The Little Higgs

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Outline

Lecture I: Basic Ideas

- Higgs as pseudo Goldstone boson
- protecting mass against 1 loop, quadratically divergent radiative corrections via "delocalization in theory space"
- obtaining Higgs interactions
  - gauge
  - quartic
  - top Yukawa
- calculation of Higgs mass squared

Selected references

- Higgs as “little” pseudo Goldstone Boson:
  Georgi and Pais, Phys. Rev.D12,1975,508

- Higgs as composite pseudo Goldstone Boson:
  Kaplan, Georgi and Dimopoulos, PLB145, 1984, 216

- Deconstructing dimensions, little Higgs models:
  Arkani-Hamed, Cohen and Georgi, hep-th/0104005; hep-ph/0105239
- “Littlest” Higgs model: *Arkani-Hamed, Cohen, Katz, A.N., hep-ph/0206021*

- Radiative corrections: *Hewett, Petriello, and Rizzo, hep-ph/0211218; Csaki, Hubisz, Kribs, Meade and Terning, hep-ph/0211124, hep-ph/0303236*

- Reviews: *Schmaltz and Tucker-Smith, Ann. Rev. Nucl. Part. Sci. 55,2005, 229; Perelstein, Prog. Part. Nucl. Phys. 58,2007,247; Georgi, Comptes Rendus Physique 8, 2007,1029*

More References

- Simplest Little Higgs: *Schmaltz, hep-ph/0407143*
- T-parity: *Cheng and Low, hep-ph/0308199, hep-ph/0405243*
- Top partners with T-parity: *Cheng, Low and Wang, hep-ph/0510225*
- Trilepton signatures in LH with T-parity: *Datta, Dey, Gupta,Mukhopadhyaya and Nyffeler, arXiv:0708.1912*
- Top partners at LHC: *Nojiri and Takeuchi, arXiv:0802.4142*
- Weakly coupled SUSY UV completion of Little Higgs with T-parity:*Csaki, Heinonen, Perelstein and Spethmann, arXiv:0804.0622*
- Composite Little Higgs: *Katz, Lee, A.N. and Walker, hep-ph/0312287*
- Dark matter: *Birkedal, Noble, Perelstein and Spray, hep-ph/0603077*

Why Higgs as pseudo Nambu-Goldstone Boson?

- Nambu-Goldstone Boson equivalence theorem: High energy $W_L, Z_L \rightarrow$ Nambu-Goldstone Bosons.
- limit $g, g' \rightarrow 0$ of Standard Model has 3 massless

  Nambu-Goldstone Boson scalars (could be pion analogues as in Technicolor)
- Only natural* mechanism known for light scalars is that they be (pseudo) Goldstone-Bosons (pions of QCD, longitudinal components of W, Z)
- PNGB $\rightarrow$ Shift Symmetry: $h \rightarrow h + \text{constant}$ protects higgs mass against radiative corrections

*Natural: found in nature

Why NOT Higgs as pseudo Nambu-Goldstone Boson?

- shift symmetry (=nonlinearly realized symmetry) eliminates $\lambda\ (H^\dagger H)^2$ from Higgs potential
- shift symmetry eliminates Yukawa couplings
- shift symmetry eliminates gauge couplings
- breaking of shift symmetry by needed couplings reintroduces quadratic divergences, fine-tuning

Inspiration from Xtra dimensions

- from 4D viewpoint, $A_{5,6}$ are scalars
- from 4D viewpoint, xtra-D gauge symmetry is shift symmetry
- $A_{5,6}$ have quartic interaction
- $A_{5,6}$ have gauge interactions
- $A_{5,6}$ have coupling to fermions
- $A_{5,6}$ have no mass
- deconstruction: KK excitations of gauge bosons= new gauge bosons of large product gauge group spontaneously broken to diagonal subgroup
- new gauge bosons soften radiative corrections to Higgs mass squared

Deconstructing New Dimensions

Arkani-Hamed, Cohen, Georgi

- treat Xtra dimensions as “theory space”
- latticized extra dimensions in finite volume are **finite** no of 4D degrees of freedom
- Little Higgs= “stripped down” version of Higgs as xtra D component of gauge boson
- extra dimensions as toy model for how to realize Higgs as PNGB
- dont need to take “theory space” literally

what deconstruction taught us

Key features of theories with new dimensions:

- Large group of approximate global symmetries
- Small number of symmetry breaking "spurions" with special property that each spurion breaks only part of global symmetry
- symmetry breaking is "delocalized in theory space"
- full symmetry breaking requires 2 or more interactions acting together
- *collective symmetry breaking* softens radiative corrections

Basic Ideas of Little Higgs

- Higgs is pseudo Nambu Goldstone Boson (PNGB), transforming nonlinearly under symmetry transformation, analogous to pions, kaons
- All symmetry breaking interactions of Higgs arise from collaboration between at least 2 terms in Lagrangian
- No single interaction breaks all the symmetry protecting the Higgs
- Each single term in Lagrangian leaves an **exact** symmetry protecting the Higgs
- generically collective symmetry breaking give more suppression of Higgs mass than Higgs quartic

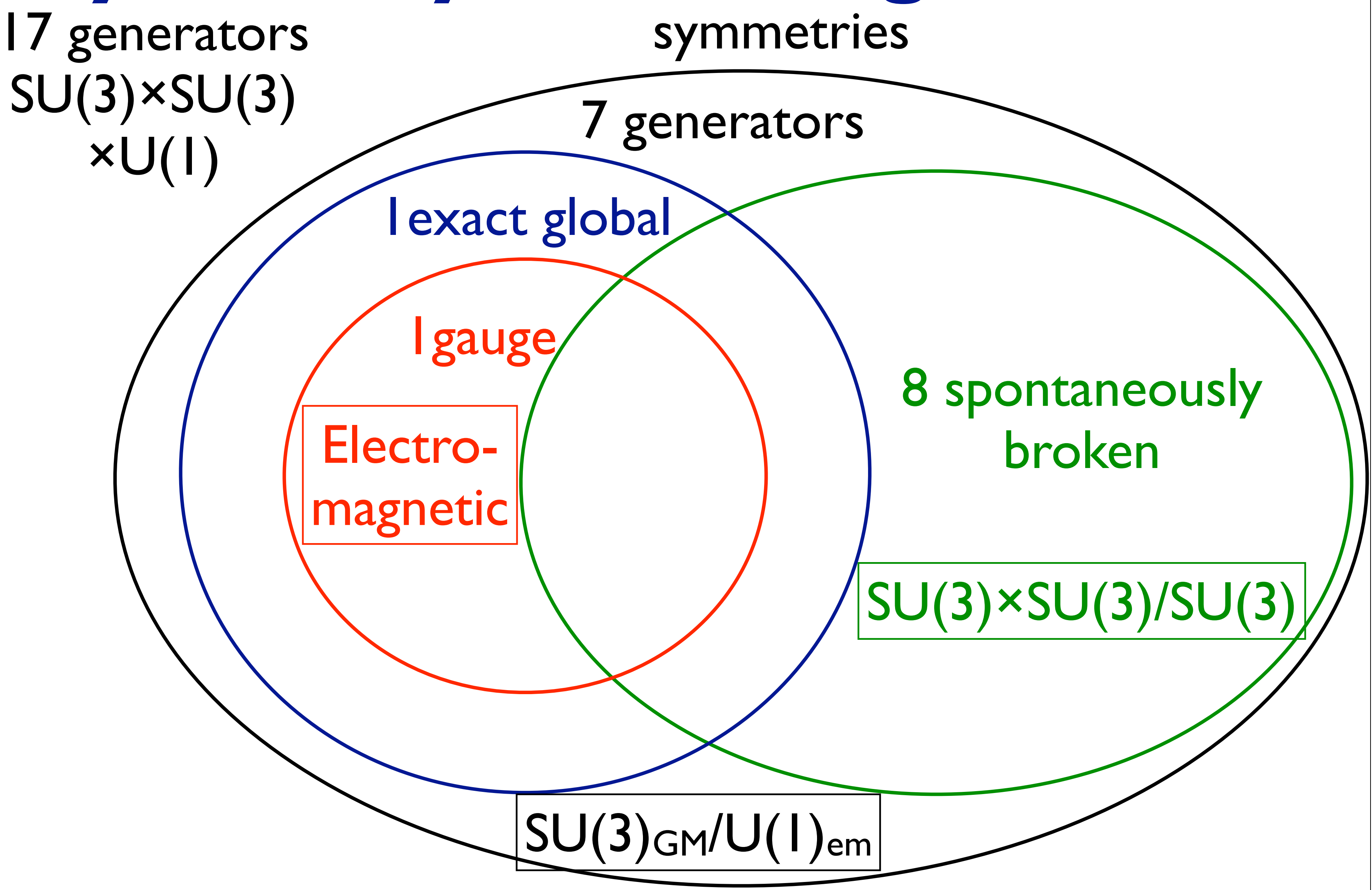
LH example

- Start with review of symmetries, effective chiral Lagrangians for PNGB, using QCD example of SU(3)×SU(3)/SU(3)
- Use effective Lagrangian for SU(5)/SO(5) to illustrate how collective symmetry breaking leads to Higgs with desired properties
  - reduced mass term relative to quartic
  - nontrivial gauge and Yukawa couplings from collective symmetry breaking

Effective Lagrangian for QCD

- QCD Low energy description is theory of pions, photon, electron, muon, neutrinos
- QCD approximate symmetry is $SU(3)_L \times SU(3)R$
- Only $SU(3)_{GM}$ is realized linearly (not spontaneously broken)
- symmetry breaking spurions: quark masses, electromagnetic interactions, weak interactions

Symmetry Venn Diagram:QCD



QCD effective Lagrangian

8 pseudo Nambu Goldstones

$$\Pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 - \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 - \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K}^0 & \sqrt{\frac{2}{3}}\eta \end{pmatrix} \propto \begin{pmatrix} u\overline{u} & u\overline{d} & u\overline{s} \\ d\overline{u} & d\overline{d} & d\overline{s} \\ s\overline{u} & s\overline{d} & s\overline{s} \end{pmatrix}$$

$$\Sigma = e^{\frac{i2\Pi}{f}} \qquad\qquad \Sigma \to U_L \Sigma U_R^\dagger$$

under $SU(3)_L$, $SU(3)_R$ transformations

$f = f_\pi$ the pion decay constant

$\Lambda = 4\pi f_\pi$ Maximum possible scale of validity of effective theory="cutoff"

Spurions

$$Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$

quark charge matrix

$$Q = Q_L \rightarrow U_L Q_L U_L^\dagger$$
$$Q = Q_R \rightarrow U_R Q_R U_R^\dagger$$

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

quark mass matrix

$$M \rightarrow U_L M U_R^\dagger$$

Leading terms

$$\mathcal{L} \supset \frac{f^2}{8}\mathrm{Tr} D_\mu \Sigma D^\mu \Sigma^\dagger + c_1 f^3 \mathrm{Tr}(\Sigma m^\dagger + h.c.) + c_2 f^4 \mathrm{Tr} Q \Sigma Q \Sigma^\dagger + \ldots$$

gives electromagnetic interactions, and pion-pion interactions ∝ momenta

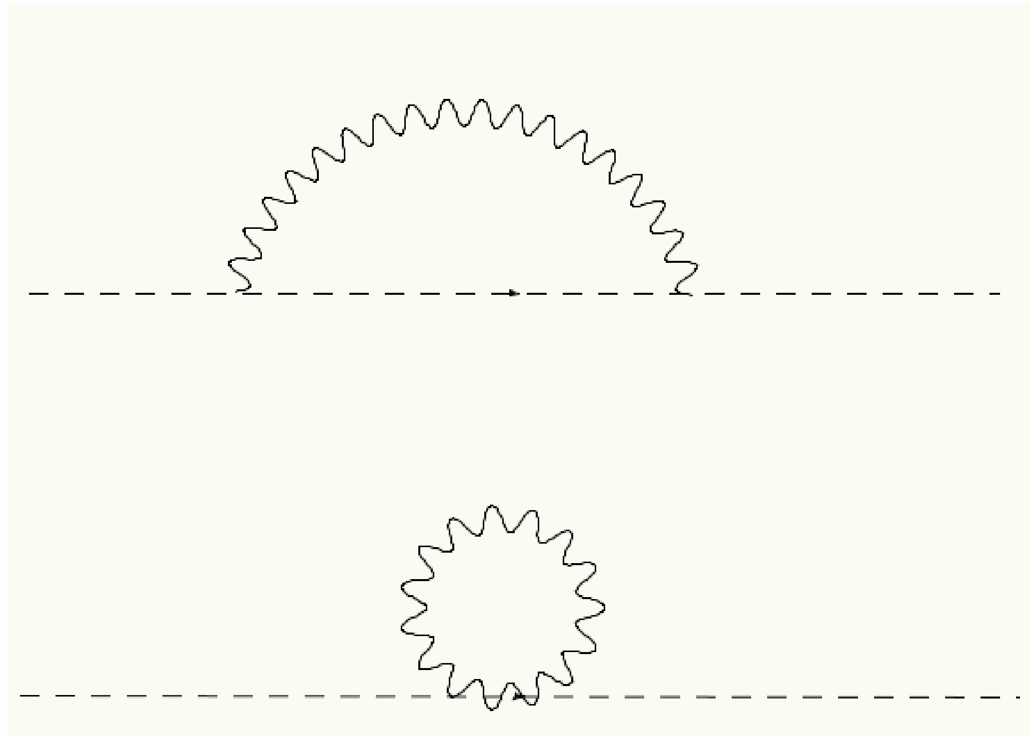
contribute to masses, potential

leading electromagnetic contribution to masses, gives positive mass squared to charged pions, kaons

$$D_\mu \Sigma = \partial_\mu \Sigma + ieA_\mu (Q_L \Sigma - \Sigma Q_R^\dagger)$$

...= more derivatives, more spurions (systematic expansion)

Loops



Divergent Correction to meson masses, absorbed in counterterm for

$$c_2 f^4 \mathrm{Tr} Q \Sigma Q \Sigma^\dagger .$$

Systematic procedure for more accuracy: more loops=add higher order terms

Original Recipe for Little Higgs theories

- Start with global symmetry G containing a weakly gauged subgroup consisting of at least two copies of SU(2) ×U(1): $G \supset G_1 \times G_2 = [SU(2) \times U(1)]^2$.
- weakly gauge $G_1 \times G_2$, explicitly breaking $G \to G_1 \times G_2$
- weak interactions are diagonal subgroup of G_1, G_2
- Each of the G_i must commute with a subgroup of G acting non-linearly on the Higgs, ⇒ Higgs potential vanishes if either g_1 or $g_2 \to 0$
- new gauge bosons cancel one loop quadratic divergence in Higgs potential from gauge interactions

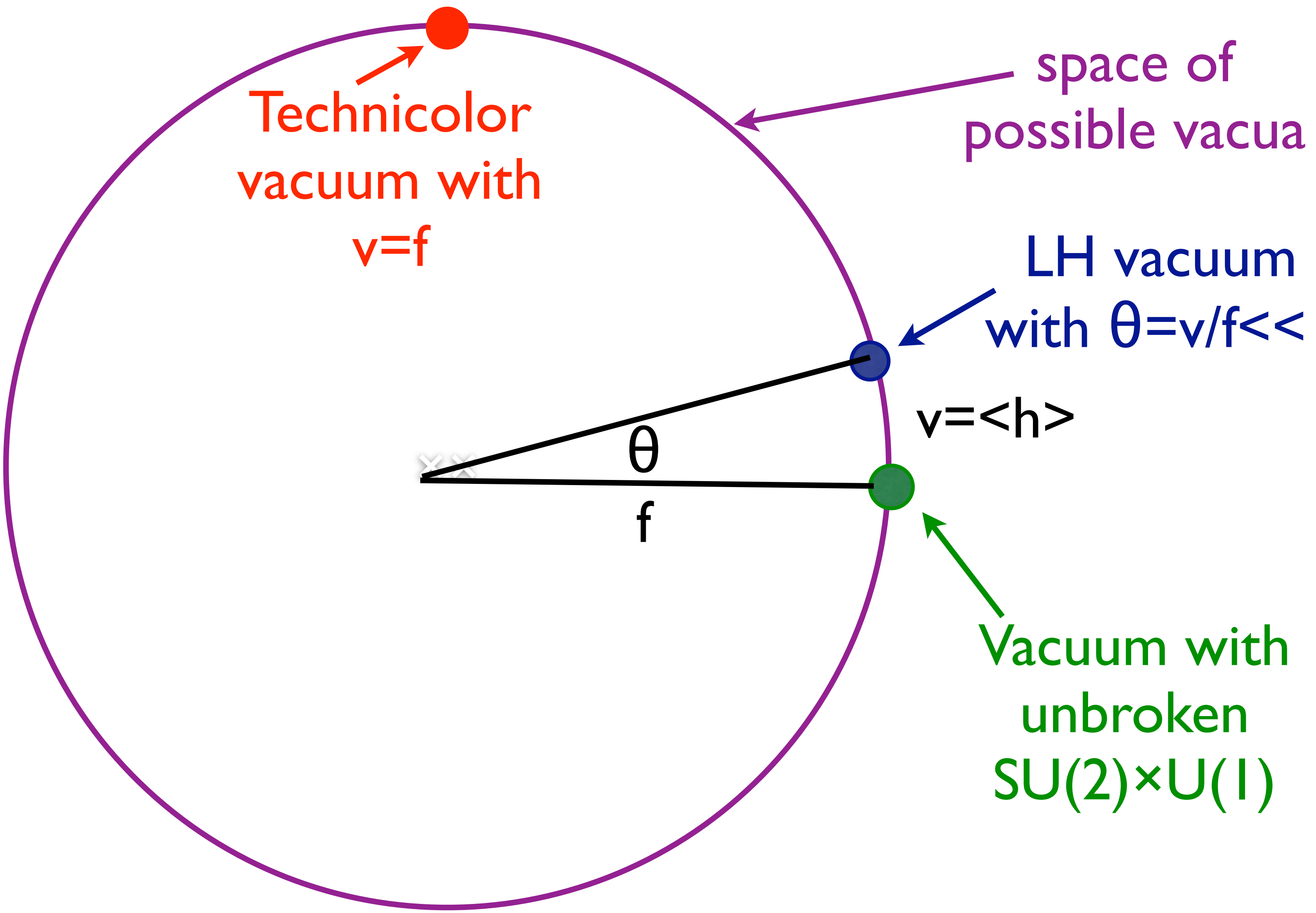
Littlest Higgs

- Global approximate symmetry group: SU(5) (24 $=5^2-1$ generators)
- SU(5) spontaneously broken to SO(5) subgroup (10 generators)
- 14=24-10 PNBGs with decay constant f
- Gauge subgroup of global symmetry group: $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$ (8 gauge bosons)
- maximal unbroken subgroup: $SU(2)_W \times U(1)_Y$
- $SU(2)_W \times U(1)_Y \rightarrow U(1)_{em}$ at scale $v < f$

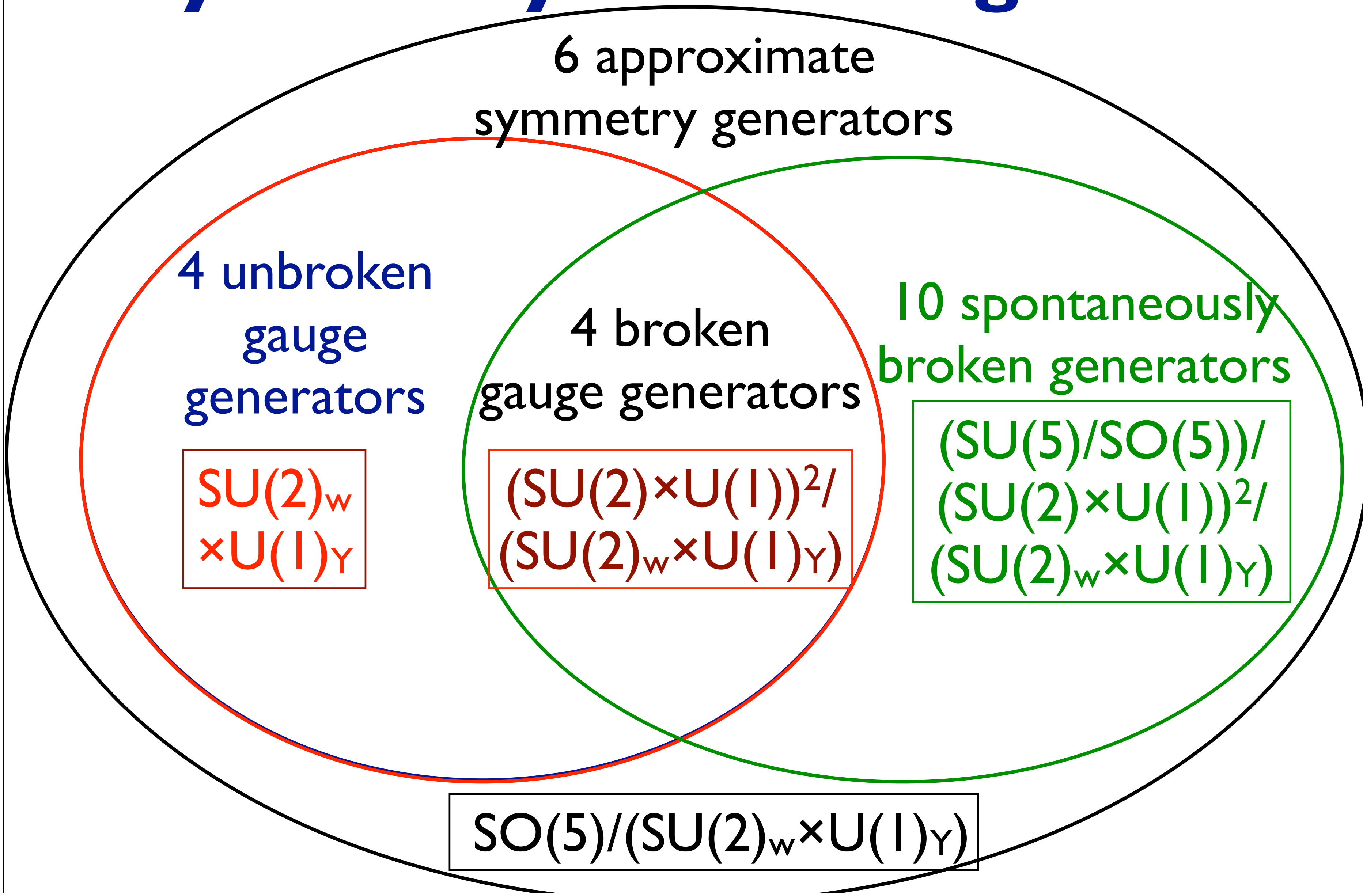
Why SU(5)/SO(5) is minimal

- want Higgs to carry SU(2) × U(1) gauge
- gauge interaction breaks shift symmetry
- **no single term** should break shift symmetry
- collective symmetry breaking: want SU(2) to be diagonal sum of SU(2)×SU(2)
- collective symmetry breaking: want U(1) to be diagonal sum of U(1)×U(1)
- SU(5) is minimal simple group with $(SU(2) \times U(1))^2$ subgroup and with global SU(3) which commutes with each SU(2) × U(1) factor

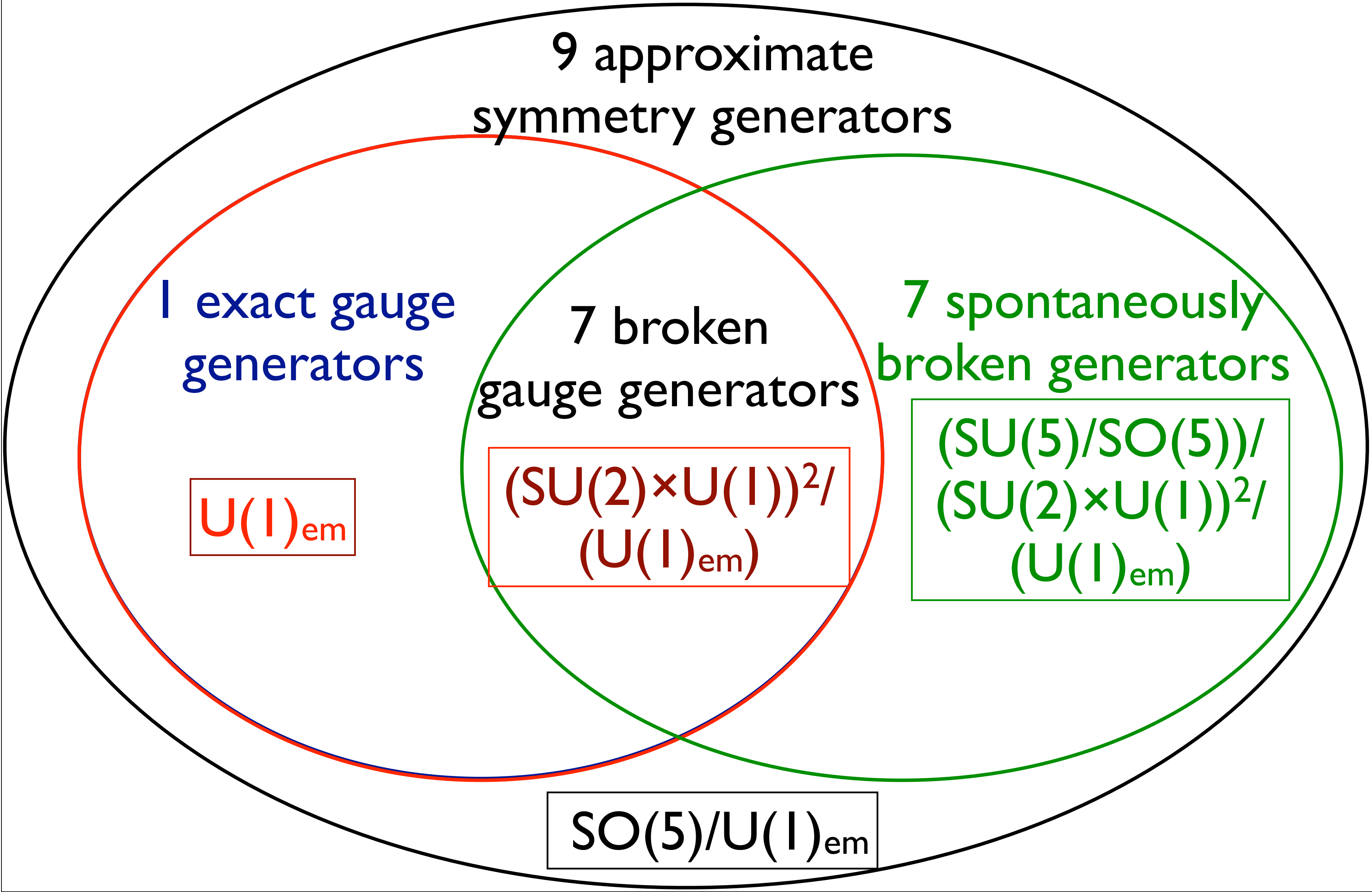
Vacuum alignment



Symmetry Venn Diagram: I



Symmetry Venn Diagram:2



Calculating Vacuum alignment

- Start with model for gauge and Yukawa interactions
- compute potential for PNGB about SU(2)×U(1) symmetric point
- find small negative contribution to Higgs mass squared from top, large positive contribution to quartic from gauge
- find small misalignment, small v/f without fine tuning
- new **same spin** weakly coupled particles cancel divergences, allow calculability in low energy effective theory

Gauge interactions

- SU(5)⊃SU(2)$_1$×SU(2)$_2$×U(1)$_1$×U(1)$_2$, weakly gauged
- weak gauge generators: $\begin{pmatrix} 2\times 2 & 2\times 1 & 2\times 2 \\ 1\times 2 & 1\times 1 & 1\times 2 \\ 2\times 2 & 2\times 1 & 2\times 2 \end{pmatrix}$

$$Q_1^a = \begin{pmatrix} \frac{\sigma_a}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad Q_1' = \begin{pmatrix} q_1 - \frac{1}{2} & 0 & 0 \\ 0 & q_1 & 0 \\ 0 & 0 & q_1 \end{pmatrix}$$

$$Q_2^a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\sigma_a^*}{2} \end{pmatrix} \qquad Q_2' = \begin{pmatrix} q_2 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_2 + \frac{1}{2} \end{pmatrix}$$

q_1= 1/5, q_2=-1/5, (or extend SU(5) to SU(5)×U(1))

PNGBs

- SU(5)→SO(5)⇒ 14 PNGBs (if SU(2)×U(1) preserved, 4 are eaten by heavy W, B, 10 are physical).
- 24 generators of SU(5)
  (5×5 traceless hermitian matrices)
- 10 generators of SO(5) (5×5 antisymmetric hermitian matrices)
- 14 PNGBs corresponding to 14 5×5 symmetric traceless hermitian matrices

Σ field

$$\begin{pmatrix} 2\times 2 & 2\times 1 & 2\times 2 \\ 1\times 2 & 1\times 1 & 1\times 2 \\ 2\times 2 & 2\times 1 & 2\times 2 \end{pmatrix}$$

$$\Sigma = e^{\frac{2i\Pi}{f}}\Sigma_0 \qquad \Sigma_0 = \begin{pmatrix} 0 & 0 & I \\ 0 & 1 & 0 \\ I & 0 & 0 \end{pmatrix}$$

preserves diagonal SU(2)×U(1)

$$\Pi = \begin{pmatrix} \xi + \eta/(2\sqrt{5}) & h/\sqrt{2} & \phi \\ h^\dagger/\sqrt{2} & -2\eta/\sqrt{5} & h^T/\sqrt{2} \\ \phi^* & h^*/\sqrt{2} & \xi^* + \eta/(2\sqrt{5}) \end{pmatrix}$$

$$\Pi = \Pi^\dagger = \Sigma_0\,\Pi^T\,\Sigma_0$$

Under SU(5) $\qquad \Sigma \to U\Sigma U^T$

ξ, η are eaten $\qquad$ h is Higgs doublet $\qquad$ Φ is charged triplet

Gauge contribution to PNGB potential

- gauge contribution to alignment determined by

$$c g_j^2 f^4 \sum_a \mathrm{tr}\left[(Q_j^a \Sigma)(Q_j^a \Sigma)^*\right] + c g_j'^2 f^4 \,\mathrm{tr}\left[(Y_j \Sigma)(Y_j \Sigma)^*\right]$$

c is analog of QCD term which gives gives positive mass squared to charged pions.
analogy with QCD suggests c positive and order 1 if UV theory is similar to QCD

Expansion to fourth order in PNGBs

$$c(g_1^2 + {g_1'}^2) f^2 |\phi_{ij} + \frac{i}{2f}(h_i h_j + h_j h_i)|^2 + c(g_2^2 + {g_2'}^2) f^2 |\phi_{ij} - \frac{i}{2f}(h_i h_j + h_j h_i)|^2$$

gauge choice: ξ, η =0

$$\begin{aligned} g &= \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \\ g' &= \frac{g_1' g_2'}{\sqrt{{g_1'}^2 + {g_2'}^2}} \end{aligned}$$

$$m_\phi = \sqrt{c(g_1^2 + {g_1'}^2 + g_2^2 + {g_2'}^2)} f$$

$$m_h = 0 \quad !$$

quartic higgs term

$$c(g_1^2 + {g_1'}^2)f^2|\phi_{ij} + \frac{i}{2f}(h_i h_j + h_j h_i)|^2 + c(g_2^2 + {g_2'}^2)f^2|\phi_{ij} - \frac{i}{2f}(h_i h_j + h_j h_i)|^2$$

- note: quartic must vanish if either $g_1 \longrightarrow 0, \quad g_1' \longrightarrow 0$ or $g_2 \longrightarrow 0, \quad g_2' \longrightarrow 0$ due to different nonlinearly realized SU(3) exact symmetries which shift *Higgs*

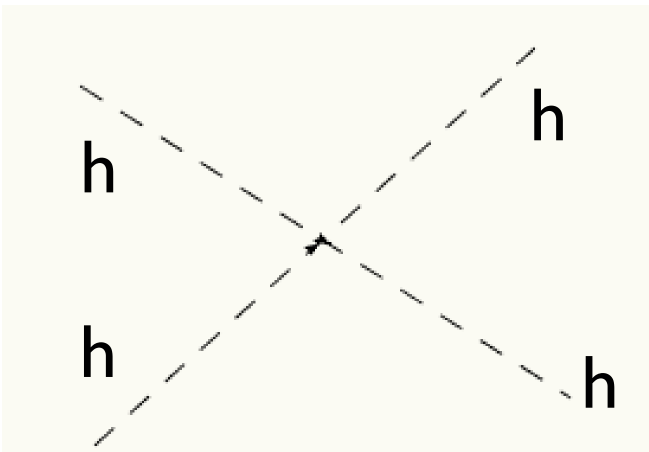
SU(3)$_1$: $h_i \rightarrow h_i + \epsilon_i + \cdots$ removes 1st terms

$$\phi_{ij} \rightarrow \phi_{ij} - i(\epsilon_i h_j + \epsilon_j h_i) + \cdots$$

SU(3)$_2$: $h_i \rightarrow h_i + \eta_i + \cdots$ removes 2nd terms

$$\phi_{ij} \rightarrow \phi_{ij} + i(\eta_i h_j + \eta_j h_i) + \cdots \ .$$

Higgs quartic after Φ integrated out



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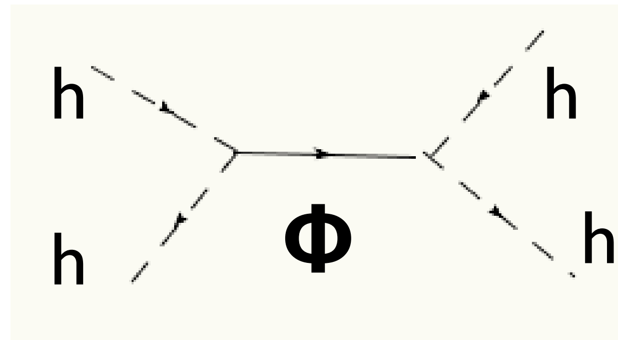
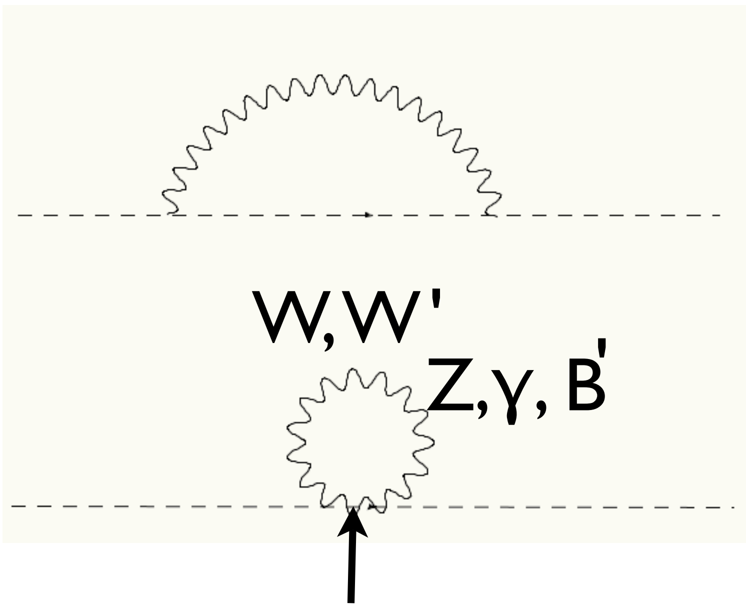


$$c(g_1^2 + g_2^2 + {g_1'}^2 + {g_2'}^2)$$

$$-c\left(\frac{\left((g_1^2 + {g_1'}^2 - (g_2^2 + {g_2'}^2)\right)^2}{g_1^2 + g_2^2 + {g_1'}^2 + {g_2'}^2}\right)$$

- quartic is nonvanishing! $\lambda = 2c\left(\frac{(g_1^2 + {g_1'}^2)(g_2^2 + {g_2'}^2)}{g_1^2 + g_2^2 + {g_1'}^2 + {g_2'}^2}\right)$

1 loop corrections



negative coupling of heavy bosons

1 loop quadratic divergences cancel, SAME SPIN, opposite sign couplings

quadratic divergence $\propto \frac{\partial^2}{\partial h^2}\left(\sum M_V^2\right)$

log correction to Higgs mass

$$\delta m_h^2 = \frac{3}{64\pi^2}\left\{3g^2 {M'_W}^2 \log\frac{\Lambda^2}{{M'_W}^2} + {g'}^2 {M'_B}^2 \log\frac{\Lambda^2}{{M'_B}^2}\right\}$$

- heavy boson masses depend on h
- log sensitivity to cutoff
- quadratic sensitivity to heavy gauge boson masses (finetuning reappears if too heavy boson masses)
- f < 2πv/g to avoid fine tuning
- model with 3 or more gauge groups would be 1 loop finite

fermion Yukawa

- quadratic divergences proportional to small Yukawas dont require fine tuning
- focus on top
- “top seesaw”: top mass from mixing with heavy fermion
- requires 2 or more couplings for top Yukawa
- requires at least 1 heavy partner of top
- will show model with 2 partners
  - with 2 top partners top contribution to higgs potential is 1 loop finite

top see saw and little Higgs

- Introduce heavy (TeV), vector-like quarks, e.g. in 5-plet of SU(5), coupled to Σ in symmetry preserving fashion
- also introduce usual quarks and leptons, usual small Yukawas from small symmetry breaking terms
- large top Yukawa arises from mixing with heavy quarks

Example of top Yukawa without quadratic divergences

- New χ quarks transform as (5,3,2/3) under $SU(5)_{global} \times SU(3)_{color} \times U(1)_Y$. Under standard model:

$$
\chi \left\{ \begin{array}{cccc}
 & SU(3)_c & SU(2)_L & U(1)_Y \\
Q & 3 & 2 & 1/6 \\
T & 3 & 1 & 2/3 \\
P & 3 & 2 & 7/6
\end{array} \right.
$$

top seesaw: Lagrangian contains

$$\lambda_1 f \bar{\chi} \Sigma \chi + \lambda_2 f \bar{t}_3 T + \lambda_3 f \bar{Q} q_3 + h.c.$$

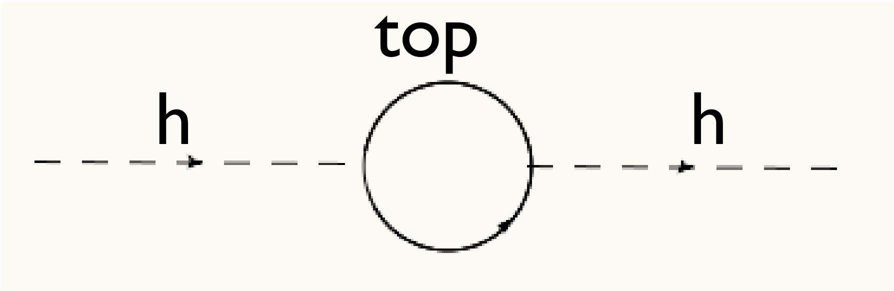
No Divergences

$$\lambda_1 f \bar{\chi} \Sigma \chi + \lambda_2 f \bar{t}_3 T + \lambda_3 f \bar{Q} q_3 + h.c.$$

$$\lambda_t = \frac{\lambda_1 \lambda_2 \lambda_3}{\sqrt{\lambda_1^2 + \lambda_2^2}\sqrt{\lambda_1^2 + \lambda_3^2}}$$

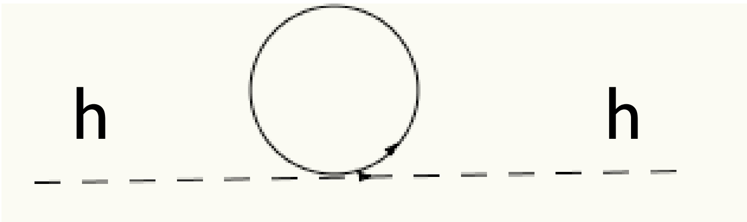
- Note that collective action of all 3 terms needed to break SU(5) symmetry which protects Higgs
- Note that collective action of all 3 terms needed for seesaw top mass
- charge 2/3 quark masses: $\sqrt{\lambda_1^2 + \lambda_2^2} f$, $\sqrt{\lambda_1^2 + \lambda_3^2} f$, $\lambda_t v$

fermion loop corrections to Higgs potential



divergences
in this diagram

heavy tops, masses a,b



cancel divergences,

result is finite negative contribution

$$\delta V(h)_{\text{eff}} = -\frac{3\lambda_t^2 h^\dagger h}{8\pi^2}\frac{a^2 b^2}{a^2-b^2}\log\frac{a^2}{b^2}$$

finetuned unless
heavy quark
lighter than 2 TeV

3 Main lessons of little Higgs

1. Electroweak symmetry breaking without (much) fine tuning can be realized with the particle content of the minimal Standard Model below a TeV.
2. A small number of new, weakly coupled particles are required at a few TeV, including new gauge bosons, new fermions and a scalar coupled to the Higgs. These cancel one loop quadratic divergences in the Higgs mass via same sign partners.
3. The old idea of dynamical electroweak symmetry breaking can manifest itself at low energies as the Standard Model with a weakly coupled light Higgs, and a strong interaction scale of several TeV.

Lecture II:

beyond the basics of Little Higgs Models

Summary of Lh theory

- **Higgs as PNGB**
  - part of multiplet corresponding to G/H
    - G is approximate symmetry of theory
    - H is symmetry of ground state
    - # generators in G/H=dimension of space of approximate ground states=# light scalars
    - Higgs parametrizes small misalignment of vacuum from SU(2)×U(1) symmetric state
- **Collective symmetry breaking**
  - gauge, top Yukawa and quartic arise as collaboration from 2 or more terms
  - no one loop quadratic divergences
  - new same spin partners of top, Higgs, gauge bosons
  - vacuum alignment from calculable Higgs potential

GB Shaw quote

"Beauty is all very well at first sight; but whoever looks at it when it has been in the house three days?"

Little Higgs issues

- light B_H
- precision electroweak
- signatures

Outline

Lecture II: Phenomenology and Variations

- precision electroweak constraints
- T-parity
- some experimental signals
- UV completions-compositeness, RS, SUSY

Spin one Boson couplings

$$W_L = \frac{g_2 W_1 + g_1 W_2}{\sqrt{g_1^2 + g_2^2}}, \quad B_L = \frac{g_2' B_1 + g_1' B_2}{\sqrt{{g_1'}^2 + {g_2'}^2}}$$

couple universally, with couplings

$$g = \frac{g_2 g_1}{\sqrt{g_1^2 + g_2^2}}, \quad g' = \frac{g_2' g_1'}{\sqrt{{g_1'}^2 + {g_2'}^2}}$$

In general, higgs vev will mix these with

$$W_H = \frac{g_1 W_1 - g_2 W_2}{g_1^2 + g_2^2}, \quad B_H = \frac{g_1' B_1 - g_2' B_2}{{g_1'}^2 + {g_2'}^2}$$

which couple differently to fermions transforming under G_1 than they do to fermions transforming under G_2

Summary of SU(5)/SO(5) spectrum

Parameters: c(Higgs mass), $g_{1,2}$ (g_W), $g'_{1,2}$ (g_Y), $\lambda_{1,2,3}$ (λ_t), f

6 undetermined (m_h, m_{W_H}, m_{B_H}, $m_{T_{1,2,3}}$), 1 mass prediction (m_Φ)

spin	standard model	partner (sm quantum no), mass
0	Higgs mass 115-600 GeV	Φ (1,3,1) $m_\phi = \sqrt{c(g_1^2 + {g'_1}^2 + g_2^2 + {g'_2}^2)}f$
1/2	q_L,t_R	Q(3,2,1/6),T(3,1,2/3),P(3,2,7/6) $\sqrt{\lambda_1^2 + \lambda_3^2}f$ $\sqrt{\lambda_1^2 + \lambda_2^2}f$ $\lambda_1 f$
1	W,Z,γ	W_H(1,3,0),B_H (1,1,0) $\frac{\sqrt{g_1^2 + g_2^2}f}{\sqrt{2}}$, $\frac{\sqrt{{g'_1}^2 + {g'_2}^2}f}{\sqrt{10}}$

Precision Electroweak

- LH models are SM at low energy
- All corrections from dim 6 or more operators and are suppressed by v^2/f^2
- Important operators: $\frac{\mathcal{O}(g^2)}{f^2}(H^\dagger D H)^2$; $\frac{\mathcal{O}(g^2)}{f^2}|D^2 H|^2$ $\frac{\mathcal{O}(g^2)}{f^2}(\bar{\psi}\gamma\psi)^2$
- Corrections to relation $M_W/M_Z=\cos\theta_W$
- Corrections to G_F lead to corrections to Z couplings

direct search+precision electroweak = fine tuning?

- Φhh trilinear coupling: $cf(g_1^2 - g_2^2 + {g'_1}^2 - {g'_2}^2)$
- <h> induces tadpole for Φ, correction to relation $M_W/M_Z = \cos\theta_W$
- trilinear vanishes if $g_1=g_2$, $g_1'=g_2'$ (best region for precision electroweak)
- $g_1=g_2$, $g_1'=g_2'$ also suppresses W_H, B_H mixing with SM bosons
- for $g_1'=g_2'$, $m_{BH} = 0.16\ f$
- for $g_1'=g_2'$, sm fermions transforming under G_1, B_H easy to rule out
- for sm fermions transforming under G_1, f> 4.5 TeV
- For f> 4.5 TeV, heavy top partners heavier than 6 TeV

Avoiding problem

- precision electroweak would be improved if $g_1=g_2$, $g_1'=g_2'$ *and* light fermions do not couple to W_H, B_H
- coupling of light fermions to W_H, B_H depends on how they transform under G_1, G_2
    - we only know how they transform under $G_1 + G_2$
- f could be as low as 500 GeV and precision electroweak would be ok (with heavy Higgs)
- Different sort of fine tuning of parameters?
- Could there be a symmetry that would do this?

"TeV-Parity"

Cheng and Low

- want symmetry $G_1 \Leftrightarrow G_2$
- reduce precision electroweak corrections to LH
- f can be as low as 500 GeV
- T-parity: New particles only contribute to loop corrections
- if exact, LTP can be Dark Matter
- T-parity may have Anomalies or not be exact and still suppress FCNC

Generic Argument for new particle parity

- Cancelling quadratic divergences requires new particles
- Higgs mass squared quadratically sensitive to new masses, typically about $g^2\ m^2_{new}/(16\ \pi^2)$
- $m_{new}\ \leq$ fewTeV
- tree level exchange of new particles gives $g^2\ v^2/m^2_{new} >$ 0.01 correction to precision electroweak observables which agree with SM at 0.002 level
- If new particles carry a Z_2 symmetry, they can still cancel divergences
- If new particles carry a Z_2 symmetry, they cannot be exchanged at tree level, give suppressed contribution to precision electroweak of order $g^4\ v^2/(m^2_{new}\ 16\ \pi^2)$ ~0.0001

Major Bonus/motivation for TeV Parity

- If exact, the lightest new particle carrying TeV parity is stable.
- We have strong evidence from dark matter that a new (neutral) stable particle exists
- WIMPS can give correct relic abundances

quote from Georgi

"We begin to be convinced that our model is more than a model only when our clever solutions start to fit together, to do more than the one job they were invented for and to produce a model that feels like more than the sum of its parts."

couplings of SM fermions

- In general, SM fermions are linear combination of fermions transforming under G_1 and G_2
- SM fermions couple universally to W_L, B_L (analogous to universal coupling to photon in SM)
- couplings to W_H,B_H depend on g_1/g_2 and on parameter describing how they transform under G_1 and G_2. (analogous to coupling to Z boson in SM, which depends on T_3)
- implementing T-parity requires SM fermions to couple equally to G_1, G_2
- T-parity $\Rightarrow$SM fermions decouple from W_H, B_H

imposing T-parity

- Require Symmetry under $$\Sigma \to \Sigma_0 \Omega \Sigma^* \Omega \Sigma_0$$ $$G_1 \leftrightarrow G_2$$

where $$\Omega = \mathrm{diag}(1, 1, -1, 1, 1)$$

which requires $$g_1 = g_2, \quad g_1' = g_2'$$

Note that Φ is odd, does not get tadpole or vev
h is even, sm fermions even

consequences of T-parity

also

$$W_H = \frac{g_1 W_1 - g_2 W_2}{\sqrt{g_1^2 + g_2^2}}, \qquad B_H = \frac{g_1' B_1 - g_2' B_2}{\sqrt{{g_1'}^2 + {g_2'}^2}}$$

are T-parity odd, while

$$W_L = \frac{g_2 W_1 + g_1 W_2}{\sqrt{g_1^2 + g_2^2}}, \qquad B_L = \frac{g_2' B_1 + g_1' B_2}{\sqrt{{g_1'}^2 + {g_2'}^2}}$$

are T-parity even.

T parity invariant fermion couplings

- need to couple sm fermions to Σ in T parity invariant way
- How do sm fermions transform under $G_1 \times G_2$? In general, in effective G/H Lagrangians, we only know how particles transform under H, as they can transform nonlinearly under G
- QCD example: Baryons

Back to QCD

- Recall, low energy QCD degrees of freedom described by pions transforming nonlinearly under $SU(3)_L \times SU(3)_R$
- we can build a Σ field $=\exp(2\,i\,\Pi/f)$ which transforms linearly as $\Sigma \to U_L \Sigma U_R^\dagger$
- Baryons transform under Gell Mann's SU(3) $B \to U_{GM} B \Sigma U_{GM}^\dagger$
- To couple Baryons, define a field $\xi = \exp(i\,\Pi/f)$ such that under $SU(3)_L \times SU(3)_R$ $\xi \to U_L \xi U_{GM}^\dagger = U_{GM} \xi U_R^\dagger$
- $\xi B \xi$ transforms as $\to U_L B U_L^\dagger$
- $\xi^\dagger B \xi^\dagger$ transforms as $\to U_R B U_R^\dagger$

QCD Baryons continued

$$\mathrm{tr}(\xi^\dagger \bar{B} \xi^\dagger \not{D}(\xi B \xi) + \xi \bar{B} \xi \not{D}(\xi^\dagger B \xi\dagger))$$

- kinetic term necessarily gives Baryon pion couplings if Baryons transform nonlinearly under $SU(3)_L \times SU(3)_R$
- cant have baryons in incomplete multiplets of SU(3)GM without badly breaking symmetries protecting pion mass
- MORAL for LH sm fermions transforming nonlinearly under G, must be in complete H multiplet?

ξ for SU(5)/SO(5)

- We want sm fermions to transform under diagonal subgroup of $G_{1,2}$
- We want to define how sm fermions transform under SO(5)
- V is SU(5) matrix
- U is SO(5) matrix

$$\Sigma(x) \to V\Sigma(x)V^T$$

$$\xi = \exp(i\Pi/f)$$

$$\xi \to V\xi U^\dagger = U\xi(\Sigma_0 V^T \Sigma_0)$$

- couple sm fermion doublet to Higgs by mixing with complete SO(5) vector $\Rightarrow$ new doublet + T odd singlet for each fermion.

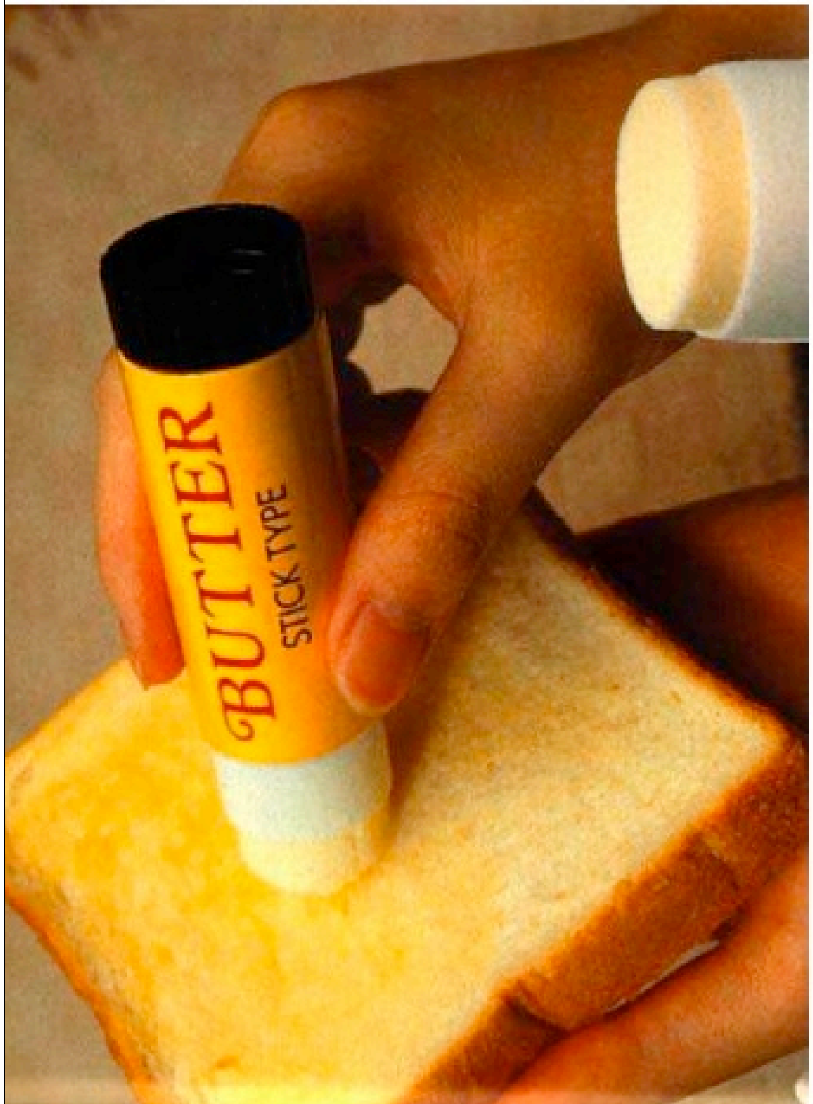
$$\psi = \begin{pmatrix} \psi_D \\ \chi \\ \tilde{\psi}_D \end{pmatrix} \to U\psi$$

Variations:

- In some models, all new particles are T-odd
- others have new T-even partners
- masses of partners of light fermions can be 500 GeV or several TeV, but should be nearly degenerate (fcnc)
- no simple UV completions
- extend symmetry to $(SU(5)\times G_3)/SO(5)$, with G_3 gauged with large coupling, and electroweak G diagonal sum of $G_1+G_2+G_3$, sm fermions charged under G_3, new heavy T-even gauge bosons

full quote from Georgi review

"We begin to be convinced that our model is more than a model only when our clever solutions start to fit together, to do more than the one job they were invented for and to produce a model that feels like more than the sum of its parts. This has not really happened for PGH models. That does not mean that Nature does not make use of the PGH. But it probably means that if it is Nature's way, we have not found quite the right way of thinking about it. Of course, none of the alternatives to PGH really satisfy this criterion either. We are just going to have to wait and see what happens at the LHC! Let me close with a telling quote from [64]. When using the lack of positive experimental results as a guideline for exploring new directions one should pay attention in not inventing a medicine that is worse than the illness: a complicated model might look less plausible than the MSSM fine-tuned at a few % level. This caveat is repeated in some form in a number of the papers on the subject of the Higgs as a pseudo-Goldstone boson. And in the papers in which it doesn't appear — it ought to!"



Georgi claims the Little Higgs reminds him of these clever Chindogu inventions

Intermediate Higgs

Katz, A.N. , Walker

- For m_h=200 GeV, finetuning avoided if M_{W_H}< 6 TeV, m_Φ < 10 TeV, m_T< 2 TeV
- For cutoff of 6 TeV don't actually need 2 copies of SU(2)×U(1) ? Only gauge SU(2)×U(1)?
- Higgs is no longer "little", receives contribution to mass^2 of order $g^2 \Lambda^2/(16\pi^2) \sim g^2 f^2$
- f~ 500 GeV, Λ~ 6 TeV avoids finetuning provided do not have top contribution of order $3 \lambda_t^2 \Lambda^2/(8\pi^2)$
- still need T quarks

Intermediate SU(5)/SO(5)

- 10 extra PNGBs (additional singlet, neutral triplet)
- custodial SU(2) in SU(5)
- keep top seesaw, top partners
- compute Higgs potential, Higgs mass as function of top partner masses, f
- f of order 500 GeV w/o finetuning
- easy to make a composite Higgs in UV
- search for $T \to t\, Z$, $t\, h$, $b\, W^+$, $P_{5/3} \to t\, W^+$, $B \to bZ$, $b\, h$, $t\, W^-$

Experimental studies of LH with T parity

Barger, Gao, Keung

- LH with T parity f=1.5 TeV m_h=200 GeV

Particle	Mass (TeV)	$\mathcal{T}$ parity
A_H	0.24	$-$
Z_H (W_H)	0.97	$-$
ϕ	1.7	$-$
T_-	1.5	$-$
$u_-, c_-, t_-, d_-, s_-, b_-$	2.1	$-$
T_+	2.1	$+$
$e_H^-, \mu_H^-, \tau_H^-, \nu_{eH}, \nu_{\mu H}, \nu_{\tau H}$	2.1	$-$

representative Production cross sections

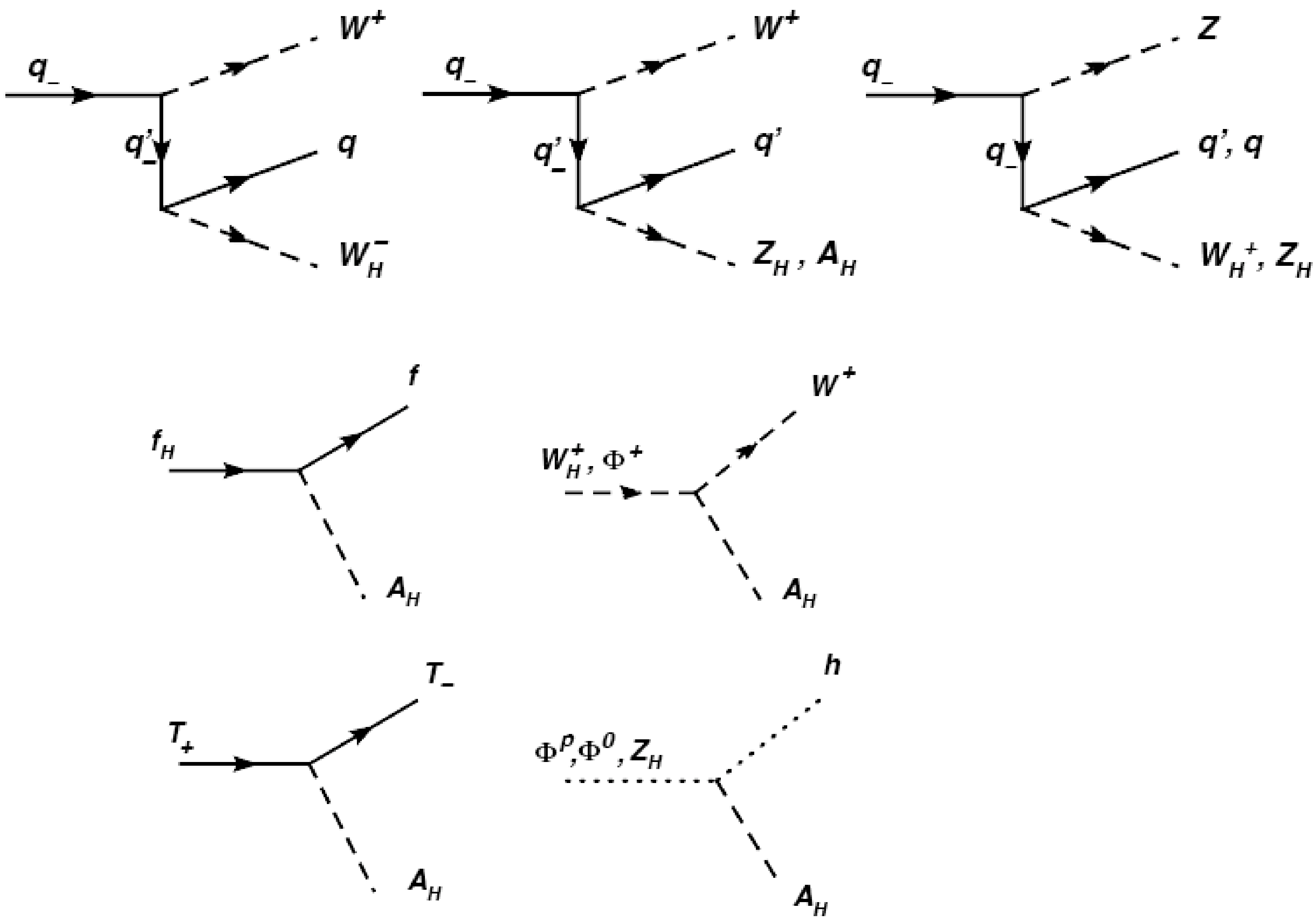
Final state	σ [fb]
$q^+ q^-$	5.2
$q^+ q^+$	2.6
$T_- \bar{T}_-$	1.5
$q_- W_H^+ + q_- W_H^-$	1.8
$q_- Z_H$	0.90
$Z_H W_H^+ \ + \ Z_H W_H^-$	1.6
$W_H^+ W_H^-$	1.0

$$q^+ = (u_-, c_-, \bar{d}_-, \bar{s}_-),$$

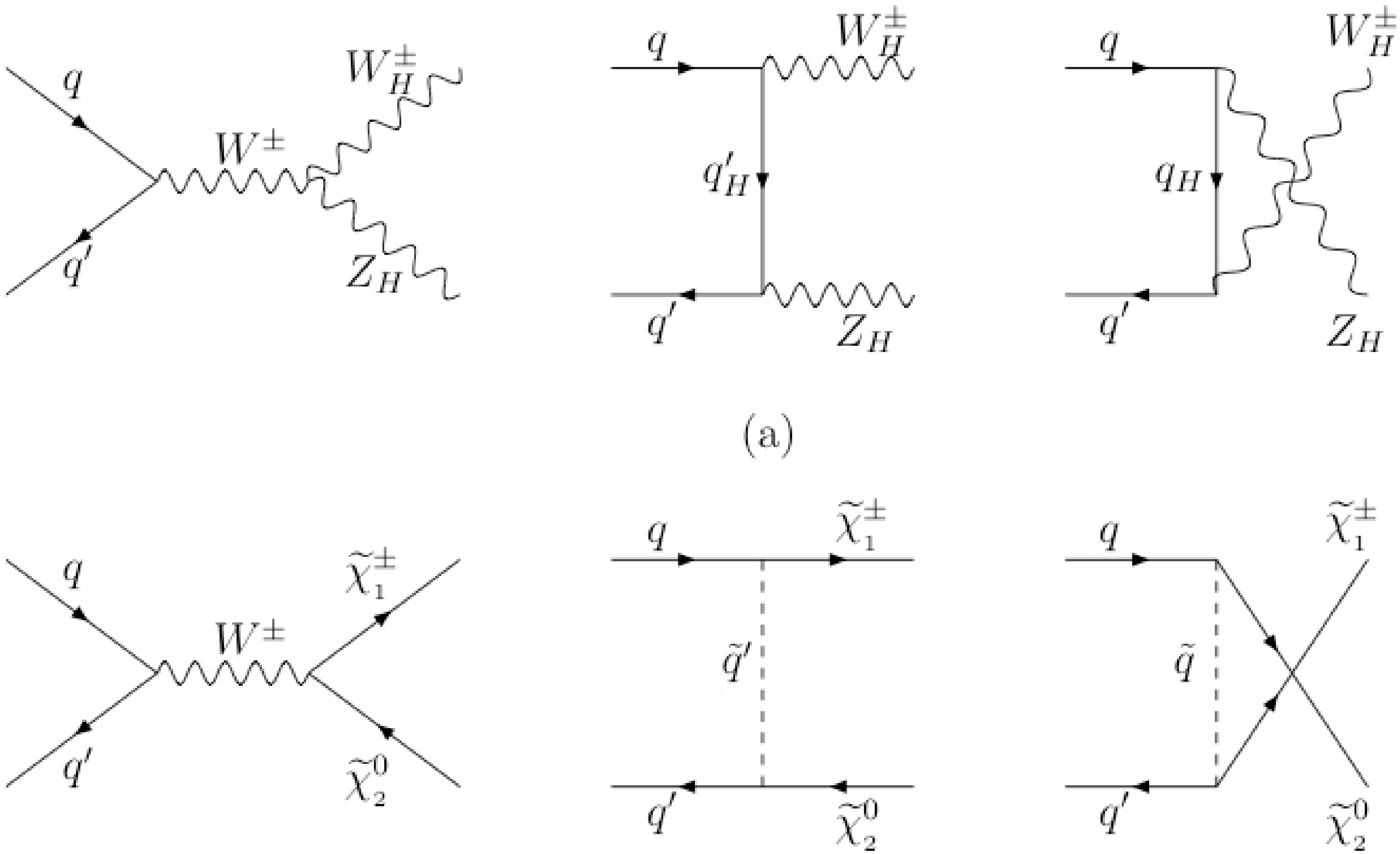
$$q^- = (\bar{u}_-, \bar{c}_-, d_-, s_-)$$

$$q_- = (u_-, c_-, t_-, d_-, s_-, b_-) ;$$

complicated decay chains



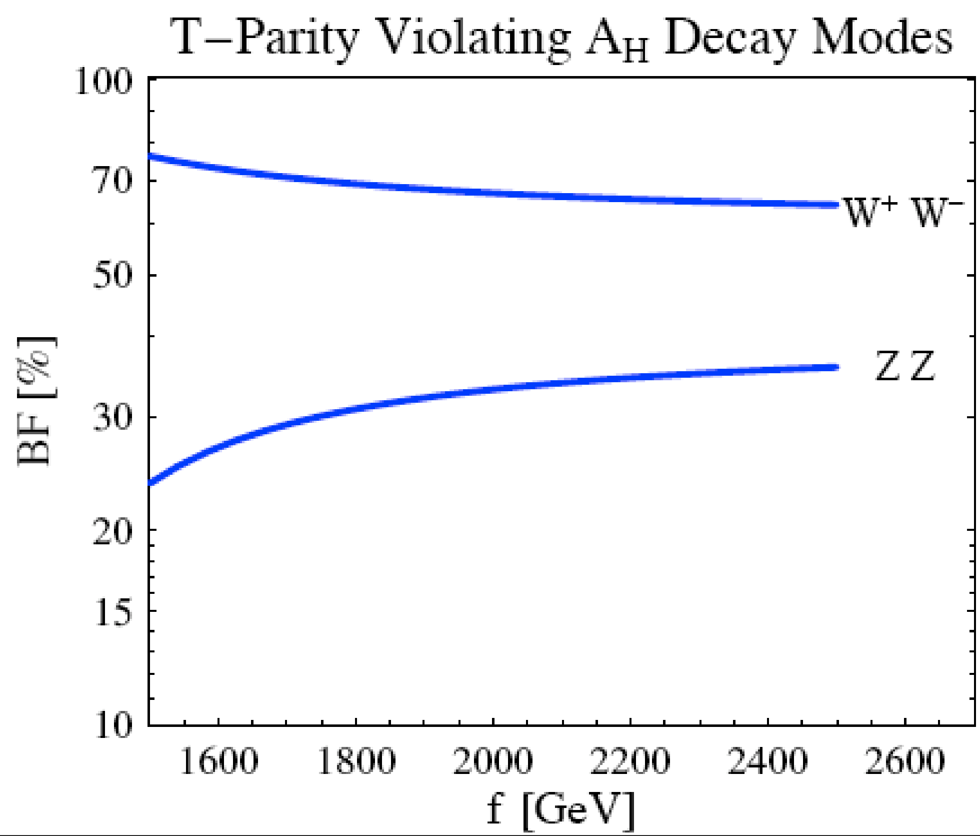
Challenge to distinguish from SUSY!



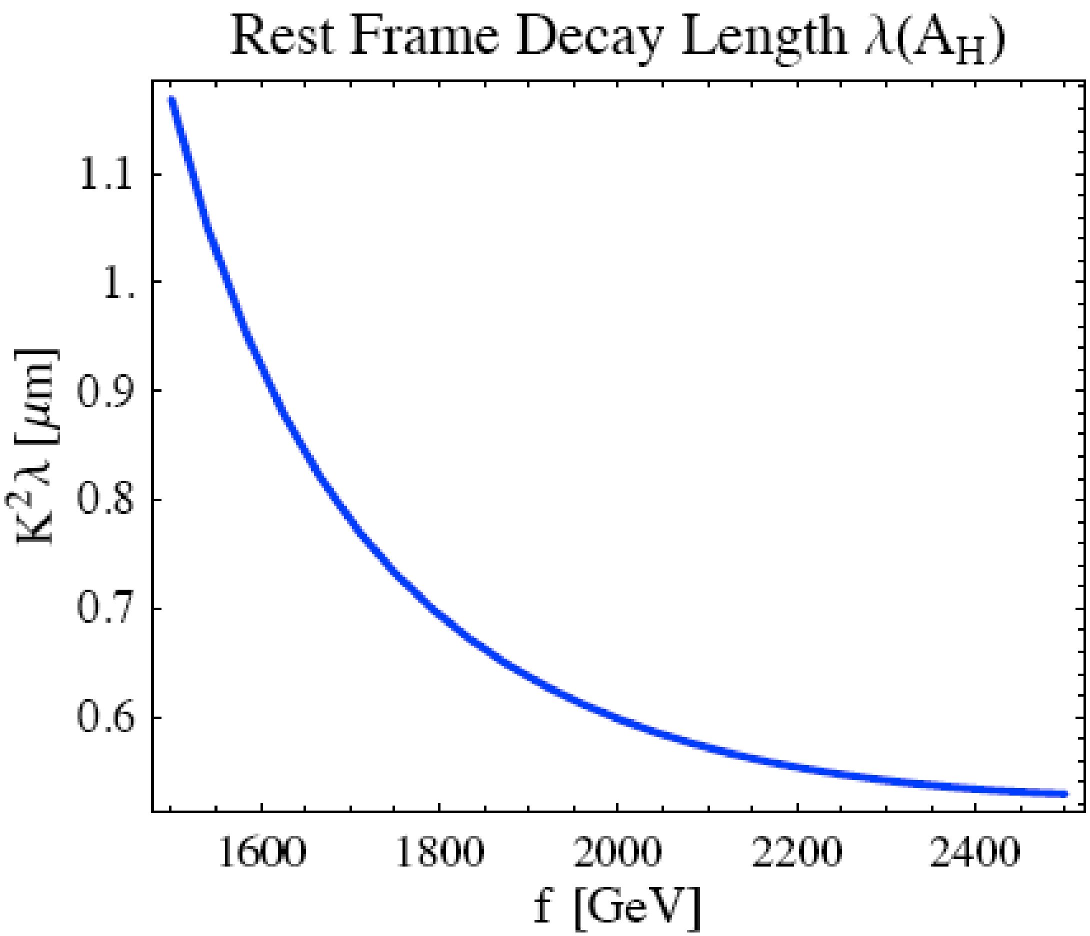
T-parity violating decays

C.Hill and R. Hill

- assume T-parity violated weakly via gauge anomaly
- still suppress radiative corrections
- LTP (A_H) can now decay



fairly prompt Decay rate



Beyond 5-10 TeV

- Many models of the BIG hierarchy (SUSY, compositeness, Randall Sundrum, strong gravity) work great until they hit constraints from low energy experiment.
- Then they have a "little hierarchy problem" of $<$ 1% fine tuning as the scale must be pushed up to satisfy constraints
- LH models are designed to address the "little hierarchy"

Possible UV completions

- weakly coupled SUSY with SUSY scale > 10 TeV
- string theory, strong gravity at 5-10 TeV
- RS with LH living on "10 TeV brane"
- composite LH (like original composite Higgs, but with naturally small v/f)
- *"Science never solves a problem without creating ten more."*

Summary

- "Collective symmetry breaking": gauge, quartic, Yukawa couplings of little Higgs each require 2 or more symmetry breaking interactions
- Spurion analysis: UV sensitivity of potential then arises only at 2 or more loops
- New weakly gauge bosons, scalars, fermions at scale g f~ TeV cancel 1 loop divergences, naturally lead to v~f/10
- Unlike SUSY, cancellations due to ***same*** spin particles
- like SUSY, discrete symmetry solves problems, produces dark matter candidate
- similar collider signals to SUSY with T-parity
- else look for resonances, e.g. T→t Z, t h,bW