by (just google the names):
Gavin Salam, Stefano Frixione, Michelangelo Mangano.

Why do we believe in QCD
[as a theory of strong interactions]?

- QCD is a non-abelian gauge theory, is renormalizable, is asymptotically free, is a one-parameter theory [Once you measure α_S you know everything fundamental about (perturbative) QCD].
- It explains the low energy properties of the hadrons, justifies the observed spectrum and catch the most important dynamical properties.
- It explains scaling (and BTW anything else we have seen up to now!!) at high energies.
- It leaves EW interaction in place since the SU(3) commutes with SU(2) x U(1). There is no mixing and there are no enhancements of parity violating effect or flavor changing currents.
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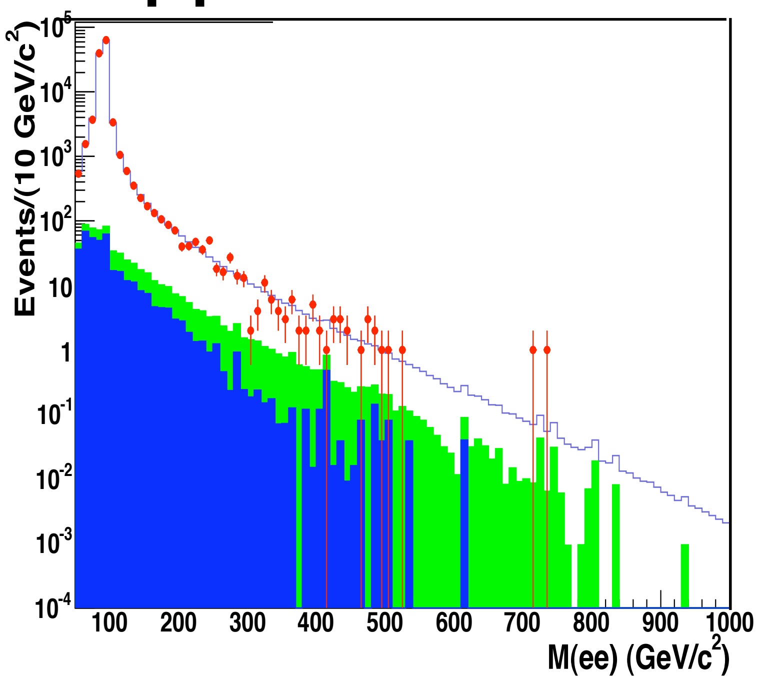
Excellent!

So are we done?

Discoveries at hadron colliders

peak

$pp \to Z' \to e^+e^-$

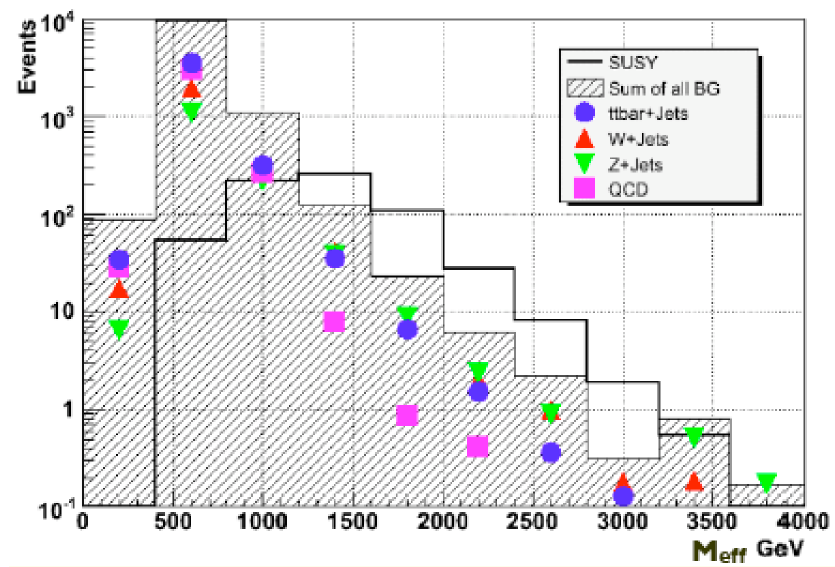


“easy”

Background directly measured from data. TH needed only for parameter extraction (Normalization, acceptance,...)

shape

$pp \to \tilde{g}\tilde{g}, \tilde{g}\tilde{q}, \tilde{q}\tilde{q} \to \text{jets} + \not{E}_T$

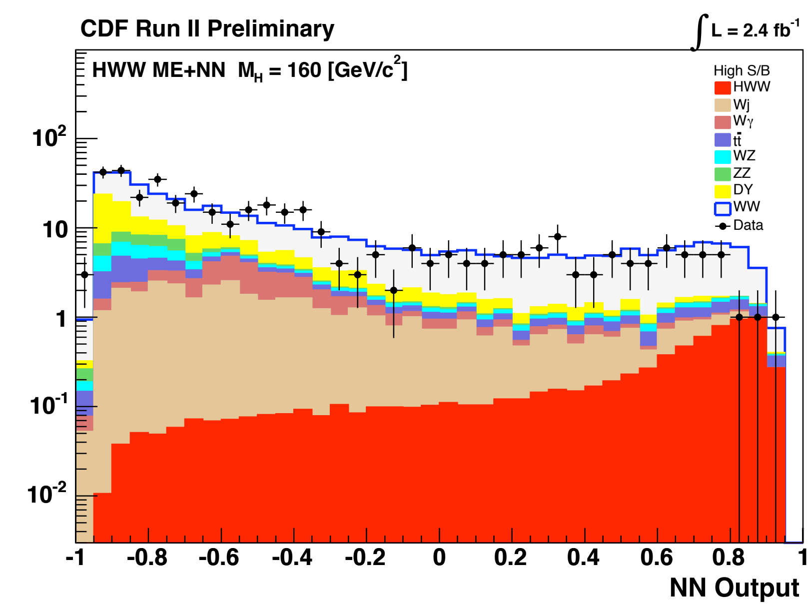


hard

Background shapes needed. Flexible MC for both signal and backgroud tuned and validated with data.

rate

$pp \to H \to W^+W^-$

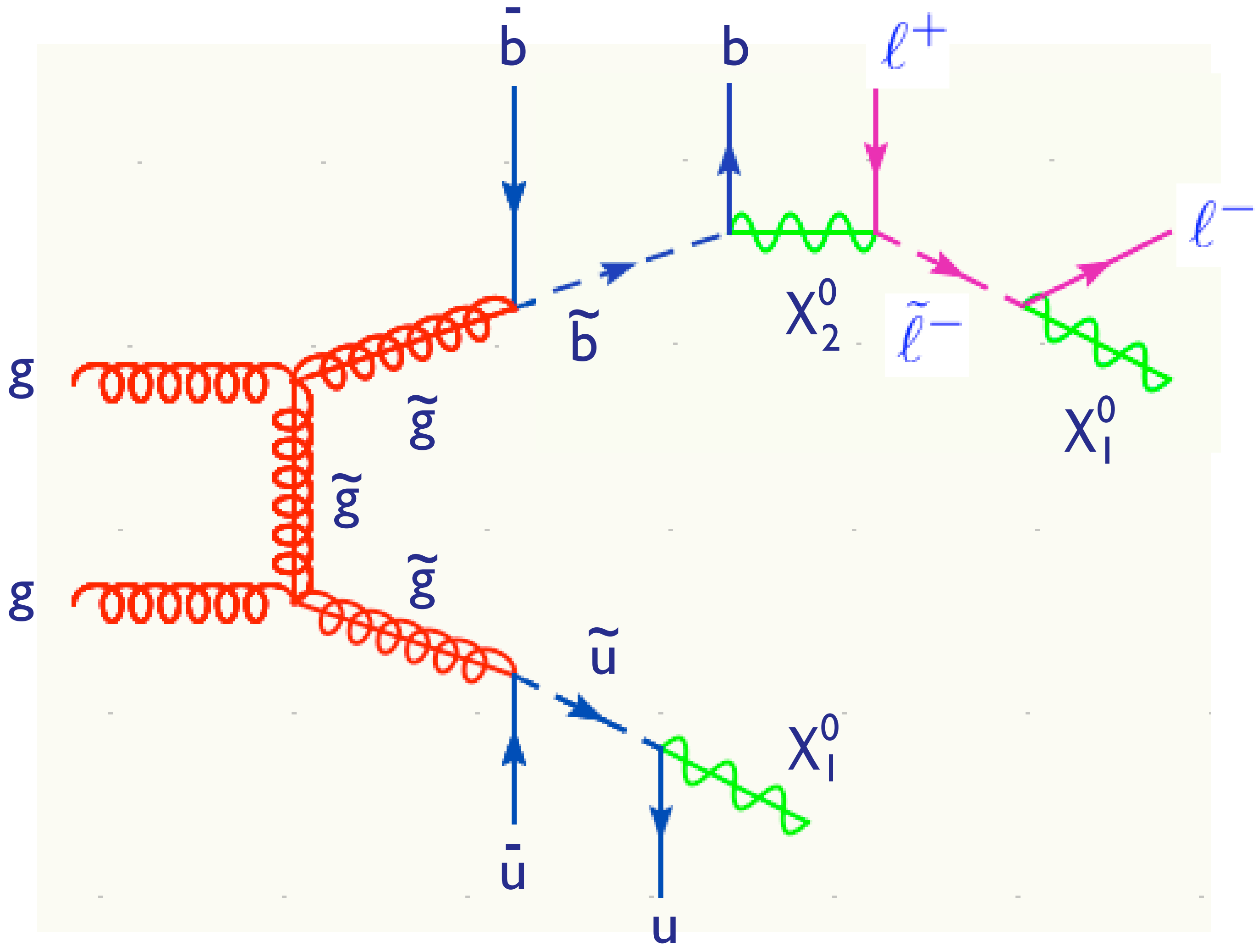


very hard

Background normalization and shapes known very well. Interplay with the best theoretical predictions (via MC) and data.

A new challenge

Consider SUSY-like inclusive searches: heavy colored states decaying through a chain into jets, leptons and missing E_T...



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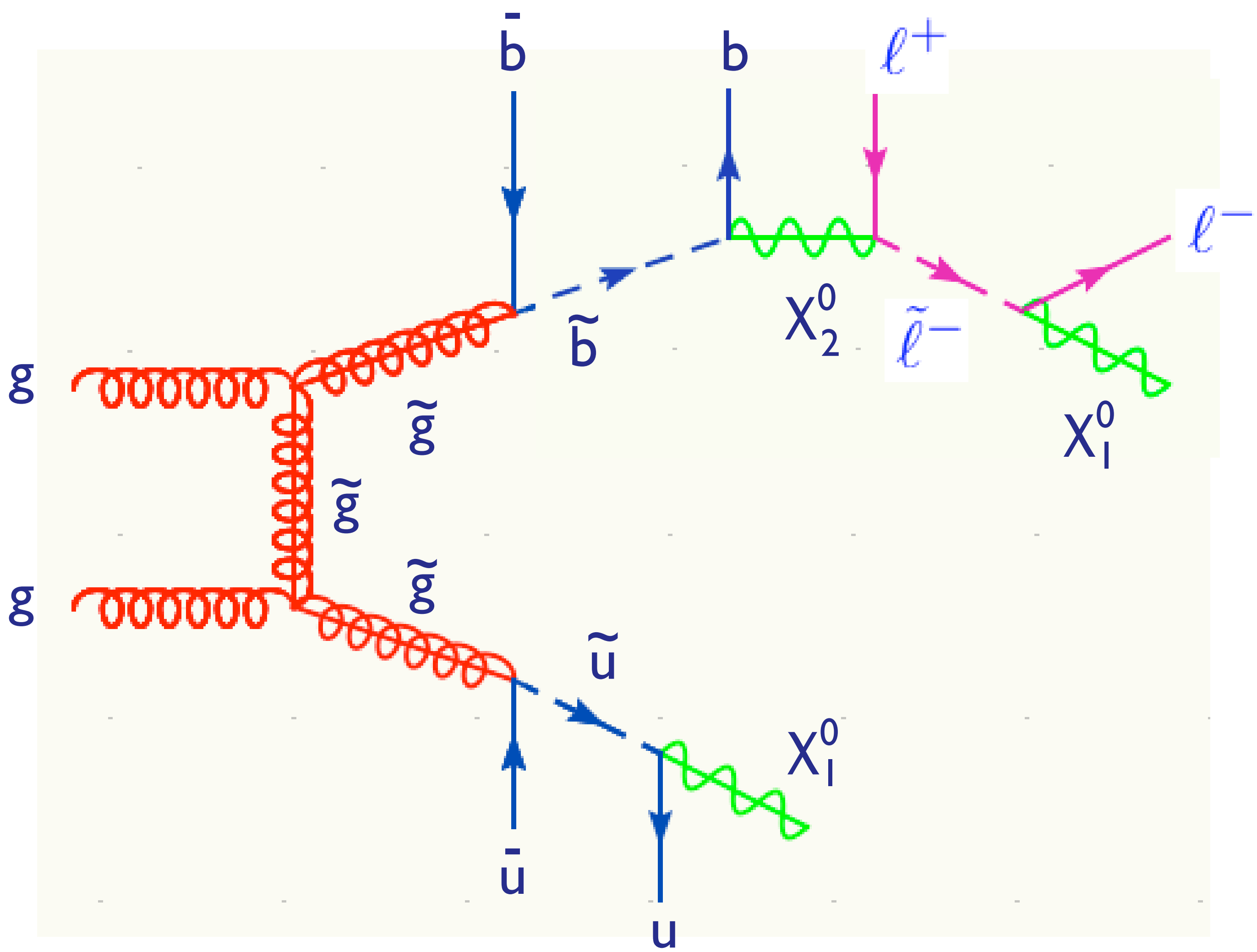
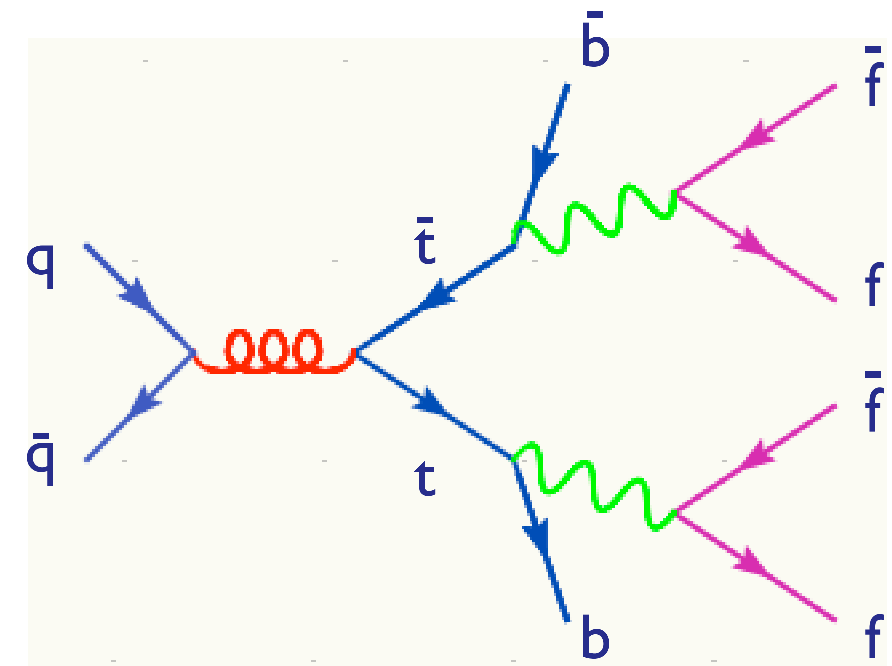
Consider SUSY-like inclusive searches: heavy colored states decaying through a chain into jets, leptons and missing E_T...We have already a very good example of a similar discovery!



VS



Follow the same approach of CDF in 1995 to establish first evidence of an excess wrt to SM-top and then consistency with SM top production [mt=174, t→blν, σ(tt)] , works for the SM Higgs, but in general beware that...

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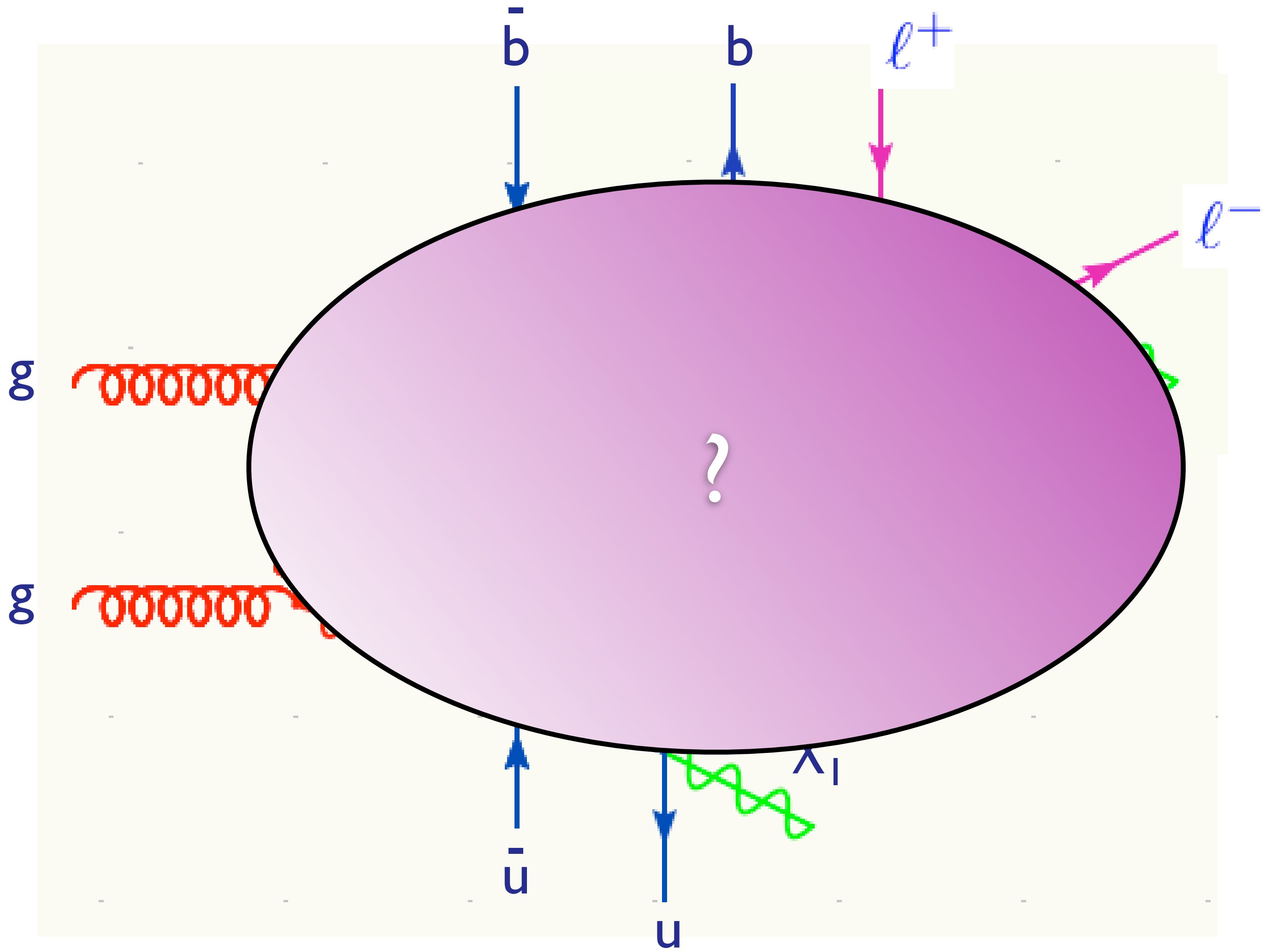

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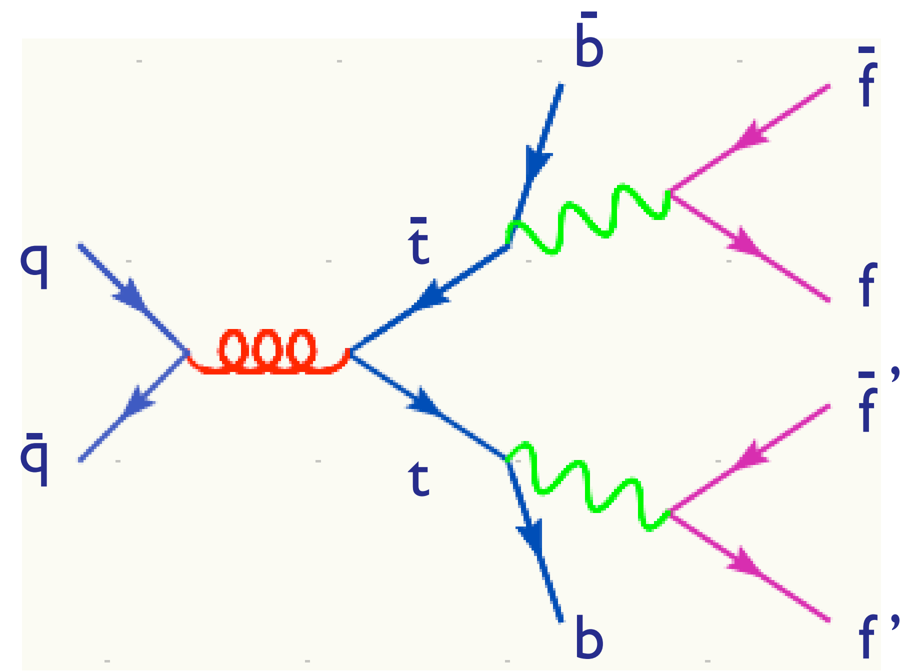
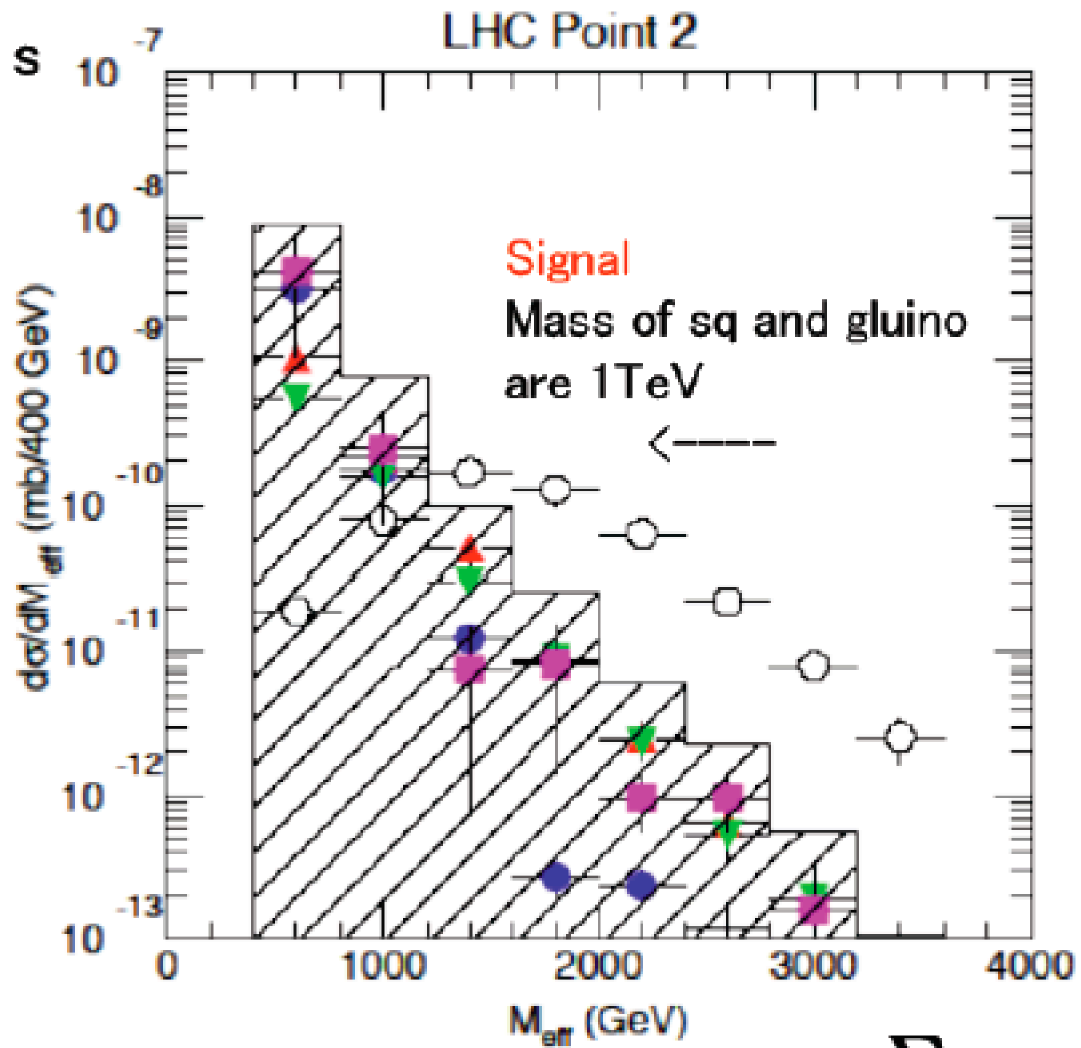


Follow the same approach of CDF in 1995 to establish first evidence of an excess wrt to SM-top and then consistency with SM top production [mt=174, t→blv, σ(tt)] , works for the SM Higgs, but in general beware that... we don't know what to expect!

Example: early discovery SuperSymmetry at the LHC



$= \not{E}_T + \sum P_T$

“Old MC”

Background: t tbar+jets,(Z,W)+jets, jets. Very difficult to estimate theoretically: many parton calculation (2 → 8 gluons = 10 millions Feynman diagrams diagrams!!). Now MC’s for this are available...

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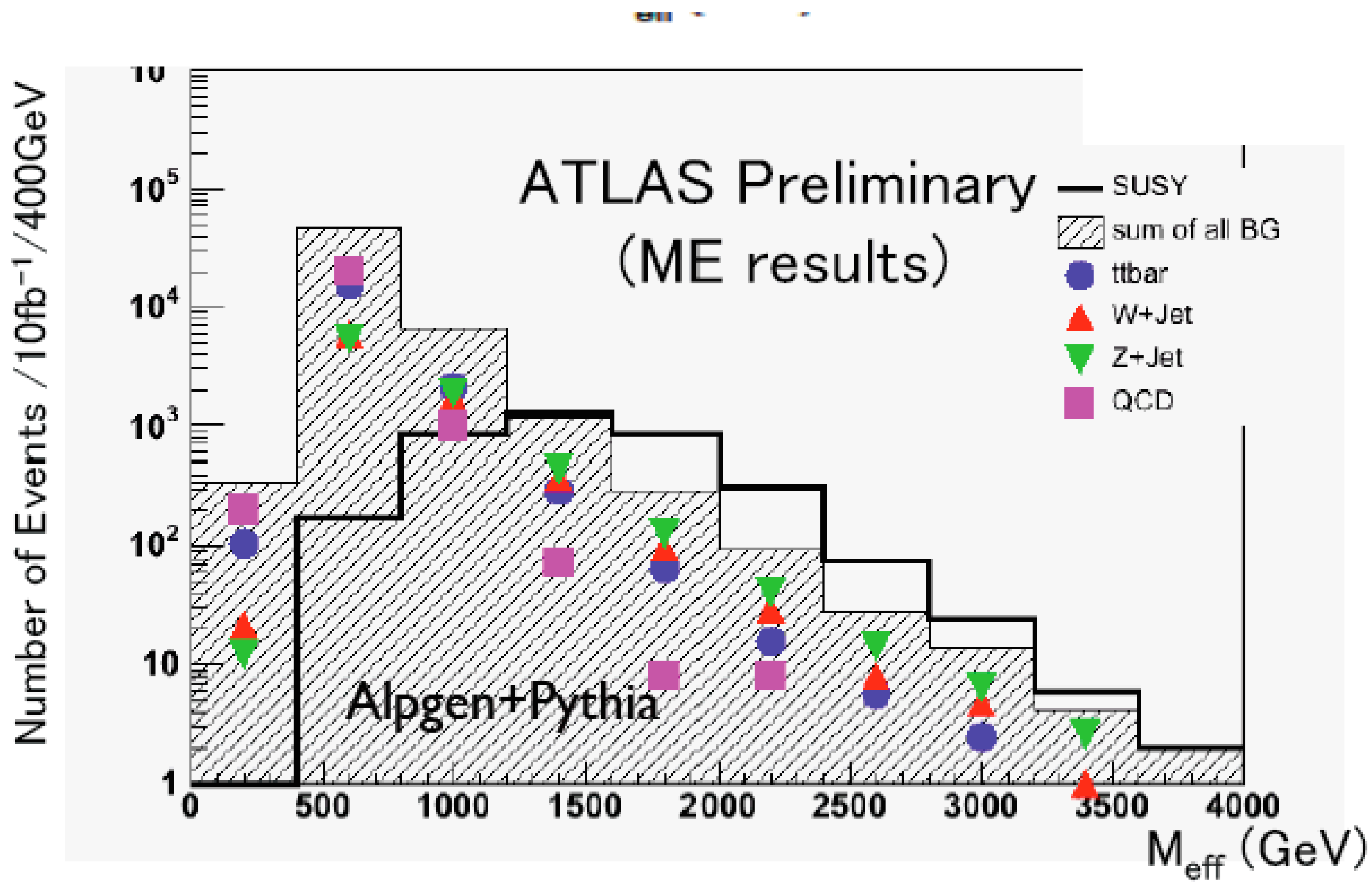

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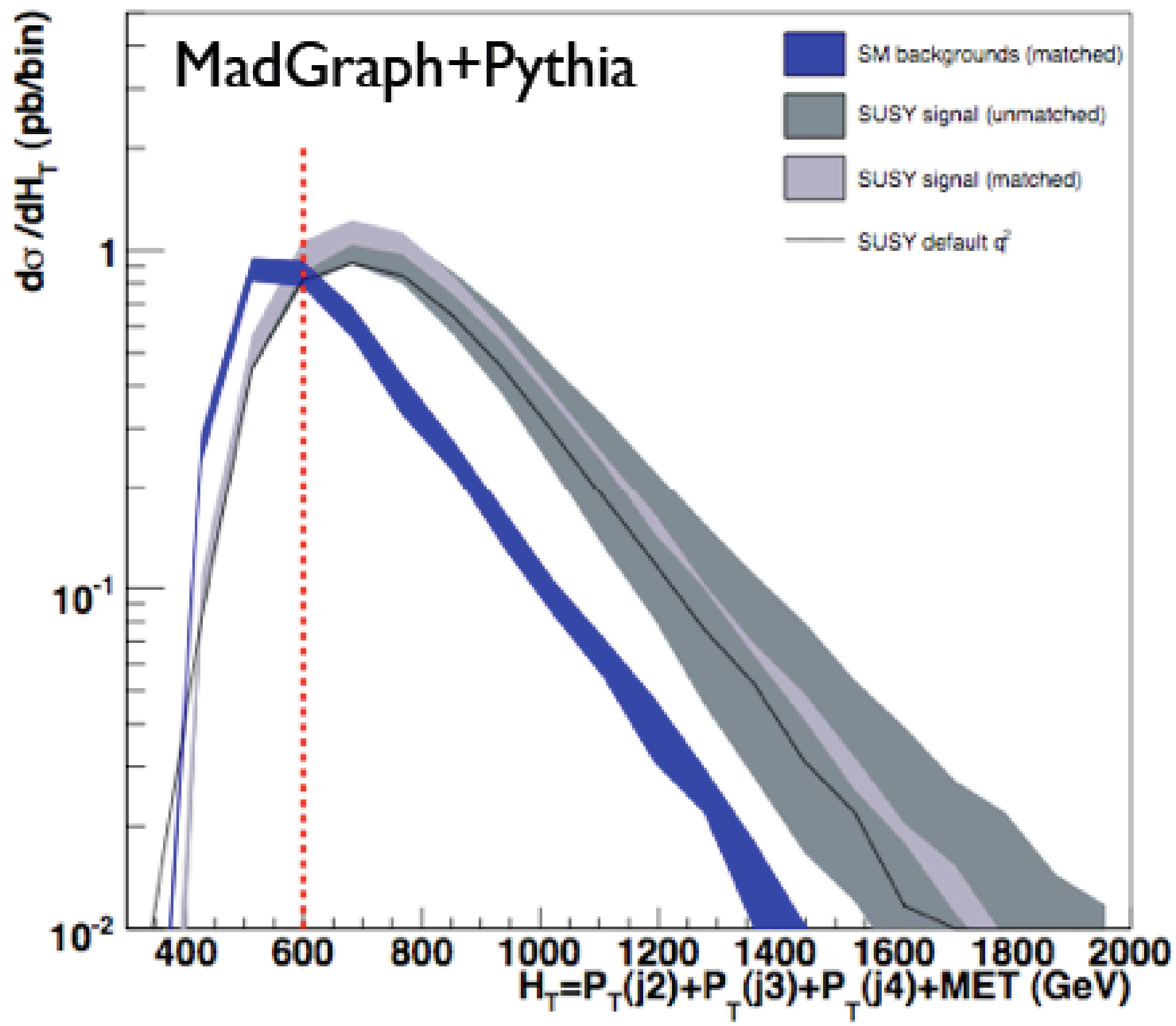


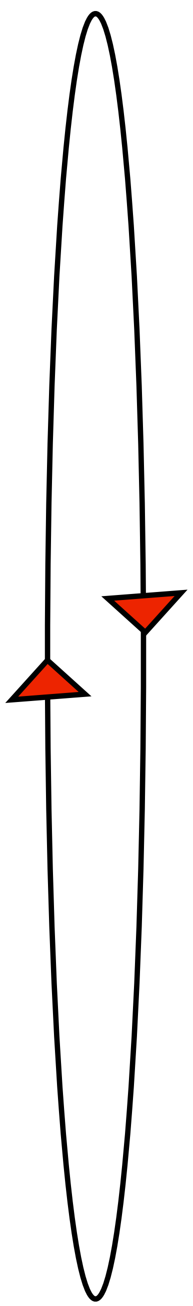
"New MC for Signal & BKG"

Background: t tbar+jets,(Z,W)+jets, jets. Very difficult to estimate theoretically: many parton calculation (2 → 8 gluons = 10 millions Feynman diagrams diagrams!!). Now MC's for this are available...

Texte: signal matched ME+PS. Predictability improved. Same theoretical status as the background.

The path towards discoveries

LHC physics = QCD + ϵ



1. Rediscover the known SM at the LHC (top's, W's, Z's) + jets.

New regime for QCD. Exclusive description for rich and energetic final states with flexible MC to be validated and tuned to control samples. Shapes for multi-jet final states and normalization for key process important. Accurate predictions (NLO,NNLO) needed only for standard candle cross sections.

2. Identify excess(es) over SM

Importance of a good theoretical description depends on the nature of the physics discovered: from none (resonances) to fundamental (inclusive SUSY).

3. Identify the nature of BSM: from coarse information to measurements of mass spectrum, quantum numbers, couplings.

Not fully worked out strategy. Several approaches proposed (MARMOSET, VISTA,...). Only in the final phase accurate QCD predictions and MC tools for SM as well as for the BSM signals will be needed.

Bottom-line

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No QCD $\Rightarrow$ No Party

A simple plan

- Intro: the LHC challenge
- Minimal QCD: basics
- Precision QCD: from NLO to NNLO
- Useful QCD: Parton Shower approach
- Best QCD: Merging Fixed Order with PS

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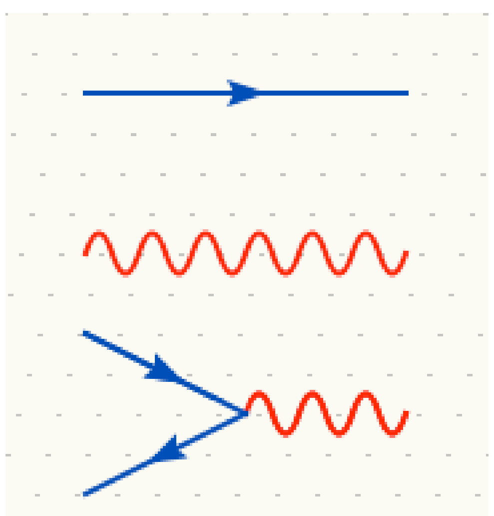
Minimal QCD: Basics

- From QED to QCD
- Color Algebra
- Helicity techniques and recursion
- Tools for tree-level calculations

From QED to QCD: abelian vs. non-abelian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\partial\!\!\!/ - m)\psi - eQ\bar{\psi}A\!\!\!/\psi$$

where $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$



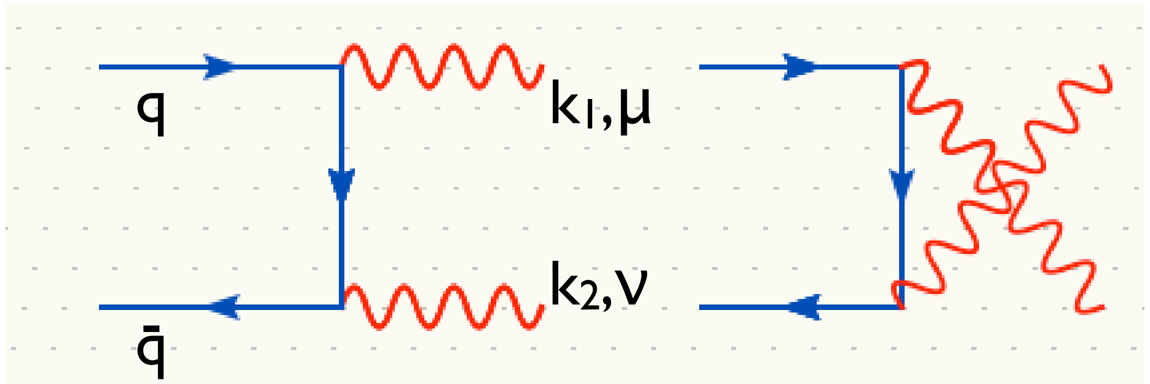
$$= \frac{i}{p\!\!\!/ - m + i\epsilon} = i\frac{p\!\!\!/ + m}{p^2 - m^2 + i\epsilon}$$

$$= -i\frac{g_{\mu\nu}}{p^2 + i\epsilon} \text{ (Feynman gauge)}$$

$$= -ie\gamma_\mu Q \quad (Q = -1 \text{ for the electron, } Q = 2/3 \text{ for the u-quark, etc}$$

From QED to QCD

We want to focus on how gauge invariance is realized in practice.
Let's start with the computation of a simple proces $e^+e^- \to \gamma\gamma$. There are two diagrams:



$$= D_1 + D_2$$

$$\frac{i}{e^2} M_\gamma \equiv D_1 + D_2 = \bar{v}(\bar{q})\, \not{\epsilon}_2 \frac{1}{\not{q} - \not{k}_1} \not{\epsilon}_1\, u(q) + \bar{v}(\bar{q}) \not{\epsilon}_1 \frac{1}{\not{q} - \not{k}_2} \not{\epsilon}_2\, n(q) \equiv M_{\mu\nu} \epsilon_1^\mu \epsilon_2^\nu$$

Gauge invariance demands that

$$\epsilon_2^\nu \partial^\mu M_{\mu\nu} = \epsilon_1^\mu \partial^\nu M_{\mu\nu} = 0$$

$M_\mu \equiv M_{\mu\nu}\epsilon_2^\nu$ is in fact the current that couples to the photon k_1. Charge conservation requires $\partial_\mu M^\mu = 0$:

$$\partial_\mu M^\mu = 0 \quad \Rightarrow \quad \frac{d}{dt}\int M^0 d^3x = \int \partial_0 M^0\, d^3x$$
$$= \int \vec{\nabla} \cdot \vec{M}\; d^3x = \int_{S\to\infty} \vec{M} \cdot d\vec{\Sigma} = 0$$

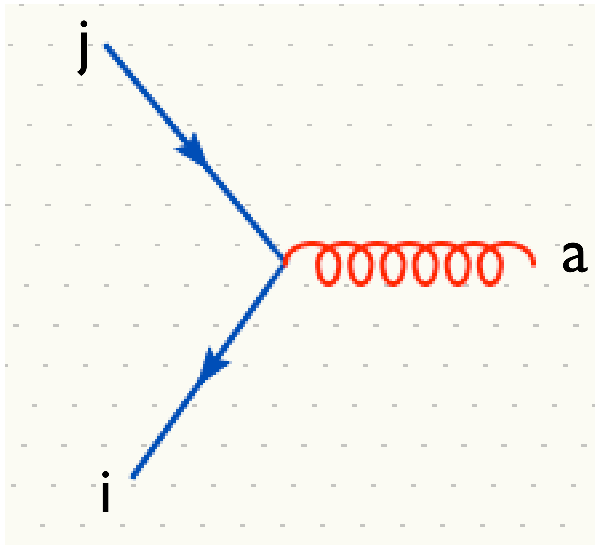
From QED to QCD

$$\begin{aligned} k_1^\mu \epsilon_2^\nu M_{\mu\nu} &= \bar{v}(\bar{q})\not{\epsilon}_2 \frac{1}{\not{q} - \not{k}_1}(\not{k}_1 - \not{q})u(q) + \bar{v}(\bar{q})(\not{k}_1 - \bar{q}) \frac{1}{k_1 - \bar{q}}\not{\epsilon}_2 u(q) \\ &= -\bar{v}(\bar{q})\not{\epsilon}_2 u(q) + \bar{v}(\bar{q})\not{\epsilon}_2 u(q) = 0 \end{aligned}$$

Only the sum of the two diagrams is gauge invariant.
For the amplitude to be gauge invariant it is enough that one of the polarizations is longitudinal. The state of the other gauge boson is irrelevant.

Let's try now to generalize what we have done for SU(3). In this case we take the (anti-)quarks
to be in the (anti-)fundamental representation of SU(3), 3 and 3^*. Then the current is in a $3 \otimes 3^* = 1 \oplus 8$. The singlet is like a photon, so we identify the gluon with the octet and generalize the QED vertex to :

$$-ig_s t^a_{ij} \gamma^\mu$$



with $[t^a, t^b] = if^{abc}t^c$

So now let's calculate qq → gg and we obtain

$$\frac{i}{g_s^2} M_g \equiv (t^b t^a)_{ij} D_1 + (t^a t^b)_{ij} D_2$$

$$M_g = (t^a t^b)_{ij} M_\gamma - g^2 f^{abc} t^c_{ij} D_1$$

From QED to QCD

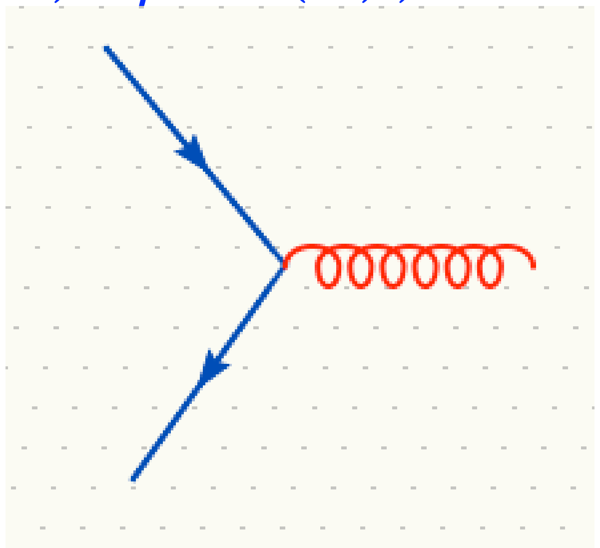
To satisfy gauge invariance we still need:

$$k_1^\mu {\epsilon_2}^\nu M_g^{\mu,\nu} = k_2^\nu \epsilon_1^\mu M_g^{\mu,\nu} = 0.$$

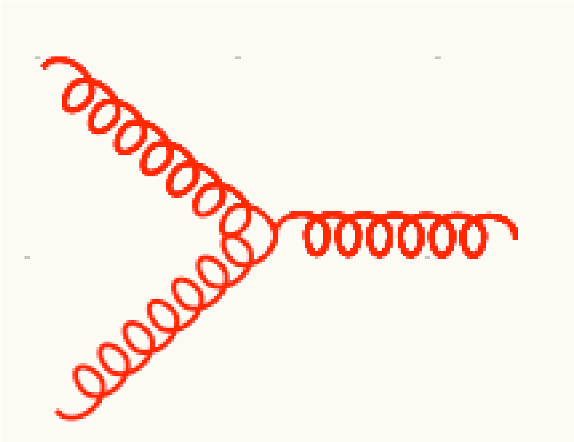
But in this case one piece is left out

$$k_{1\mu} M_g^\mu = -g_s^2 f^{abc} t_{ij}^c \bar{v}_i(\bar{q}) \not{\epsilon}_2 u_i(q)$$

$$k_{1\mu} M_g^\mu = i(-g_s f^{abc} \epsilon_2^\mu)(-i g_s t_{ij}^c \bar{v}_i(\bar{q}) \gamma_\mu u_i(q))$$

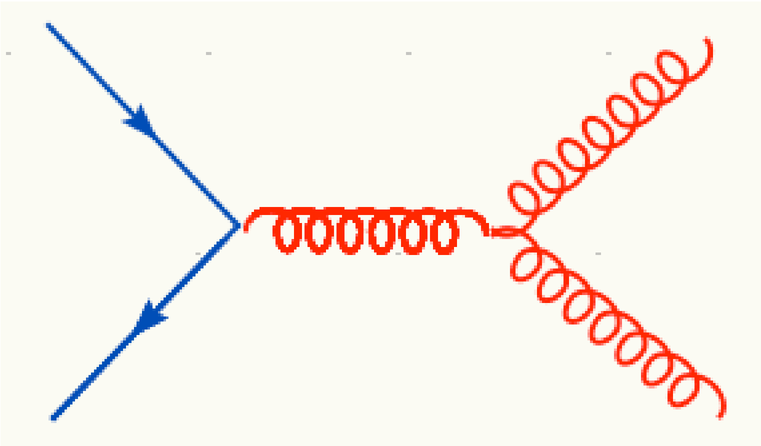


We indeed see that we interpret as the normal vertex times a new 3 gluon vertex:



$$-g_s f^{abc} V_{\mu_1 \mu_2 \mu_3}(p_1, p_2, p_3)$$

From QED to QCD



$$-ig_s^2 D_3 = \left(-ig_s t^a_{ij} \bar{v}_i(\bar{q})\gamma^\mu u_j(q)\right) \times \left(\frac{-i}{p^2}\right) \times$$

$$\left(-g f^{abc} V_{\mu\nu\rho}(-p, k_1, k_2)\epsilon_1^\nu(k_1)\epsilon_2^\rho(k_2)\right)$$

How do we write down the Lorentz part for this new interaction? We can impose
1. Lorentz invariance : only structure of the type $g_{\mu\nu}\, p_\rho$ are allowed
2. fully anti-symmetry : only structure of the type remain $g_{\mu1\mu2}\ (k_1)_{\mu3}$ are allowed...
3. dimensional analysis : only one power of the momentum.

that uniquely constrain the form of the vertex:

$$V_{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = V_0\left[(p_1 - p_2)_{\mu_3} g_{\mu_1\mu_2} + (p_2 - p_3)_{\mu_1} g_{\mu_2\mu_3} + (p_3 - p_1)_{\mu_2} g_{\mu_3\mu_1}\right]$$

With the above expression we obtain a contribution to the gauge variation:

$$k_1 \cdot D_3 = g^2 f^{abc} t^c V_0 \left[\bar{v}(\bar{q})\not{\epsilon}_2 u(q) - \frac{k_2 \cdot \epsilon_2}{2k_1 \cdot k_2}\bar{v}(\bar{q})\not{k}_1 u(q)\right]$$

The first term cancels the gauge variation of $D_1 + D_2$ if $V_0=1$, the second term is zero IFF the other gluon is physical!!

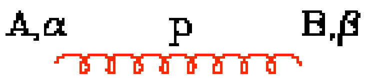
[EXERCISE]: Derive the form of the four-gluon vertex using the same heuristic method

The QCD Lagrangian

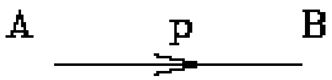
By direct inspection and by using the form non-abelian covariant derivation, we can check that indeed non-abelian gauge symmetry implies self-interactions. This is not surprising since the gluon itself is charged (In QED the photon is not!)

$$\mathcal{L} = \underbrace{-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}}_{\text{Gauge Fields and their interact.}} + \sum_f \underbrace{\bar{\psi}_i^{(f)}(i\partial\!\!\!/ - m_f)\psi_i^{(f)}}_{\text{Matter}} - \underbrace{\bar{\psi}_i^{(f)}(g_s t^a_{ij} A\!\!\!/_a)\psi_j^{(f)}}_{\text{Interaction}}$$

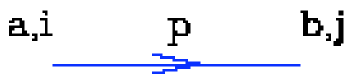
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$



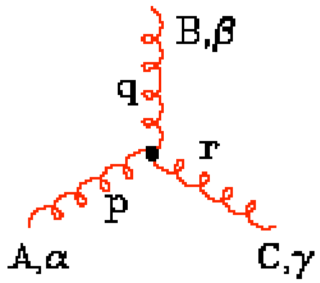
$$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^{\alpha}p^{\beta}}{p^{2}+i\epsilon}\right]\frac{i}{p^{2}+i\epsilon}$$



$$\delta^{AB}\,\frac{i}{(p^{2}+i\epsilon)}$$

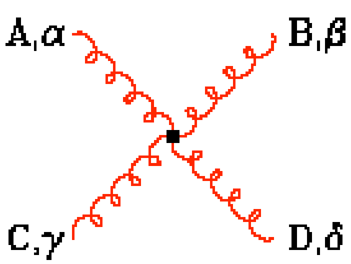


$$\delta^{ab}\,\frac{i}{(\not{p}-m+i\epsilon)_{ji}}$$

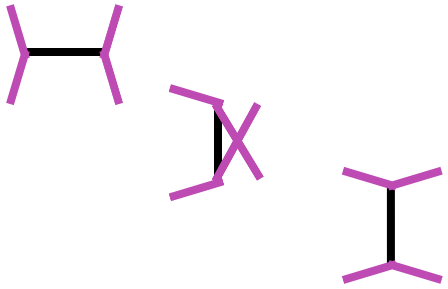


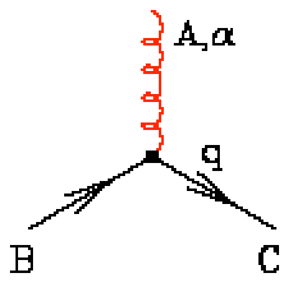
$$-g\ f^{ABC}[(p-q)^{\gamma}g^{\alpha\beta}+(q-r)^{\alpha}g^{\beta\gamma}+(r-p)^{\beta}g^{\gamma\alpha}]$$

(all momenta incoming)

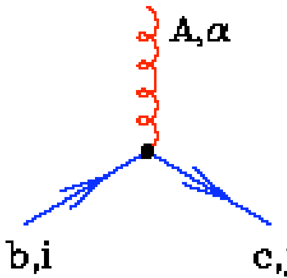


$$-ig^{2}\ f^{XAC}f^{XBD}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$
$$-ig^{2}\ f^{XAD}f^{XBC}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$$
$$-ig^{2}\ f^{XAB}f^{XCD}\ [g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$$





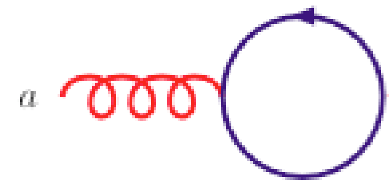
$$g\ f^{ABC}q^{\alpha}$$



$$-ig\ (t^{A})_{cb}\ (\gamma^{\alpha})_{ji}$$

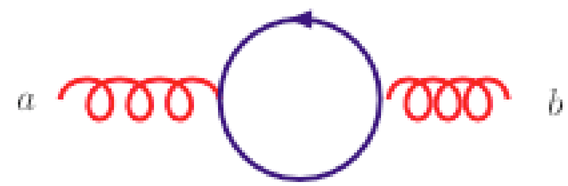
Color algebra

$$\mathrm{Tr}(t^a) = 0$$



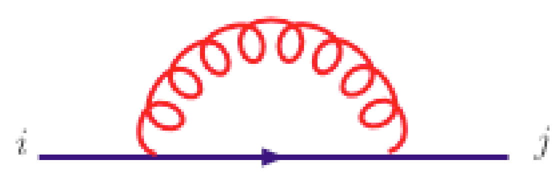
= 0

$$\mathrm{Tr}(t^a t^b) = T_R \delta^{ab}$$



= T_R *

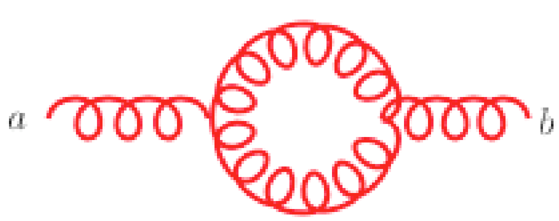
$$(t^a t^a)_{ij} = C_F \delta_{ij}$$



= C_F *

$$\sum_{cd} f^{acd} f^{bcd}$$

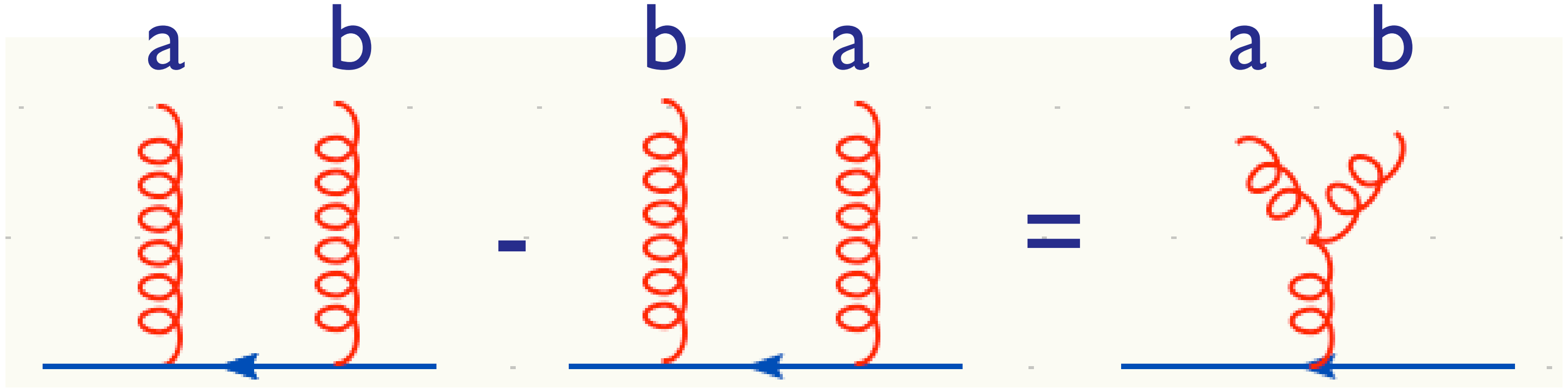
$$= (F^c F^c)_{ab} = C_A \delta_{ab}$$



= C_A*

Color algebra

$$[t^a, t^b] = i f^{abc} t^c$$



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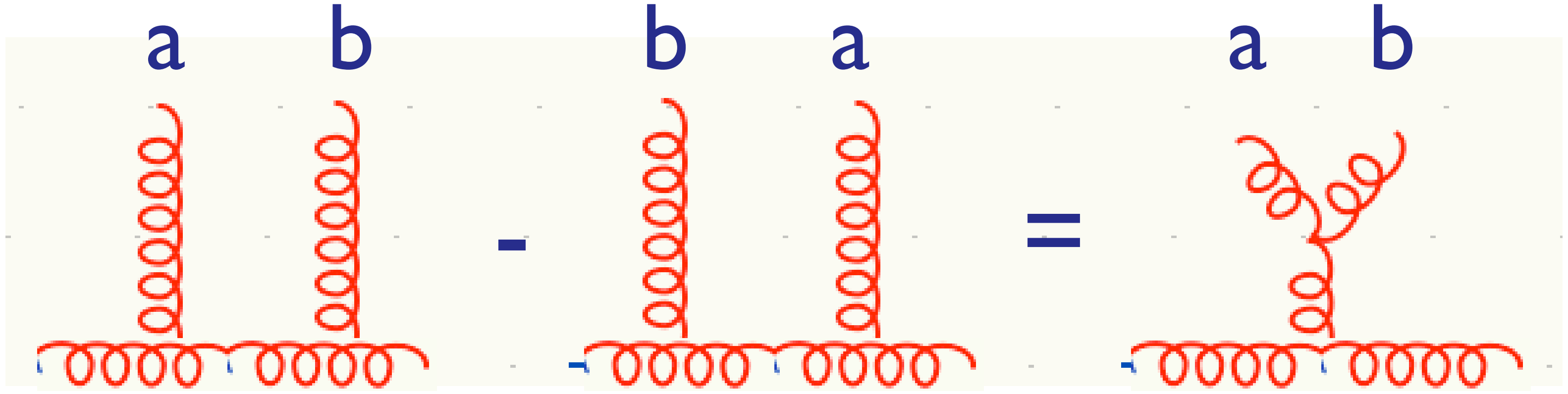

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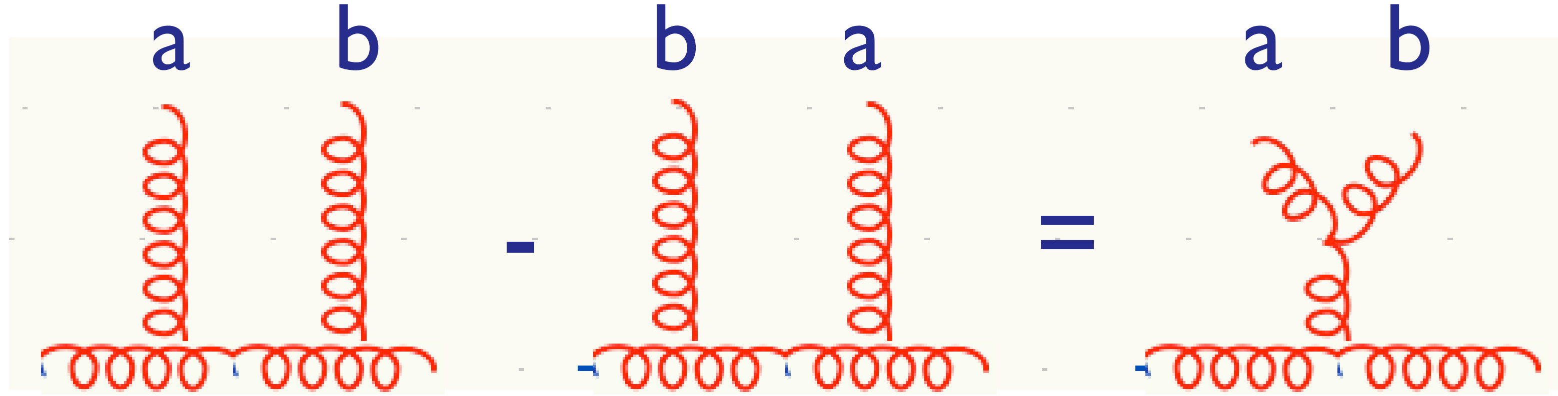

1-loop verteces

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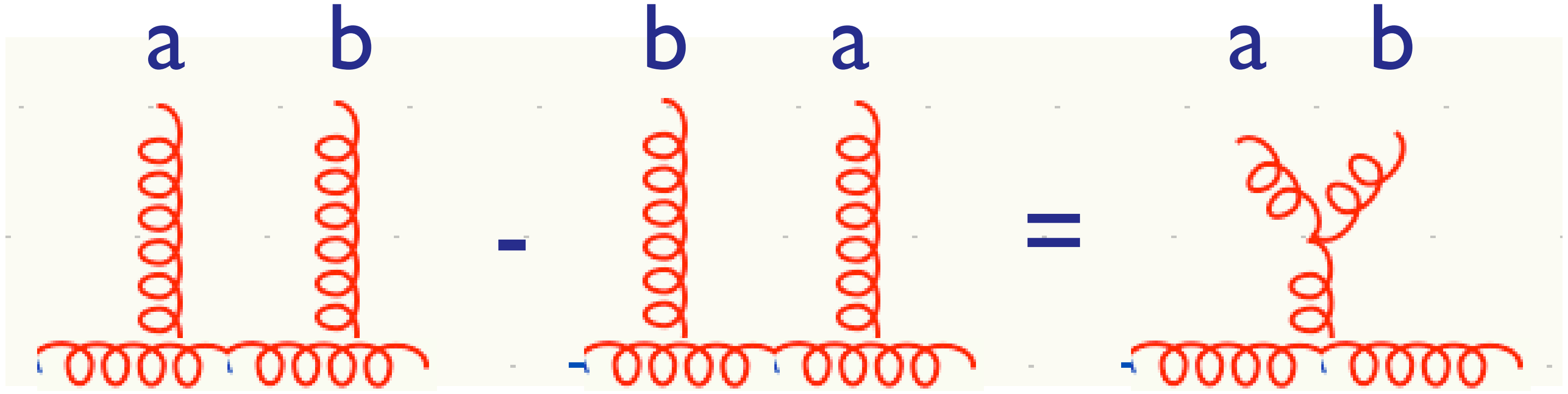

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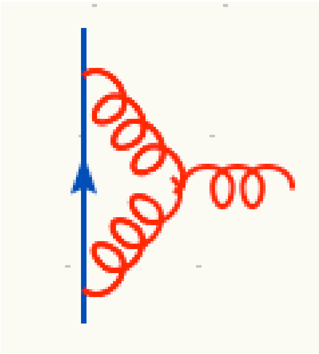
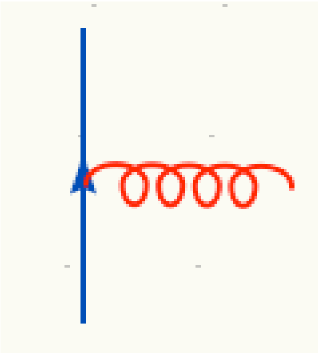
1-loop verteces

$$i f^{abc} (t^b t^c)_{ij} = \frac{C_A}{2} t^a_{ij}$$



= $C_A/2$ *



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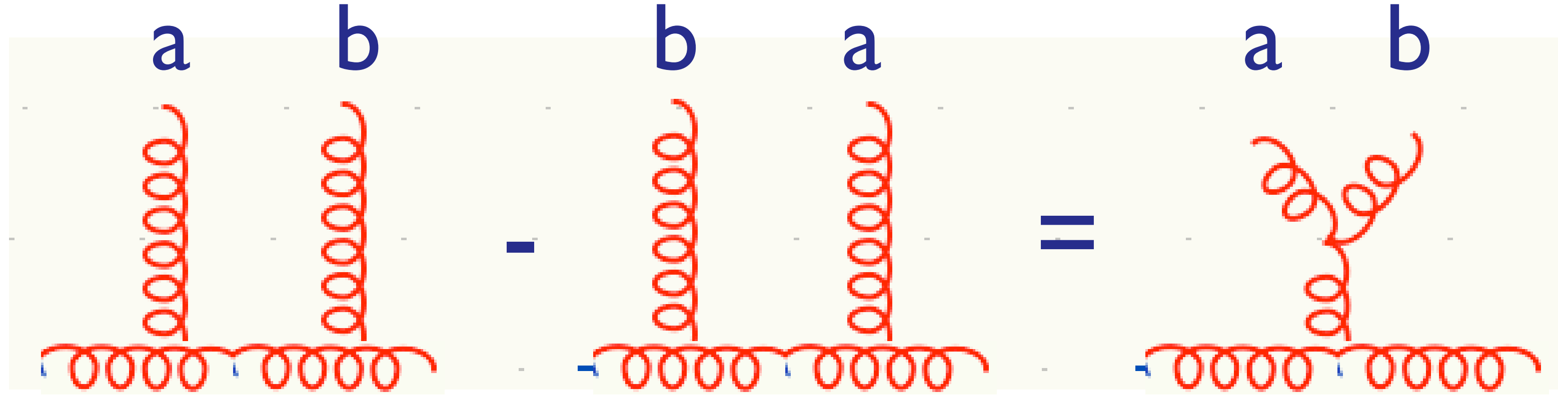

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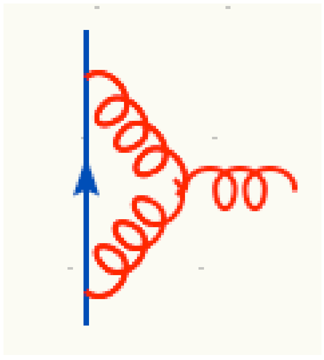

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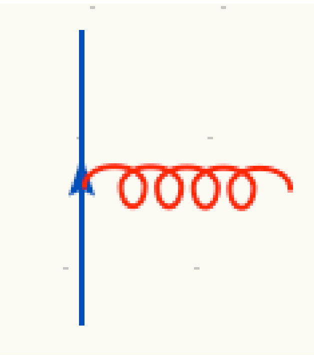
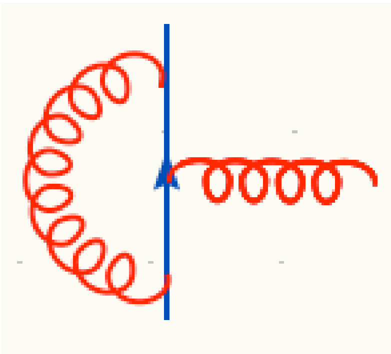
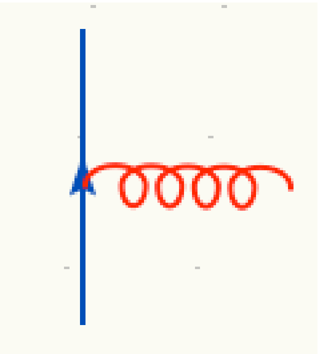


$$(t^b t^a t^b)_{ij} = (C_F - \frac{C_A}{2}) t^a_{ij}$$

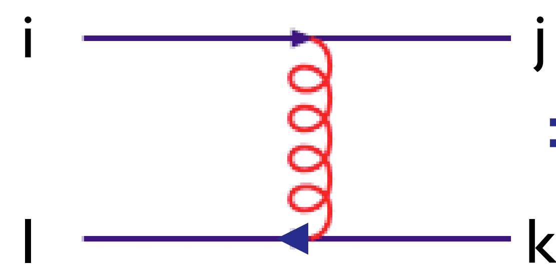


= -1/2/Nc *

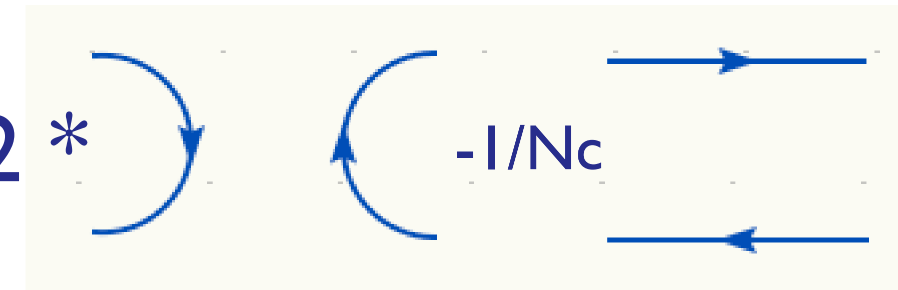


Color algebra: The Fierz identity

$$t^a_{ij} t^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)$$



= 1/2 *



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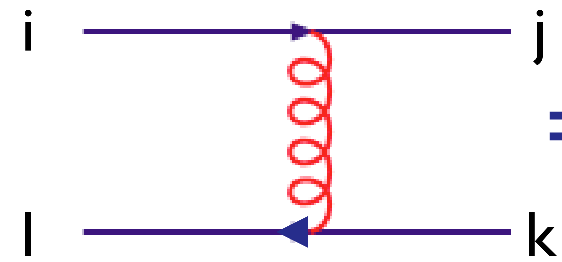

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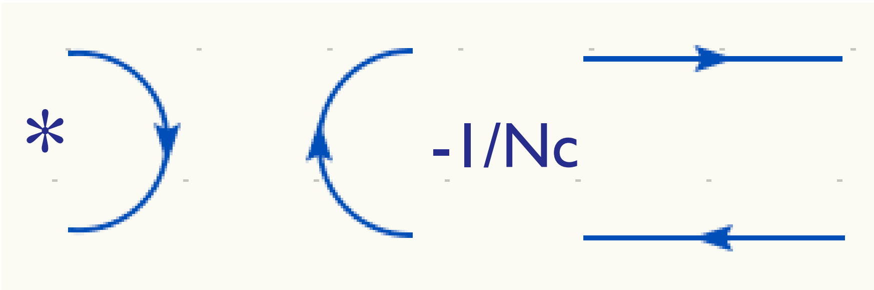

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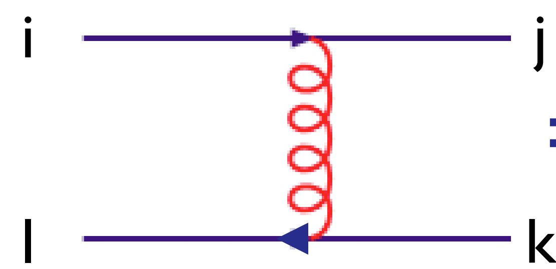

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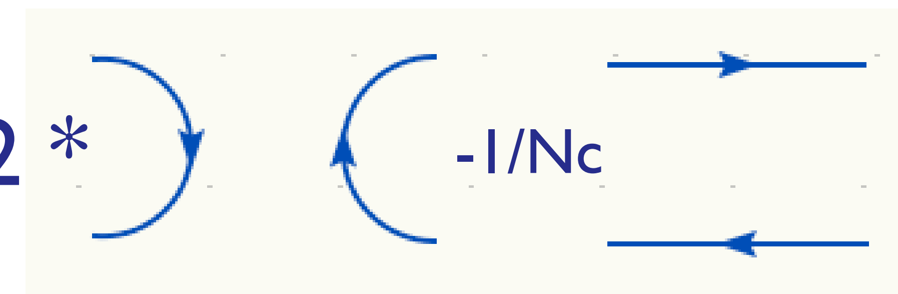

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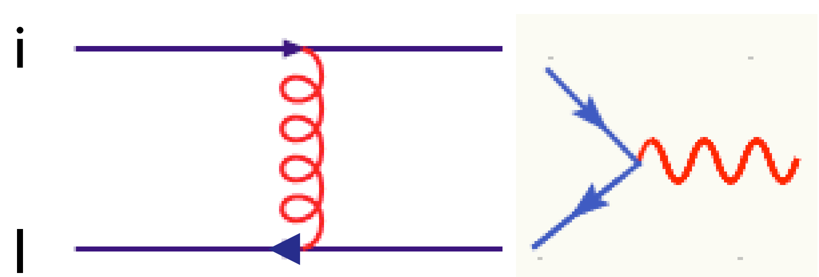
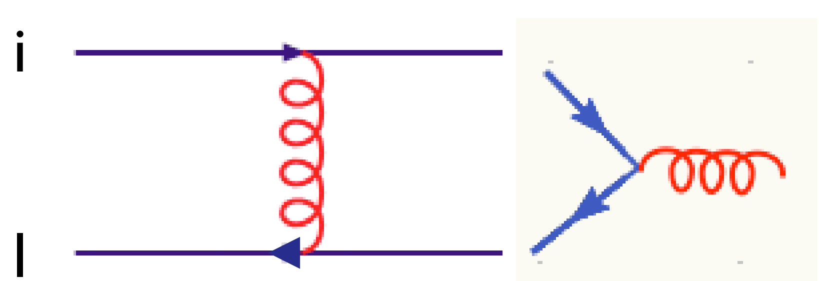


$$\frac{1}{2}(\delta_{ik}\delta_{lj} - \frac{1}{N_c}\delta_{ij}\delta_{lk})\delta_{ki} = \frac{1}{2}\delta_{lj}(N_c - \frac{1}{N_c}) = C_F\delta_{lj}$$



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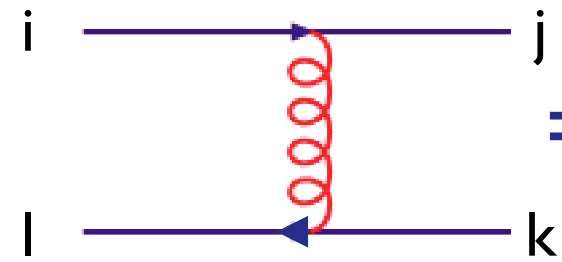
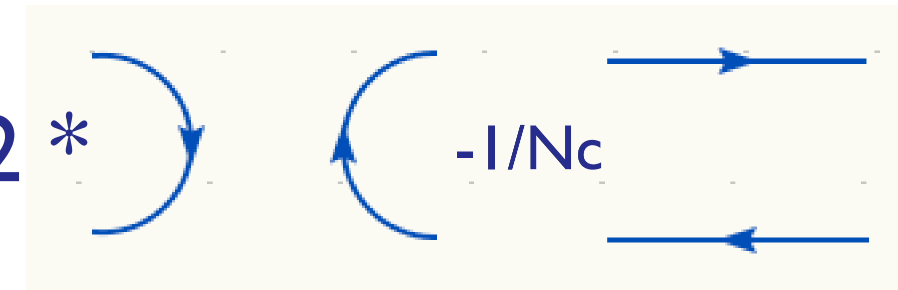


= 1/2 *



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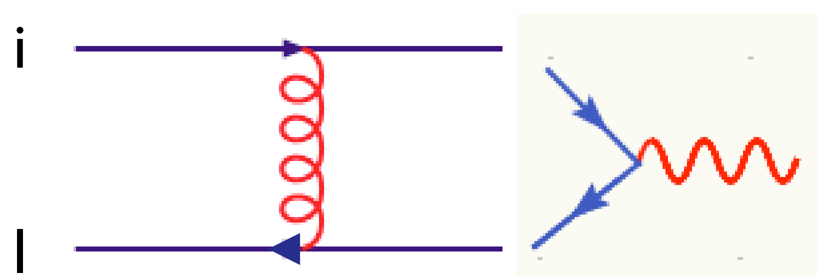

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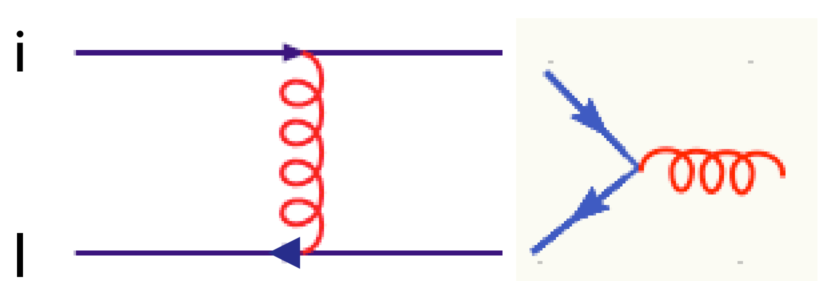

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Example: WBF fusion

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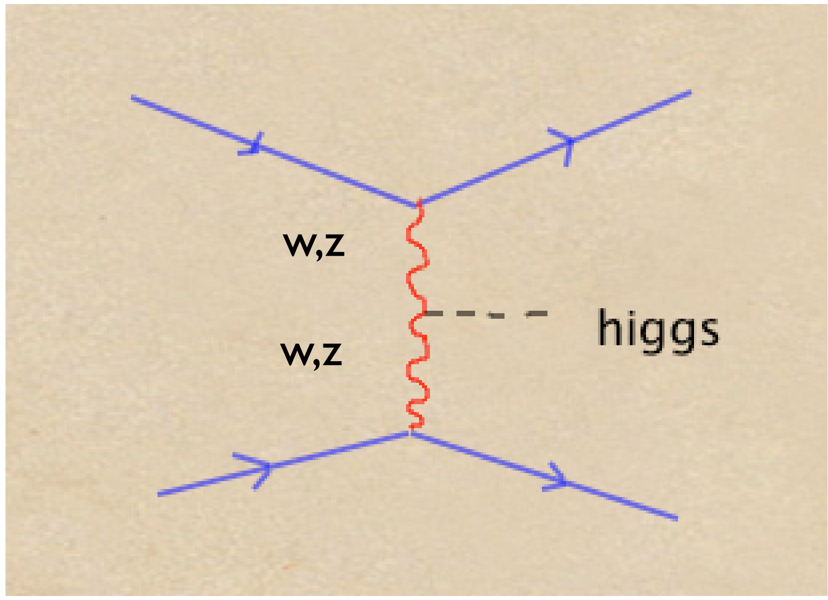

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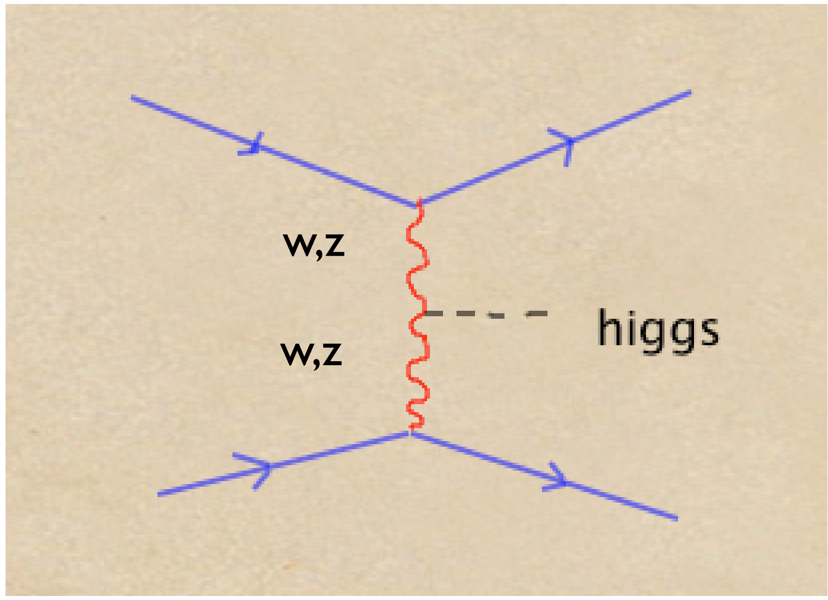

Facts:

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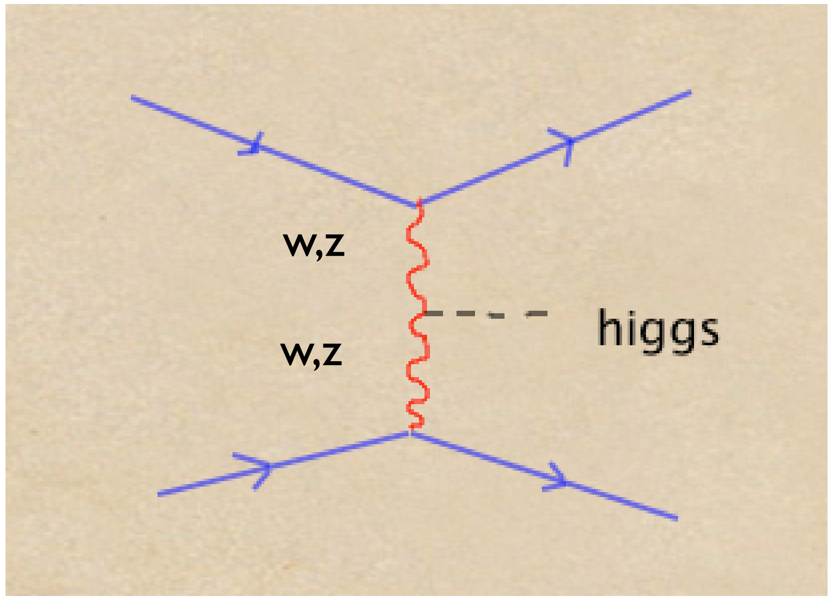

Facts:

1. Important channel for light Higgs both for discovery and measurement

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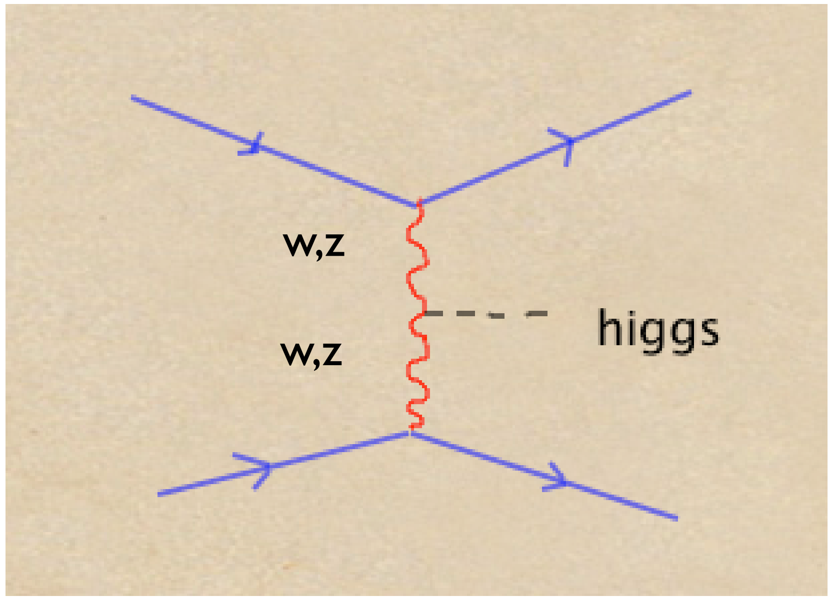

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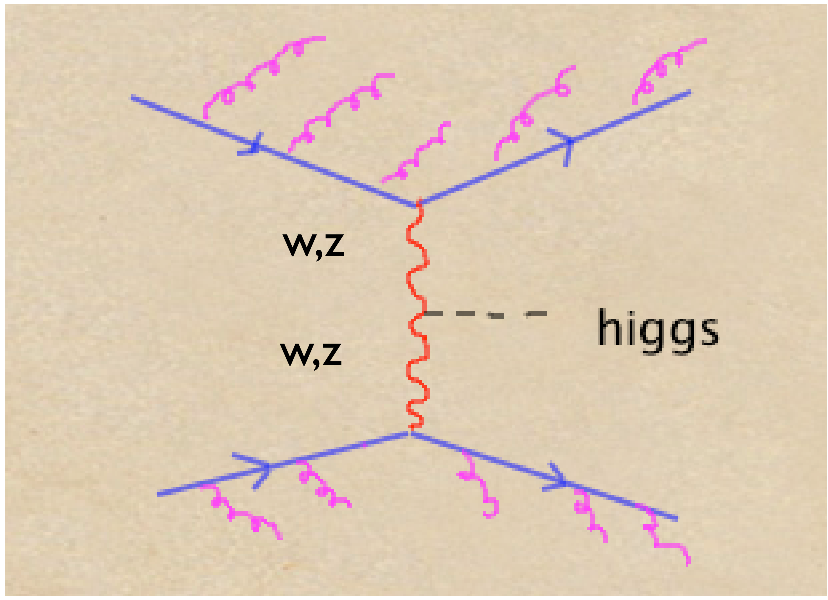

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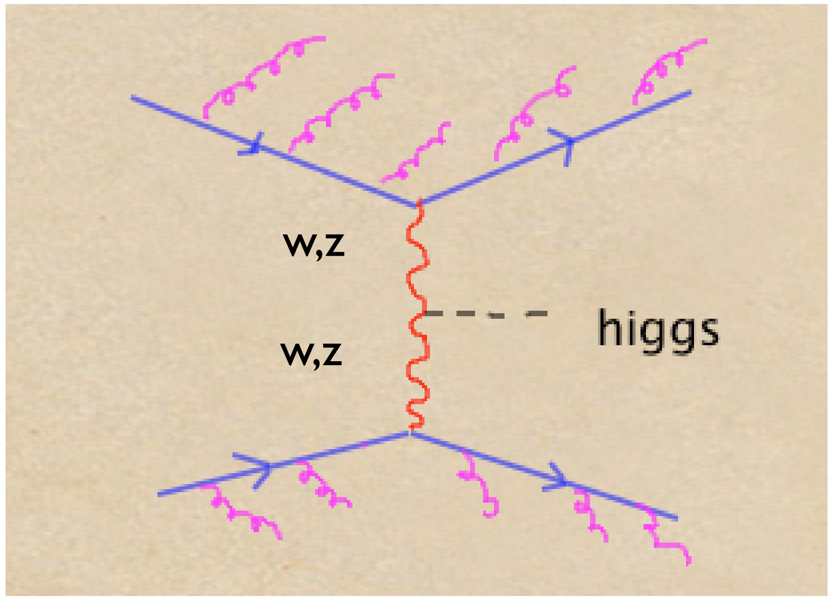

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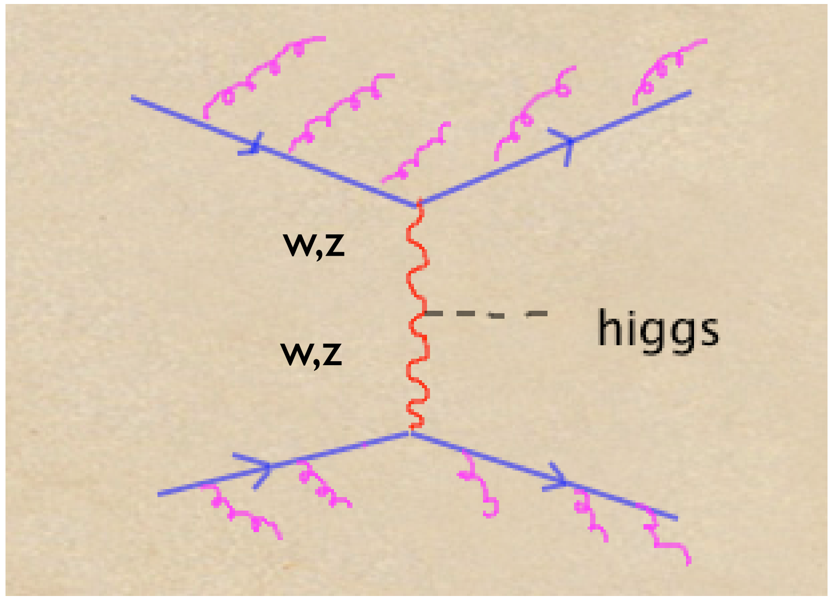

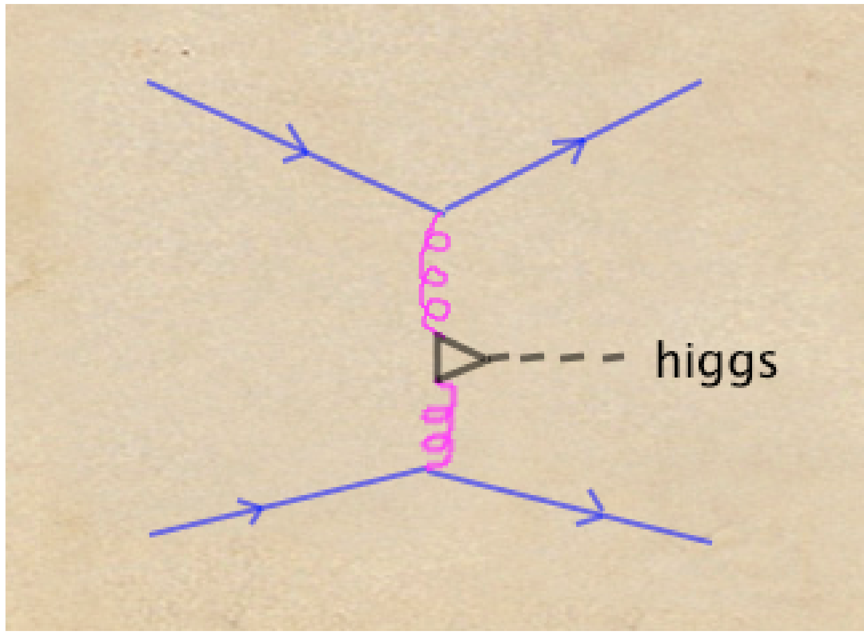

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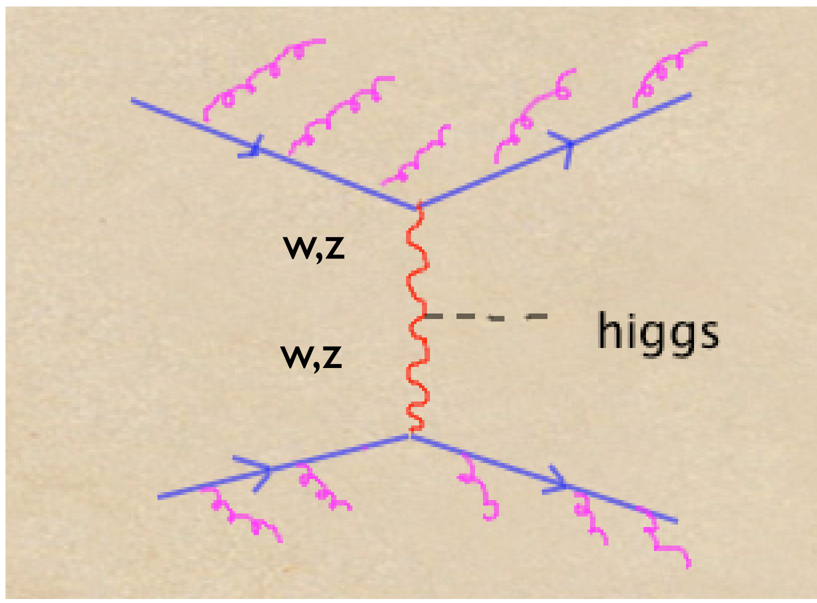

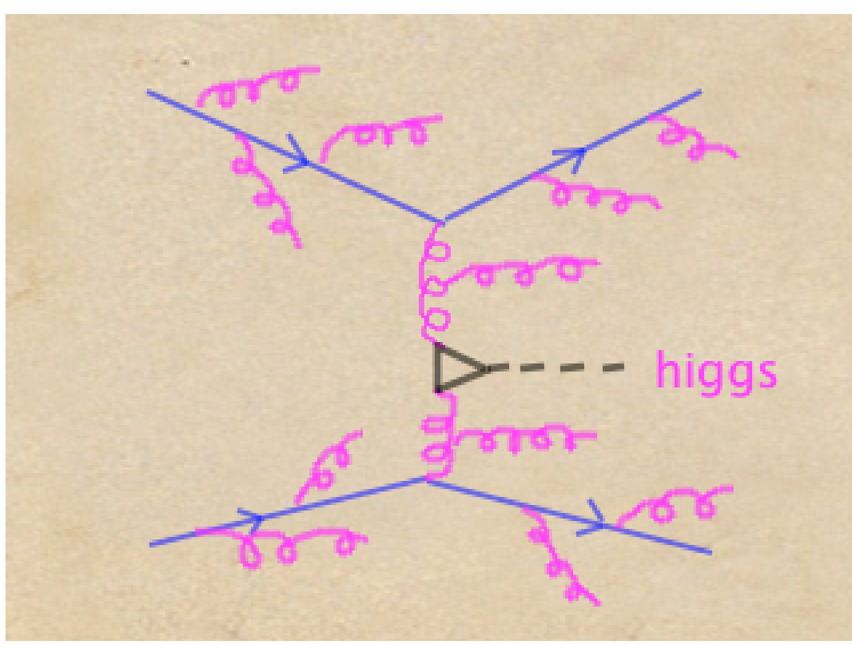

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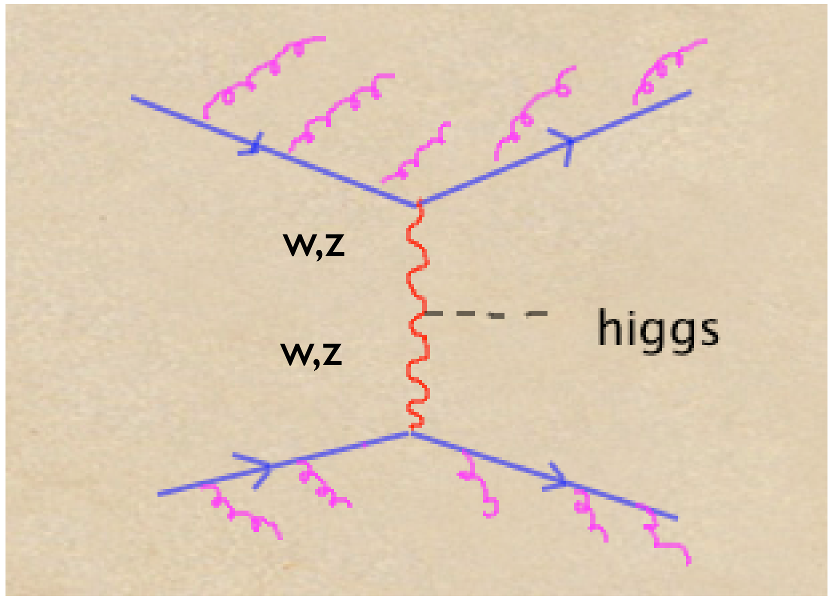

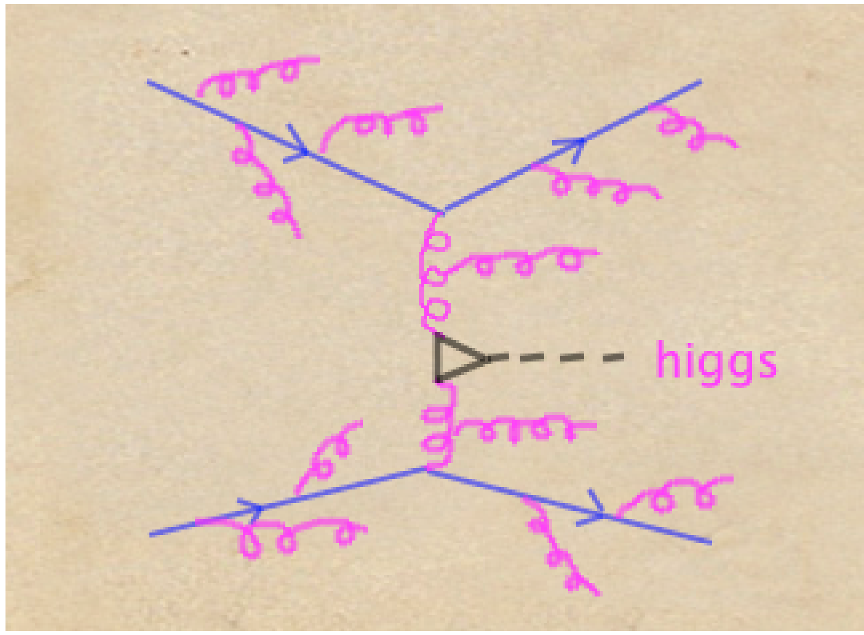

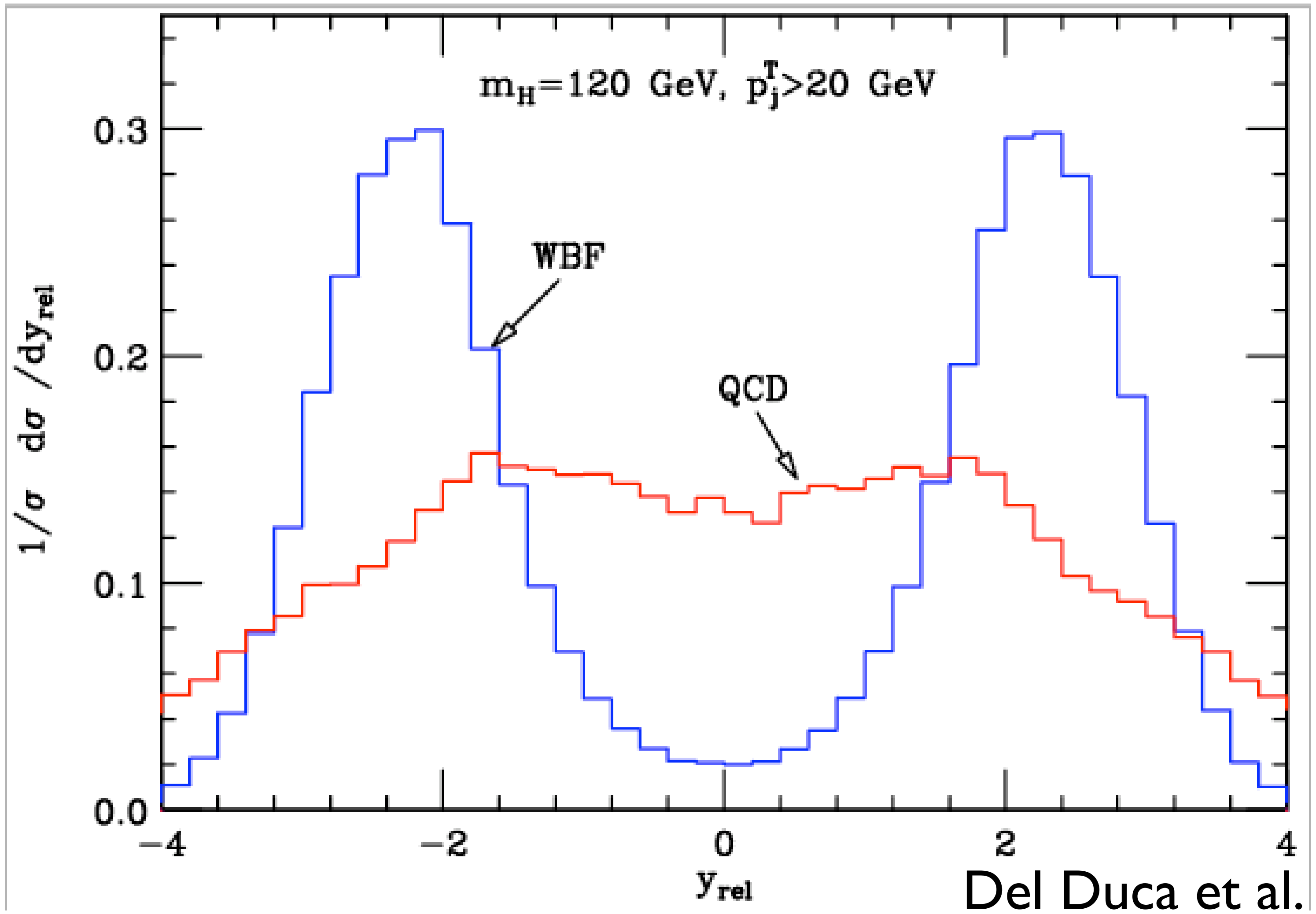

Del Duca et al.

Third jet distribution

Example: WBF fusion

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Consider WBF: at LO there is no exchange of color between the quark lines:

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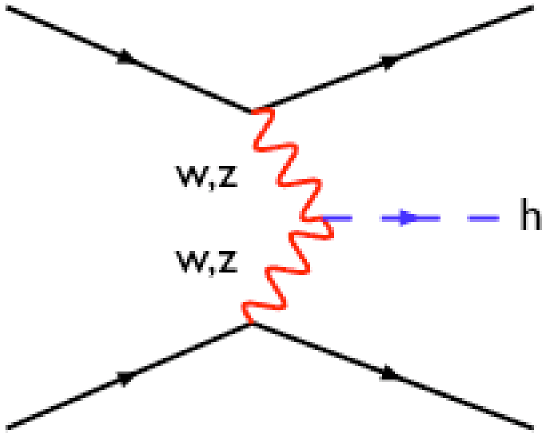

$$\delta_{ij}\delta_{kl}$$

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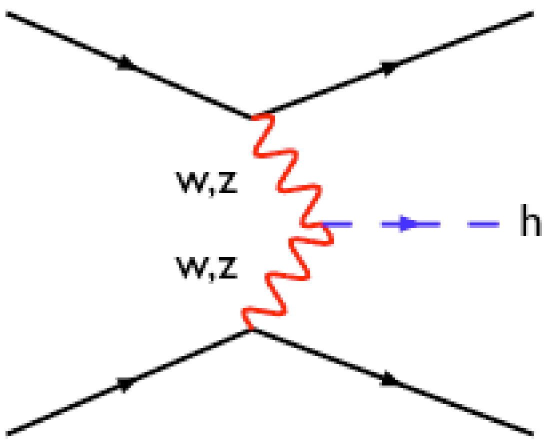

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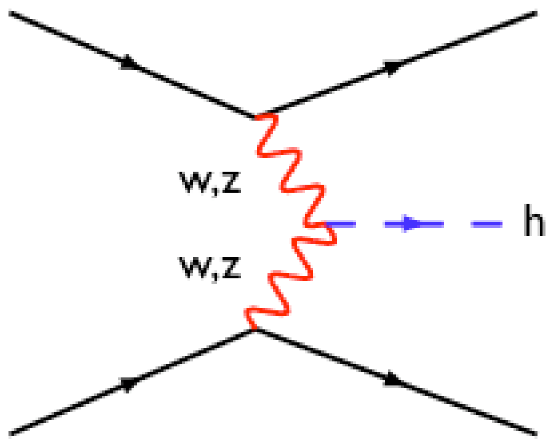

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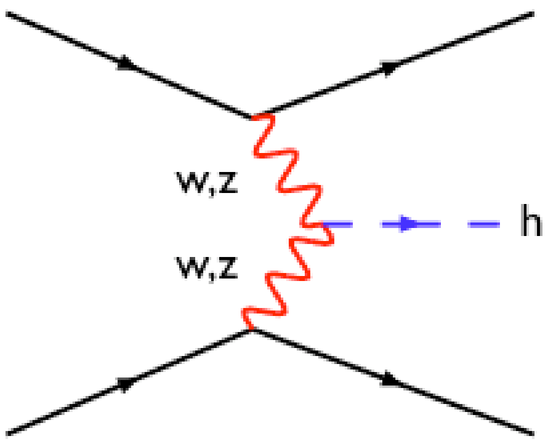

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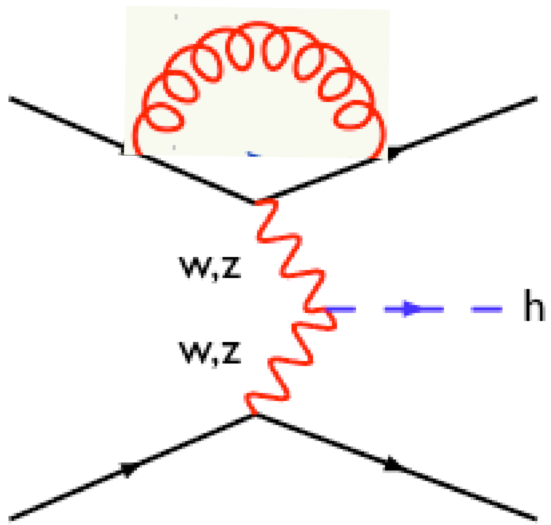

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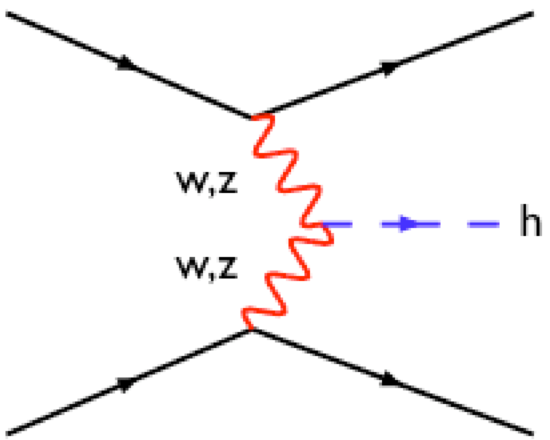

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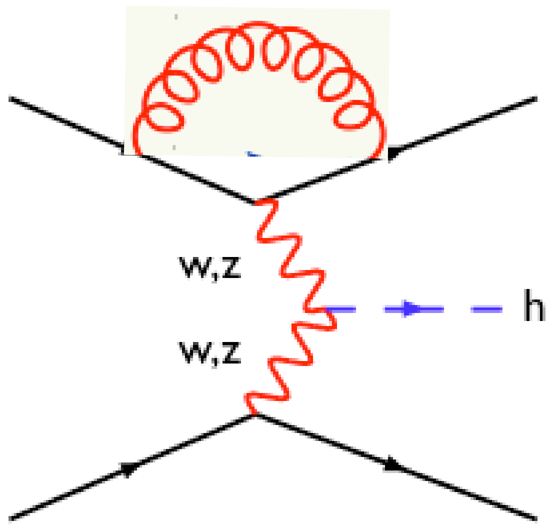

$C_F \delta_{ij}\delta_{kl} \Rightarrow$

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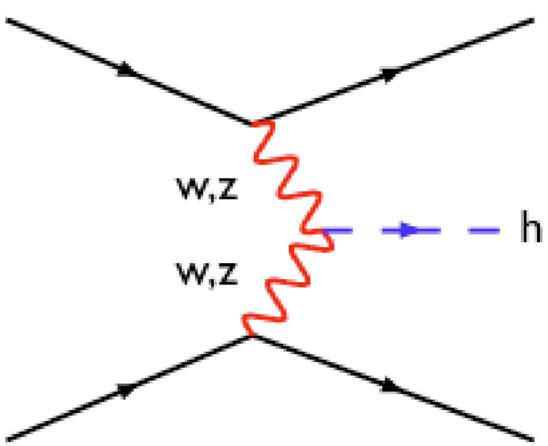

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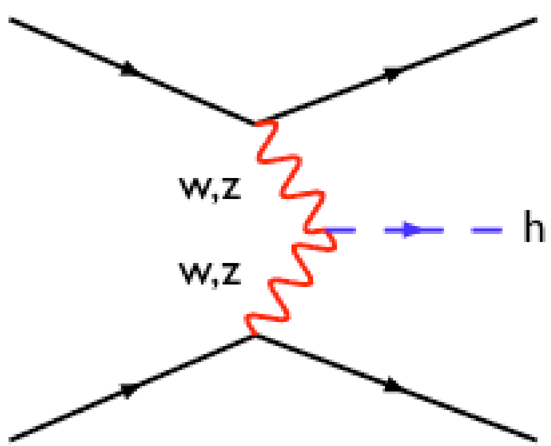

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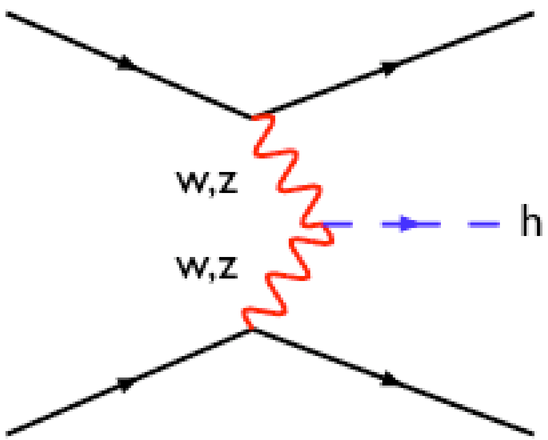

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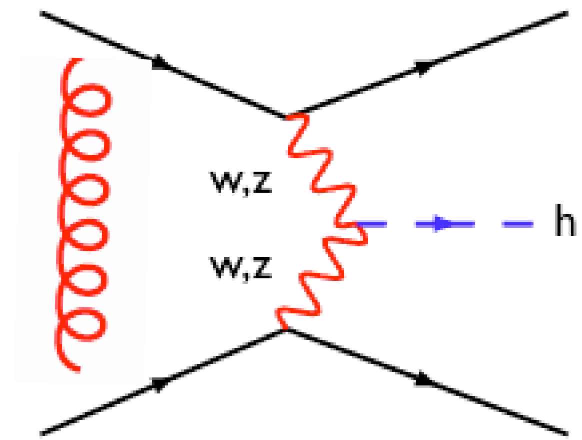

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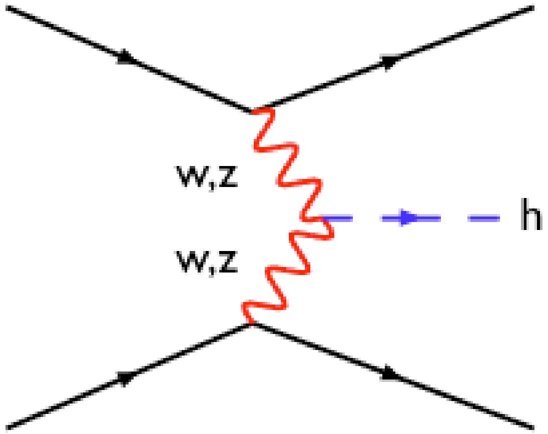

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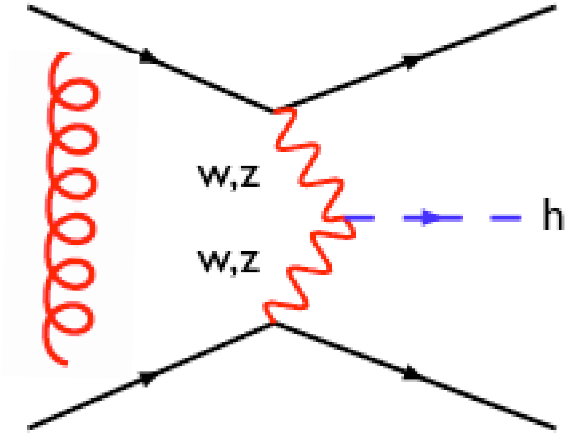

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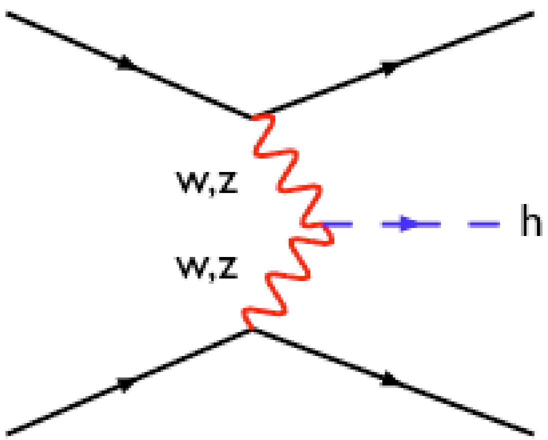

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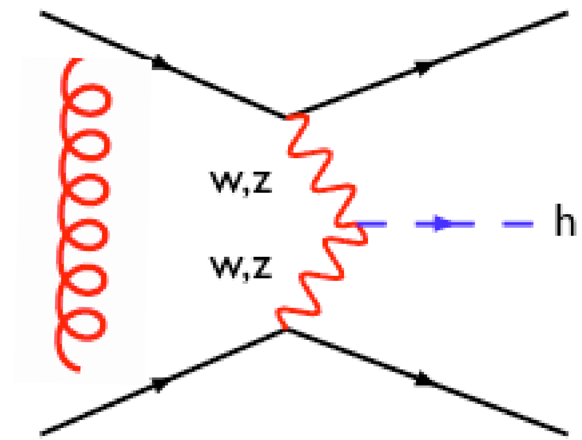

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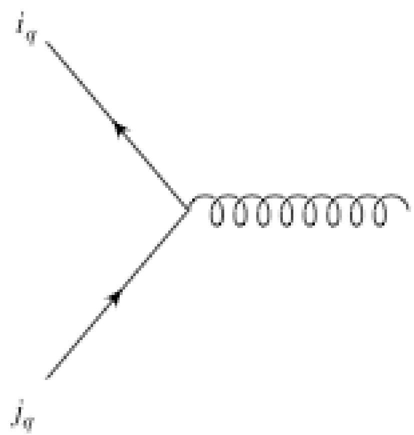
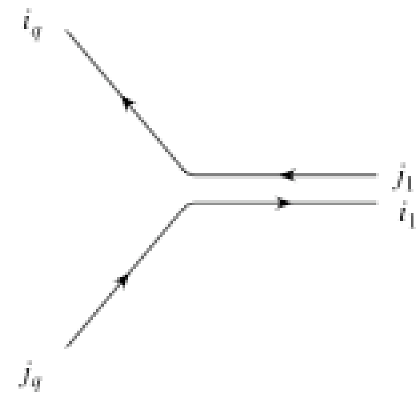
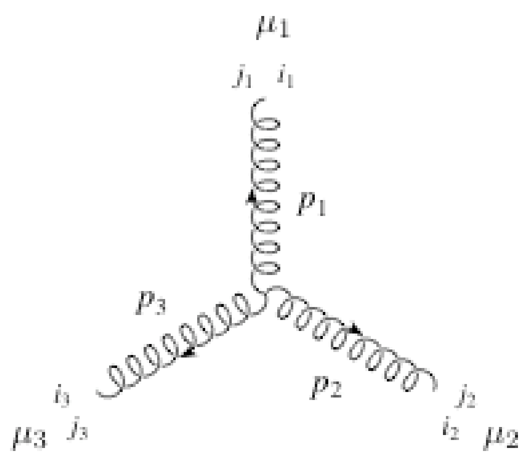
Also at NLO there is no color exchange! With one little exception....

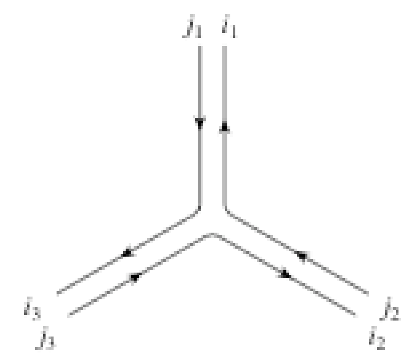
Color algebra: ‘t Hooft double line





$$i\frac{g}{\sqrt{2}}\gamma_1^{\mu}\delta^{i_q}_{j_1}\delta^{i_1}_{j_q}$$

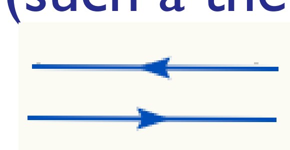




$$i\frac{g}{\sqrt{2}}\sum K^{\mu_1\mu_2\mu_3}\delta^{i_3}_{j_1}\delta^{i_1}_{j_2}\delta^{i_2}_{j_3}$$

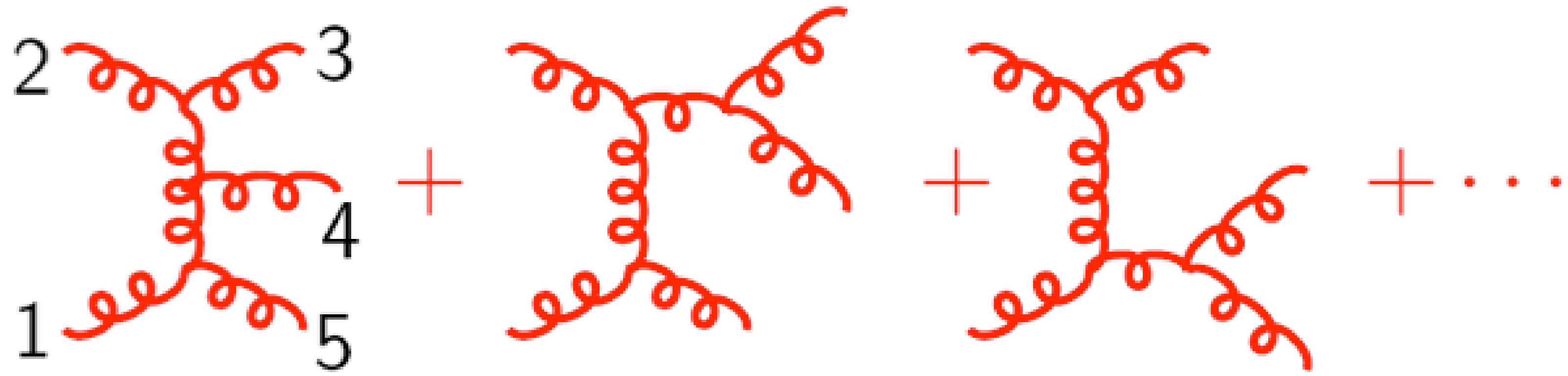
This formulation leads to a graphical representation of the simplifications occuring in the large Nc limit, even though it is exactly equivalent to the usual one.

In the large Nc limit, a gluon behaves as a quark-antiquark pair. In addition it behaves classically, in the sense that quantum interference, which are effects of order $1/Nc^2$ are neglected. Many QCD algorithms and codes (such a the parton showers) are based on this picture.

 ≈ 1/2 

Example: a simple calculation?

Consider a simple 5 gluon amplitude:



There are 25 diagrams with a complicated tensor structure, so you get....

Example: a simple calculation?

```
A(k1,e1,k2,e2,k3,e3,k4,e4,k5,e5) = + Tr(Ta1,Ta2,Ta3,Ta4,Ta5) * ( 1/2*den(2*k1.k2)*k1.e2*e1.e3*e4.e5 - den( 2*k1.k2)*k1.e2*e1.e4*e3.e5
+ 1/2*den(2*k1.k2)*k1.e2*e1.e5*e3.e4 - 1/4 *den(2*k1.k2)*k1.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*k1.e4*e1.e2*e3.e5 - 1/4*den(2*k1.k2)*k1.e5*e1.e2*e3.e4
- 1/2*den(2*k1.k2)*k2.e1*e2.e3* e4.e5 + den(2*k1.k2)*k2.e1*e2.e4*e3.e5 - 1/2*den(2*k1.k2)*k2.e1*e2.e5 *e3.e4 + 1/4*den(2*k1.k2)*k2.e3*e1.e2*e4.e5
- 1/2*den(2*k1.k2)*k2.e4* e1.e2*e3.e5 + 1/4*den(2*k1.k2)*k2.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)* den(2*k3.k4)*k1.k3*k1.e2*e1.e5*e3.e4
- 1/2*den(2*k1.k2)*den(2*k3.k4)* k1.k3*k2.e1*e2.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k2.e5* e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k3.e4*e1.e2*e3.e5
+ 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k4.e3*e1.e2*e4.e5 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.k3*k4.e5*e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*
k3.k4)*k1.k4*k1.e2*e1.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4* k2.e1*e2.e5*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4*k2.e5*e1.e2*
e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4*k3.e4*e1.e2*e3.e5 + 1/2* den(2*k1.k2)*den(2*k3.k4)*k1.k4*k3.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*
den(2*k3.k4)*k1.k4*k4.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*den(2*k3.k4)* k1.k5*k3.e4*e1.e2*e3.e5 - 1/4*den(2*k1.k2)*den(2*k3.k4)*k1.k5*k3.e5*
e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k5*k4.e3*e1.e2*e4.e5 + 1/4*den(2*k1.k2)*den(2*k3.k4)*k1.k5*k4.e5*e1.e2*e3.e4 -
den(2* k1.k2)*den(2*k3.k4)*k1.e2*k1.e3*k3.e4*e1.e5 + den(2*k1.k2)*den(2* k3.k4)*k1.e2*k1.e4*k4.e3*e1.e5 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k1.e5*k3.e1*e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k3.e4*e1.e3 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k4.e1*e3.e4
- den(2* k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k4.e3*e1.e4 + 1/2*den(2*k1.k2)*den(2* k3.k4)*k1.e2*k2.k3*e1.e5*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k2.k4*e1.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e3*k3.e4*e1.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e4*k4.e3*e1.e5 - 1/2*den(2*
k1.k2)*den(2*k3.k4)*k1.e2*k2.e5*k3.e1*e3.e4 + den(2*k1.k2)*den(2* k3.k4)*k1.e2*k2.e5*k3.e4*e1.e3 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k2.e5*k4.e1*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e5*k4.e3*e1.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.k5*e1.e5*e3.e4 +
den(2* k1.k2)*den(2*k3.k4)*k1.e2*k3.e1*k3.e4*e3.e5 - den(2*k1.k2)*den(2* k3.k4)*k1.e2*k3.e1*k4.e3*e4.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k3.e1*k4.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.e4*k3.e5*e1.e3 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.e4*k4.e1*e3.e5 - den(2*k1.k2)*
den(2*k3.k4)*k1.e2*k3.e4*k4.e5*e1.e3 - den(2*k1.k2)*den(2*k3.k4)* k1.e2*k3.e4*k5.e1*e3.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.e4*k5.e3
*e1.e5 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.e5*k4.e1*e3.e4 + den(2* k1.k2)*den(2*k3.k4)*k1.e2*k3.e5*k4.e3*e1.e4 + 1/2*den(2*k1.k2)*den(2*
k3.k4)*k1.e2*k3.e5*k5.e1*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2* k4.k5*e1.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e1*k4.e3*e4.e5
+ den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e3*k4.e5*e1.e4 + den(2*k1.k2)* den(2*k3.k4)*k1.e2*k4.e3*k5.e1*e4.e5 - den(2*k1.k2)*den(2*k3.k4)*
k1.e2*k4.e3*k5.e4*e1.e5 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e5* k5.e1*e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e3*k2.e1*k3.e4*e2.e5
- den(2*k1.k2)*den(2*k3.k4)*k1.e3*k2.e5*k3.e4*e1.e2 + 1/2*den(2*k1.k2)* den(2*k3.k4)*k1.e3*k3.e4*k3.e5*e1.e2 + 1/2*den(2*k1.k2)*den(2*k3.k4)*
k1.e3*k3.e4*k4.e5*e1.e2 - den(2*k1.k2)*den(2*k3.k4)*k1.e4*k2.e1*k4.e3 *e2.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e4*k2.e5*k4.e3*e1.e2
- 1/2*den( 2*k1.k2)*den(2*k3.k4)*k1.e4*k3.e5*k4.e3*e1.e2 - 1/2*den(2*k1.k2)*den( 2*k3.k4)*k1.e4*k4.e3*k4.e5*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*
k1.e5*k2.k3*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.k4* e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k3.e2*e3.e4
- den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k3.e4*e2.e3 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k4.e2*e3.e4 + den(2*k1.k2)*den(2*
k3.k4)*k1.e5*k2.e1*k4.e3*e2.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e5* k2.e3*k3.e4*e1.e2 - den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e4*k4.e3*e1.e2
+ 1/4*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k3.k5*e1.e2*e3.e4 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.e5*k3.e4*k5.e3*e1.e2 - 1/4*den(2*k1.k2)*den(2*
k3.k4)*k1.e5*k4.k5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5* k4.e3*k5.e4*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k3*k2.e1*e2.e5*
e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k3*k3.e4*e1.e2*e3.e5 - 1/2* den(2*k1.k2)*den(2*k3.k4)*k2.k3*k4.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*
den(2*k3.k4)*k2.k3*k4.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)* k2.k4*k2.e1*e2.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k3.e4*
e1.e2*e3.e5 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k3.e5*e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k4.e3*e1.e2*e4.e5 - 1/2*den(2*
k1.k2)*den(2*k3.k4)*k2.k5*k3.e4*e1.e2*e3.e5 + 1/4*den(2*k1.k2)*den(2* k3.k4)*k2.k5*k3.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k5*
k4.e3*e1.e2*e4.e5 - 1/4*den(2*k1.k2)*den(2*k3.k4)*k2.k5*k4.e5*e1.e2* e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e3*k3.e4*e2.e5 -
den(2* k1.k2)*den(2*k3.k4)*k2.e1*k2.e4*k4.e3*e2.e5 + 1/2*den(2*k1.k2)*den(2* k3.k4)*k2.e1*k2.e5*k3.e2*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k2.e1*
k2.e5*k3.e4*e2.e3 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e5*k4.e2* e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e5*k4.e3*e2.e4
```

•••

Example: a simple calculation?

A(k1,e1,k2,e2,k3,e3,k4,e4,k5,e5) = + Tr(Ta1,Ta2,Ta3,Ta4,Ta5) * (1/2*den(2*k1.k2)*k1.e2*e1.e3*e4.e5 - den(2*k1.k2)*k1.e2*e1.e4*e3.e5
+ 1/2*den(2*k1.k2)*k1.e2*e1.e5*e3.e4 - 1/4*den(2*k1.k2)*k1.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*k1.e4*e1.e2*e3.e5 - 1/4*den(2*k1.k2)*k1.e5*e1.e2*e3.e4
- 1/2*den(2*k1.k2)*k2.e1*e2.e3* e4.e5 + den(2*k1.k2)*k2.e1*e2.e4*e3.e5 - 1/2*den(2*k1.k2)*k2.e1*e2.e5 *e3.e4 + 1/4*den(2*k1.k2)*k2.e3*e1.e2*e4.e5
- 1/2*den(2*k1.k2)*k2.e4* e1.e2*e3.e5 + 1/4*den(2*k1.k2)*k2.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)* den(2*k3.k4)*k1.k3*k1.e2*e1.e5*e3.e4
- 1/2*den(2*k1.k2)*den(2*k3.k4)* k1.k3*k2.e1*e2.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k2.e5* e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k3.e4*e1.e2*e3.e5
+ 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k4.e3*e1.e2*e4.e5 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.k3*k4.e5*e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*
k3.k4)*k1.k4*k1.e2*e1.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4* k2.e1*e2.e5*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4*k2.e5*e1.e2*
e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4*k3.e4*e1.e2*e3.e5 + 1/2* den(2*k1.k2)*den(2*k3.k4)*k1.k4*k3.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*

+ Tr(Ta1,Ta2,Ta3,Ta4,Ta5) * (1/2*den(2*k1.k2)*k1.e2*e1.e3*e4.e5

k1.e5*k3.e1*e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k3.e4*e1.e3 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k4.e1*e3.e4
- den(2* k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k4.e3*e1.e4 + 1/2*den(2*k1.k2)*den(2* k3.k4)*k1.e2*k2.k3*e1.e5*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k2.k4*e1.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e3*k3.e4*e1.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e4*k4.e3*e1.e5 - 1/2*den(2*
k1.k2)*den(2*k3.k4)*k1.e2*k2.e5*k3.e1*e3.e4 + den(2*k1.k2)*den(2* k3.k4)*k1.e2*k2.e5*k3.e4*e1.e3 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k2.e5*k4.e1*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e5*k4.e3*e1.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.k5*e1.e5*e3.e4 +
den(2* k1.k2)*den(2*k3.k4)*k1.e2*k3.e1*k3.e4*e3.e5 - den(2*k1.k2)*den(2* k3.k4)*k1.e2*k3.e1*k4.e3*e4.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k3.e1*k4.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.e4*k3.e5*e1.e3 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.e4*k4.e1*e3.e5 - den(2*k1.k2)*
den(2*k3.k4)*k1.e2*k3.e4*k4.e5*e1.e3 - den(2*k1.k2)*den(2*k3.k4)* k1.e2*k3.e4*k5.e1*e3.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.e4*k5.e3
*e1.e5 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.e5*k4.e1*e3.e4 + den(2* k1.k2)*den(2*k3.k4)*k1.e2*k3.e5*k4.e3*e1.e4 + 1/2*den(2*k1.k2)*den(2*
k3.k4)*k1.e2*k3.e5*k5.e1*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2* k4.k5*e1.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e1*k4.e3*e4.e5
+ den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e3*k4.e5*e1.e4 + den(2*k1.k2)* den(2*k3.k4)*k1.e2*k4.e3*k5.e1*e4.e5 - den(2*k1.k2)*den(2*k3.k4)*
k1.e2*k4.e3*k5.e4*e1.e5 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e5* k5.e1*e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e3*k2.e1*k3.e4*e2.e5
- den(2*k1.k2)*den(2*k3.k4)*k1.e3*k2.e5*k3.e4*e1.e2 + 1/2*den(2*k1.k2)* den(2*k3.k4)*k1.e3*k3.e4*k3.e5*e1.e2 + 1/2*den(2*k1.k2)*den(2*k3.k4)*
k1.e3*k3.e4*k4.e5*e1.e2 - den(2*k1.k2)*den(2*k3.k4)*k1.e4*k2.e1*k4.e3 *e2.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e4*k2.e5*k4.e3*e1.e2
- 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e4*k3.e5*k4.e3*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e4*k4.e3*k4.e5*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*
k1.e5*k2.k3*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.k4* e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k3.e2*e3.e4
- den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k3.e4*e2.e3 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k4.e2*e3.e4 + den(2*k1.k2)*den(2*
k3.k4)*k1.e5*k2.e1*k4.e3*e2.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e5* k2.e3*k3.e4*e1.e2 - den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e4*k4.e3*e1.e2
+ 1/4*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k3.k5*e1.e2*e3.e4 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.e5*k3.e4*k5.e3*e1.e2 - 1/4*den(2*k1.k2)*den(2*
k3.k4)*k1.e5*k4.k5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5* k4.e3*k5.e4*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k3*k2.e1*e2.e5*
e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k3*k3.e4*e1.e2*e3.e5 - 1/2* den(2*k1.k2)*den(2*k3.k4)*k2.k3*k4.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*
den(2*k3.k4)*k2.k3*k4.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)* k2.k4*k2.e1*e2.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k3.e4*
e1.e2*e3.e5 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k3.e5*e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k4*k4.e3*e1.e2*e4.e5 - 1/2*den(2*
k1.k2)*den(2*k3.k4)*k2.k5*k3.e4*e1.e2*e3.e5 + 1/4*den(2*k1.k2)*den(2* k3.k4)*k2.k5*k3.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k5*
k4.e3*e1.e2*e4.e5 - 1/4*den(2*k1.k2)*den(2*k3.k4)*k2.k5*k4.e5*e1.e2* e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e3*k3.e4*e2.e5 -
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k2.e5*k3.e4*e2.e3 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e5*k4.e2* e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e5*k4.e3*e2.e4

Example: a simple calculation?

```
A(k1,e1,k2,e2,k3,e3,k4,e4,k5,e5) = + Tr(Ta1,Ta2,Ta3,Ta4,Ta5) * ( 1/2*den(2*k1.k2)*k1.e2*e1.e3*e4.e5 - den(2*k1.k2)*k1.e2*e1.e4*e3.e5
+ 1/2*den(2*k1.k2)*k1.e2*e1.e5*e3.e4 - 1/4*den(2*k1.k2)*k1.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*k1.e4*e1.e2*e3.e5 - 1/4*den(2*k1.k2)*k1.e5*e1.e2*e3.e4
- 1/2*den(2*k1.k2)*k2.e1*e2.e3* e4.e5 + den(2*k1.k2)*k2.e1*e2.e4*e3.e5 - 1/2*den(2*k1.k2)*k2.e1*e2.e5 *e3.e4 + 1/4*den(2*k1.k2)*k2.e3*e1.e2*e4.e5
- 1/2*den(2*k1.k2)*k2.e4* e1.e2*e3.e5 + 1/4*den(2*k1.k2)*k2.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)* den(2*k3.k4)*k1.k3*k1.e2*e1.e5*e3.e4
- 1/2*den(2*k1.k2)*den(2*k3.k4)* k1.k3*k2.e1*e2.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k2.e5* e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k3.e4*e1.e2*e3.e5
+ 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k3*k4.e3*e1.e2*e4.e5 - 1/2*den(2* k1.k2)*den(2*k3.k4)*k1.k3*k4.e5*e1.e2*e3.e4 - 1/2*den(2*k1.k2)*den(2*
k3.k4)*k1.k4*k1.e2*e1.e5*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4* k2.e1*e2.e5*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4*k2.e5*e1.e2*
e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.k4*k3.e4*e1.e2*e3.e5 + 1/2* den(2*k1.k2)*den(2*k3.k4)*k1.k4*k3.e5*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*
```

+ Tr(Ta1,Ta2,Ta3,Ta4,Ta5) * (1/2*den(2*k1.k2)*k1.e2*e1.e3*e4.e5

```
k1.e5*k3.e1*e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k3.e4*e1.e3 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k4.e1*e3.e4
- den(2* k1.k2)*den(2*k3.k4)*k1.e2*k1.e5*k4.e3*e1.e4 + 1/2*den(2*k1.k2)*den(2* k3.k4)*k1.e2*k2.k3*e1.e5*e3.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k2.k4*e1.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e3*k3.e4*e1.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e4*k4.e3*e1.e5 - 1/2*den(2*
k1.k2)*den(2*k3.k4)*k1.e2*k2.e5*k3.e1*e3.e4 + den(2*k1.k2)*den(2* k3.k4)*k1.e2*k2.e5*k3.e4*e1.e3 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k2.e5*k4.e1*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k2.e5*k4.e3*e1.e4 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k3.k5*e1.e5*e3.e4 +
den(2* k1.k2)*den(2*k3.k4)*k1.e2*k3.e1*k3.e4*e3.e5 - den(2*k1.k2)*den(2* k3.k4)*k1.e2*k3.e1*k4.e3*e4.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e2*
k3.e1*k4.e5*e3.e4 - den(2*k                                                                   k1.k2)*
den(2*k3.k4)*k1.e2*k3.e4*                                                                     *k5.e3
*e1.e5 - den(2*k1.k2)*den(2                                                                   )*den(2*
k3.k4)*k1.e2*k3.e5*k5.e1*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2* k4.k5*e1.e5*e3.e4 - den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e1*k4.e3*e4.e5
+ den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e3*k4.e5*e1.e4 + den(2*k1.k2)* den(2*k3.k4)*k1.e2*k4.e3*k5.e1*e4.e5 - den(2*k1.k2)*den(2*k3.k4)*
k1.e2*k4.e3*k5.e4*e1.e5 - 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e2*k4.e5* k5.e1*e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e3*k2.e1*k3.e4*e2.e5
- den(2*k1.k2)*den(2*k3.k4)*k1.e3*k2.e5*k3.e4*e1.e2 + 1/2*den(2*k1.k2)* den(2*k3.k4)*k1.e3*k3.e4*k3.e5*e1.e2 + 1/2*den(2*k1.k2)*den(2*k3.k4)*
k1.e3*k3.e4*k4.e5*e1.e2 - den(2*k1.k2)*den(2*k3.k4)*k1.e4*k2.e1*k4.e3 *e2.e5 + den(2*k1.k2)*den(2*k3.k4)*k1.e4*k2.e5*k4.e3*e1.e2
- 1/2*den( 2*k1.k2)*den(2*k3.k4)*k1.e4*k3.e5*k4.e3*e1.e2 - 1/2*den(2*k1.k2)*den( 2*k3.k4)*k1.e4*k4.e3*k4.e5*e1.e2 - 1/2*den(2*k1.k2)*den(2*k3.k4)*
k1.e5*k2.k3*e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.k4* e1.e2*e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e1*k3.e2*e3.e4
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k3.k4)*k1.e5*k2.e1*k4.e3*e2.e4 + den(2*k1.k2)*den(2*k3.k4)*k1.e5* k2.e3*k3.e4*e1.e2 - den(2*k1.k2)*den(2*k3.k4)*k1.e5*k2.e4*k4.e3*e1.e2
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e3.e4 + 1/2*den(2*k1.k2)*den(2*k3.k4)*k2.k3*k3.e4*e1.e2*e3.e5 - 1/2* den(2*k1.k2)*den(2*k3.k4)*k2.k3*k4.e3*e1.e2*e4.e5 + 1/2*den(2*k1.k2)*
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k4.e3*e1.e2*e4.e5 - 1/4*den(2*k1.k2)*den(2*k3.k4)*k2.k5*k4.e5*e1.e2* e3.e4 + den(2*k1.k2)*den(2*k3.k4)*k2.e1*k2.e3*k3.e4*e2.e5 -
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```

Brute force is not an option!

Solution

Keep track of all the quantum numbers,
(momenta, spin and color)
and organize them in
efficient way, by choosing appropriate basis.

The helicity method

Pioneering work of Berends, Gastmans, Troost, Wu in the '80, where they introduce the techniques of helicity amplitudes

$$u_\pm(k) = \frac{1}{2}(1 \pm \gamma_5)u(k)$$

$$\overline{u_-(k_i)}u_+(k_j) = \langle k_i - | k_j + \rangle \equiv \langle ij \rangle = \sqrt{s_{ij}}e^{-i\phi}$$

$$\overline{u_+(k_i)}u_-(k_j) = \langle k_i + | k_j - \rangle \equiv [ij] = -\sqrt{s_{ij}}e^{i\phi}$$

Using these objects, Xu, Zhang and Chang (1987) introduced simple vector polarizations

$$\varepsilon_\mu^+(k;q) = \frac{\langle q^- | \gamma_\mu | k^- \rangle}{\sqrt{2}\,\langle q\,k \rangle}, \qquad \varepsilon_\mu^-(k,q) = \frac{\langle q^+ | \gamma_\mu | k^+ \rangle}{\sqrt{2}\,[k\,q]}$$

gauge vector

It's just a more sophisticated version of the circular polarization. Choosing appropriately the gauge vector, expressions simplify dramatically.

Stripping color out

Inspired by the way gauge theories appear as the zero-slope limits of (open) string theories, it has been suggested to decompose the full amplitude as a sum of gauge invariant Subamplitudes times color coefficients:

$$\mathcal{A}_n(g_1,\ldots,g_n) = g^{n-2} \sum_{\sigma\in S_{n-1}} \mathrm{Tr}(\mathsf{t}^{a_1}\,\mathsf{t}^{a_{\sigma_2}}\cdots\mathsf{t}^{a_{\sigma_n}})A_n(1,\sigma_2,\ldots,\sigma_n)$$

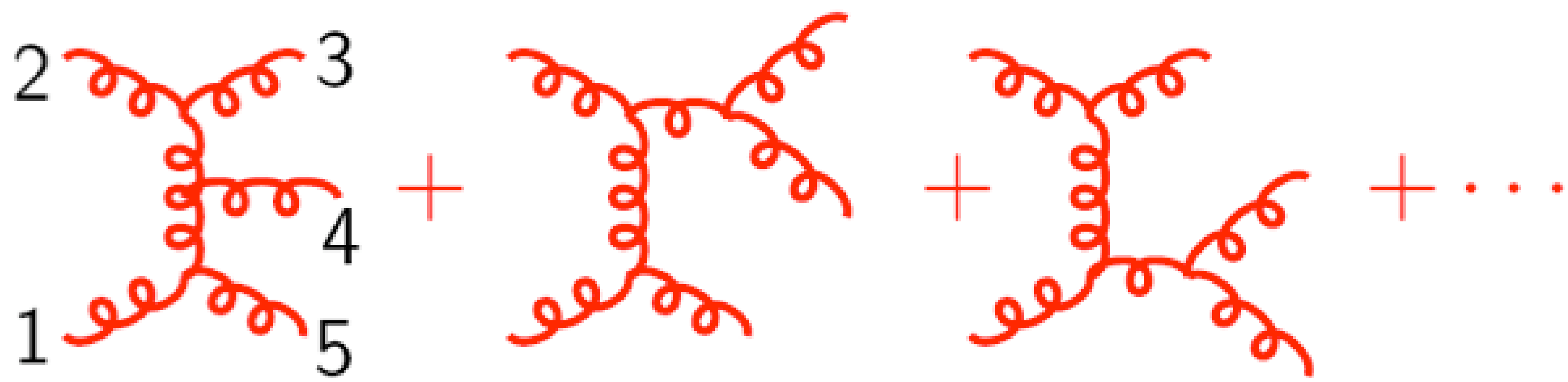
where the formula $if^{abc} = \mathrm{Tr}(\mathsf{t}^{a},[\mathsf{t}^{b},\mathsf{t}^{c}])$ has been repeatedly used to reduce the f's into traces of lambdas and the Fierz identities to cancel traces of length l<n.

Analogously for quarks:

$$\mathcal{A}_n(q_1,g_2,\ldots,g_{n-1},\bar{q}_n) = g^{n-2} \sum_{\sigma\in S_{n-2}} (\mathsf{t}^{a_{\sigma_2}}\cdots\mathsf{t}^{a_{\sigma_{n-1}}})^i_j A_n(1_q,\sigma_2,\ldots,\sigma_{n-2},n_{\bar{q}})$$

Example

Consider a simple 5 gluon amplitude:



There are 25 diagrams with a complicated tensor structure, but only 10 for a color flow and even less w/ helicities

$$A_5(1^{\pm},2^{+},3^{+},4^{+},5^{+}) = 0$$

MHV amplitude

$$A_5(1^{-},2^{-},3^{+},4^{+},5^{+}) = i\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}$$

Number of diagrams for a n-gluon amplitude

n	full Amp	partial Amp
4	4	3
5	25	10
6	220	36
7	2485	133
8	34300	501
9	559405	1991
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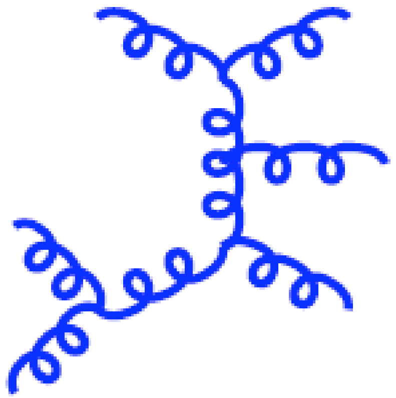
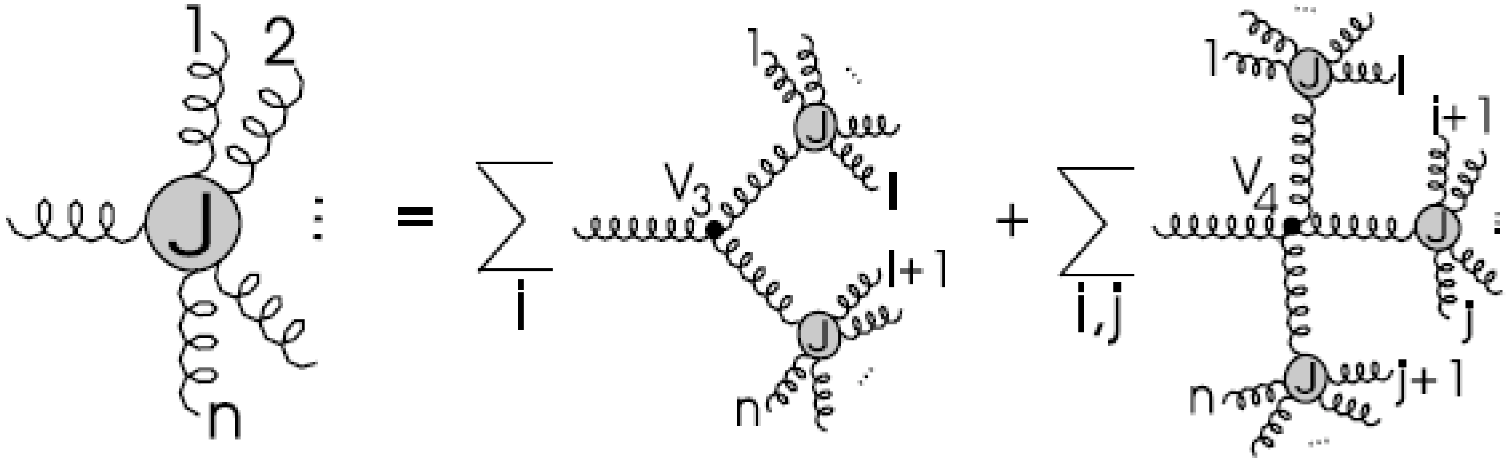
$(2n)!$ $\qquad 3.8^n$

Recursive relations

Feynman diagram beg to be evaluated recursively





J^μ is the Berends-Giele current. For MHV can solve analytically!

$$J^\mu(1^-,2^+,\ldots,n^+) \;=\; \frac{\langle 1^-|\gamma^\mu \not{P}_{2,n}|1^+\rangle}{\sqrt{2}\,\langle 1\,2\rangle\cdots\langle n\,1\rangle}\sum_{m=3}^{n}\frac{\langle 1^-|\not{k}_m \not{P}_{1,m}|1^+\rangle}{P^2_{1,m-1}P^2_{1,m}}\,,$$

Dotting with ε^- on the free leg and cleaning up gives:

$$A_n^{\text{tree}}(1^-,2^-,3^+,4^+,\ldots,n^+) \;=\; i\,\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\cdots\langle n\,1\rangle}$$

Parke-Taylor amplitude is proven!

Infinite number of Feynman diagrams solved at once!

Number of diagrams for n-gluon amplitudes

n	full Amp	partial Amp	BG
4	4	3	3
5	25	10	10
6	220	36	35
7	2485	133	70
8	34300	501	126
9	559405	1991	210
10	10525900	7335	330
11	224449225	28199	495
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$(2n)!$ 3.8^n n^4

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Note, however, one still needs to sum over color, an operation which sets the complexity back to exponential.

LO : the technical challenges

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easy

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$$\sigma(pp \to 3j) = \sum_{ijk} \int f_i(x_1) f_j(x_2) \hat{\sigma}(ij \to k_1 k_2 k_3)$$

easy

II. For each one, calculate the amplitude:

$$\mathcal{A}(\{p\},\{h\},\{c\}) = \sum_i D_i$$

difficult

III. Square the amplitude, sum over spins & color, integrate over the phase space (D ~ 3n)

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LO : the technical challenges

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$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1,\mu_F^2) f_b(x_2,\mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

Two ingredients necessary:

1. Parton Distribution functions (from exp, but evolution from th).

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$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \dots$$

Leading order

Next-to-leading order

Next-to-next-to-leading order

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from integration to event generation

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General and flexible method is needed

Phase Space

Phase Space

$$d\Phi_n = \left[\Pi_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 (2E_i)}\right] (2\pi)^4 \delta^{(4)}(p_0 - \sum_{i=1}^{n} p_i)$$

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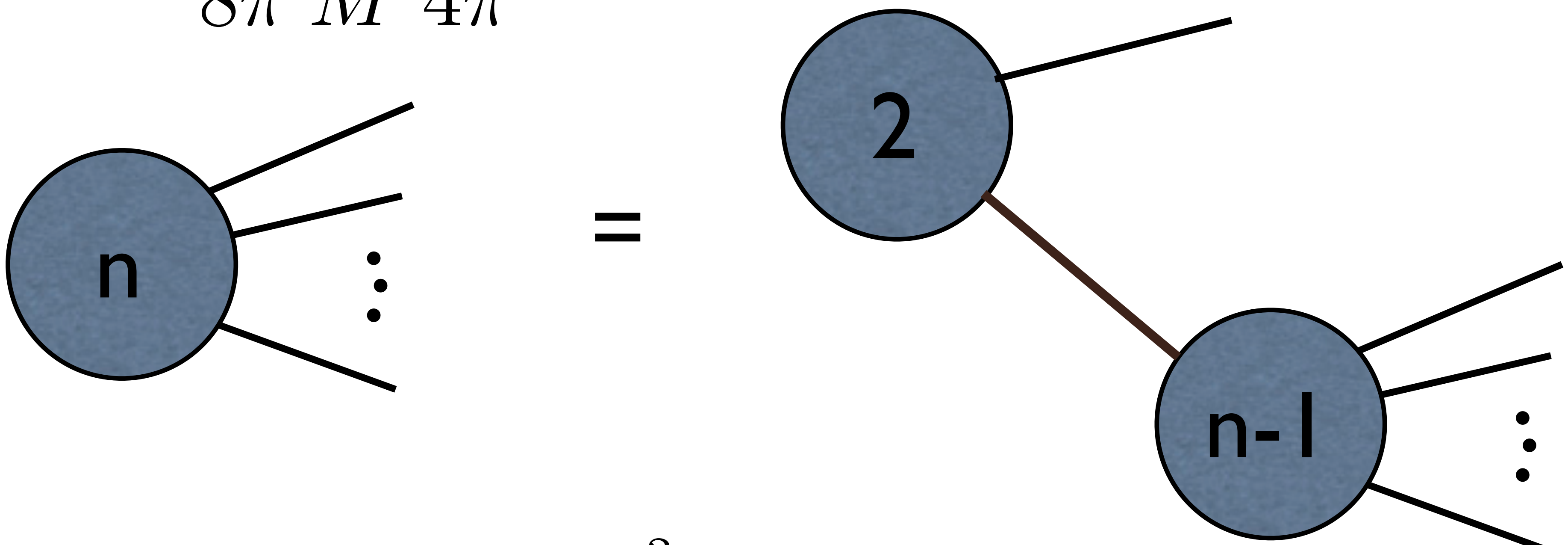

$$d\Phi_n(M) = \frac{1}{2\pi} \int_0^{(M-\mu)^2} d\mu^2 d\Phi_2(M) d\Phi_{n-1}(\mu)$$

Integrals as averages

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$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

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- Convergence is slow but it can be estimated easily
- Error does not depend on # of dimensions!
- Improvement by minimizing V_N.
- Optimal/Ideal case: $f(x)=C \Rightarrow V_N=0$

Importance Sampling

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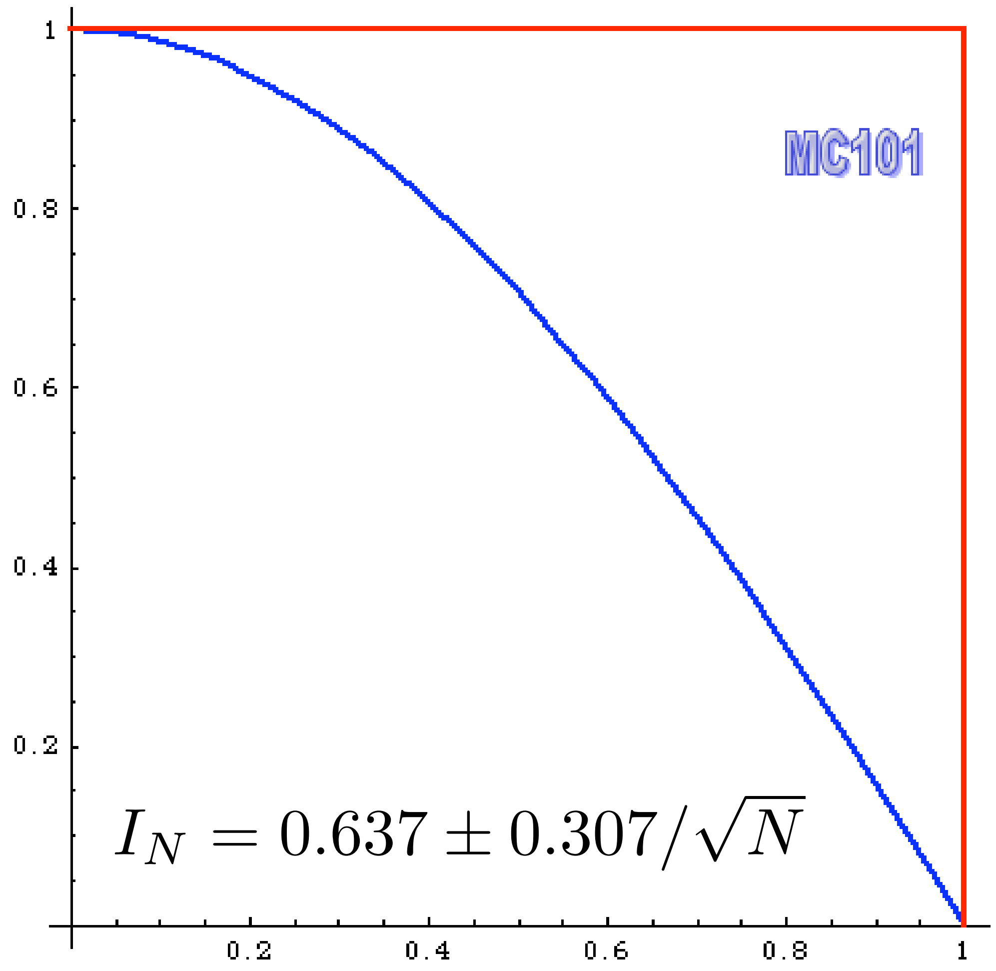

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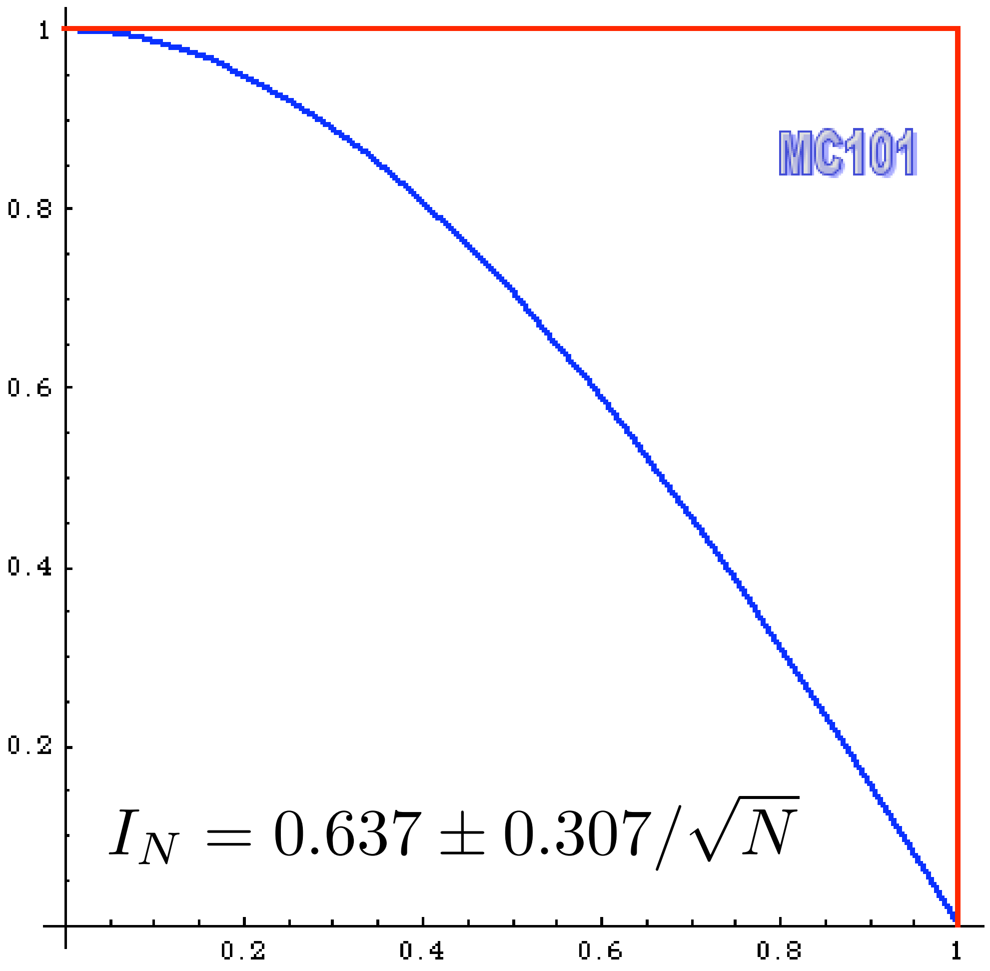

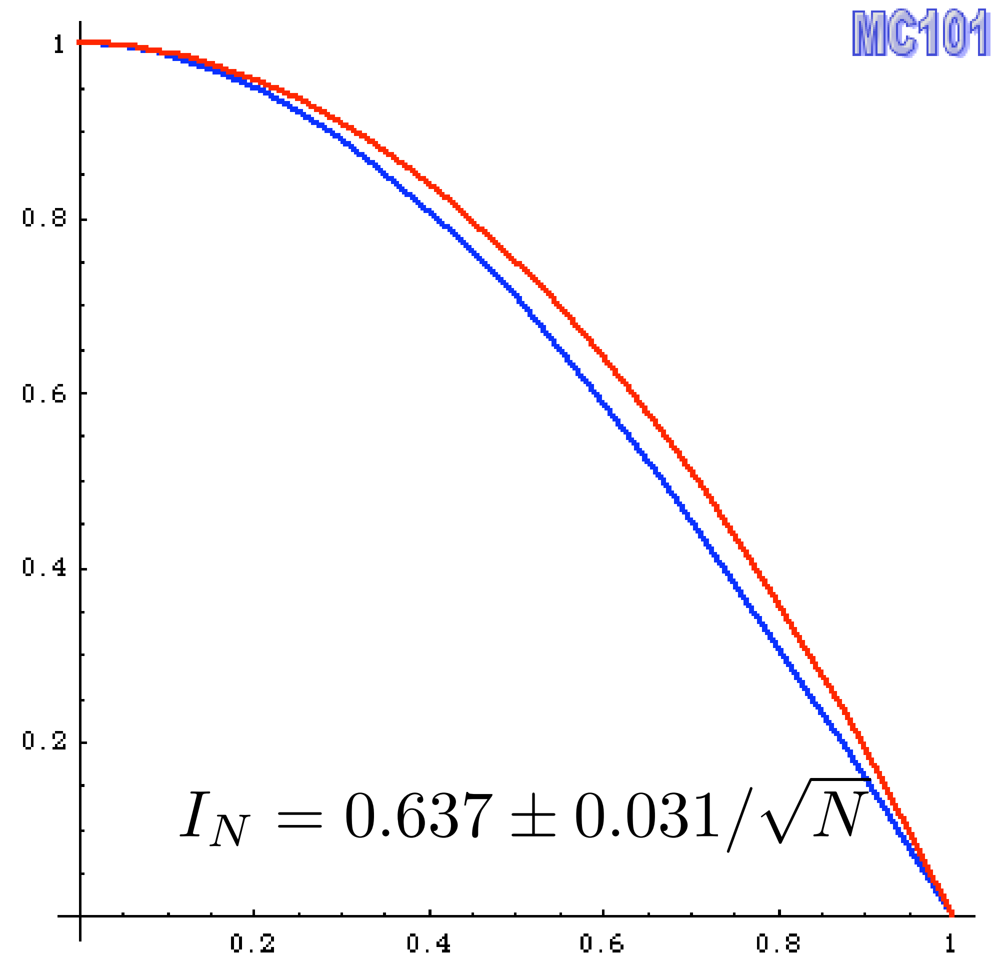

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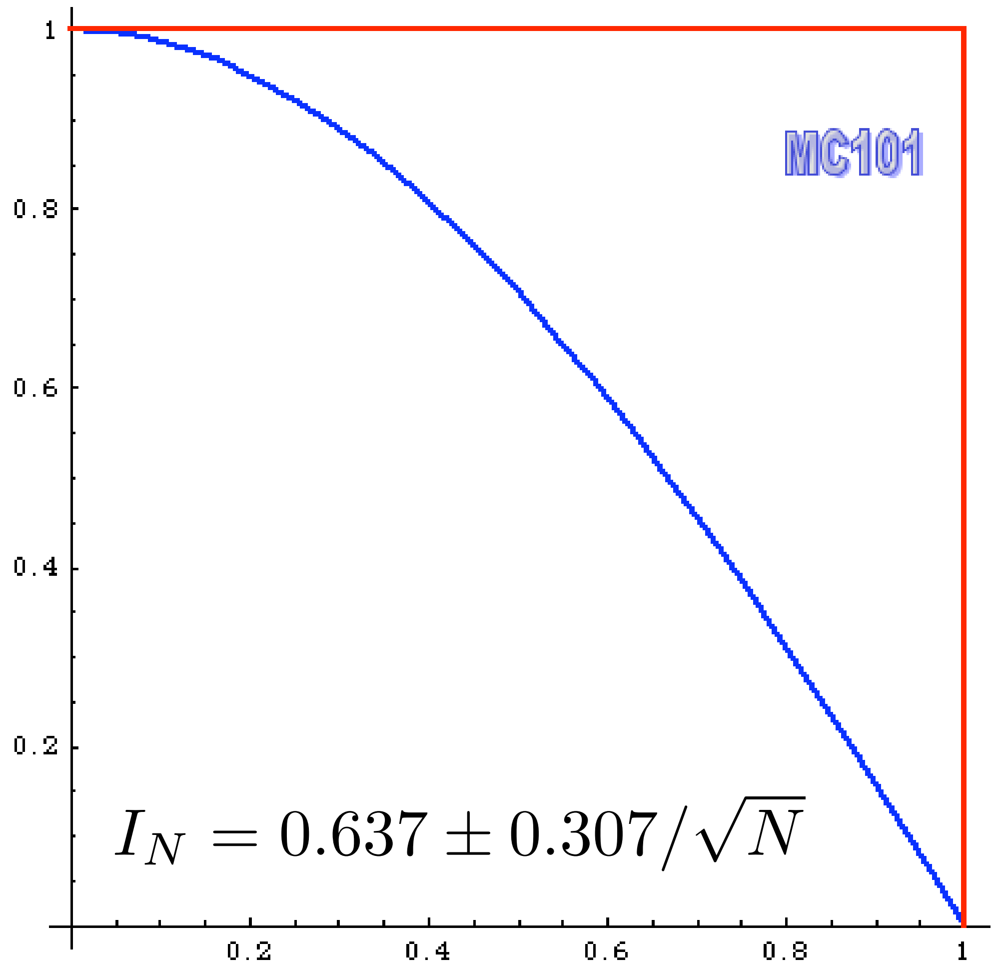

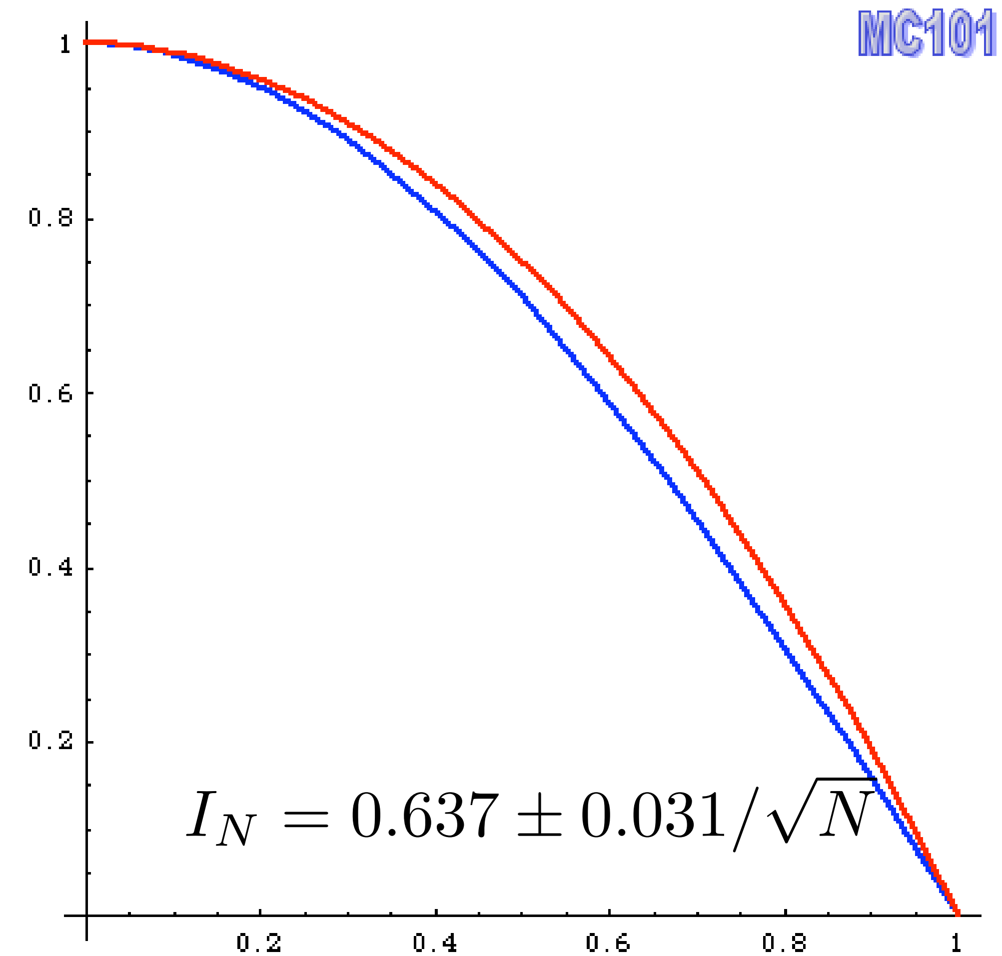

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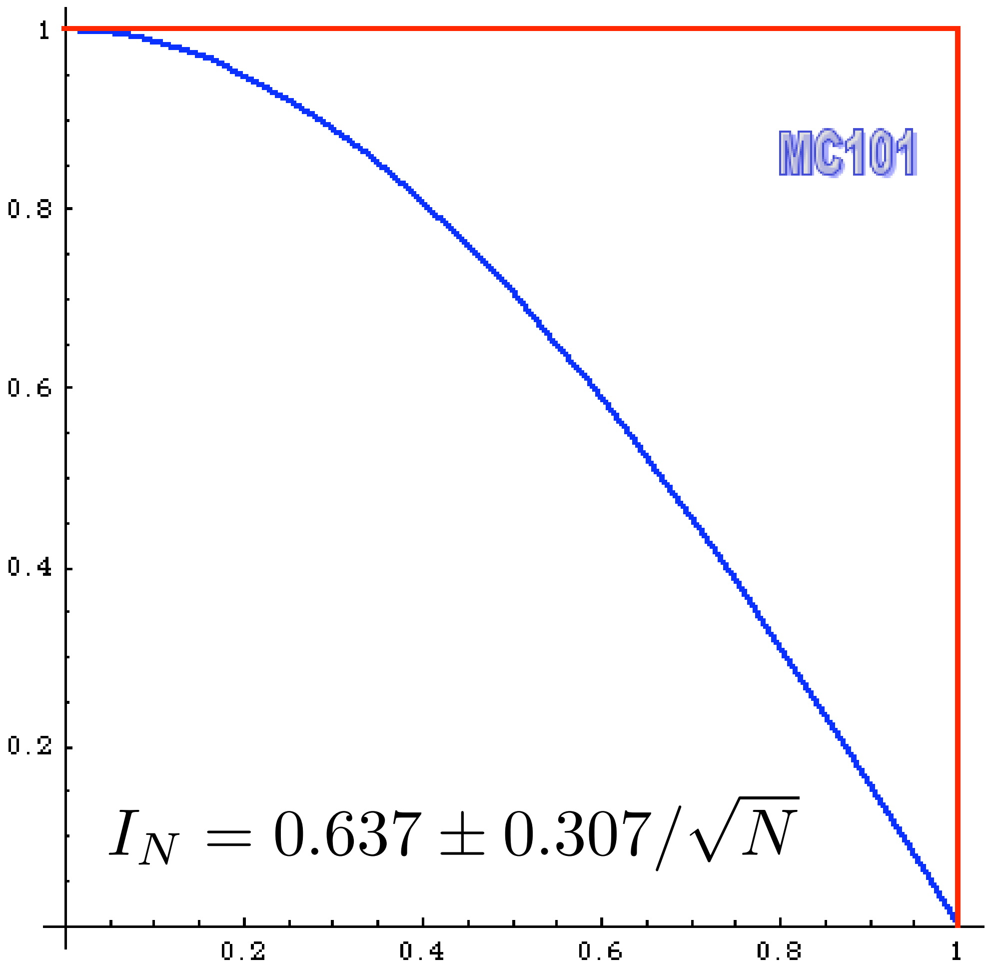

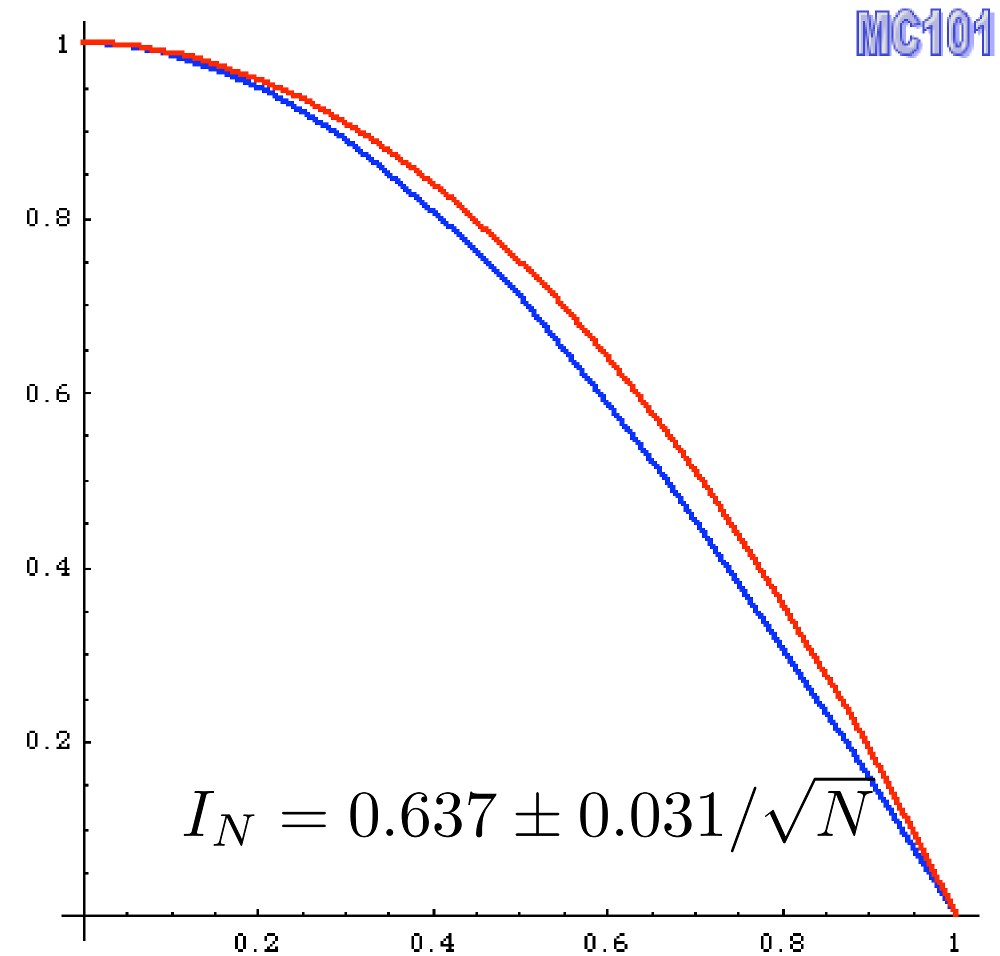

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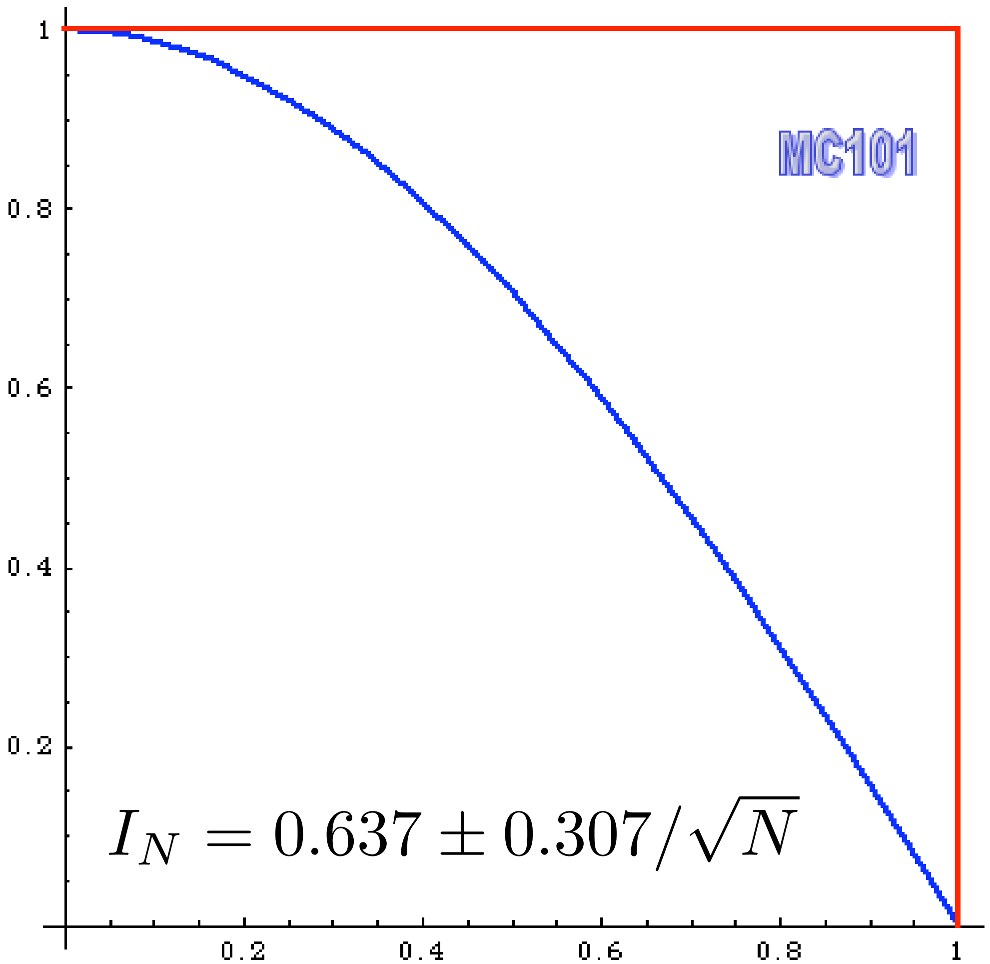

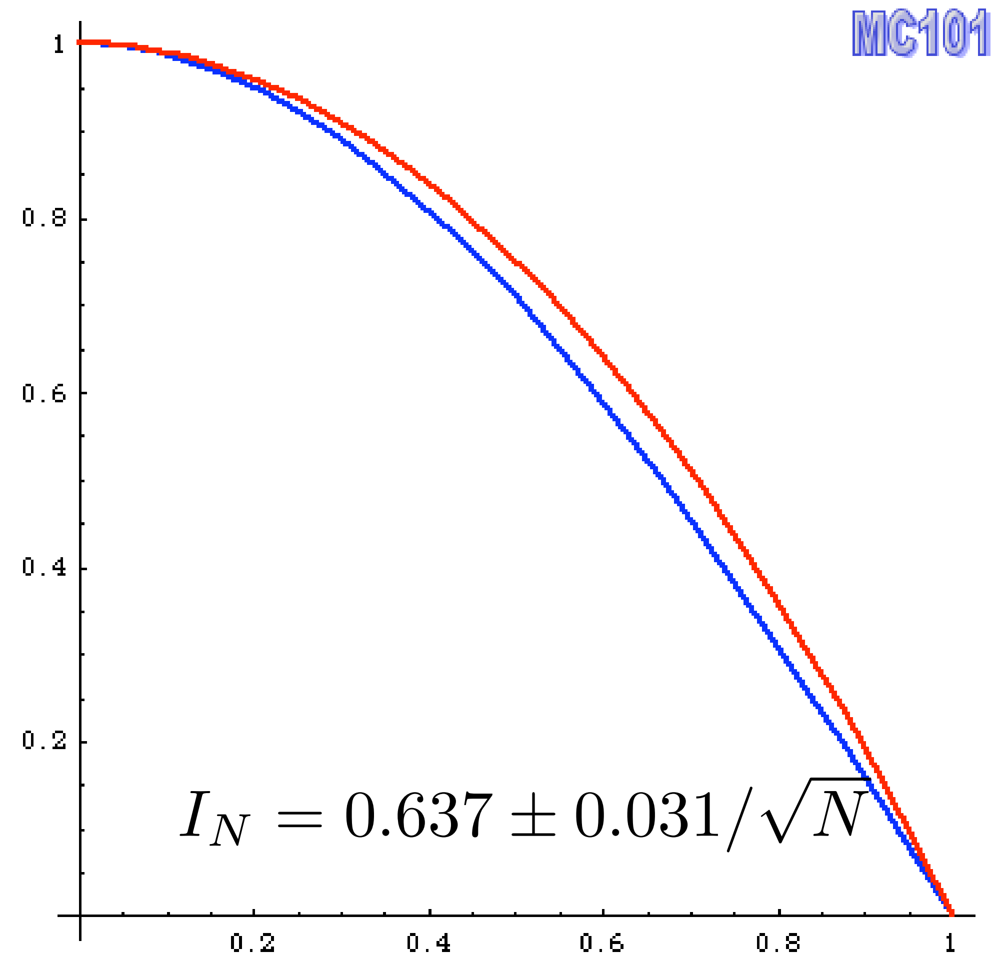

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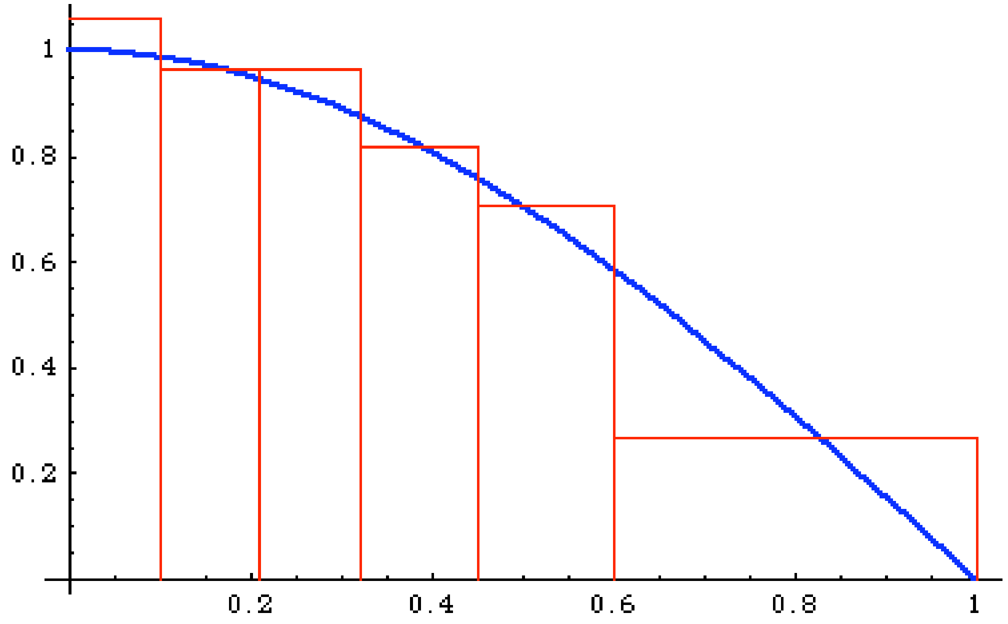
MC101



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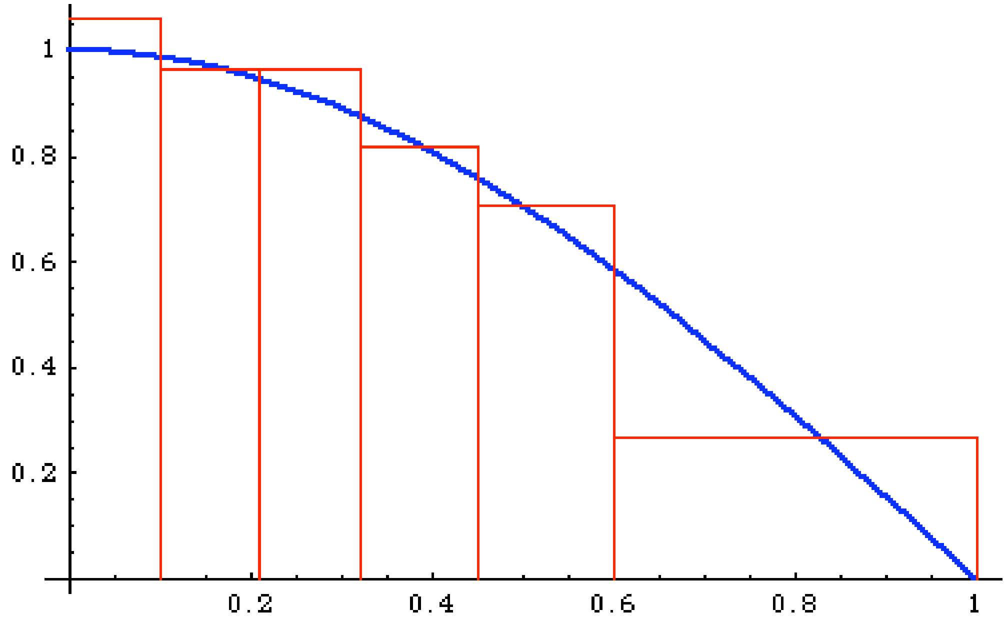

many bins where f(x) is large

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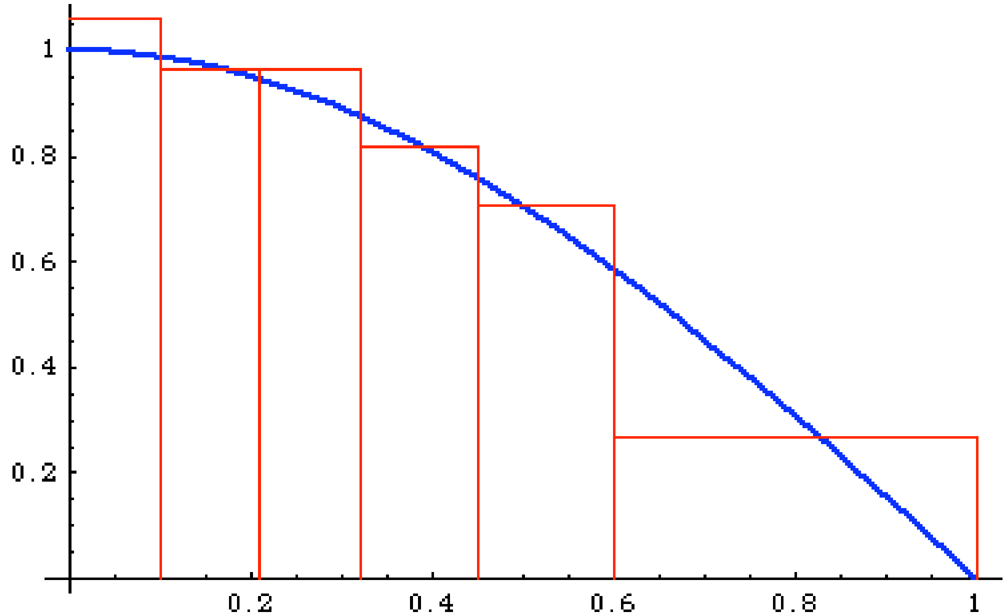

many bins where f(x) is large

$$p(x) = \frac{1}{N_b \Delta x_i}, \quad x_i - \Delta x_i < x < x_i$$

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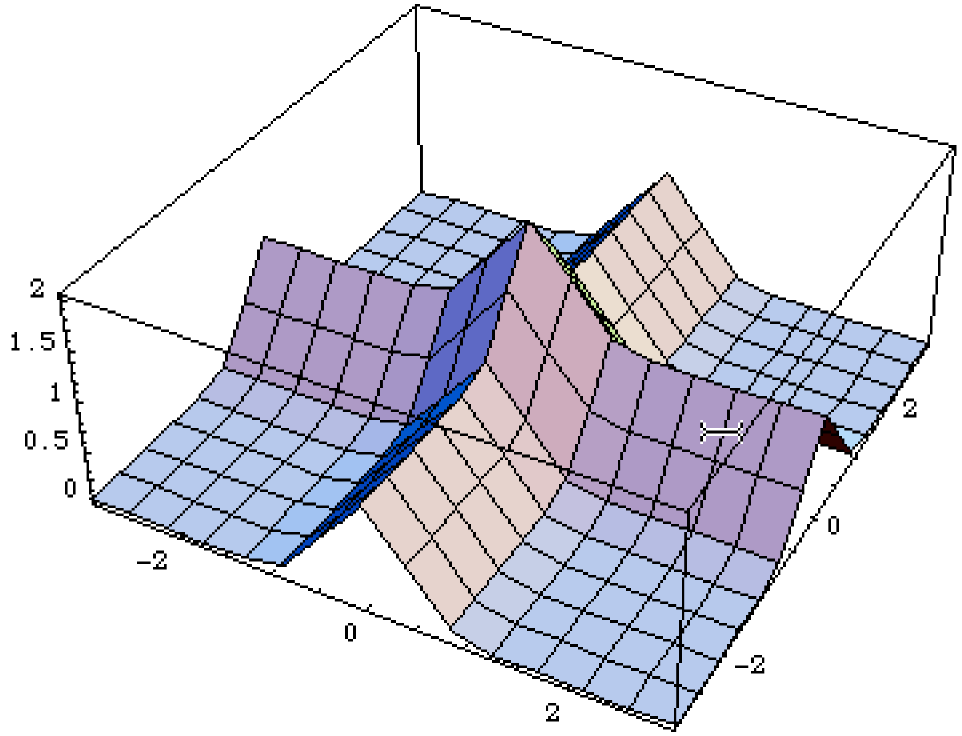

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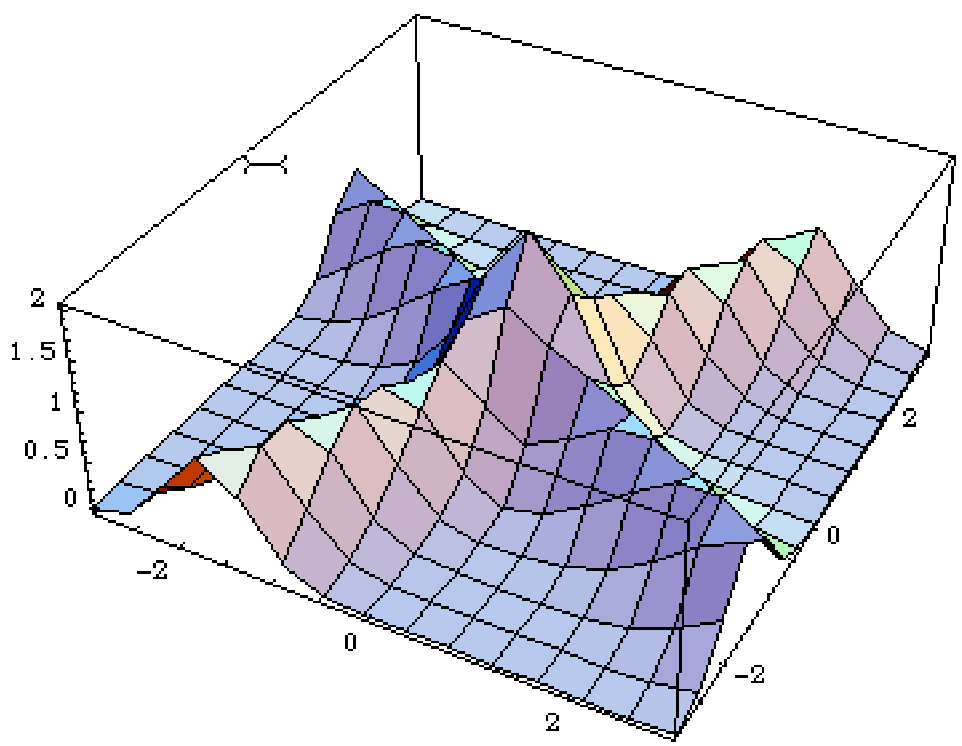

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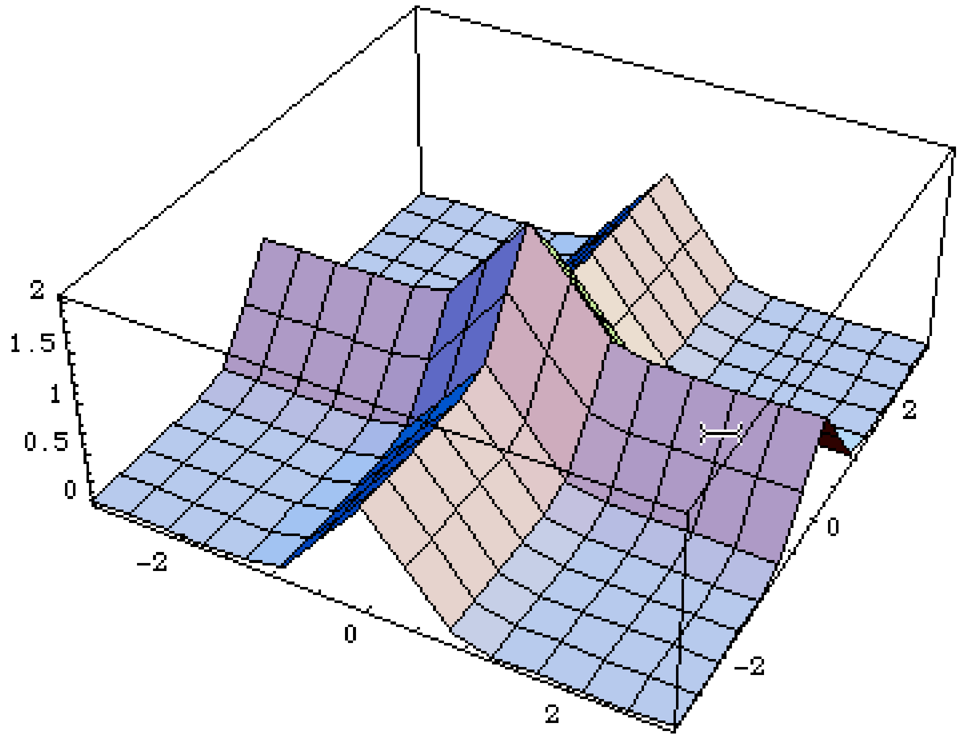

but it is sufficient to make a change of variables!

Multi-channel

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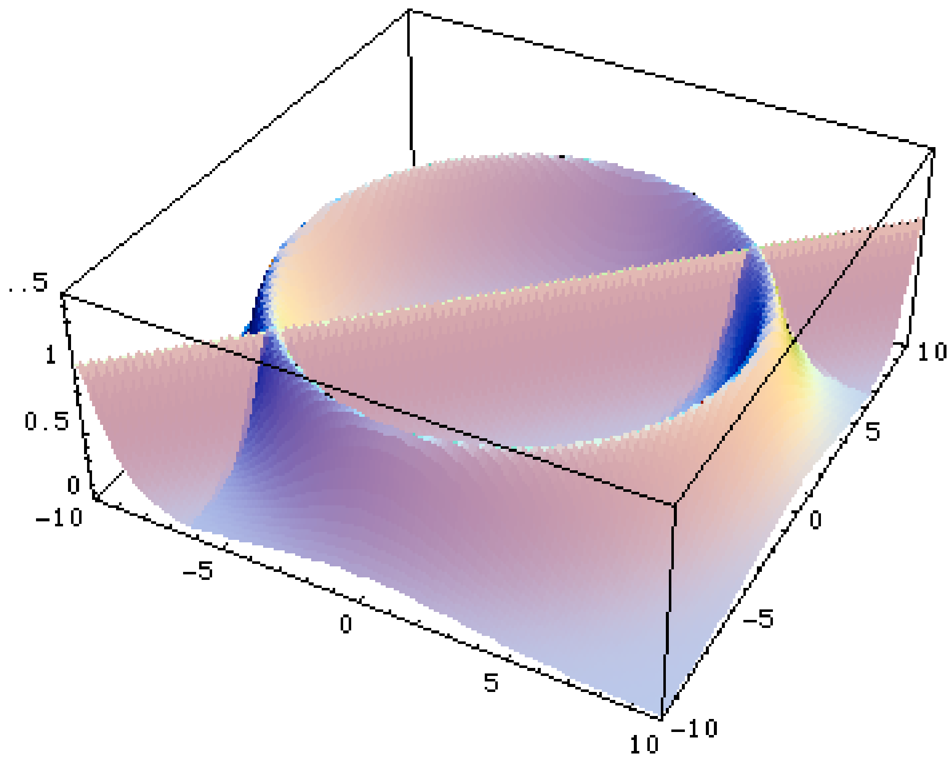

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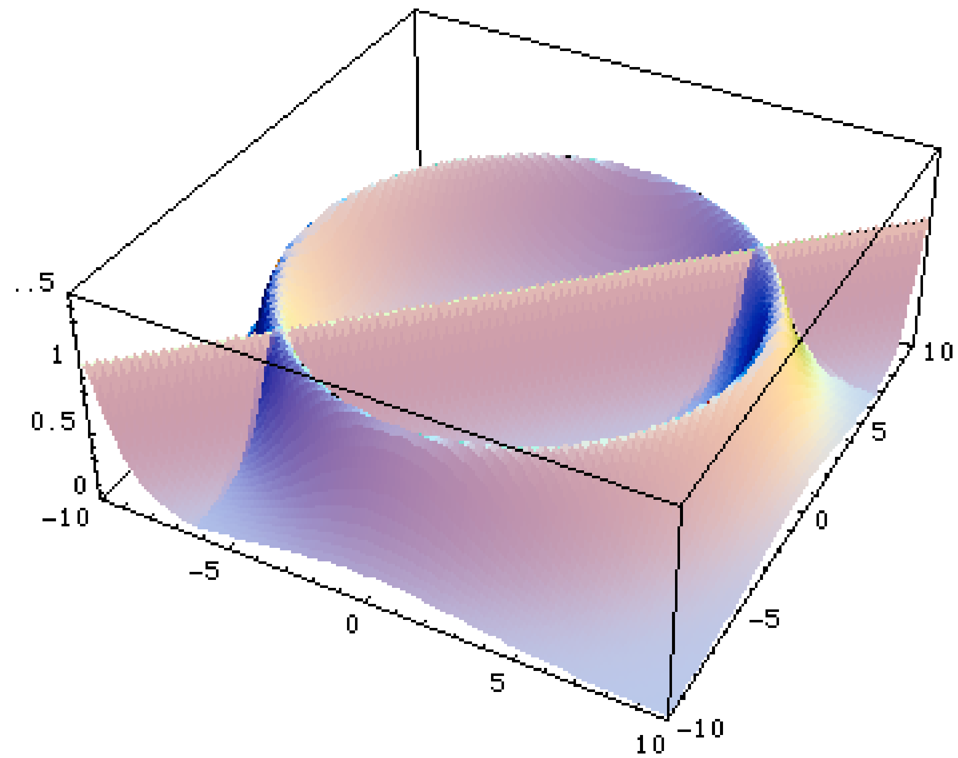

In this case there is no unique tranformation: Vegas is bound to fail!

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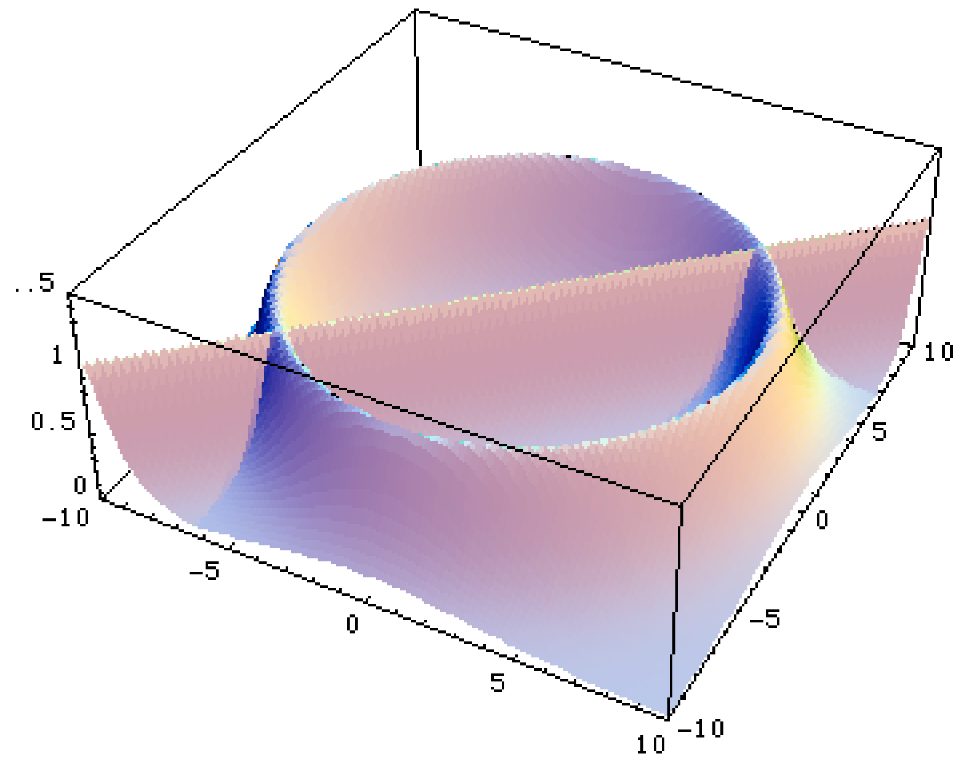

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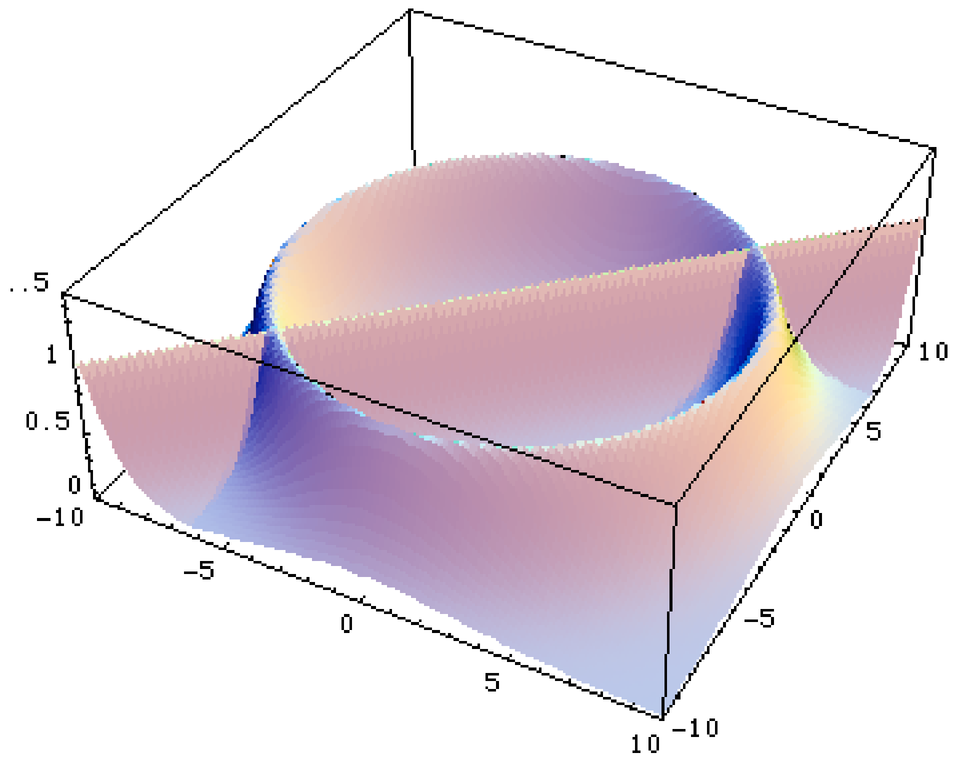
Solution: use different transformations= channels

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

with each $p_i(x)$ taking care of one "peak" at the time

Multi-channel



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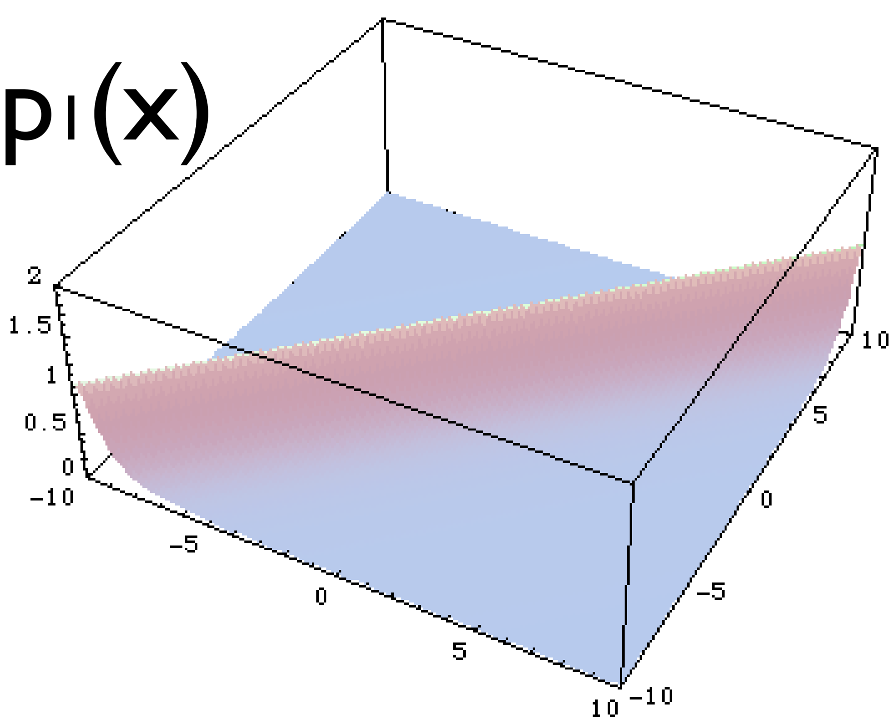
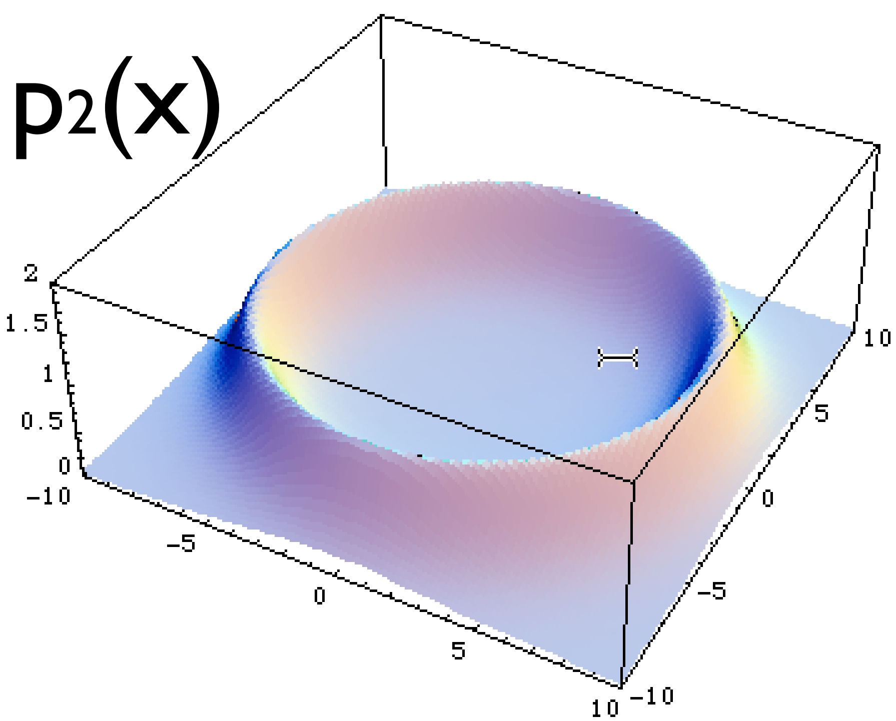
$p_1(x)$



$p_2(x)$



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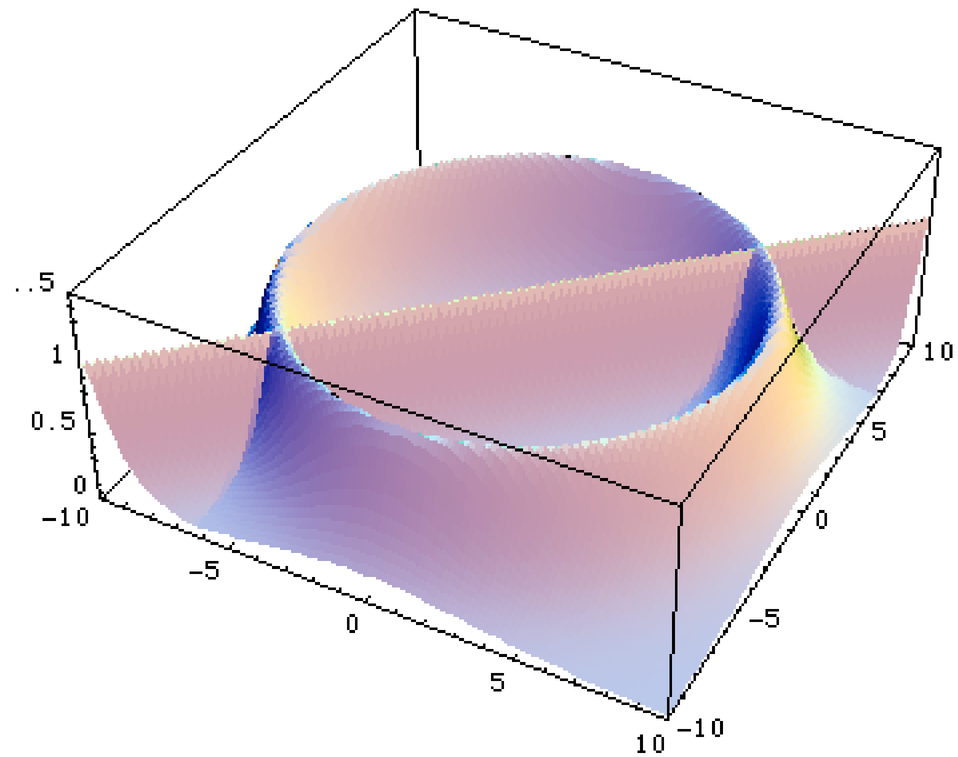

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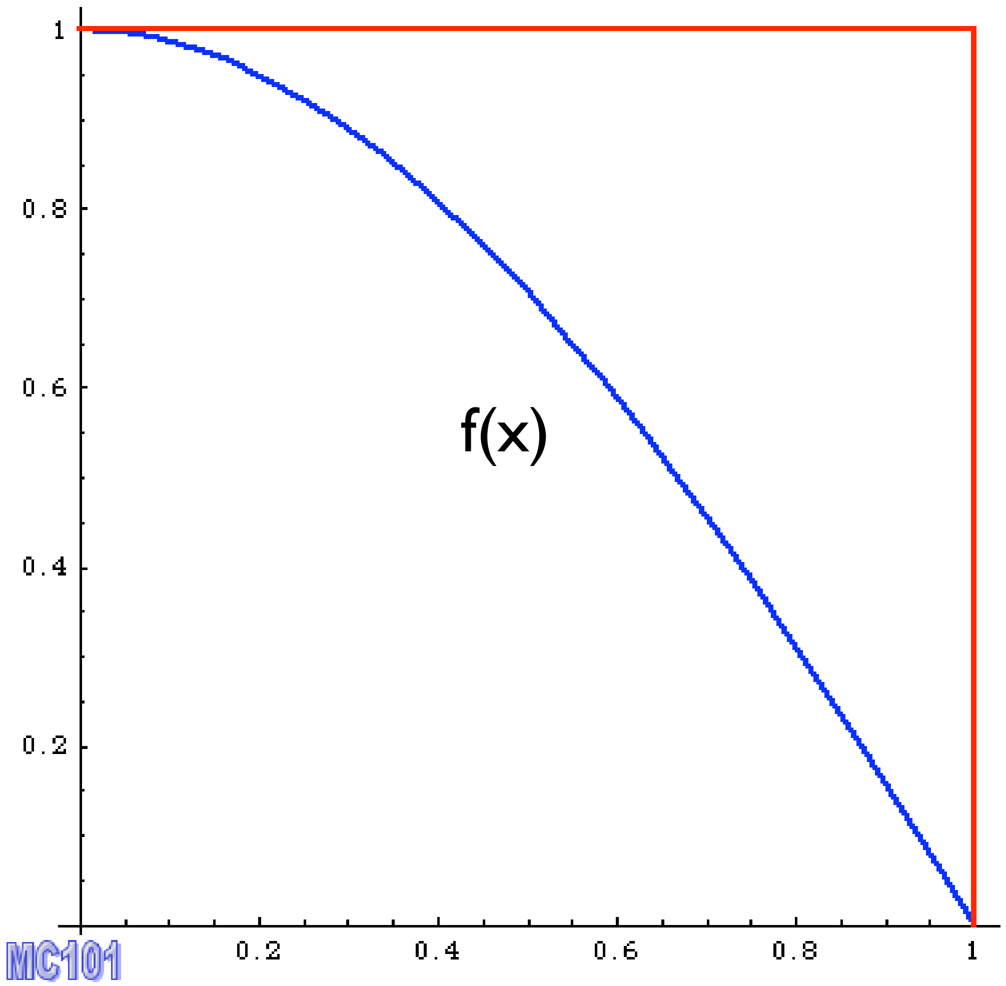
But if you know where the peaks are (=in which variables) we can use different transformations= channels:

$$p(x) = \sum_{i=1}^{n} \alpha_i p_i(x) \qquad \text{with} \qquad \sum_{i=1}^{n} \alpha_i = 1$$

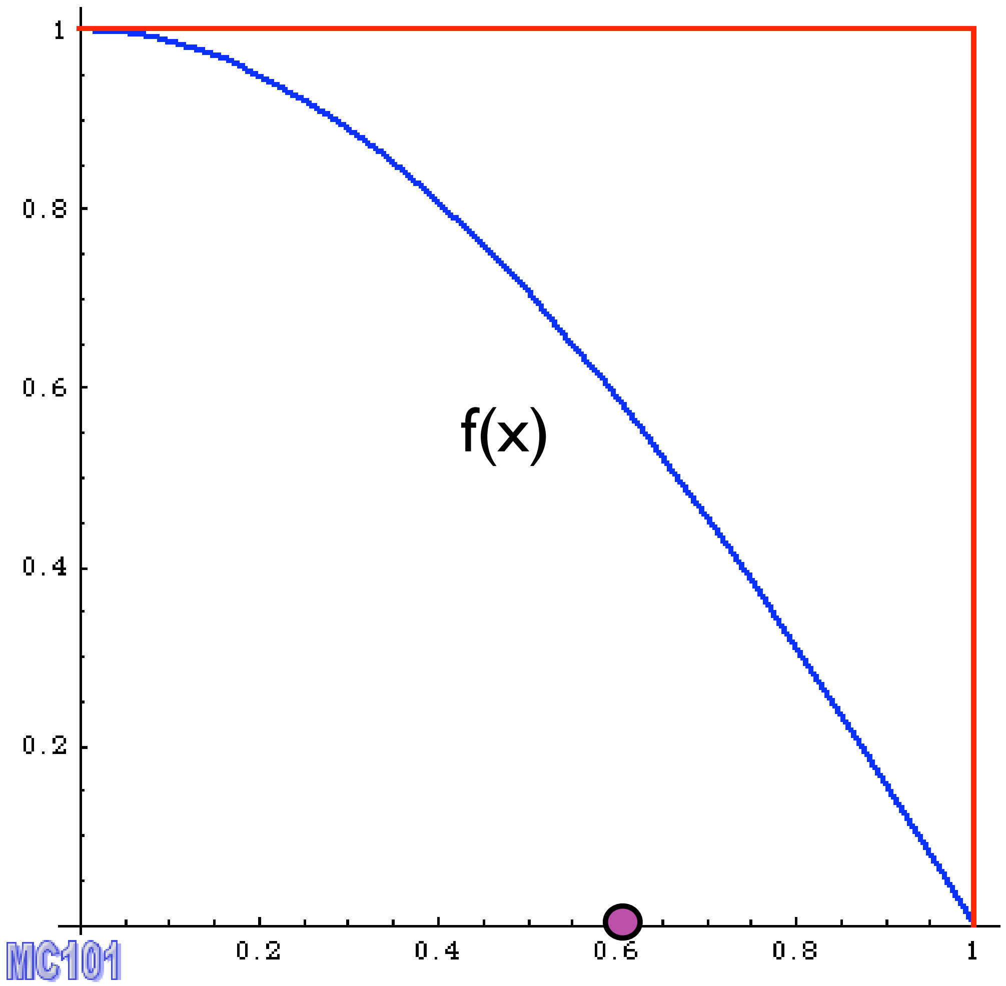
$$I = \int f(x)dx = \sum_{i=1}^{n} \alpha_i \int \frac{f(x)}{p(x)} p_i(x) dx$$

Event generation



Alternative way

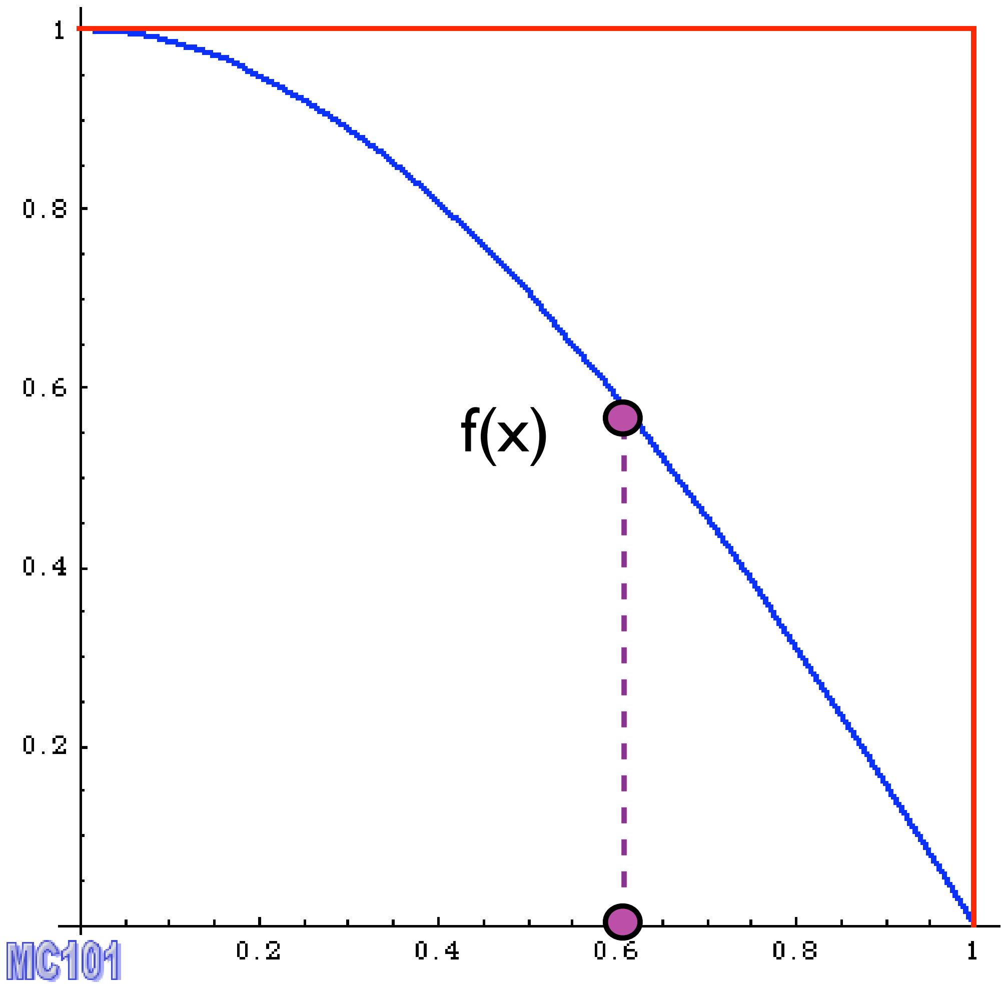
Event generation



Alternative way

1. pick x

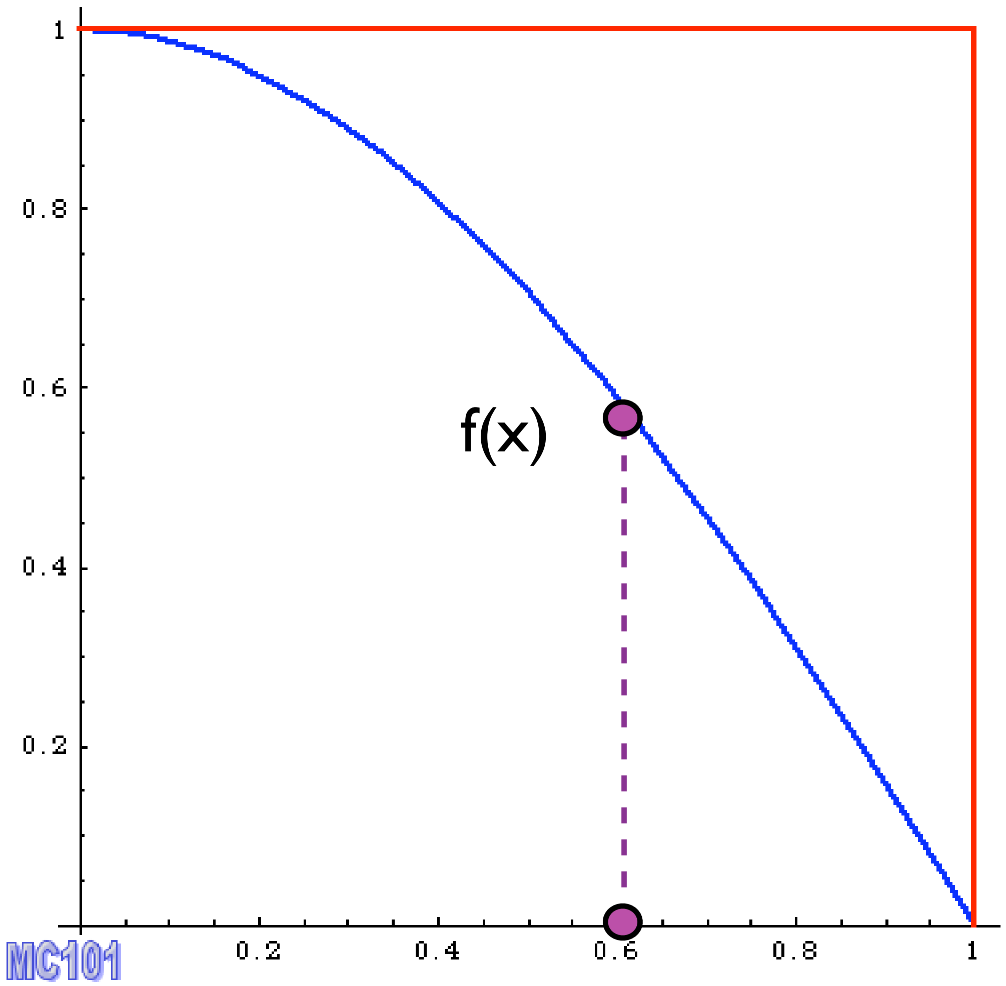
Event generation



Alternative way

1. pick x
2. calculate f(x)

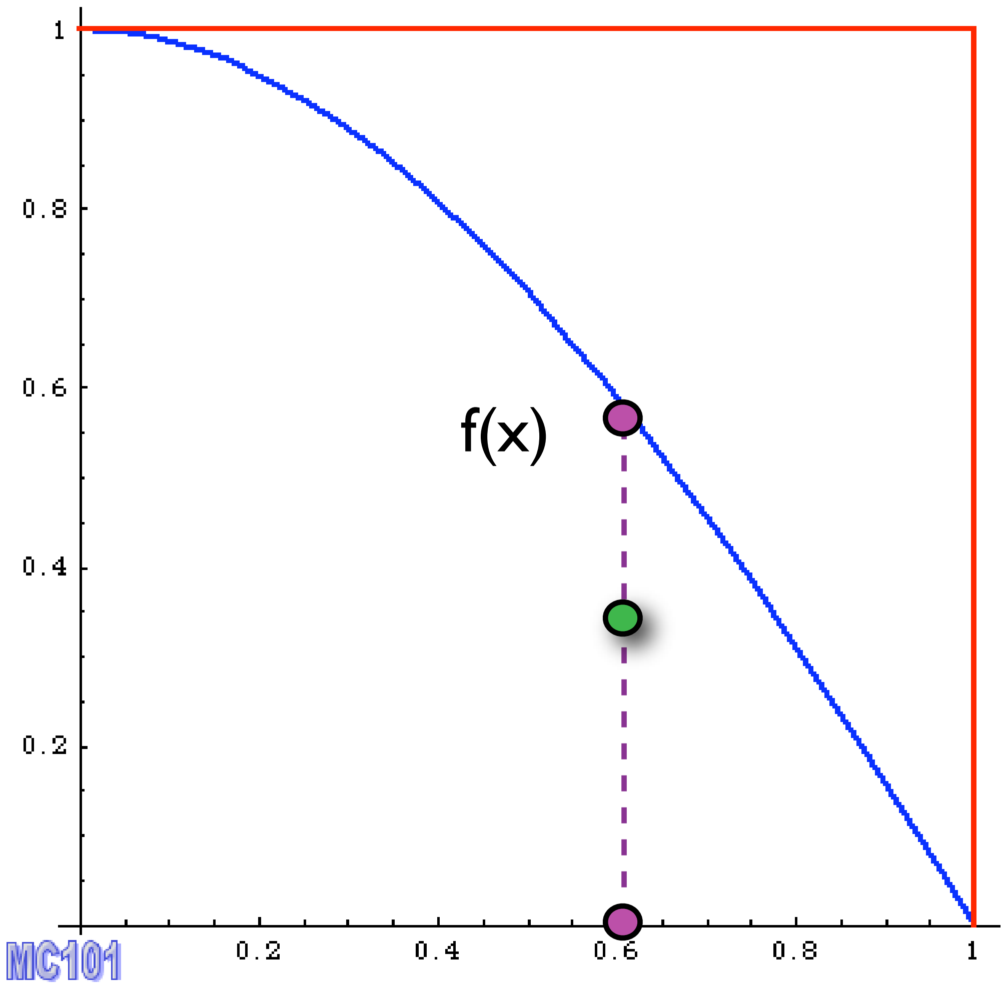
Event generation



Alternative way

1. pick x
2. calculate f(x)
3. pick 0<y<fmax

Event generation



Alternative way

1. pick x
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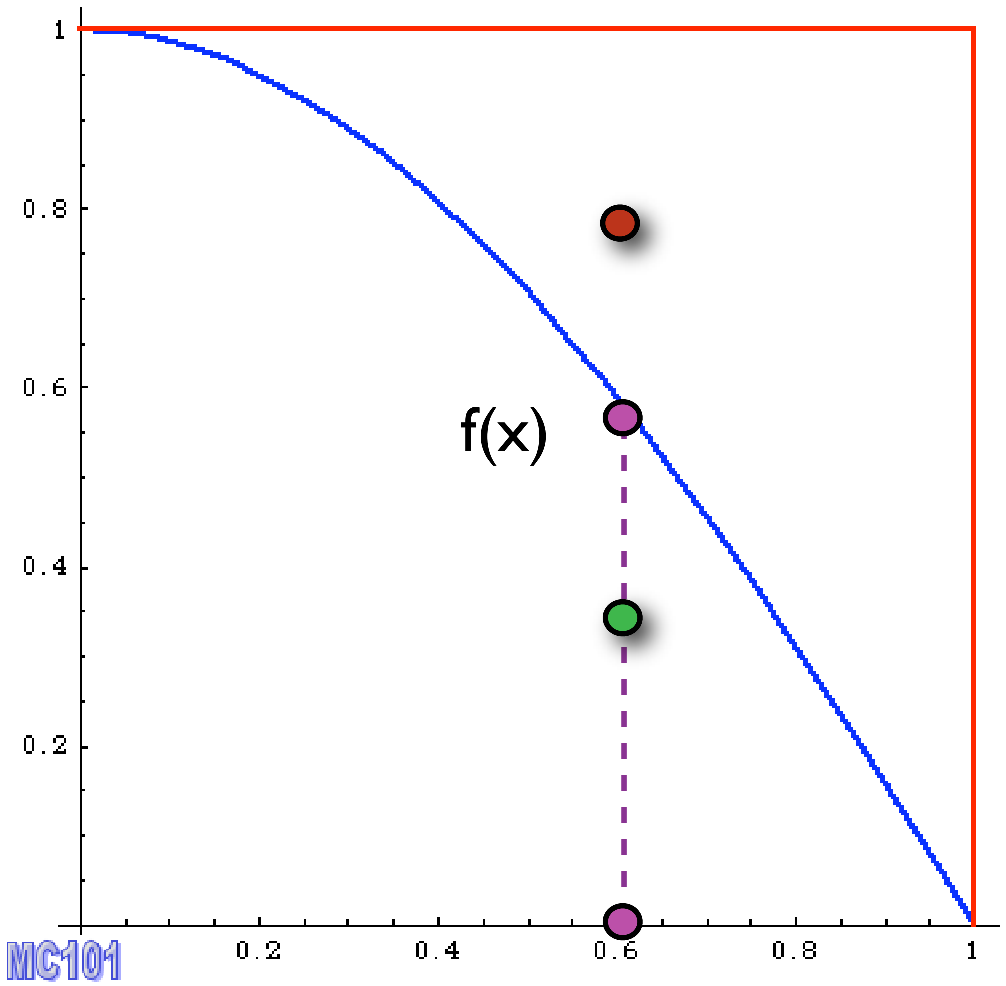
4. Compare:
   if f(x)>y accept event,

Event generation



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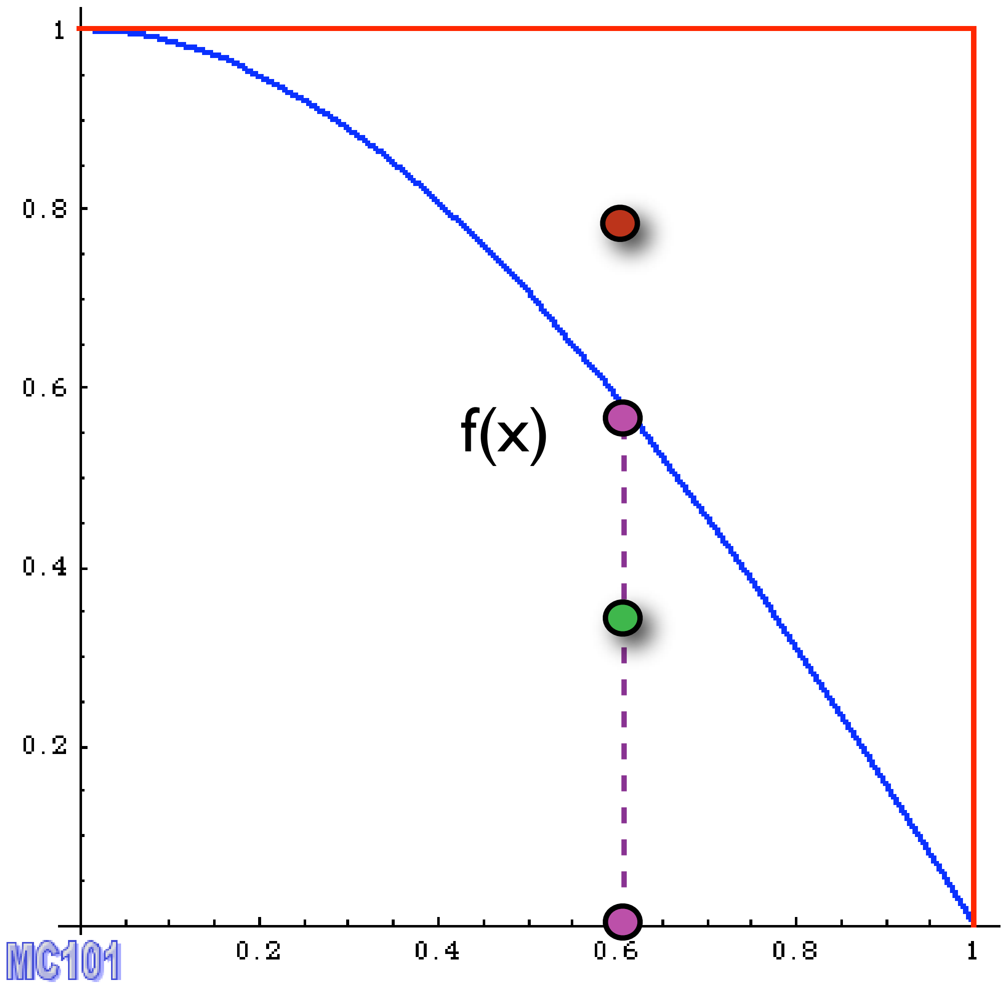

Alternative way

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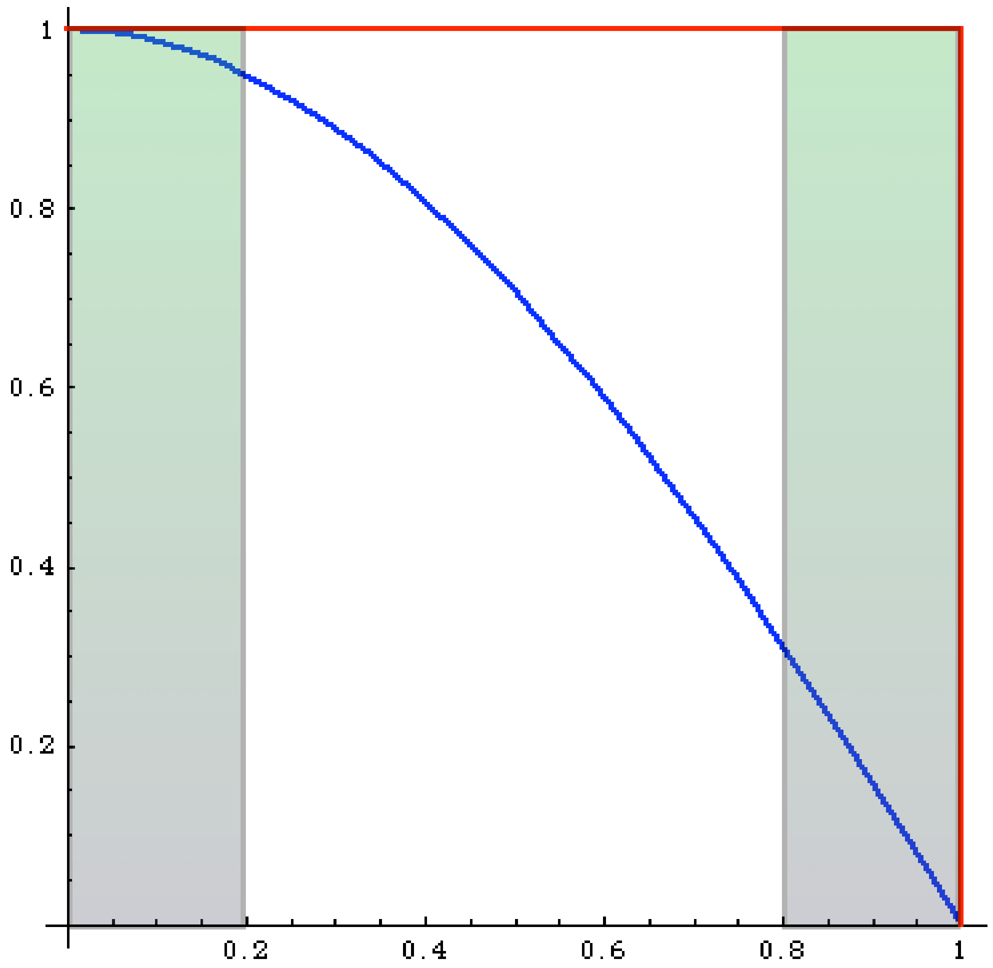
4. Compare: if f(x)>y accept event, else reject it.

$$I = \frac{\text{accepted}}{\text{total tries}} = \text{efficiency}$$

Event generation

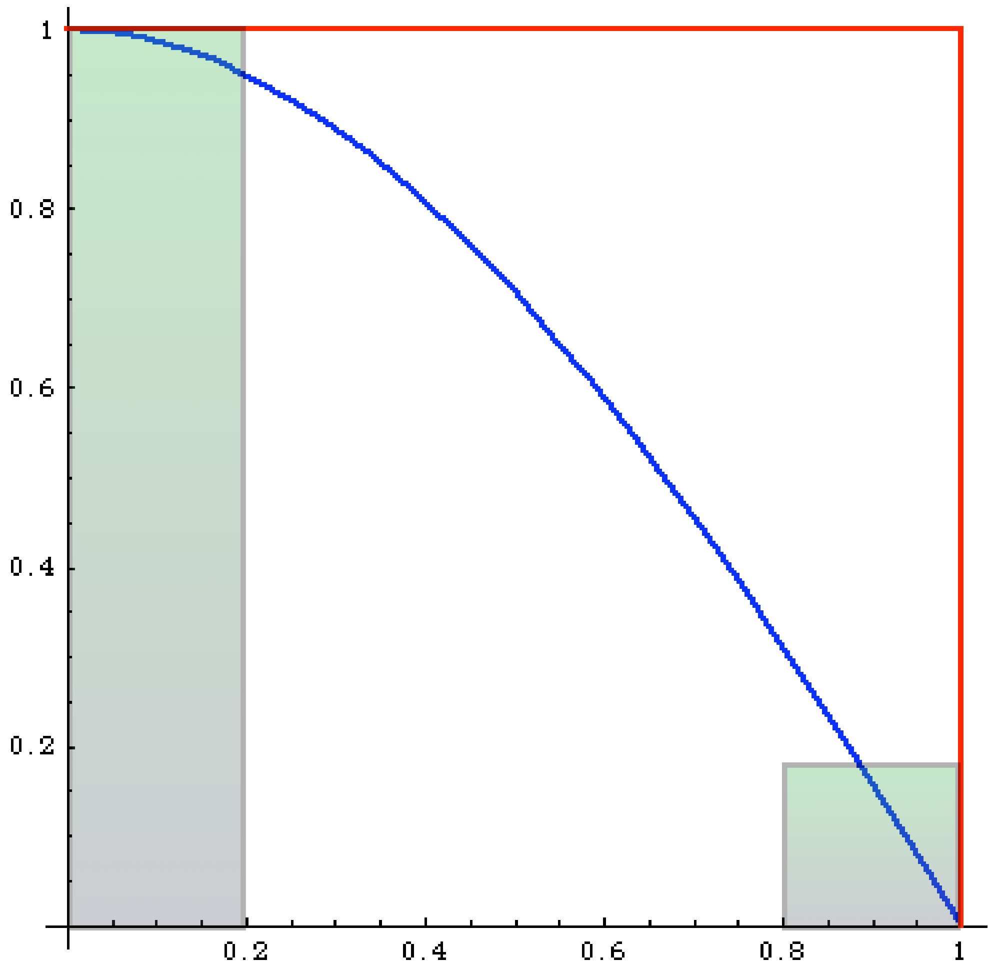


What's the difference?

before:

same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



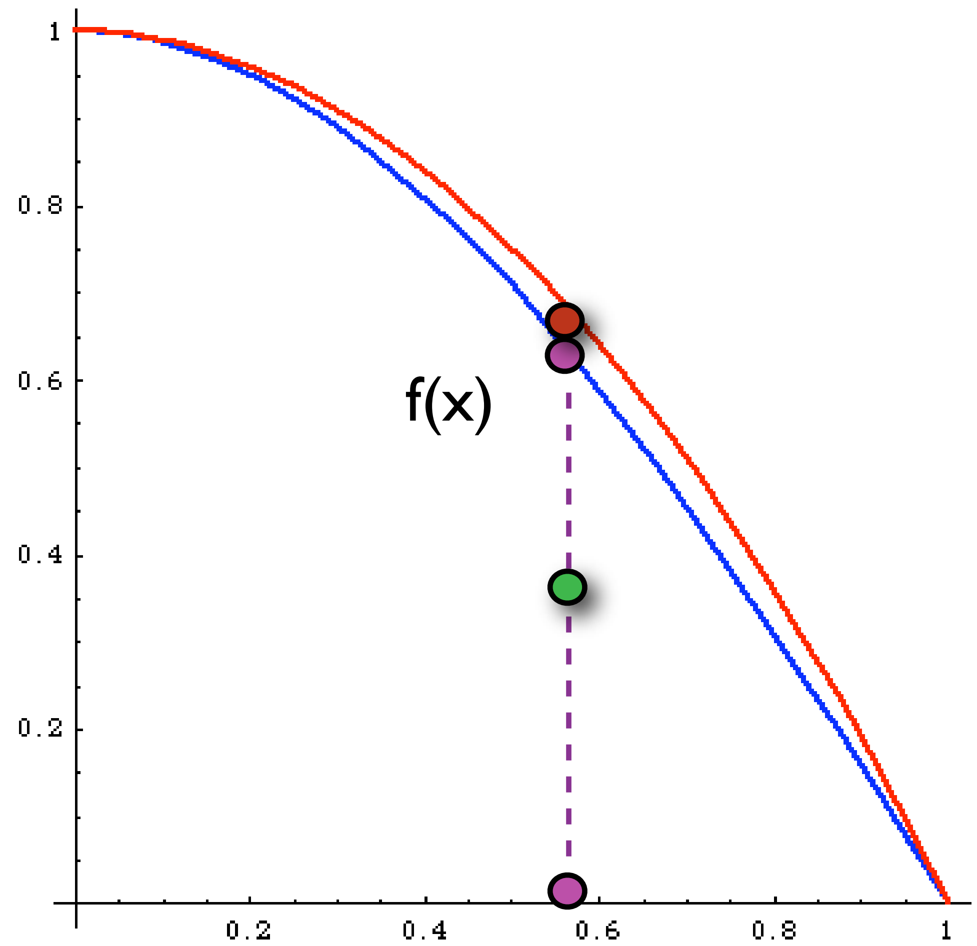
What's the difference?

after:

events is proportional to the probability of areas of phase space:
events have all the same weight ("unweighted")

Events distributed as in Nature

Event generation



Improved

1. pick x distributed as p(x)
2. calculate f(x) and p(x)
3. pick 0<y<1
4. Compare:
   if f(x)>y p(x) accept event,

else reject it.

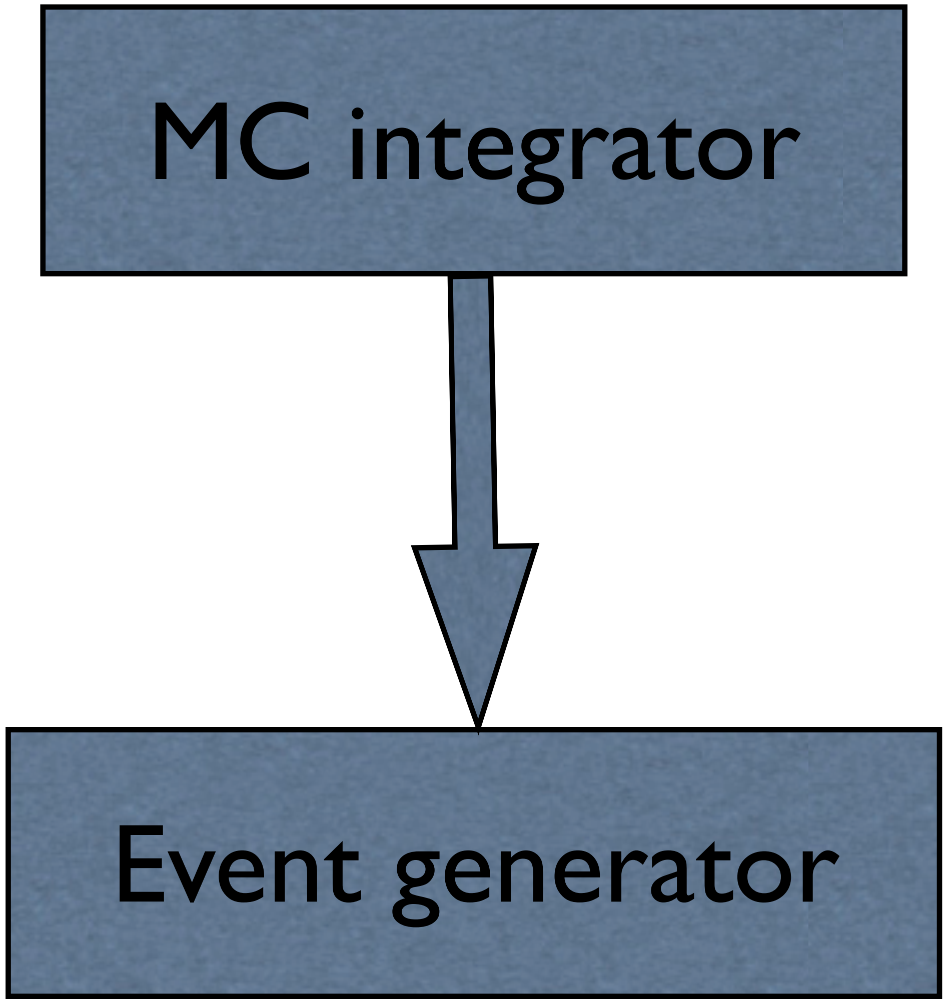
much better efficiency!!!

Event generation

Event generation

MC integrator

Event generation



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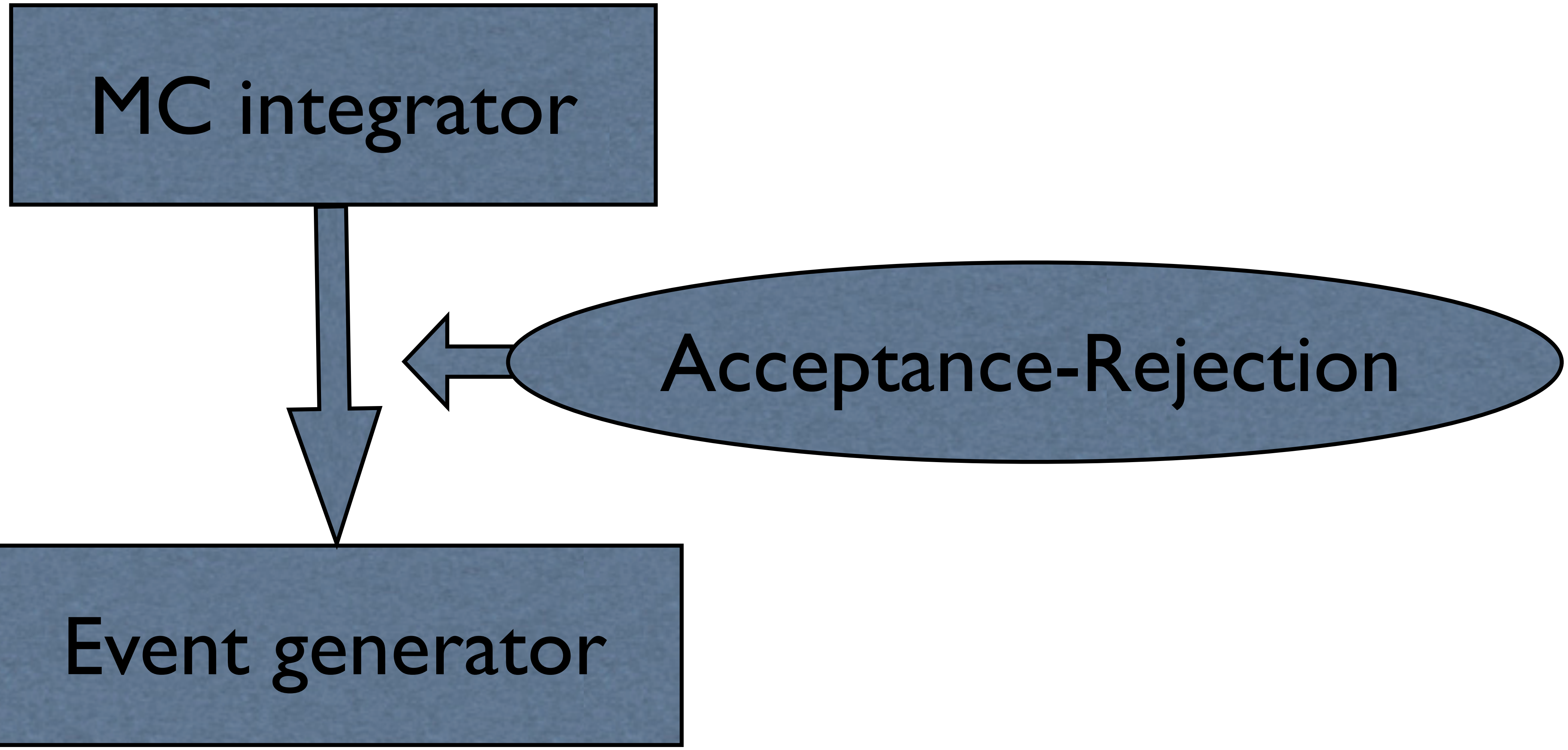

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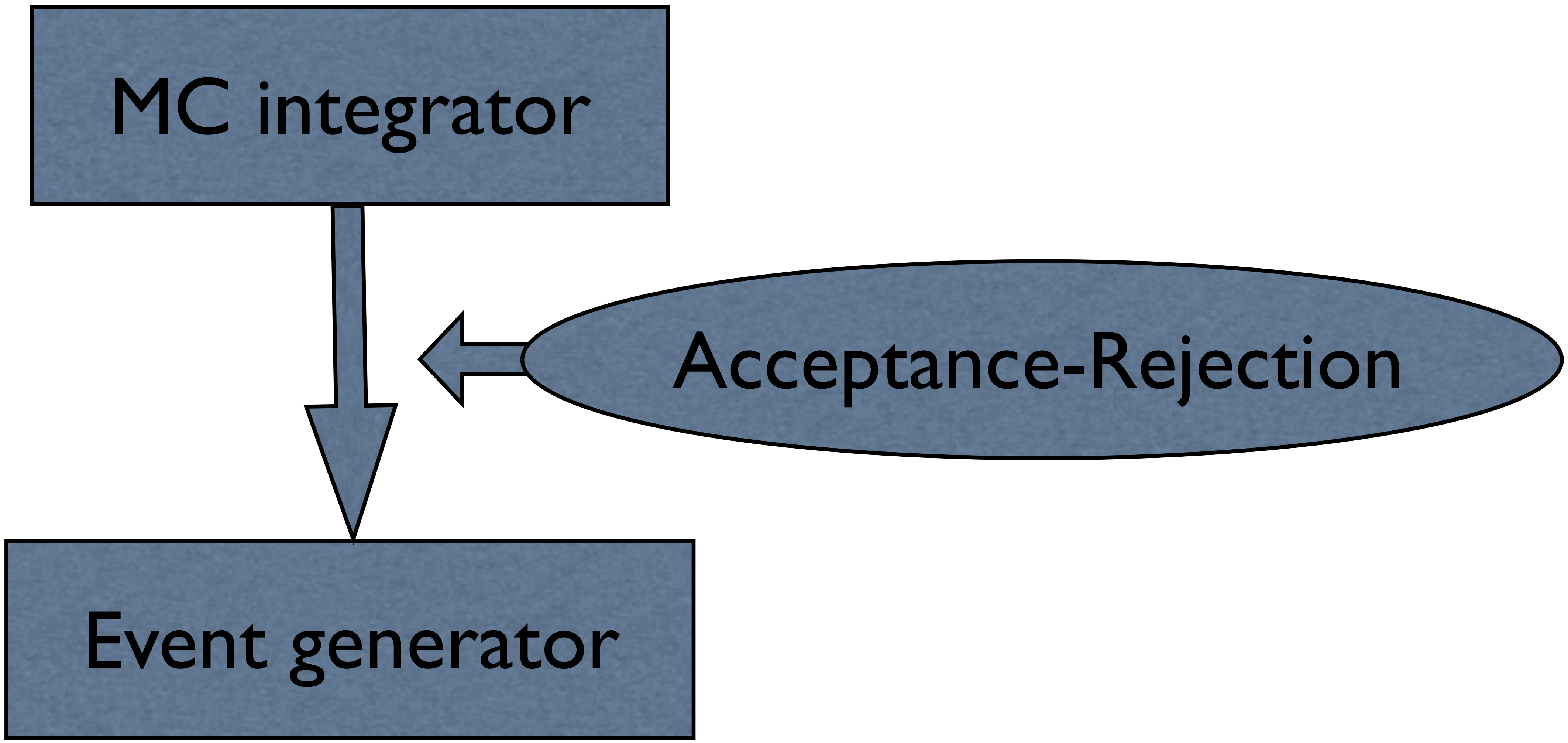


☞ This is possible only if $f(x)<\infty$ AND has definite sign!

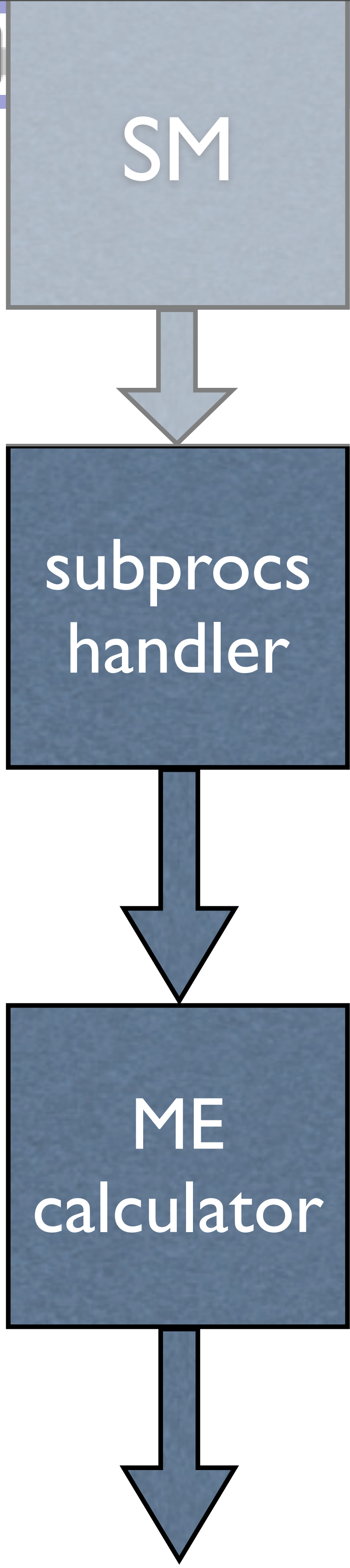
Monte Carlo Event Generator: definiton

At the most basic level a Monte Carlo event generator is a program which produces particle physics events with the same probability as they occur in nature (virtual collider).

In practice it performs a large number of (sometimes very difficult) integrals and then unweights to give the four momenta of the particles that interact with the detector (simulation).

Note that, at least among theorists, the definition of a "Monte Carlo program" also includes codes which don't provide a fully exclusive information on the final state but only cross sections or distributions at the parton level, even when no unweighting can be performed. I will refer to these kind of codes as "MC integrators".

General structure



Includes all possible subprocess leading to a given multi-jet final state automatically or manually (done once for all)

d~ d -> a a u u~ g
d~ d -> a a c c~ g
s~ s -> a a u u~ g
s~ s -> a a c c~ g

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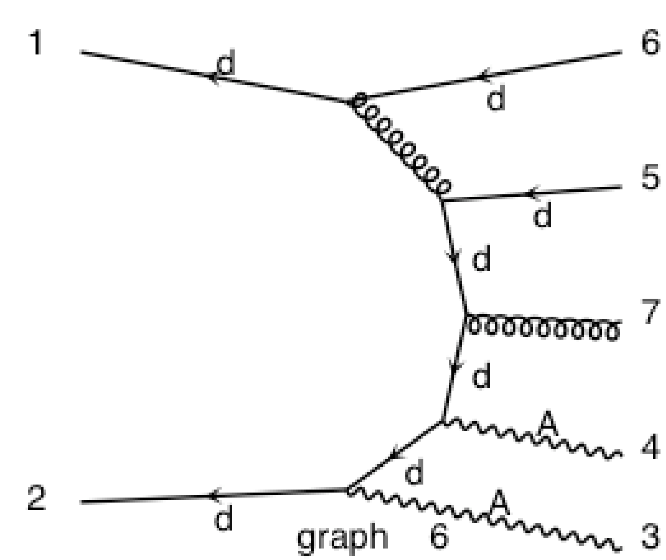
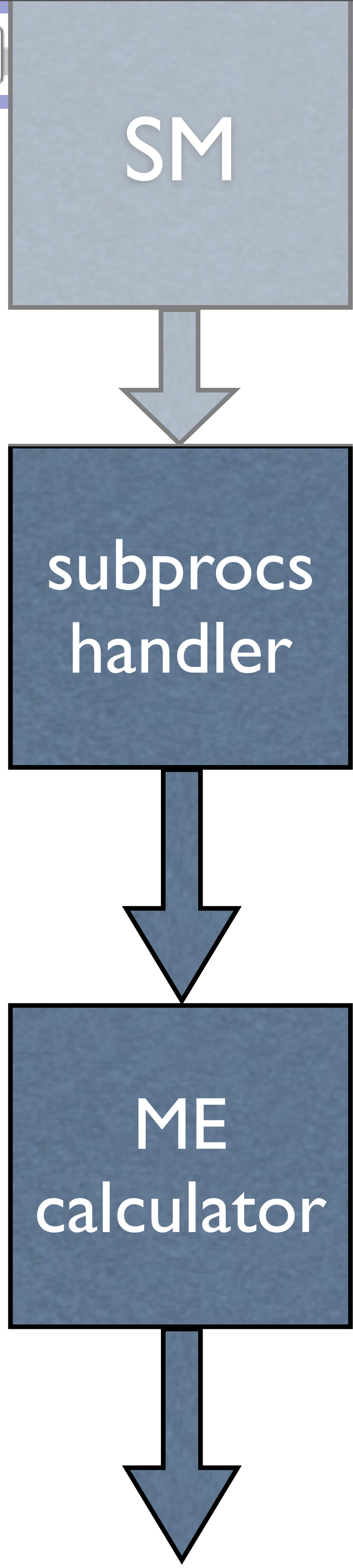
Most use Feynman diagrams w/ tricks to reduce the factorial growth, others have recursive relations to reduce the complexity to exponential. ☺





General structure

Includes all possible subprocess leading to a given multi-jet final state automatically or manually (done once for all)

d~ d -> a a u u~ g
d~ d -> a a c c~ g
s~ s -> a a u u~ g
s~ s -> a a c c~ g

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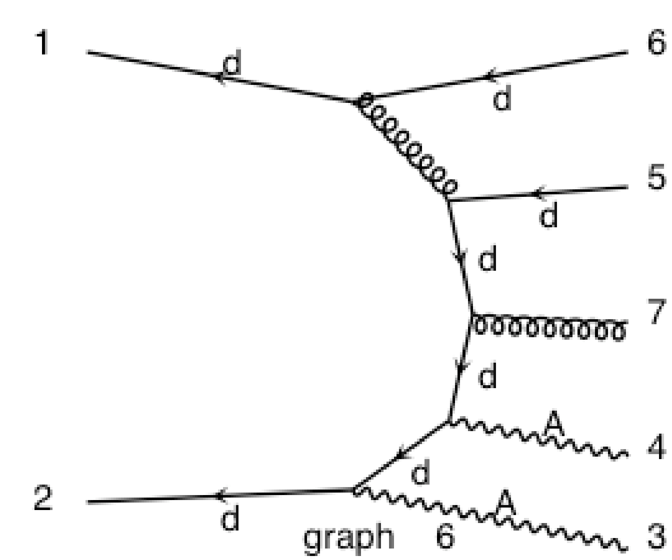

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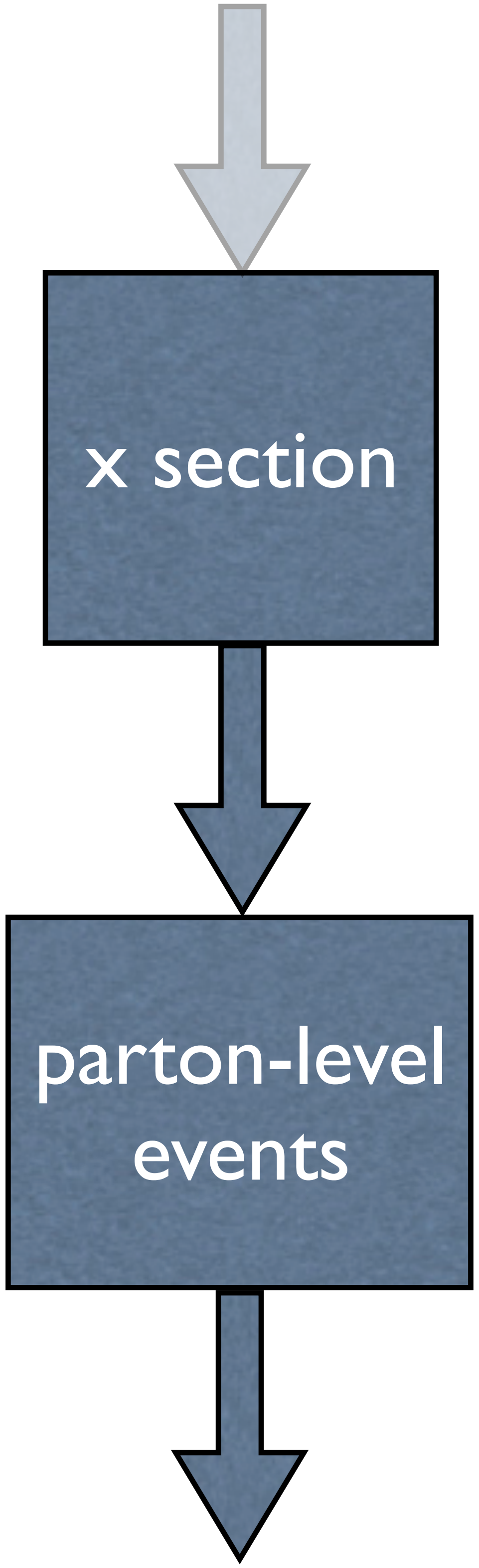
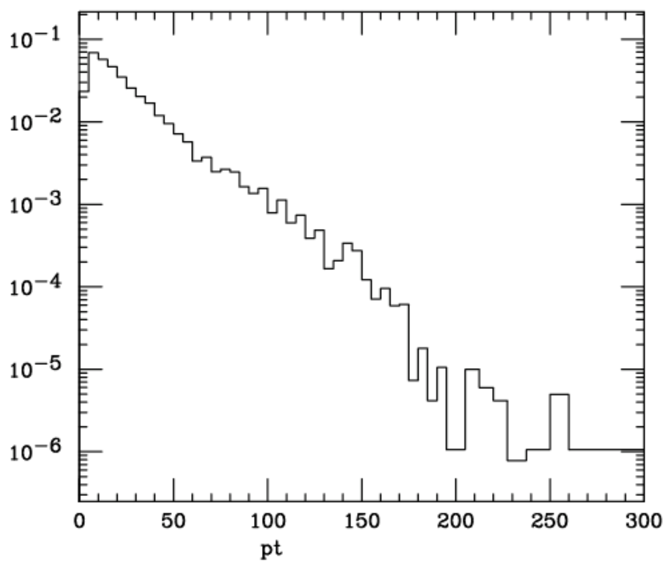
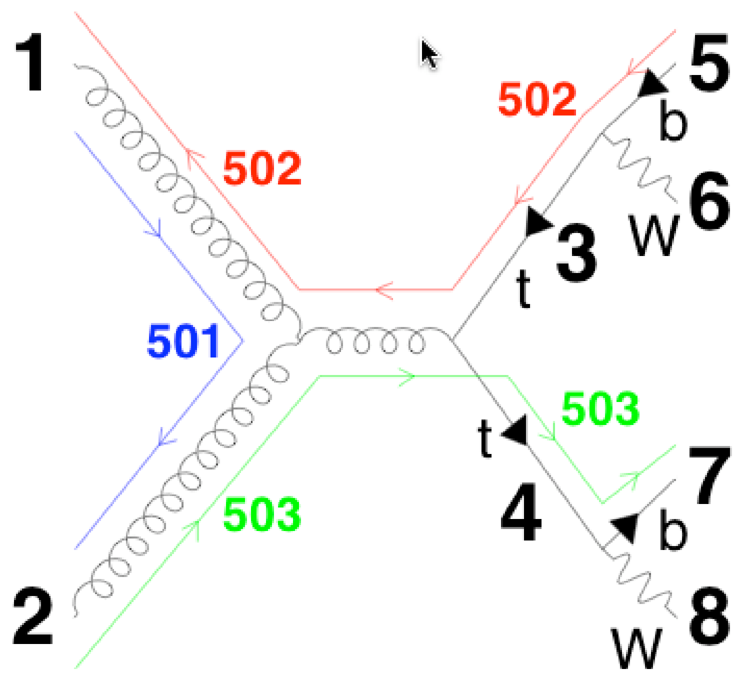


Integrate the matrix element over the phase space using a multi-channel technique and using parton-level cuts.



Events are obtained by unweighting. These are at the parton-level. Information on particle id, momenta, spin, color is given in the Les Houches format.



Summary of tree-level computations

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$

- Matrix element calculators provide our first estimation of rates for inclusive final states.

- Extra radiation is included: it is described by the PDF's in the initial state and by the definition of a final state parton, which at LO represents all possible final state evolutions.

- Due to the above approximations a cross section at LO can strongly depend on the factorization and renormalization scales.

- Any tree-level calculation for a final state F can be promoted to the exclusive F + X through a shower. More on this soon...

A simple plan

- Intro: the LHC challenge
- Minimal QCD: basics
- Precision QCD: from NLO to NNLO
- Useful QCD: Parton Shower approach
- Best QCD: Merging Fixed Order with PS

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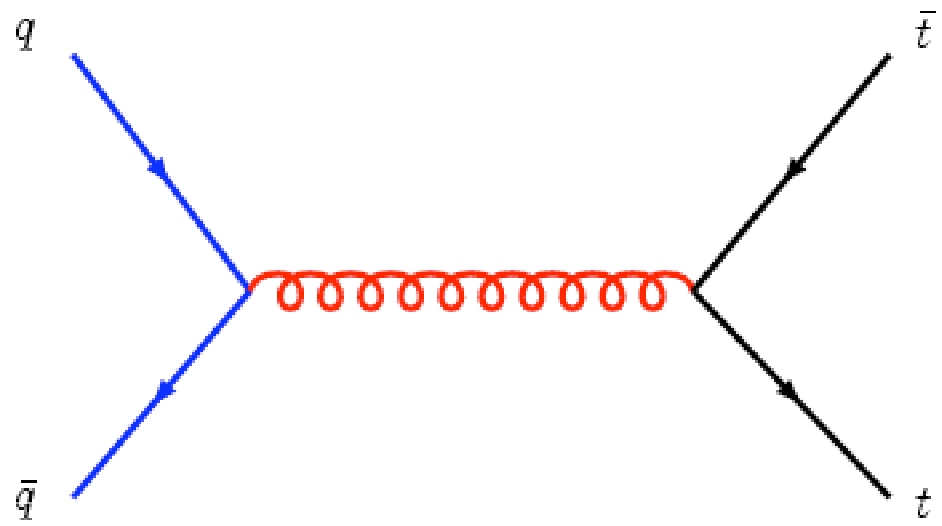
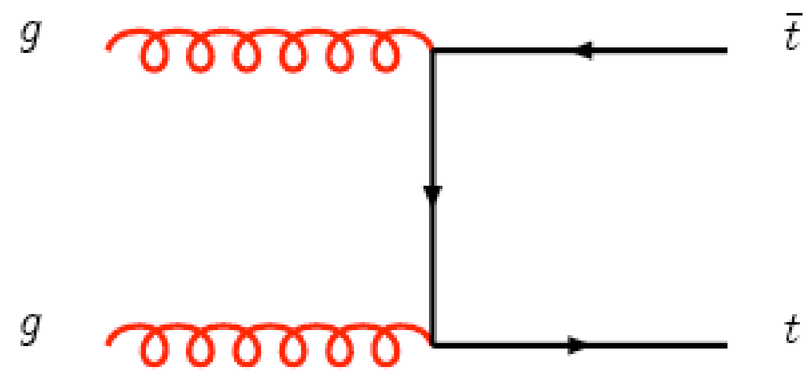
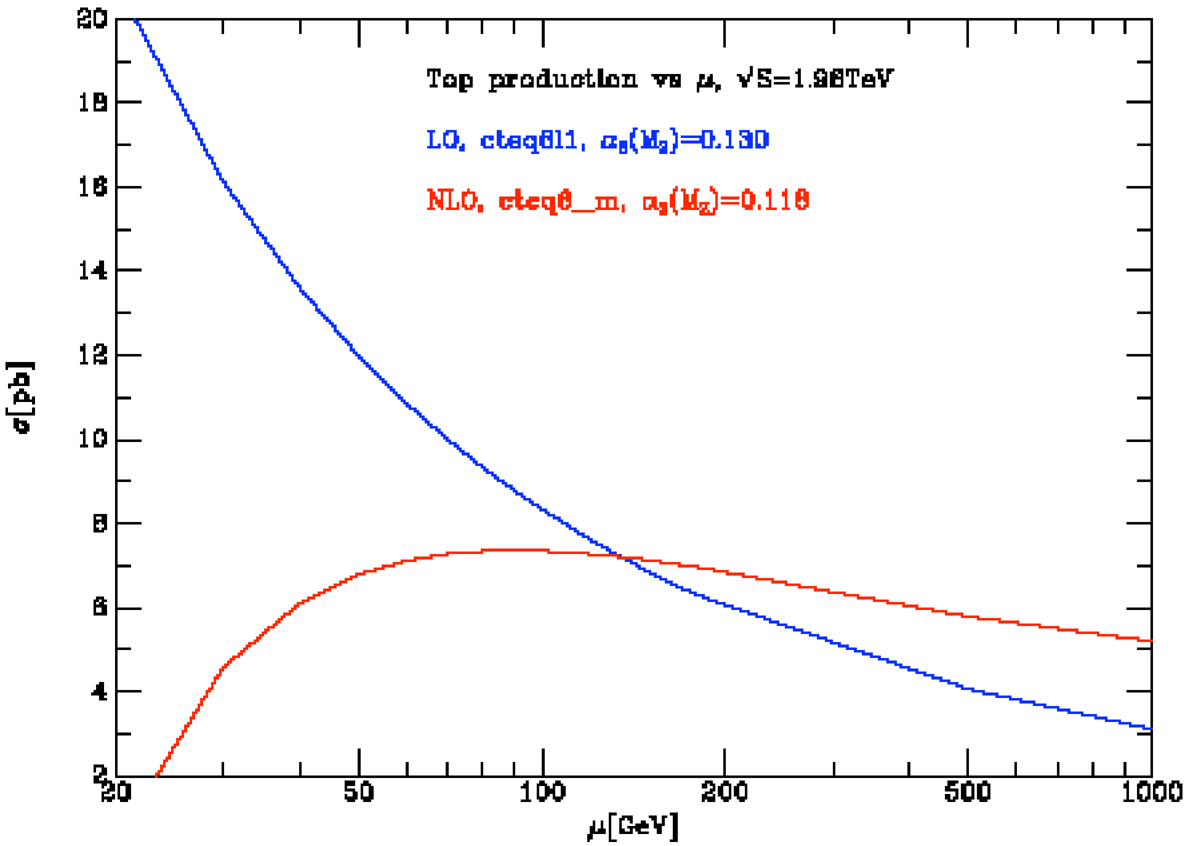
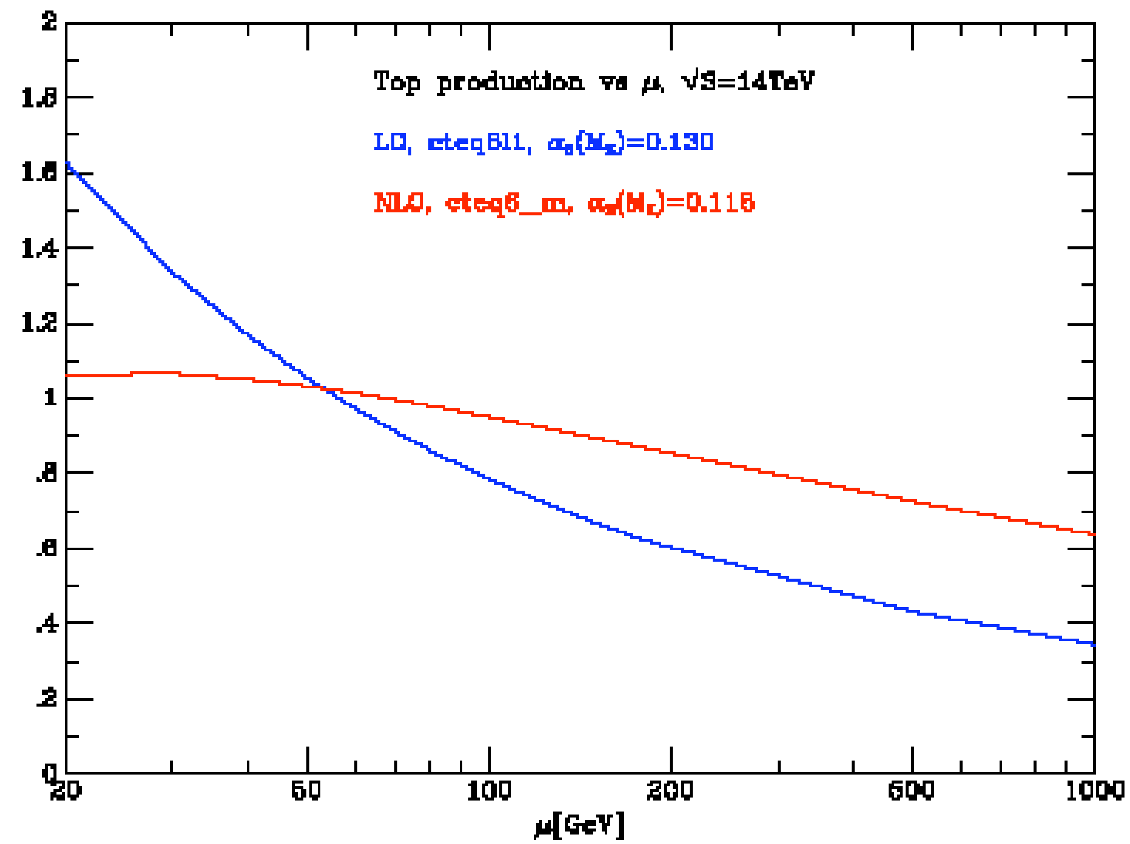
Tevatron vs LHC



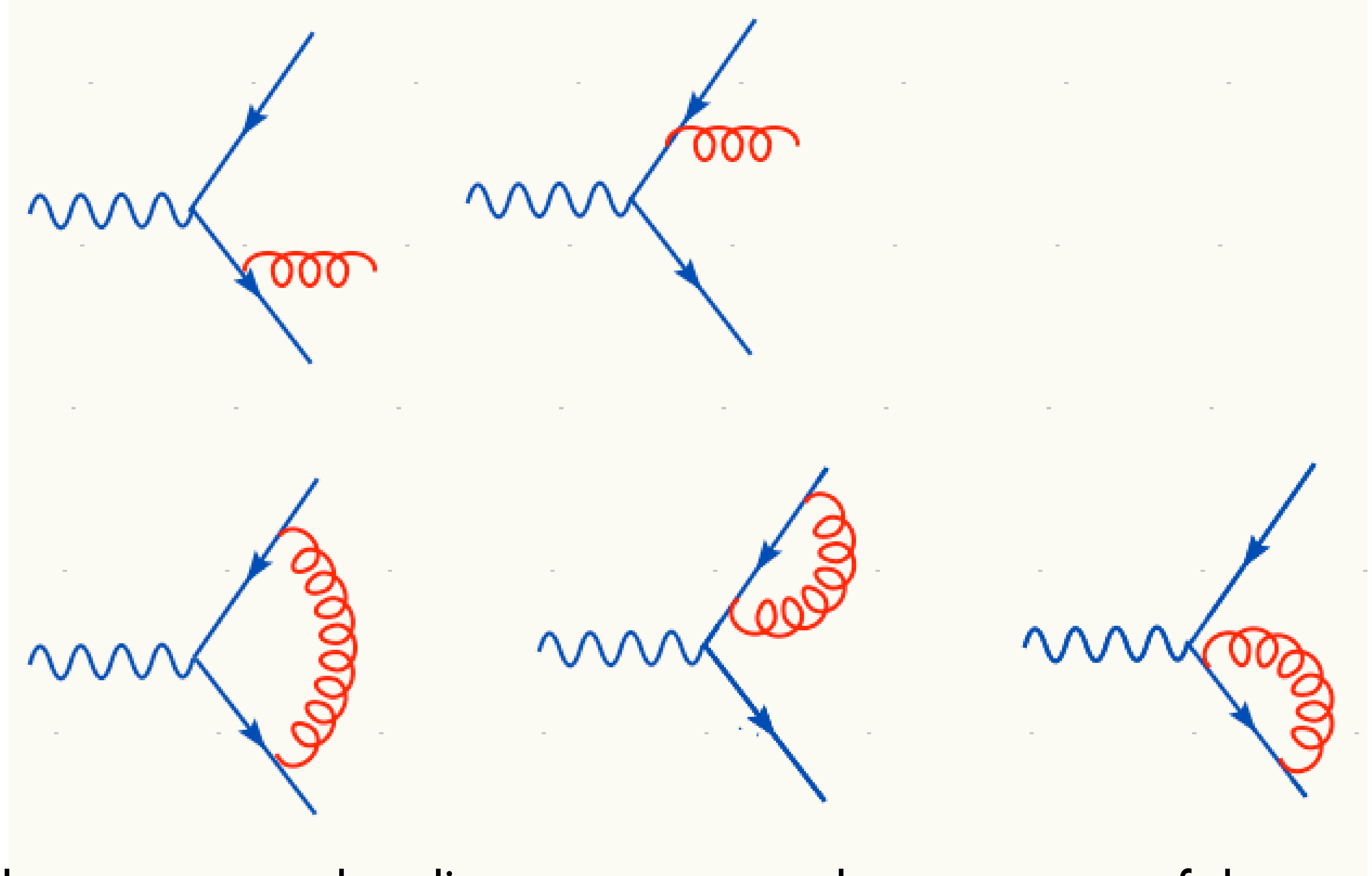






Inclusion of higher order corrections leads to a stabilization of the prediction.
At the LHC scale dependence is more difficult to estimate.

The elements of NLO calculation



Real

Virtual

The KLN theorem states that divergences appear because some of the final state are physically degenerate but we treated them as different. A final state with a soft gluon is nearly degenerate with a final state with no gluon at all (virtual).

$$\int \frac{d^d k}{(2\pi)^d} \ldots$$

$$\sigma^{\rm NLO} = \int_R |M_{real}|^2 d\Phi_3 + \int_V 2Re\left(M_0 M^*_{virt}\right) d\Phi_2 = \text{finite!}$$

Infrared divergences

Infrared divergences arise from interactions that happen a long time after the creation of the quark/antiquark pair.

When distances become comparable to the hadron size of ~1 Fermi, quasi-free partons of the perturbative calculation are confined/hadronized non-perturbatively.

We have seen that in total cross sections such divergences cancel. But what about for other quantities?

Well obviously the only possibility is to try to use the pQCD calculations for quantities that are not sensitive to the to the long-distance physics.

Can we formulate a criterium that is valid in general?

YES! It is called INFRARED SAFETY

Infrared-safe quantities

DEFINITION: quantities are that are insensitive to soft and collinear branching.

For these quantities, an extension of the general theorem (KLN) exists which proves that infrared divergences cancel betwen real and virtual or are simply removed by kinematic factors.

Such quantities are determined primarly by hard, short-distance physics. Long-distance effects give power corrections, suppressed by the inverse power of a large momentum scale (which must be present in the first place to justify the use of PT).

EXAMPLES: total rates & cross sections, jet distrubutions, shape variables...

NLO codes calculate IR safe quantities and return histograms (calculators)

Something to remember well

Calling a code "a NLO code" is an abuse of language and can be confusing.

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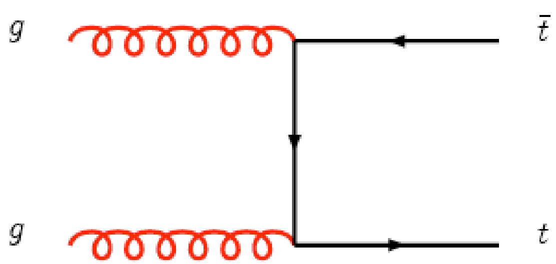

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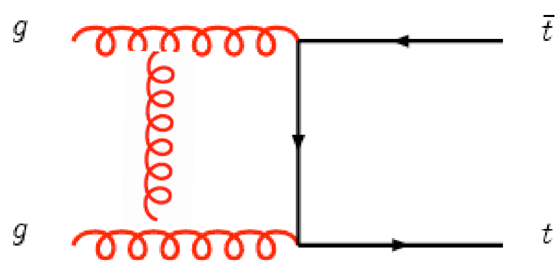
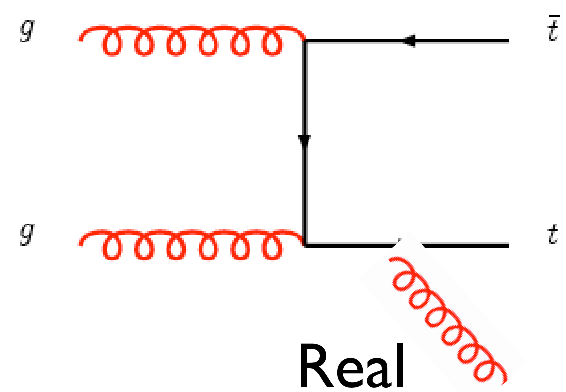


Virt



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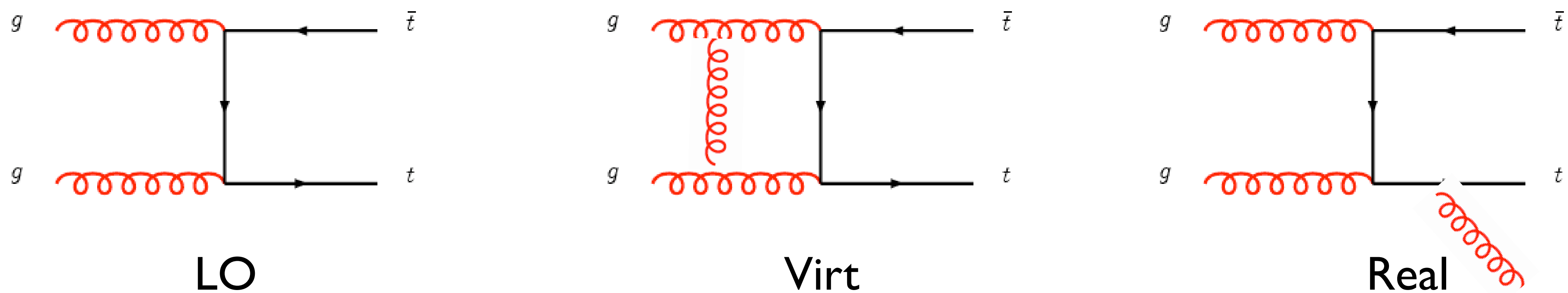

- ☞ Total cross section, σ(tt)
- ☞ P_T of one top quark
- ☞ P_T of the tt pair
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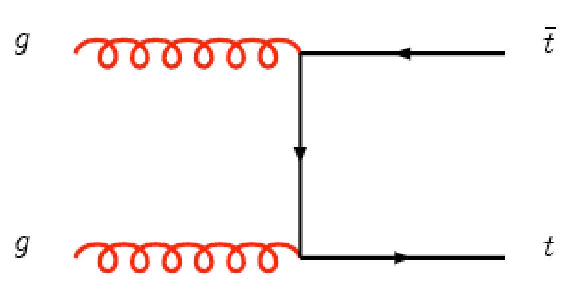

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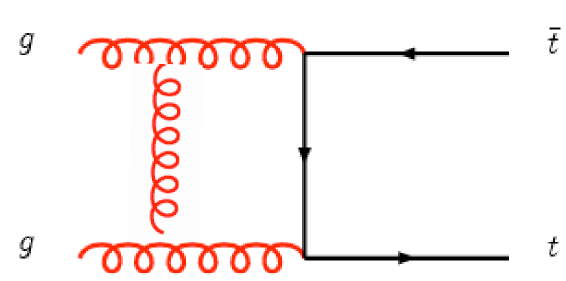

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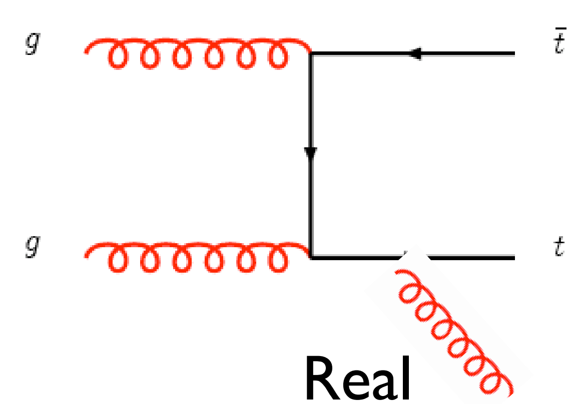

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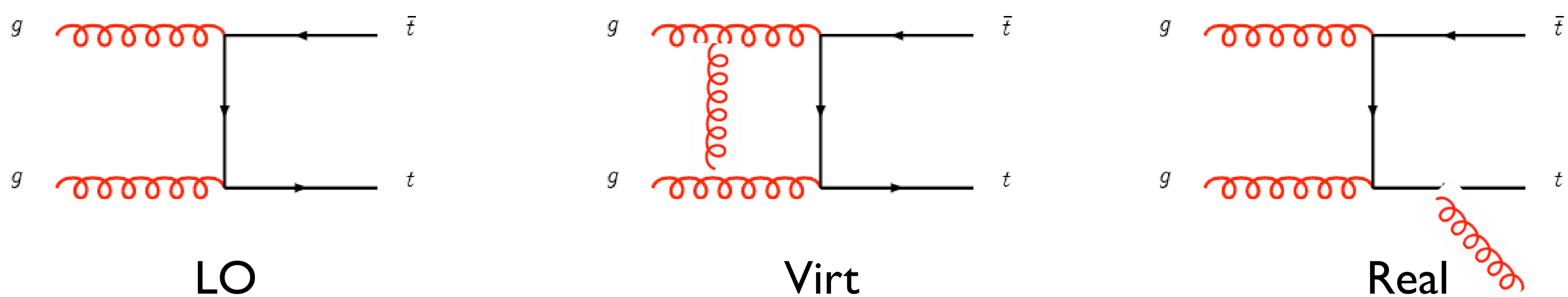

LO Virt Real

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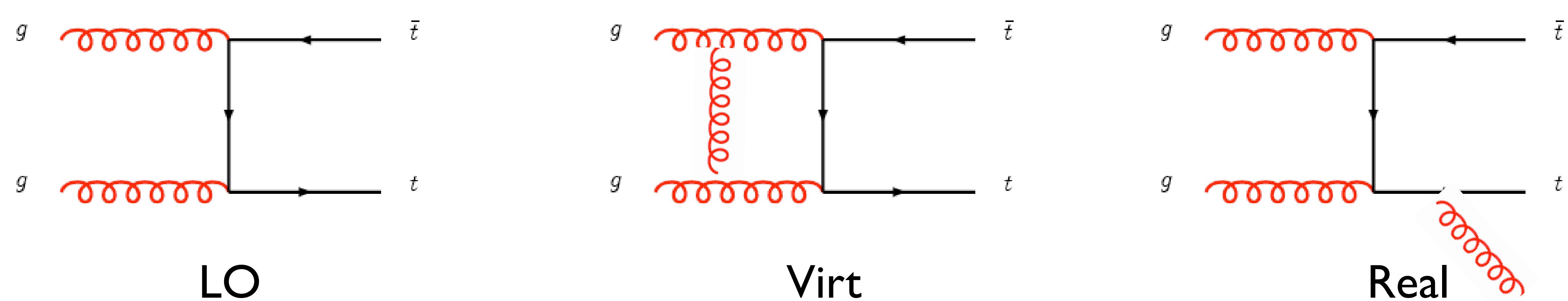

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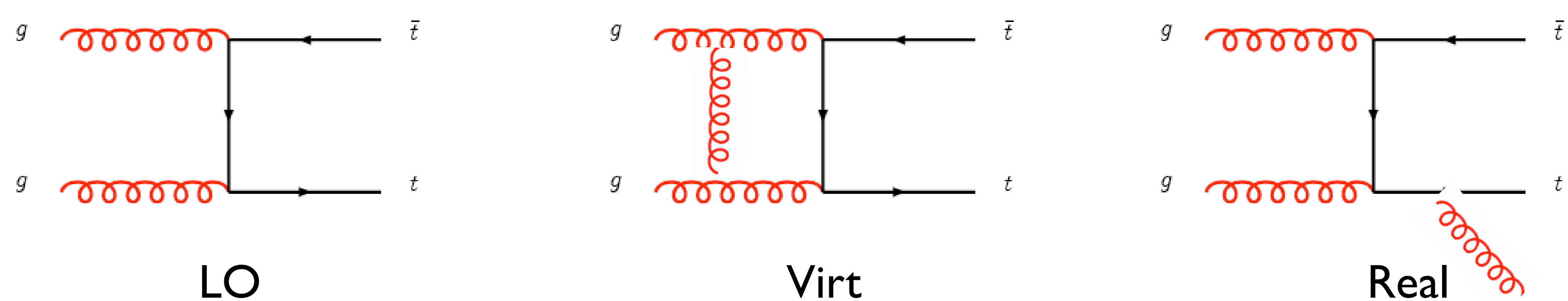

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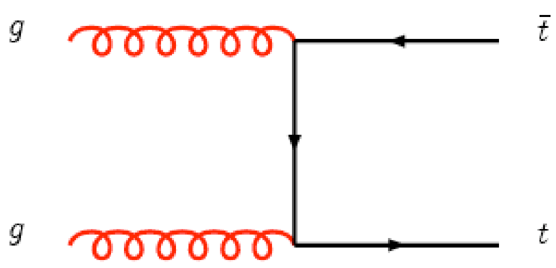

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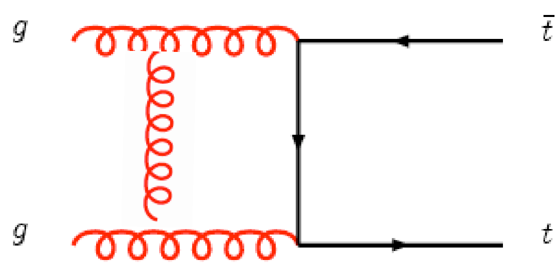

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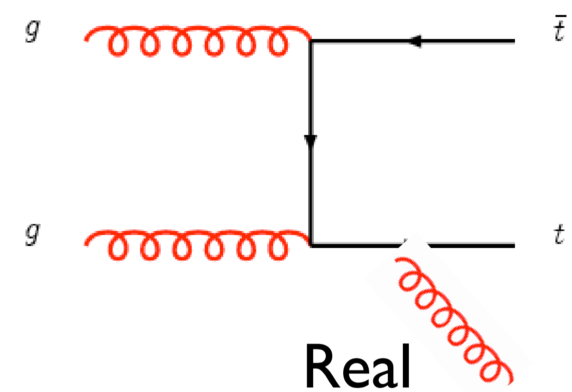

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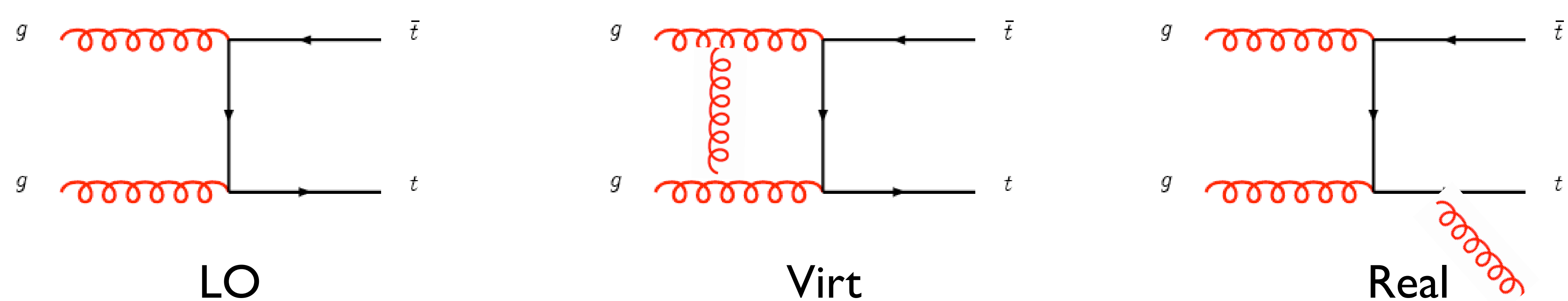

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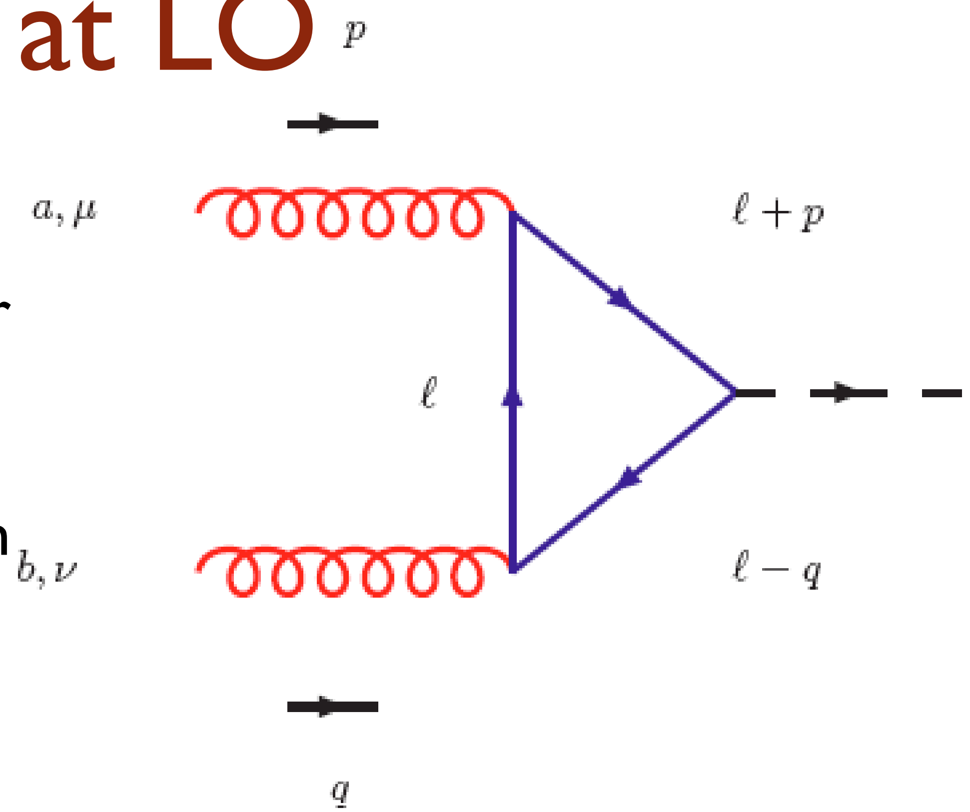
Anatomy of pp→Higgs at NLO

- LO : 1-loop calculation and HEFT
- NLO in the HEFT
  - Virtual corrections and renormalization
  - Real corrections and IS singularities
- Cross sections at the LHC

pp→H at LO



This is a “simple” 2→1 process.

However, at variance with pp→W, the LO order process already proceeds through a loop.

In this case, this means that the loop calculation has to give a finite result!

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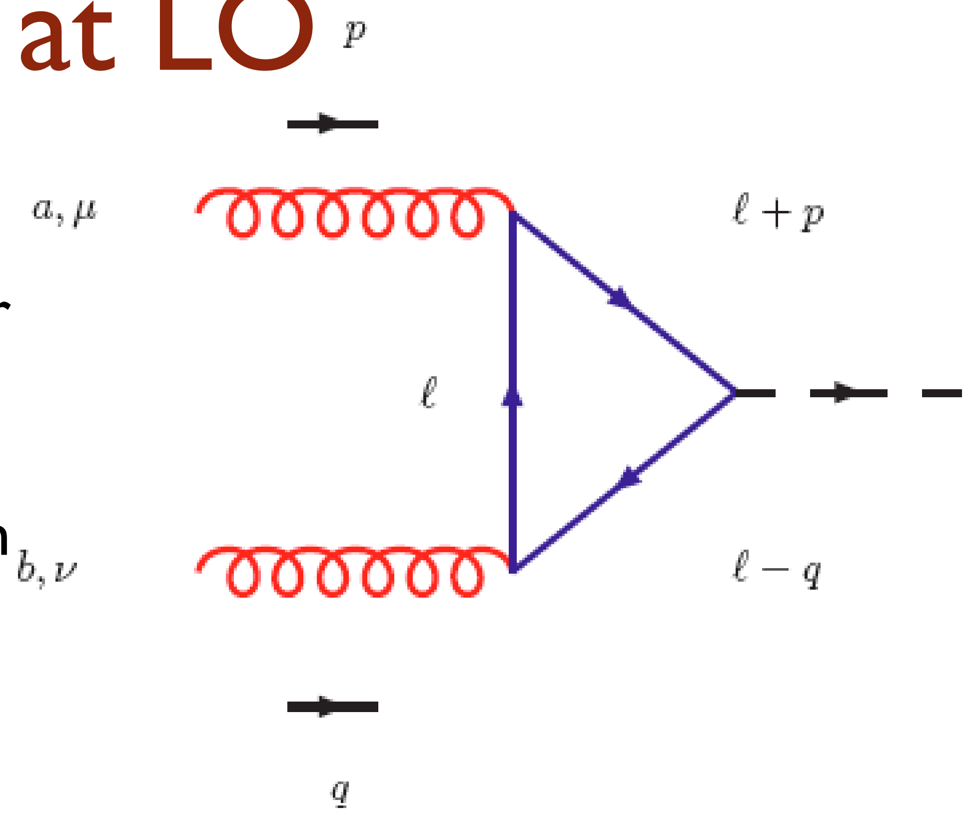

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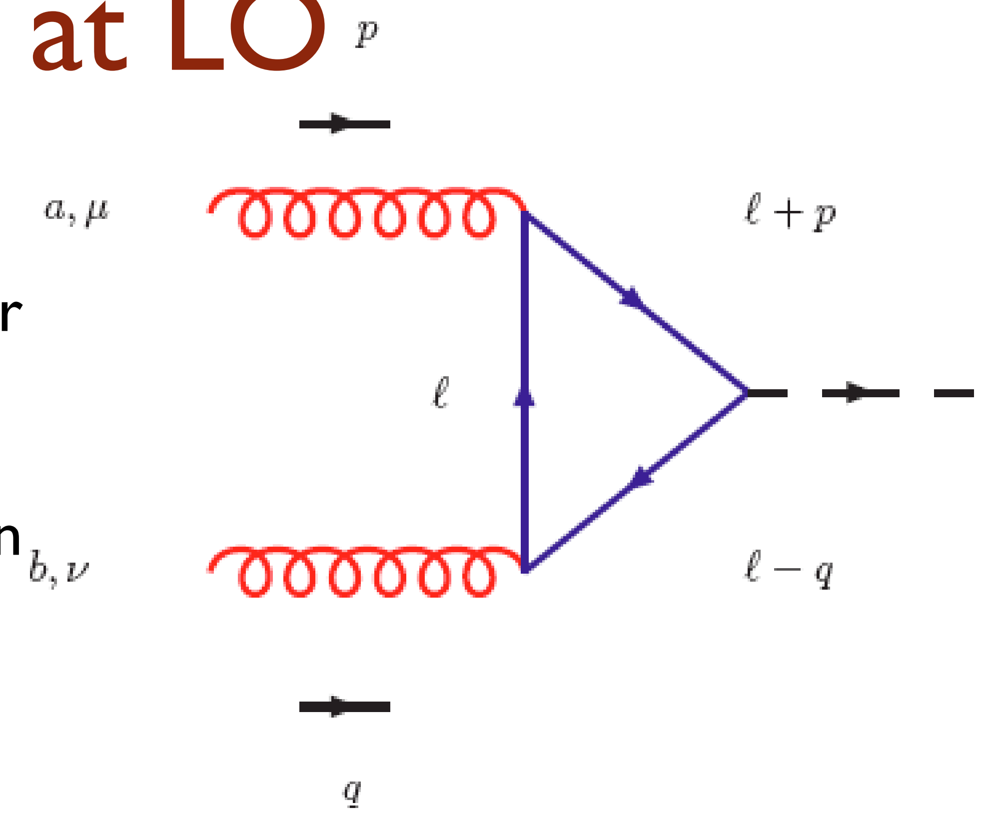

$$i\mathcal{A} = -(-ig_s)^2 \mathrm{Tr}(t^a t^b)\left(\frac{-im_t}{v}\right)\int \frac{d^d\ell}{(2\pi)^n}\frac{T^{\mu\nu}}{\mathrm{Den}}(i)^3\epsilon_\mu(p)\epsilon_\nu(q)$$

where

$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$$

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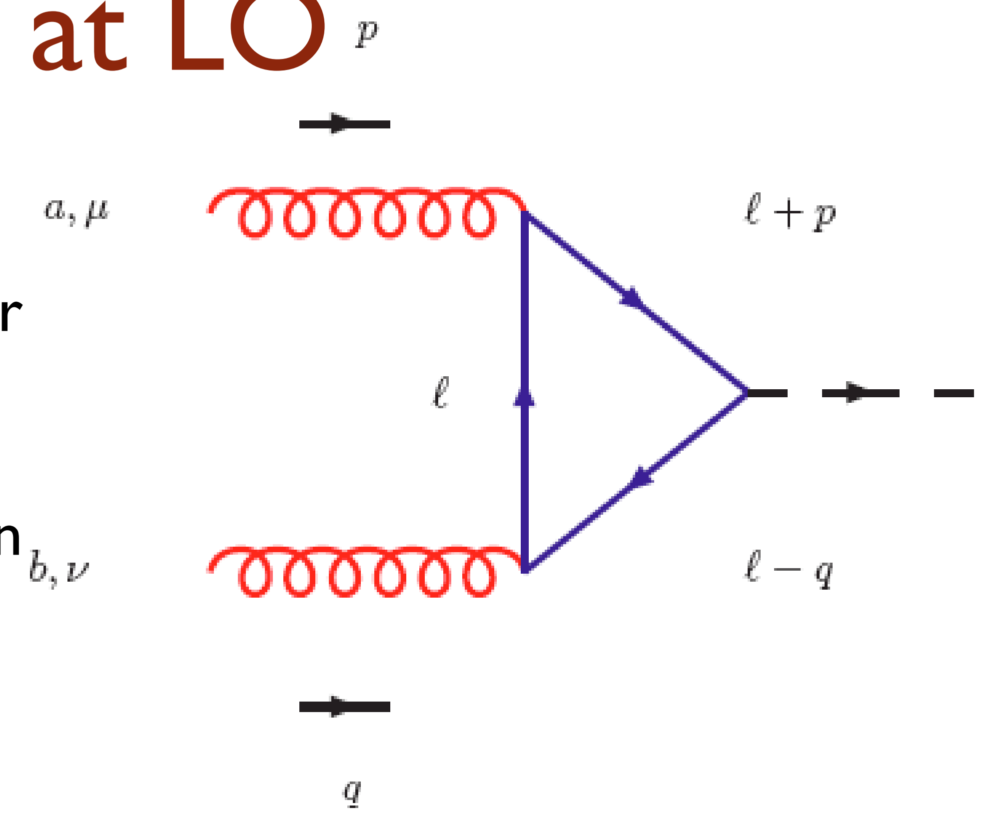

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$$\mathrm{Den} = (\ell^2 - m_t^2)[(\ell+p)^2 - m_t^2][(\ell-q)^2 - m_t^2]$$

We combine the denominators into one by using $\frac{1}{ABC} = 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[Ax + By + C(1-x-y)]^3}$

$$\frac{1}{\mathrm{Den}} = 2\int dx\, dy \frac{1}{[\ell^2 - m_t^2 + 2\ell \cdot (px - qy)]^3}.$$

pp→H at LO

$$\int \frac{d^dk}{(2\pi)^d} \frac{k^2}{(k^2-C)^3} = \frac{i}{32\pi^2}(4\pi)^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon}(2-\epsilon)C^{-\epsilon}$$

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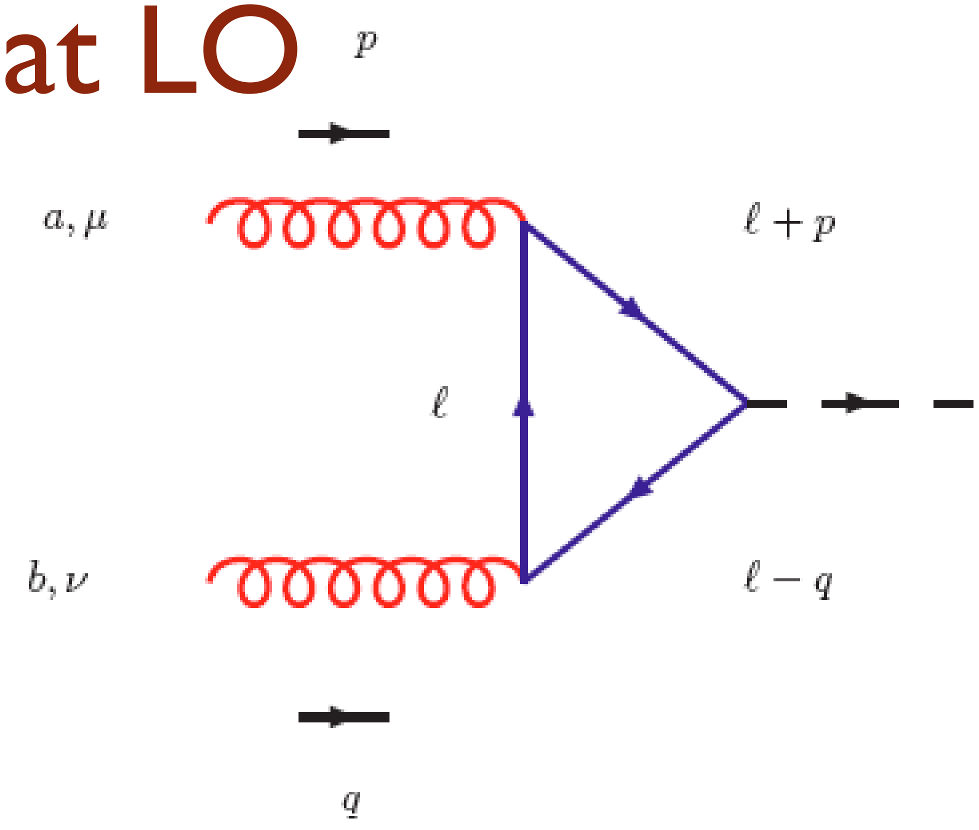

where d=4-2eps. By substituting we arrive at a very simple final result!!

$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dx dy \left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

Comments:

* The final dependence of the result is mt^2 : one from the Yukawa coupling, one from the spin flip.
* The tensor structure could have been guessed by gauge invariance.
* The integral depends on mt and mh.

LO cross section

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 \, g(x_1, \mu_f) g(x_2, \mu_f) \, \hat{\sigma}(gg \to H)$$

$$x_1 \equiv \sqrt{\tau} e^{y} \quad x_2 \equiv \sqrt{\tau} e^{-y} \quad \tau = x_1 x_2 \qquad \tau_0 = M_H^2/S \quad z = \tau_0/\tau$$

$$= \frac{\alpha_S^2}{64\pi v^2} \mid I\left(\frac{M_H^2}{m^2}\right) \mid^2 \tau_0 \int_{\log\sqrt{\tau_0}}^{-\log\sqrt{\tau_0}} dy g(\sqrt{\tau_0} e^{y}) g(\sqrt{\tau_0} e^{-y})$$

The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

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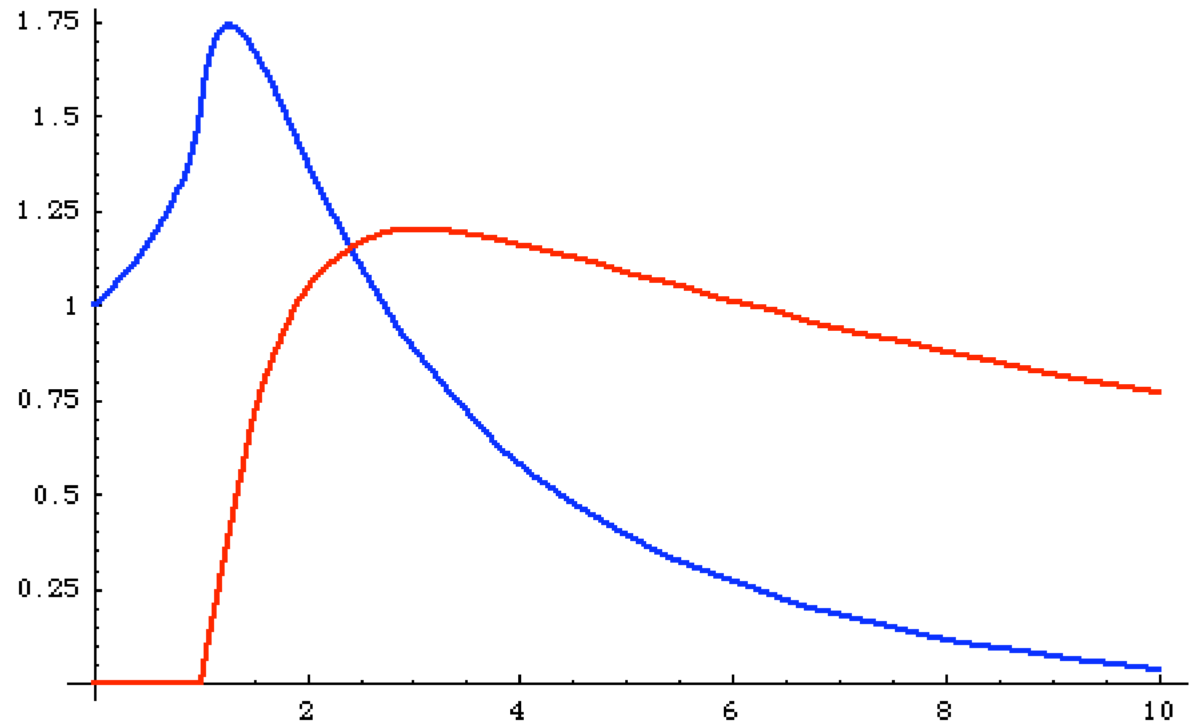

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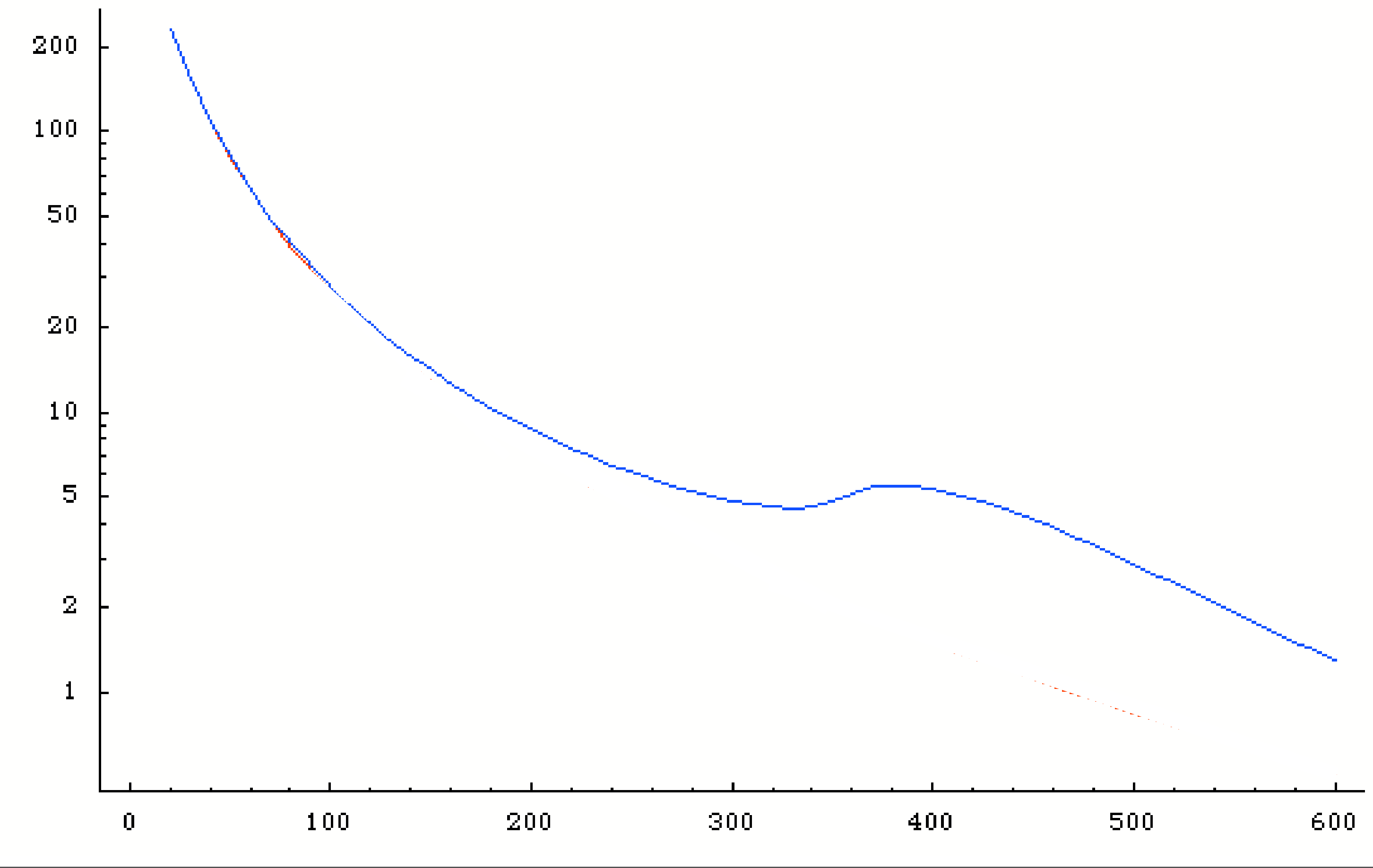
The hadronic cross section can be expressed a function of the gluon-gluon luminosity.

I(x) has both a real and imaginary part, which develops at mh=2mt.

This causes a bump in the cross section.



pp →H @ NLO

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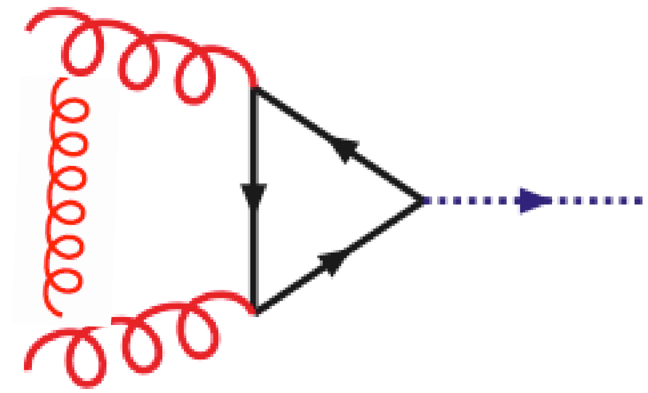

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Can we avoid that?

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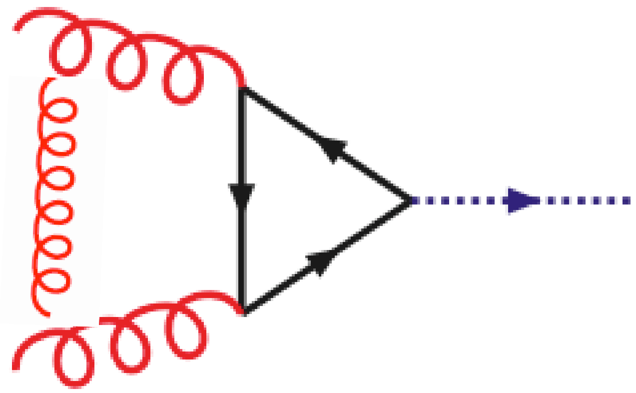

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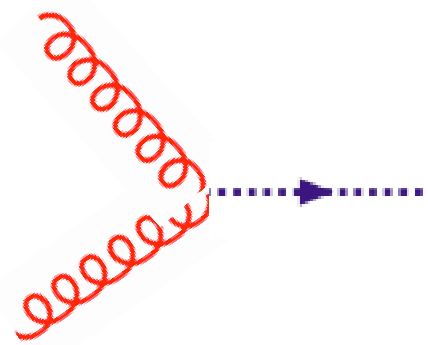
Can we avoid that?

Let's consider the case where the Higgs is light:

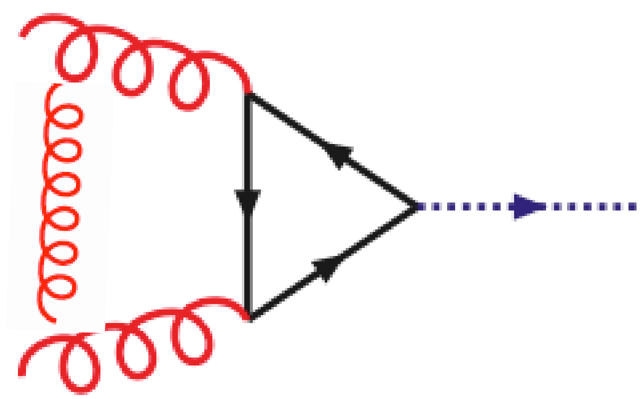
$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dx dy \left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

$$\xrightarrow{m \gg M_H} \quad -\frac{\alpha_S}{3\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\epsilon_\mu(p)\epsilon_\nu(q).$$



pp →H @ NLO

At NLO we have to include an extra parton (virtual or real).



The virtuals will become a two-loop calculation!!

Can we avoid that?

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$$\mathcal{A}(gg \to H) = -\frac{\alpha_S m_t^2}{\pi v}\delta^{ab}\left(g^{\mu\nu}\frac{M_H^2}{2} - p^\nu q^\mu\right)\int dxdy\left(\frac{1-4xy}{m_t^2 - m_H^2 xy}\right)\epsilon_\mu(p)\epsilon_\nu(q).$$

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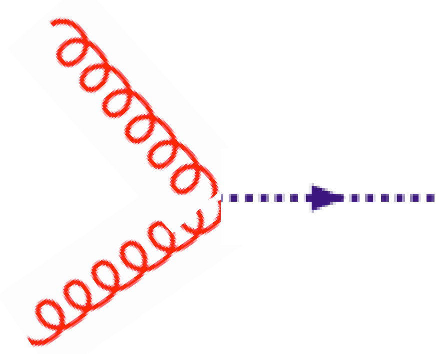


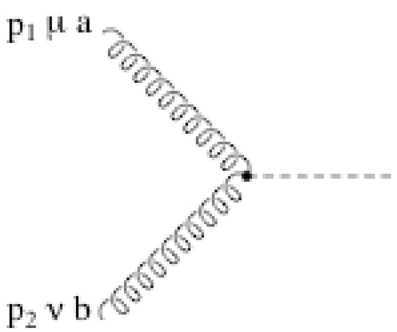
This looks like a local vertex, ggH.

The top quark has disappeared from the low energy theory but it has left something behind (non-decoupling).

Higgs effective field theory

$$\mathcal{L}_{\rm eff} = -\frac{1}{4}\left(1 - \frac{\alpha_S}{3\pi}\frac{H}{v}\right) G^{\mu\nu}G_{\mu\nu}$$

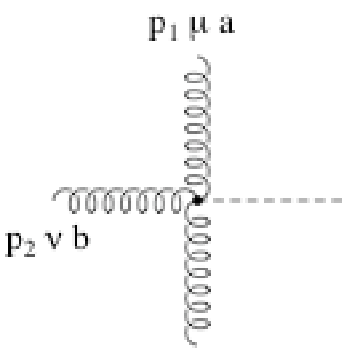
This is an effective non-renormalizable theory (no top) which describes the Higgs couplings to QCD.



(a)

$iA\delta^{ab}H^{\mu\nu}(p_1,p_2)$

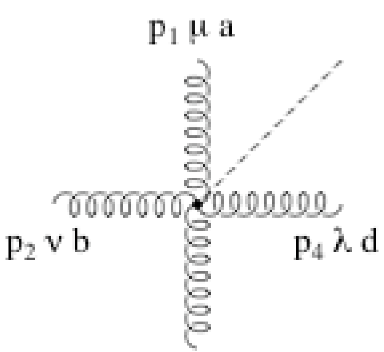
$$H^{\mu\nu}(p_1,p_2) = g^{\mu\nu}p_1\cdot p_2 - p_1^\nu p_2^\mu.$$



(b)

$-Agf^{abc}V^{\mu\nu\sigma}(p_1,p_2,p_3)$

$$V^{\mu\nu\rho}(p_1,p_2,p_3) = (p_1-p_2)^\rho g^{\mu\nu} + (p_2-p_3)^\mu g^{\nu\rho} + (p_3-p_1)^\nu g^{\rho\mu},$$



(c)

$-iAg^2X^{abcd}_{\mu\nu\sigma\lambda}$

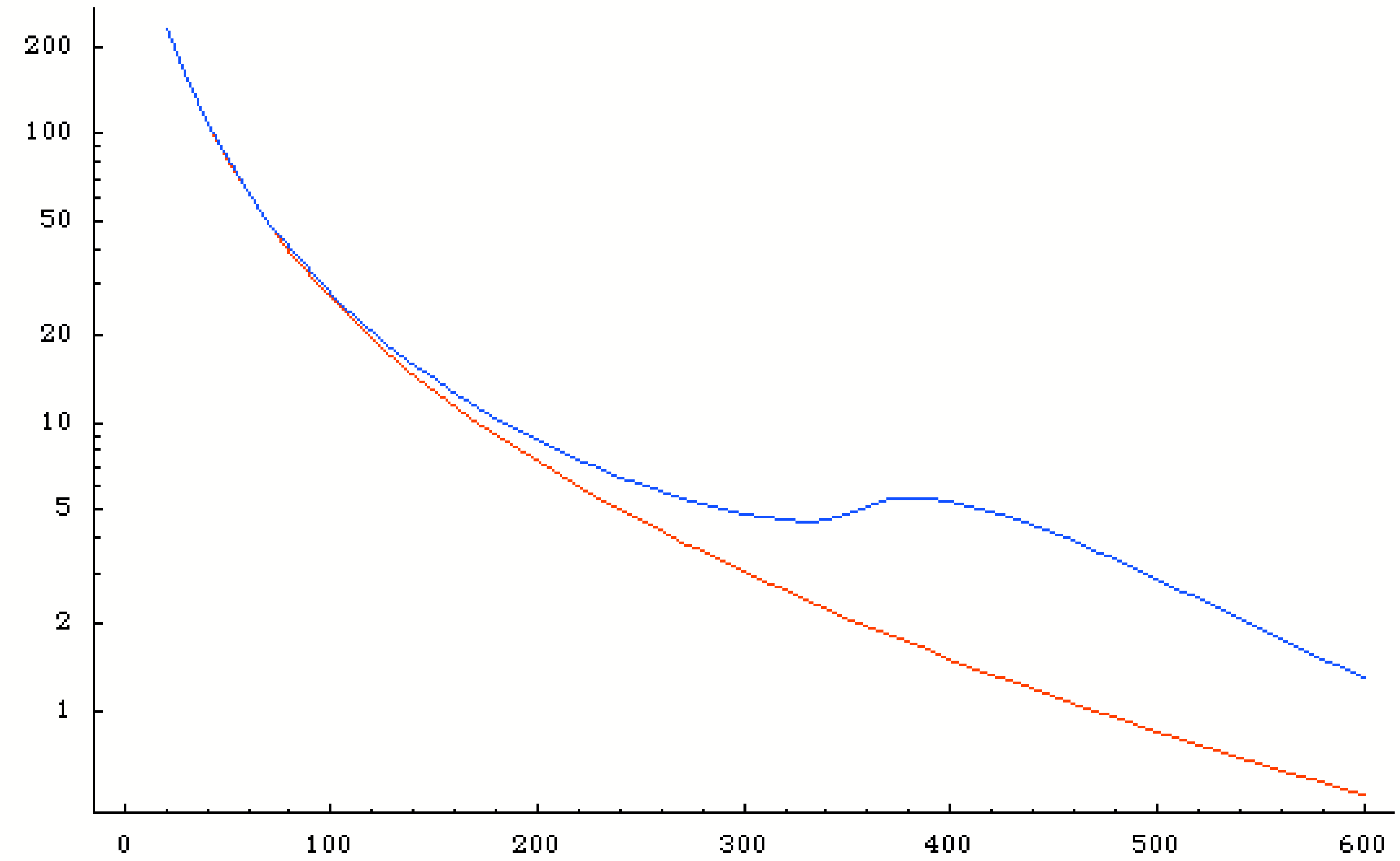
$$\begin{aligned} X^{\mu\nu\rho\sigma}_{abcd} &= f_{abe}f_{cde}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f_{ace}f_{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\ &+ f_{ade}f_{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma}). \end{aligned}$$

LO cross section: full vs HEFT

$$\sigma(pp \to H) = \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2\, g(x_1,\mu_f)g(x_2,\mu_f)\,\hat{\sigma}(gg \to H)$$

The accuracy of the calculation in the HEFT calculation can be directly assessed by taking the limit m→∞.

For light Higgs is better than 10%.

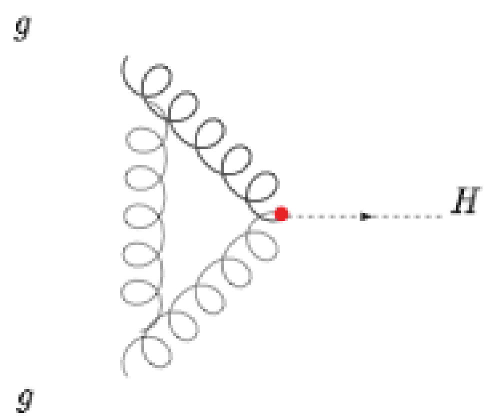


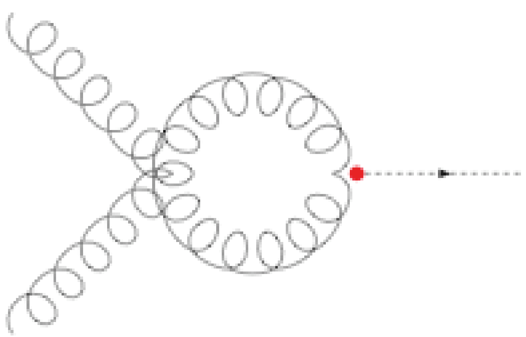
So, if we are interested in a light Higgs we use the HEFT and simplify our life.
If we do so, the NLO calculation becomes a standard 1-loop calculation, similar to Drell-Yan at NLO.

We can do it!!

Virtual contributions

Virtual contributions

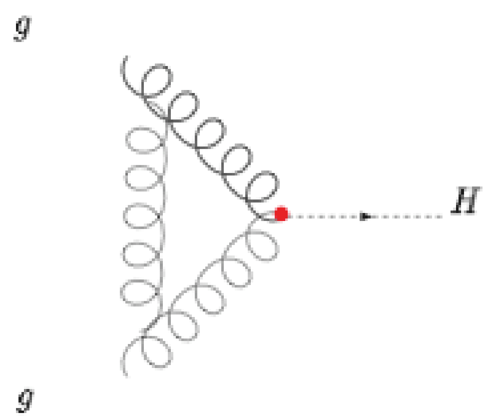


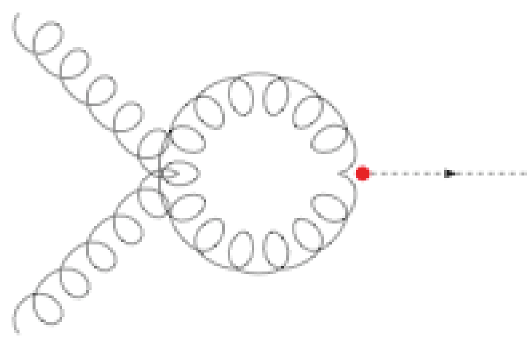


Out of 8 diagrams, only two are non-zero (in dimensional regularization), a bubble and a triangle.

They can be easily written down by hand.

Virtual contributions





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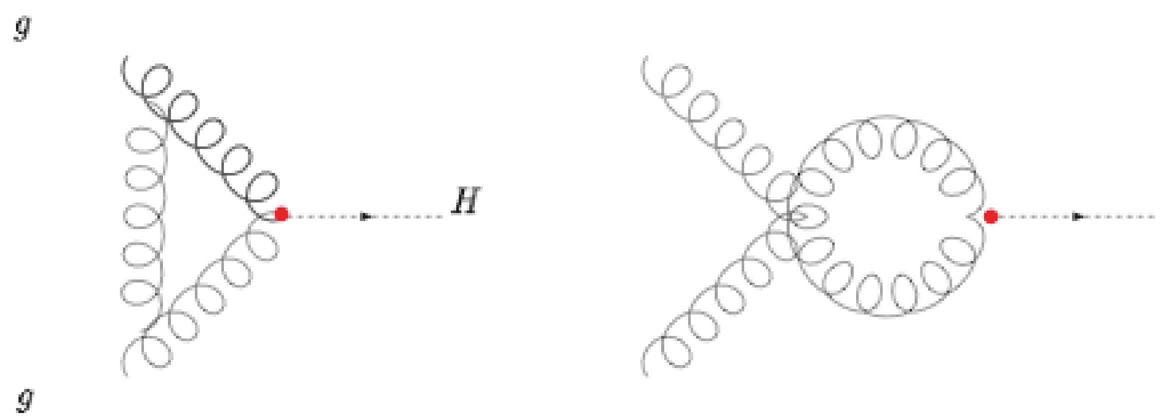

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$$\mathcal{L}_{\rm eff}^{\rm NLO} = \left(1 + \frac{11}{4}\frac{\alpha_S}{\pi}\right)\frac{\alpha_S}{3\pi}\frac{H}{v}G^{\mu\nu}G_{\mu\nu}$$

One also have to consider that the coefficient of the HEFT receive corrections which have to be included in the result.

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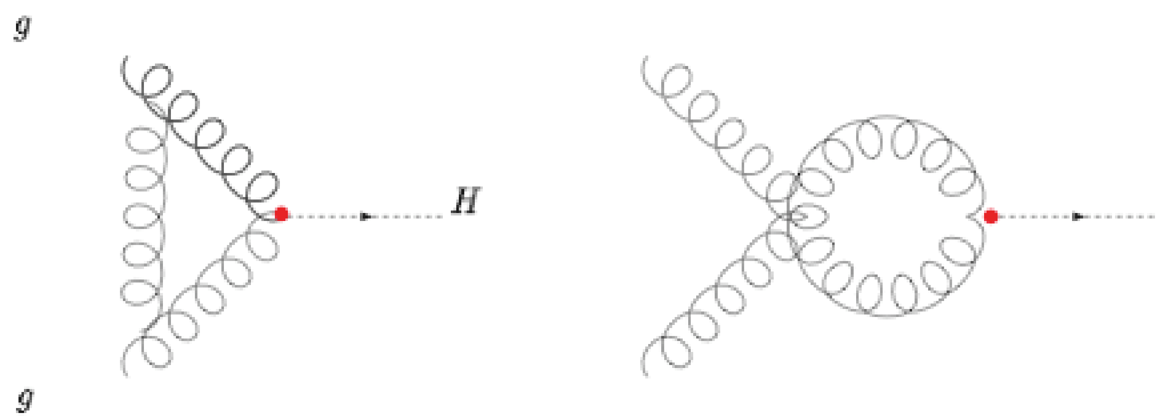

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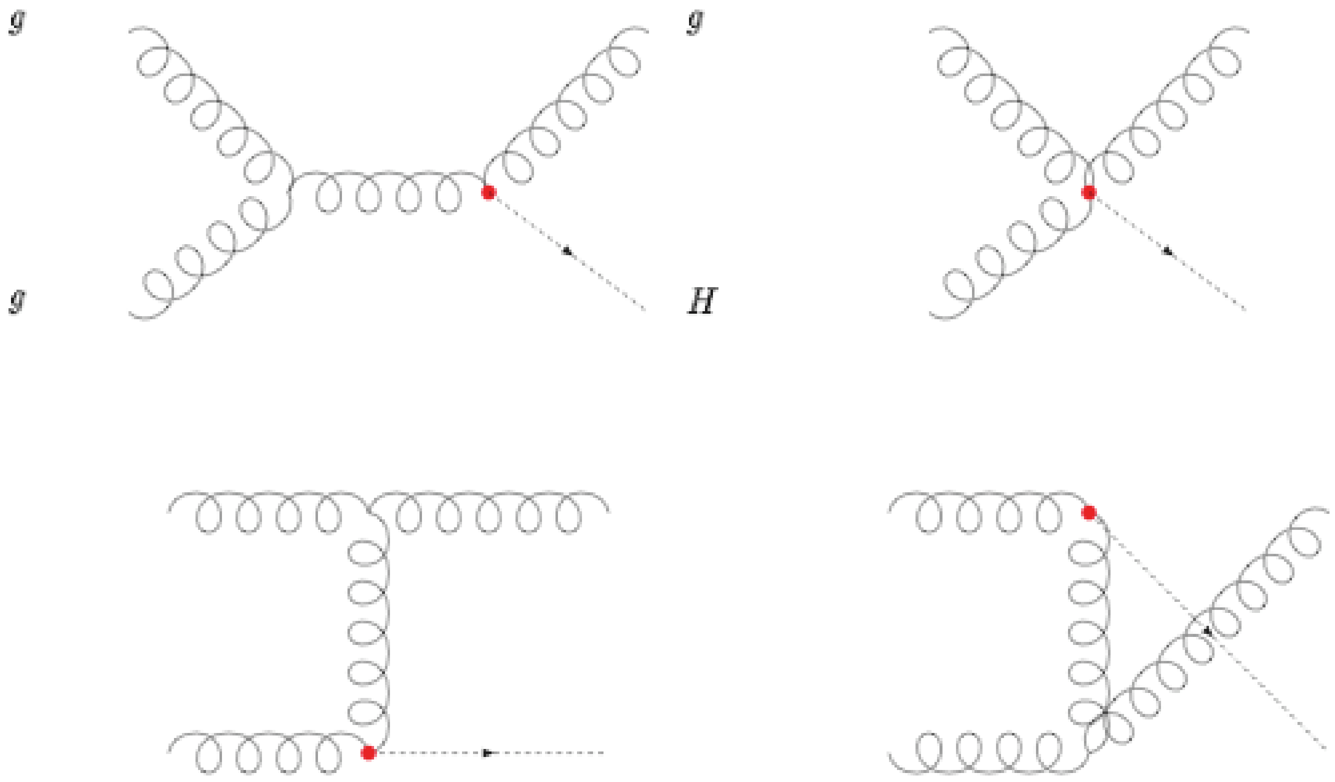
The result is:

$$\sigma_{\rm virt} = \sigma_0\,\delta(1-z)\,\left[1 + \frac{\alpha_S}{2\pi}C_A\left(\frac{\mu^2}{m_H^2}\right)^\epsilon c_\Gamma\left(-\frac{2}{\epsilon^2} + \frac{11}{3} + \pi^2\right)\right],$$

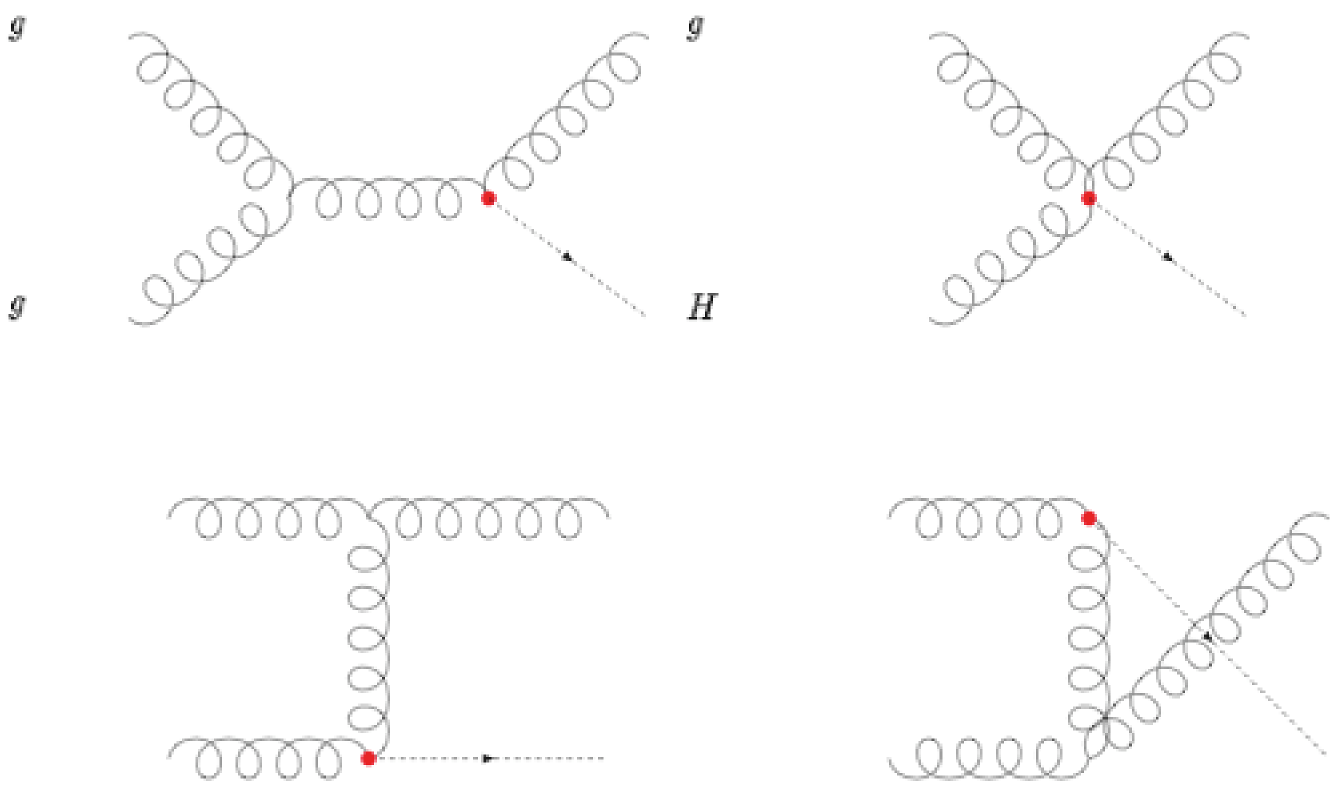
$$\sigma_{\rm Born} = \frac{\alpha_S^2}{\pi}\frac{m_H^2}{576 v^2 s}(1+\epsilon+\epsilon^2)\mu^{2\epsilon}\,\delta(1-z) \equiv \sigma_0\,\delta(1-z) \qquad z = m_H^2/s$$

Real contributions



This is the last piece: the result at the end must be finite!

Real contributions



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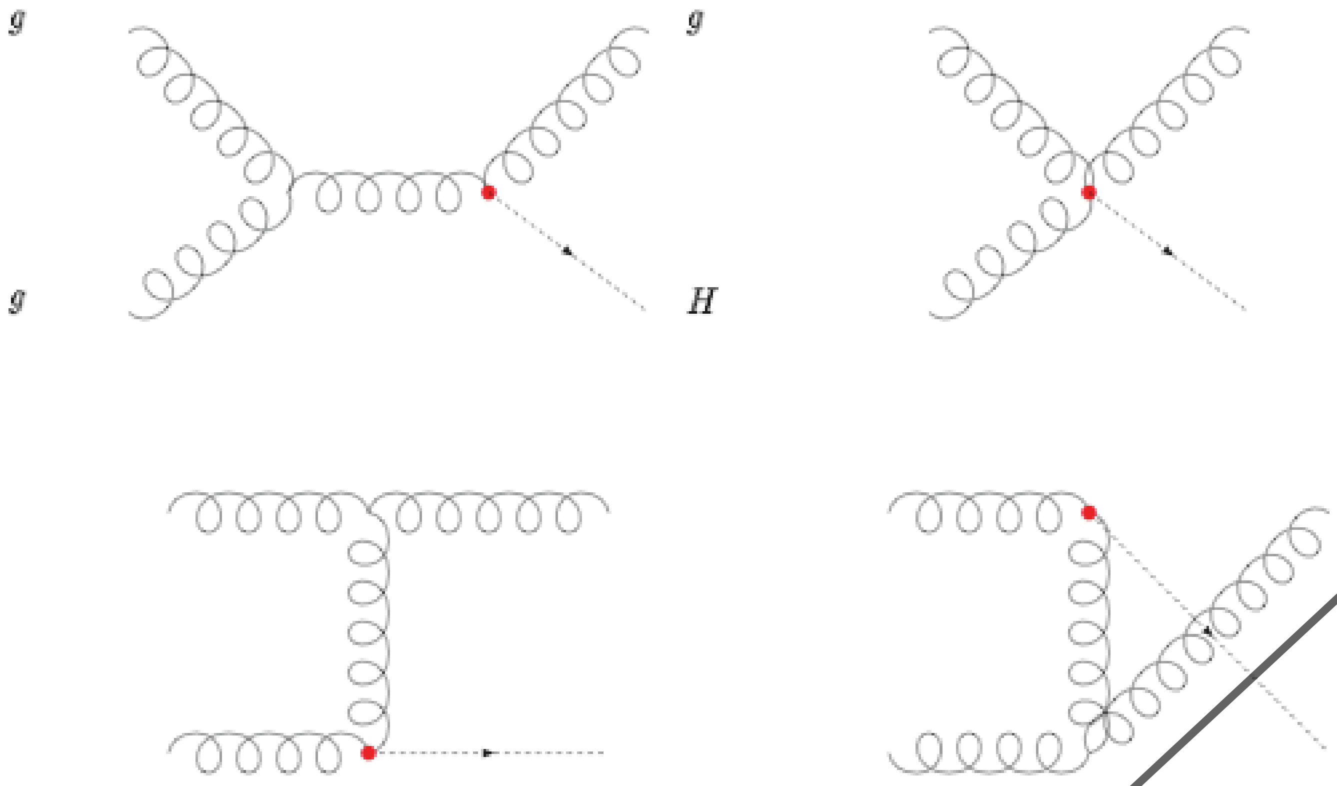
$$
\begin{aligned}
\sigma_{\rm real} &= \sigma_0 \, \frac{\alpha_S}{2\pi} C_A \left(\frac{\mu^2}{m_H^2}\right)^\epsilon c_\Gamma \left[\left(\frac{2}{\epsilon^2} + \frac{2}{\epsilon}\frac{b_0}{C_A} - \frac{\pi^2}{3} \right) \delta(1-z) \right. \\
& -\frac{2}{\epsilon} p_{gg}(z) - \frac{11}{3}\frac{(1-z)^3}{z} - 4\frac{(1-z)^2(1+z^2)+z^2}{z(1-z)} \log z \\
& \left. + 4 \frac{1+z^4+(1-z)^4}{z} \left(\frac{\log(1-z)}{1-z} \right)_+ \right] .
\end{aligned}
$$

Real contributions



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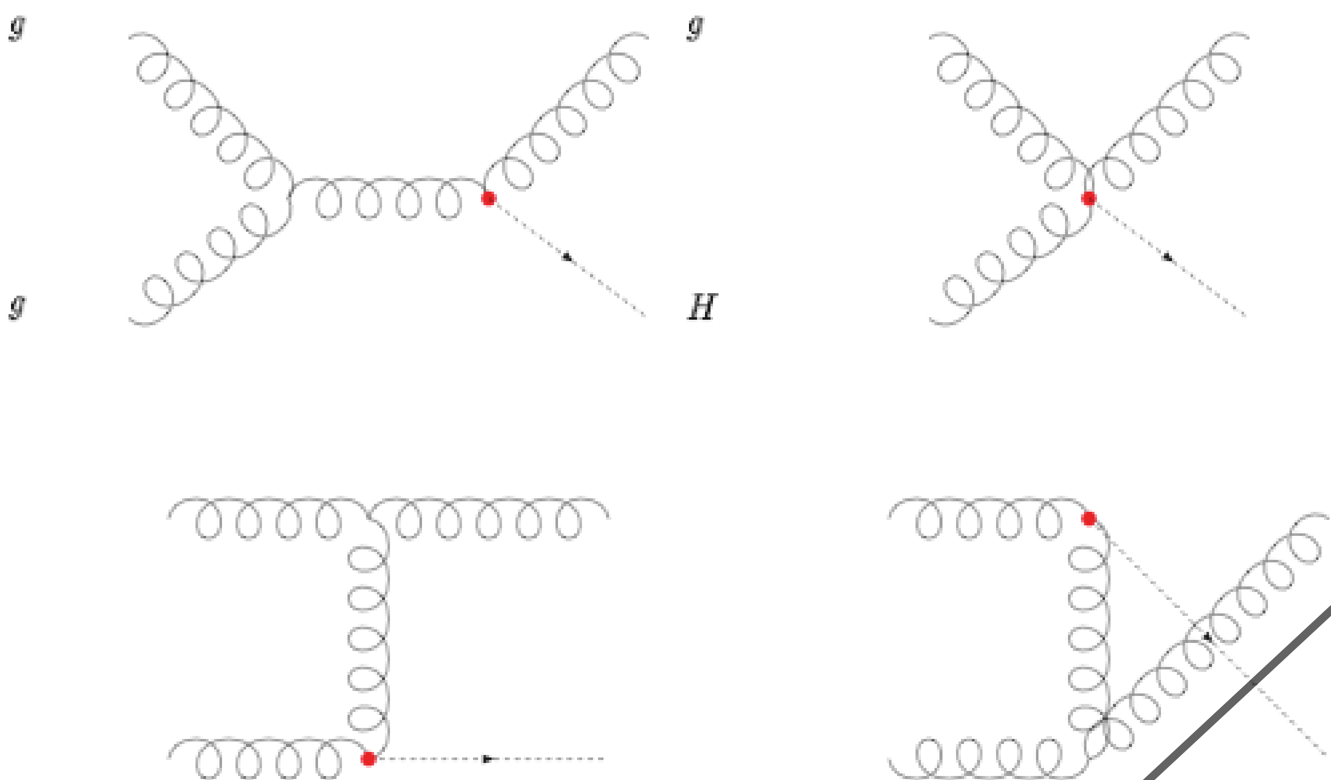

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This is the renormalization of the coulping!!

$$\sigma^{\rm UV}_{\rm c.t.} = 2\,\sigma_{\rm Born}\frac{\alpha_S}{2\pi}\left[-\left(\frac{\mu^2}{\mu^2_{\rm UV}}\right)^{\epsilon} c_\Gamma \frac{b_0}{\epsilon}\right] \checkmark$$

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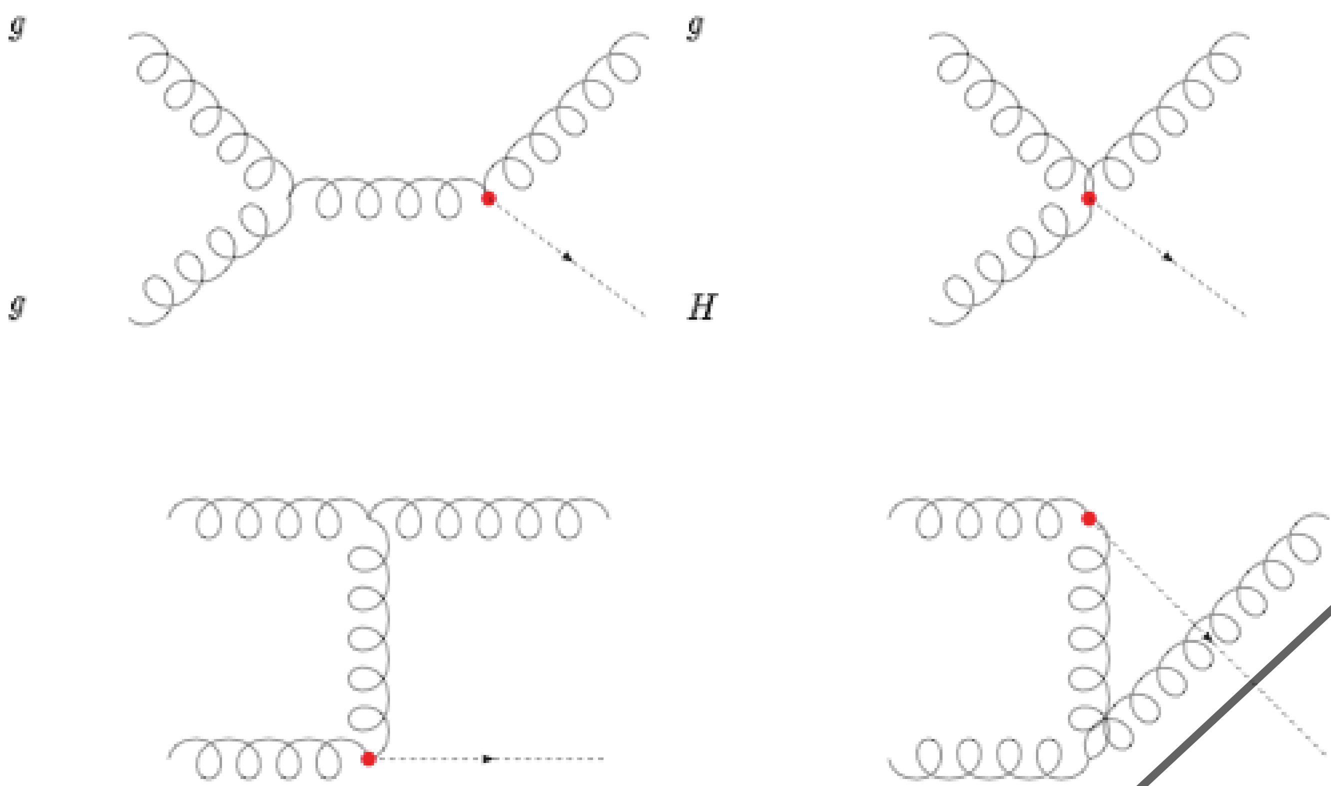

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This is an initial-state divergence to be reabsorbed in the pdf

$$\sigma^{\mathrm{coll.}}_{\mathrm{c.t.}} = 2\,\sigma_0\frac{\alpha_S}{2\pi}\left[\left(\frac{\mu^2}{\mu^2_F}\right)^{\epsilon}\frac{c_\Gamma}{\epsilon}P_{gg}(z)\right] \checkmark$$

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Final results = we made it!!

$$\sigma(pp \to H) = \sum_{ij} \int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 f_i(x_1, \mu_f) f_j(x_2, \mu_f) \hat{\sigma}(ij)[\mu_f/m_h, \mu_r/m_h, \alpha_S(\mu_r)]$$

The final cross section is the sum of three channels: q qbar, q g, and g g.

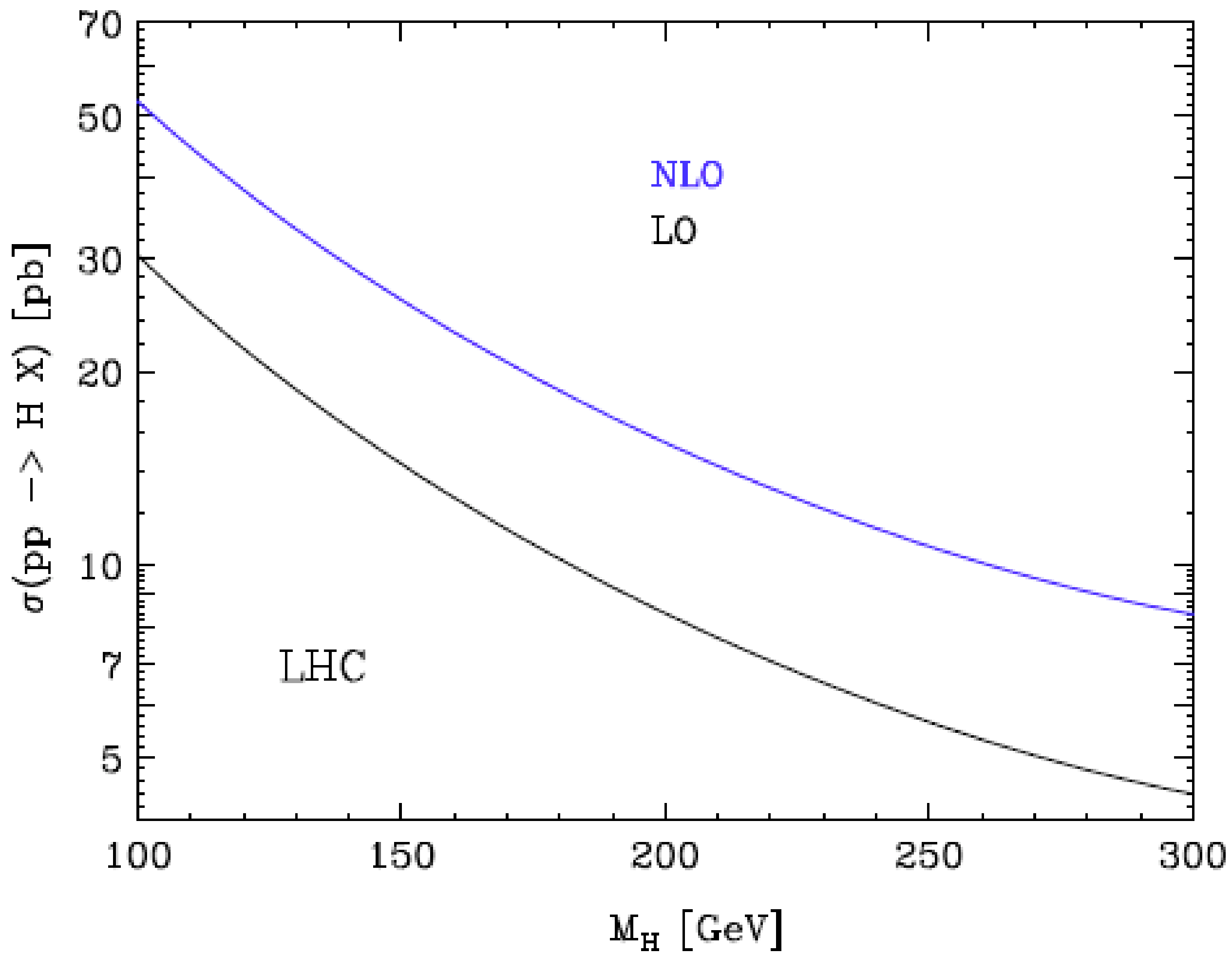
The short distance cross section at NLO depends explicitly on the subtraction scales (renormalization and factorization).

The explicit integration over the pdf's is trivial (just mind the plus distributions).

The result is that the corrections are huge!

K factor is ~2 and scale dependence not really very much improved.

Is perturbation theory valid?
NNLO is mandatory...



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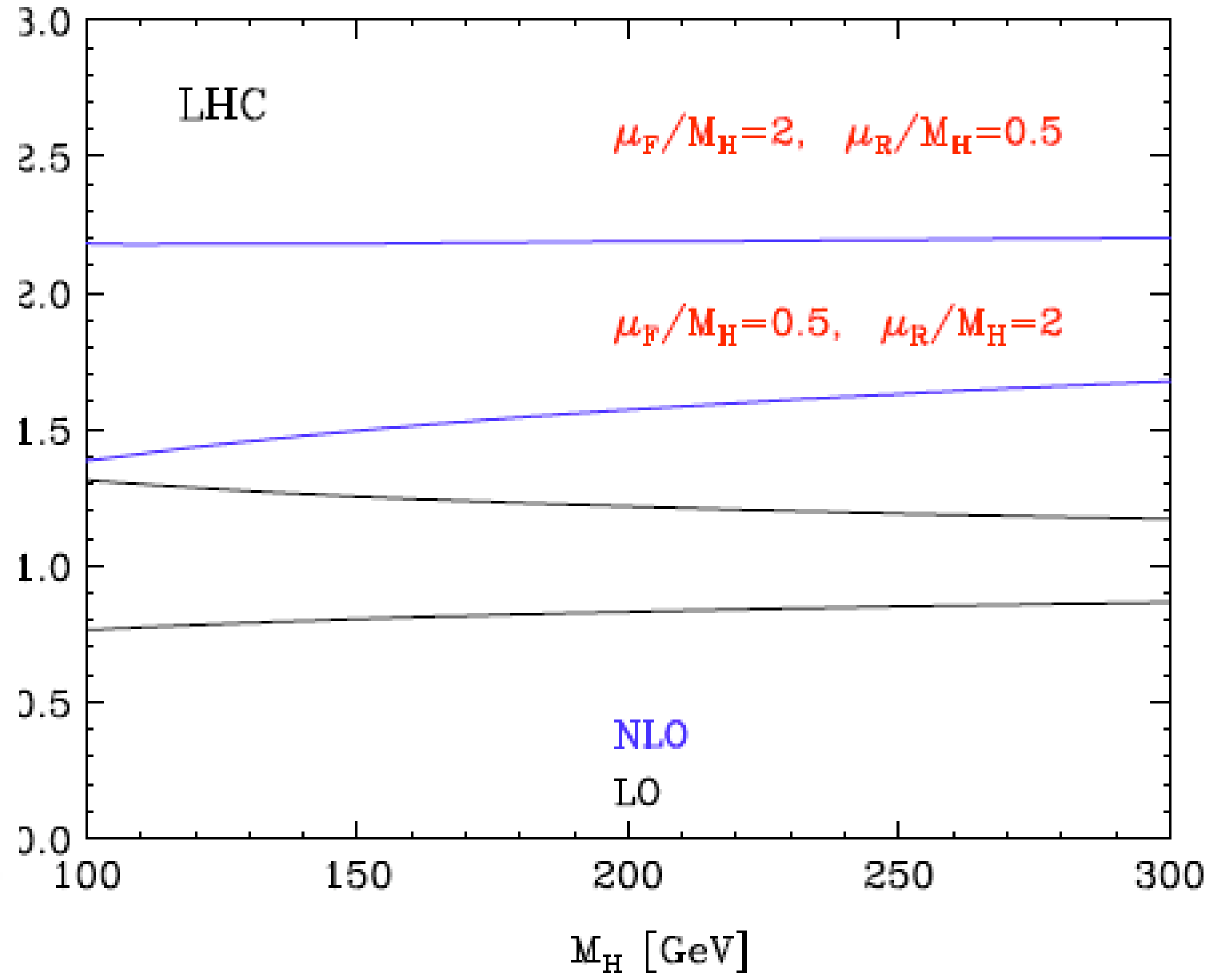

M_H [GeV]

A simple plan

- Intro: the LHC challenge
- Minimal QCD: basics
- Precision QCD: from NLO to NNLO
- Useful QCD: Parton Shower approach
- Best QCD: Merging Fixed Order with PS

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Summary of last lecture

The adjective "NLO" refers to IR-safe observables which are calculable in pQCD.

General algorithm for calculations of observables at NLO

As we discussed, the form of the soft and collinear terms are UNIVERSAL, i.e., they don't depend on the short distance coefficients, but only on the color and spin of the partons partecipating soft or collinear limit.

Therefore it is conceivable to have an algorithm that can handle any process, once the real and virtual contributions are computed.

There are several such algorithms avaiable, but the conceptually simplest is the Subtraction Method [Catani & Seymour ; Catani, Dittmaier, Seymour, Trocsanyi]

$$
\begin{aligned}
\sigma_{ab}^{LO} &= \int_m d\sigma_{ab}^B \\
\sigma_{ab}^{NLO} &= \int_{m+1} d\sigma_{ab}^R + \int_m d\sigma_{ab}^V
\end{aligned}
$$

General algorithm for calculations of observables at NLO

One can use the universality to construct a set of counterterms

$$d\sigma^{ct} = \sum_{ct} \int_m d\sigma^B \otimes \int_1 dV_{ct}$$

which only depend on the partons involved in the divergent regions, $d\sigma^B$ denotes the approriate colour and spin projection of the Born-level cross section and the counter terms are independent on the process under considerations.
These counter terms cancell all non-integrable singularities in $d\sigma^R$, so that one can write

$$\sigma_{ab}^{NLO} = \int_{m+1} [d\sigma_{ab}^R - d\sigma_{ab}^{ct}] + \int_{m+1} d\sigma_{ab}^{ct} + \int_m d\sigma_{ab}^V$$

where the space integration in the first term can be performed numerically in four dimensions and the integral of the counter terms can be done once for all.

An (incomplete) list of NLO codes

- NLOJET++ [Nagy] $pp \to$ (2,3) jets
- AYLEN/EMILIA [de Florian, Dixon, Kunszt, Signer] $pp \to (W,Z) + (W,Z,\gamma)$
- DIPHOX/EPHOX [Aurenche, Binoth, Fontannaz, Guillet, Heinrich, Pilon, Werlen] $pp \to \gamma + 1$ jet, $pp \to \gamma\gamma$, $\gamma^* p \to \gamma + 1$ jet
- MCFM [Campbell, Ellis] $pp \to (W,Z) + (0,1,2)$ jets, $pp \to (W,Z) + b\bar{b}, \ldots$
- heavy-quark production [Mangano, Nason, Ridolfi] $pp \to Q\bar{Q}$
- single-top production [Harris, Laenen, Phaf, Sullivan, Weinzierl] $pp \to Q\bar{q}$
- associated Higgs production with $t\bar{t}$ [Dawson, Jackson, Orr, Reina, Wackeroth, Beenakker, Dittmaier, Kramer, Plumper, Spira, Zerwas] $pp \to HQ\bar{Q}$
- VBFNLO [Figy, Zeppenfeld, C.O.] $pp \to (W,Z,H,WW,ZZ,WZ) + 2$ jets, QCD corrections to electroweak production, when typical vector-boson fusion cuts are applied
- di-photon production [del Duca, Maltoni, Nagy, Trocsanyi] $pp \to \gamma\gamma + 1$ jet

For a more complete list, and the corresponding web pages, see:

`http://www.cedar.ac.uk/hepcode`

Example:MCFM

Downloadable general purpose NLO code (Campbell & Ellis)

$p\bar{p} \to W^\pm/Z$	$p\bar{p} \to W^+ + W^-$
$p\bar{p} \to W^\pm + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^\pm + \gamma$	$p\bar{p} \to W^\pm/Z + H$
$p\bar{p} \to W^\pm + g^\star\ (\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^\pm/Z + \text{1 jet}$	$p\bar{p} \to W^\pm/Z + \text{2 jets}$
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H + \text{1 jet}$
$p\bar{p}(VV) \to H + \text{2 jets}$	$p\bar{p} \to t + q$
$p\bar{p} \to H + b$	$p\bar{p} \to Z + b$

☞ Plus all single-top channels, Wc, WQJ, ZQJ,...

☞ Extendable/sizeable library of processes,

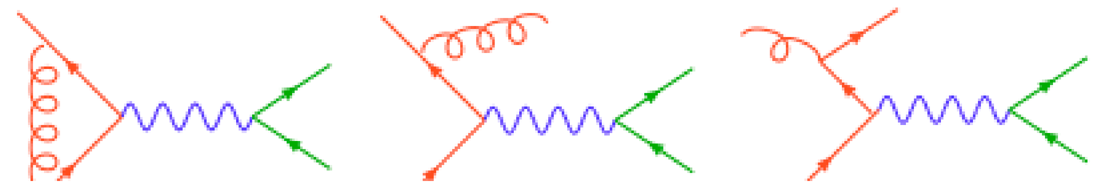
relevant for signal and background studies, including spin correlations.

☞ Cross sections and distributions at NLO are provided

☞ Easy and flexible choice of parameters/cuts (input card).

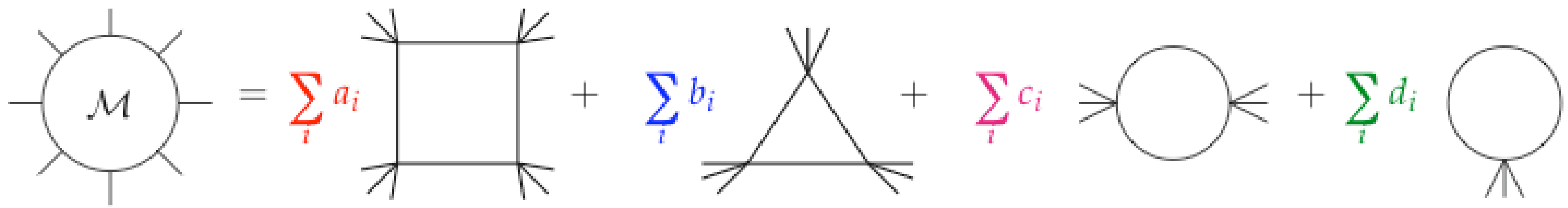
Next-to-leading order : Loops

$\hat{\sigma}^{(1)}$



virtual real

Any one-loop amplitude can be written as (PV decomposition):



$$\mathcal{M} = \sum_i a_i(D)\,\text{Boxes}_i + \sum_i b_i(D)\,\text{Triangles}_i + \sum_i c_i(D)\,\text{Bubbles}_i + \sum_i d_i(D)\,\text{Tadpoles}_i$$

* All the scalar loop integrals are known and now easily available [Ellis, Zanderighi]

* Open issue is to compute the D-dimensional coefficient in the expansion:
  large number of terms forbid a direct evaluation with symbolic algebra. In addition normally large gauge cancellation, inverse Gram determinants, spurious phace-space singularities lead to numerical instabilities.

Sometimes it is better to calculate

$$\mathcal{M} = \sum_i a_i(4)\,\text{Boxes}_i + \sum_i b_i(4)\,\text{Triangles}_i + \sum_i c_i(4)\,\text{Bubbles}_i + \sum_i d_i(4)\,\text{Tadpoles}_i + R$$

Where R is a rational function

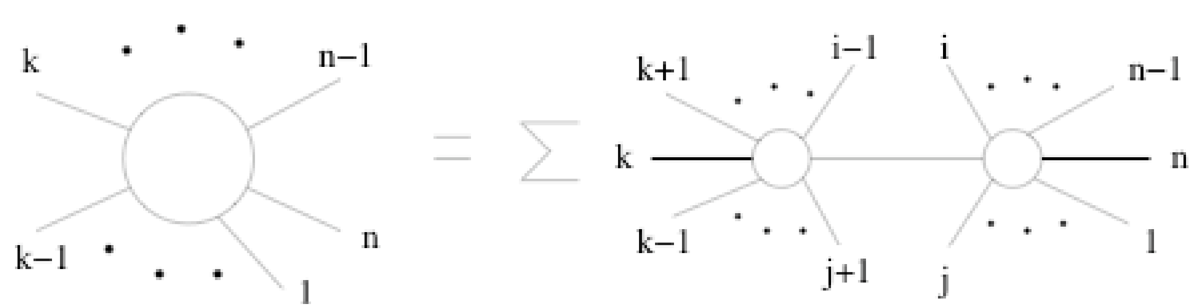
Progress in loops

Several new developments coming from the idea

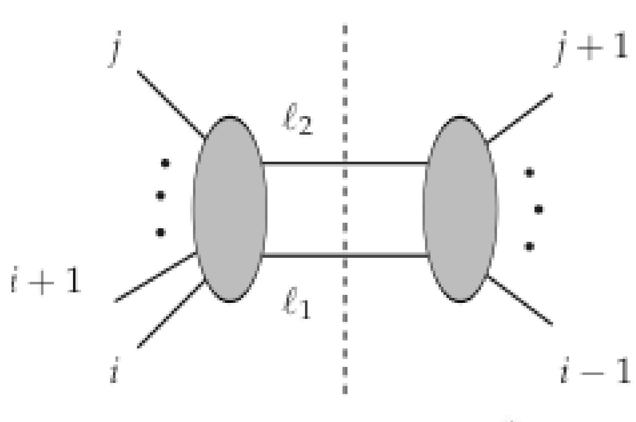
> A scattering amplitude is an analytic function of the external momenta and (most) its structure can be reconstructed from the poles and the branch cuts.
>
> LOOPS can be calculated from tree-level amplitudes

✓ POLES : lower number of external lines. Cauchy residue theorem

[Cachazo, Svreck, Witten]
[Witten]
[Britto, Cachazo, Feng]



✓ BRANCH CUTS : lower number of loops



$$\mathrm{Disc} = \int d^4\Phi \, A^{\mathrm{tree}}(\ell_1, i, \ldots, j, \ell_2) \, A^{\mathrm{tree}}(-\ell_2, j+1, \ldots, i-1, -\ell_1)$$

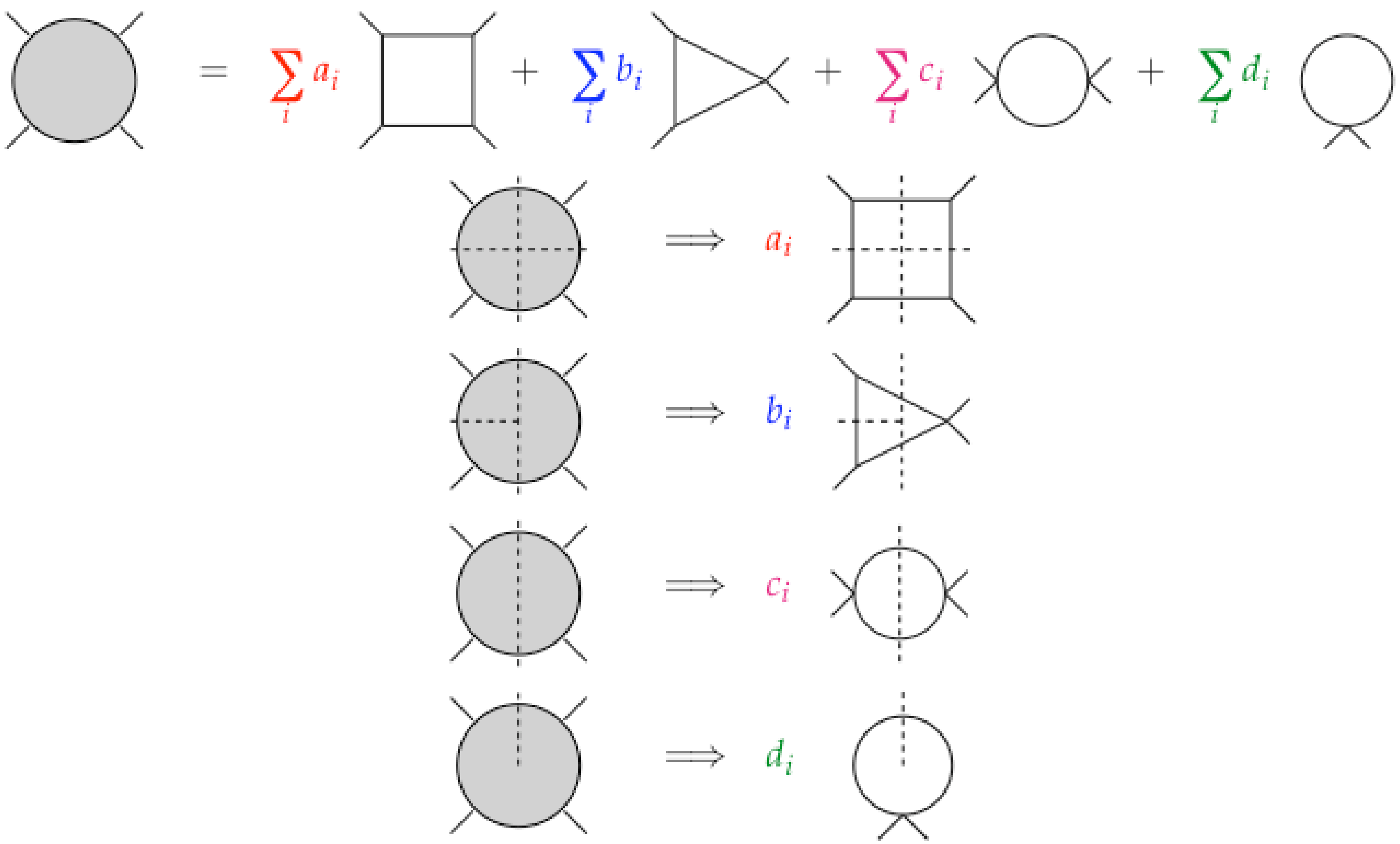
$$d^4\Phi = d^4\ell_1 \, d^4\ell_2 \, \delta^{(4)}(\ell_1 + \ell_2 - P_{ij}) \, \delta^{(+)}(\ell_1^2) \, \delta^{(+)}(\ell_2^2)$$

$$\delta^{(+)}(p^2) = \delta(p^2)\,\theta(p_0) \qquad \text{on-shell condition}$$

[Vermaseren, van Neerven]
[Bern, Dixon, Dunbar, Kosower]
[Britto, Cachazo, Feng]

Generalized unitarity

[Bern, Dixon, Kosower]
[Britto, Cachazo, Feng]
[Anastasiou, Kunszt, Mastrolia]



Three and four particle cuts are non zero due to the continuation of momenta into complex values!

NLO : summary

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- NLO calculations are needed to perform measurements where the knowledge of total and differential rates is essential. This is true not only for the signal but also for the backgrounds.

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- Standard NLO programs do not produce unweighted events and therefore are not suitable for direct experimental analysis.

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- Suggestion: always consult with the authors of the code in case of doubts...

What about NNLO?

- At present only $2\to1$ calculations available, all of them (parton) exclusive final state.
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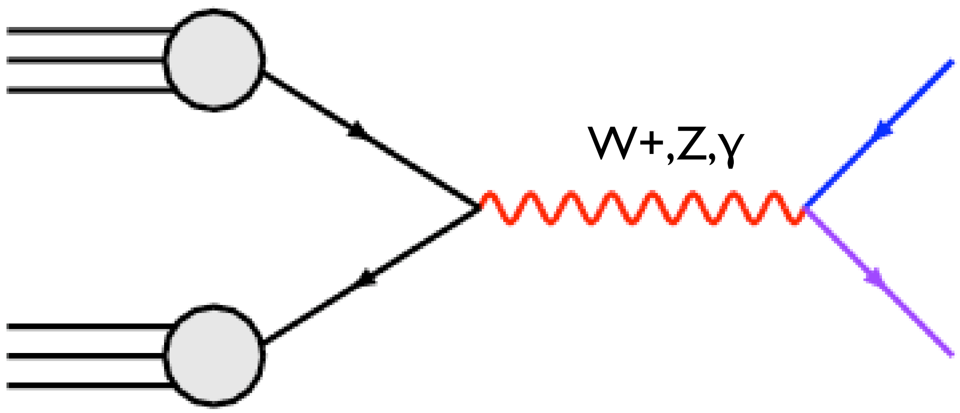
Let's consider two physics cases:

a. Drell-Yan
b. Higgs

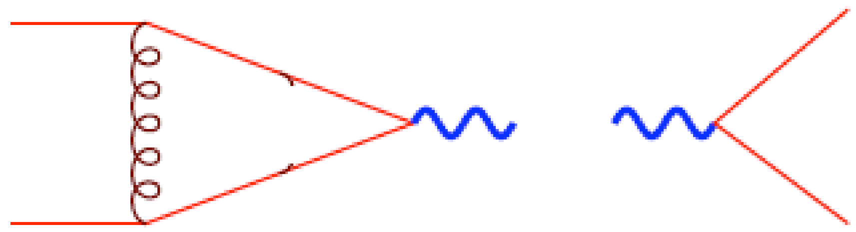
Drell-Yan



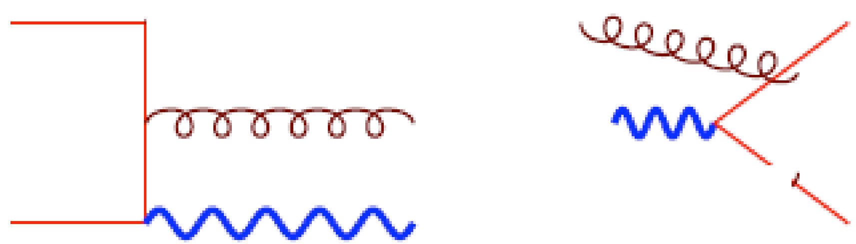
- Clean final state (no hadrons from the hard process).
- Nice test of QCD and EW interactions. The cross sections are known up to NNLO (QCD) and at NLO (EW).
- Measure m_W to be used in the EW fits together with the top mass to guess the Higgs mass.
- Constraint the PDF
- Channel to search for new heavy gauge bosons or new kind of interactions

Elements of pp→W NLO calculation

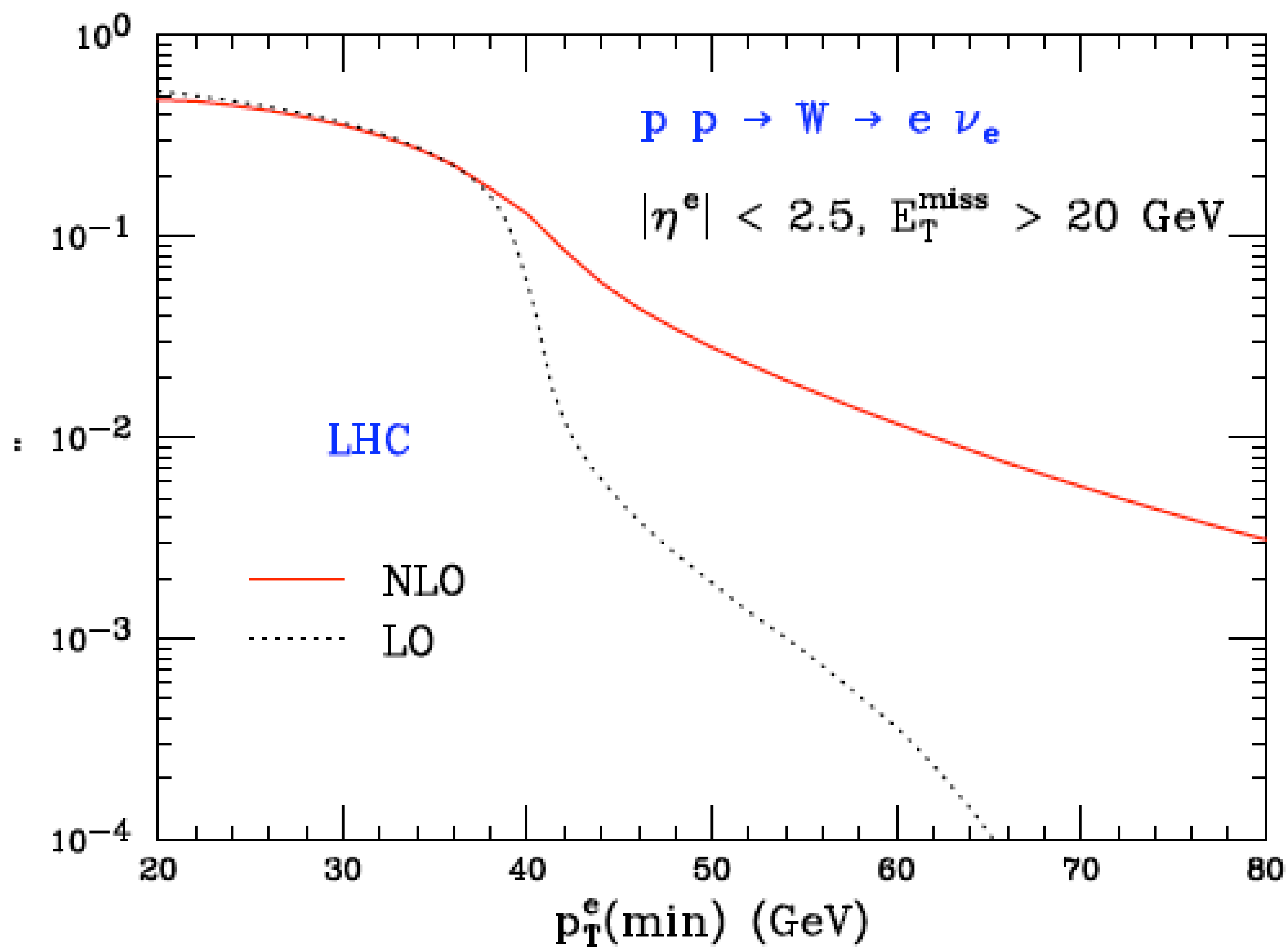
- Virtual



- Real



Drell-Yan @ NLO



✓ $A_W = \frac{1}{\sigma^{(tot)}} \int_{p_T^e(\mathrm{min})}^{\sqrt{S}/2} dp_T^e \frac{d\sigma}{dp_T^e}(\mathrm{cuts})$

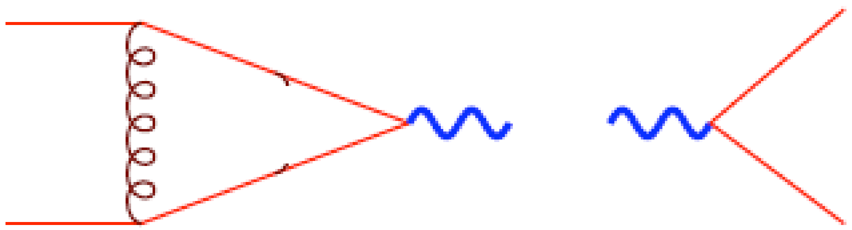
✓ $K(x) = \frac{d\sigma_{NLO}/dx}{d\sigma_{LO}/dx}$

K factors STRONGLY phase-space dependent.

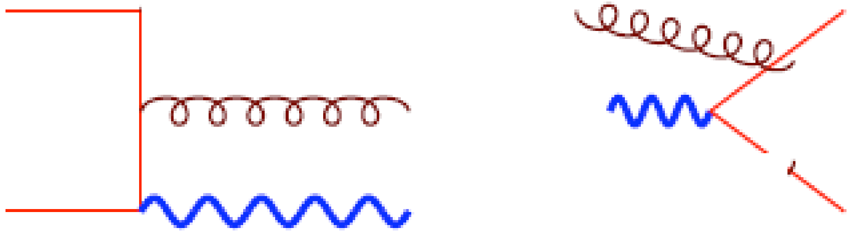
Lepton spin correlations have to be taken account correctly!

Elements of pp→W NLO calculation

- Virtual

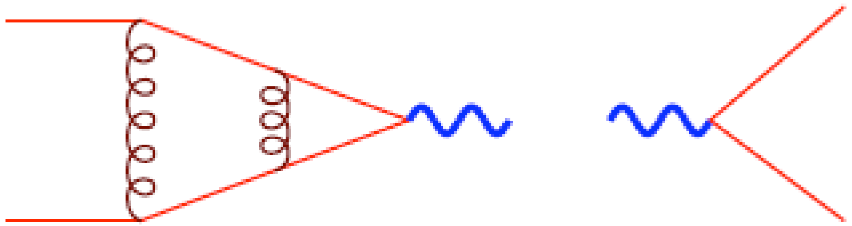


- Real



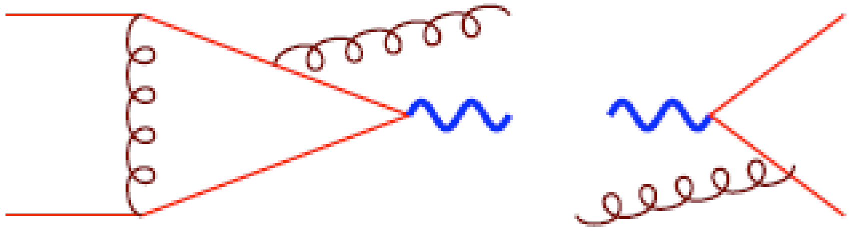
Elements of pp→W NNLO calculation

- Virtual-Virtual



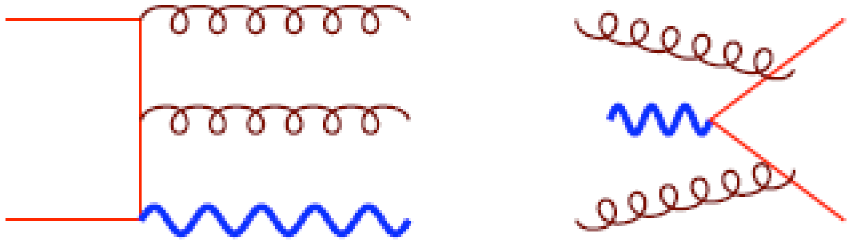
+ 100 terms

- Real-Virtual



+ 300 terms

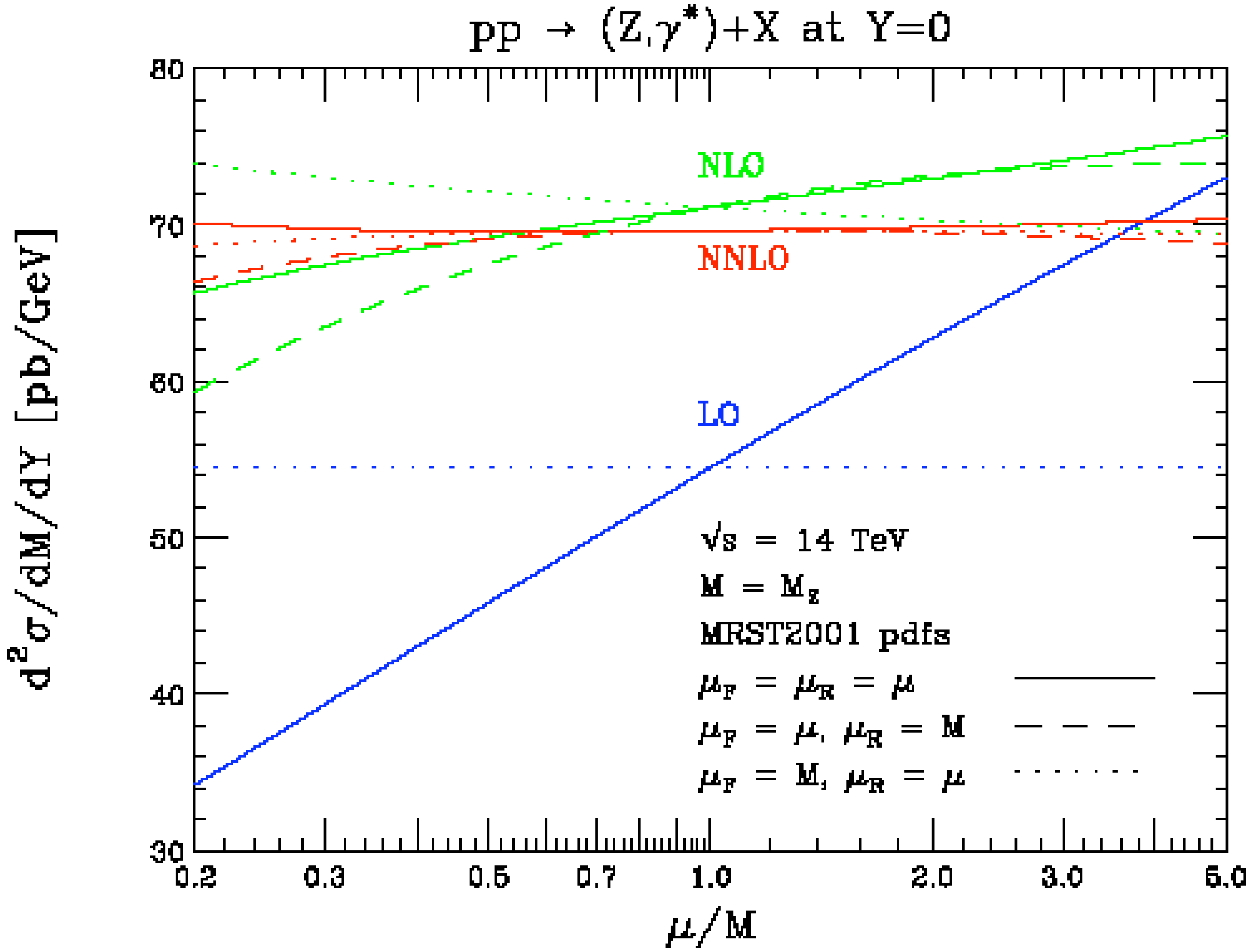
- Real-Real



+ 500 terms

⇒ Need clever algorithms to handle!

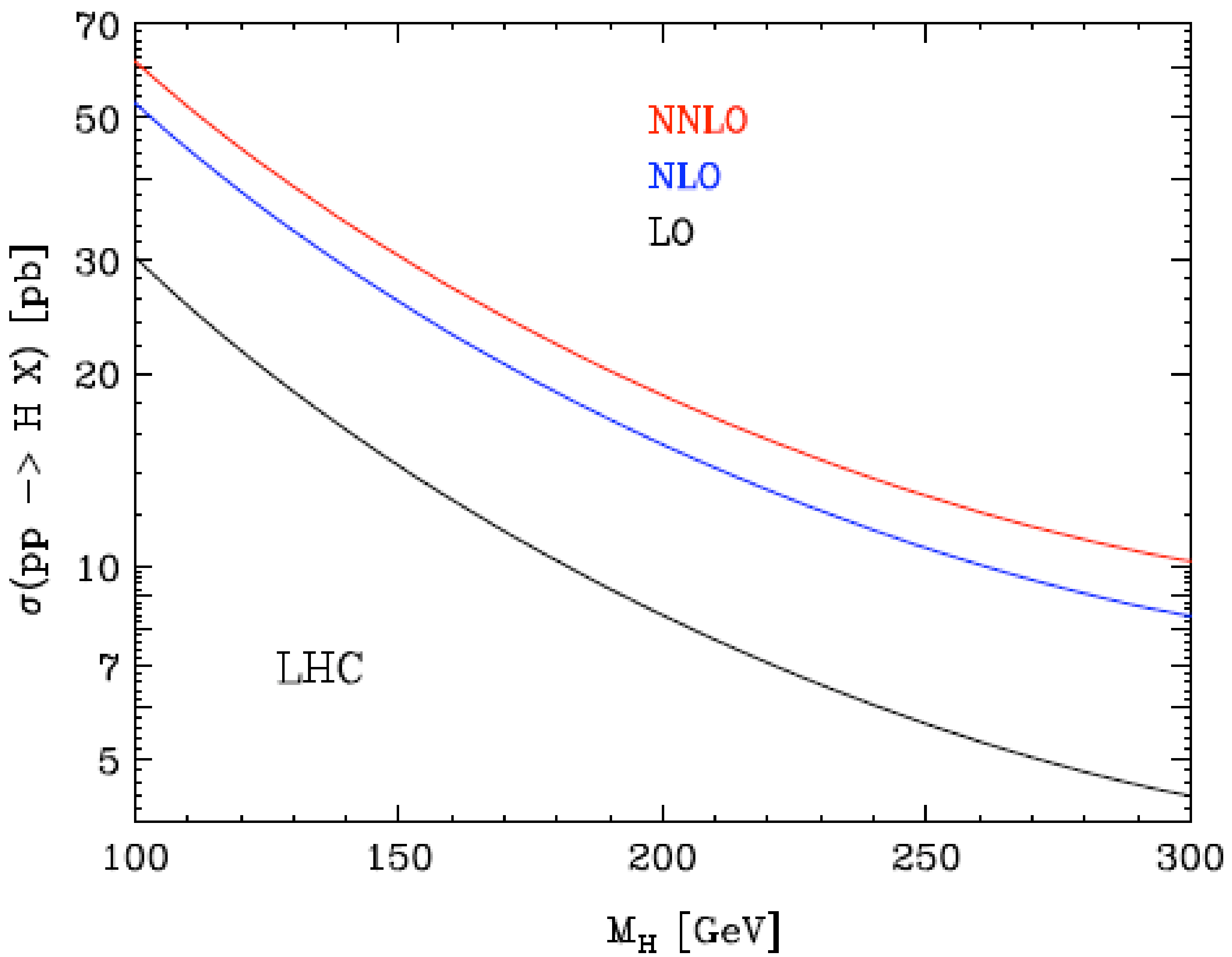
The NNLO result

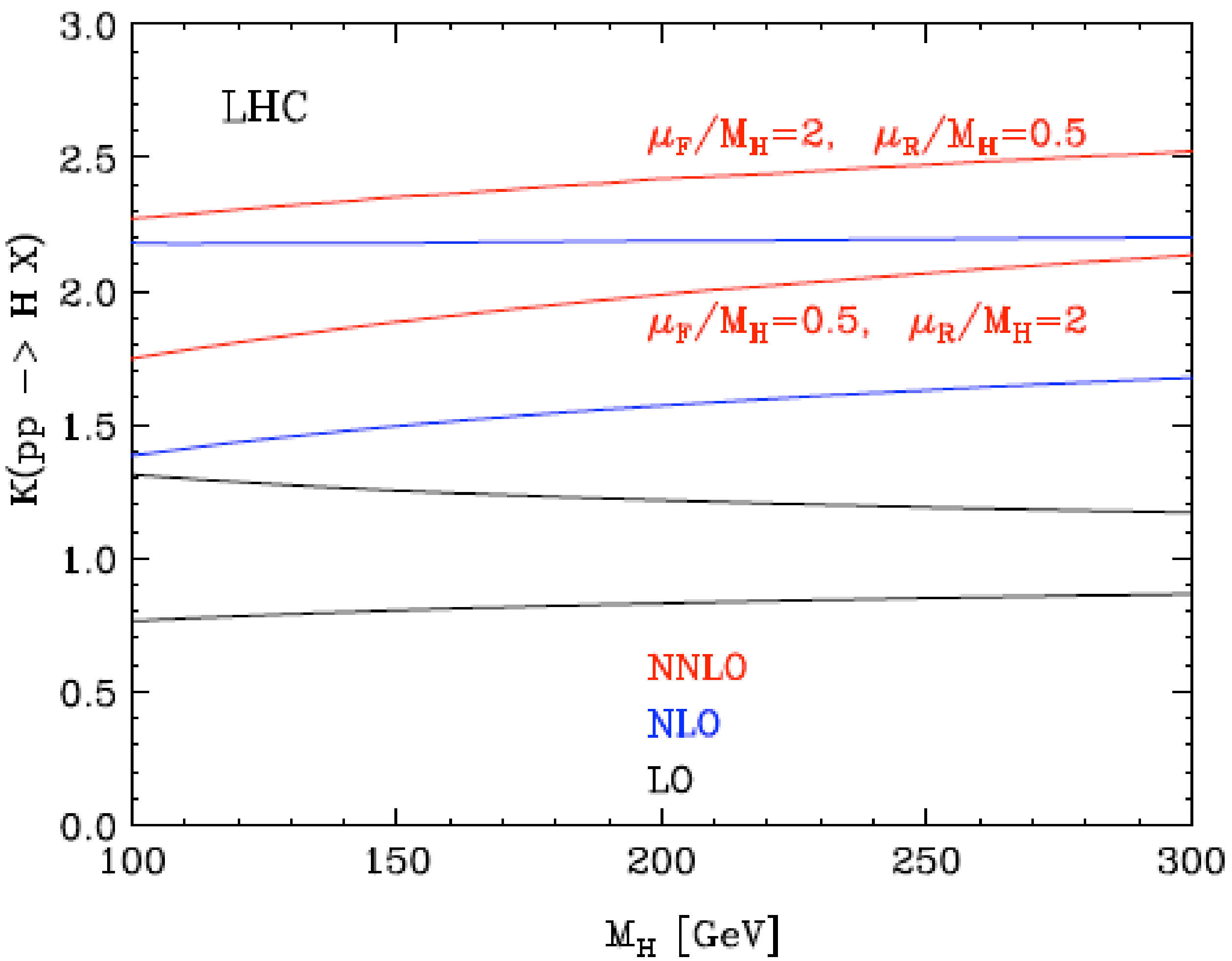


- Precision predictions at NNLO
- Also miss qualitative effects at lower orders
  - Few initial channels open; sensitivity to pdfs underestimated
  - Few jets in final state
  - Jets modeled by too few partons
  - Incorrect kinematics, e.g., no p_T

[Anastasiou, Dixon, Melnikov, Petriello. 2004]

pp→H at NNLO





Is the series well behaved? $\Longrightarrow$ YES NNLO 15%

The current TH QCD uncertainty on the total cross section is about 10%.

What about our predictions for limited areas of the phase space?

NNLO : summary

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- Frontier of precision QCD calculations.
- NNLO calculations are needed for very special cases, such as standard candles and/or precision physics.
- Still an art. General algorithm not yet in place.
- Handful of results available, mostly in private codes (few exceptions!).

A simple plan

- Intro: the LHC challenge
- Minimal QCD: basics
- Precision QCD: from NLO to NNLO
- Useful QCD: Parton Shower approach
- Best QCD: Merging Fixed Order with PS

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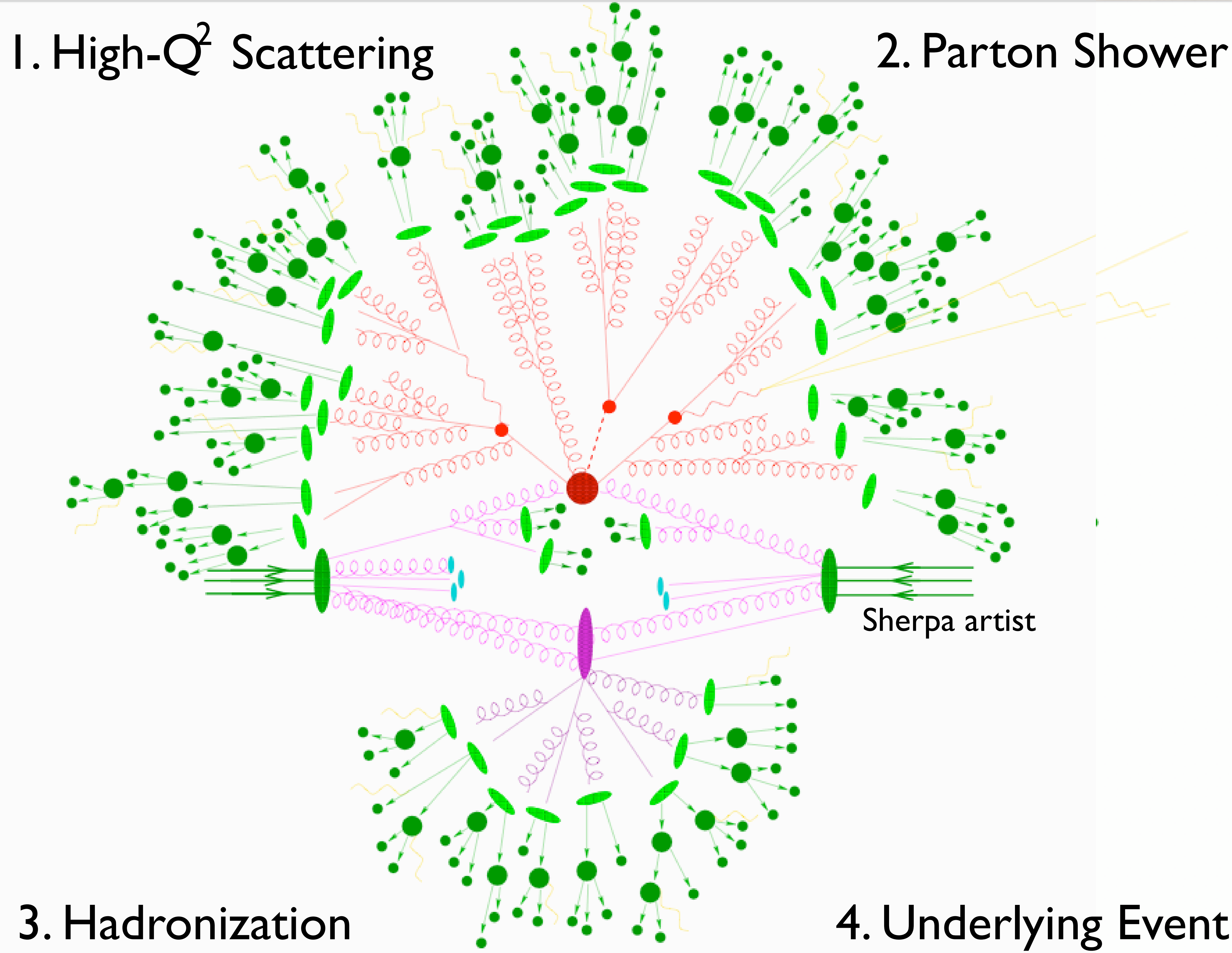
1. High-Q^2 Scattering
2. Parton Shower
3. Hadronization
4. Underlying Event
Sherpa artist

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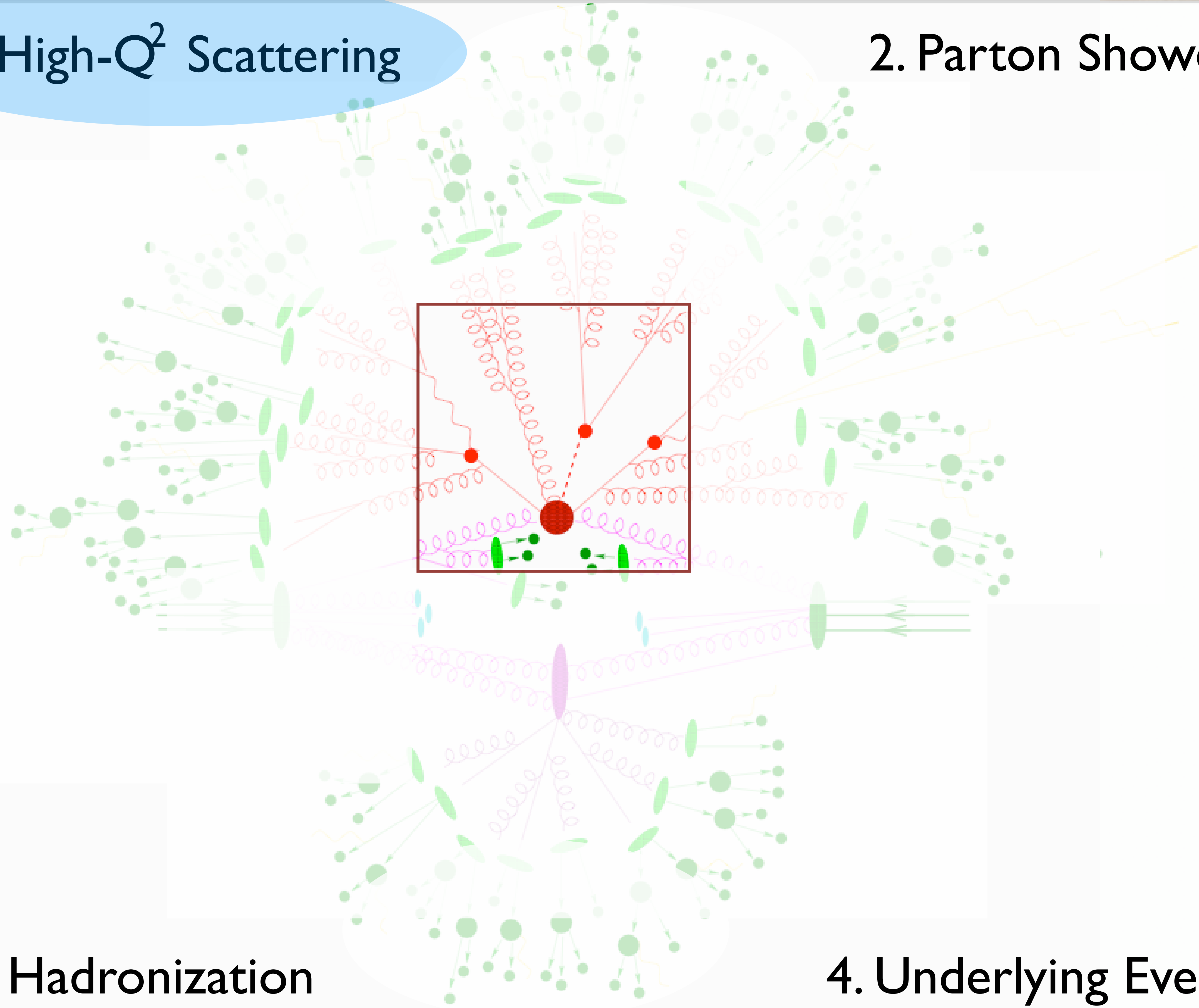

3. Hadronization

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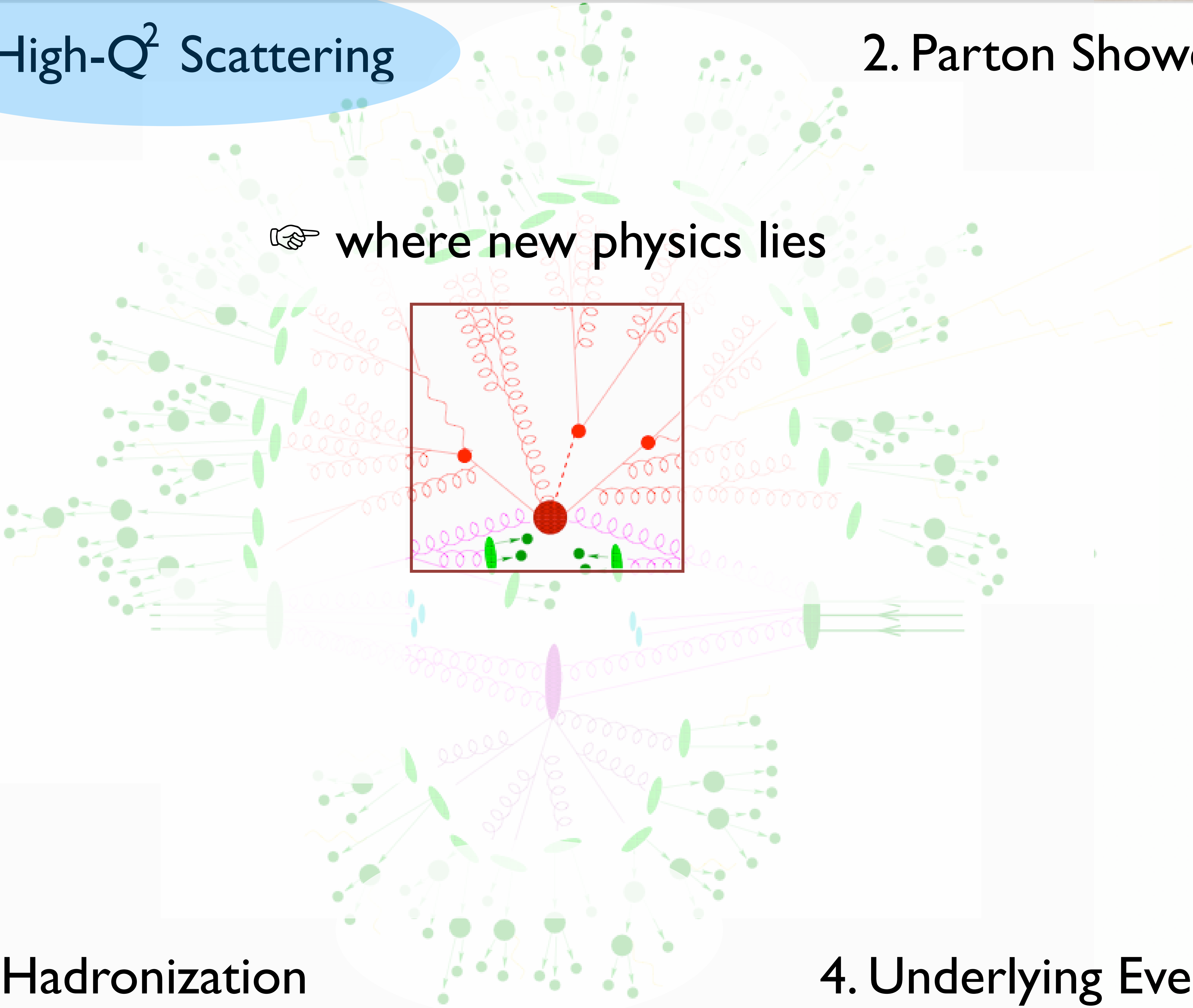
☞ where new physics lies



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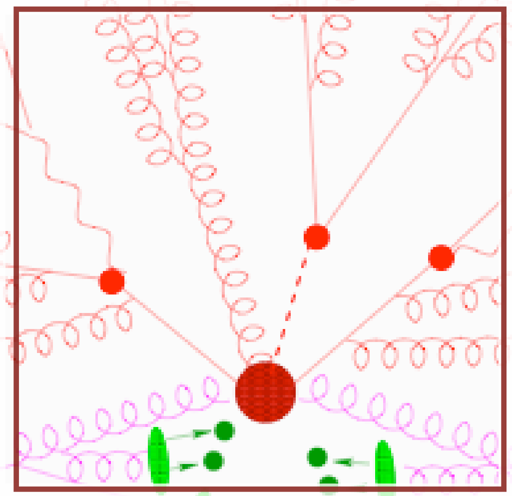

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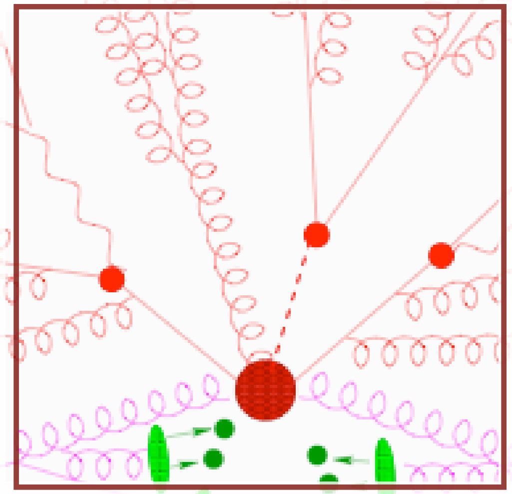

☞ process dependent

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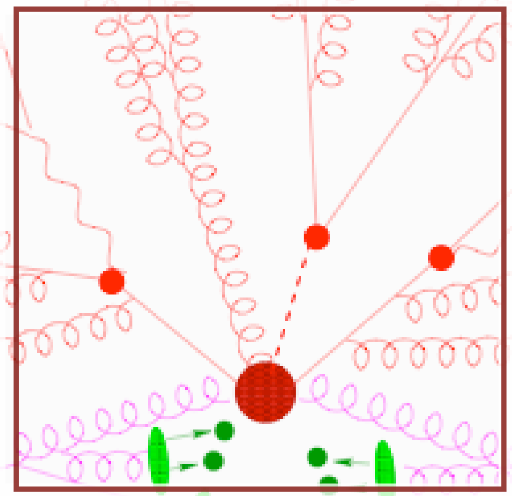

- ☞ process dependent
- ☞ first principles description
- ☞ it can be systematically improved

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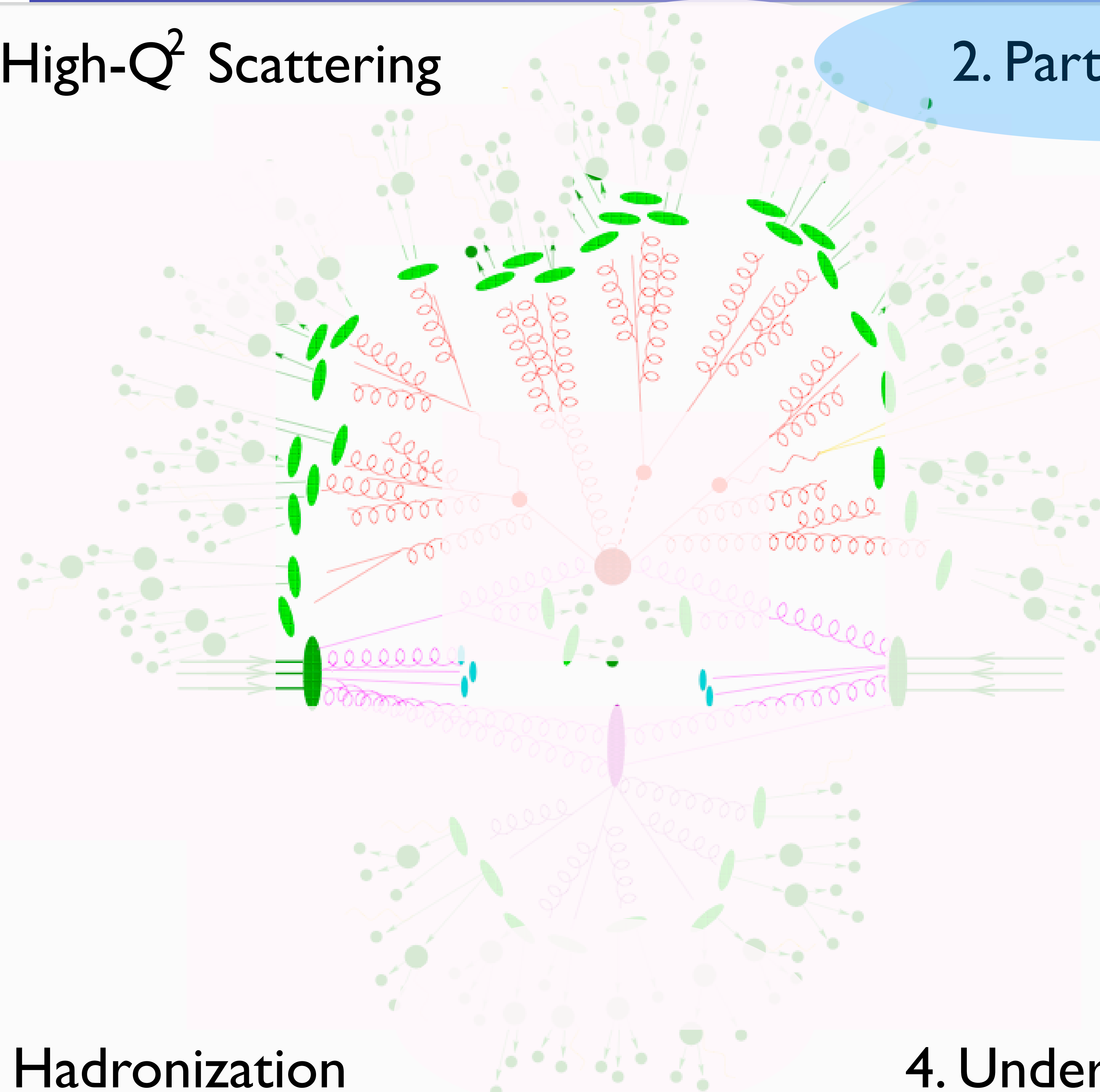

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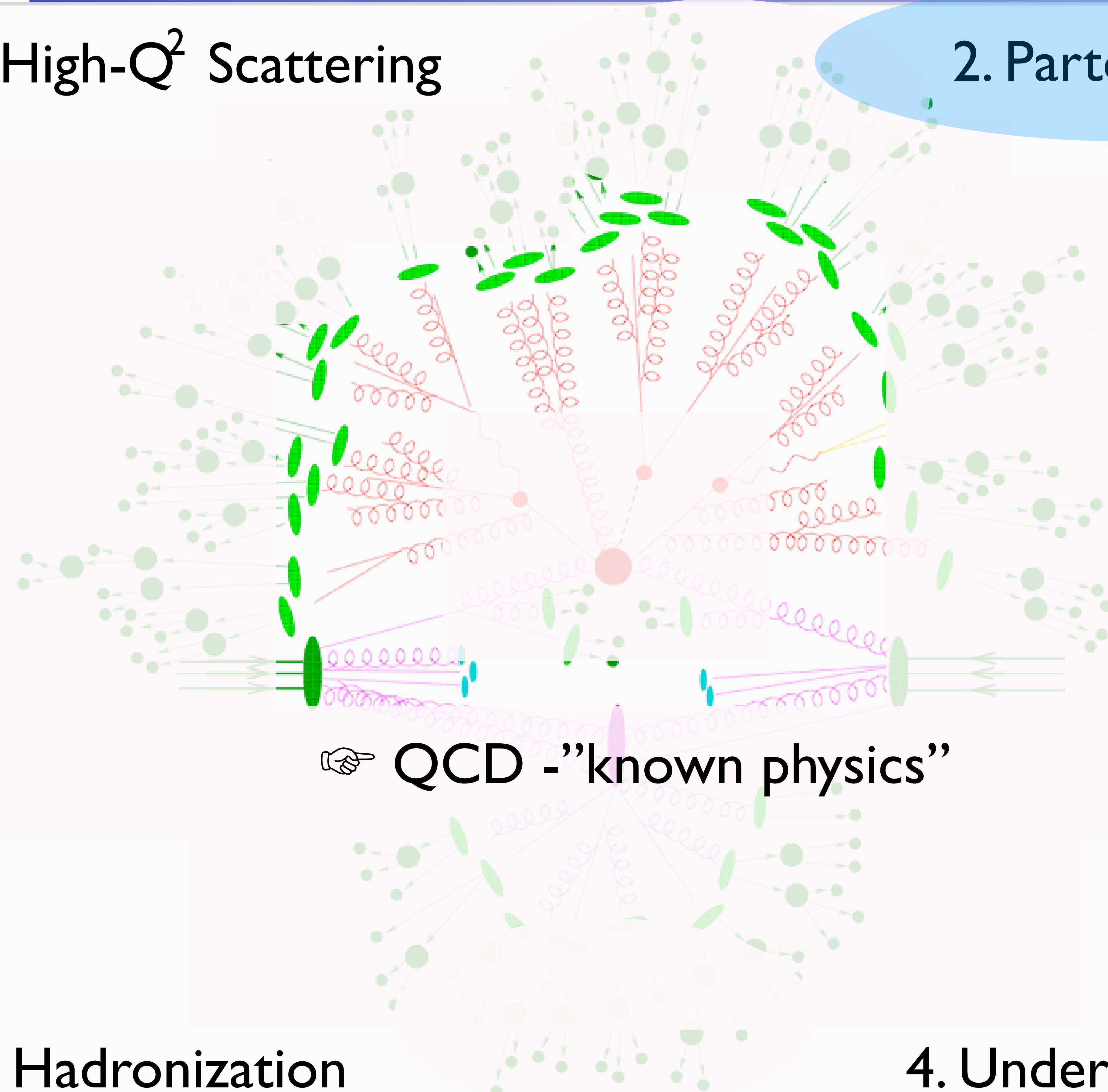

☞ QCD -"known physics"

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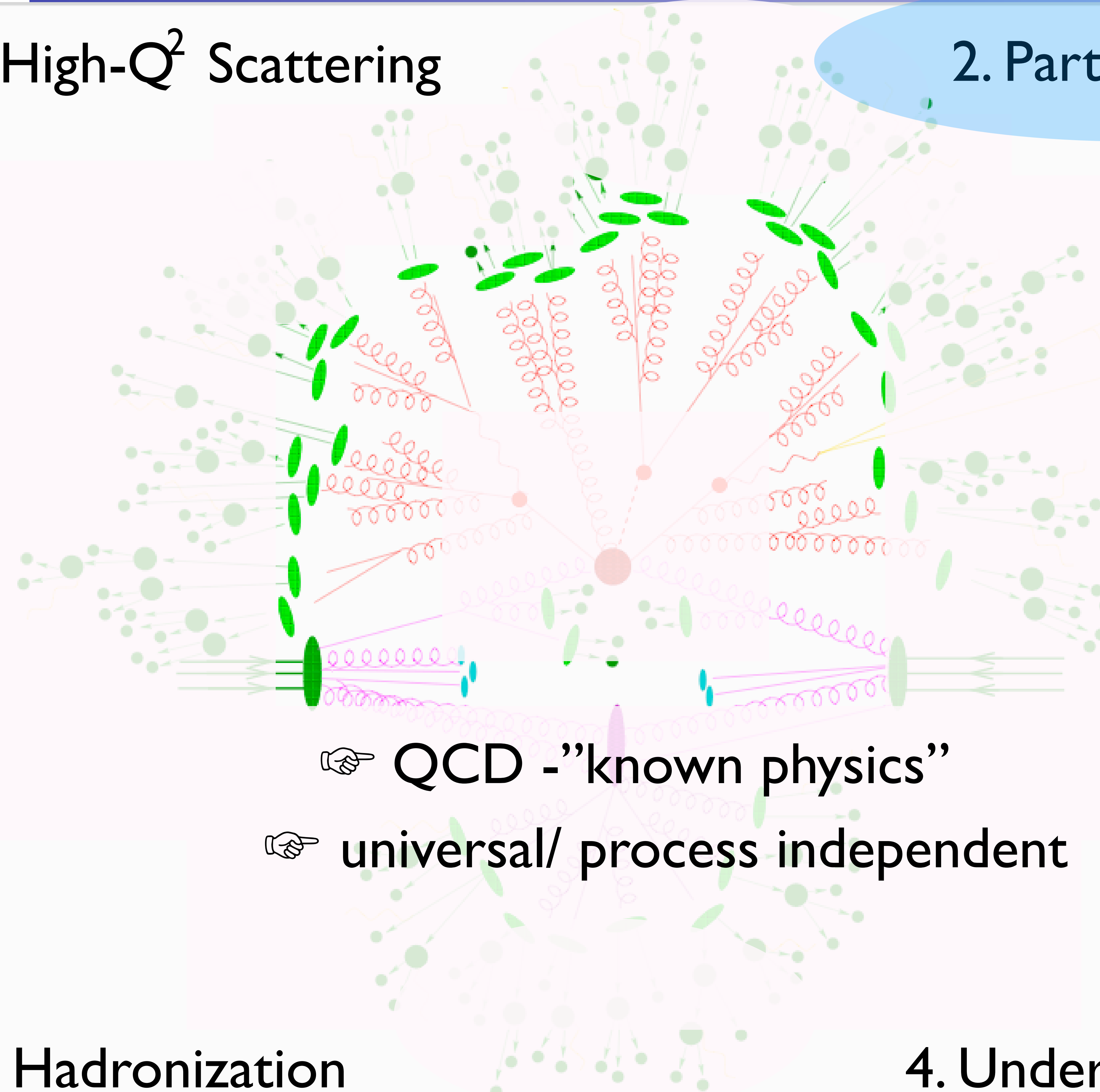

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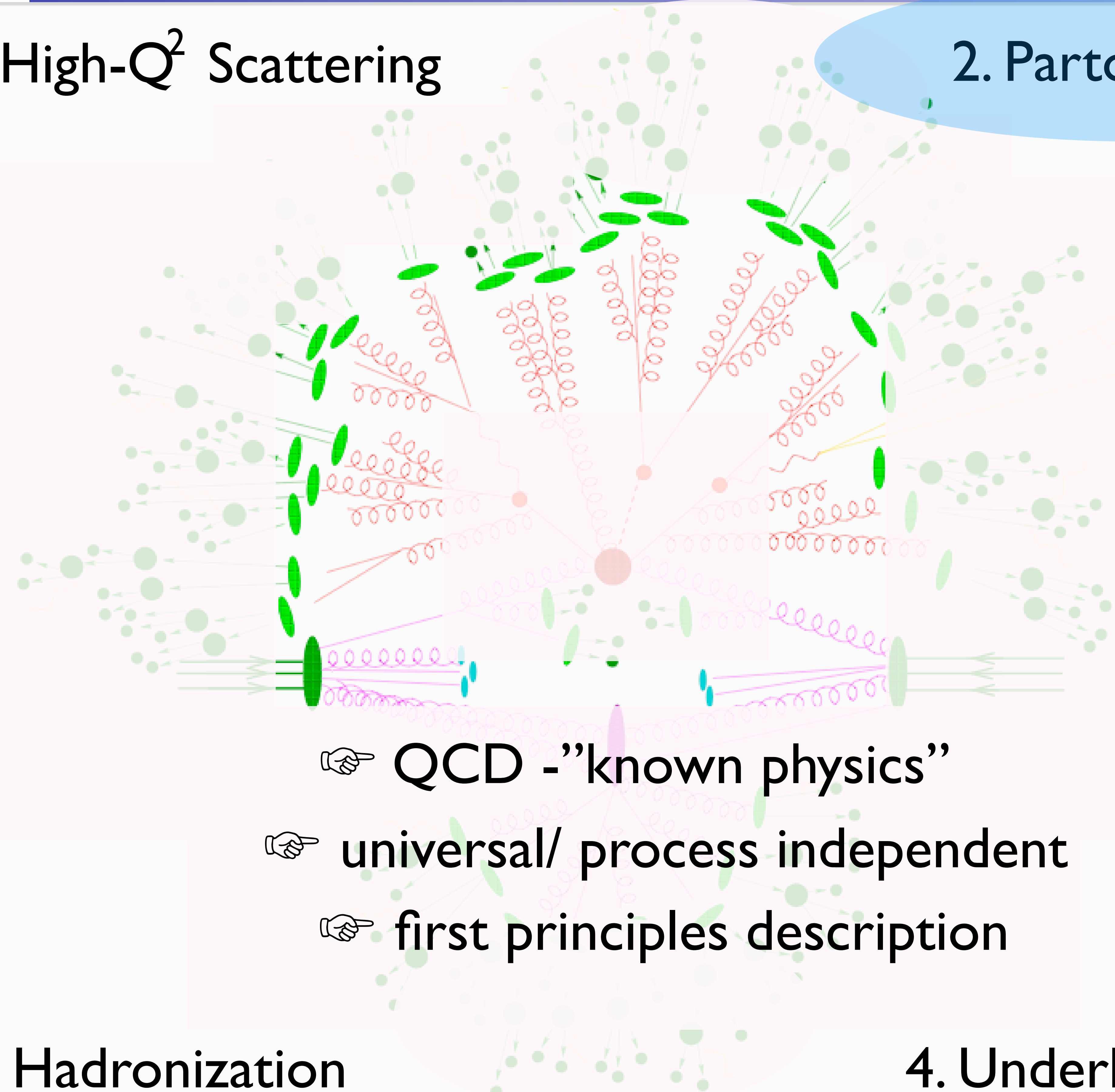

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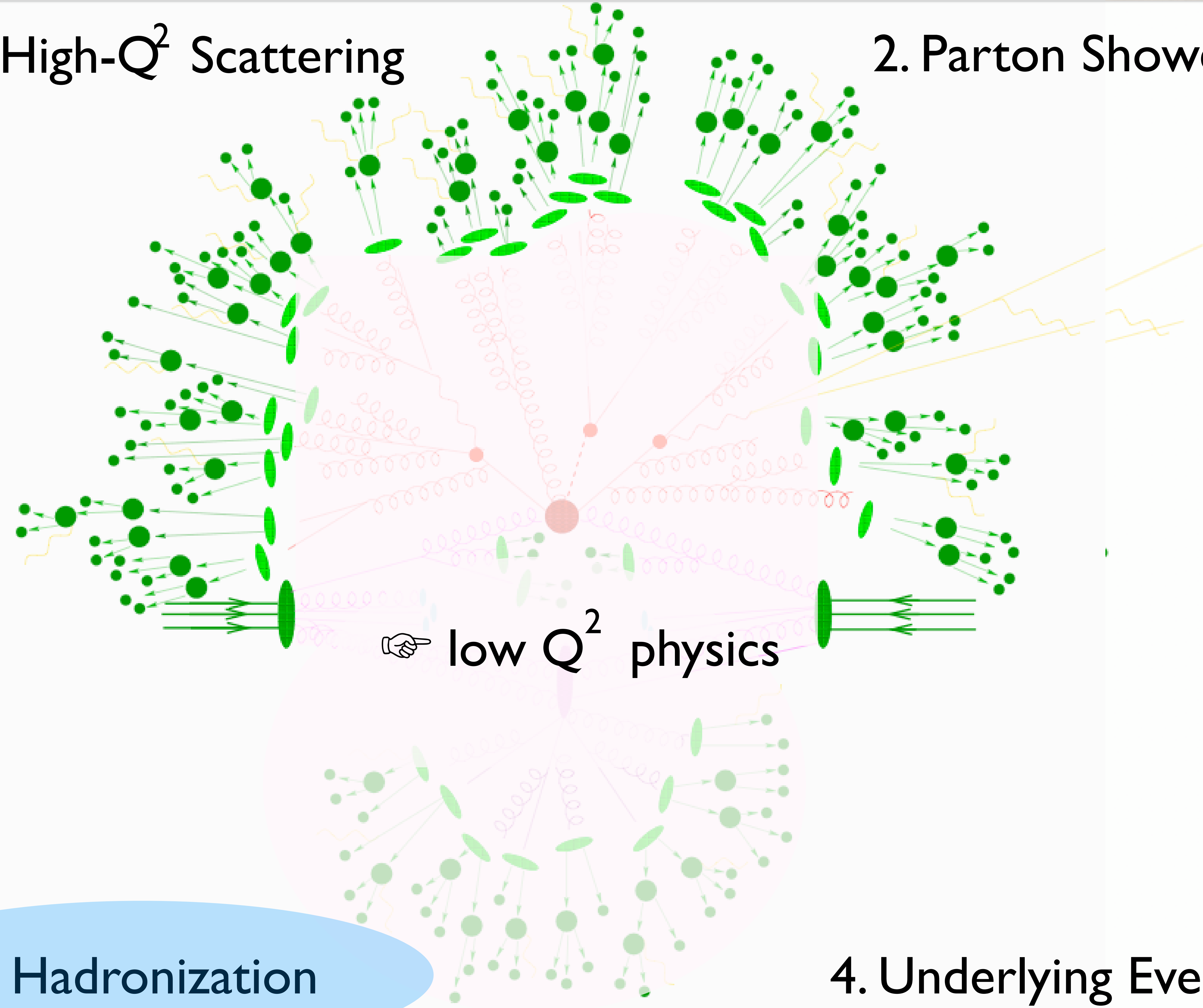

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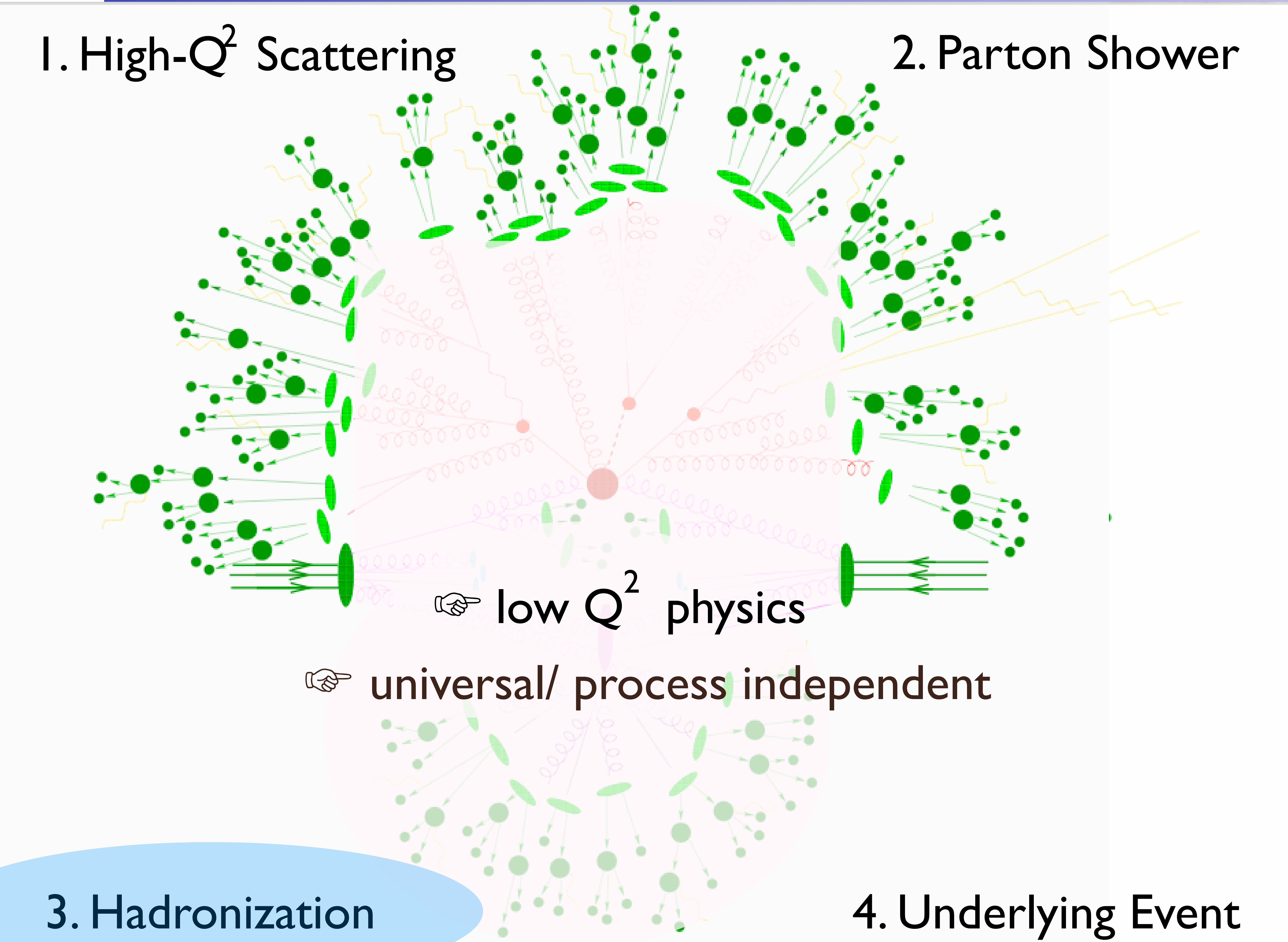
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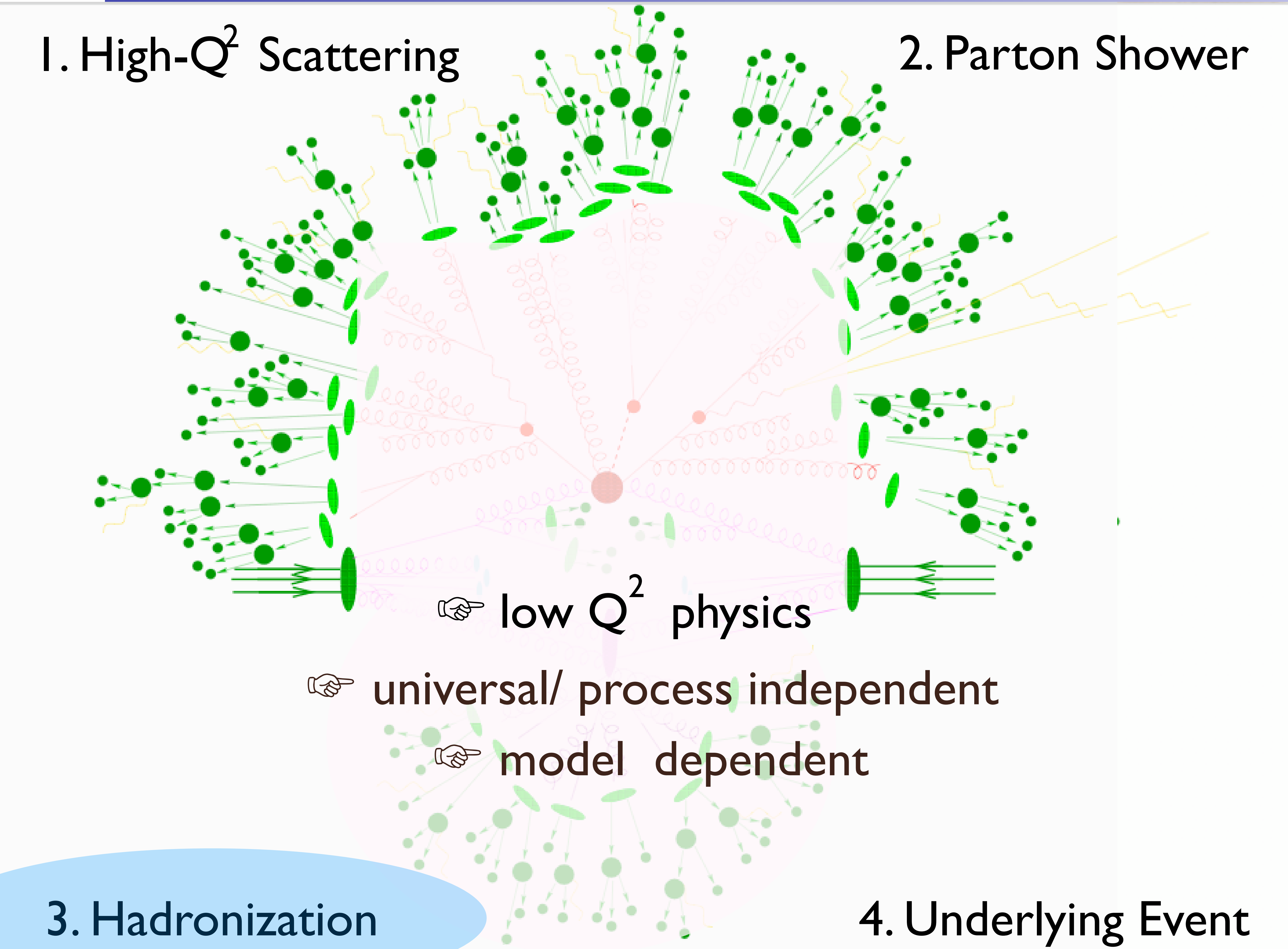

☞ low Q^2 physics

☞ universal/ process independent

☞ model dependent

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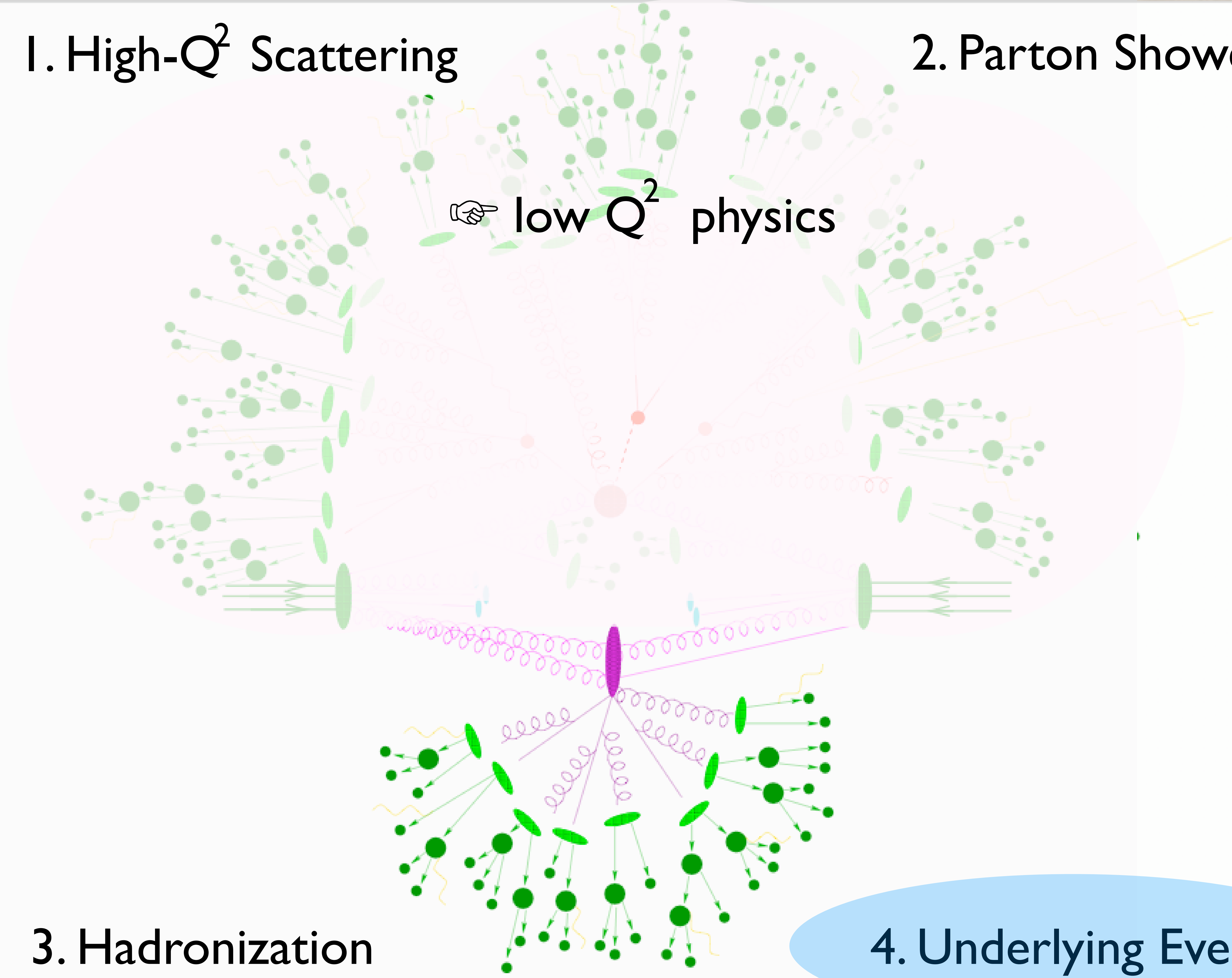

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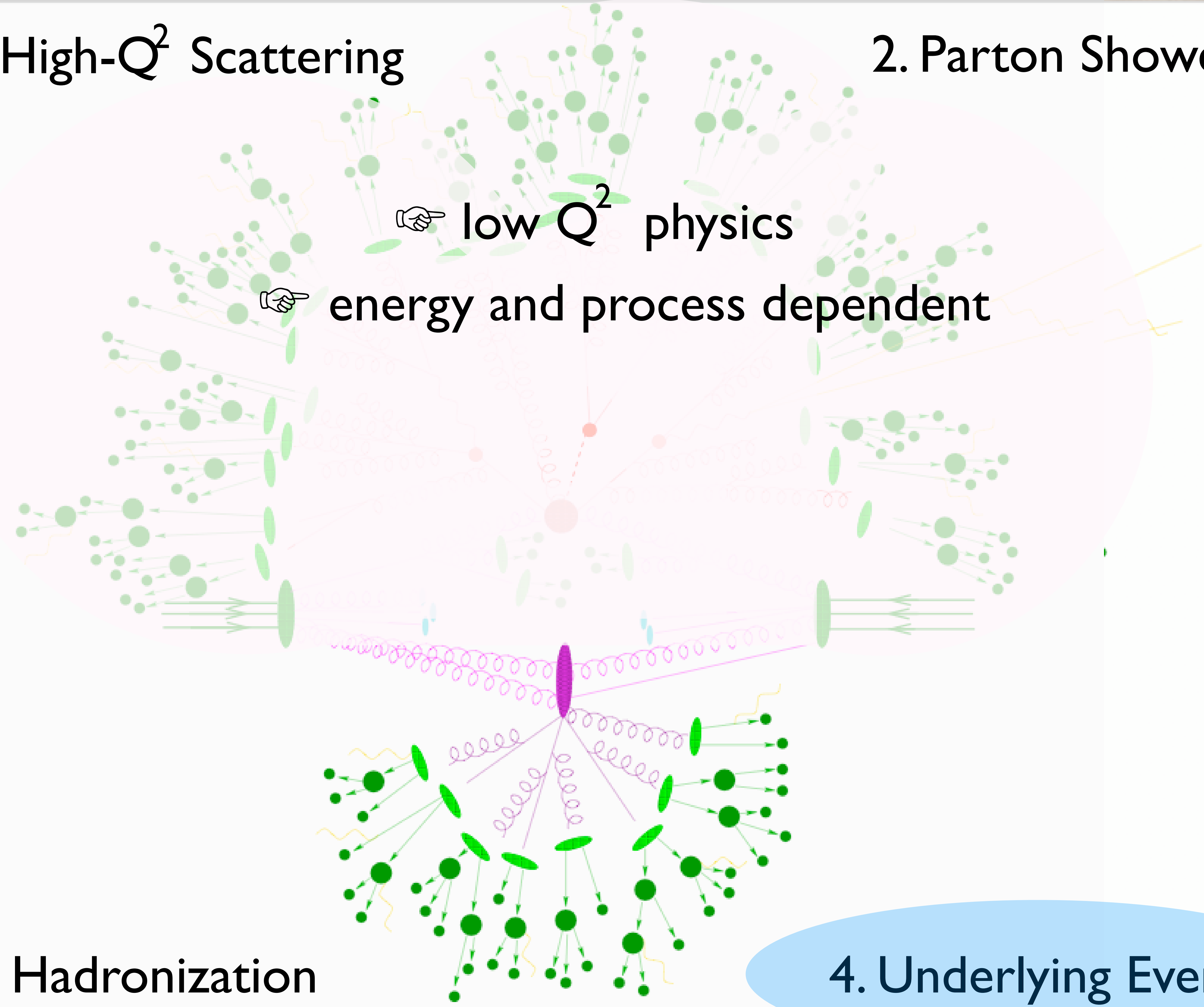

☞ low Q^2 physics

☞ energy and process dependent

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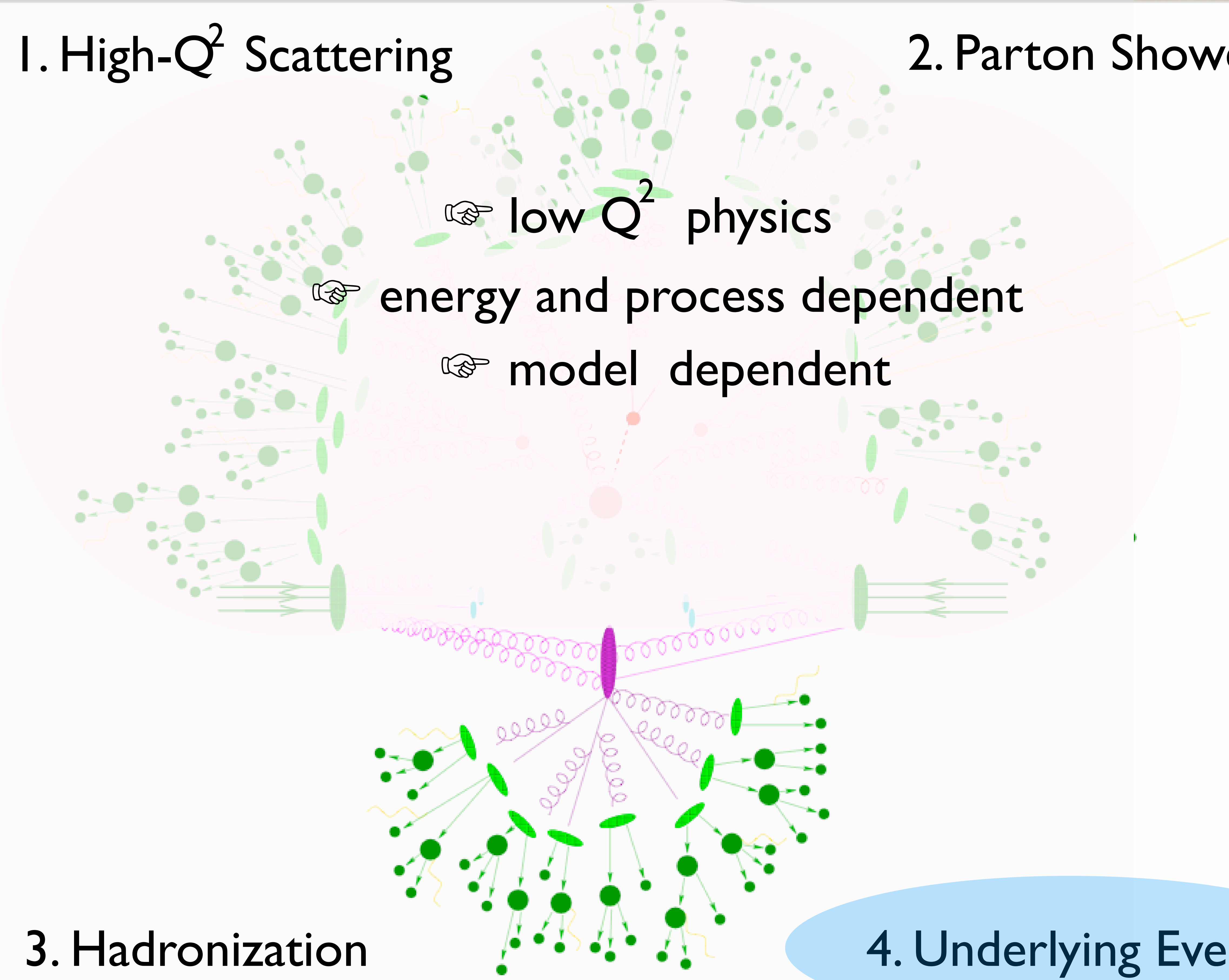

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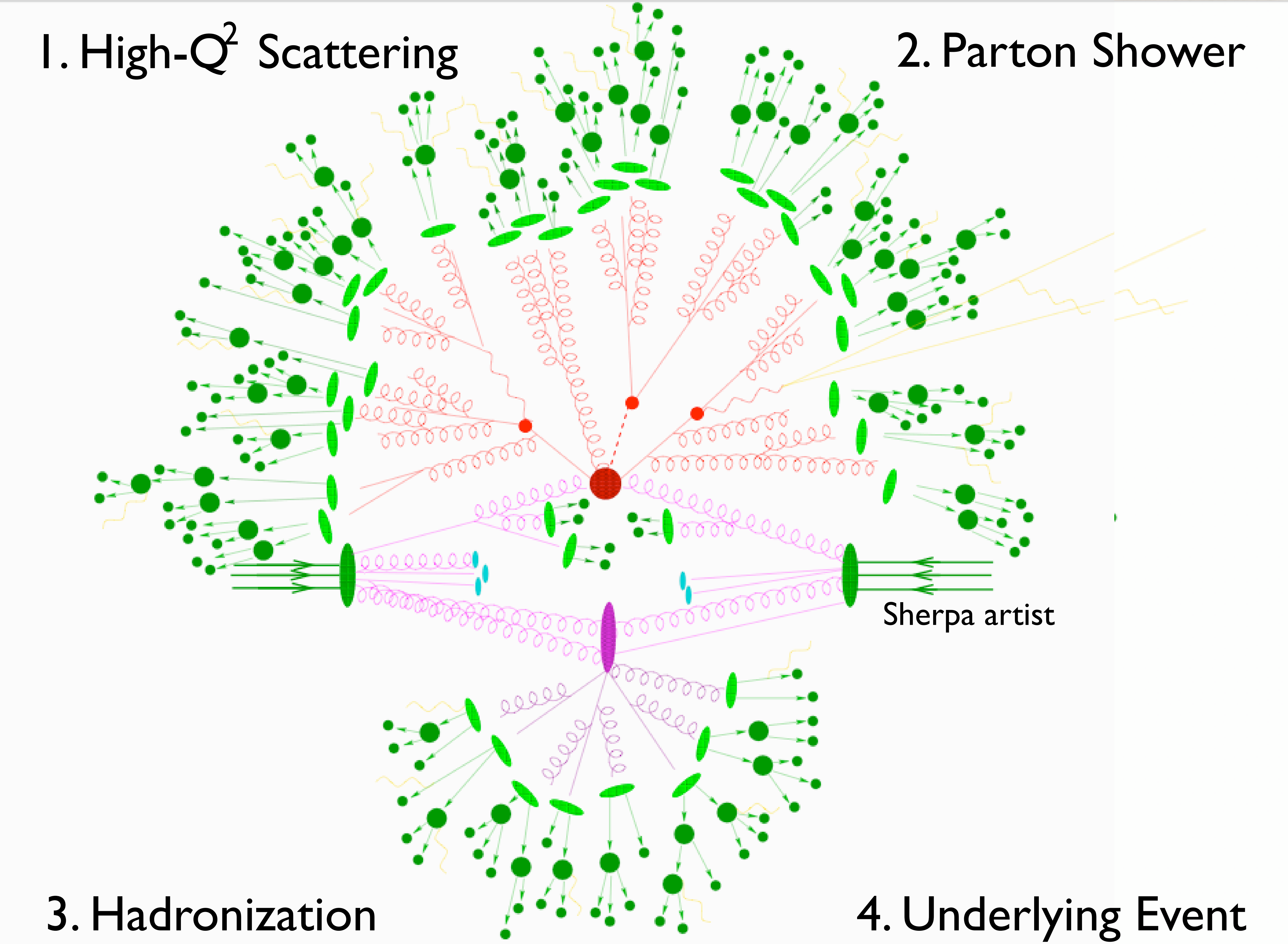
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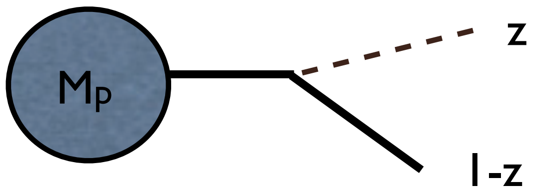
Parton Shower MC event generators

ME involving q →q g (or g → gg) are strongly enhanced when they are close in the phase space:

$$\frac{1}{(p_q+p_g)^2} \simeq \frac{1}{2E_qE_g(1-\cos\theta)}$$



Collinear factorization:

$$|M_{p+1}|^2 d\Phi_{p+1} \simeq |M_p|^2 d\Phi_p \frac{dt}{t}\frac{\alpha_S}{2\pi}P(z)dzd\phi$$

1. Allows for a parton shower (Markov process) evolution
2. The evolution resums the dominant leading-log contributions
3. By adding angular ordering the main quantum (interference) effects are also included

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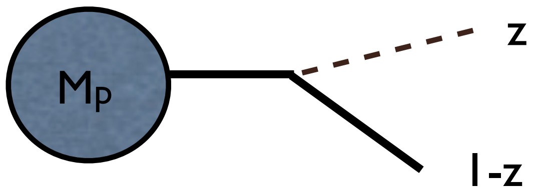

Both soft

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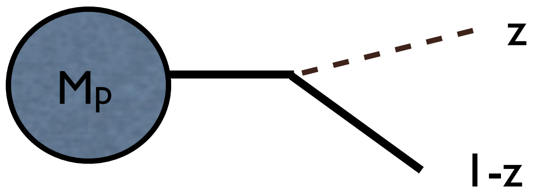

Both soft and collinear divergences: very different nature!

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Parton branching

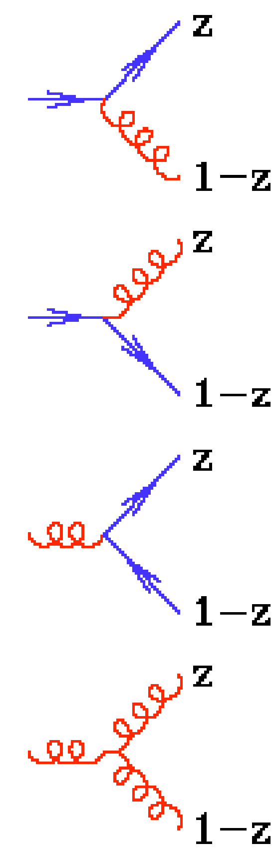
The spin averaged (unregulated) splitting functions for the various types of branching are:

$$\hat{P}_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)}\right],$$

$$\hat{P}_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right],$$

$$\hat{P}_{qg}(z) = T_R\left[z^2+(1-z)^2\right],$$

$$\hat{P}_{gg}(z) = C_A\left[\frac{z}{(1-z)}+\frac{1-z}{z}+z\,(1-z)\right].$$



$$C_F = \frac{4}{3}, C_A = 3, T_R = \frac{1}{2}.$$

Comments:

* Gluons radiate the most
* There soft divergences in z=1 and z=0.
* P_{qg} has no soft divergences.

Sudakov Form factor

Conservation of total probability:

$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$

"multiplicativeness" in "time" evolution:

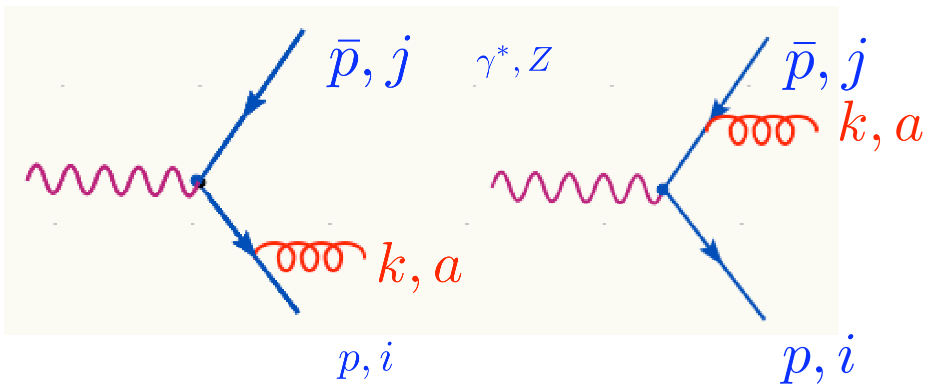
$\mathcal{P}_{\text{nothing}}(0 < t \leq T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1)\, \mathcal{P}_{\text{nothing}}(T_1 < t \leq T)$

Subdivide further, with $T_i = (i/n)T$, $0 \leq i \leq n$:

$$
\begin{aligned}
\mathcal{P}_{\text{nothing}}(0 < t \leq T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \leq T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} \left(1 - \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1})\right) \\
&= \exp\left(-\lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1})\right) \\
&= \exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right) \quad = \Delta(T)
\end{aligned}
$$

$$
\Longrightarrow \quad \mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T) \exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)
$$

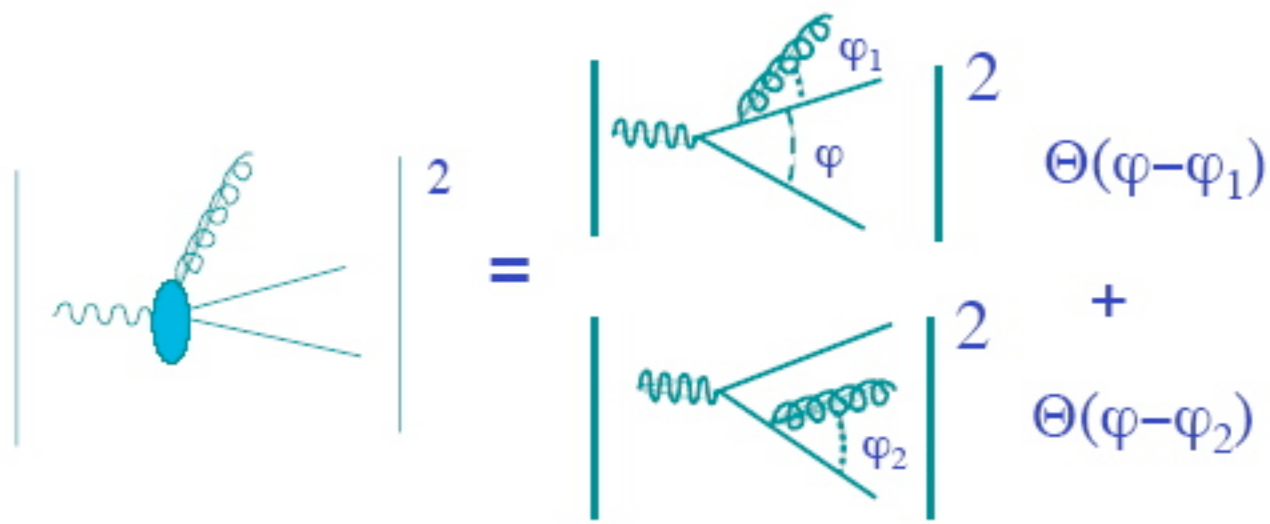
Angular ordering



$$d\sigma_g = \sum |A_{soft}|^2 \frac{d^3k}{(2\pi)^3 2k^0} \sum |A_0|^2 \frac{-2p^\mu \bar{p}^\nu}{(pk)(\bar{p}k)} g^2 \sum \varepsilon_\mu \varepsilon_\nu^* \frac{d^3k}{(2\pi)^3 2k^0}$$

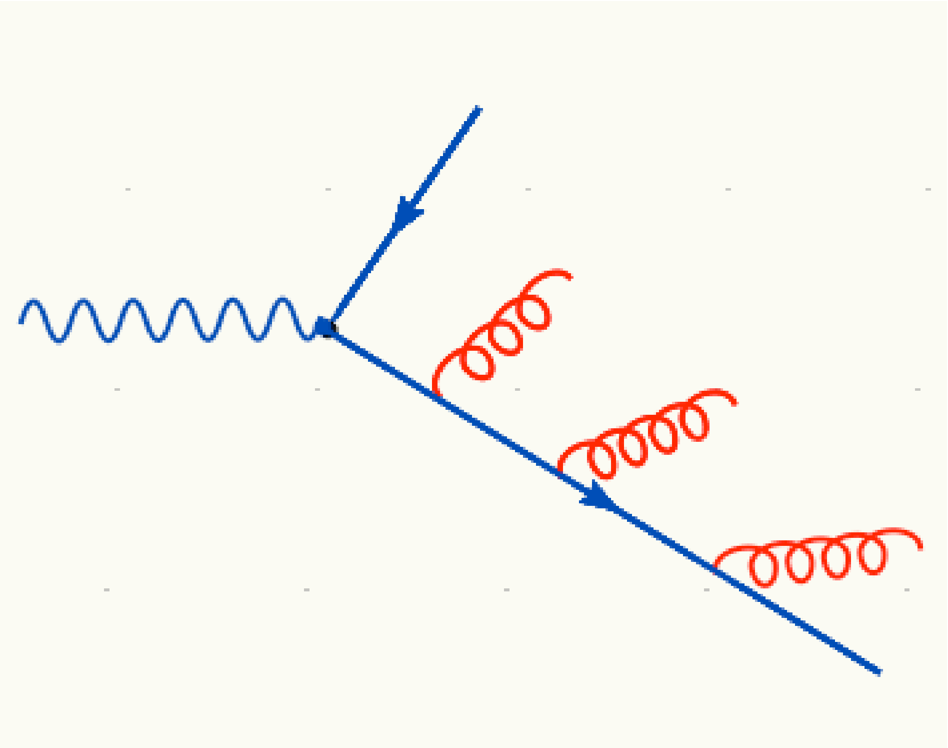
$$= d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{ik})(1-\cos\theta_{jk})} d\cos\theta$$

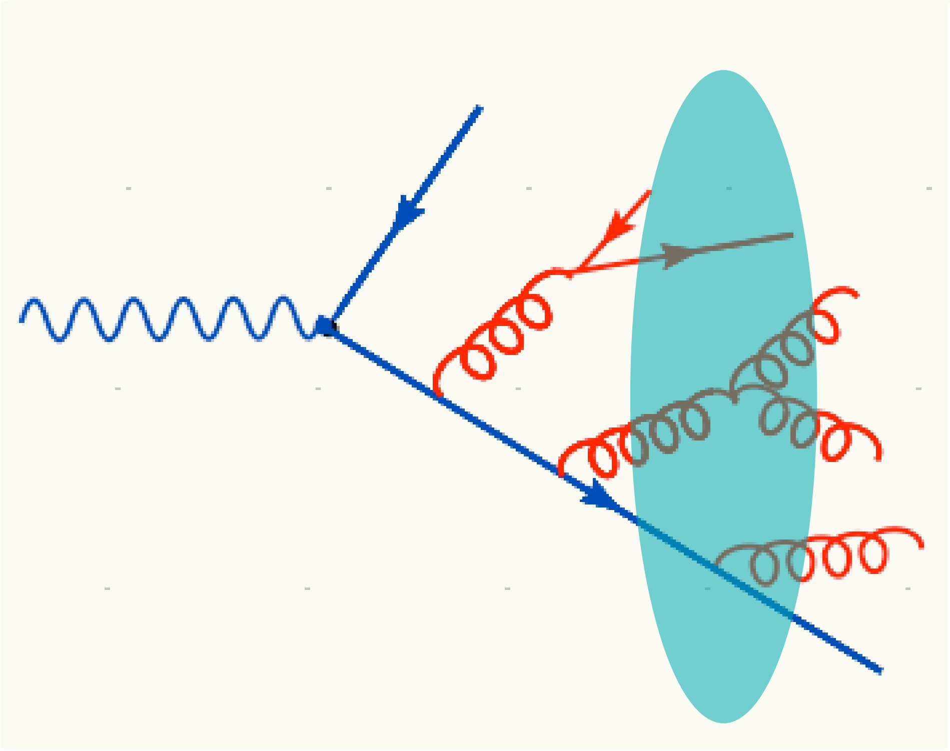
You can easily prove that:



Radiation happens only for angles smaller than the color connected (antenna) opening angle!

Angular ordering





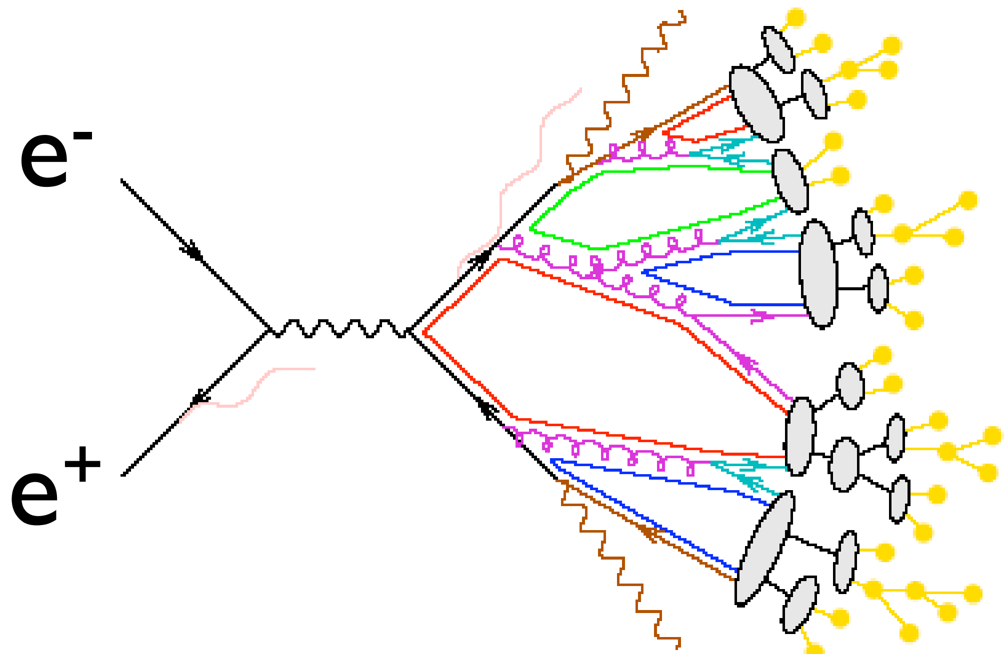
The construction can be iterated to the next emission, with the result that the emission angles keep getting smaller and smaller.

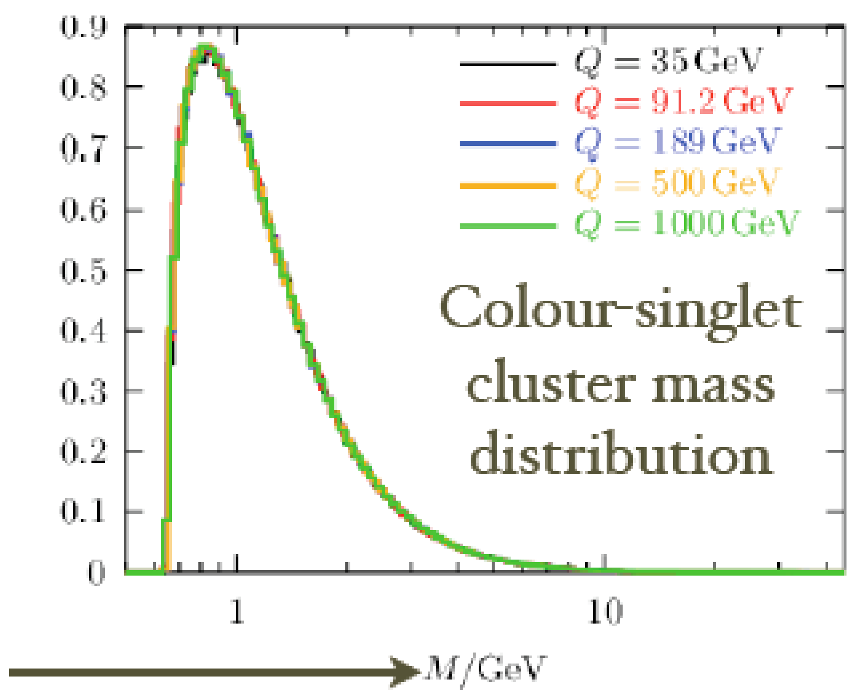
In fact one can generalize the treatment before to a generic parton of color charge Q_k splitting into two partons i and j , $Q_k=Q_i+Q_j$. The result is that inside the cones i and j emit as independent charges, and outside their angular-order cones the emission is coherent and can be treated as if it was directly from color charge Q_k.

This has an effect on the multiplicity of hadrons in jets (INTRAjet radiation), since the radiation is more suppressed with respect to the total phase space available, which one would get from an incoherent radiation. Color ordering enforces coherence and leads to the proper evolution with energy of particle multiplicities.

Monte Carlo approach to PS

The structure of the perturbative evolution, including angular ordering, leads naturally to the clustering in phase-space of color-singlet parton pairs (preconfinement). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass color singlet clusters.





Parton Shower MC event generators

- General-purpose tools
- Always the first exp choice
- Complete exclusive description of the events: hard scattering, showering & hadronization, underlying event
- Reliable and well tuned tools.

most famous: PYTHIA, HERWIG, SHERPA

- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD
  [Nagy, Soper, 2005; Giele, Kosower, Skands, 2007; Krauss, Schumman, 2007]

How we (used to) make predictions?

First way:

- For low multiplicity include higher order terms in our fixed-order calculations (LO→NLO→NNLO...)

  $\Rightarrow \quad \hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_S \sigma_1 + \alpha_S^2 \sigma_2 + \ldots$

- For high multeplicity use the tree-level results

TH

Comments:

1. The theoretical errors systematically decrease.
2. Pure theoretical point of view.
3. A lot of new techniques and universal algorithms are developed.
4. Final description only in terms of partons and calculation of IR safe observables ⇒ not directly useful for simulations

How we (used to) make predictions?

Second way:

- Describe final states with high multiplicities starting from $2 \to 1$ or $2 \to 2$ procs, using parton showers, and then an hadronization model.

EXP

Comments:

1. Fully exclusive final state description for detector simulations
2. Normalization is very uncertain
3. Very crude kinematic distributions for multi-parton final states
4. Improvements are only at the model level.

[Mangano]
[Catani, Krauss, Kuhn, Webber]
[Frixione, Nason, Webber]

ME vs PS

ME	Shower MC
1. parton-level description	1. hadron-level description
2. fixed order calculation	2. resums large logs
3. quantum interference exact	3. quantum interference through angular ordering
4. valid when partons are hard and well separated	4. valid when partons are collinear and/or soft
5. needed for multi-jet description	5. nedeed for realistic studies

Difficulty: avoid double counting

[Mangano]
[Catani, Krauss, Kuhn, Webber]
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5. nedeed for realistic studies

Approaches are complementary: merge them!

Difficulty: avoid double counting

How to improve our predictions?

New trend: TH & EXP

Match fixed-order calculations and parton showers to obtain the most accurate predictions in a detector simulation friendly way!

Two directions:

1. Get fully exclusive description of many parton events correct at LO (LL) in all the phase space. ME+PS

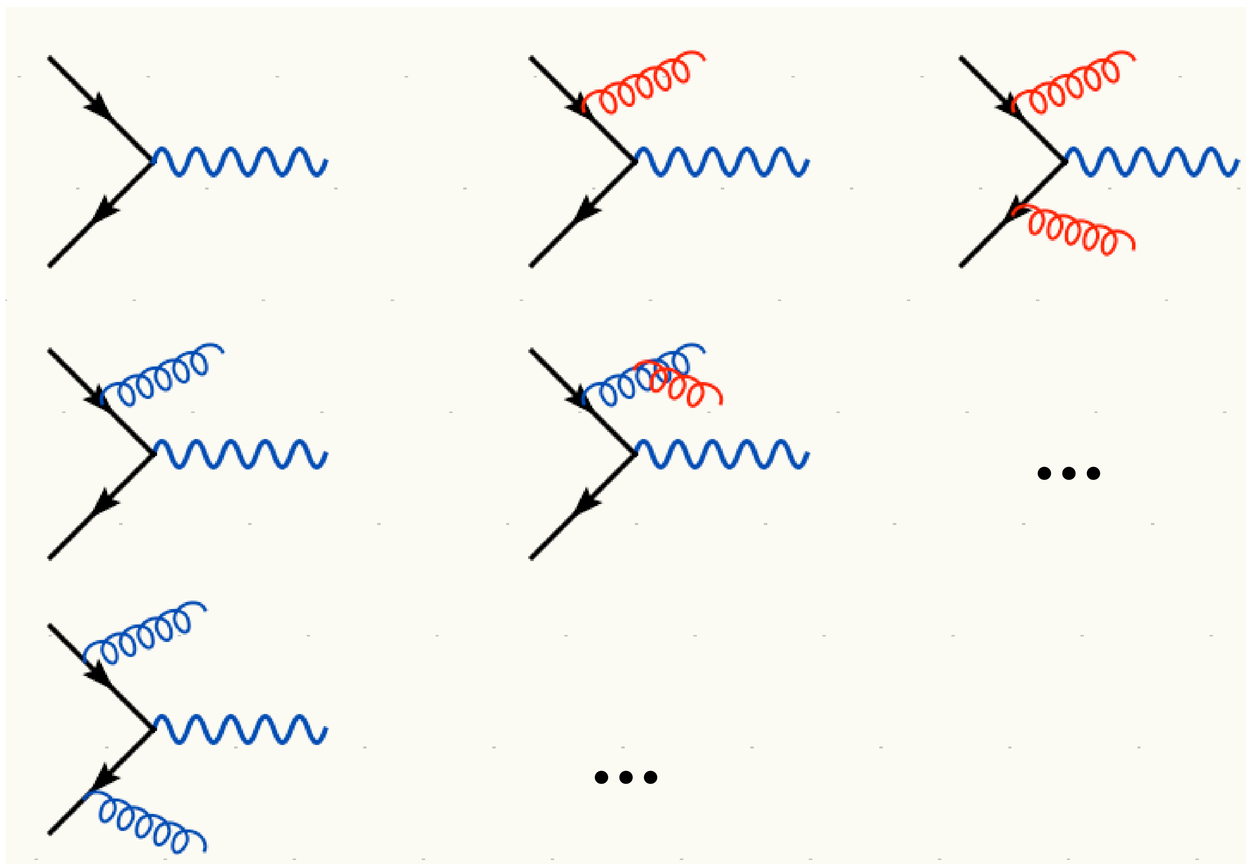
2. Get fully exclusive description of events correct at NLO in the normalization and distributions. NLOwPS

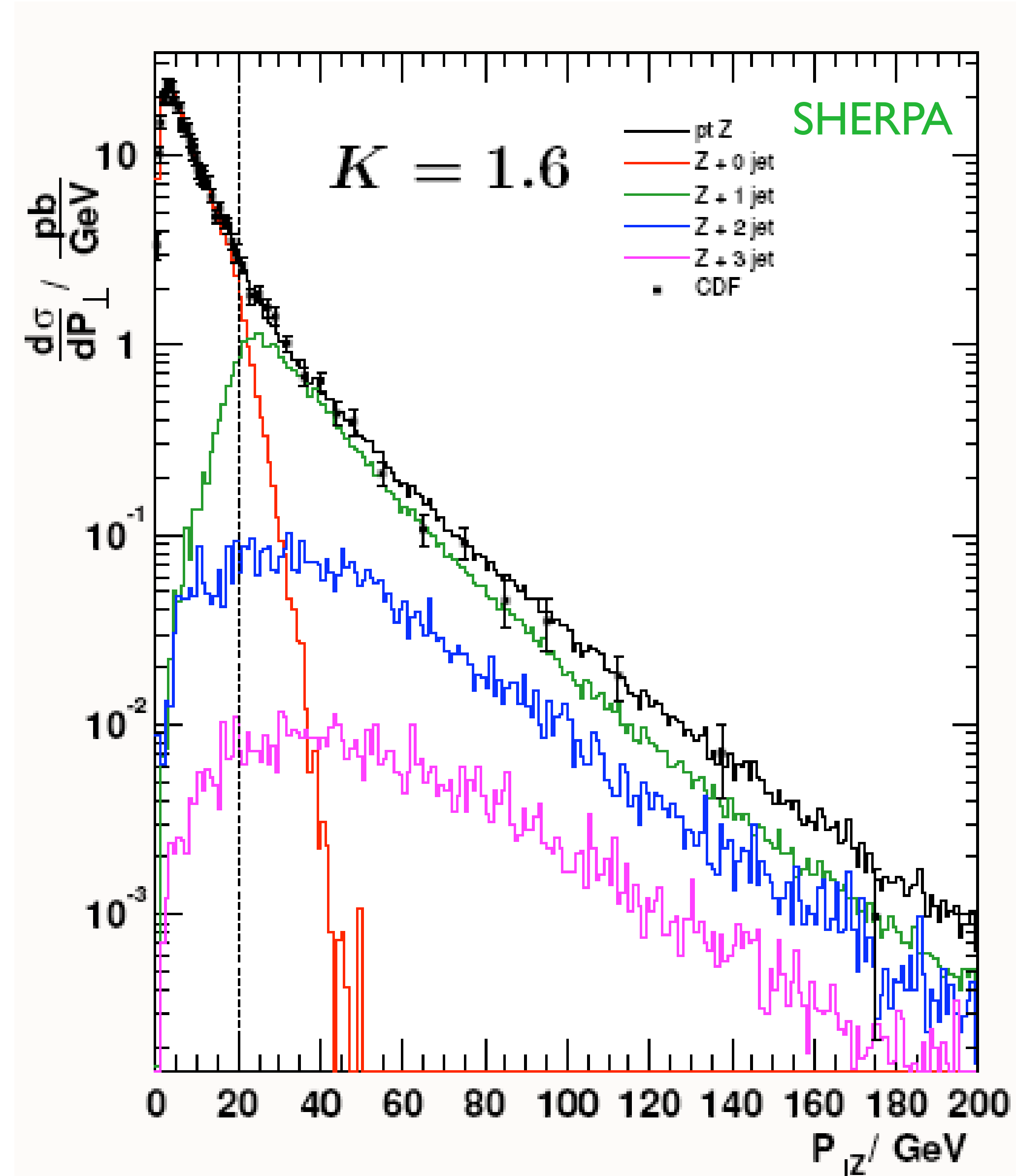
Merging fixed order with PS

[Mangano]
[Catani, Krauss, Kuhn, Webber]

PS →

ME ↓

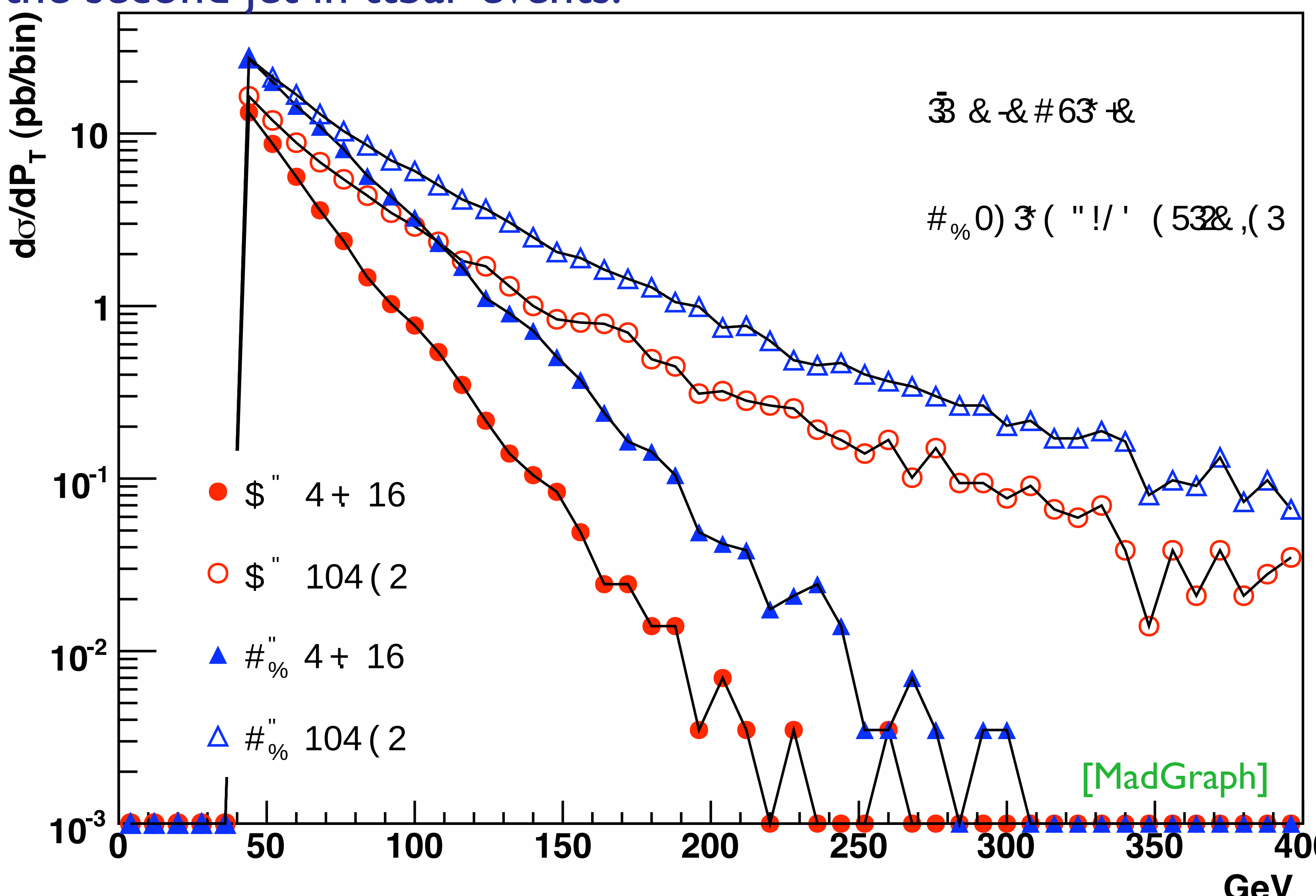




Double counting of configurations that can be obtained in different ways (histories). All the matching algorithms (CKKW, MLM,...) apply criteria to select only one possibility based on the hardness of the partons. As the result events are exclusive and can be added together into an inclusive sample. Distributions are accurate but overall normalization still "arbitrary".

PS alone vs matched samples

A MC Shower like Pythia produces inclusive samples covering all phase space. However, there are regions of the phase space (ex. high pt tails) which cannot be described well by the log enhanced (shower) terms in the QCD expansion and lead to ambiguities. Consider for instance the high-pt distribution of the second jet in ttbar events:



Changing some choices/parameters leads to huge differences ⇒ self diagnosis. Trying to tune the log terms to make up for it is not a good idea ⇒ mess up other regions/shapes, process dependence.

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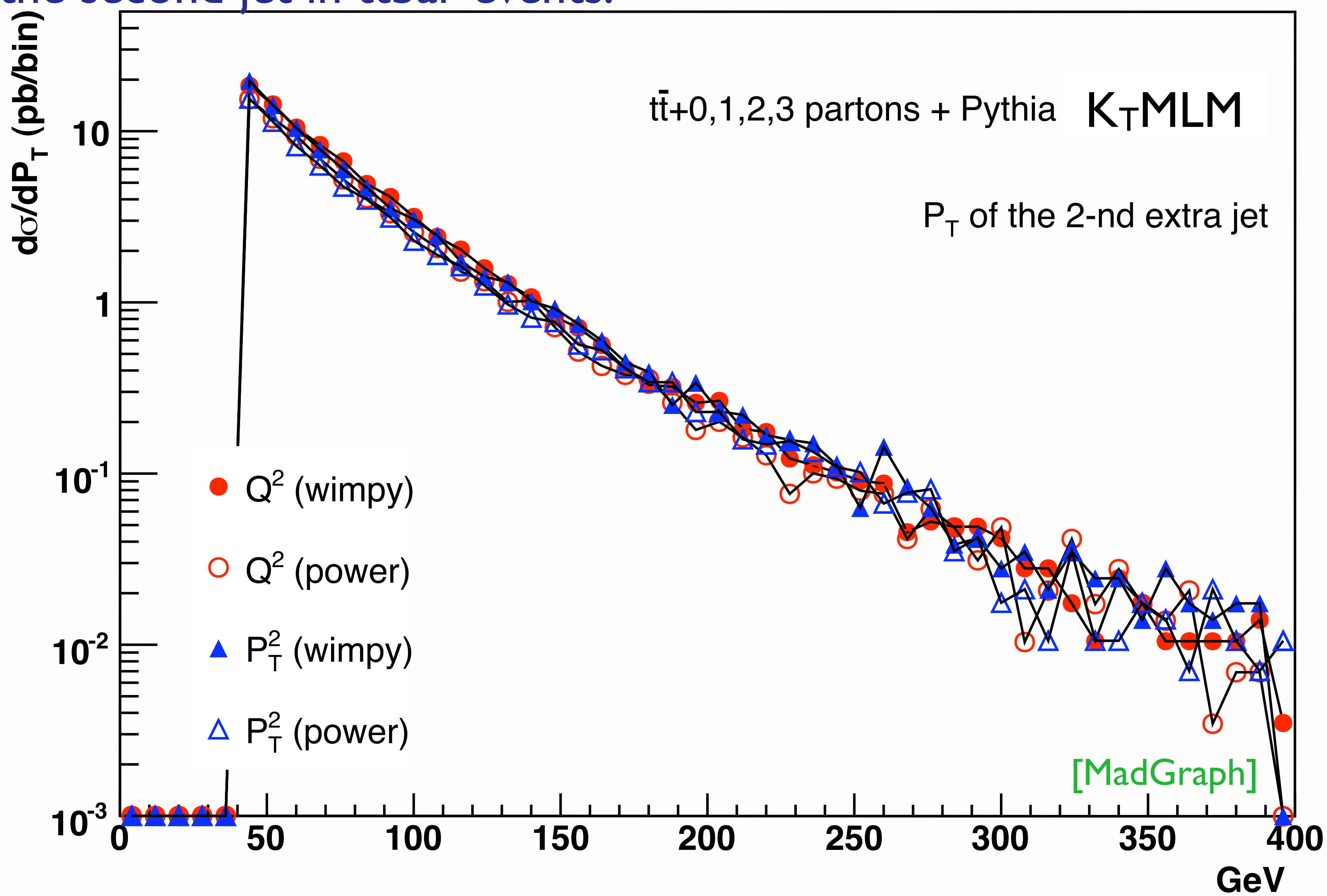
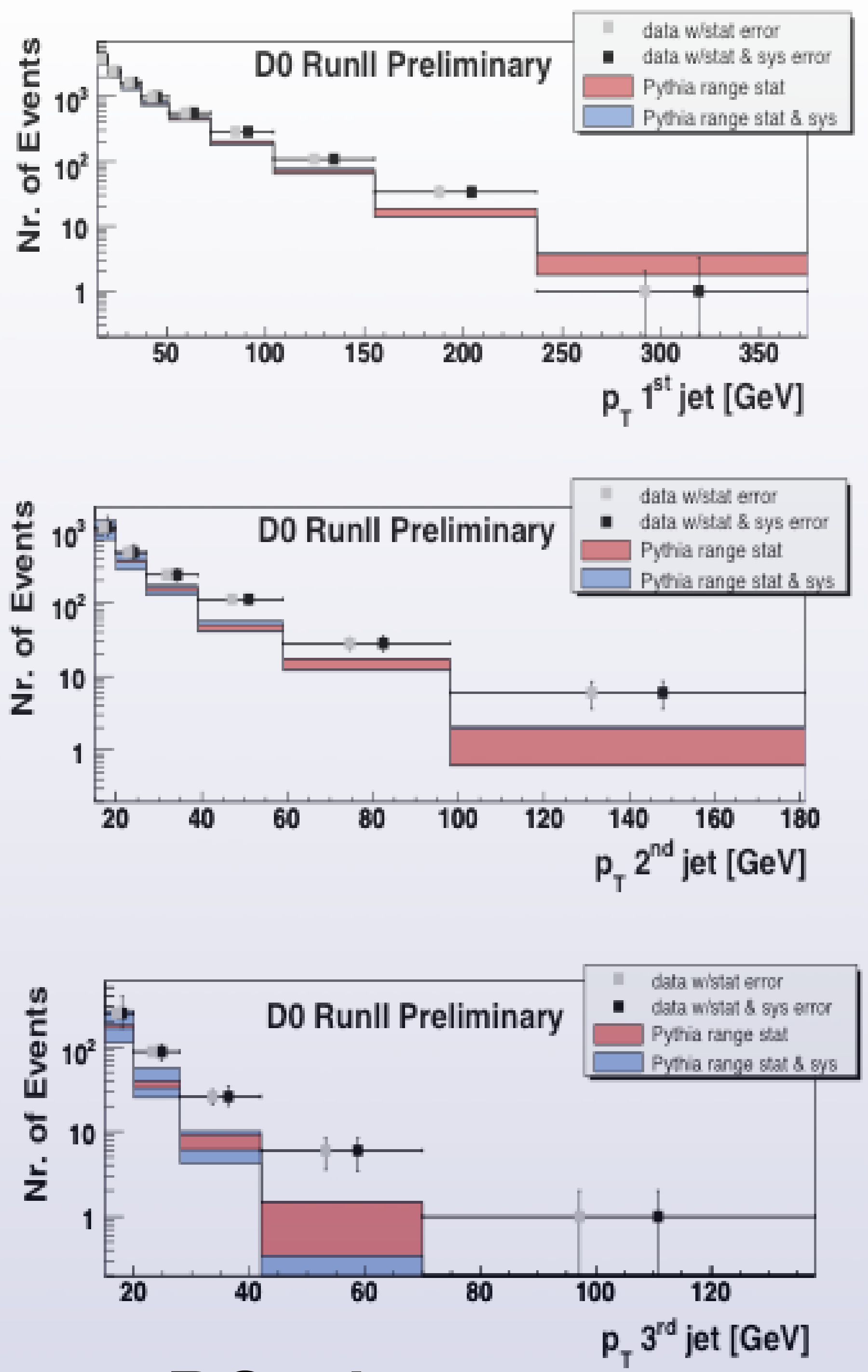
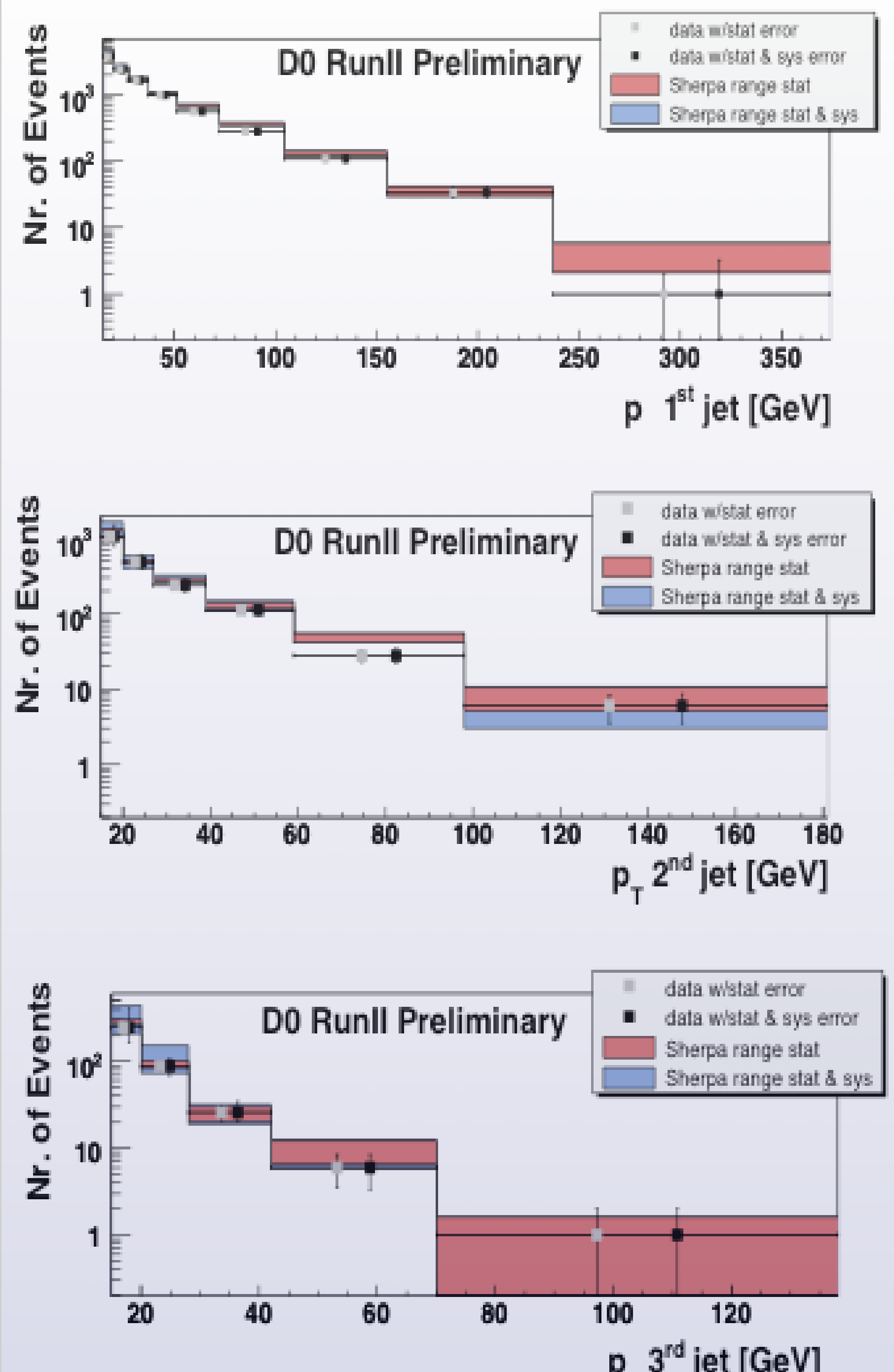


In a matched sample these differences are irrelevant since the behaviour at high pt is dominated by the matrix element. LO+LL is more reliable. (Matching uncertaintes not shown.)

PS alone vs matched samples : Z+jets at D0

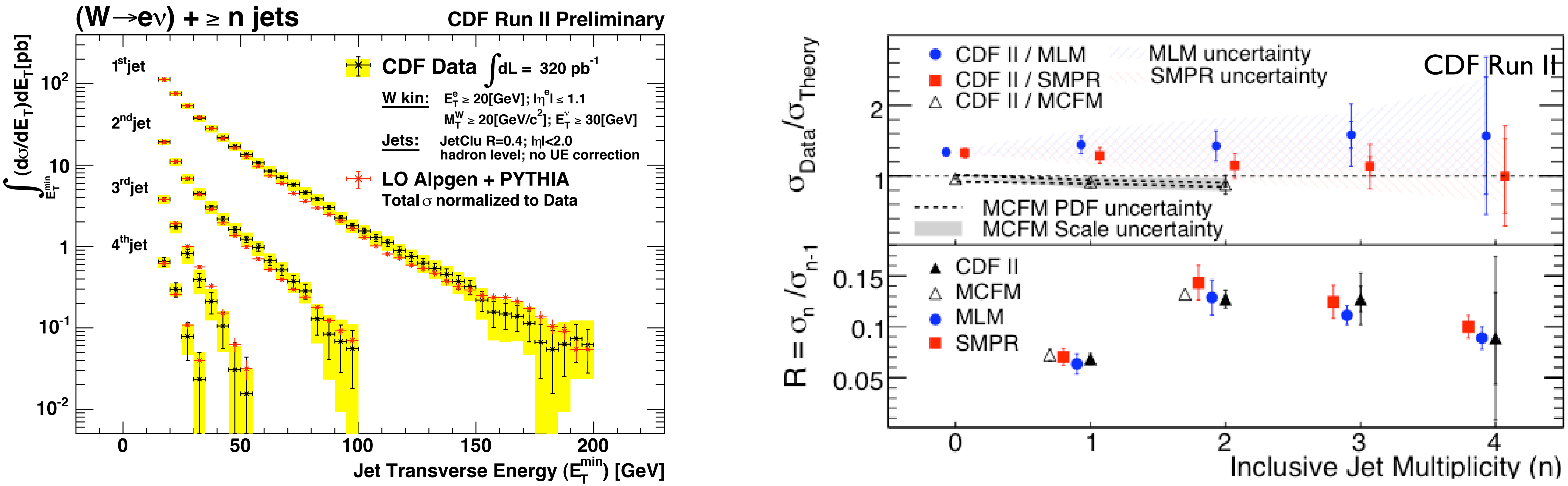


PS alone (Pythia)



ME+PS (SHERPA)

W+jets at CDF



* Very good agreement in shapes (left) and in relative normalization (right).

* NLO rates in outstanding agreement with data.

* Matched samples obtained via different matching schemes (MLM and CKKW) consistent within the expected uncertaintes. Differences might arise in more exclusive quantities.

NLOwPS

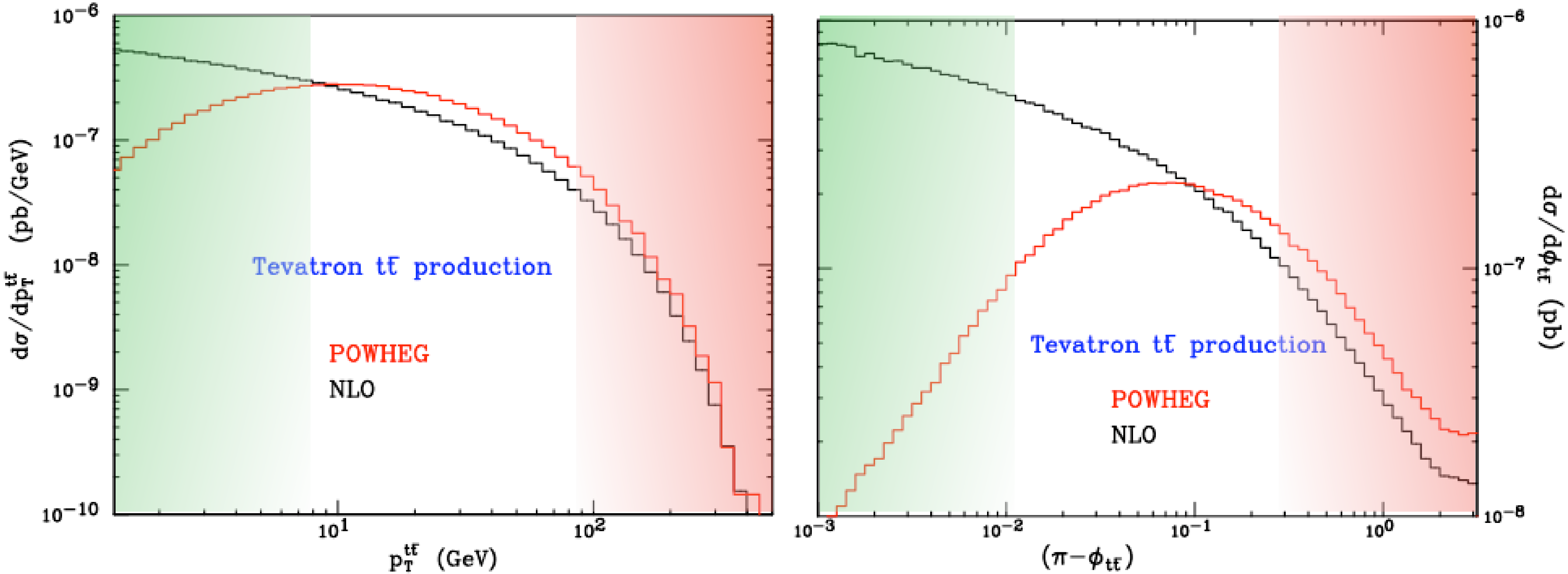
Problem of double counting becomes even more severe at NLO
* Real emission from NLO and PS has to be counted once
* Virtual contributions in the NLO and Sudakov should not overlap

Current available (and working) solutions:

MC@NLO [Frixione, Webber, 2003; Frixione, Nason, Webber, 2003]
- Matches NLO to HERWIG angular-ordered PS.
- "Some" work to interface an NLO calculation to HERWIG. Uses only FKS subtraction scheme.
- Some events have negative weights.
- Sizable library of procs now.

POWHEG [Nason 2004; Frixione, Nason, Oleari, 2007]
- Is independent from the PS. It can be interfaced to PYTHIA or HERWIG.
- Can use existing NLO results.
- Generates only positive unit weights.
- For top only ttbar (with spin correlations) is available so far.

ttbar : NLOwPS vs NLO



* Soft/Collinear resummation of the $p_T(tt) \to 0$ region.
* At high $p_T(tt)$ it approaches the tt+parton (tree-level) result.
* When $\Phi(tt) \to 0$ ($\Phi(tt) \to \pi$) the emitted radiation is hard (soft).
* Normalization is FIXED and non trivial!!

NLOwPS : Summary

"Best" tools when NLO calculation is available (i.e. low jet multiplicity).

* Main points:
  * NLOwPS provide a consistent to include K-factors into MC's
  * Scale dependence is meaningful
  * Allows a correct estimates of the PDF errors.
  * Non-trivial dynamics beyond LO included for the first time.

* Status

  * POWHEG Box simplifies the implementation of new processes
  * Only SM*.
  * Only available for low multiplicity.

* Future

  * Full automatization of NLO calculations interfaced with showers (~ Pythia@NLO) imminent.

Status : SM

pp$\rightarrow$ n particles

Status : SM

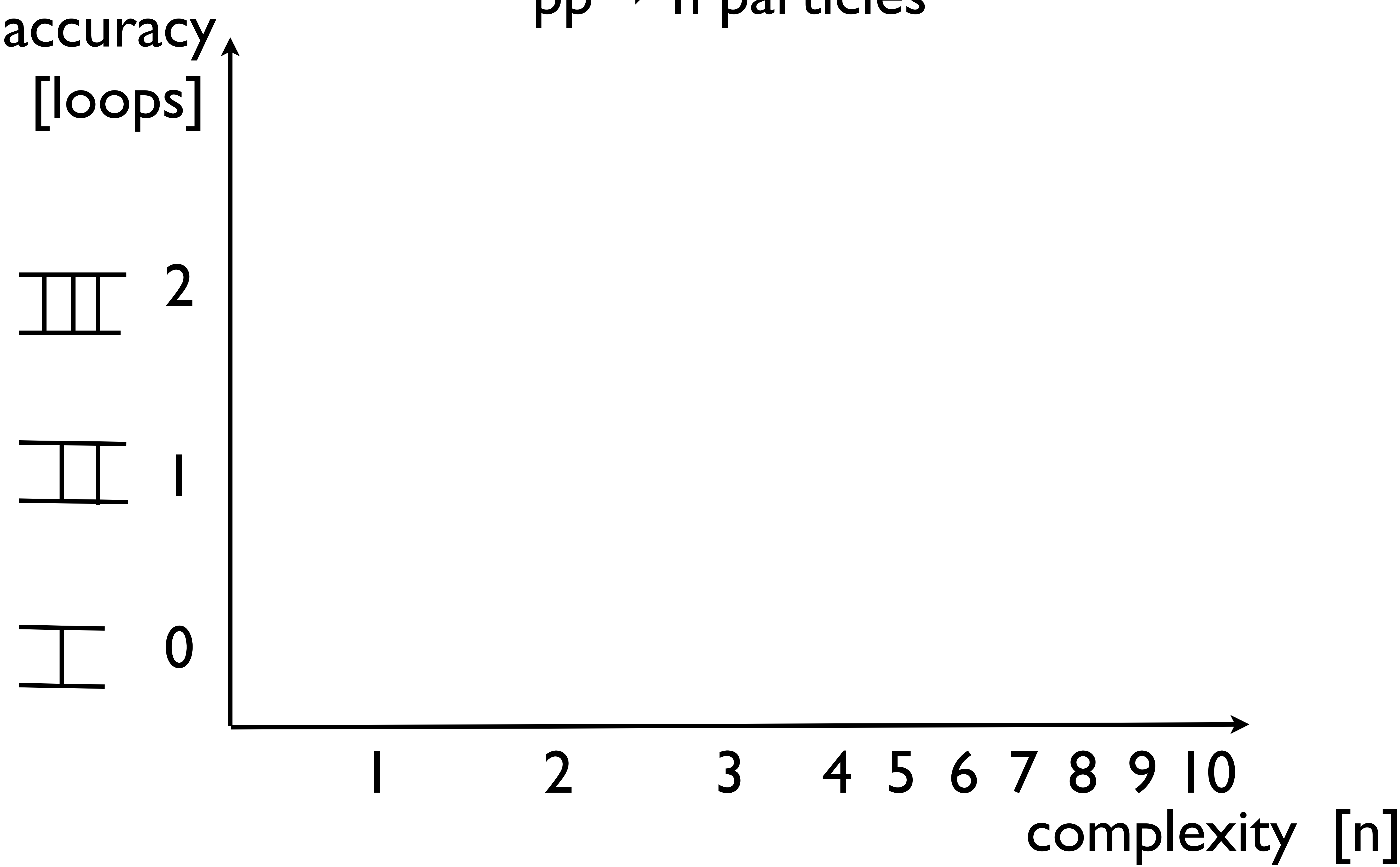
pp → n particles

1 2 3 4 5 6 7 8 9 10

complexity [n]

Status : SM

pp→ n particles



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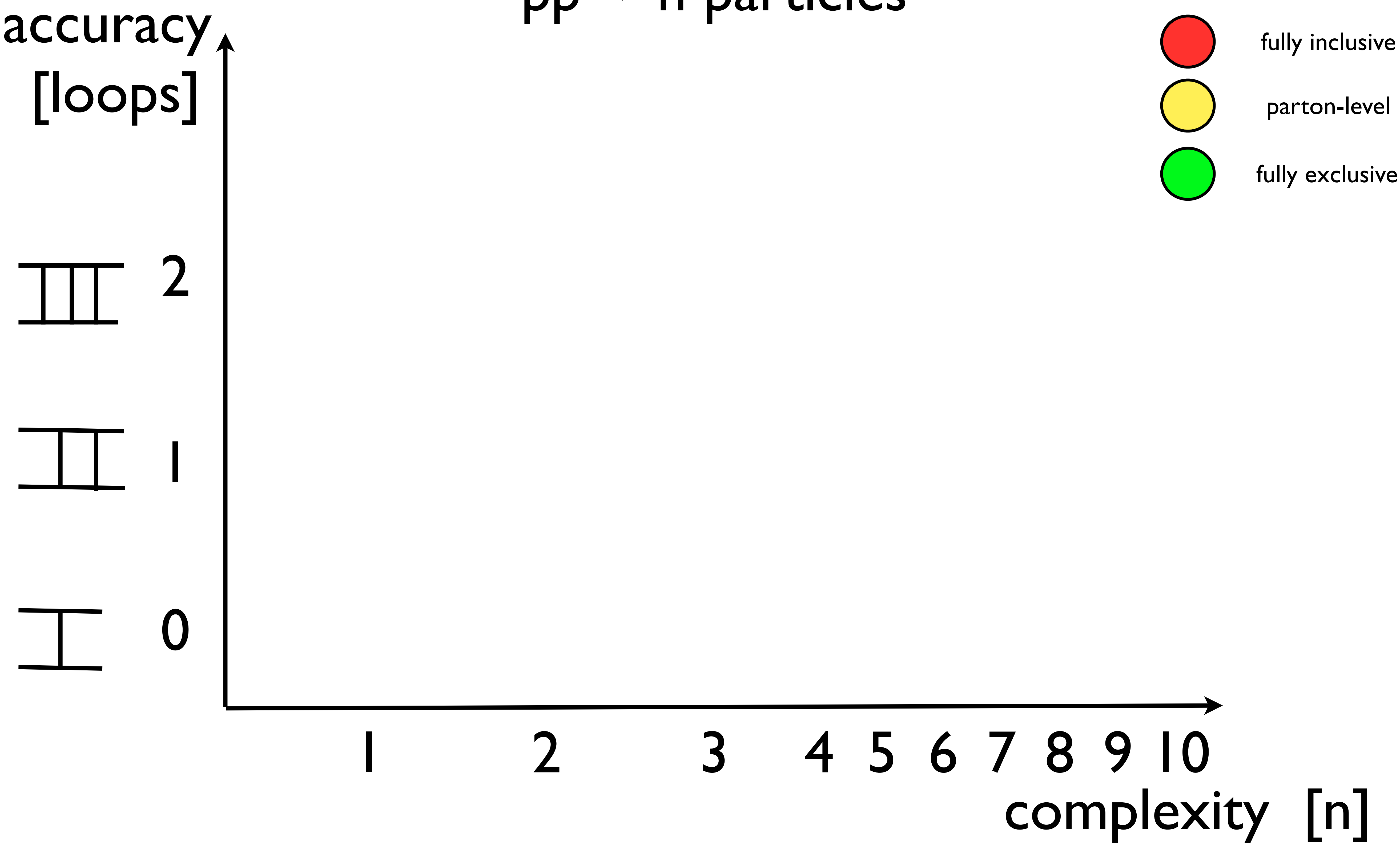

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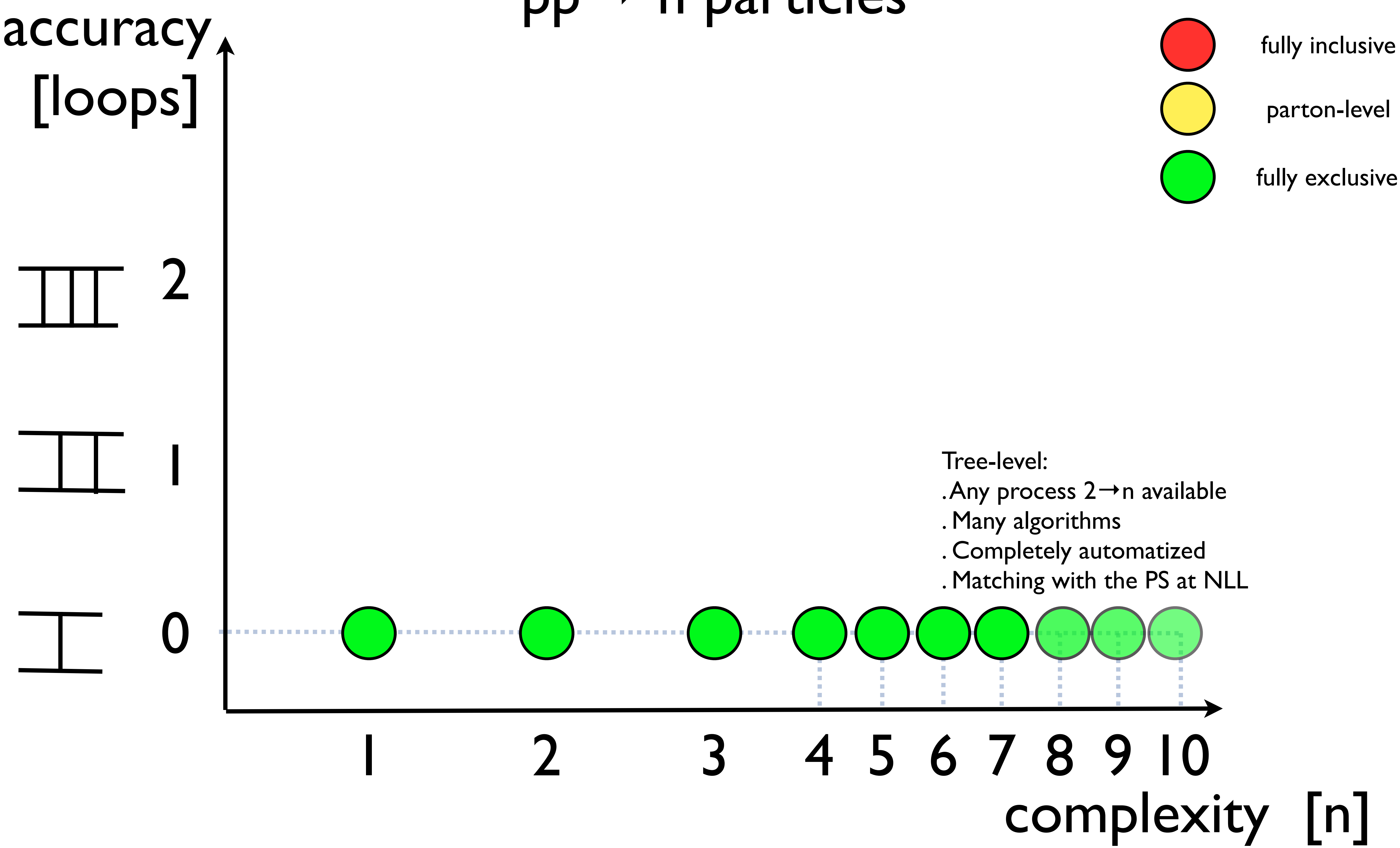

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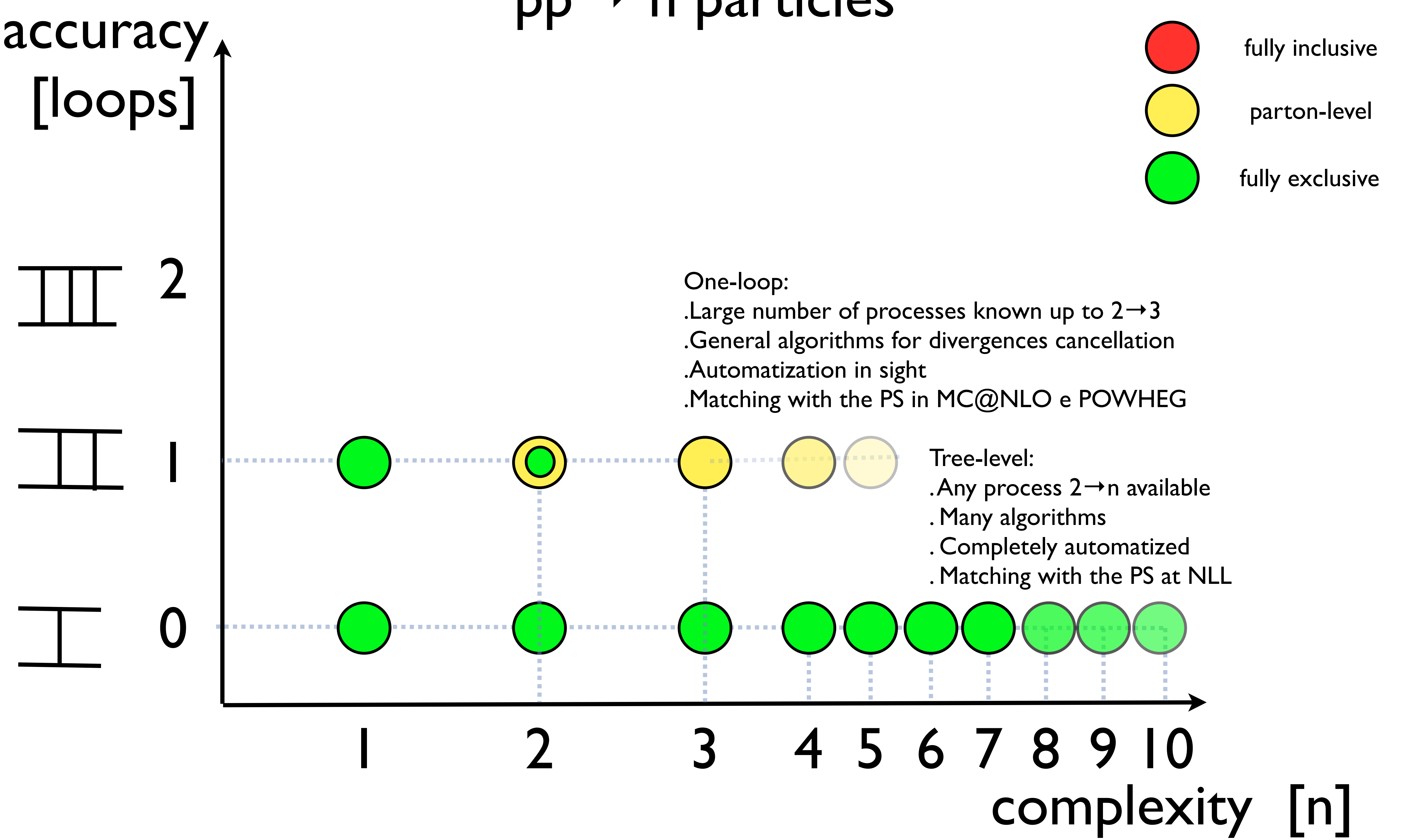

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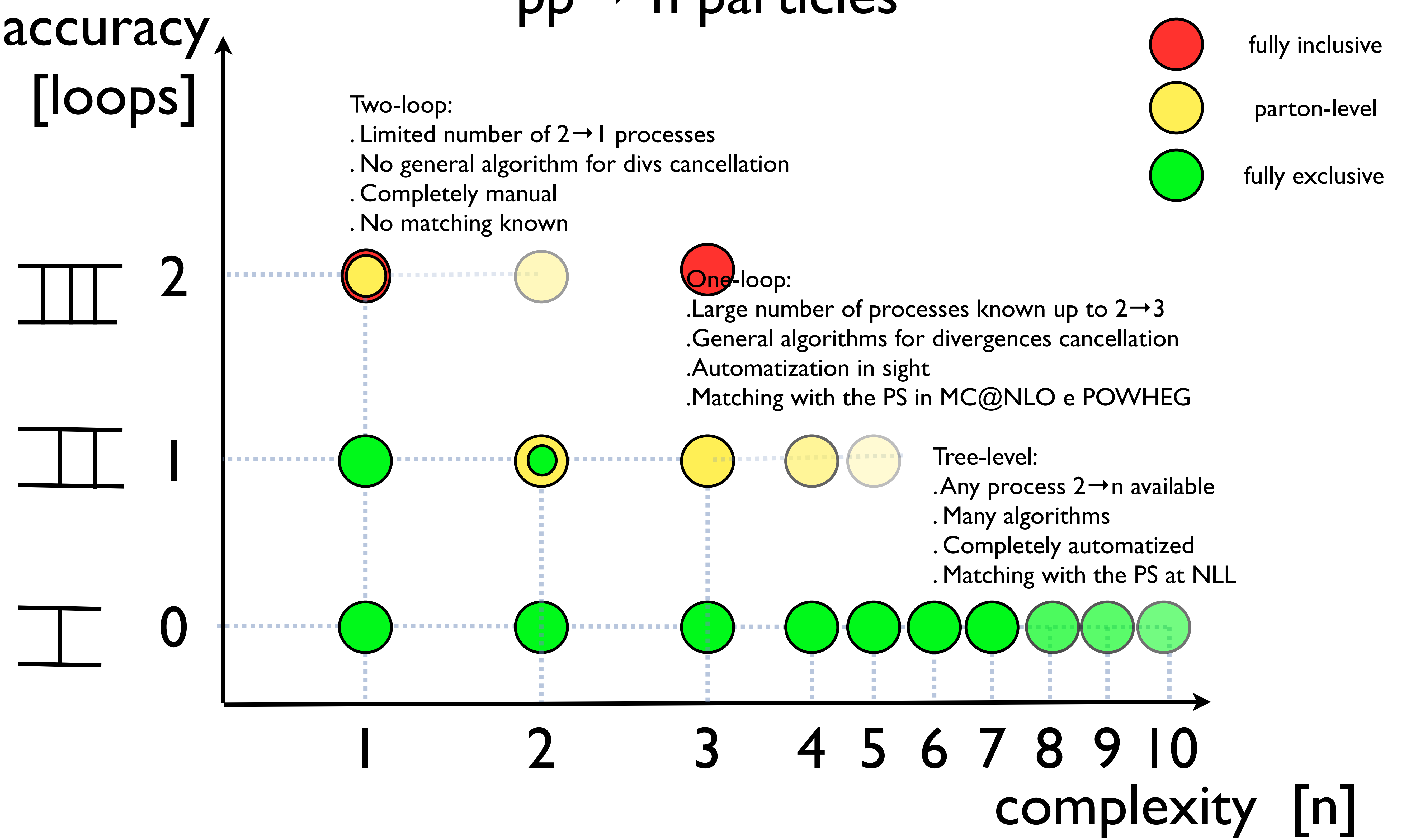
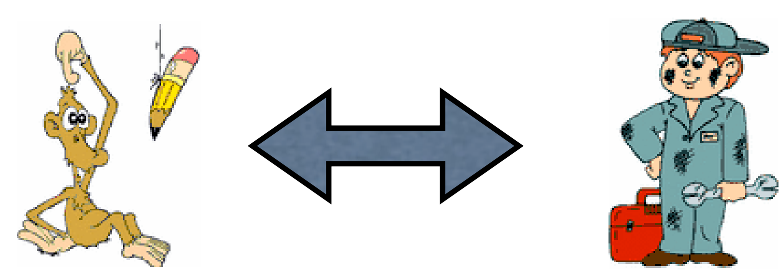


What about BSM?

Two main (related) issues:

1. A plethora of BSM proposals exist to be compared with data. It will be essential to have an efficient, validated MC framework for theorists to communicate with experimentalits their idea (and viceversa).



2. Once models are available in multipurpose MC's, new detailed studies are possible that allow to bring to the BSM signatures the same level of sophistication achieved for the SM.

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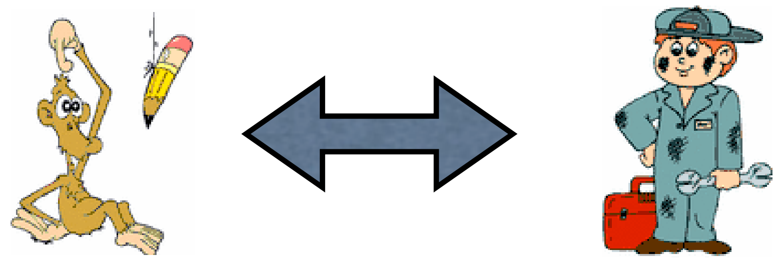

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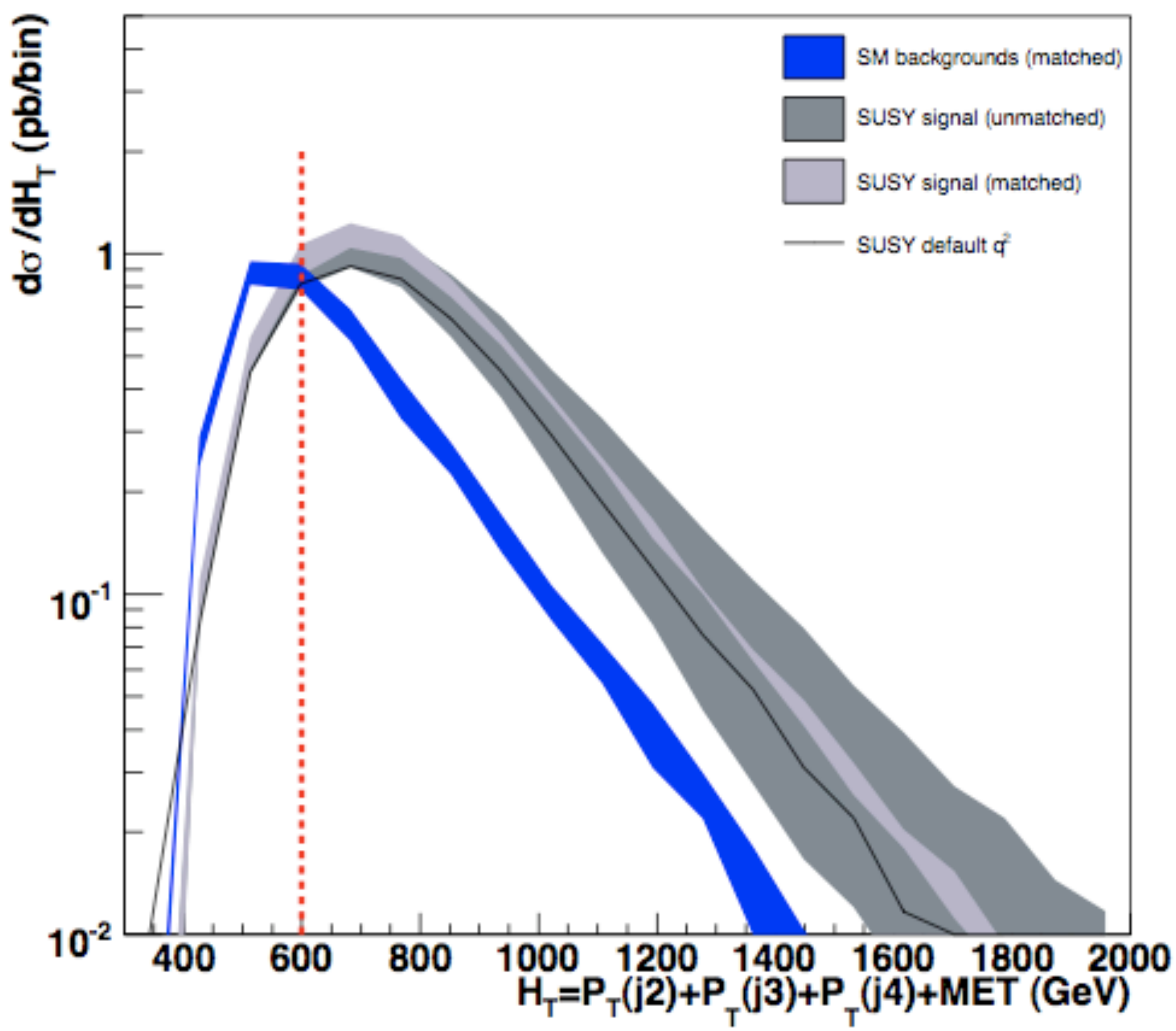
BSM @ LHC : present

[Alwall, de Visscher, FM, 2009]



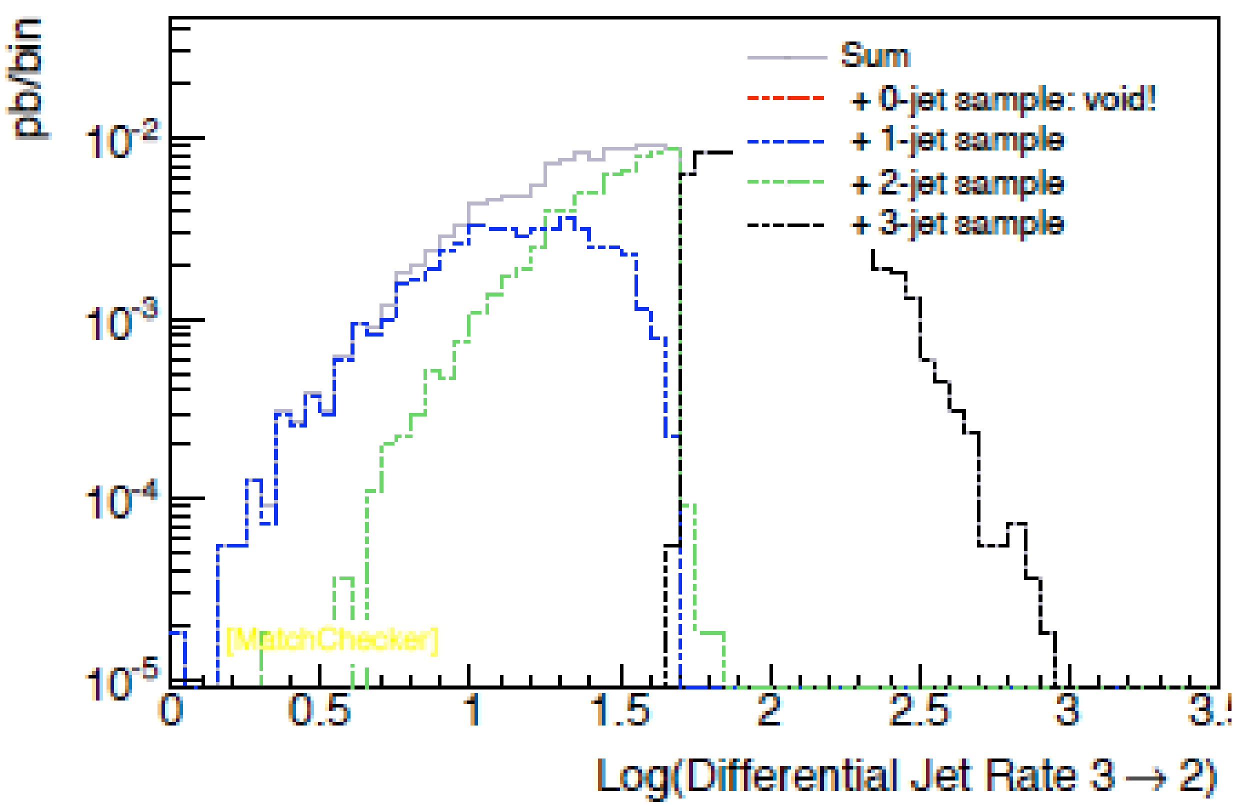
Both signal and background matched!

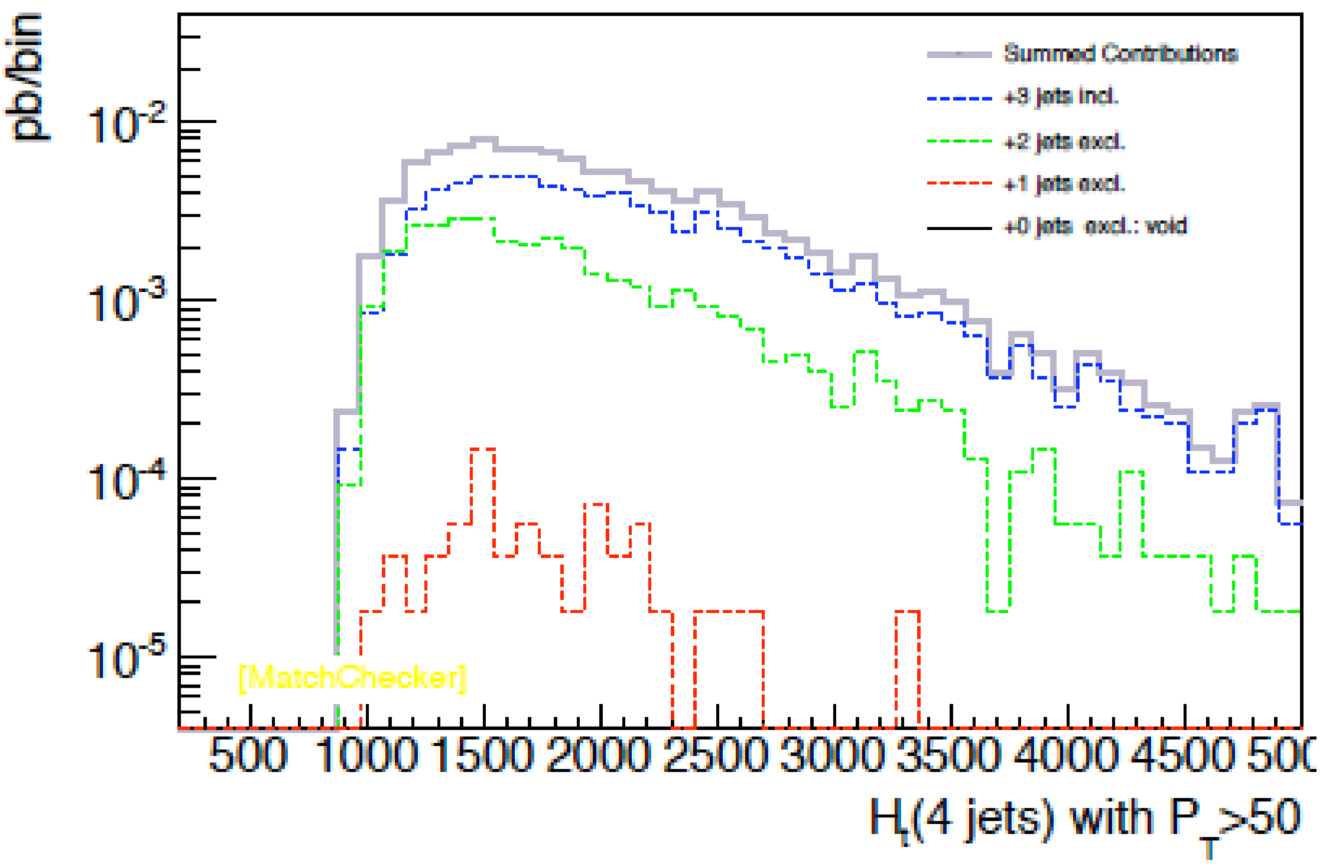
Sizable reduction of the uncertainties. Overall picture unchanged for SPS1a.

Gravitons

[K. Hagiwara, J. Kanzaki, Q. Li and K. Mawatari, 2009]
[P. de Aquino, K. Hagiwara, Q. Li, F. M.]

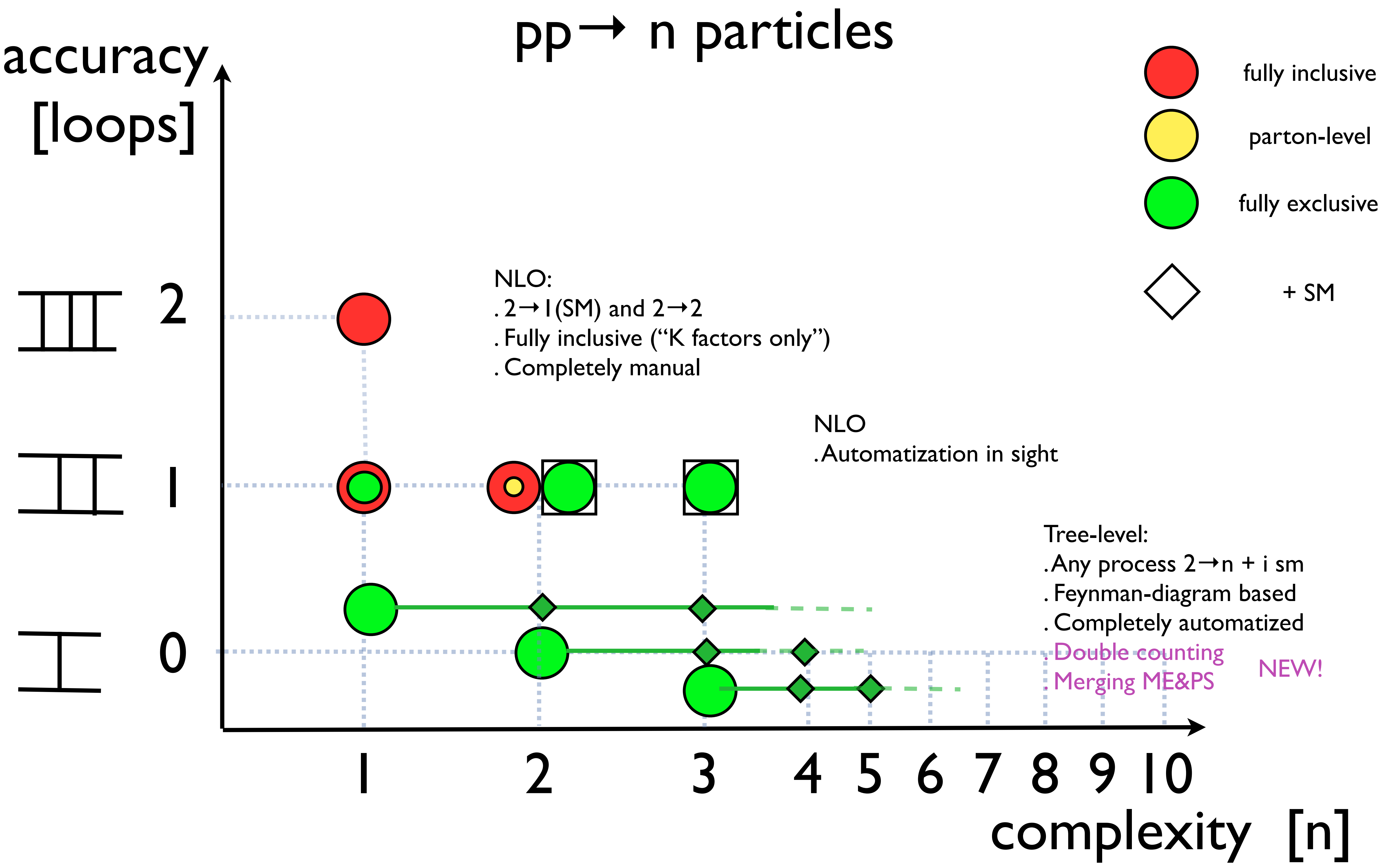
- Fixed mass gravitons (RS and also mG=0)
- ADD gravitons also available : challenging due peculiar "propagator" : this is automatically handled in MG now.





Works out of the box..

BSM : status and outlook



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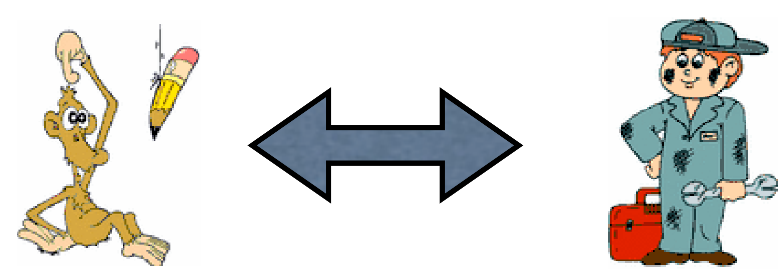

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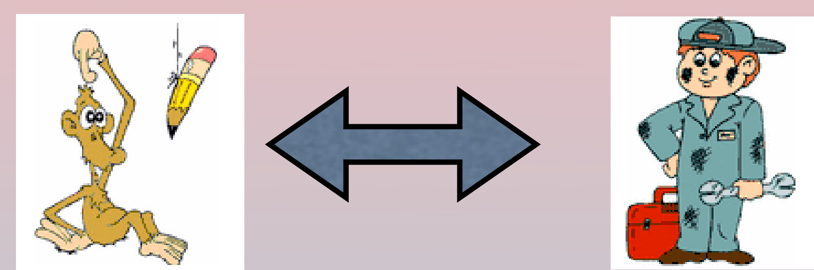

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A Roadmap (with roadblocks) for BSM @ the LHC

TH

EXP

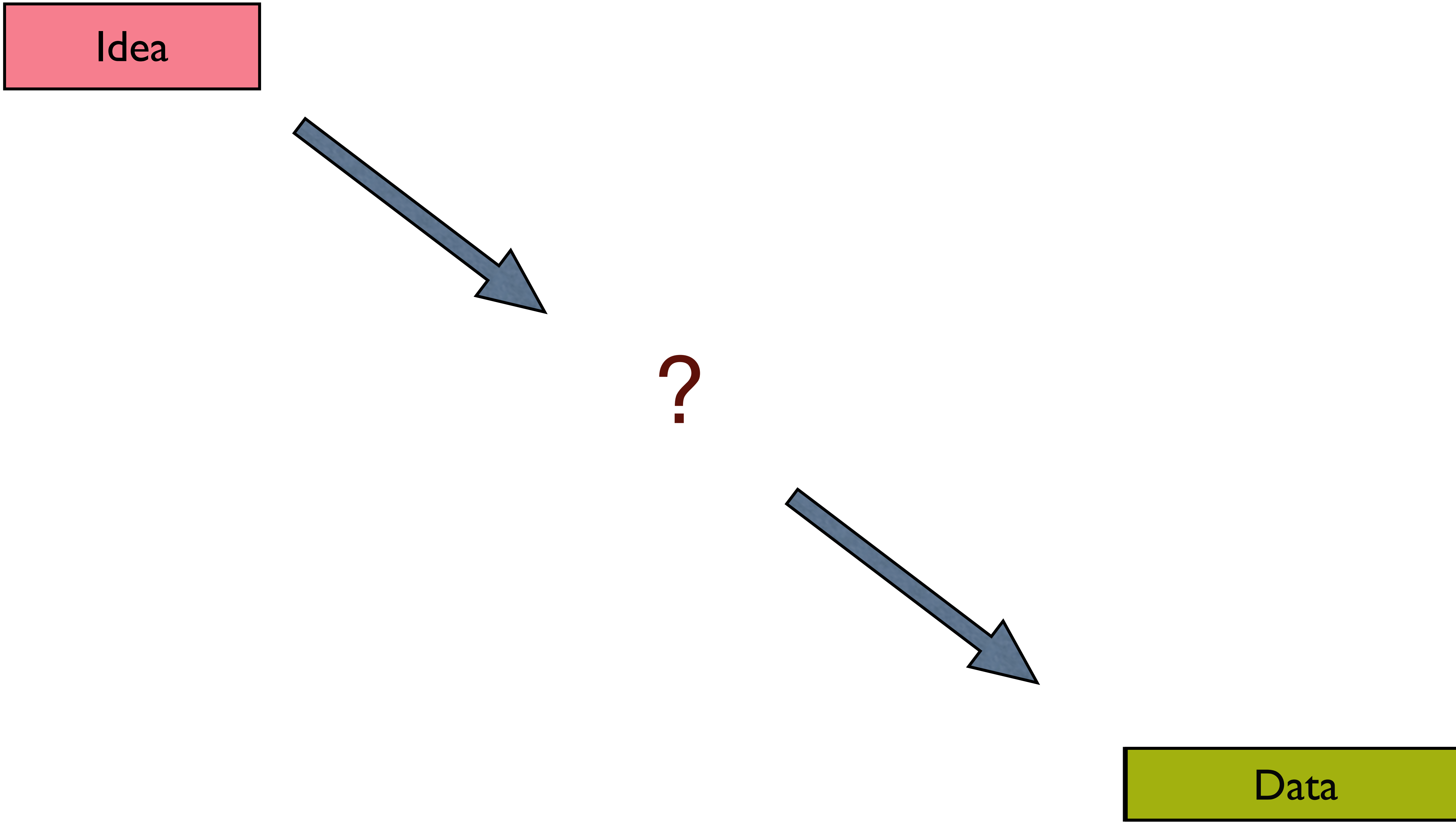
Idea

Data

A Roadmap (with roadblocks) for BSM @ the LHC

TH

EXP



A Roadmap (with roadblocks) for BSM @ the LHC

TH

Idea

A Roadmap (with roadblocks) for BSM @ the LHC

TH

Idea

Lagrangian

Feyn. Rules

Amplitudes

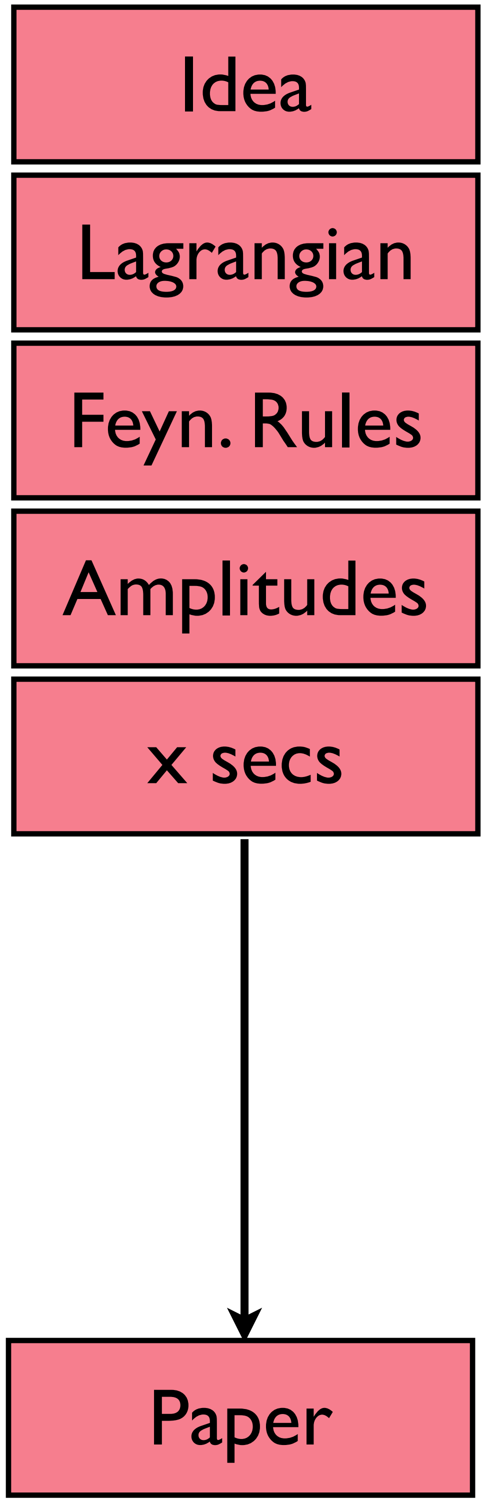
x secs

↓

Paper

A Roadmap (with roadblocks) for BSM @ the LHC

TH PHENO



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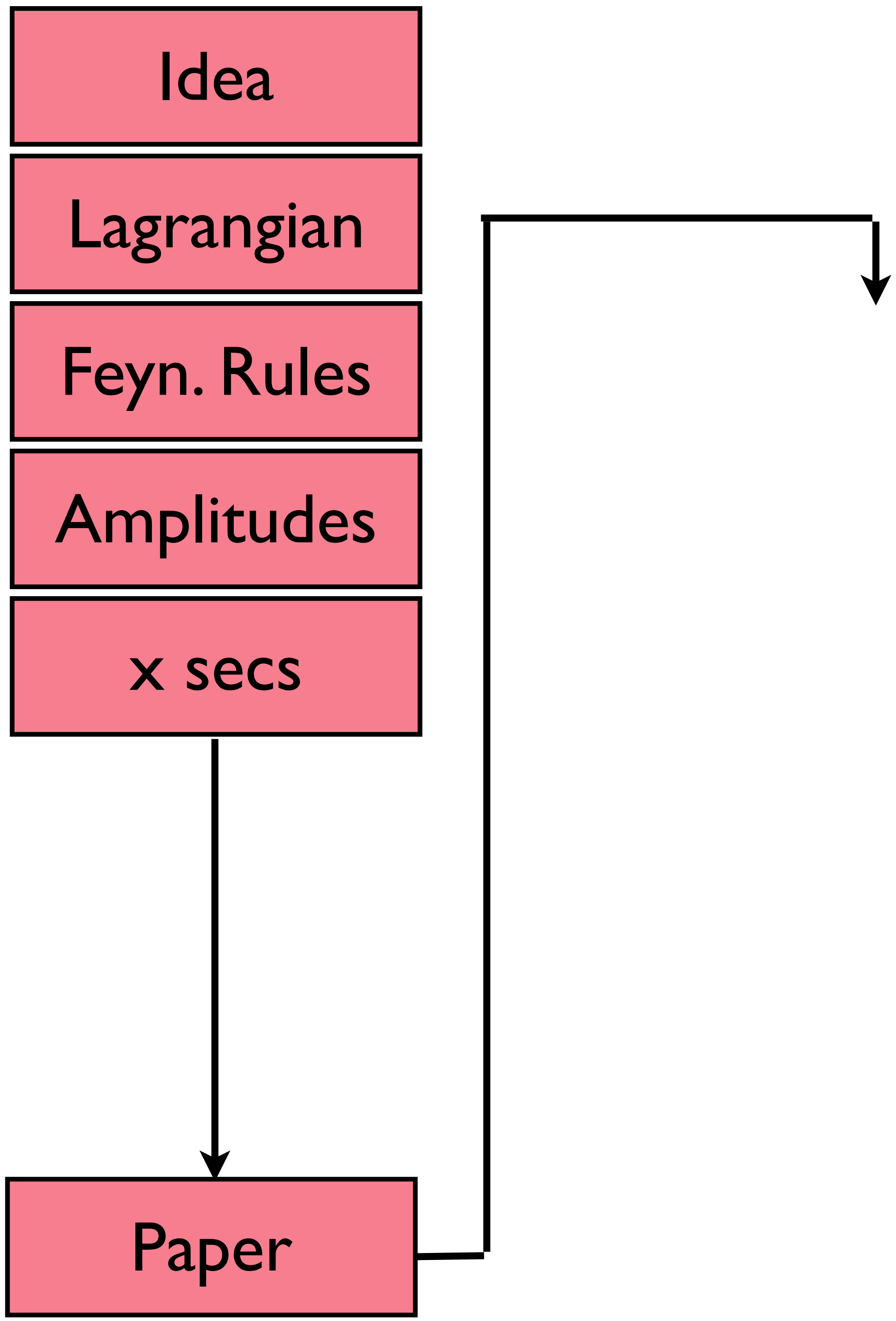
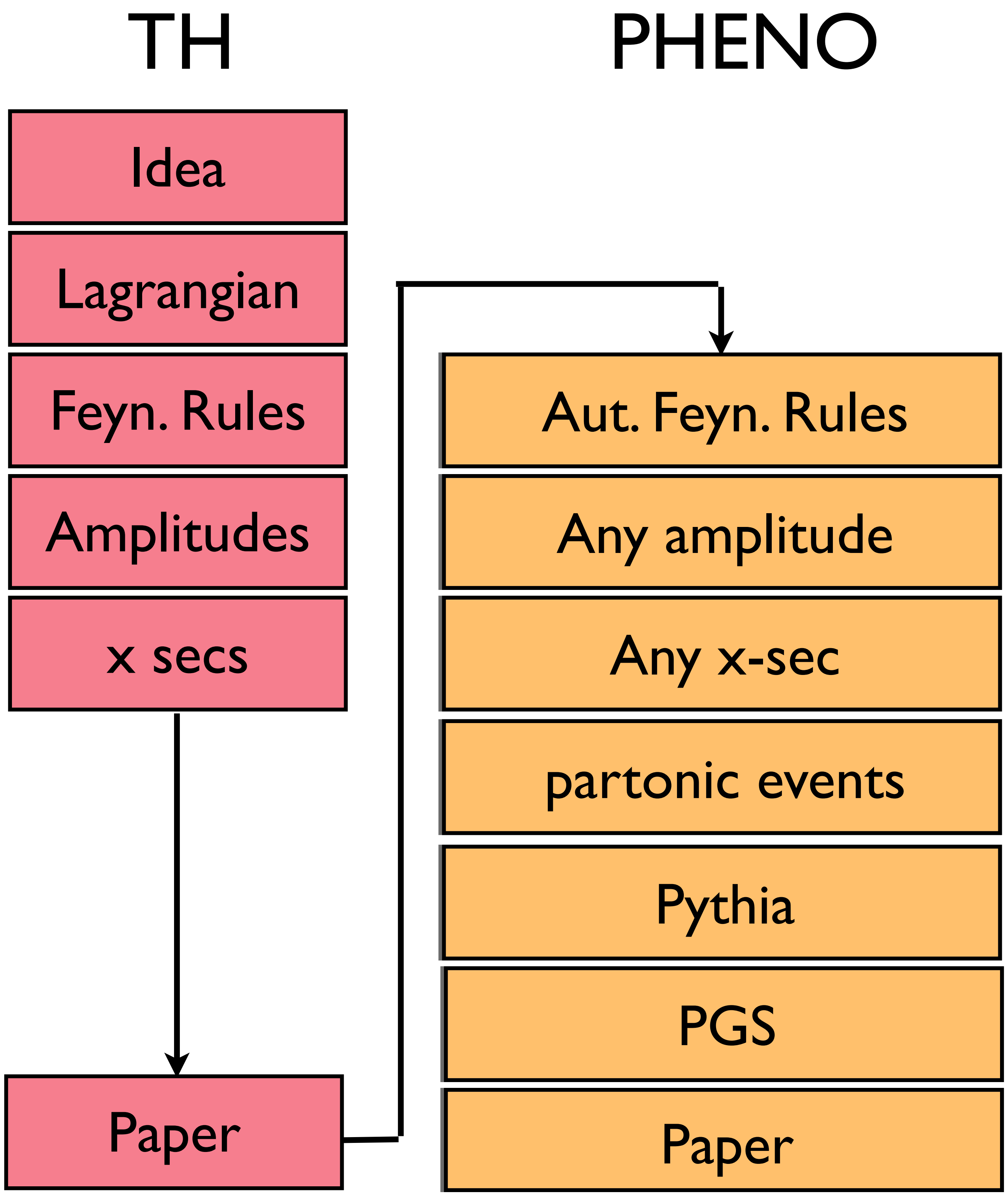
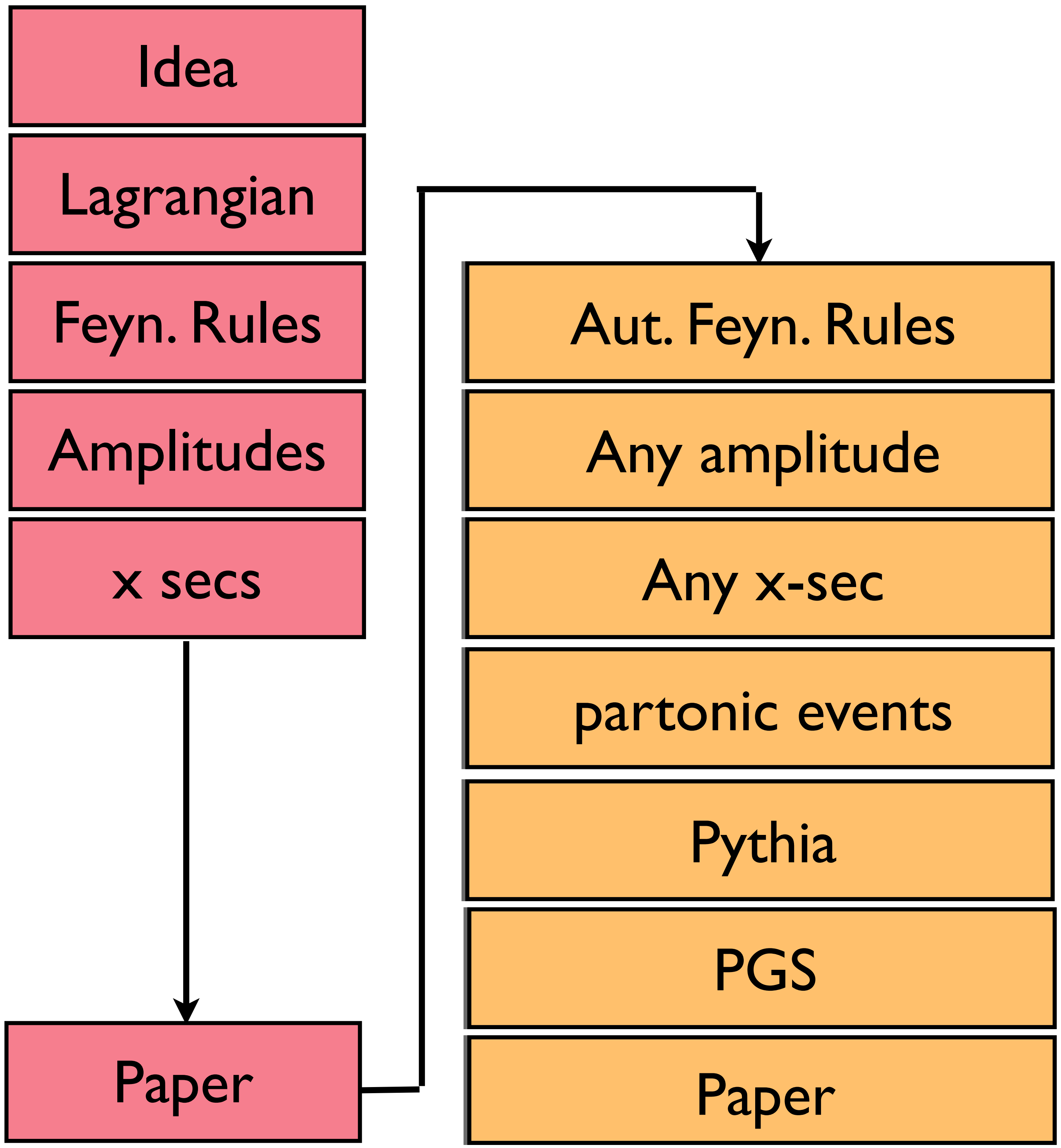


A Roadmap (with roadblocks) for BSM @ the LHC



A Roadmap (with roadblocks) for BSM @ the LHC

TH PHENO EXP



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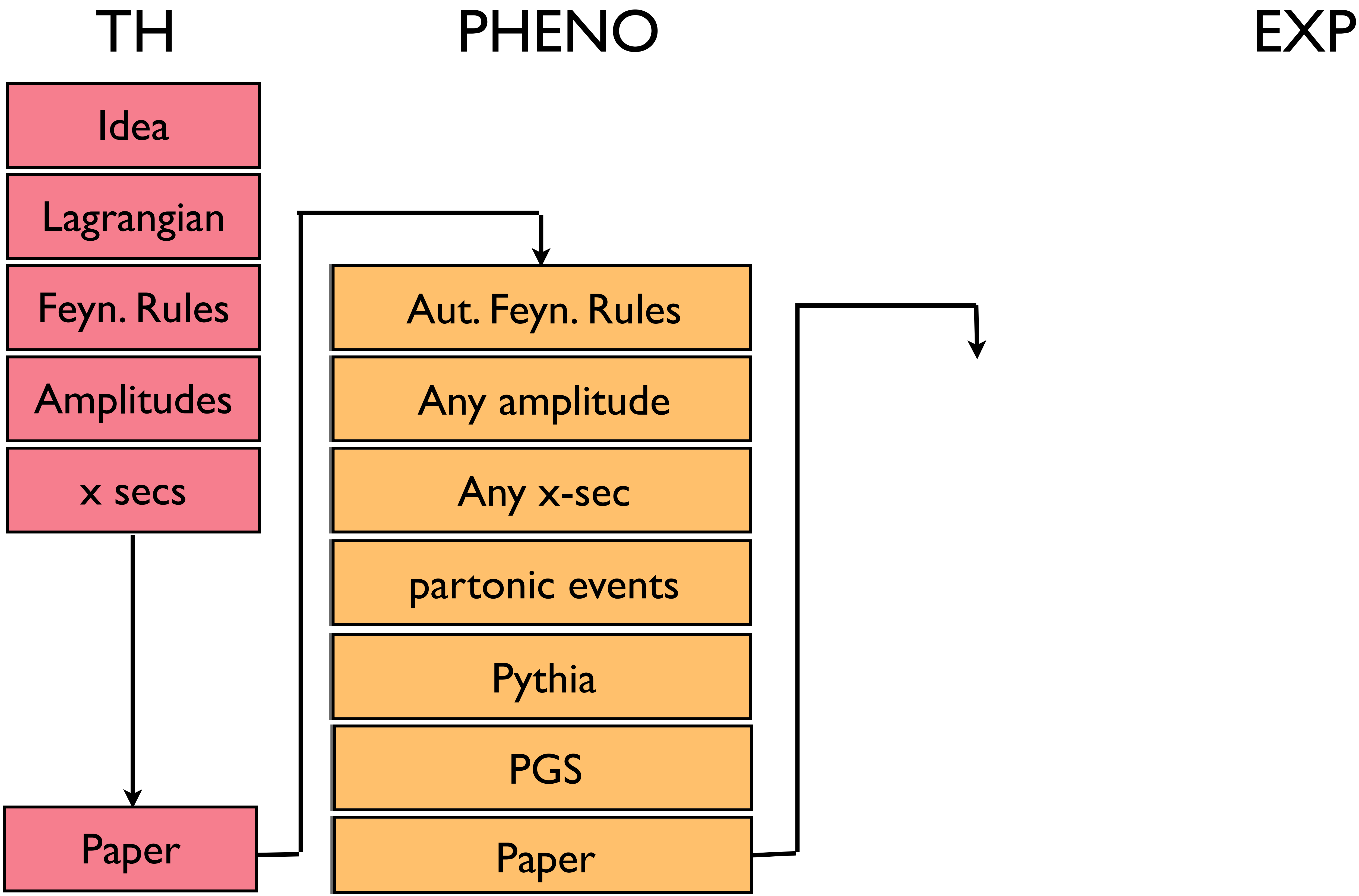

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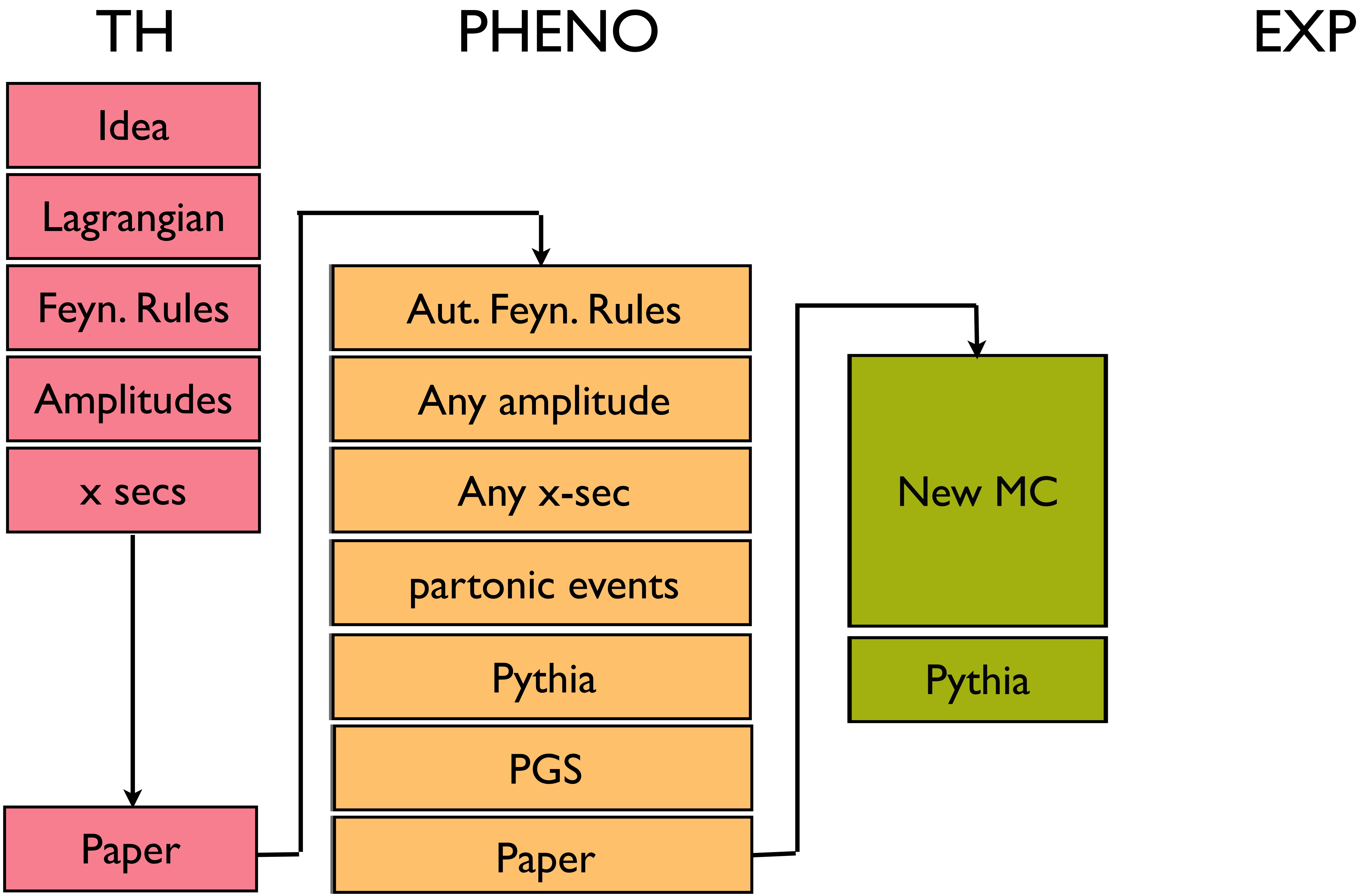

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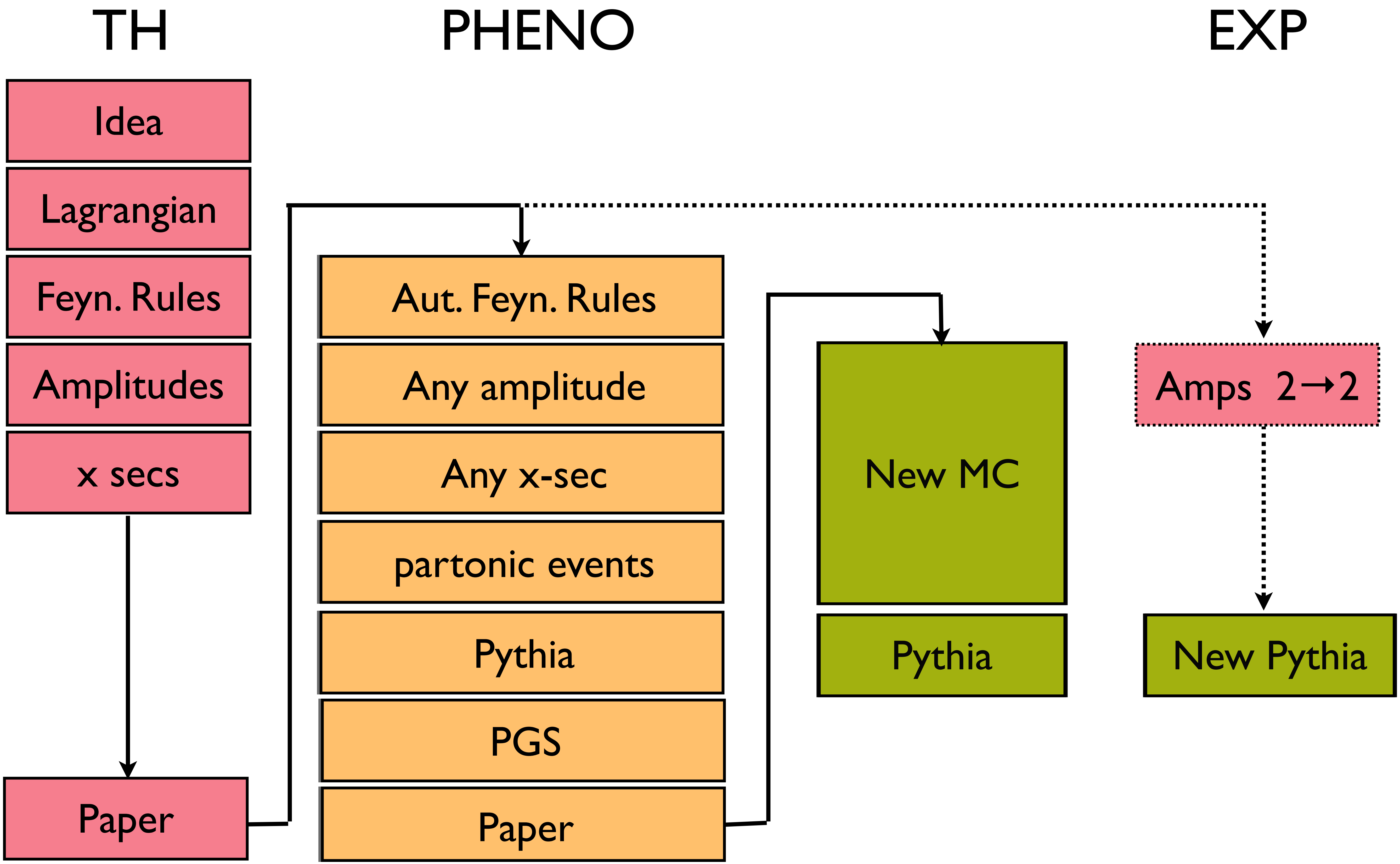

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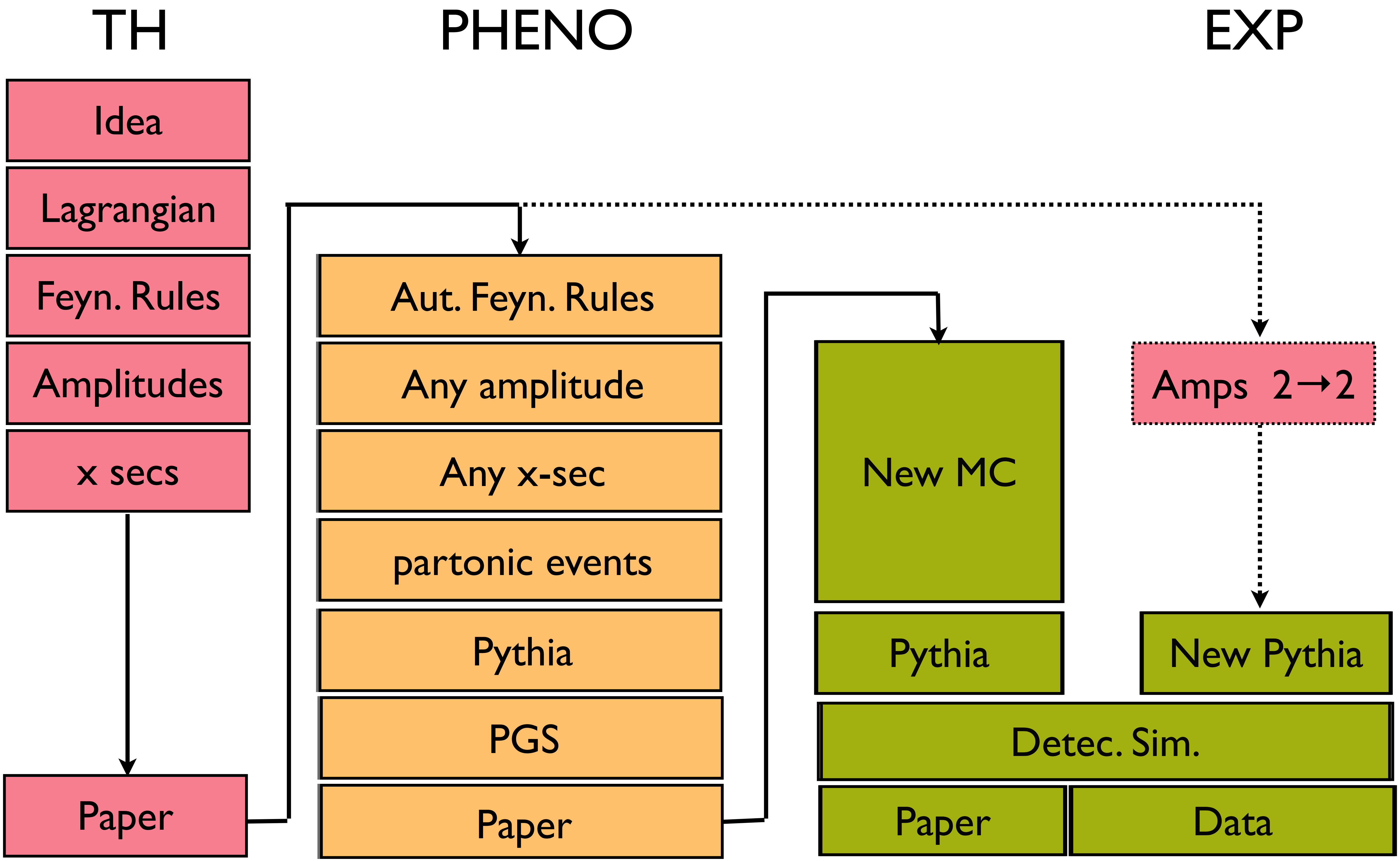
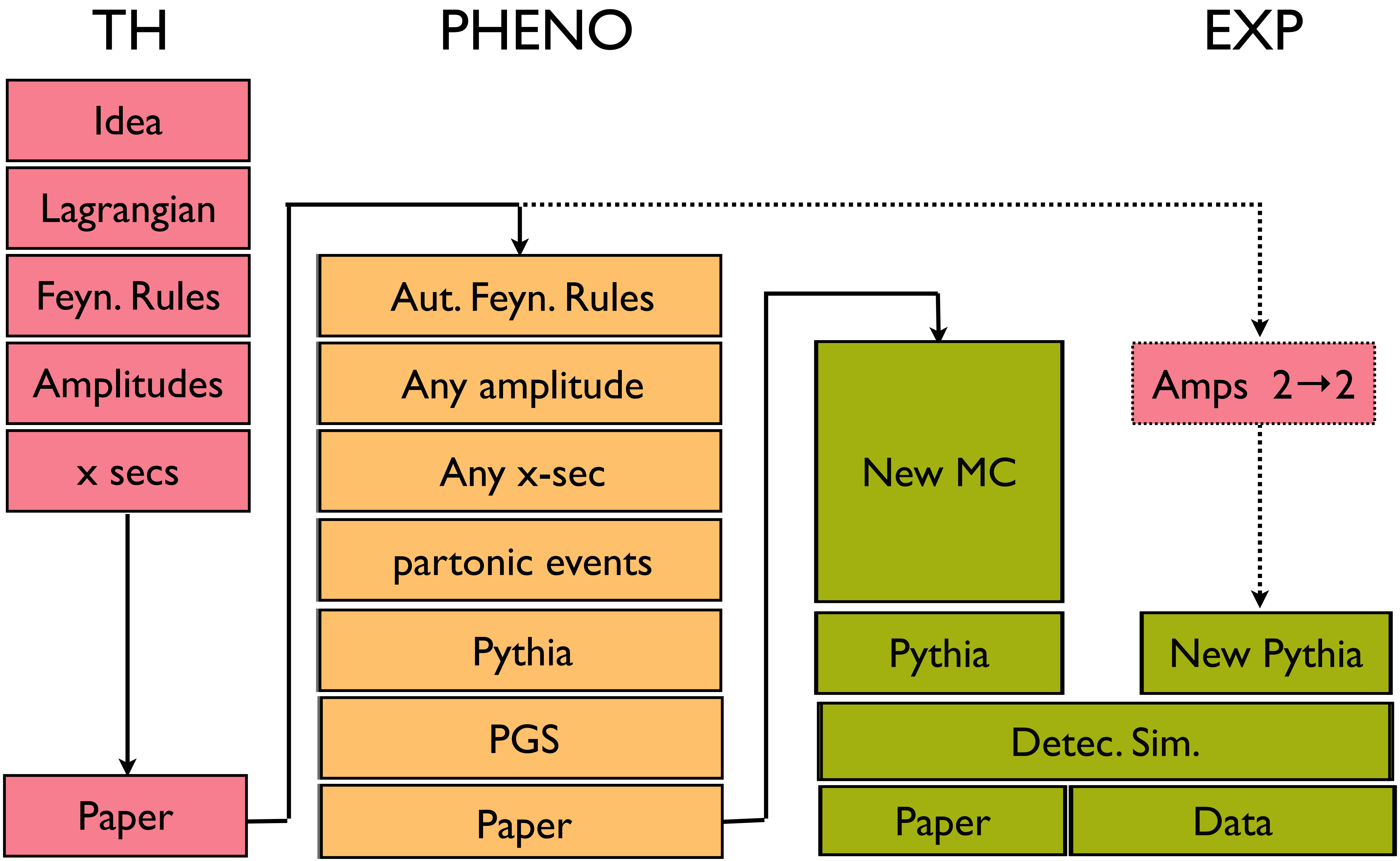


A Roadmap (with roadblocks) for BSM @ the LHC

- Workload is tripled!
- Long delays due to localized expertises and error prone. Painful validations are necessary at each step.
- It leads to a proliferation of private MC tools/ sample productions impossible to maintain, document and reproduce on the mid- and long-term.
- Just publications is a very inefficient way of communicating between TH/PHENO/EXP.

A Roadmap (with roadblocks) for BSM @ the LHC



A Roadmap (with roadblocks) for BSM @ the LHC

TH

PHENO

EXP

Idea

Lagrangian

Aut. Feyn. Rules

Any amplitude

Any x-sec

partonic events

Pythia

Detec. Sim.

Data

A Roadmap for BSM @ the LHC

TH

EXP

Idea

Lagrangian

FeynRules

ME Generator

Signal & Bkg

Events

Pythia

PGS

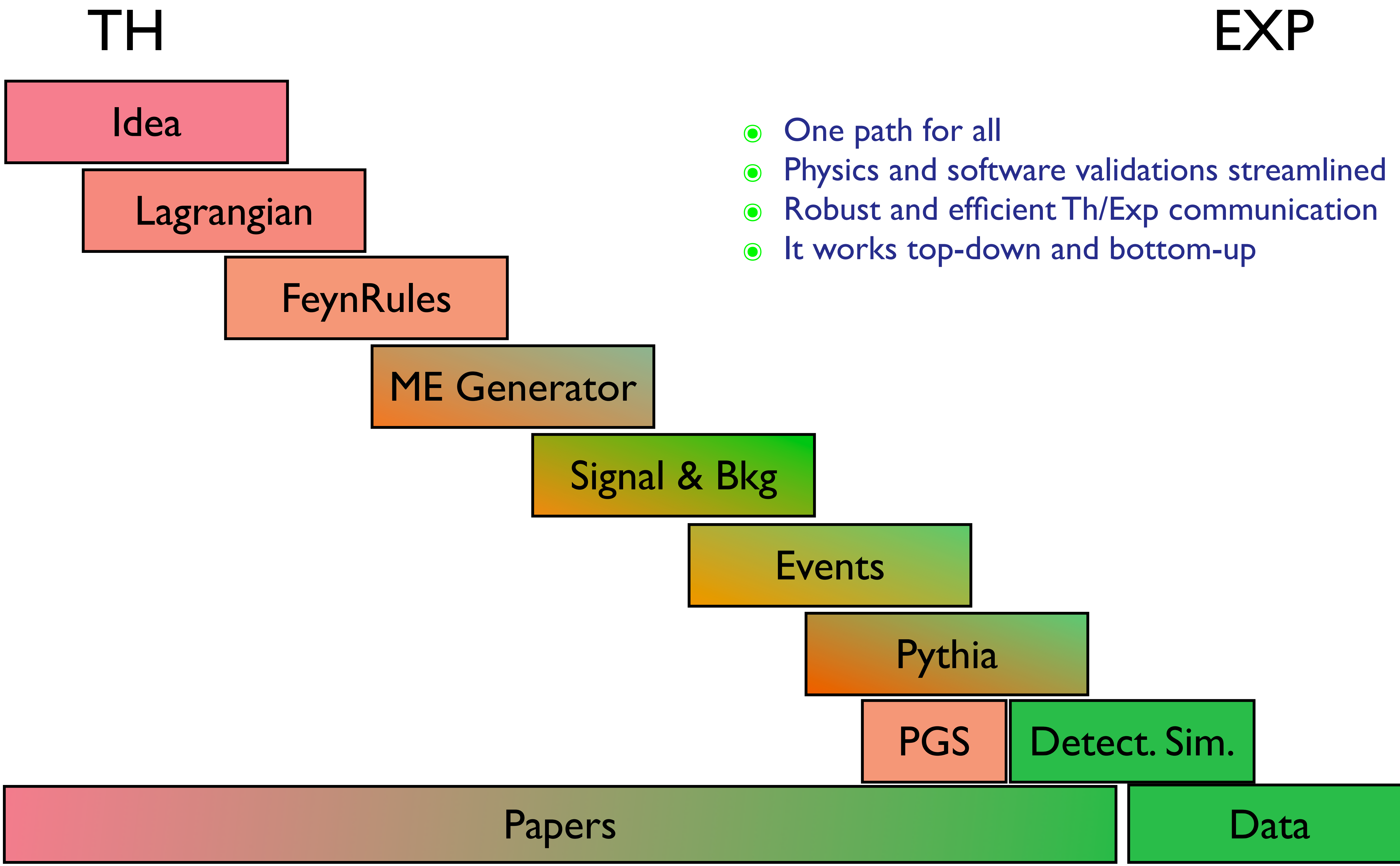
Detect. Sim.

Papers

Data

A Roadmap for BSM @ the LHC

- One path for all
- Physics and software validations streamlined
- Robust and efficient Th/Exp communication
- It works top-down and bottom-up



A Roadmap for BSM @ the LHC

TH

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Detect. Sim.

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Data

- One path for all
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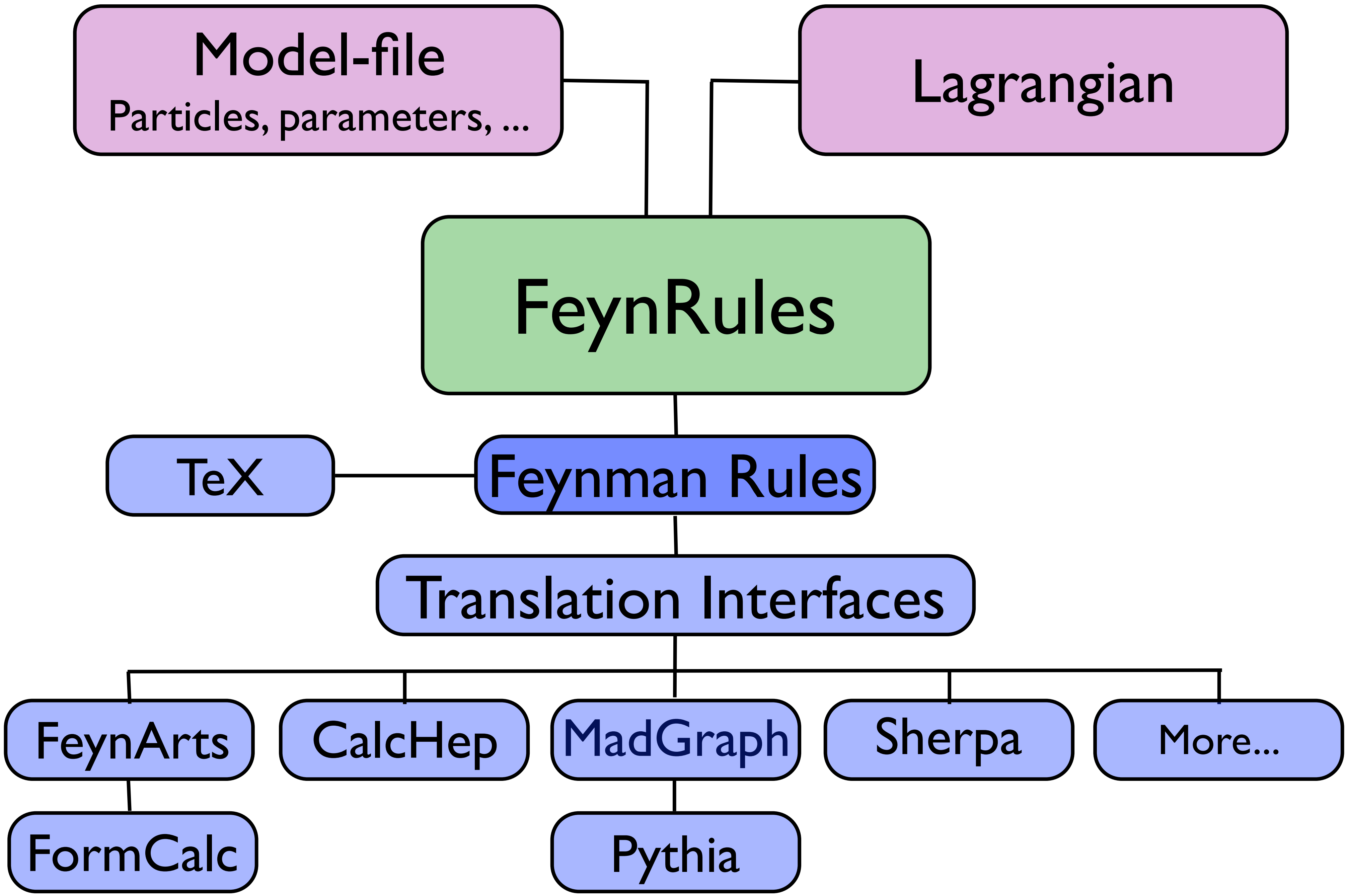
Complete automatization for tree-level based calculations available, including merging with the parton shower in multi-jet final states, for SM as well as for BSM physics. Automatization of NLO is very promising now...

The FeynRules Project

[Christensen, Duhr, 2008; Christensen, et al.2009]



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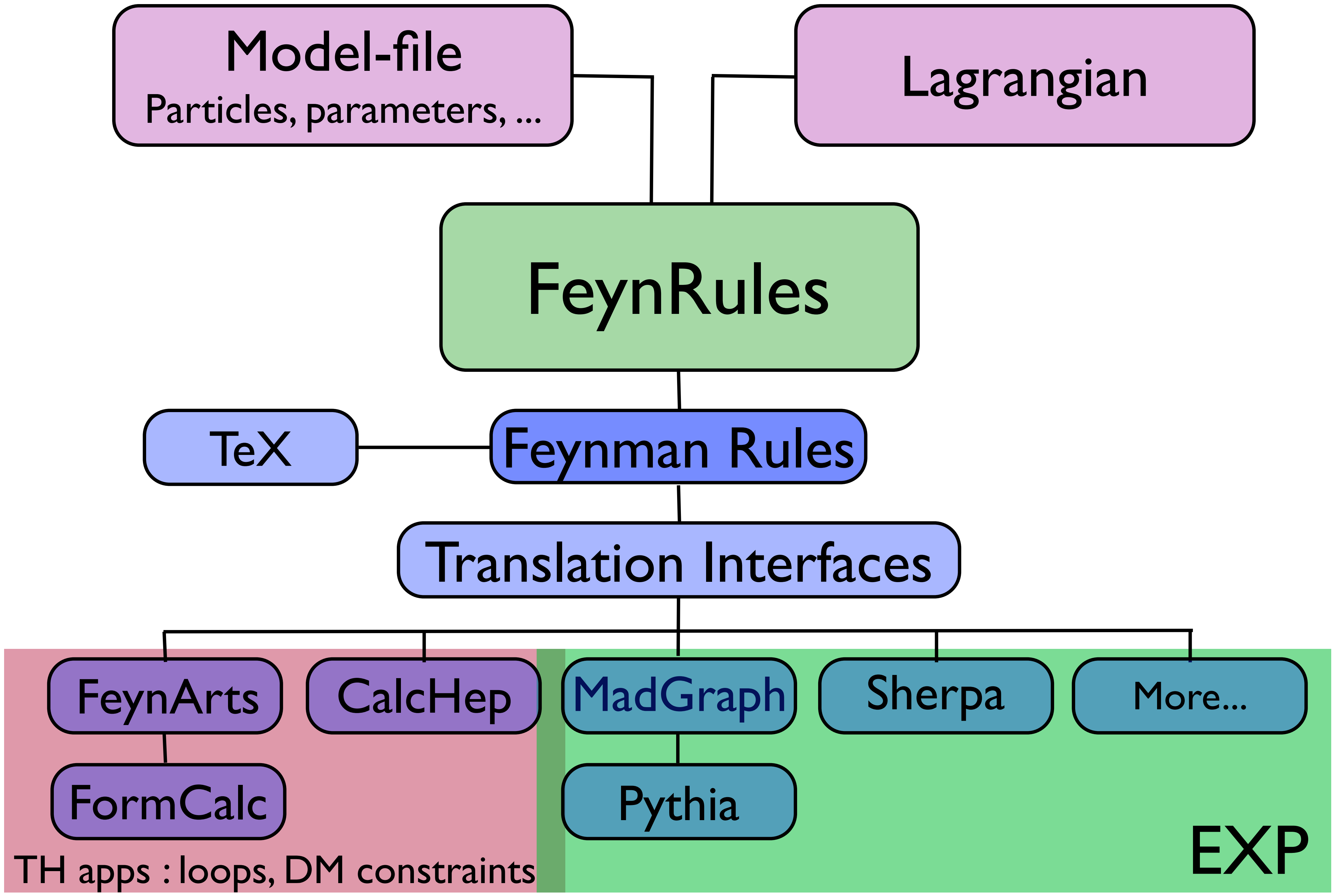

Conclusions

- The need for better description and more reliable predictions for SM processes for the LHC has motivated a significant increase of theoretical and phenomenological activity in the last years, leading to several important achievements.
- A new generation of tools and techniques has been is available. Among the most useful is the matching between fixed-order and parton-shower both at tree-level and at NLO.
- Fully efficient and flexible BSM simulation chain being completed. Same level of sophistication as SM processes attained.
- Shift in paradigm: useful TH predictions in the form of tools that can be used by EXP's. Communication and collaboration between THs & EXPs easier $\Rightarrow$ emergence of an integrated LHC community.