W_{ADS}: mass terms

masses for all flavors

$$W_{\text{exact}} = C_{N,F} \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{1/(N-F)} + m^i_j M^j_i$$

where m^i_j is the quark mass matrix. Equation of motion for M

$$M^j_i \quad = \quad (m^{-1})^j_i \left(\frac{\Lambda^{3N-F}}{\det M} \right)^{1/(N-F)} \qquad\qquad (**)$$

taking the determinant and plugging the result back in to (**) gives

$$\bar{\Phi}^j \Phi_i = M^j_i = (m^{-1})^j_i \left(\det m \, \Lambda^{3N-F} \right)^{1/N}$$

SUSY QCD for $F \geq N$

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Φ, Q	$\square$	$\square$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{\Phi}$, $\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	-1	$\frac{F-N}{F}$

$$\text{define } d^n_m \equiv \langle \Phi^{*in} \Phi_{mi} \rangle$$
$$\overline{d}^n_m = \langle \overline{\Phi}^{in} \overline{\Phi}^*_{mi} \rangle$$

maximal rank N. SUSY vacua are D-flat:

$$D^a = T^{am}_n (d^n_m - \overline{d}^n_m) = 0$$

Flat directions $F \geq N$

d^n_m and $\overline{d}^n_m$ are $N \times N$ positive semi-definite Hermitian matrices of maximal rank N in a SUSY vacuum :

$$d^n_m - \overline{d}^n_m = \rho I \ .$$

d^n_m can be diagonalized by an $SU(N)$ gauge transformation:

$$d = \begin{pmatrix} |v_1|^2 & & & \\ & |v_2|^2 & & \\ & & \ddots & \\ & & & |v_N|^2 \end{pmatrix}$$

In this basis, $\overline{d}^n_m$ must also be diagonal, with eigenvalues $|\overline{v}_i|^2$, so

$$|v_i|^2 = |\overline{v}_i|^2 + \rho \ .$$

Flat directions $F \geq N$

Since d^n_m and $\overline{d}^n_m$ are invariant under flavor transformations, we can use $SU(F) \times SU(F)$ transformations to put $\langle \Phi \rangle$ and $\langle \overline{\Phi} \rangle$ in the form

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & & & 0 & \ldots & 0 \\ & \ddots & & \vdots & & \vdots \\ & & v_N & 0 & \ldots & 0 \end{pmatrix} , \quad \langle \overline{\Phi} \rangle = \begin{pmatrix} \overline{v}_1 & & \\ & \ddots & \\ & & \overline{v}_N \\ 0 & \ldots & 0 \\ \vdots & & \vdots \\ 0 & \ldots & 0 \end{pmatrix} .$$

At a generic point in the moduli space the $SU(N)$ is completely broken vacua are physically distinct, different VEVs correspond to different masses for the gauge bosons

Classical moduli space for $F \geq N$

generic point in the moduli space: $SU(N)$ completely broken
$2NF - (N^2 - 1)$ massless chiral supermultiplets

gauge-invariant description "mesons," "baryons" and superpartners:

$$
\begin{aligned}
M_i^j &= \overline{\Phi}^{jn} \Phi_{ni} \\
B_{i_1,\ldots,i_N} &= \Phi_{n_1 i_1} \ldots \Phi_{n_N i_N} \epsilon^{n_1,\ldots,n_N} \\
\overline{B}^{i_1,\ldots,i_N} &= \overline{\Phi}^{n_1 i_1} \ldots \overline{\Phi}^{n_N i_N} \epsilon_{n_1,\ldots,n_N}
\end{aligned}
$$

constraints relate M and B, since the M has F^2 components, B and $\overline{B}$ each have $\begin{pmatrix} F \\ N \end{pmatrix}$ components, and all three constructed out of the same $2NF$ underlying squark fields
classically

$$
B_{i_1,\ldots,i_N} \overline{B}^{j_1,\ldots,j_N} = M_{[i_1}^{j_1} \ldots M_{i_N]}^{j_N}
$$

where $[\,]$ denotes antisymmetrization

Classical moduli space for $F \geq N$

up to flavor transformations:

$$\begin{aligned}
\langle M \rangle &= \begin{pmatrix} v_1\overline{v}_1 & & & & & \\ & \ddots & & & & \\ & & v_N\overline{v}_N & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix} \\
\langle B_{1,\ldots,N} \rangle &= v_1 \ldots v_N \\
\langle \overline{B}^{1,\ldots,N} \rangle &= \overline{v}_1 \ldots \overline{v}_N
\end{aligned}$$

all other components set to zero
rank $M \leq N$, if less than N, then B or $\overline{B}$ (or both) vanish
if the rank of M is k, then $SU(N)$ is broken to $SU(N-k)$
with $F-k$ massless flavors

Quantum moduli space for $F \geq N$

classical constraints between M, B, and $\overline{B}$ may be modified

parameterize the quantum moduli space by M, B, and $\overline{B}$
VEVs $\gg \Lambda$ perturbative regime
M, B, and $\overline{B} \to 0$ strong coupling
naively expect a singularity from gluons becoming massless

IR fixed points

$F \geq 3N$ lose asymptotic freedom: weakly coupled low-energy effective theory

For F just below $3N$ we have an IR fixed point (Banks-Zaks)

exact NSVZ β function:

$$\beta(g) \quad = \quad -\tfrac{g^3}{16\pi^2}\tfrac{(3N-F(1-\gamma))}{1-Ng^2/8\pi^2}$$

where γ is the anomalous dimension of the quark mass term

$$\gamma \quad = \quad -\tfrac{g^2}{8\pi^2}\tfrac{N^2-1}{N} + \mathcal{O}(g^4)$$

$$16\pi^2\beta(g) \quad = \quad -g^3\,(3N-F) - \tfrac{g^5}{8\pi^2}\left(3N^2 - 2FN + \tfrac{F}{N}\right) + \mathcal{O}(g^7)$$

IR fixed points

Large N with $F = 3N - \epsilon N$

$$16\pi^2\beta(g) \quad = \quad -g^3\epsilon N - \tfrac{g^5}{8\pi^2}\left(3(N^2-1) + \mathcal{O}(\epsilon)\right) + \mathcal{O}(g^7)$$

approximate solution of $\beta = 0$ where there first two terms cancel at

$$g_*^2 = \tfrac{8\pi^2}{3}\tfrac{N}{N^2-1}\,\epsilon$$

$\mathcal{O}(g^7)$ terms higher order in ϵ

without masses, gauge theory is scale-invariant for $g = g_*$

scale-invariant theory of fields with spin ≤ 1 is conformally invariant

SUSY algebra $\rightarrow$ superconformal algebra

particular R-charge enters the superconformal algebra, denote by $R_{\rm sc}$ dimensions of scalar component of gauge-invariant chiral and antichiral superfields:

$$\begin{aligned} d &= \tfrac{3}{2}R_{\rm sc}, \quad \text{for chiral superfields} \\ d &= -\tfrac{3}{2}R_{\rm sc}, \quad \text{for antichiral superfields} \end{aligned}$$

Chiral Ring

charge of a product of fields is the sum of the individual charges:

$$R_{\rm sc}[\mathcal{O}_1\mathcal{O}_2] = R_{\rm sc}[\mathcal{O}_1] + R_{\rm sc}[\mathcal{O}_2]$$

so for chiral superfields dimensions simply add:

$$D[\mathcal{O}_1\mathcal{O}_2] = D[\mathcal{O}_1] + D[\mathcal{O}_2]$$

More formally we can say that the chiral operators form a chiral ring.

ring: set of elements on which addition and multiplication are defined, with a zero and an a minus sign

in general, the dimension of a product of fields is affected by renormalizations that are independent of the renormalizations of the individual fields

Fixed Point Dimensions

R-symmetry of a SUSY gauge theory seems ambiguous since we can always form linear combinations with other $U(1)$'s

for the fixed point of SUSY QCD, $R_{\rm sc}$ is unique since we must have

$$R_{\rm sc}[Q] = R_{\rm sc}[\overline{Q}]$$

denote the anomalous dimension at the fixed point by γ_* then

$$D[M] = D[\Phi\overline{\Phi}] = 2 + \gamma_* \quad = \quad \tfrac{3}{2} 2 \tfrac{(F-N)}{F} = 3 - \tfrac{3N}{F}$$

and the anomalous dimension of the mass operator at the fixed point is

$$\gamma_* = 1 - \tfrac{3N}{F}$$

check that the exact β function vanishes:

$$\beta \propto 3N - F(1 - \gamma_*) = 0$$

Fixed Point Dimensions

For a scalar field in a conformal theory we also have

$$D(\phi) \geq 1 \; ,$$

with equality for a free field
Requiring $D[M] \geq 1 \Rightarrow$

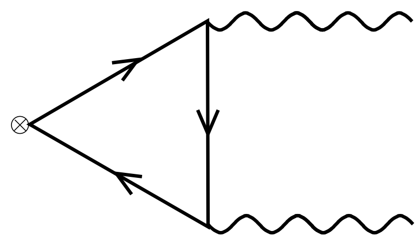
$$F \geq \tfrac{3}{2} N$$

IR fixed point (non-Abelian Coulomb phase) is an interacting conformal theory for $\frac{3}{2}N < F < 3N$

no particle interpretation, but anomalous dimensions are physical quantities

Anomalies: triangle diagram

fermion triangle with the global current and two gauge currents



linearly divergent: depends on how the momentum is routed

$$\int d^4k \, \frac{k^\mu}{(k-p)^2-m^2} = -\frac{i\pi^2}{2} p^\mu + \int d^4k \, \frac{k^\mu + p^\mu}{k^2-m^2} \; .$$

-gauge invariance fixes the correct choice of loop momentum.
-contracting with external momenta gives a nonzero result
-the global current is not conserved

‘t Hooft

‘t Hooft’s anomaly matching

asymptotically free gauge theory, with a global symmetry group G compute the anomaly for three global G currents in the UV: A^{UV}

imagine that we weakly gauge G with a gauge coupling $g \ll 1$
If $A^{UV} \neq 0$, add spectators that only have G gauge couplings, such that their G anomaly is $A^S = -A^{UV}$

construct the effective theory at a scale below strong interaction scale
If G is not spontaneously broken by the strong interactions

$$0 = A^{IR} + A^S$$

Thus

$$A^{IR} = A^{UV}$$

taking $g \to 0$ decouples the weakly coupled gauge bosons but does not change the three-point functions of currents

Seiberg

SUSY QCD for $F \geq N$

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Φ, Q	$\square$	$\square$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{\Phi}$, $\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	-1	$\frac{F-N}{F}$

Duality

conformal theory global symmetries unbroken
'‘t Hooft anomaly matching should apply to low-energy degrees of freedom

anomalies of the M, B, and $\overline{B}$ do not match to quarks and gaugino

Seiberg found a nontrivial solution to the anomaly matching using a "dual" $SU(F-N)$ gauge theory with a "dual" gaugino, "dual" quarks and a gauge singlet "dual mesino":

	$SU(F-N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
q	$\square$	$\overline{\square}$	$\mathbf{1}$	$\frac{N}{F-N}$	$\frac{N}{F}$
$\overline{q}$	$\overline{\square}$	$\mathbf{1}$	$\square$	$-\frac{N}{F-N}$	$\frac{N}{F}$
mesino	$\mathbf{1}$	$\square$	$\overline{\square}$	0	$2\,\frac{F-N}{F}$

Anomaly Matching

global symmetry	anomaly = dual anomaly
$SU(F)^3$	$-(F-N)+F=N$
$U(1)SU(F)^2$	$\frac{N}{F-N}(F-N)\frac{1}{2}=\frac{N}{2}$
$U(1)_R SU(F)^2$	$\frac{N-F}{F}(F-N)\frac{1}{2}+\frac{F-2N}{F}F\frac{1}{2}=-\frac{N^2}{2F}$
$U(1)^3$	$0=0$
$U(1)$	$0=0$
$U(1)U(1)_R^2$	$0=0$
$U(1)_R$	$\left(\frac{N-F}{F}\right)2(F-N)F+\left(\frac{F-2N}{F}\right)F^2+(F-N)^2-1$ $=-N^2-1$
$U(1)_R^3$	$\left(\frac{N-F}{F}\right)^3 2(F-N)F+\left(\frac{F-2N}{F}\right)^3 F^2+(F-N)^2-1$ $=-\frac{2N^4}{F^2}+N^2-1$
$U(1)^2U(1)_R$	$\left(\frac{N}{F-N}\right)^2\frac{N-F}{F}2F(F-N)=-2N^2$

Dual Superpotential

$$W = \lambda \widetilde{M}_i^j \phi_j \overline{\phi}^i$$

where ϕ represents the "dual" squark and $\widetilde{M}$ is the dual meson

ensures that the two theories have the same number of degrees of freedom, $\widetilde{M}$ eqm removes the color singlet $\phi\overline{\phi}$ degrees of freedom

dual baryon operators:

$$\begin{aligned} b^{i_1,\ldots,i_{F-N}} &= \phi^{n_1 i_1} \ldots \phi^{n_{F-N} i_{F-N}} \epsilon_{n_1,\ldots,n_{F-N}} \\ \overline{b}_{i_1,\ldots,i_{F-N}} &= \overline{\phi}_{n_1 i_1} \ldots \overline{\phi}_{n_{F-N} i_{F-N}} \epsilon^{n_1,\ldots,n_{F-N}} \end{aligned}$$

moduli spaces have a simple mapping

$$\begin{aligned} & M \leftrightarrow \widetilde{M} \\ & B_{i_1,\ldots,i_N} \leftrightarrow \epsilon_{i_1,\ldots,i_N,j_1,\ldots j_{F-N}} b^{j_1,\ldots,j_{F-N}} \\ & \overline{B}^{i_1,\ldots,i_N} \leftrightarrow \epsilon^{i_1,\ldots,i_N,j_1,\ldots j_{F-N}} \overline{b}_{j_1,\ldots,j_{F-N}} \end{aligned}$$

Dual β function

$$\beta(\widetilde{g}) \propto -\widetilde{g}^3(3\widetilde{N} - F) = -\widetilde{g}^3(2F - 3N)$$

dual theory loses asymptotic freedom when $F \leq 3N/2$
the dual theory leaves the conformal regime to become IR free at exactly the point where the meson of the original theory becomes a free field

strong coupling $\leftrightarrow$ weak coupling

Integrating out a flavor

give a mass to one flavor

$$W_{\text{mass}} = m\overline{\Phi}^F \Phi_F$$

In dual theory

$$W_d = \lambda \widetilde{M}_i^j \overline{\phi}^i \phi_j + m\mu\lambda \widetilde{M}_F^F$$

common to write

$$\lambda \widetilde{M} = \frac{M}{\mu}$$

trade the coupling λ for a scale μ and use the same symbol, M, for fields in the two different theories

$$W_d = \frac{1}{\mu} M_i^j \overline{\phi}^i \phi_j + mM_F^F$$

Integrating out a flavor

The equation of motion for M^F_F is:

$$\frac{\partial W_d}{\partial M^F_F} = \frac{1}{\mu}\overline{\phi}^F \phi_F + m = 0$$

dual squarks have VEVs:

$$\overline{\phi}^F \phi_F = -\mu m$$

along such a D-flat direction we have a theory with one less color, one less flavor, and some singlets

Integrating out a flavor

	$SU(F-N-1)$	$SU(F-1)$	$SU(F-1)$
q'	$\square$	$\overline{\square}$	$\mathbf{1}$
$\overline{q}'$	$\overline{\square}$	$\mathbf{1}$	$\square$
M'	$\mathbf{1}$	$\square$	$\overline{\square}$
q''	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$
$\overline{q}''$	$\mathbf{1}$	$\mathbf{1}$	$\square$
S	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
M_j^F	$\mathbf{1}$	$\square$	$\mathbf{1}$
M_F^j	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$
M_F^F	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

$$W_{\rm eff} = \frac{1}{\mu}\left(\langle\overline{\phi}^F\rangle M_F^j\phi_j'' + \langle\phi_F\rangle M_i^F\overline{\phi}''^i + M_F^F S\right) + \frac{1}{\mu}M'\overline{\phi}'\phi'$$

integrate out M_F^j, ϕ_j'', M_i^F, $\overline{\phi}''^i$, M_F^F, and S since, leaves just the dual of $SU(N)$ with $F-1$ flavors which has a superpotential

$$W = \frac{1}{\mu}M'\overline{\phi}'\phi'$$

Consistency Checks

- global anomalies of the quarks and gauginos match those of the dual quarks, dual gauginos, and "mesons."

- Integrating out a flavor gives $SU(N)$ with $F-1$ flavors, with dual $SU(F-N-1)$ and $F-1$ flavors. Starting with the dual of the original theory, the mapping of the mass term is a linear term for the "meson" which forces the dual squarks to have a VEV and Higgses the theory down to $SU(F-N-1)$ with $F-1$ flavors.

- The moduli spaces have the same dimensions and the gauge invariant operators match.

Classically, the final consistency check is not satisfied

Consistency Checks

moduli space of complex dimension

$$2FN - (N^2 - 1)$$

$2FN$ chiral superfields and $N^2 - 1$ complex D-term constraints

dual has F^2 chiral superfields (M) and the equations of motion set the dual squarks to zero when M has rank F

duality: weak $\leftrightarrow$ strong also classical $\leftrightarrow$ quantum

original theory: rank$(M) \leq N$ classically

dual theory: $F_{eff} = F -$ rank(M) light dual quarks

If rank$(M) > N$ then $F_{eff} < \widetilde{N} = F - N$, $\Rightarrow$ ADS superpotential $\Rightarrow$ no vacuum with rank$(M) > N$

in dual, rank$(M) \leq N$ is enforced by nonperturbative quantum effects

Consistency Checks

rank constraint $\Rightarrow$ number of complex dof in M is $F^2 - \widetilde{N}^2$ since rank N $F \times F$ matrix can be written with an $(F-N) \times (F-N)$ block set to zero.

when M has N large eigenvalues, $F_{eff} = \widetilde{N}$ light dual quarks
$2F_{eff}\widetilde{N} - (\widetilde{N}^2 - 1) = \widetilde{N}^2 + 1$ complex dof
M eqm removes $\widetilde{N}^2$ color singlet dof
dual quark equations of motion enforce that an $\widetilde{N} \times \widetilde{N}$ corner of M is set to zero

two moduli spaces match:

$$2FN - (N^2 - 1) = F^2 - \widetilde{N}^2 + \widetilde{N}^2 + 1 - \widetilde{N}^2 = F^2 - \widetilde{N}^2 + 1$$

once nonperturbative effects are taken into account

$F = N$: confinement with χSB

For $F = N$ 't Hooft anomaly matching works with just M, B, and $\overline{B}$ confining: all massless degrees of freedom are color singlet particles

For $F = N$ flavors the baryons are flavor singlets:

$$\begin{aligned} B &= \epsilon^{i_1,\ldots,i_F} B_{i_1,\ldots,i_F} \\ \overline{B} &= \epsilon_{i_1,\ldots,i_F} \overline{B}^{i_1,\ldots,i_F} \end{aligned}$$

classical constraint:

$$\det M = B\overline{B}$$

With quark masses:

$$\langle M_i^j \rangle = (m^{-1})_i^j \left(\det m \Lambda^{3N-F}\right)^{1/N}$$

Confinement with χSB

Taking a determinant of this equation (using $F = N$)

$$\det\langle M\rangle = \det\left(m^{-1}\right)\det m\Lambda^{2N} = \Lambda^{2N}$$

independent of the masses

$\det m \neq 0$ sets $\langle B\rangle = \langle\overline{B}\rangle = 0$, can integrate out all the fields that have baryon number

classical constraint is violated!

Holomorphy and the Symmetries

flavor invariants are:

	$U(1)_A$	$U(1)$	$U(1)_R$
$\det M$	$2N$	0	0
B	N	N	0
$\overline{B}$	N	$-N$	0
Λ^{2N}	$2N$	0	0

R-charge of the squarks, $(F-N)/F$, vanishes since $F = N$ generalized form of the constraint with correct $\Lambda \to 0$ and B,$\overline{B} \to 0$ limits is

$$\det M - \overline{B}B = \Lambda^{2N}\left(1 + \sum_{pq} C_{pq} \frac{\left(\Lambda^{2N}\right)^p (\overline{B}B)^q}{(\det M)^{p+q}}\right)$$

with $p, q > 0$. For $\langle \overline{B}B \rangle \gg \Lambda^{2N}$ the theory is perturbative, but with $C_{pq} \neq 0$ we find solutions of the form

$$\det M \approx \left(\overline{B}B\right)^{(q-1)/(p+q)}$$

which do not reproduce the weak coupling $\Lambda \to 0$ limit

Quantum Constraint

For $F = N$:

$$\det M - \overline{B} B = \Lambda^{2N}$$

Instanton Action

The Euclidean action of an instanton configuration can be bounded

$$0 \quad \leq \quad \int d^4x Tr \left(F_{\mu\nu} \pm \widetilde{F}_{\mu\nu} \right)^2 = \int d^4x Tr \left(2F^2 \pm 2F\widetilde{F} \right)$$

$$\int d^4x Tr F^2 \geq | \int d^4x Tr F \widetilde{F} | = 16\pi^2 |n|$$

one instanton effects are suppressed by

$$e^{-S_{\rm int}} = e^{-(8\pi^2/g^2(\mu)) + i\theta_{\rm YM}} = \left(\frac{\Lambda}{\mu} \right)^b$$

Quantum Constraint

For $F = N$:

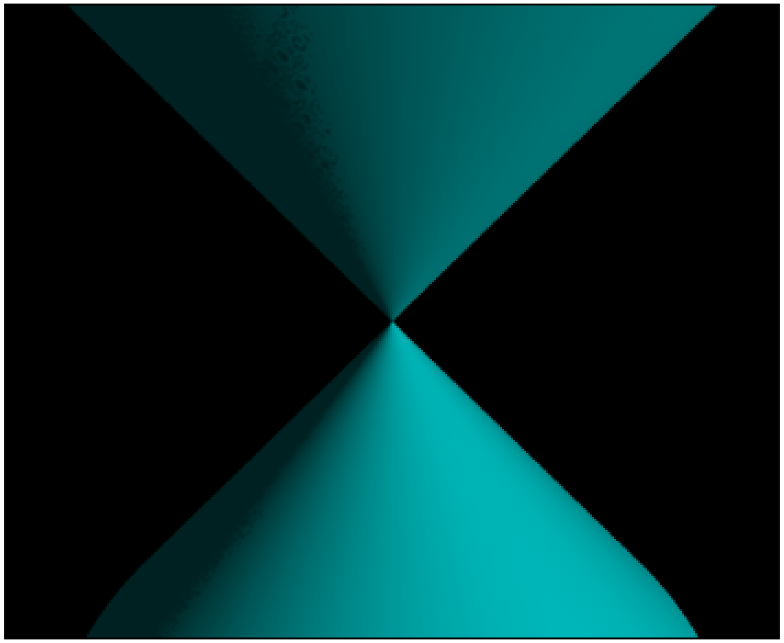
$$\det M - \overline{B} B = \Lambda^{2N}$$

correct form to be an instanton effect

$$e^{-S_{\mathrm{inst}}} \propto \Lambda^b = \Lambda^{2N}$$

Quantum Constraint

cannot take $M = B = \overline{B} = 0$

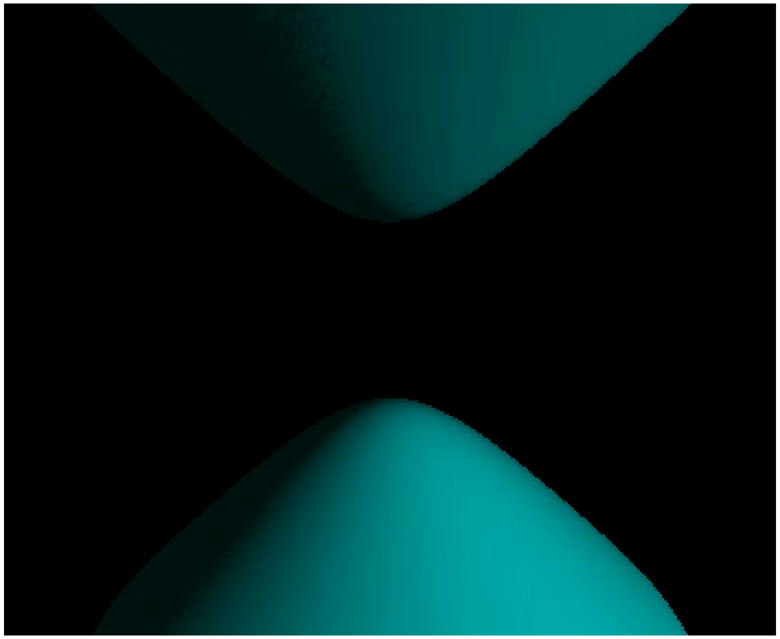


cannot go to the origin of moduli space ("deformed" moduli space)

global symmetries are at least partially broken everywhere

Quantum Constraint

cannot take $M = B = \overline{B} = 0$



cannot go to the origin of moduli space (“deformed” moduli space)

global symmetries are at least partially broken everywhere

Enhanced Symmetry Points

$M_i^j = \Lambda^2 \delta_i^j$, $B = \overline{B} = 0$

$SU(F) \times SU(F) \times U(1) \times U(1)_R \to SU(F)_d \times U(1) \times U(1)_R$

chiral symmetry breaking, as in non-supersymmetric QCD

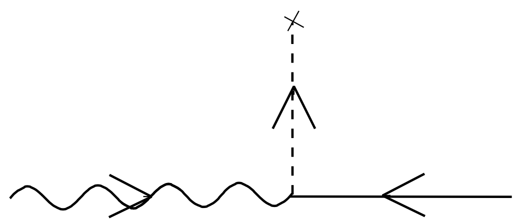
$M = 0$, $B\overline{B} = -\Lambda^{2N}$

$SU(F) \times SU(F) \times U(1) \times U(1)_R \to SU(F) \times SU(F) \times U(1)_R$

baryon number spontaneously broken

Smooth Moduli Space

For large VEVs : perturbative Higgs phase, squark VEVs give masses to quarks and gauginos



no point in the moduli space where gluons become light
$\Rightarrow$ no singular points

theory exhibits "complementarity": can go smoothly from a Higgs phase (large VEVs) to a confining phase (VEVs of $\mathcal{O}(\Lambda)$) without going through a phase transition

Enhanced Symmetry Point

$M_i^j = \Lambda^2 \delta_i^j$, $B = \overline{B} = 0$
Φ and $\overline{\Phi}$ VEVs break $SU(N) \times SU(F) \times SU(F) \to SU(F)_d$
quarks transform as $\Box \times \overline{\Box} = \mathbf{1} + \mathbf{Ad}$ under $SU(F)_d$
gluino transforms as $\mathbf{Ad}$ under $SU(F)_d$

	$SU(F)_d$	$U(1)$	$U(1)_R$
$M - \mathrm{Tr}M$	$\mathbf{Ad}$	0	0
$\mathrm{Tr}M$	$\mathbf{1}$	0	0
B	$\mathbf{1}$	N	0
$\overline{B}$	$\mathbf{1}$	$-N$	0

$\mathrm{Tr}M$ gets a mass with the Lagrange multiplier field X

Enhanced Symmetry Points: Anomalies

global symmetry	elem. anomaly	=	comp. anomaly
$U(1)^2U(1)_R$	$-2FN$	$=$	$-2N^2$
$U(1)_R$	$-2FN+N^2-1$	$=$	$-(F^2-1)-1-1$
$U(1)_R^3$	$-2FN+N^2-1$	$=$	$-(F^2-1)-1-1$
$U(1)_R SU(F)_d^2$	$-2N+N$	$=$	$-N$

agree because $F=N$

Enhanced Symmetry Points

At $M = 0$, $B\overline{B} = -\Lambda^{2N}$ only the $U(1)$ symmetry is broken

	$SU(F)$	$SU(F)$	$U(1)_R$
M	$\square$	$\overline{\square}$	0
B	$\mathbf{1}$	$\mathbf{1}$	0
$\overline{B}$	$\mathbf{1}$	$\mathbf{1}$	0

linear combination $B + \overline{B}$ gets mass with Lagrange multiplier field X

global symmetry	elem. anomaly	=	comp. anomaly
$SU(F)^3$	N	=	F
$U(1)_R SU(F)^2$	$-N\frac{1}{2}$	=	$-F\frac{1}{2}$
$U(1)_R$	$-2FN + N^2 - 1$	=	$-F^2 - 1$
$U(1)_R^3$	$-2FN + N^2 - 1$	=	$-F^2 - 1$

agree because $F = N$

$F = N + 1$: s-confinement

For $F = N + 1$ 't Hooft anomaly matching works with M, B, and $\overline{B}$ confining

does not require χSB, can go to the origin of moduli space

theory develops a dynamical superpotential

	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
M	$\square$	$\overline{\square}$	0	$\frac{2}{F}$
B	$\overline{\square}$	$\mathbf{1}$	N	$\frac{N}{F}$
$\overline{B}$	$\mathbf{1}$	$\square$	$-N$	$\frac{N}{F}$

For $F = N + 1$ baryons are flavor antifundamentals since they are antisymmetrized in $N = F - 1$ colors:

$$\begin{aligned} B^i &= \epsilon^{i_1,\ldots,i_N,i} B_{i_1,\ldots,i_N} \\ \overline{B}_i &= \epsilon_{i_1,\ldots,i_N,i} \overline{B}^{i_1,\ldots,i_N} \end{aligned}$$

$F = N + 1$: Classical Constraints

$$(M^{-1})^i_j \mathrm{det} M = B^i \overline{B}_j$$
$$M^j_i B^i = M^j_i \overline{B}_j = 0$$

with quark masses:

$$\begin{aligned} \langle M^j_i \rangle &= (m^{-1})^j_i \left(\mathrm{det} m \Lambda^{2N-1}\right)^{1/N} \\ \langle B^i \rangle &= \langle \overline{B}_j \rangle = 0 \end{aligned}$$

taking determinant gives

$$(M^{-1})^i_j \mathrm{det} M = m^i_j \Lambda^{2N-1} \ .$$

Thus, we see that the classical constraint is satisfied as $m^i_j \to 0$
taking limit in different ways covers the classical moduli space
classical and quantum moduli spaces are the same
chiral symmetry remains unbroken at $M = B = \overline{B} = 0$

Most General Superpotential

$$W = \frac{1}{\Lambda^{2N-1}} \left[\alpha B^i M_i^j \overline{B}_j + \beta \mathrm{det} M + \mathrm{det} M \, f\left(\frac{\mathrm{det} M}{B^i M_i^j \overline{B}_j} \right) \right]$$

where f is an as yet unknown function
only $f = 0$ reproduces the classical constraints:

$$\begin{aligned}
\frac{\partial W}{\partial M_i^j} &= \frac{1}{\Lambda^{2N-1}} \left[\alpha B^i \overline{B}_j + \beta (M^{-1})_j^i \mathrm{det} M \right] = 0 \\
\frac{\partial W}{\partial B^i} &= \frac{1}{\Lambda^{2N-1}} \alpha M_i^j \overline{B}_j = 0 \\
\frac{\partial W}{\partial \overline{B}_j} &= \frac{1}{\Lambda^{2N-1}} \alpha B^i M_i^j = 0
\end{aligned}$$

provided that $\beta = -\alpha$

$F = N + 1$ Superpotential

to determine α, add a mass for one flavor

$$W = \frac{\alpha}{\Lambda^{2N-1}} \left[B^i M_i^j \overline{B}_j - \det M \right] + mX$$

$$M \quad = \quad \begin{pmatrix} M'^{i}_{j} & Z^i \\ Y_j & X \end{pmatrix} \;, \quad B = \left(U^i, B' \right) \;, \quad \overline{B} = \begin{pmatrix} \overline{U}_j \\ \overline{B}' \end{pmatrix}$$

$$\begin{aligned}
\frac{\partial W}{\partial Y} &= \frac{\alpha}{\Lambda^{2N-1}} \left(B'\overline{U} - \mathrm{cof}(Y) \right) = 0 \\
\frac{\partial W}{\partial Z} &= \frac{\alpha}{\Lambda^{2N-1}} \left(U\overline{B}' - \mathrm{cof}(Z) \right) = 0 \\
\frac{\partial W}{\partial U} &= \frac{\alpha}{\Lambda^{2N-1}} Z\overline{B}' = 0 \\
\frac{\partial W}{\partial \overline{U}} &= \frac{\alpha}{\Lambda^{2N-1}} B'\overline{Y} = 0 \\
\frac{\partial W}{\partial X} &= \frac{\alpha}{\Lambda^{2N-1}} \left(B'\overline{B}' - \det M' \right) + m = 0
\end{aligned}$$

$F = N + 1$ Superpotential

solution of eqms:

$$\begin{aligned} Y &= Z = U = \overline{U} = 0 \\ \det M' - B'\overline{B}' &= \frac{m\Lambda^{2N-1}}{\alpha} = \frac{1}{\alpha}\Lambda_{N,N}^{2N} \end{aligned}$$

correct quantum constraint for $F = N$ flavors if and only if $\alpha = 1$

Plugging back in superpotential with $m\Lambda^{2N-1} = \Lambda_{N,N}^{2N}$:

$$W_{\rm eff} = \frac{X}{\Lambda^{2N-1}}\left(B'\overline{B}' - \det M' + \Lambda_{N,N}^{2N}\right)$$

Holding $\Lambda_{N,N}$ fixed as $m \to \infty \Rightarrow \Lambda \to 0$
X becomes Lagrange multiplier
reproduce the superpotential for $F = N$

$F = N + 1$ Superpotential

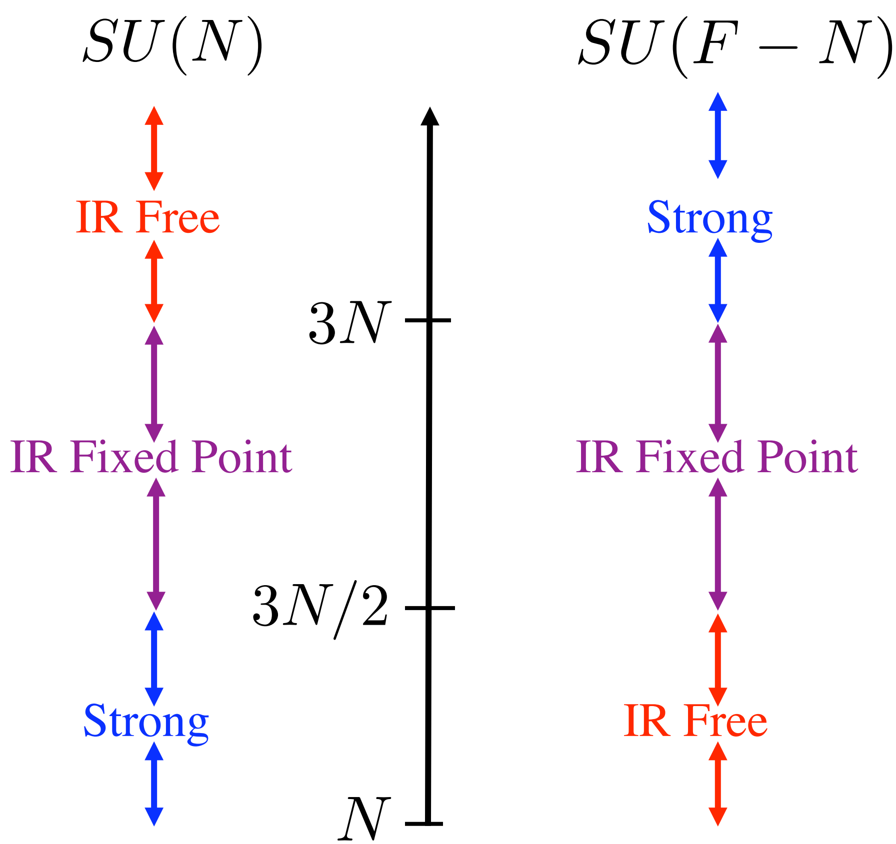
superpotential for confined SUSY QCD with $F = N + 1$ flavors is:

$$W = \frac{1}{\Lambda^{2N-1}} \left[B^i M_i^j \overline{B}_j - \det M \right]$$

$M = B = \overline{B} = 0$ is on the quantum moduli space, possible singular behavior since naively gluons and gluinos should become massless

actually M, B, $\overline{B}$ become massless: confinement without χSB

Duality for SUSY QCD



special cases:

F=N+1 $\rightarrow$ confinement without χSB

F=N $\rightarrow$ confinement with χSB

Toy-Model of EWSB

	$SU(2)_{\rm SC}$	$SU(2)_L$	$SU(2)_R$	$U(1)$	$U(1)_R$
T_L	$\square$	$\square$	$\mathbf{1}$	1	0
T_R	$\square$	$\mathbf{1}$	$\square$	-1	0
H	$\mathbf{1}$	$\square$	$\square$	0	1
S_L	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	-2	2
S_R	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	2	2

$$W = \lambda_L S_L T_L T_L + \lambda_R S_R T_R T_R + \lambda_H H T_L T_R + \frac{1}{2}\mu H H$$

$$U(1)_Y \subset SU(2)_R,\ Y \propto \tau_{3R}$$

Two colors with Two flavors

Confinement

	$SU(2)_L$	$SU(2)_R$	$U(1)$	$U(1)_R$
$\Pi \sim (T_L T_R)$	$\Box$	$\Box$	0	0
$B_L \sim (T_L T_L)$	$\mathbf{1}$	$\mathbf{1}$	2	0
$B_R \sim (T_R T_R)$	$\mathbf{1}$	$\Box$	-2	0
H	$\Box$	$\Box$	0	1
S_L	$\mathbf{1}$	$\mathbf{1}$	-2	2
S_R	$\mathbf{1}$	$\mathbf{1}$	2	2

$$W_{\text{eff}} = f\left[\lambda_L S_L B_L + \lambda_R S_R B_R + \lambda_H H \Pi\right] + \frac{1}{2}\mu H H$$

Confinement with χSB

$$\det(\Pi) - B_L B_R = \frac{1}{2} f^2$$

$$f = \frac{\Lambda}{4\pi}$$

$$W_{\text{eff}} = f\left[\lambda_L S_L B_L + \lambda_R S_R B_R + \lambda_H H \Pi\right] + \frac{1}{2}\mu H H$$

$$\Pi^j{}_k = \frac{1}{\sqrt{2}}(\Pi_0 \mathbf{1}_2 + i\Pi_A \tau_A)^j{}_k$$

$$\det(\Pi) = \frac{1}{2}\left(\Pi_0^2 + \Pi_A \Pi_A\right)$$

$$\Pi_0 = \left(f^2 + 2B_L B_R - \Pi_A \Pi_A\right)^{1/2}$$

Confinement with χSB

equations of motion:

$$
\begin{aligned}
H_0 &= -\frac{\lambda_H f}{\mu}\Pi_0 \\
f\lambda_H \Pi_A &= -\mu H_A \\
H_0 \Pi_A &= H_A \Pi_0
\end{aligned}
$$

3 linear combinations of Π_A and H_A are undetermined: Goldstone Bosons

Fat Higgs

$$W = \lambda_L S_L T_L T_L + \lambda_R S_R T_R T_R + \lambda_H H T_L T_R + \frac{1}{2} y (S_L + S_R) H H$$

$$\begin{aligned}
\langle H_0 \rangle &= \left(\frac{2\lambda_L \lambda_R}{9y^2} \right)^{1/4} f \\
\langle S_L \rangle &= \langle S_R \rangle = \pm \lambda_H \left(\frac{2}{9y^2 \lambda_L \lambda_R} \right)^{1/4} f \\
\langle B_L \rangle &= -\left(\frac{\lambda_R}{18y^2 \lambda_L} \right)^{1/2} f \\
\langle B_R \rangle &= -\left(\frac{\lambda_L}{18y^2 \lambda_R} \right)^{1/2} f
\end{aligned}$$

Luty, Terning, Grant hep-ph/0006224

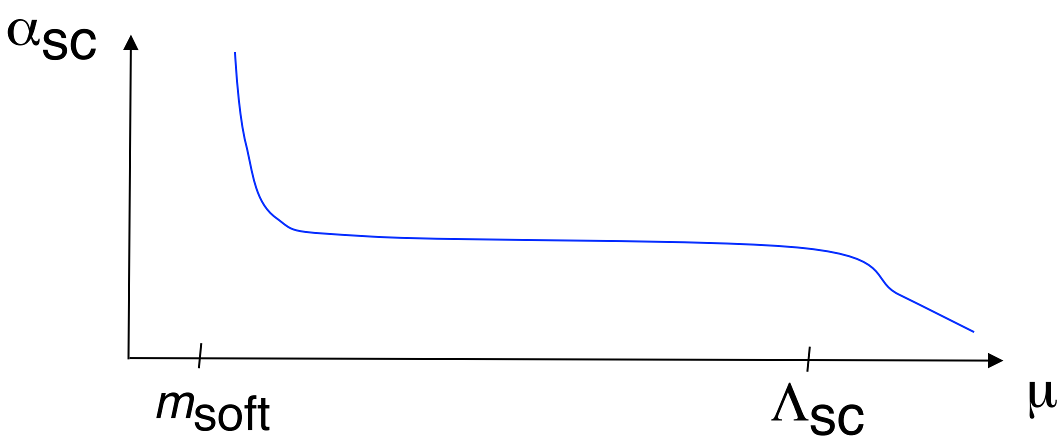
Murayama, Harnik, Kribs, Larsen hep-ph/0311349

Why is $f \approx m_{\mathrm{soft}}$?

$$W = \frac{\lambda}{3} X^3$$

$$V = -m^2_{\mathrm{soft}} X^\dagger X + |\lambda X|^2$$

$$X^\dagger X = \frac{m^2_{\mathrm{soft}}}{\lambda}$$



S-parameter

$$\frac{1}{\Lambda^2}\int d^2\theta\,(W^\alpha)^j{}_k(W_\alpha)^\ell{}_m\epsilon_{j\ell np}M^{kn}M^{mp}+h.c.$$

would contribute to S but cannot appear
due to holomorphy and the weak coupling limit

$$\frac{1}{\Lambda^2}\int d^4\theta\,\mathrm{Tr}\left[\overline{\nabla}\nabla\left(M^\dagger e^V\right)\overline{\nabla}\nabla M\right]+h.c.$$

Kahler term contributes to S
but the sign is unknown

in principle it could be calculated in some models
eg. the Klebanov-Strassler model