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Pion nucleon interaction

Pion nucleon (πN) interaction is an example for the application of an effective field theory (EFT)
here: chiral perturbation theory XPT in its twofold meaning:

- It deals with mesons and baryons instead of quarks
- It is effective in the sense of being efficient

Much of the development of XPT has its roots historically in the study of the πN interaction → highly advanced both in experiment as well as in theory.

XPT and related subjects had been subject of many ZUOZ schools (1972-2000): Straumann, Scheck, Leutwyler, Gasser, Sainio, Kambor, …. Also: Menu 1999 (Zuoz).

Outline

- Topics in πN physics <-> scattering lengths/volumes

- Connection χPT $\leftrightarrow$ πN (historical development)

- Meson-Baryon (πN) observables :

  scattering experiments

  exotic atoms experiments

Topics in πN interaction :

- Extension of soft pion theorems (Goldberger Treiman, Weinberg-Tomozawa, etc....) with χPT
- isospin (non)conservation
- $\sigma_{\pi N}$ -term
- πNN coupling constant

Scattering lengths: notation

scattering amplitudes T: $T_{l\pm}(j = l \pm \frac{1}{2}) = \frac{1}{2i}\left\{\eta_l e^{2i\delta_{l\pm}} - 1\right\}$

Pion energy < 80 MeV: $\eta \approx 1 \qquad T_{l\pm} = e^{i\delta_{l\pm}} \sin\delta_{l\pm}$

Scattering lengths (volumes) a_l
Pion momentum q ->0: $a_l = \lim_{q \to 0}\left(\mathrm{Re}\ T_l / q^{2l+1} \right)$

A_+ : $\pi^+ p \to \pi^+ p$ $\leftrightarrow$ $a_{3/2}$

A_- : $\pi^- p \to \pi^- p$ $\leftrightarrow$ $(2a_{1/2}+a_{3/2})/3$

A_0 : $\pi^- p \to \pi^0 n$ $\leftrightarrow$ $-\ 2^{1/2}(a_{1/2}-a_{3/2})/3$

Isospin triangle:
$A_+ - A_- = 2^{1/2} A_0$

Höhler's notation:

$a^+_{0+} \equiv a^+ = 1/3(a_{1/2}+2a_{3/2})$ isoscalar (: isospineven)

$a^-_{0+} \equiv a^- = 1/3\ (a_{1/2}-a_{3/2})$ isovector (: isospinodd)

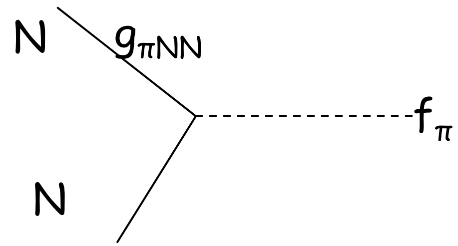
$\pi^+ p \to \pi^+ p$	$\sim a^+ - a^-$
$\pi^- p \to \pi^- p$	$\sim a^+ + a^-$
$\pi^- p \to \pi^0 n$	$\sim a^-$

Scattering lengths (volumes)

- πNN coupling constant: $g_{\pi NN}$

  Strength of coupling of a pion to a nucleon.

  Obtainable from a^- (isovector) + GMO sumrule



- Isospin (non) conservation

  Difference in quark masses + Coulomb effects

  Obtainable from $\pi^+ p$, $\pi^- p$ scattering (isospin triangle); pionic deuterium atom

- Pion nucleon σ-term: $\sigma_{\pi N}$

  Response of the nucleon mass to a change in the quark masses.

  strange quark content of the nucleon

  Obtainable from πN scattering lengths(volumes)

Historical overview *

Pre QCD:

PCAC: Lead to first ideas for chiral symmetry. App. I

$$\partial^{\mu} A_{\mu}^{a} = f_{\pi} m_{\pi}^{2} \Phi_{a}$$

Axial current would be conserved if $m_\pi = 0$

Goldberger Treiman: Links weak and strong interaction terms under the assumption $m_\pi = 0$. App. II
Valid on the % level
$\rightarrow$ corroborates assumption $m_\pi = 0$

* Based mainly on review articles by St. Scherer, J. Gasser, H. Leutwyler cited in the appendices as well as A. Thomas, W. Weise, The Structure of the Nucleon, WILEY-VCH 2000

Historical development ctd.

QCD:

Massless quarks lead to chiral symmetry which in turn would require a parity doubling of hadronic states: not observed in nature.

App. III

Symmetry is hidden (spontaneously broken) ↔ Goldstone theorem applies:

1: massless particle exist with quantum numbers of the field
2: Its coupling to the current does not vanish

GOR relation, chiral condensate,

App. IV

$$m_\pi^2 = -\frac{1}{2f_0^2}(m_u + m_d)\langle 0|\bar{u}u + \bar{d}d|0\rangle + O(m_{u,d}^2, \ldots)$$

Cf. σ-term physics

Isospin symmetry breaking

$$L^{mass} = -\bar{q}Mq = -(\bar{q}_R M q_L + \bar{q}_L M q_L) \quad m_u \bar{u}u + m_d \bar{d}d = \frac{m_u + m_d}{2}(\bar{u}u + \bar{d}d) + \frac{m_u - m_d}{2}(\bar{u}u - \bar{d}d)$$

The most general chiral invariant Lagrangian leads e.g. to PCAC;
Including the mass term provides predictive power

App. V, VI

EFT to higher orders („True" EFT)

S. Weinberg, Physica 96A (1979)327

Phenomenological Lagrangians, (from chapter: current algebra without current algebra)

...phenomenological Lagrangians themselves can be used to justify the calculation of soft-pion matrix elements from tree graphs, without any use of operator algebra.

This remark is based on a „theorem", which as far as I know has never been proven, but which I cannnot imagine could be wrong. The „theorem" says .... This can be put more precisely in the context of perturbation theory: if one writes down the most general possible Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. As I said, this has not been proved, but any counterexamples would be of great interest, and I do not know of any.

H. Leutwyler, Annals Phys. 235 (1994) 165

On the foundations of chiral perturbation theory

Abstract:

The properties of the effective field theory relevant for the low energy structure generated by the Goldstone bosons of a spontaneously broken symmetry are reexamined. It is shown that anomaly free, Lorentz invariant theories are characterized by a gauge invariant effective Lagrangian, to all orders of the low energy expansion. The paper includes a discussion of anomalies and approximate symmetries, but does not cover nonrelativistic effective theories.

Higher orders ctd.

J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984); Nucl. Phys. B **250**, 465(1985)

The work of Gasser, Leutwyler is the basis for a
renormalizable EFT:
chiral perturbation theory
(CHPT or χPT).

It used a path integral representation of the QCD generating functional and introduced external fields (v_μ,a_μ ,s, ps-fields).
Expansion in powers of external momenta and quark masses.
The renormalization constants of Leff are the so-called

LEC constants

to be determined from experiment or lattice QCD or....

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Topics in πN interaction :

Low energy scattering experiments

- Extension of soft pion theorems (Goldberger Treiman, Weinberg-Tomozawa, etc....) with χPT
- **isospin (non)conservation**
- **$\sigma_{\pi N}$ -term**
- πNN coupling constant

Some of next slides from R. Meier's talk at „Graduiertenkolloquium Basel-Tübingen" Basel 2003

Mass term ΔL in the QCD-Lagrangian breaks symmetries!

$L = L_0 - \Delta L$ — L_0 chirally invariant, contains quark- and gluon fields

$$\Delta L = m_u\bar{u}u + m_d\bar{d}d = \underbrace{\tfrac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d)}_{} + \underbrace{\tfrac{1}{2}(m_u - m_d)(\bar{u}u - \bar{d}d)}_{}$$

breaking of — chiral symmetry — isospin symmetry

Measure of explicit χSB:

Sigma term

Extracted from πp elastic scattering data

Strangeness content of the nucleon

If isospin is conserved:

$A_+ = A(\pi^+ p \to \pi^+ p)$

$A_- = A(\pi^- p \to \pi^- p) \quad \to \quad A_0 = \dfrac{A_+ - A_-}{\sqrt{2}}$

$A_0 = A(\pi^- p \to \pi^0 n)$

Test by comparison of πp elastic scattering and SCX

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The pion nucleon σ term

$$\sigma_N = \tfrac{1}{2}(m_u + m_d)\langle N|\bar{u}u + \bar{d}d|N\rangle \quad = m_q\langle N|\bar{u}u + \bar{d}d|N\rangle$$

σ-term of the nucleon from baryon masses

Dynamic mass

$$M_N = M_0 + \underbrace{m_q\langle N|\bar{u}u + \bar{d}d|N\rangle}_{\sigma_N} + m_s\langle N|\bar{s}s|N\rangle$$

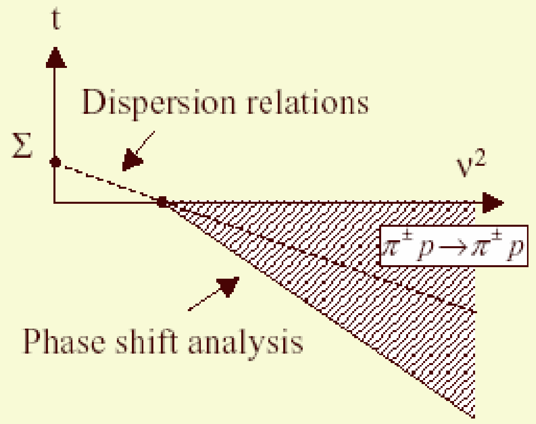
Mass from u,d quark masses

$$\sigma_N(1-y) = \frac{m_q}{m_s - m_q}(M_{\Xi^0} + M_{\Sigma^0} - 2M_n) \quad + o(m_q)$$

Strange sea quark content $\quad y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}$

$$\sigma_N(M_{Baryon}) = \frac{36 \pm 7\ MeV}{1-y}$$

σ term of the nucleon from πp scattering



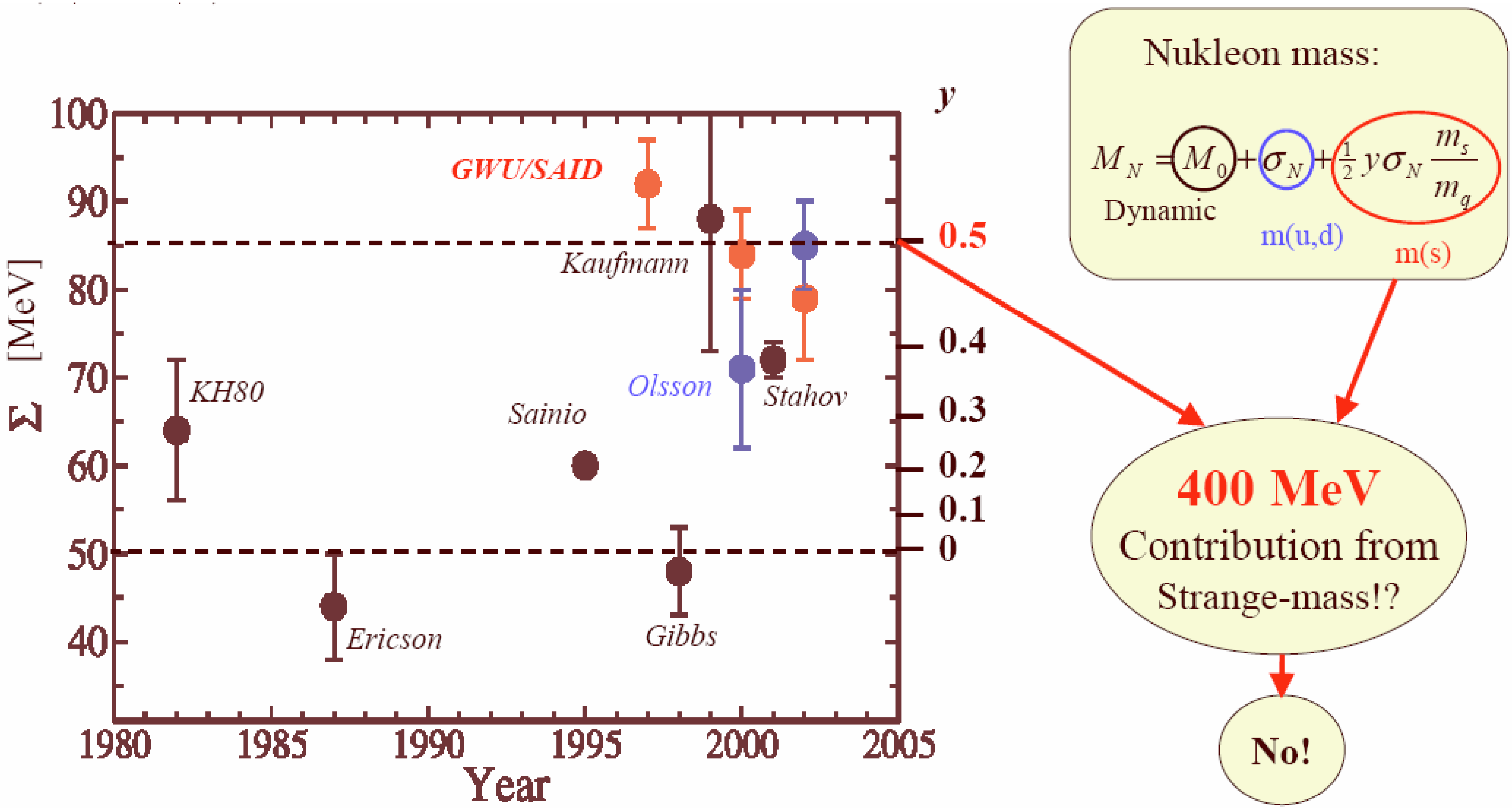
Σ from scat. ampl. at Cheng-Dashen point

$$\sigma_N(\pi\, p) = \Sigma - \underbrace{15 MeV}_{\chi PT}$$

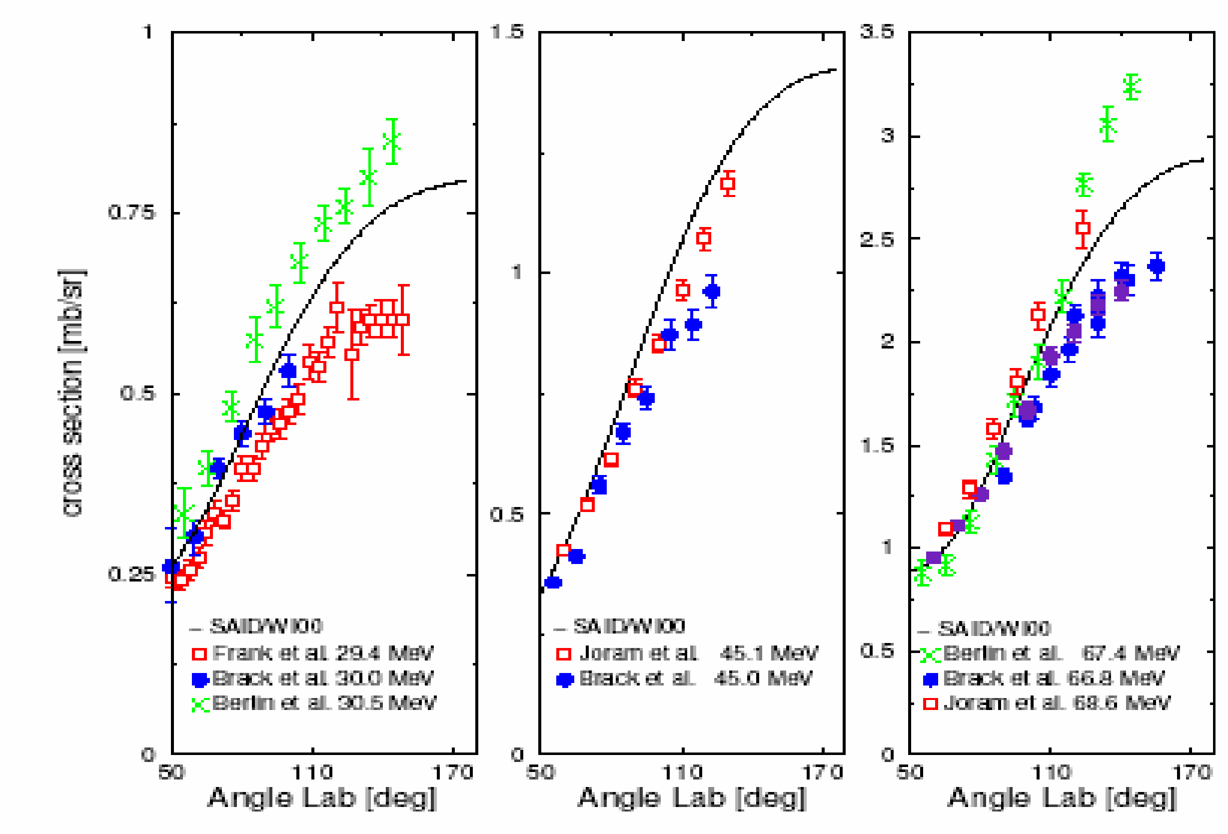
$$\frac{36 \pm 7\ MeV}{1-y} = \Sigma - 15 MeV$$

Gasser Ann. Phys (1981)
Borasoy, Meißner Ann Phys (1997)

Bernard et al. Z. Phys. (1993)
Becher et al. EurPhys.J. (1999)


Σ [MeV]
Year
y
GWU/SAID
Kaufmann
KH80
Sainio
Olsson
Stahov
Ericson
Gibbs
0.5
0.4
0.3
0.2
0.1
0
Nukleon mass:
$M_N = M_0 + \sigma_N + \frac{1}{2} y \sigma_N \frac{m_s}{m_q}$
Dynamic
m(u,d)
m(s)
400 MeV
Contribution from
Strange-mass!?
No!

Data base suboptimal: → CHAOS @ TRIUMF, LEPS, CEX @PSI



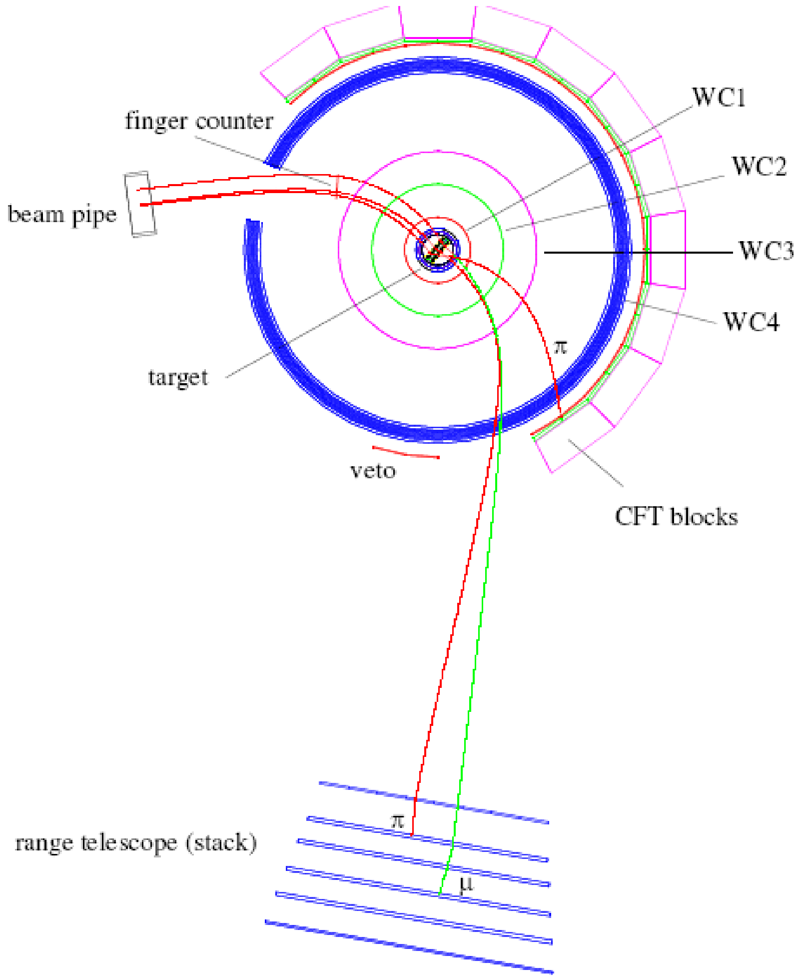
Extension of data base with CHAOS/TRIUMF

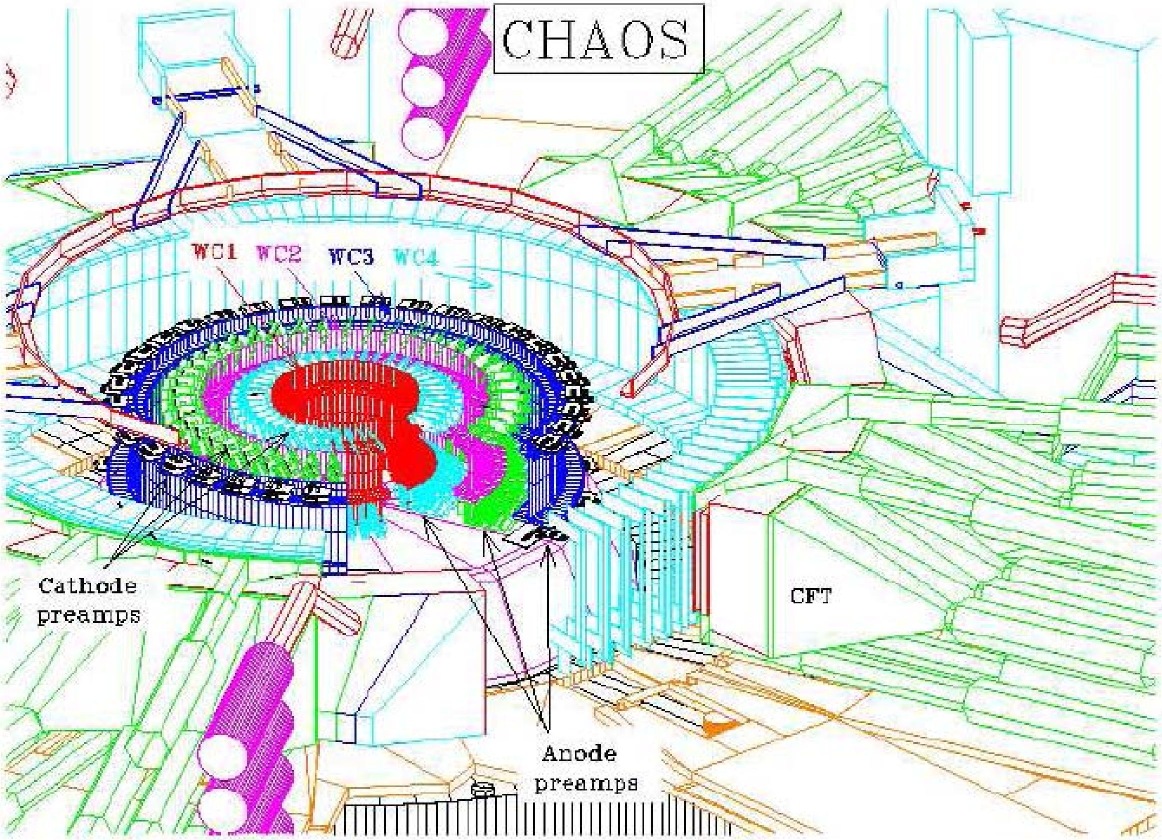
$\pi^{\pm} p$ differential cross sections at low energies

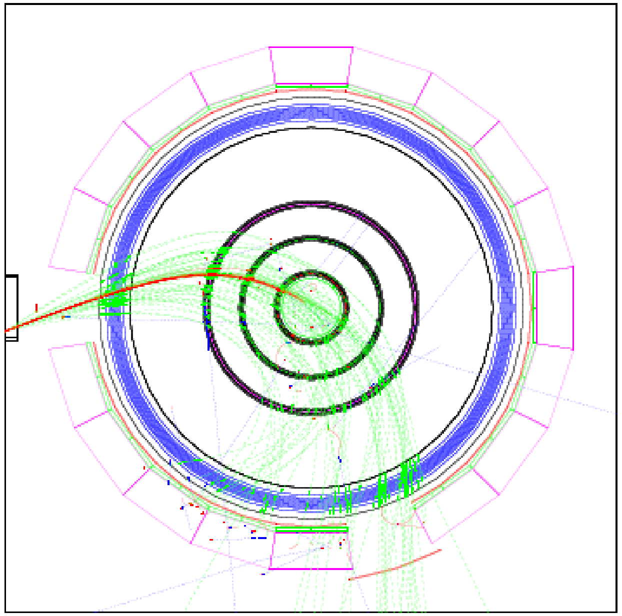
H. Denz [a,*], P. Amaudruz [b], J.T. Brack [c], J. Breitschopf [a], P. Camerini [d,e], J.L. Clark [f], H. Clement [a], L. Felawka [b], E. Fragiacomo [e], E.F. Gibson [g], N. Grion [e], G.J. Hofman [b,h], B. Jamieson [b], E.L. Mathie [h], R. Meier [a], G. Moloney [f], D. Ottewell [b], O. Patarakin [i], J.D. Patterson [c], M.M. Pavan [b], S. Piano [d,e], K. Raywood [b], R.A. Ristinen [c], R. Rui [d,e], M.E. Sevior [f], G.R. Smith [b,j], J. Stahov [k], R. Tacik [h], G.J. Wagner [a], F. von Wrochem [a], D.M. Yeomans [h]

[a] *Physikalisches Institut, Universität Tübingen, D-72076 Tübingen, Germany*
[b] *TRIUMF, Vancouver, British Columbia, Canada V6T 2A3*
[c] *University of Colorado, Boulder, CO 80309-0446, USA*
[d] *Dipartimento di Fisica dell'Università di Trieste, I-34127 Trieste, Italy*
[e] *Istituto Nazionale di Fisica Nucleare, I-34127 Trieste, Italy*
[f] *School of Physics, University of Melbourne, Parkville, Victoria 3052, Australia*
[g] *California State University, Sacramento, CA 95819, USA*
[h] *University of Regina, Regina, Saskatchewan, Canada S4S 0A2*
[i] *Kurchatov Institute, Moscow, Russia*
[j] *Jefferson Lab, Newport News, VA 23006, USA*
[k] *University Tuzla, Faculty of Science, 35000 Tuzla, Bosnia and Herzegovina*

Physics Letters B633 209 (2006)

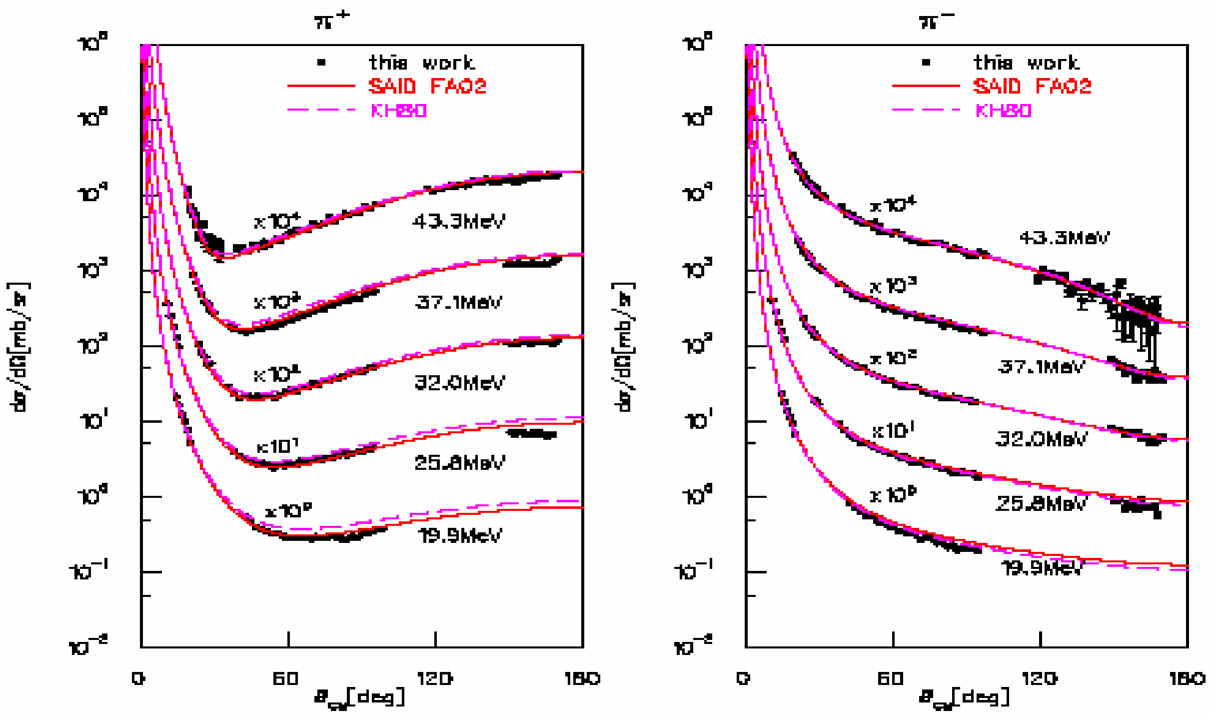

WC1
WC2
WC3
WC4
finger counter
beam pipe
target
π
veto
CFT blocks
range telescope (stack)
π
μ


CHAOS
WC1
WC2
WC3
WC4
Cathode
preamps
CFT
Anode
preamps



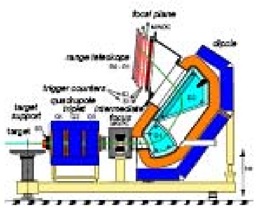
Drastic improvement of low energy scattering data base

CHAOS: H. Denz et al. Phys. Lett. B 633, 299 (2006)



Up to now a qualitative statement only:

. Although these shifted values of the scattering length correspond to a πN-sigma term at the low end of the range currently being discussed, it is very important to recognize that such extracted physics quantities are best determined through a full PSA, also making use of the complementary data available at energies above those of this work.



PSI - LEPS Collaboration

Low Energy Pion Spectrometer

R. Bilger, J. Breitschopf, H. Clement, M. Cröni, H. Denz,
R. Meier, J. Pätzold, G. J. Wagner
Physikalisches Institut, Univ. Tübingen, Tübingen, Germany

B. van den Brandt, P. Hautle, J. A. Konter, S. Mango
Paul Scherrer Institut, Villigen, Switzerland

G. J. Hofman, M. Pavan
TRIUMF, Vancouver, British Columbia, Canada

J. Comfort
Arizona State University, Tempe, Arizona, USA

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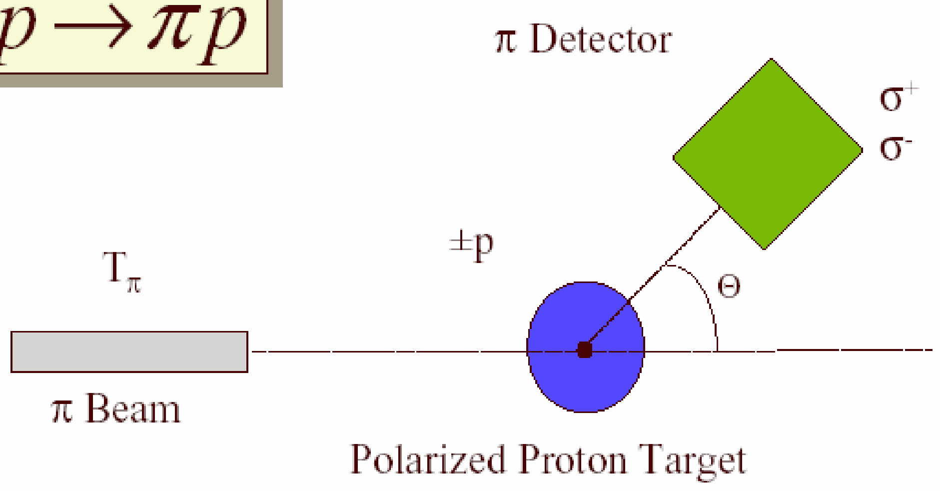
School of Physics, Univ. of Edinburgh, UK

E. Friedman
Racah Inst. Of Physics, The Hebrew University, Jerusalem, Israel

Analyzing power measurement at PSI: principle

$\pi p \rightarrow \pi p$



$$A_y = \frac{1}{p} \cdot \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$$

Background suppression:

Active target signal

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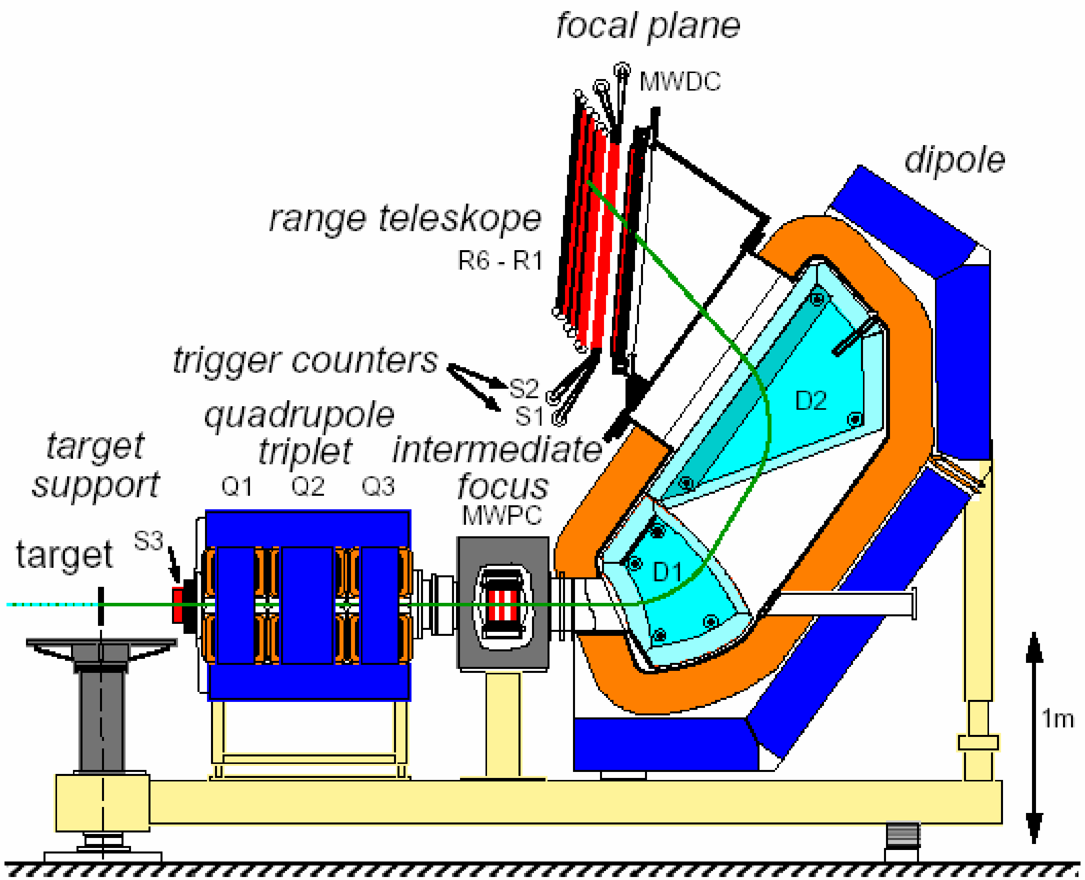
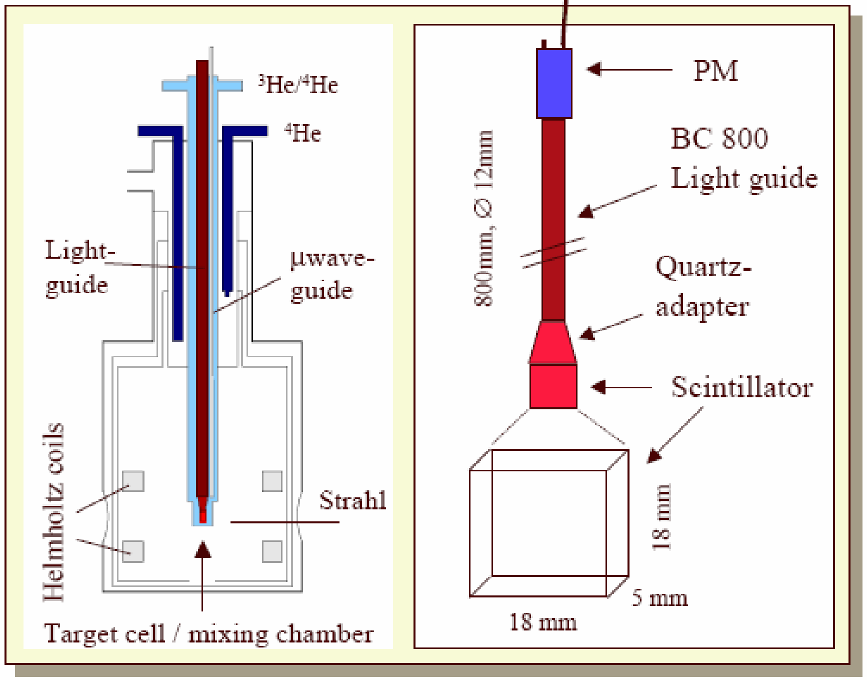
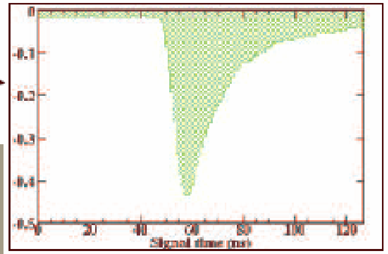
Low Energy Pion Spectrometer



Polarized target at PSI

B. van den Brandt et al., NIM A446 (2000)

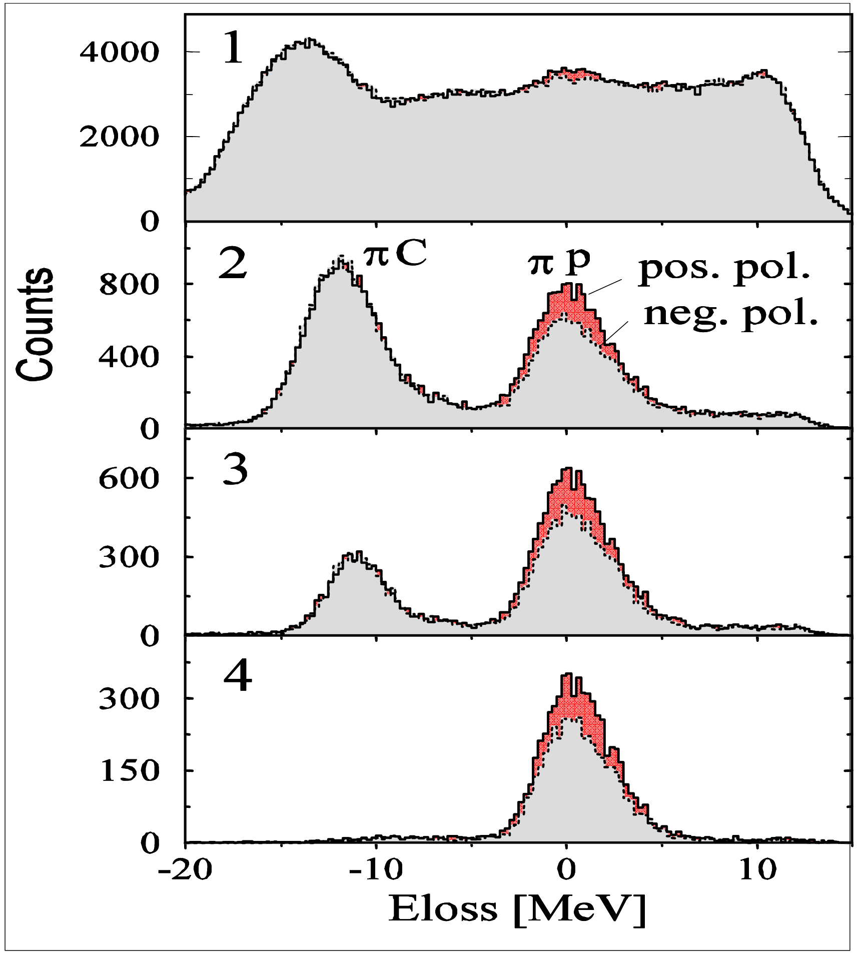




Focal plane spectrum

1. without constraint

2.-4. with increasing size of active target signal.



π^+ scattering at $T_\pi = 68.6$ MeV, $\theta_{cm} = 81.3°$

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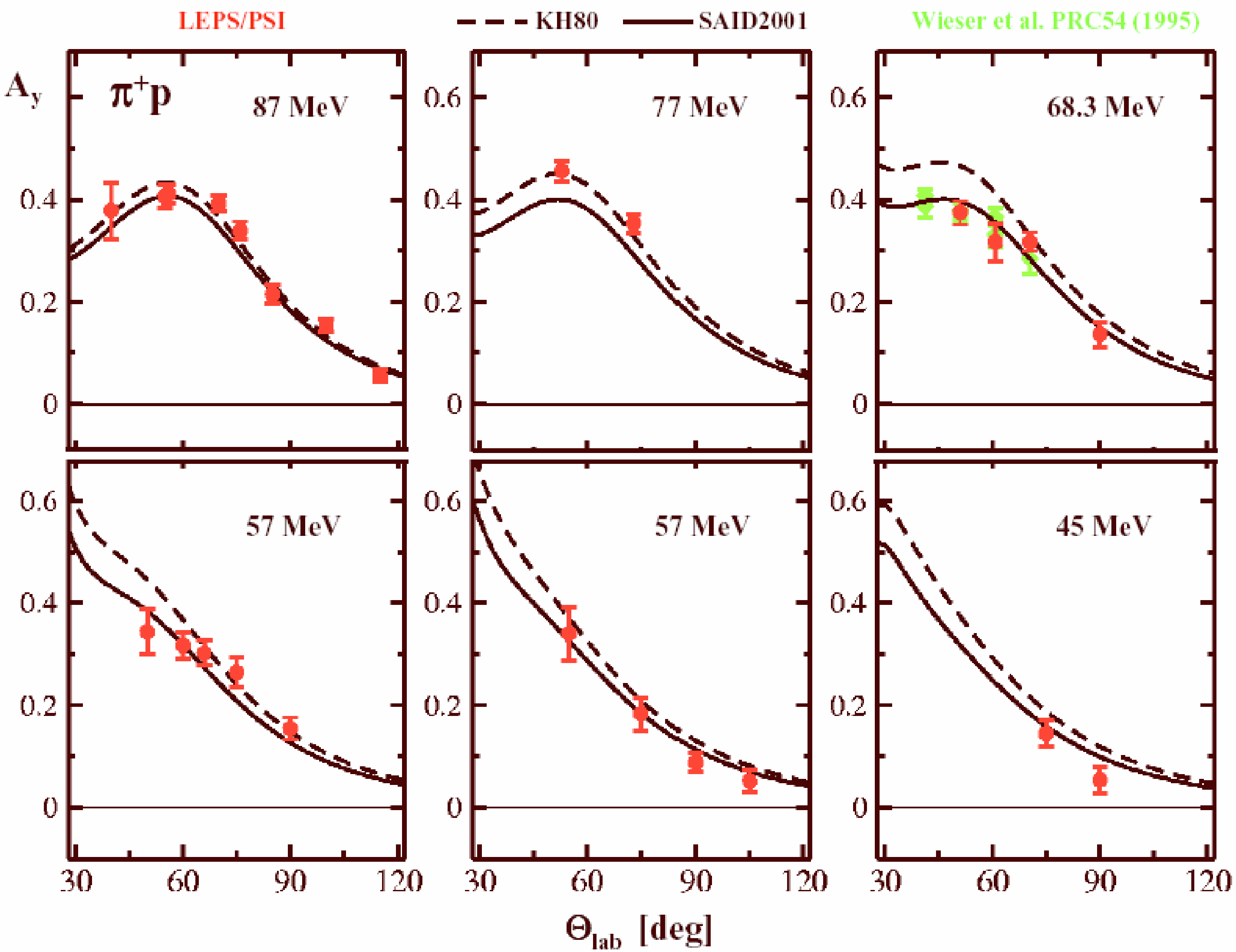
PSI



CEX experiment at PSI: isospin violation

Pionic Charge Exchange on the Proton from 40 to 250 MeV

J. Breitschopf,[1] M. Bauer,[1] H. Clement,[1] M. Cröni,[1] H.Denz,[1]
E. Friedman,[2] E.F. Gibson,[3] R. Meier,[1] and G.J. Wagner[1,*]

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[2]*Racah Institute of Physics, The Hebrew University, Jerusalem, Israel*
[3]*California State University, Sacramento, California 95819, U.S.A.*

(Dated: May 22, 2006)

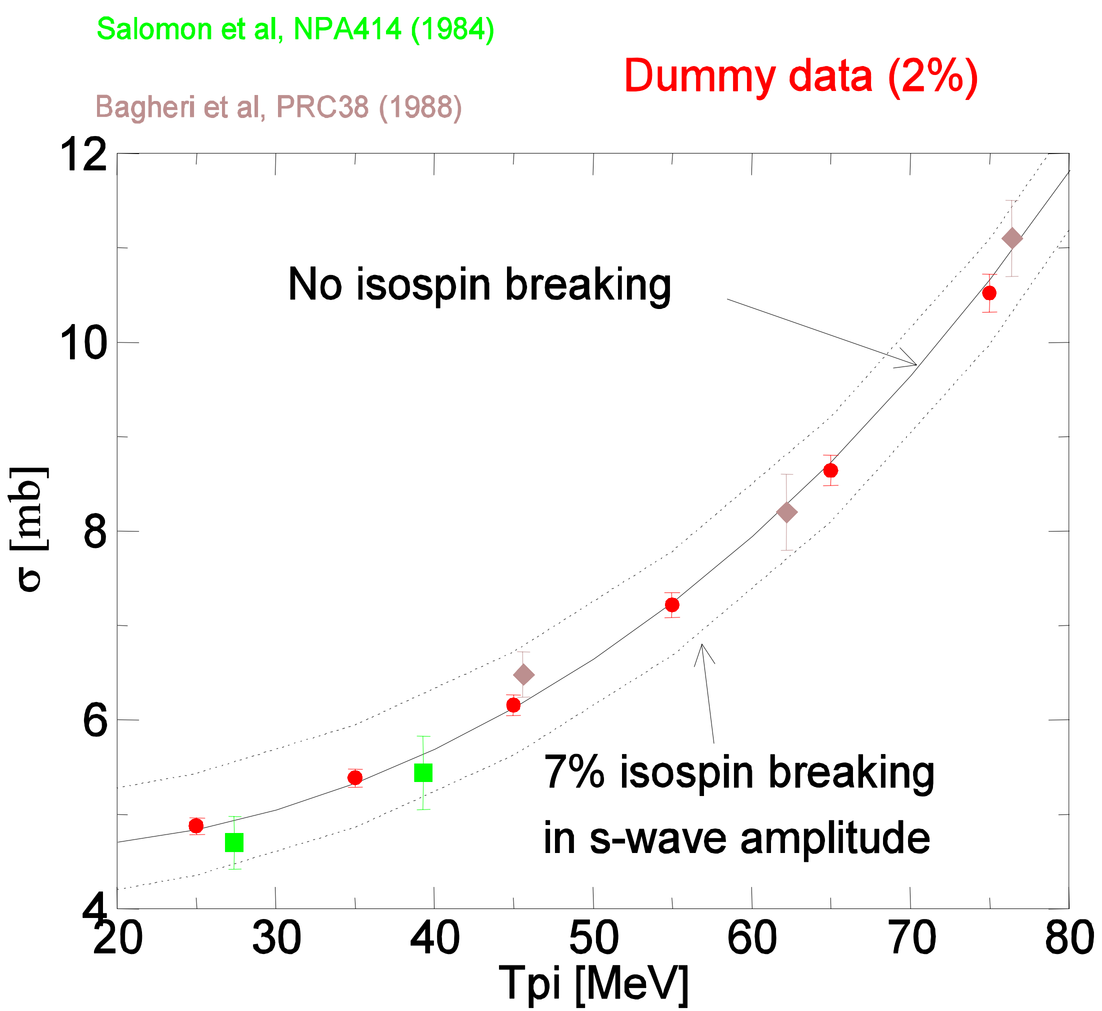
Abstract

The total cross sections for pionic charge exchange on hydrogen were measured using a transmission technique on thin CH_2 and C targets. Data were taken for π^- lab energies from 39 to 247 MeV with total errors of typically 2 % over the Δ-resonance and up to 10 % at the lowest energies. Deviations from the predictions of the SAID phase shift analysis in the 60-80 MeV region are interpreted as evidence for isospin-symmetry breaking in the *s*-wave amplitudes. The charge dependence of the Δ-resonance properties appears to be smaller than previously reported.

W.R. Gibbs et al. Phys. Rev. Lett. **74** 3740 (1995)
E. Matsinos, Phys. Rev. **C 56** 3014 (1997)

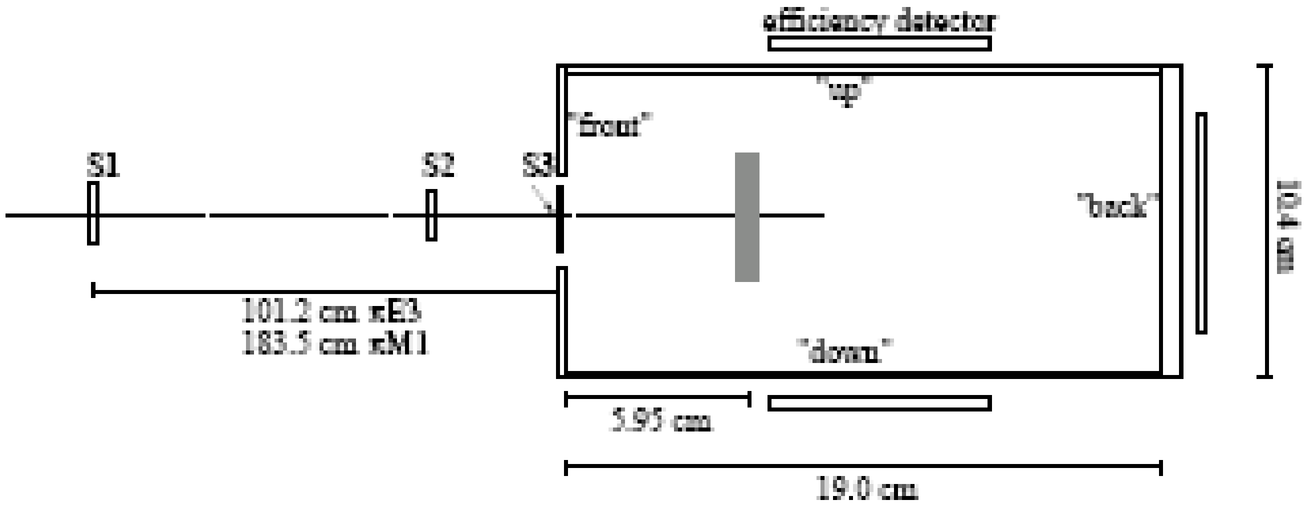
Isospin breaking of about 7% in s-wave amplitudes near Tp=50 MeV

Suspicion: Charge exchange reaction may be the culprit (particularly scarce data base)



Salomon et al, NPA414 (1984)
Bagheri et al, PRC38 (1988)
Dummy data (2%)
No isospin breaking
7% isospin breaking
in s-wave amplitude
σ [mb]
Tpi [MeV]
12
10
8
6
4
20
30
40
50
60
70
80

Transmission technique to measure CEX total cross section:
$T_i/T_0 = \exp(-(\alpha_i \sigma_i))$; α: thickness; i: C, CH2



Features: Detection efficiency monitored > 99%
Targets interchanged every 20-30 minutes, equivalent Carbon amount
measurements at two different beams at PSI
Checks with positive pions (as well as muons/electrons: zero result)

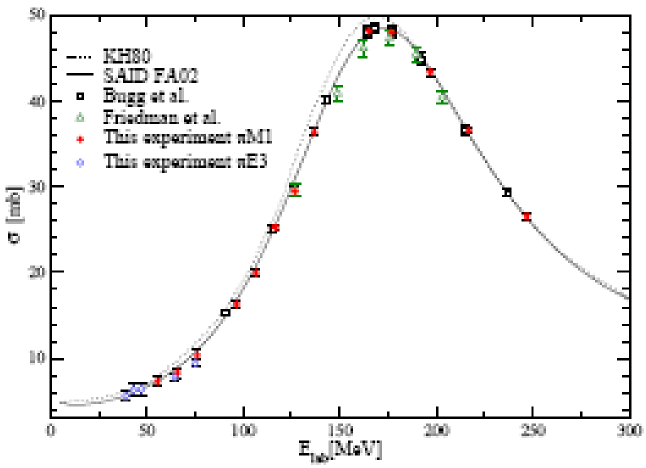


FIG. 2: Total CX cross sections from this and preceding [10, 11] transmission experiments. The error bars represent the total errors. Results from both pion beam lines used in the present experiment are shown separately. The solid and dashed curves represent the results from the phase shift analyses SAID-FA02 [9] and KH80 [18], respectively.

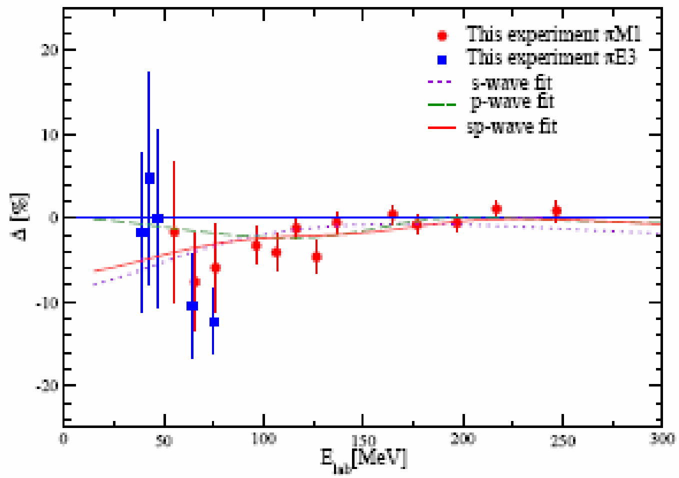


FIG. 3: Cross sections from this experiment with total errors, plotted as percent deviation from the SAID-FA02 [9] predictions. The curves represent the results of fit procedures with a slight modification of the S-amplitudes (dotted), the P_{33}-amplitude (dashed) or both (solid).

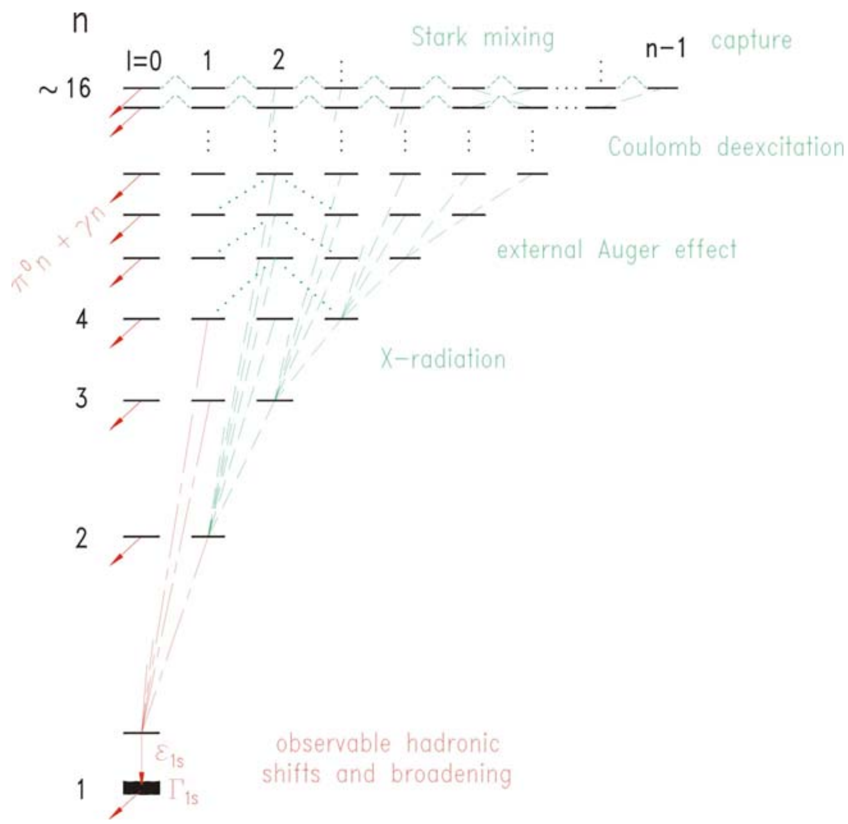
Main results: Discrepancy between two previous transmission experiments resolved.
Isospin symmetry breaking smaller than assumed by Gibbs, Matsinos, but still existing

Topics in πN interaction :

Experiment with exotic atoms: pionic hydrogen and deuterium

- **Extension of soft pion theorems (Goldberger Treiman, Weinberg-Tomozawa, etc....) with χPT**
- **isospin (non)conservation**
- $\sigma_{\pi N}$ -term: needs scattering volume **not** provided by πp, πd
- **πNN coupling constant**

Pionic hydrogen experiment at PSI



Deser-Trueman formula

S. Deser et al.. Phys. Rev. 96, 774 (1954)
G. Rasche and W.S. Woolcock, NP A381 405 (1982)

$$\varepsilon_{1s} = -4E_{1s}\frac{1}{r_B}\left(a_{\pi^- p\to\pi^- p}\right)\left(1+\delta_\varepsilon\right) \qquad E_{1s} \approx 7\ \text{eV}$$

$$\Gamma_{1s} = 8E_{1s}\frac{Q_0}{r_B}\left(1+\frac{1}{P}\right)\left(a_{\pi^- p\to\pi^0 n}\left(1+\delta_\Gamma\right)\right)^2 \qquad \Gamma_{1s} \approx 1\ \text{eV}$$

E_{1s}: e.m. binding energy of ground state: 3238 eV
r_B : Bohr radius pionic hydrogen: 222.56 fm
Q_0 : kinematic factor: 0.142 fm^{-1}
P : Panofsky ratio: 1.546±0.009
$\delta_{\varepsilon,\Gamma}$: e. m. corrections: under debate

Goals:

$\varepsilon_{1s} \rightarrow$ $a^+ + a^-$ 0.2%
$\Gamma_{1s} \rightarrow$ $(a^-)^2$ 1%

Debrecen – Coimbra – Ioannina – Jülich – Leicester – Paris – PSI - Vienna

H.-Ch.- Schröder et al., Eur. Phys. J. C21,473 (2001): ε_{1s}=-7.105±0.013$_{stat.}$±0.034$_{syst.}$, Γ_{1s} =0.868±0.04$_{stat.}$±0.038$_{syst.}$eV

Motivation?

S. Weinberg, Phys. Rev. Lett. 17, 616 (1966)
Y. Tomozawa, Nuovo Cimento A, 707, (1966)

$$a^- = \frac{M_\pi}{8\pi(1+\mu)F_\pi^2}(1+........) \quad ; \qquad \mu = \frac{M_\pi}{M_N} \qquad : 79\ [10^{-3}\ m_\pi^{-1}]$$

$$a^+ = 0 \qquad\qquad + ...$$

Higher orders

E. Jenkins and A.V. Manohar, Phys. Lett. B255, 558(1991) : HBCHPT

T. Becher and H. Leutwyler, JHEP0106,017(2001): manifestly Lorentz invariant

ππ : expansion to 6th order in chiral dim. (no. of derivatives and/or quark masses)
πN : expansion to 4th order in chiral dim.

$$L_{\pi\pi} = L^{(2)}{}_{\pi\pi} + L^{(4)}{}_{\pi\pi} + L^{(6)}{}_{\pi\pi}$$

	$L^{(2)}_{\pi\pi}$	$L^{(4)}_{\pi\pi}$	$L^{(6)}_{\pi\pi}$
Number of LEC	2	7	53

$$L_{\pi N} = L^{(1)}{}_{\pi N} + L^{(2)}{}_{\pi N} + L^{(3)}{}_{\pi N} + L^{(4)}{}_{\pi N}$$

	$L^{(1)}_{\pi N}$	$L^{(2)}_{\pi N}$	$L^{(3)}_{\pi N}$	$L^{(4)}_{\pi N}$
Number of LEC	2	7	23	118

N. Fettes and U. G. Meissner, Nucl. Phys. **A676**, 311 (2000)

2. order 3. order

$$a_{0+}^{+} = \frac{M_\pi^2[-g_A^2 + 8m(-2c_1 + c_2 + c_3)]}{16\pi(m+M_\pi)F_\pi^2} + \frac{3g_A^2 m M_\pi^3}{256\pi^2(m+M_\pi)F_\pi^4}$$

$$- \frac{g_A^2 M_\pi^4}{64\pi(m+M_\pi)m^2 F_\pi^2} - \frac{4M_\pi^4 c_1 c_2}{\pi(m+M_\pi)F_\pi^2} + \frac{2mM_\pi^4 c_1 \ell_3}{\pi(m+M_\pi)F_\pi^4}$$

$$- \frac{g_A M_\pi^4(2\bar{d}_{16} - \bar{d}_{18})}{4\pi(m+M_\pi)F_\pi^2}$$

$$+ \frac{2M_\pi^4 m(2\bar{e}_{14} + 2\bar{e}_{15} + 2\bar{e}_{16} + 2\bar{e}_{19} + 2\bar{e}_{20} + 2\bar{e}_{35} - \bar{e}_{36} - 4\bar{e}_{38})}{\pi(m+M_\pi)F_\pi^2}$$

$$- \frac{M_\pi^4[8 - 3g_A^2 + 2g_A^4 + 4m(2c_1 - c_3)]}{256\pi^3(m+M_\pi)F_\pi^4},$$

4. order

1. order

$$a_{0+}^{-} = \frac{mM_\pi}{8\pi(m+M_\pi)F_\pi^2} + \frac{M_\pi^3[g_A^2 + 32m^2(\bar{d}_1 + \bar{d}_2 + \bar{d}_3 + 2\bar{d}_5)]}{32\pi m(m+M_\pi)F_\pi^2}$$

$$+ \frac{M_\pi^3 m}{64\pi^3(m+M_\pi)F_\pi^4},$$

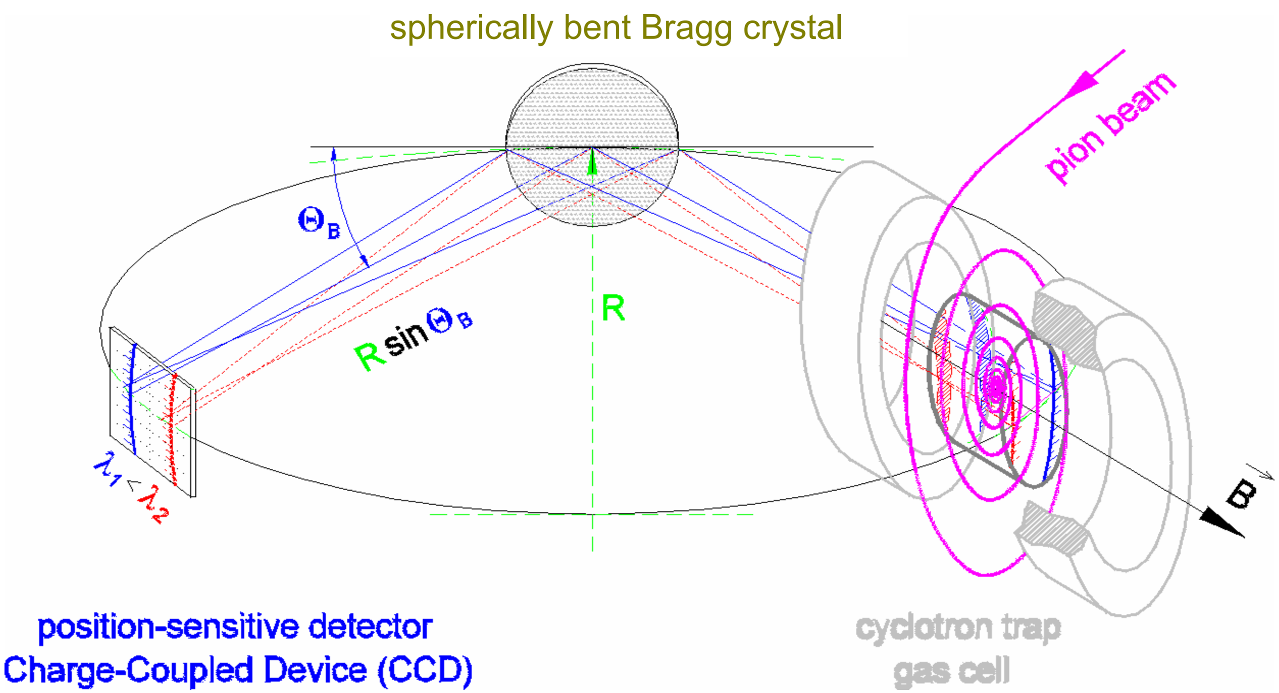
3. order; 4. order vanishes exactly

Bernard, Kaiser, Meissner Phys. Rev C 52, 2185 (1995)

PAUL SCHERRER INSTITUT

Principle of the X-ray experiment at PSI

ultimate energy resolution



position & energy resolution

⇒ **background reduction** *by analysis of hit pattern*

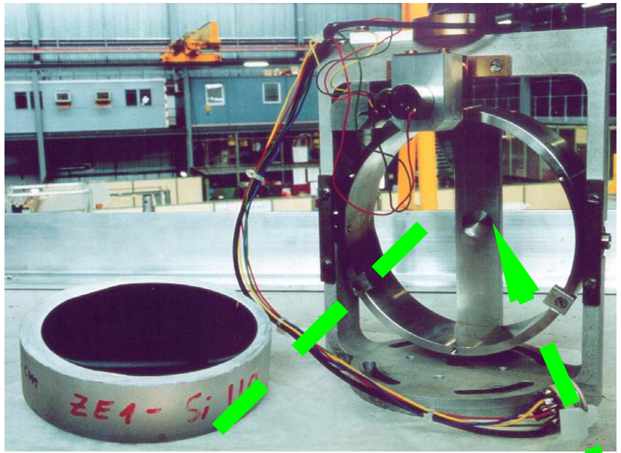
high stop density

⇒ **high X - ray line yields**

⇒ **bright X - ray source**

Spherically curved Bragg crystal

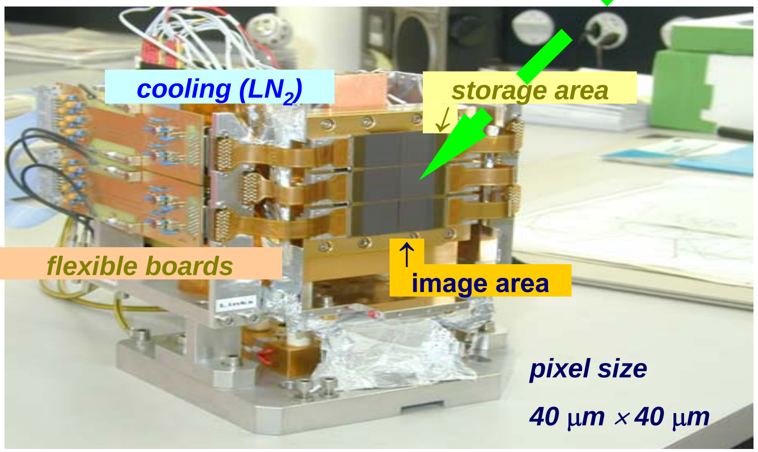
R = 3 m



Large - Area Focal Plane Detector

CYCLOTRON TRAP

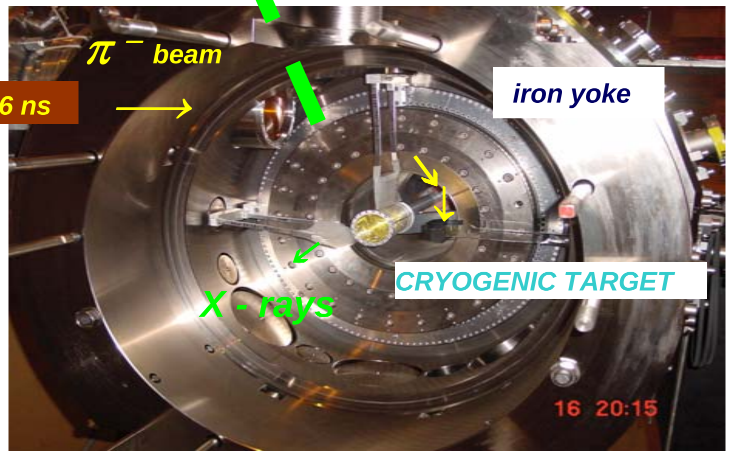
one coil removed



N. Nelms et al., Nucl. Instr. Meth 484 (2002) 419

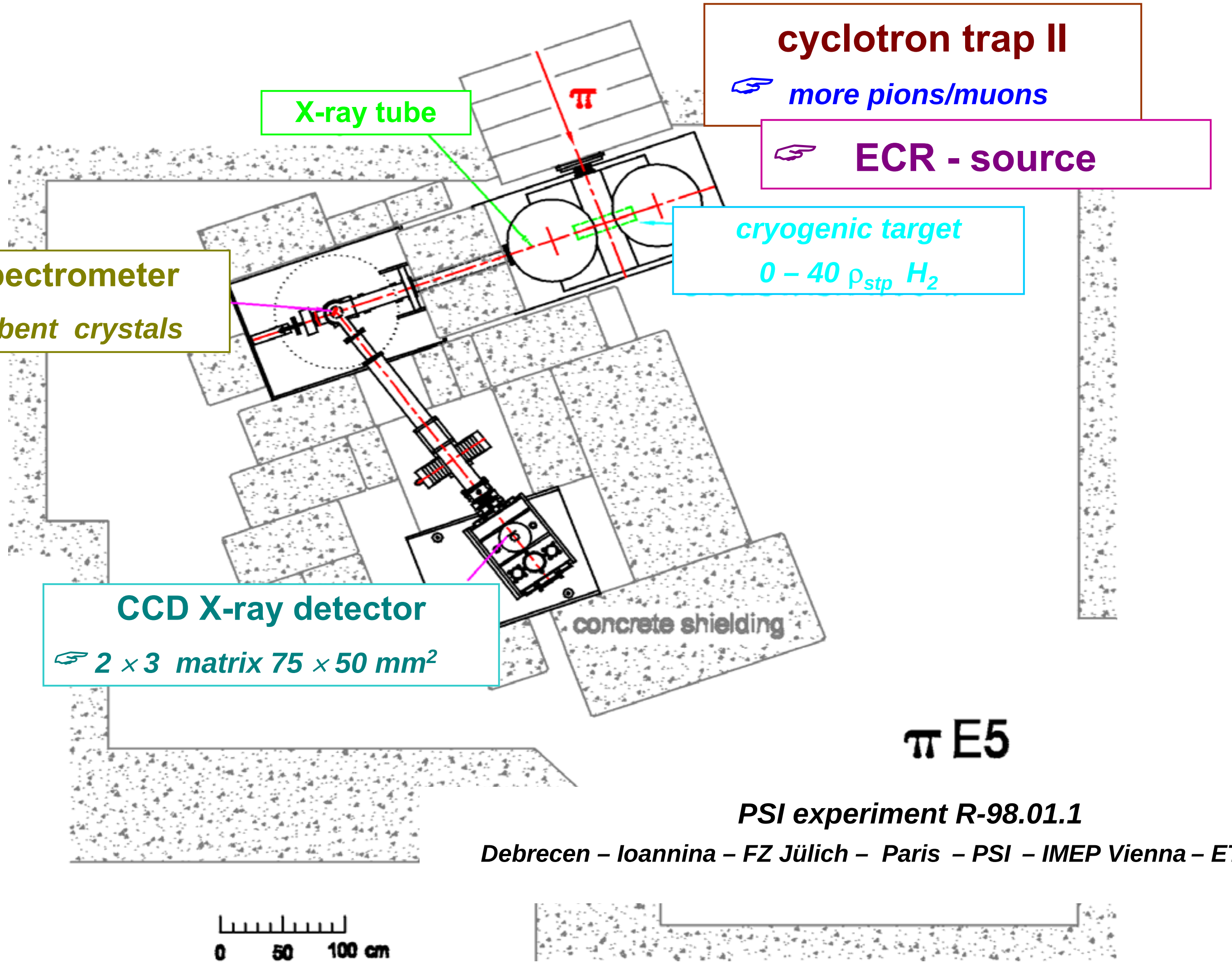
10^9 π/s

$\tau_{\pi\pm}$ = 26 ns



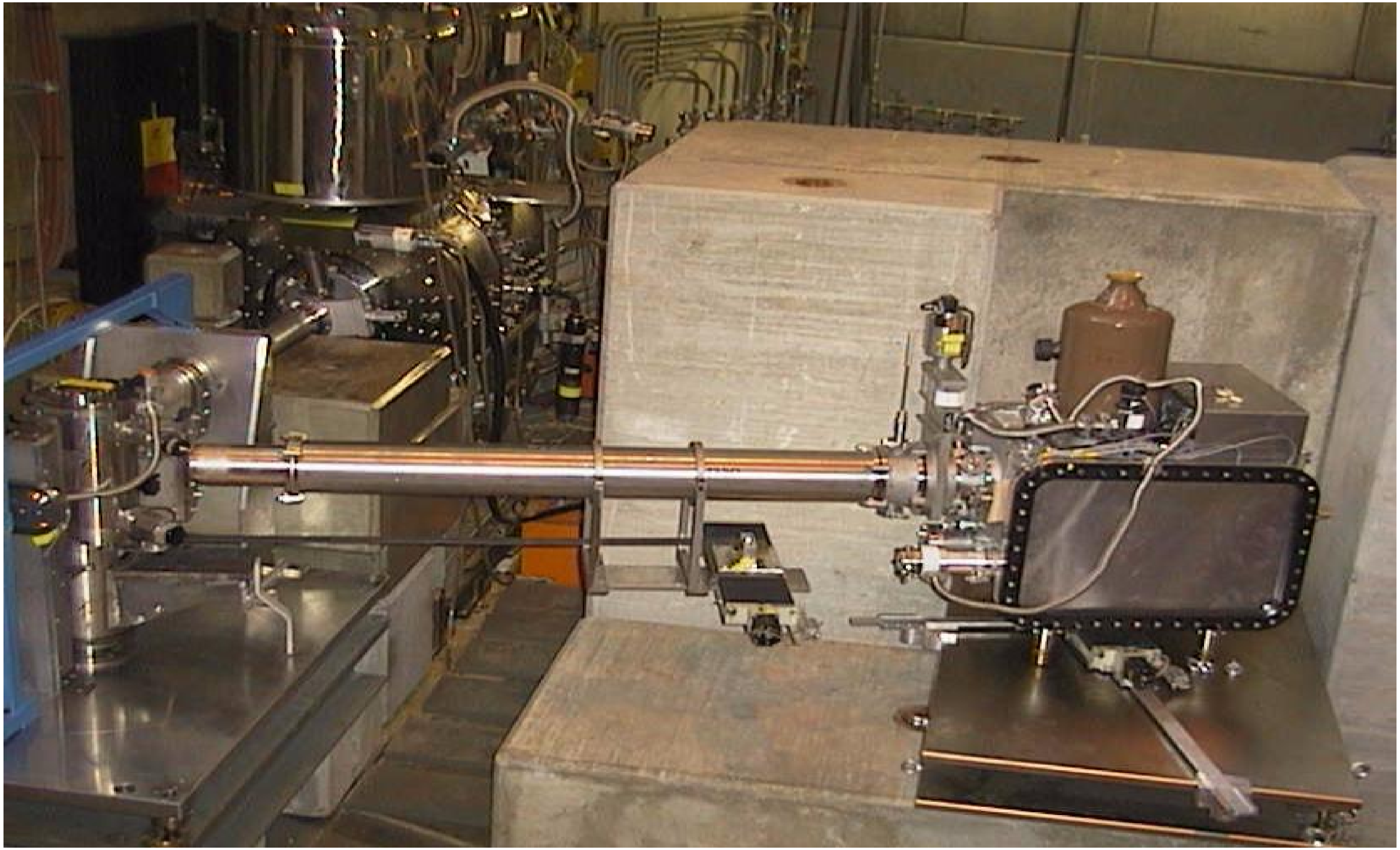
L. M. Simons, Hyperfine Interactions 81 (1993) 253

Arrangement



PSI experiment R-98.01.1

Debrecen – Ioannina – FZ Jülich – Paris – PSI – IMEP Vienna – ETH Zürich



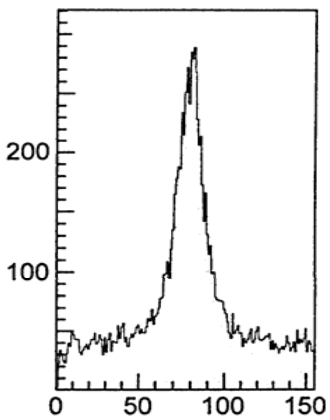
„Trivial“ difficulties

- Statistics: cyclotron trap (6) + spherically bent crystals (3-4)

  Up to now 68000 events accumulated (12)
- Background : New CCD detectors + much improved shielding (10)

In green: improvement factors compared to PSI experiment ETHZ-Neuchâtel-PSI

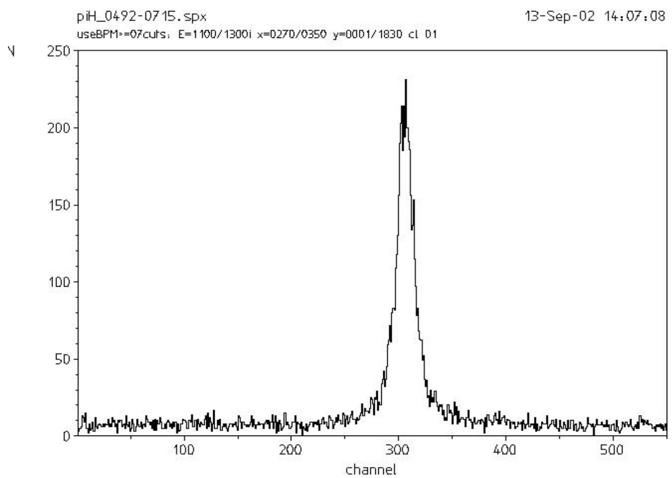
Small fit range:
→ error in Γ
(~ 100meV too small)



← Width: 12 eV →

↑

ETHZ-Neuchâtel-PSI



← Width: 72 eV →

↑

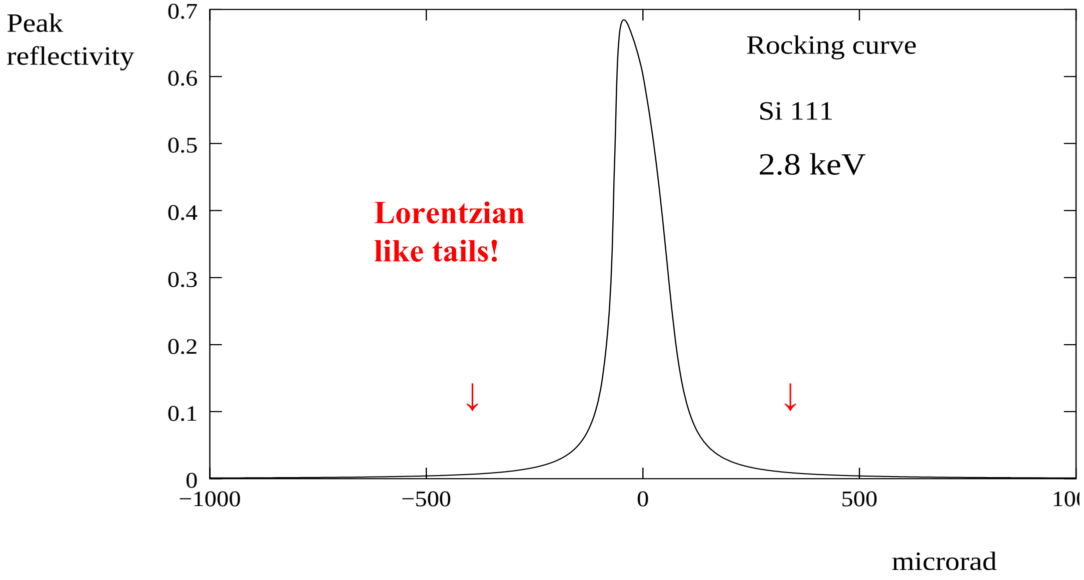
R98-01.1

From Monte Carlo simulation:
Fit error for this spectrum is
12%

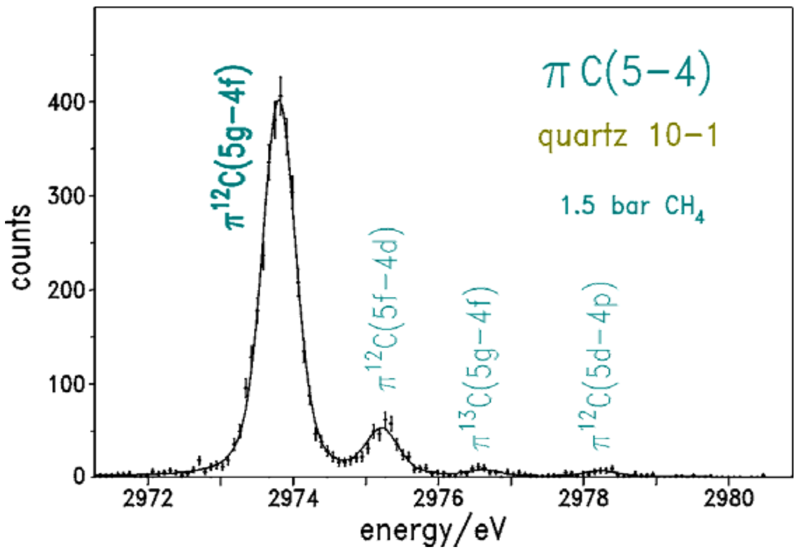
From Monte Carlo simulations:
Fit error is
5 %

Less trivial : line shape

There are Lorentzian tails in the response function.
$\Delta\Gamma/\Gamma$ < 1% requires a good knowledge of the resp.fct.:
Needed: > 30000 events + „no“ background.
Previous experiment (calibration πBe-1400 cts): $\Delta\Gamma/\Gamma$ = 8%
Present experiment: First round (πC-4500cts): $\Delta\Gamma/\Gamma$ =3.5%



First trial: response function via exotic atoms



Drawback: missing intensity

CH_4 1500 mbar @ T = 295K

πC(5g-4f) - @ 2974 eV

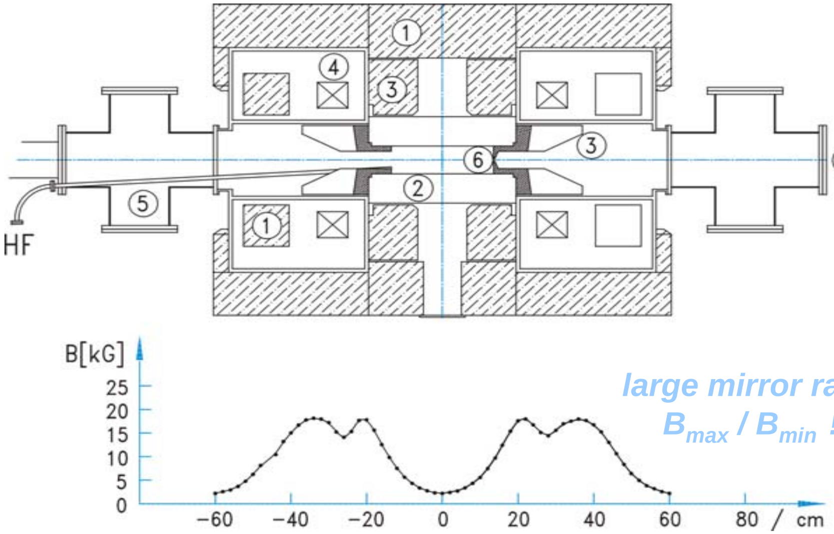
quartz 10-1

$\Delta E = 478 \pm 29$ meV (FWHM)

Si 111

$\Delta E = 504 \pm 16$ meV (FWHM)

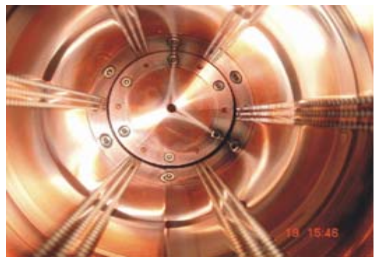
PSI ECRIT - Electron – Cyclotron – Resonance Ion Trap



large mirror ratio
B_{max} / B_{min} !

FIRST PLASMA

INSIDE HEXAPOLE



argon / oxygen (1/9)

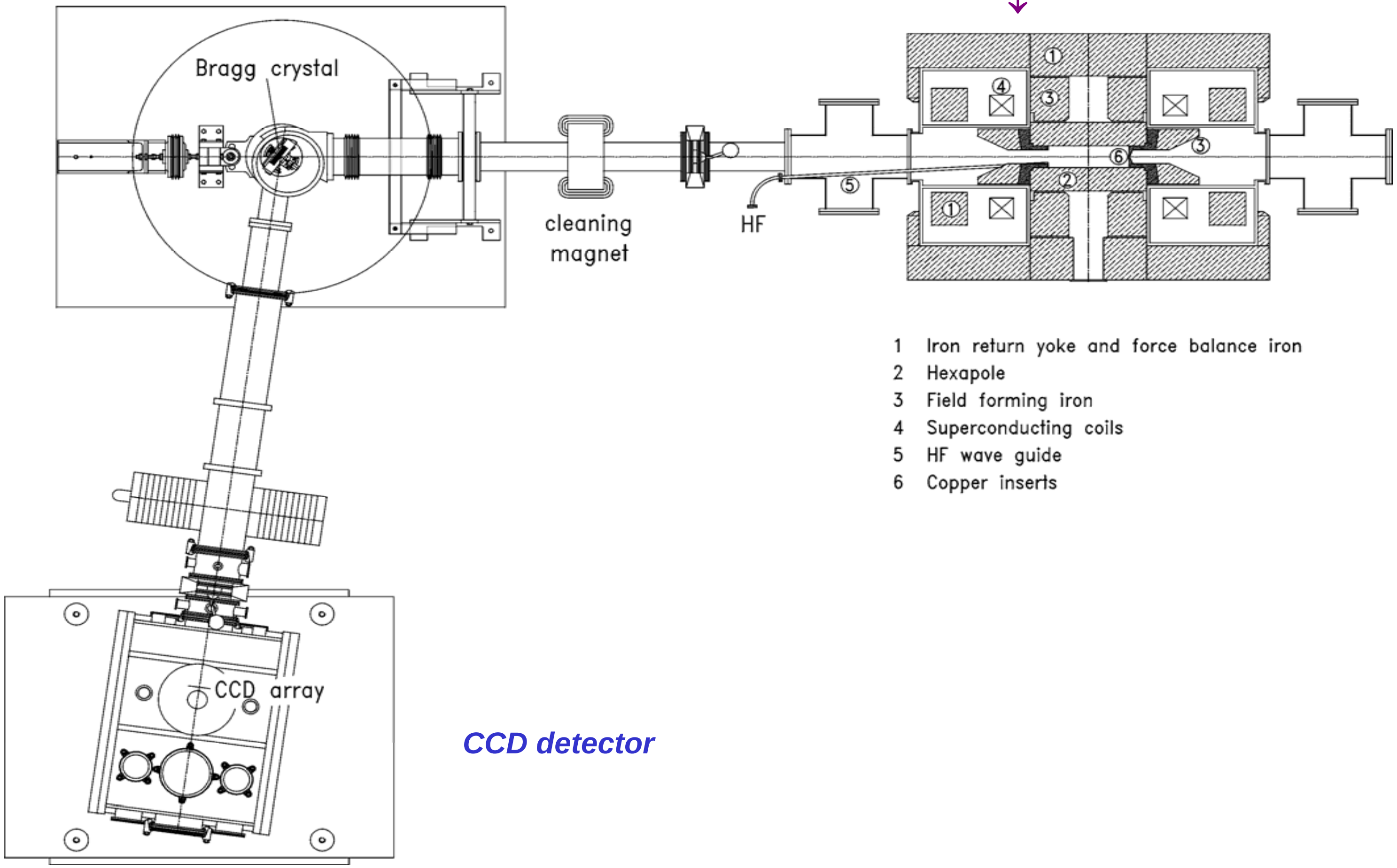
$1.4 \cdot 10^{-6}$ mbar

HF 6.4 Ghz

ECRIT and CRYSTAL SPECTROMETER

crystals *silicon 111*
quartz 10-1

aperture ↓



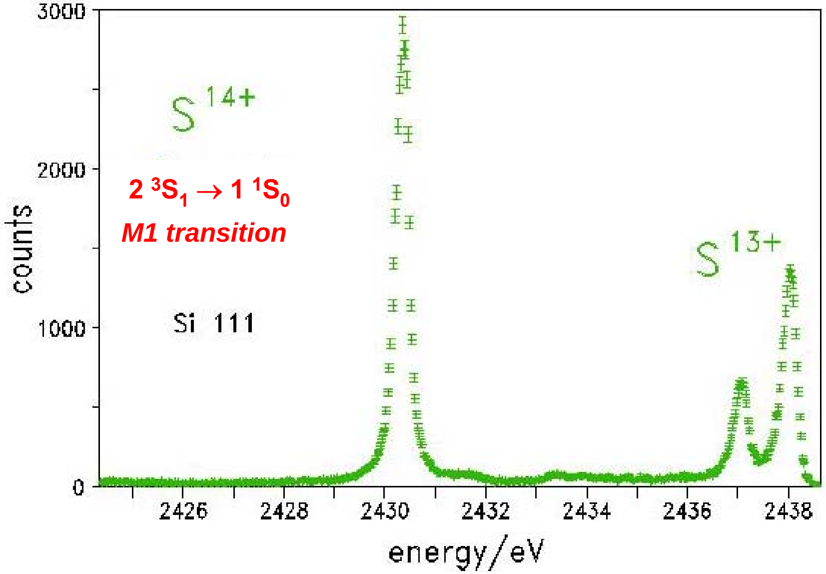
1 Iron return yoke and force balance iron
2 Hexapole
3 Field forming iron
4 Superconducting coils
5 HF wave guide
6 Copper inserts

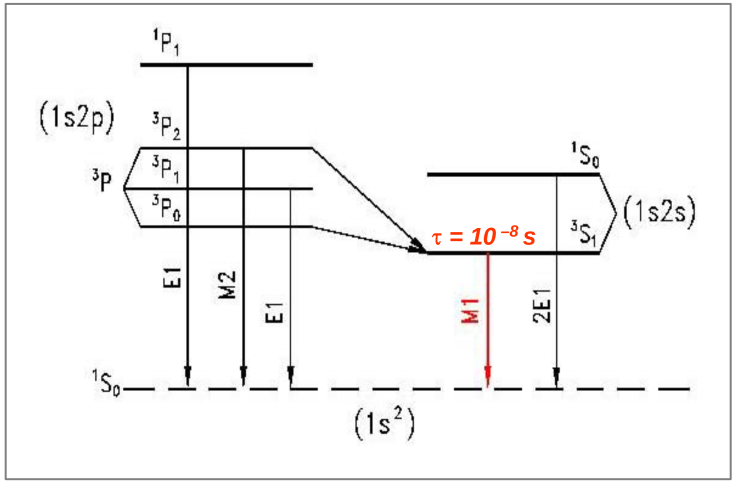
CCD detector

ECRIT measurements 2004

M1 transitions **in He-like**

S	↔	**πH(2p-1s)**
Cl	↔	**πH(3p-1s)**
Ar	↔	**πH(4p-1s)**

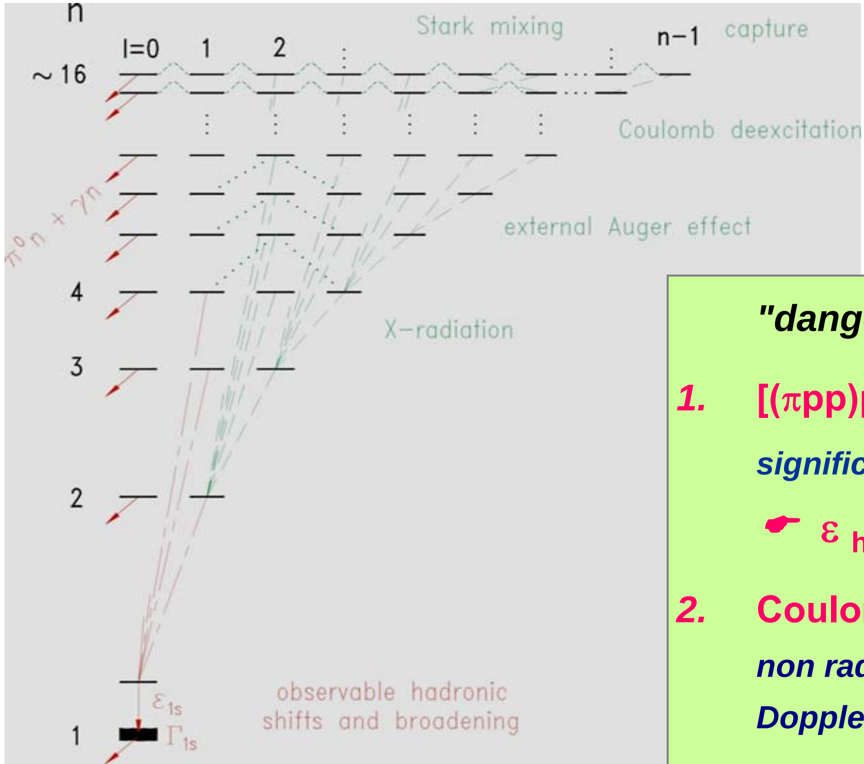




30000 events in line ↔ tails can be fixed with sufficient accuracy

Show stopper?   πp NOT formed in vacuum!

Theoretical input: V.M. Markushin PSI
T. S. Jensen PSI/Paris
V. Popov, V. Pomerant'sev
Moscow State University



"dangerous" processes

1. **[(πpp)p]ee – molecule formation („DH“) ?**

significant radiative decay modes ?

☛ ε **had**

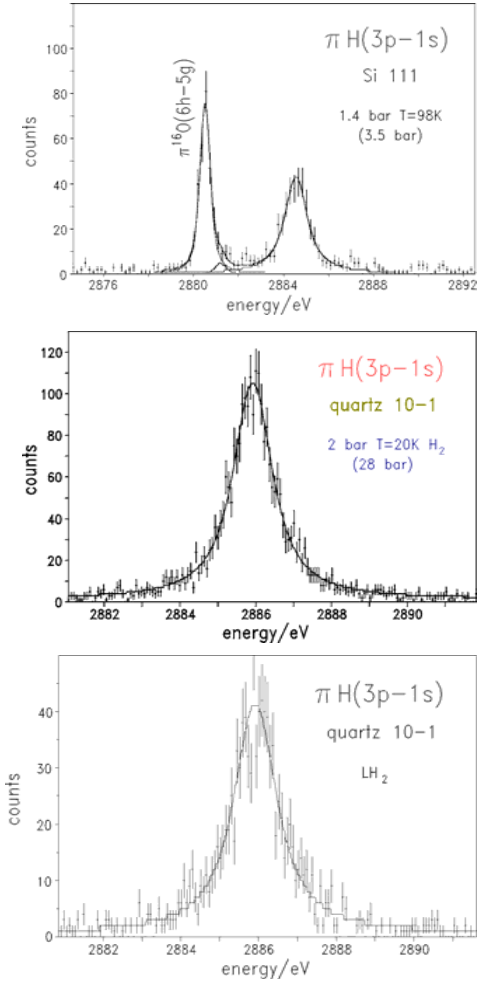
2. **Coulomb - de-excitation !**

non radiative process $n_i \rightarrow n_f$ + kinetic energy

Doppler broadening

☛ Γ_{had}

Strategy of shift measurement: density dependence



mixture H_2 / $^{16}O_2$

(98%/2%)

85K at 1.2 bar

≈ 4 bar equivalent density

H_2

20K at 2 bar

≈ 28.5 bar equivalent density

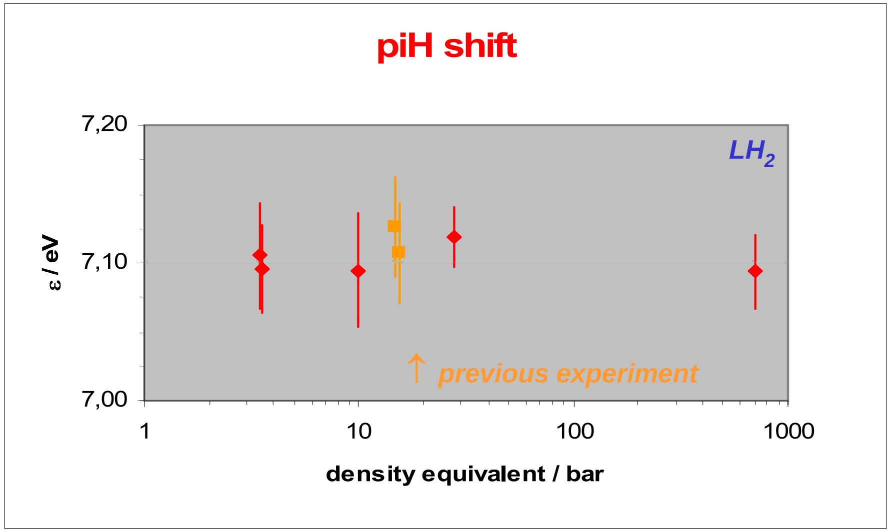
H_2

17K at 1 bar

LH_2

first time

πH(3p-1s) energy no density dependence identified



previous experiment – Ar Kα
ETHZ-PSI H.-Ch.Schröder et al.
Eur.Phys.J.C 1(2001)473

R-98.01

Maik Hennebach, *thesis Cologne 2003*

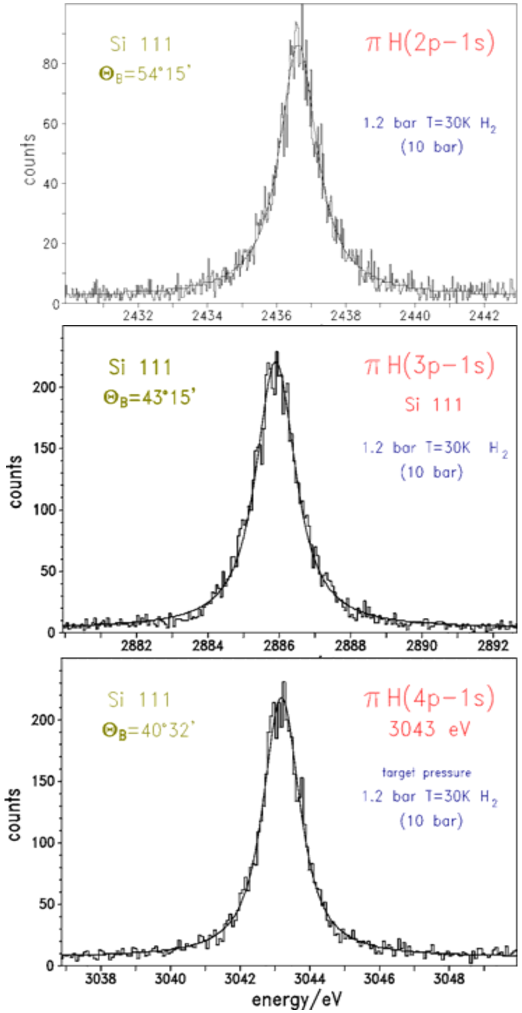
$\varepsilon_{1s} = -7.120 \pm 0.008 \pm 0.009$ eV

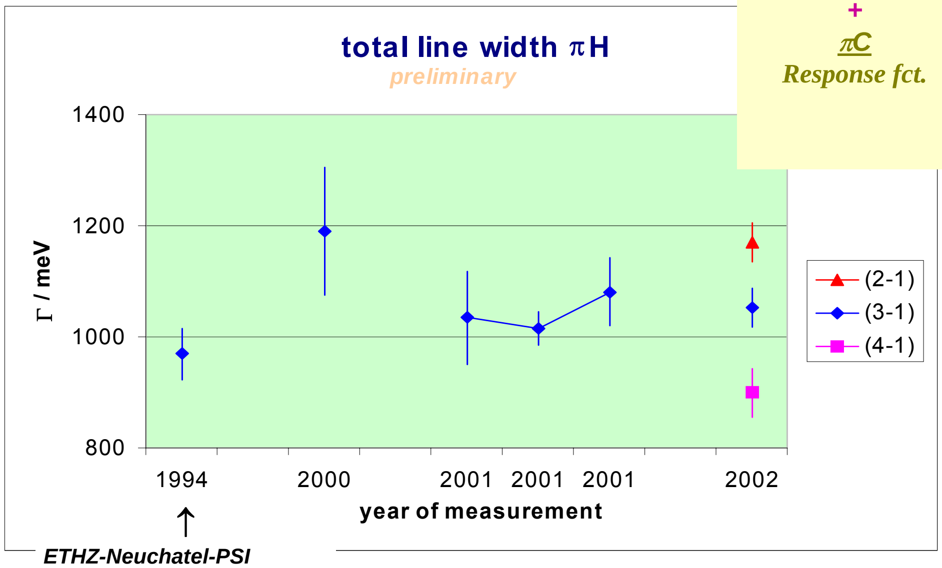
$\Delta E_{QED} = \pm 0.006$ eV !
P. Indelicato, priv. comm.

! πD prediction *radiative decay from molecule increases*

! πT " " " " " ***dominates***

Stategy of width measurement: different initial states





***not* corrected for Coulomb de-excitation**
+
***πC* Response fct.**

Γ_{1s} < 850 meV

Maik Hennebach, *thesis Cologne 2003*

PEAK-TO-BACKGROUND ratio improved by one order of magnitude !

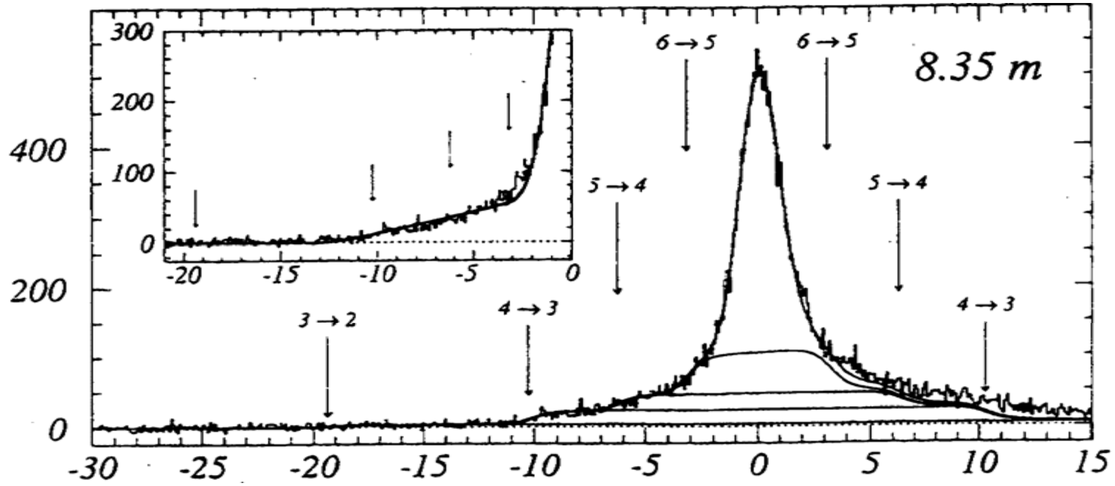
Proof for deceleration process from a different PSI experiment!

$(\pi^- H)_n + H{=}H \rightarrow (\pi^- H)_{n-1} + H + H +$ ***kinetic energy***

NEUTRON - TOF

$(\pi^- p)_{ns} \rightarrow \pi^0 n$

A. Badertscher et al., Eur. Phys. Lett. 54 (2001) 313

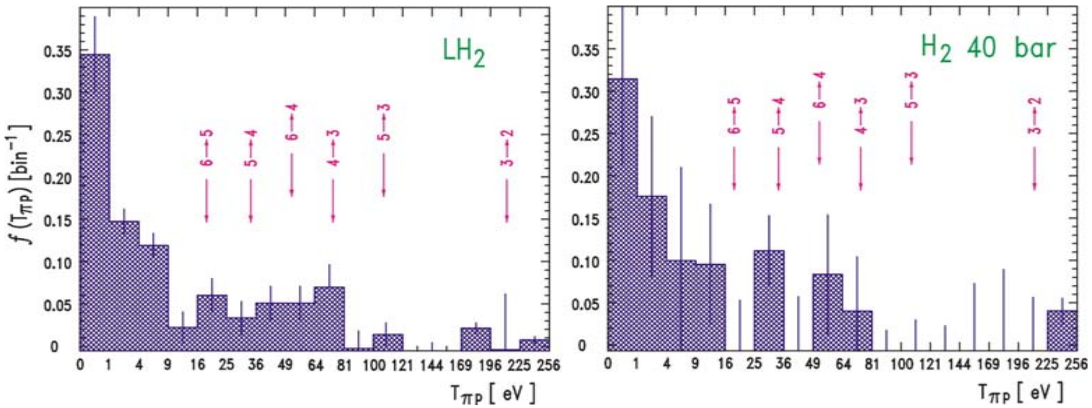


non-radiative transitions

⇓

quasi-discrete

velocity profile

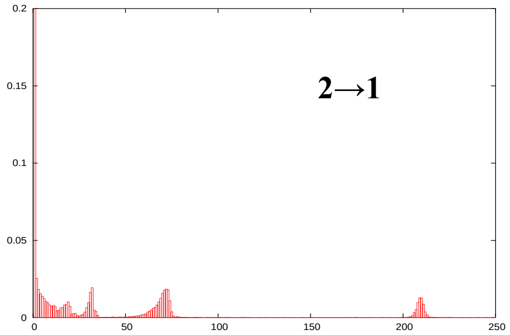


Kinetic energy distributions (Cascade theory)

Fit results without Doppler effect

Fit results including boxes with FREE weights

2→1



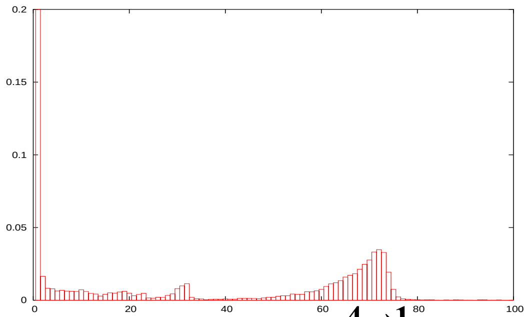
250 eV

Si 111 10b
1170±60 meV

Boxes:
0-2 eV
2-20 eV
29-33 eV
65-75 eV
210-220 eV

907±34 meV

3→1



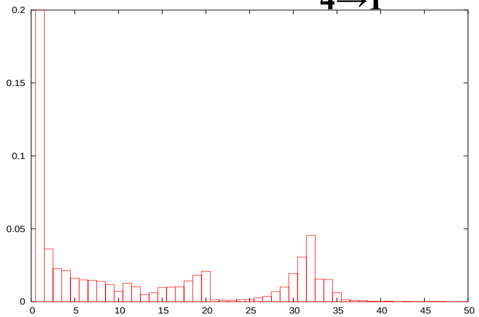
100 eV

Si 111 10b
1053± 40 meV

Boxes:
0-2 eV
2-20 eV
29-33 eV
65-75 eV

775±40 meV

4→1



50 eV

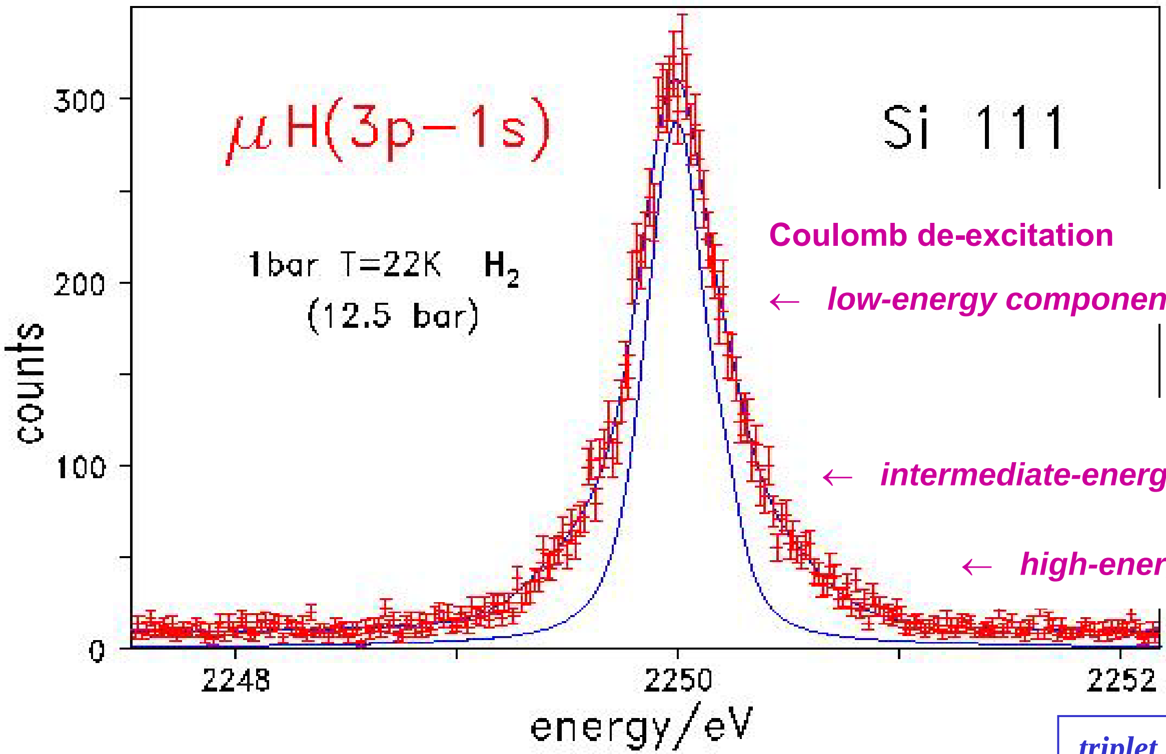
Si 111 10b
899±50 meV

Boxes
0-2eV
3-15 eV
27-34 eV

812±60 meV

$\Gamma_{1s} \approx$ **823±19 meV (2.3%)**

Muonic hydrogen: assumptions about cascade supported



Coulomb de-excitation

← ***low-energy component***

← ***intermediate-energy component***

← ***high-energy component***

----- crystal response ECRIT 2004

triplet / singlet = 3.02±0.2

HFS splitting: 191±2 meV
(cf. 184.6 meV)

PAUL SCHERRER INSTITUT PSI

ε_{1s} = -7.120 ± 0.008 ± 0.006 eV (± 0.2%); Γ_{1s} = 823±19 meV (2.3%)

ε_{1s} : **Extraction of scattering lengths dominated by**

χPT correction δ_ε = (-7.2 ± 2.9)% *

J. Gasser et al., Eur. Phys. J. C 26, 13 (2003)

$$a^+ + a^- = (93.2 \pm 2.9)\ [10^{-3} m_\pi^{-1}]$$

---

result 2005: Γ_{1s} = **823±19 meV (2.3%)** ***preliminary***

χPT correction δ_Γ = (0.6±0.2)% *

P. Zemp, hadatom05

$$a^- = (86.4\ {}^{+0.099}_{-1.02})\ [10^{-3} m_\pi^{-1}]$$

$$a^+ = (6.8 \pm 3.1)\ [10^{-3} m_\pi^{-1}]$$

* δ_ε depend on three LEC: c_1, f_1, f_2; f_1 badly known, δ_Γ depends on f_2 only

a^+ and a^- in units of $[10^{-3} m_\pi^{-1}]$

ε_{1s} = - 7.120 ± 0.008 ± 0.006 eV (± 0.2%)

Γ_{1s} = 823±19 meV (± 2.3%)

Adapted from: St. Scherer, Advances in Nucl. Physics. 27, 277 (2003) ; hep-ph/0210398

		a^+	a^-
Experiment	R98-01.1	a^+ =+(6.8±3.1)	a- = (86.4 ±1)
W - T		0	79
HBChPT O(p^4) I	Fet(00)	-9.6	90.29
HBChPT O(p^4) II	Fet(00)	+4.5	77.03
HBChPT O(p^4) III	Fet(00)	+2.7	86.7
RChPT O(p^4)	Bec(01)	-8.4 ↔-13.1	91.41

Fet(00): N. Fettes and U.-G. Meissner, Nucl. Phys. A676, 311 (2000) KA85, Matsinos, VPI/GW98
Bec(01): T. Becher and H. Leutwyler, JHEP 0106,017 (2001) KA85

PAUL SCHERRER INSTITUT PSI

Pion nucleon coupling constant

GMO sum rule

M.L. Goldberger, H. Miyazawa, R. Oehme
Phys. Rev. 99, 986 (1955)

$$(1+\frac{m_\pi}{M})\frac{a^-}{m_\pi} = \frac{2f_{\pi N}^2}{m_\pi^2-(m_\pi^2/2M)^2} + \frac{1}{2\pi^2}\int_0^\infty \frac{\sigma_{\pi^- p}^{tot}(k_\pi)-\sigma_{\pi^+ p}^{tot}(k_\pi)}{2\omega(k_\pi)}dk_\pi$$

J

J = -(1.082±0.032)mb T. E. O. Ericson et al. Phys. Rev. C 66, 014005(2002)

$f^2_{\pi N}$ = 0.5712 $a^-[m_\pi]$ +0.02488 J[mb^{-1}] = 0.0763(+9,-10)

$g_{\pi NN}$ = 47.66 $f_{\pi N}$ = 13.165(+0.077,-0.087)

[$g^2_{\pi NN}/4\pi = 13.79 \pm^{0.164}_{0.180}$]

From M. Sainio's talk, Meson Nucleon99 ZUOZ, πN Newsletter 15, 156 (1999)

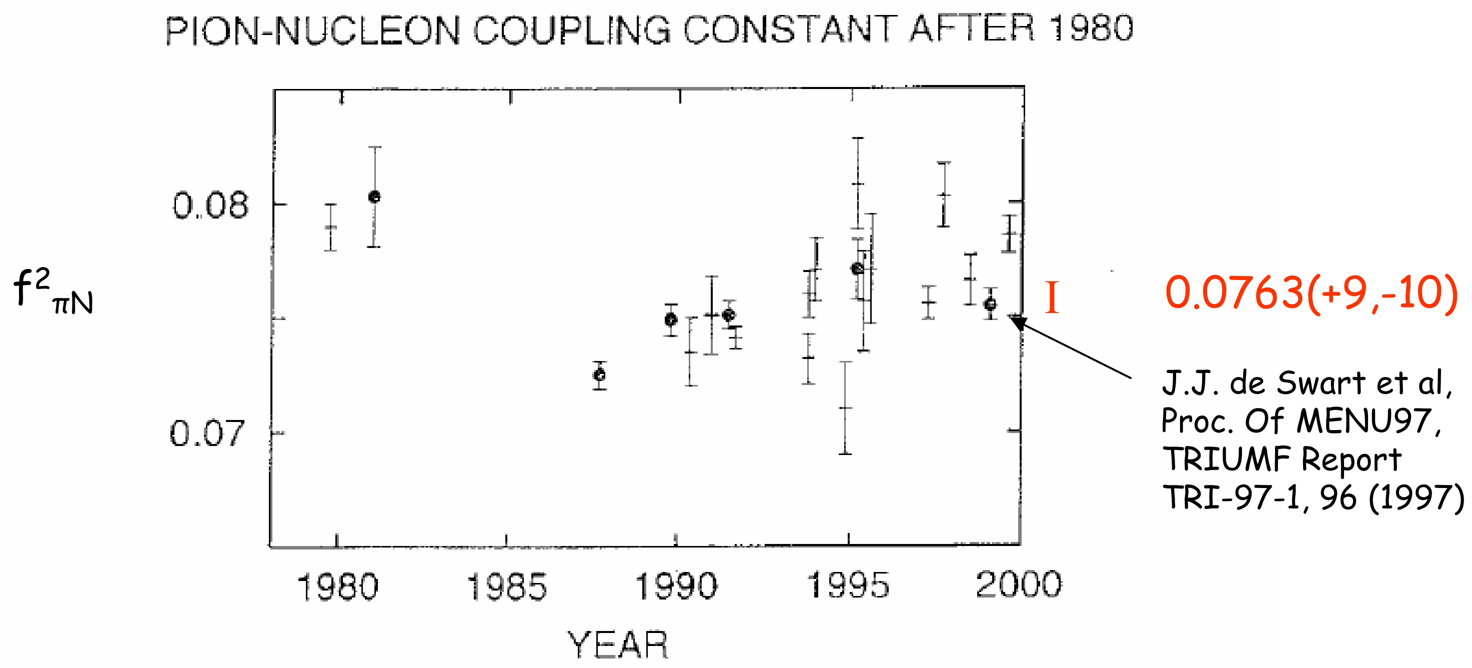


Figure 2 The values of the pion-nucleon coupling constant f^2 after 1980 until the prèsent. Neutral pion couplings are denoted by the solid dots, the remaining points refer to charged pion couplings or charge independent determinations

Recent publications

VPI-GWU Analysis (FA02) http://gwdac.phys.gwu.edu/
R. Arndt et al. Phys. Rev. **C69**, 035213 (2004) GMO **NOT** used!!!

$a^- = (88.3 \pm 0.47)\ [10^{-3}\ m_\pi^{-1}]$; $g_{\pi NN} = 13.145 \pm 0.048$* (constrained by earlier PSI result)
No constraints: $g_{\pi NN} = 13.08$; claimed to be robust

D, V. Bugg, Eur. Phys. J **C33**, 505 (2004) GMO **NOT** used

a^- = (85.0 (Fit I)to 86.6 (Fit II)) $[10^{-3}\ m_\pi^{-1}]$; $g_{\pi NN}$: 13.09 to13.168

T.E.O. Ericson et al. Phys. Lett. **B 594**,76 (2004)
Analysis of earlier PSI result, GMO sum rule **USED**

$a^- = (88.39 \pm 0.3)\ [10^{-3}\ m_\pi^{-1}]$ $g_{\pi NN} = 13.28 \pm 0.08$

M. Döring et al. (nucl-th/0402086): $a^- = (88.1 \pm 0.48)\ [10^{-3}\ m_\pi^{-1}]$

* SP06 nucl-th/0605082: $g\pi NN = 13.149 \pm 0.005$!!

So what?

I. Goldberger Treiman relation:

$g_{\pi NN} = M_N G_A / F_\pi (1+\Delta_{GT})$; $\Delta GT \approx m_q$

$\Delta_{GT} = c\, M^2_\pi + O(M^4_\pi)$, $c \approx 1/\mathrm{GeV}^2$; T. Becher, hep-ph/0206165

⇓

$\Delta_{GT}^{theor} \approx 2\%$

R98-01.1: $g_{\pi NN} \approx 13.165$; together with $M_N G_A / F_\pi \approx 12.9$

⇓

$\Delta_{GT}^{exp.} = 2.05(+0.60,-0.67)\%$ ⟶ values for LEC's

---

II. Induced pseudoscalar coupling constant g_P (muon capture: least well known)

$$g_P = \frac{2m_\mu g_{\pi NN} F_\pi}{m_\pi^2 + 0.88 m_\mu^2} - \frac{1}{3} g_A m_\mu m_N < r_A >^2$$

V. Bernard, N. Kaiser, U.-G. Meissner, Phys. Rev.D **50**,6899 (1994)
N. Kaiser, Phys. Rev. C **67**, 027002 (2003)

$\langle r_A \rangle^2$: axial radius of nucleon (0.44±0.02) fm^2

With $g_{\pi NN}$ from R98-01.1:

$g_P = 8.3 \pm 0.07$

III. S-wave electric dipole multipoles E in charged pion photoproduction: Corrections to LET (Kroll-Ruderman) to $O(m_\pi^3)$

V. Bernard, N. Kaiser, U.-G. Meissner, Phys. Lett. **B383**, 116 (1996) „BKM"
V. Bernard, Proc. of Chiral Dynamics 1997, Mainz, Springer Lecture notes in Physics 513, (Springer, Berlin, 1998) hep-ph/9710430

With $g_{\pi NN}$ = 13.165 : Values in units of $[10^{-3} m_\pi^{-1}]$

	LET	BKM	DA*	Experiment
E^{thr}_{0+} $(\gamma p \to \pi^+ n)$	27.3	27.9±0.6	27.99	$28.06 \pm 0.27_{stat.} \pm 0.45_{syst.}$ **
E^{thr}_{0+} $(\gamma n \to \pi^- p)$	-31.4	-32.4±0.6	-31.7	-31.5±0.8 ***

* „Dispersion theor. analysis" O. Hanstein et al, Phys. Lett. **B399**, 13 (1999)
** E. Korkmaz et al., Phys. Rev. Lett. 18, 3609 (1999)
*** M.A. Kovash et al., πN Newsletter 12, 55 (1997)

Independently: $(a^-)^2 = q/k_o \, P \, |(E^{thr}_{0+}(\pi^- p))|^2$; q, k_0 CMS momenta of γ, π

With R98-01.1 value of a^-: $E^{thr}_{0+}(\pi^- p) = -32.46 \pm 0.39$ $[10^{-3} m_\pi^{-1}]$

Pionic deuterium ↔ Pionic hydrogen: isospin breaking?

Starting point: a^+ is small (pion does NOT scatter);
Consequences for pionic deuterium scattering length:

$$\mathrm{Re}\, a_{\pi d} = \mathrm{Re}\, \bar{a}_{\pi d} + \Delta a_{\pi d}\,,$$
$$\Delta a_{\pi d} = A_1 \alpha + A_2 (m_d - m_u) + O(\delta^2)$$

Exp.: Re $a_{\pi d}$=-(0.0261±0.0005) M_π^{-1}

$$\alpha \sim (m_d - m_u) \sim \delta$$

$\pi^- p \rightarrow \pi^- p$ $\quad \mathcal{T}_p = 4\pi(1+\mu)(a^+ + a^-) + \delta\mathcal{T}_p\,,$

$\pi^- n \rightarrow \pi^- n$ $\quad \mathcal{T}_n = 4\pi(1+\mu)(a^+ - a^-) + \delta\mathcal{T}_n\,,$

$\pi^- p \rightarrow \pi^0 n$ $\quad \mathcal{T}_x = 4\pi(1+\mu)(\sqrt{2}a^-) + \delta\mathcal{T}_x\,,$

$$\delta\mathcal{T}_p = \frac{4(M_\pi^2 - M_{\pi^0}^2)}{F_\pi^2} c_1 - \frac{e^2}{2}(4f_1 + f_2) + O(p^3)$$
$$\delta\mathcal{T}_n = \frac{4(M_\pi^2 - M_{\pi^0}^2)}{F_\pi^2} c_1 - \frac{e^2}{2}(4f_1 - f_2) + O(p^3)\,,$$
$$\delta\mathcal{T}_x = -\sqrt{2}\left(\frac{g_A^2(M_\pi^2 - M_{\pi^0}^2)}{4m_p F_\pi^2} + \frac{e^2 f_2}{2}\right) + O(p^3)\,,$$

c_1, f_2 rather well known, f1 is the problem

$$\Delta a_{\pi d} = \Delta a_{\pi d}^{\mathrm{LO}} + O(p^3) \quad \Delta a_{\pi d}^{\mathrm{LO}} = (4\pi(1+\mu/2))^{-1}(\delta\mathcal{T}_p + \delta\mathcal{T}_n)\,. \longrightarrow$$ f_1 (c1 known)

$$\Delta a_{\pi d}^{\mathrm{LO}} = -(0.0110^{+0.0081}_{-0.0058})\, M_\pi^{-1}\,,$$
$$\Delta a_{\pi d}^{\mathrm{LO}} / \mathrm{Re}\, a_{\pi d}^{\mathrm{exp}} = 0.42 \ \text{(central values)}$$

Combination πH, πD: values for a MUST overlap

Last remarks

Scattering data await a proper phase shift analysis
before σ-term problem can be settled.

Isospin breaking still an item

$g_{\pi NN}$: overall agreement

Pionic hydrogen/deuterium have (almost) reached the desired precision,
Still hampered by molecular effects.
Even now: important constraint.

Nuclear physics at low energies (light nuclei) is nowadays
on a firm theoretical basis

Appendix I: Pions and PCAC *

Consider pion decay matrix element for decay: $\pi \rightarrow \mu + \nu$:

$$\langle \mu\nu | H | \pi \rangle = \langle 0 | J_{hadron} | \pi \rangle \langle \mu\nu | j_{lepton} | 0 \rangle$$

- j_{lepton} has V-A structure $\rightarrow$ hadron part should be V or A or both, must be constructed from kinematical quantities $q = k_\nu + p_\mu$:
  $\langle 0 | J_{hadron} | \pi \rangle = i\, f_\pi q$
  Pion pseudoscalar (0^-), $\rightarrow$ (parity) J_{hadron} is an axial vector
  $A_\lambda(0)$: axial current

- Translational invariance:
  $\langle 0 | A_\lambda(x) | \pi(q) \rangle = \langle 0 | A_\lambda(0) | \pi(q) \rangle \exp(-iqx)$
  $= i\, f_\pi\, q \exp(-iqx)$

Appendix I ctd.

Question: Is the axial Current conserved (CAC valid)?
Would require
$$\partial_\mu \langle 0|\, A_\lambda(x)\, |\pi(q)\rangle \;=\; \exp(-iqx)\, f_\pi \;\; m^2_\pi = 0 \;\;?$$

Answer: If $m_\pi \rightarrow 0$, then YES
$m_\pi = 0.15\ m_{proton}$: rather small → CAC is almost valid:

PCAC

Zuoz (5.-15. 4. 1972) :
Spring school on weak interactions and nuclear structure, talk by N. Straumann:
...Many theoreticians nowadays idealize strong interaction physics by lettering m_π → 0. In this limit the axial current could also be conserved(chiral symmetry). As long as SIN has not yet produced any pions, we should, however, better not consider them as being massless.

* Literature: e.g. U. Mosel: Fields, Symmetries and Quarks, Springer 1988

Appendix II: Goldberger Treiman relation *

CAC would be valid for $m_\pi \to 0$, which are the consequences for neutron decay ?
Axial part of the matrix element for neutron β decay:

$$n(p) \to p(p') + e + \nu;$$

Tranlational invariance: $\langle p'|A_\lambda(x)|p\rangle = \langle p'|A_\lambda(0)|p\rangle \exp(-iqx)$, $q=p'-p$

$$\langle p'|\partial^\lambda A_\lambda(x)|p\rangle = -i(p'-p)\langle p'|A_\lambda(0)|p\rangle \exp(-i(p'-p)x)$$

Symmetry considerations:

$$\langle p'|A_\lambda(0)|p\rangle = \bar{u}(p')\left[\gamma^\mu\gamma^5 F(q^2) + q^\mu\gamma^5 G(q^2)\right]u(p)$$

$$(p'-p)_\lambda\langle p'|A_\lambda(0)|p\rangle = \bar{u}(p')\left[\gamma^\mu(p'-p)_\mu F(q^2) + q^2 G(q^2)\right]\gamma^5 u(p)$$

Assume CAC: $2M_N F(q^2)+q^2G(q^2)=0$, M_N: nucleon mass

$$\Downarrow$$

$$G(q^2) = \frac{\sqrt{2} f_\pi}{m_\pi^2 - q^2}\sqrt{2} g_{\pi NN}(q^2) \to \frac{-2 f_\pi g_{\pi NN}(q^2)}{q^2} \quad for \quad m_\pi^2 \to 0$$

Goldberger Treiman relation $F(q^2) \to f_\pi$, $G(q^2) \to G_A$

$g_{\pi NN} = M_N G_A / f_\pi$ is valid on the % level

Question: CAC, CVC seem to be established, how to implement this in QCD?
Answer: A chiral symmetric Lagrangian for QCD can be constructed for quark masses $m_q=0$.
i.e. invariant against chiral transformation
↓ Noether's theorem
Vector and **Axial Vecor currents** are **conserved** for $m_q=0$

* Literature E. D. Commins & P. H. Bucksbaum, Weak interactions of leptons and quarks

Appendix III: Chiral symmetry and its consequences *

Lagrangian density of QCD : $$L = \bar{q}(i\gamma^{\mu}D_{\mu} - M)q - \frac{1}{2}Tr(G^{\mu\nu}G_{\mu\nu}),$$

quark- gluon part

$$L = L^0 + L^{mass}, \qquad L^{mass} = -\bar{q}Mq\,, \qquad M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

$m_u/Mp \simeq 0.005$
$m_d/Mp \simeq 0.01$
$m_s/Mp \simeq 0.2$

L^0 has an extra symmetry related to conserved right- or left-handedness (chirality) of zero mass spin ½ particles. One introduces right- and left-handed quark fields:

$$q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$$

Separate global unitary transformations of these fields leave L^0 invariant.

$$q_R \rightarrow \exp(i\Theta_R^a \frac{\lambda_a}{2})q_R \qquad q_L \rightarrow \exp(i\Theta_L^a \frac{\lambda_a}{2})q_L, \qquad \lambda_a : \; Gell-Mann \;\; matrices, \; flavour\, index \;\; a = 1,8$$

*Right- and left-handed components of **massless** quark fields do not commute. $SU(3)_R x SU(3)_L$ transformations leave the Lagrangian L^0 invariant, this is called:*

Chiral symmetry of QCD

⇩ Noether's theorem

Conserved currents

$$J^{\mu}_{R,a} = \bar{q}_R\gamma^{\mu}\frac{\lambda_a}{2}q_R \qquad J^{\mu}_{L,a} = \bar{q}_L\gamma^{\mu}\frac{\lambda_a}{2}q_L \qquad with \quad V^{\mu}_a = J^{\mu}_{R,a} + J^{\mu}_{L,a} \quad and \;\; A^{\mu}_a = J^{\mu}_{R,a} - J^{\mu}_{L,a} :$$

$$V^{\mu}_a = \bar{q}\gamma^{\mu}\frac{\lambda_a}{2}q \quad and \quad A^{\mu}_a = \bar{q}\gamma^{\mu}\gamma_5\frac{\lambda_a}{2}q \quad with \;\; the \;\; corresponding \;\; Q^V_a \;\; and \;\; Q^V_a$$

Appendix III ctd.

Assumption: The axial and vector charges are conserved , they commute with the the hamiltonian $H^0 = H_{QCD}(m_q=0)$

$$[H^0, Q_{V,a}]=0=[H^0, Q_{A,a}]$$

Eigenstates: $H^0 |\psi\rangle = E |\psi\rangle$

Then the states $Q_{V,a}|\psi\rangle$ and $Q_{A,a}|\psi\rangle$ have the **same energy** but they should have **opposite parity**.

Meson and baryon mass spectra show: this is **NOT the case!**

Way out: $Q_{A,a}|0\rangle \neq 0$

The ground state does not have the symmetry of the Lagrangian.

Chiral symmetry is **hidden** (spontaneously broken).

Goldstone's theorem applies:

1: massless particle exist with quantum numbers of the field

2: Its coupling to the current does not vanish :

$$\langle 0|\, A_\lambda(0)\, |\pi(\mathbf{q})\rangle = i\, f_\pi\, q_\mu \neq 0$$

To summarize:

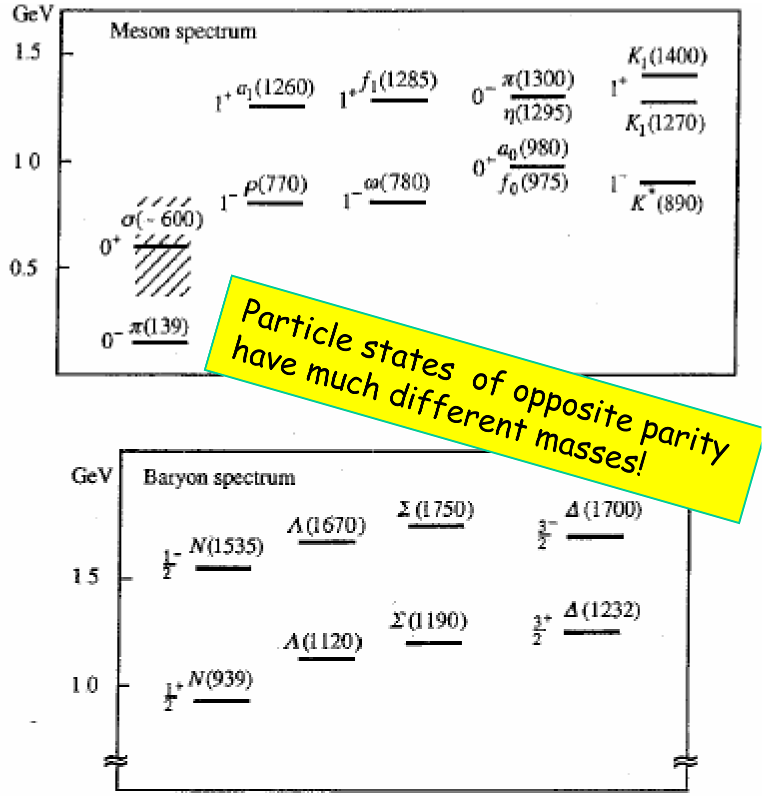
$$Q_{A,a}|0\rangle \neq 0 \qquad Q_{V,a}|0\rangle = 0$$

Axial vector symmetry is spontaneously broken. 8 massless Goldstone particles are associated with the axial charges: (π,K,η)

They are not exactly massless as L^{mass} is not chirally invariant.

Vector symmetry not broken spontaneously. Hadrons occur in nearly degenerate multiplets which constitute representations of $SU(3)_V$.

* Literature: J. Gasser, Light quark dynamics hep-ph/0312367



Masses of mesons and baryons of opposite parity.

Graphs taken from:
A. Hosaka, H. Toki, Quarks, Baryons and Chiral Symmetry, World Sc. 2001

Appendix IV: Chiral condensate, GOR relation, Isospin breaking *

Finite value of m_π from **explicit** breaking of chiral symmetry by quark masses. Calculating the divergence of Noether currents (only flavour part u,d) from the

variation of $$L^{mass} = -\bar{q}Mq = -(\bar{q}_R M q_L + \bar{q}_L M q_L) \;\; under \;\; U_{L,R}$$

results in
$$\partial_\mu V_a^\mu = i\bar{q}\left[M, \frac{\lambda_a}{2}\right]q \qquad \propto e.g.(m_u - m_d)$$

$$\partial_\mu A_a^\mu = i\bar{q}\left\{M, \frac{\lambda_a}{2}\right\}\gamma_5 q, \propto e.g.(m_u + m_d) \;\; a = 1 \qquad (I)$$

$$\left[Q_a^1, \bar{q}\gamma_5\lambda_1 q\right] = -\langle 0|\bar{u}u + \bar{d}d|0\rangle \qquad (II)$$

e.g. St. Scherer hep-ph/0210398

$$(I)+(II): \quad \langle 0|\left[Q_a^1, \partial_\mu A^\mu\right]|0\rangle = -\frac{i}{2}(m_u + m_d)\langle 0|\bar{u}u + \bar{d}d|0\rangle$$

Using PCAC results in: $$m_\pi^2 = -\frac{1}{2f_0^2}(m_u + m_d)\langle 0|\bar{u}u + \bar{d}d|0\rangle + O(m_{u,d}^2, \ldots)$$

explicit spontaneous symmetry breaking

Mass term: $$m_u\bar{u}u + m_d\bar{d}d = \frac{m_u + m_d}{2}(\bar{u}u + \bar{d}d) + \frac{m_u - m_d}{2}(\bar{u}u - \bar{d}d)$$

Chiral symmetry breaking. Isospin symmetry breaking

* Literature: Conference contributions by H. Leutwyler (especially hep-ph/9409423)

Appendix V: Lagrangian of lowest order (Goldstone bosons only) *

The most general, chirally invariant, Lagrangian density with a minimal number of derivative (equivalent to the „nonlinear σ model) reads:

$$L_{eff} = \frac{F_0^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger)\, with\, U(x) = \exp(i\frac{\Phi(x)}{F_0}),\ \Phi(x) = \sum_{a=1}^{8} \lambda_a \Phi_a(x) \equiv \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\overline{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

- L_{eff} is invariant under the global $SU(3)_L \times SU(3)_R$ transformations $U \to RUL^\dagger$

$$L = \exp(-i\Theta_a^L \frac{\lambda_a}{2}), \qquad R = \exp(-i\Theta_a^R \frac{\lambda_a}{2})$$

- $F^2_0/4$ is chosen to generate the standard form of the kinetic term $\frac{1}{2}\,\partial_\mu\Phi_a\, \partial^\mu\Phi_a$

- Axial vector Noether current :

$$J_A^{\mu,a} = -i\frac{F_0^2}{4} Tr(\lambda_a [U, \partial^\mu U^\dagger]) = -i\frac{F_0^2}{4} Tr(\lambda_a \left\{1 + \ldots..,-i\frac{\lambda_b \partial^\mu \Phi_b}{F_0} + \ldots..\right\}) = -F_0 \partial^\mu \Phi_a + \ldots$$

$$\langle 0 | -F_0 \partial^\mu \Phi_a(x) | \Phi^b(p) \rangle = F_0 \partial^\mu \exp(-ipx)\delta^{ab} = ip^\mu F_0 \exp(-ipx)\delta^{ab}$$ **PCAC**

* Literature: St. Scherer, in Advances in Nuclear Physics, Vol 27, 277 (2003), hep-ph/0210398

Appendix VI: Including the mass term *

Including the mass term even provides predictive power (see below):

$$L_{eff} = \frac{F_0^2}{4} Tr(\partial_\mu U^\dagger \partial^\mu U) + \frac{F_0^2}{2} BTr(M(U + U^\dagger))$$

From this expression soft pion theorems can be derived.
It may be considered to be the starting point of „true" effective theories i.e. of a Lagrangians leading to renormalizable solutions and containing higher orders.

For L_{eff} above::
Mass formulae of Gell-Mann Okubo can be derived
GOR relation is readily obtained: $\mathbf{M^2=(m_u+m_d)B}$

B is proportional to the quark condensate:
It is the leading order parameter of the spontaneously broken chiral symmetry.
Corroborated by Kl4 decays in terms of the isoscalar scattering length a^0_0
S. Pislak et al. Phys. Rev. Lett. **87** (2001) 221801.
G. Colangelo, J. Gasser, and H. Leutwyler, Phys. Rev. Lett **86** (2001) 5008

* Monograph by St. Scherer (hep-ph/0210398)