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Supersymmetry and Flavour

1. Flavour in the S.M.

"Repetition" of similar particles (fields)

$u \to u, c, t \qquad d \to d, s, b$

$e \to e, \mu, \tau \qquad \nu_e \to \nu_e, \nu_\mu, \nu_\tau$

- four 'kinds' of particles, defined by their interactions (e.g. charge)
- grouped into generations $(u, e, d, \nu_e), \ldots$

observed: three generations

- Understand why 4 kinds from basic symmetries of S.M (e.g. $3Q_u + 3Q_d + Q_e + Q_\nu = 0$)

The three flavours of each kind have identical interactions (undistinguished)

only their masses are different

Do not know why 3 generations ($\not{C}\not{P}$?), why masses hierarchical, ... $\Rightarrow$ Flavour problem

2. Flavour change in SM

Notation: $u^L, c^L, t^L \to u_1^L, u_2^L, u_3^L \sim u_i^L$
$d, s, b \to d_1, d_2, d_3 \sim d_i^L$
etc.

$$L^* = g W^\mu \bar{u}_i^L \gamma_\mu (1-\gamma_5) d_j^L \delta_{ij}$$ charged

$$+ e_u A^\mu \bar{u}_i \gamma_\mu u_j \delta_{ij} + e_d \bar{A}^\mu \bar{d}_i \gamma_\mu d_j \delta_{ij}$$

neutral

$$+ \ldots$$

$$+ \underbrace{m_{ij}^u \bar{u}_i^L u_j^R + m_{ij}^d \bar{d}_i^L d_j^R}_{\text{mass-term (not-diagonal)}} + \ldots$$

→ Mass eigenstates (physical states)
diagonalize $M^U = (m_{ij}^u)$, $M^d = (m_{ij}^d)$

Change of basis $u \to u' = U_u u$
$d \to d' = U_d d$

Result: • Neutral couplings have form $\bar{u}_i' \Gamma_\mu u_j' \delta_{ij}$
no flavour violation, GIM

• charged current
$\bar{u}_i' \Gamma_\mu d_j' (U_u U_d^{-1})_{ij}$

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* Not complete, just to make the point

Flavour violation only in charged current, and $(U_u U_d^{-1}) \neq \mathbb{1}$ required.

$$(U_u U_d^{-1}) = U_{CKM} \sim (V_{ij})$$

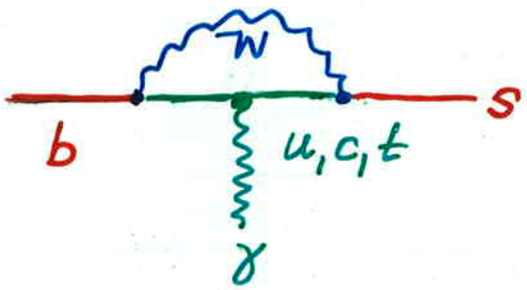
Kobayashi-Maskawa-Cabibbo-Matrix

<u>Rule:</u> Flavour violation requires two kinds of particles (here u, d) which interact (here by W-exchange) and which have different U-matrices (here $[M_u, M_d] \neq 0$). "Mismatch"

Comments:

- U_{CKM} : 3×3 unitary matrix : 4 parameters (3 real, 1 phase after redefinition)
  phase* → CP-violation

- Flavour violation in "Neutral couplings" only via loops and GIM supressed



$C \cdot A^\mu (\bar{s}\, \Gamma_\mu\, b)$
↳ small (BR ~ 10^{-4} ...)
"FCNC"

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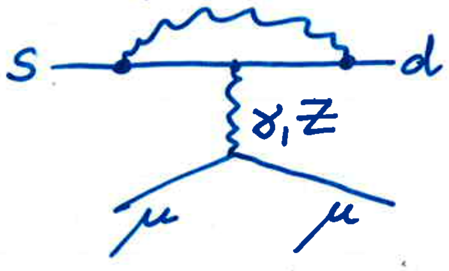
* No phase for 1, 2 generations

- Degeneracies ($m_u = m_c = m_t$) imply the absence of flavour changes
- Same true for leptons; $m_{\nu_e} \approx m_{\nu_\mu} \approx m_{\nu_\tau}$ (no flavour violations).
- GIM is intimately connected to deeper structure of SM (no anomalies,..)

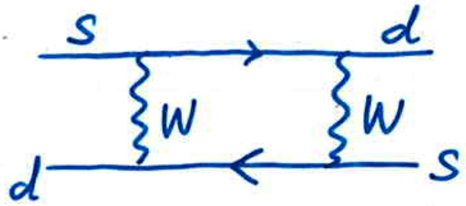
3. Flavour physics, CP-violation

- weak decays like $K \to \pi \ell \nu$, $B \to D \ell \nu$
- more interesting:

* $K \to \mu\mu$, Δm_K, $K \to \pi e e$, $K \to \pi \nu\nu$



$\Delta S = 1$



$\Delta S = 2$

* $D \to \mu\mu$ etc.

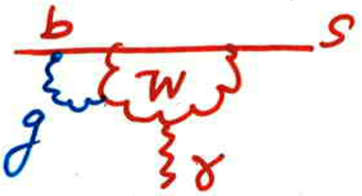
!

* $B \to \mu\mu$, $B \to X_s \gamma$, $B \to X_s \ell^+ \ell^-$
Δm_{B_d}, Δm_{B_s}, CP-violation

* $\mu \to e\gamma, \dots$

* d_n, d_e (el. dipole moments)

$B \to X_s \gamma$ particularly powerful
SM to NLL; errors $\sim 10-15\,\%$



4. Flavour in susy

From H. Haber

$$\begin{pmatrix} e_i^L \\ \nu_i^L \end{pmatrix} \longleftrightarrow \begin{pmatrix} \tilde{e}_i^L \\ \tilde{\nu}_i^L \end{pmatrix} \qquad \begin{matrix} e_i^R \longleftrightarrow \tilde{e}_i^R \\ \nu_i^R \longleftrightarrow \tilde{\nu}_i^R \end{matrix}$$

m_ν

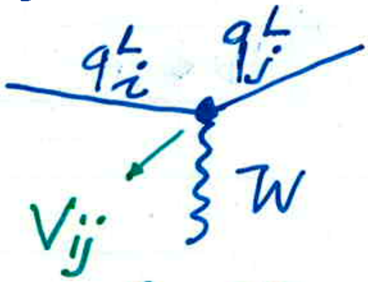
$$\begin{pmatrix} u_i^L \\ d_i^L \end{pmatrix} \longleftrightarrow \begin{pmatrix} \tilde{u}_i^L \\ \tilde{d}_i^L \end{pmatrix} \qquad \begin{matrix} u_i^R \longleftrightarrow \tilde{u}_i^R \\ d_i^R \longleftrightarrow \tilde{d}_i^R \end{matrix}$$

Compare to SM:

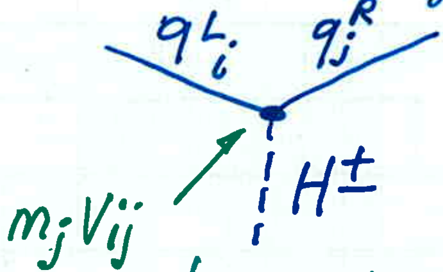
- more kinds of particles
- flavour also in scalars ($\tilde{q}$, $\tilde{\ell}$, ...)

Many more possibilities for flavour violations.

- Flavour violations via charged bosons

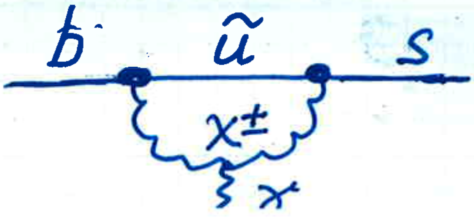


also SM



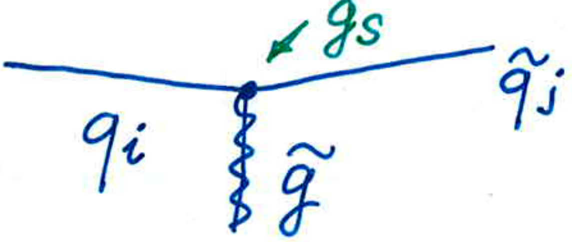
charged Higgs

- Flavour violation via charginos (Loops)



χ : $\tilde{W}$, $\tilde{H}$

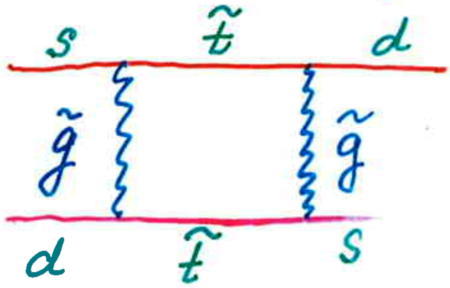
- Flavour violation via Neutralinos and gauginos (gluinos!)



The Rule: if $[M_q^2, M_{\tilde{q}}] \neq 0 \to$ flavour violation
violation

possible to have "strong" flavour violation

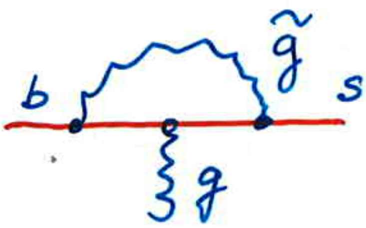
- Δm_K :



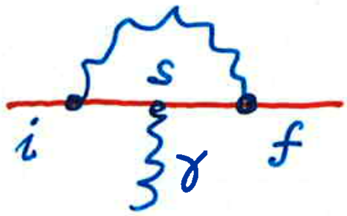
$$\sim \frac{1}{m_{\tilde{t}}^2, m_{\tilde{g}}^2} (\ast)^4 \cdot g_s^4$$

- $m_{\tilde{t}}^2, M_{\tilde{g}}^2 \longrightarrow \infty$
- $(\ast) \to 0$ (no mismatch of $q, \tilde{q}$)

- $b \to sg$:



- CP-violation



phase

$$\sim (V_{is})(V_{fs}^*)$$

and μ-term

Many sources of flavour violation + CP

SM quite accurate $\Longrightarrow$ bounds on SUSY

5. Squark mass matrix $M_{\tilde{q}}$

Most important for flavour violation

From H. H. Lecture: *

$$W = H_d H_1 Q D + H_u H_2 Q U + H_e H_1 L E$$

$$\to (H_d \cdot v) \to m_d \quad \text{etc.}$$

$$(H_d v)^2 \to m^2_{\tilde{d}} \quad \text{etc.}$$

at this, $q, \tilde{q}$ are diagonalized by same matrix → no flavour changes by gluinos, but no susy breaking

Need SUSY breaking → soft terms

$$\mathcal{L}_{soft} \sim m_1^2 \hat{H}_1^2 + m_2^2 \hat{H}_2^2 - (B\mu \tilde{H}_1 \tilde{H}_2 + h.c.)^{**}$$
$$+ m_Q^2 \hat{Q}^2 + m_L^2 \hat{L}^2 + m_D^2 \tilde{D}^2$$
$$+ A_e H_e \tilde{H}_1 \hat{L} \tilde{E} + A_d H_d \hat{H}_1 \tilde{Q} \hat{D} + \ldots$$

A_e, A_d, m_Q^2, etc. arbitrary

$$\to M^2_{\tilde{Q}} \simeq \begin{pmatrix} m^2_{LL} & m^2_{LR} \\ m^2_{LR} & m^2_{RR} \end{pmatrix}$$

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* $Q \sim \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ etc. ** $\hat{Q}, \ldots$ scalar fields

D-term

$$m_{LL}^2 \cong m_Q^2 + m_L^{D^2} + m_q^2$$

$$m_{RR}^2 \simeq m_D^2 + m_R^{D2} + m_q^2$$

$$m_{LR}^2 \cong A_d\, m_d + \cdots$$

Fermions

Presence of M_Q^2, etc. lead to flavour violation (if $[M_Q^2, m_{\tilde{q}}] \neq 0$).

6. Idea for supressing flavour changes:

$m_Q^2 \propto \mathbb{1}$ "universality"

Can be built in into the model

→ conditions for symmetry breaking
conditions for beyond MSSM

Running of masses:

because of R.G. running, the conditions on m_Q^2 can only be imposed at one scale

① Universality at high scale
e.g. at 10^{16} GeV (MSUGRA)

$$m_{\text{gauginos}} \equiv m_{1/2}$$

$$m_{\tilde{q}} \equiv m_0$$

$$A_d = A_u = A_e \equiv A_0\, \delta_{ij}$$

Higgs

H-parameters
$\tan\beta$ sgn μ

Flavour violation through new exchanges and gluinos (small running effects)

② Universality at "low" scale $\sim M_W$ (no gluino effects); not 'natural'

Results:

F1: Analysis of MSUGRA (Situation ①)

Includes various effects like running, renormalization, etc.

③ Case ① + small additional non universal terms

$$\delta = \delta m_q^2 / m_0^2 \, *$$

Various possibilities, according to entry in $M^2_{\tilde{Q}}$:

δ_{LL}, δ_{LR}, etc. — real part: BR; imag part: EDM, etc.

Typical results (selection)

$m_0 \simeq 500$ GeV, $m_{\tilde{g}}^2 / m_{\tilde{q}}^2 \approx 1$

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* usually defined at M_W

F1

MSUGRA

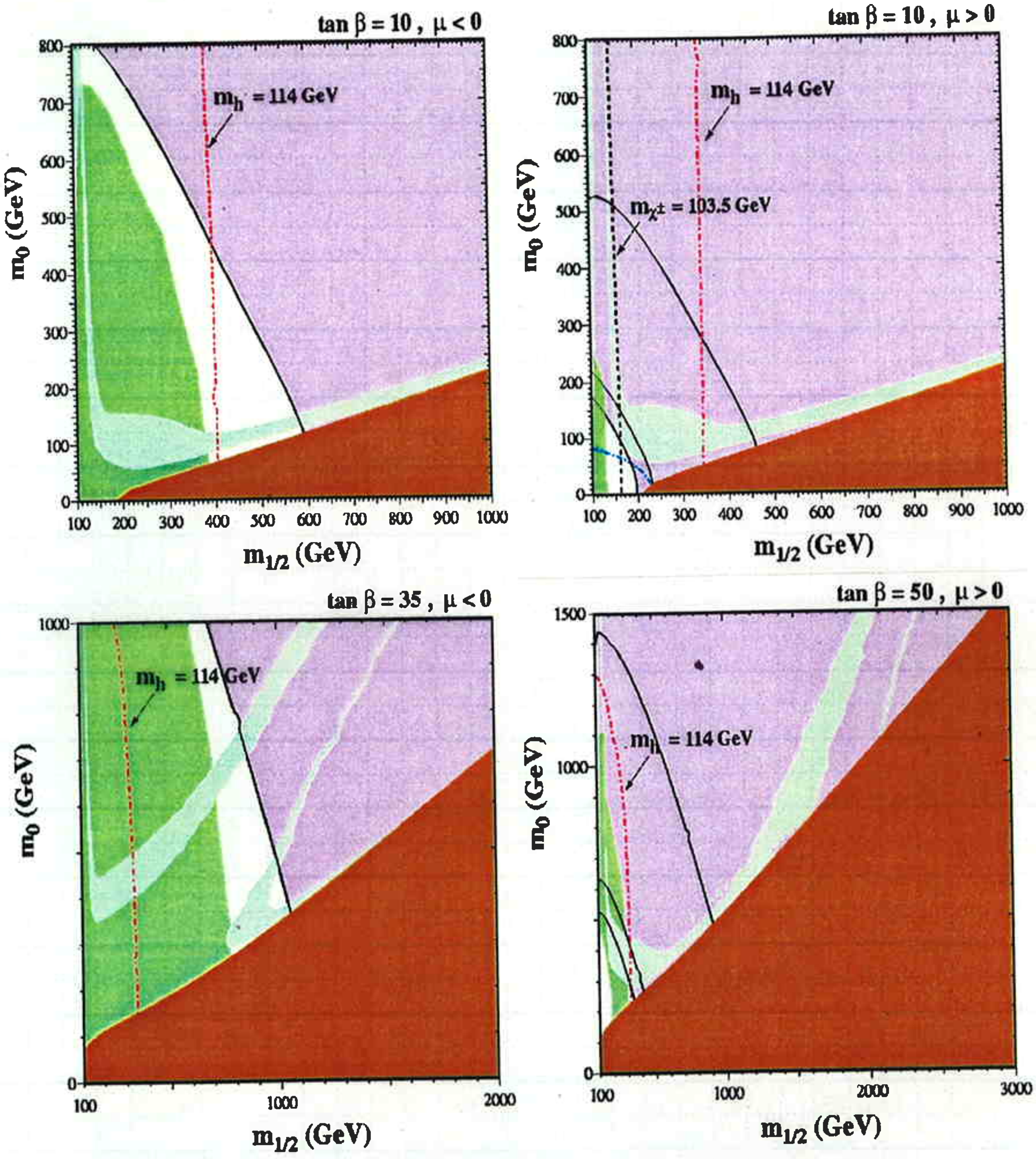

Fig. 11: Compilations of phenomenological constraints on the CMSSM for (a) $\tan\beta = 10, \mu < 0$, (b) $\tan\beta = 10, \mu > 0$, (c) $\tan\beta = 35, \mu < 0$ and (d) $\tan\beta = 50, \mu > 0$, assuming $A_0 = 0, m_t = 175$ GeV and $m_b(m_b)^{\overline{MS}}_{SM} = 4.25$ GeV [61]. The near-vertical lines are the LEP limits $m_{\chi^\pm} = 103.5$ GeV (dashed and black) [59], shown in (b) only, and $m_h = 114.1$ GeV (dotted and red) [20]. Also, in the lower left corner of (b), we show the $m_{\tilde{e}} = 99$ GeV contour [60]. In the dark (brick red) shaded regions, the LSP is the charged $\tilde{\tau}_1$, so this region is excluded. The light (turquoise) shaded areas are the cosmologically preferred regions with $0.1 \leq \Omega_\chi h^2 \leq 0.3$ [61]. The medium (dark green) shaded regions that are most prominent in panels (a) and (c) are excluded by $b \to s\gamma$ [62]. The shaded (pink) regions in the upper right regions delineate the $\pm 2\,\sigma$ ranges of $g_\mu - 2$. For $\mu > 0$, the $\pm 1\,\sigma$ contours are also shown as solid black lines.

from J. Ellis, Beatenberg 0203114
(others)

- $\not{C}\!P$ in K-system : ϵ_K

$\mathrm{Im}\,\delta^{12}_{LL} \lesssim 10^{-3}$ $\mathrm{Im}\,\delta^{12}_{LR} \lesssim 10^{-4}$

- EDMs

$\mathrm{Im}\,\delta^{11}_{LR} \lesssim 10^{-6}$

Masiero, Vives
Porod et. al.

- $b \to s\gamma$

$\delta^{23}_{LL} \lesssim 10$ $\delta_{LR} \lesssim 10^{-2}$

Note different sensitivities to δ_{LL}, δ_{LR} due to structure of operator *

Needs a theoretical explanation for the δ's.

- Interesting possibilities in $\not{C}\!P$ - B

Example (maybe outdated):

$$\text{SM}: \; a_{CP}(B \to J/\psi K_S) = a_{CP}(B \to \phi K_S)$$

$$\text{SUSY}: \; a_{CP}(B \to J/\psi K_S) \underset{\propto \delta\text{'s}}{\neq} a_{CP}(B \to \phi K_S)$$

SM - predictions for CP-asymmetries (relations) can be modified.

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* $b \to s\gamma \to \bar{s}\,\sigma_{\mu\nu}\, b\, F^{\mu\nu}$ is a $b_R - s_L$ transition

7. Specific lepton effects

- Running induces deviations from universality (p. 9).

See-saw affects running strongly

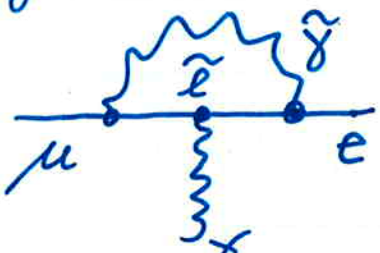
(Borzumati, Masiero)

Recall: Seesaw natural mechanism for ν-masses *

$$(m^2_{\tilde{e}})_{ij} \sim \frac{3m_0 + A_0}{8\pi^2} (h^\nu h^{\nu +})_{ij} \log \left(\frac{10^{16}}{M} \right)$$

ν-mixings point to off-diagonal h^ν and M $\rightarrow$ non-diagonal $m_{\tilde{e}}^2$

gives rise to $\mu \rightarrow e\gamma$



$$BR \sim \frac{\alpha^3 (m^4_{\tilde{e}})_{12}}{G_F^2 \, m_0^8} \tan^2\beta$$

Size of effect depends on model

① "sensible" SO(10), "minimal"

Masiero, Vempati, Vives 0405017

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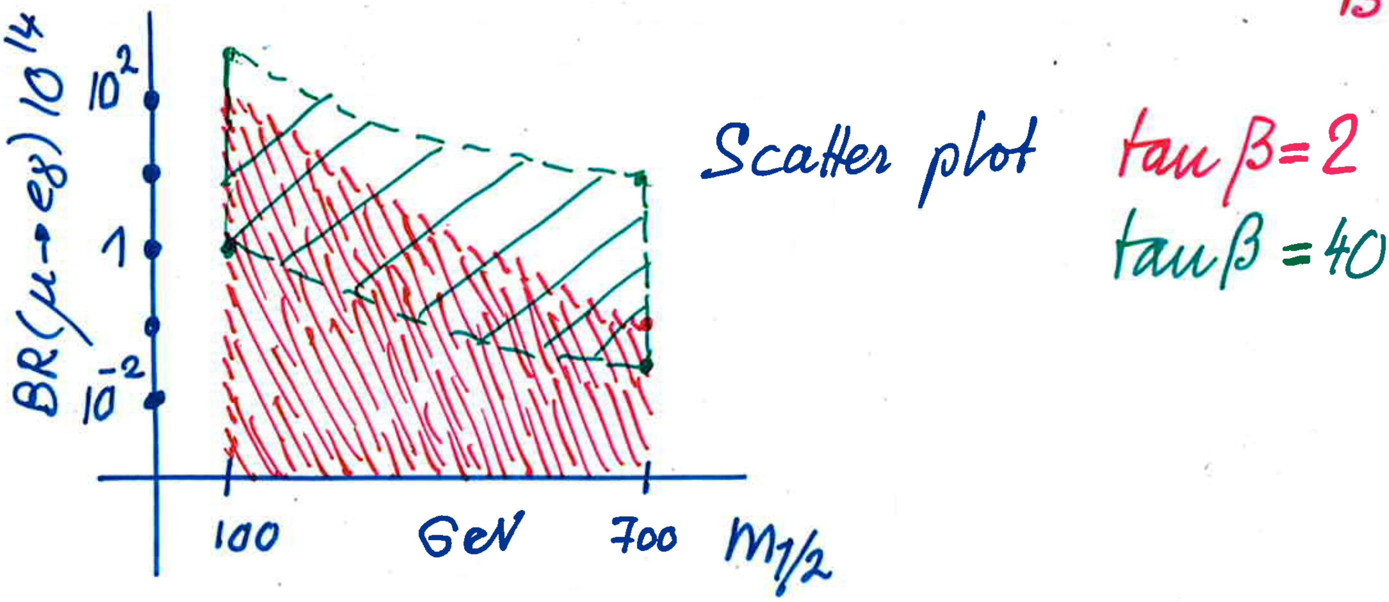
* $L_M \cong \nu_L \nu_R h_\nu v + M \nu_R \nu_R \rightarrow m_\nu \simeq m_e^2 / M$
  M large



for $\tan\beta = 40$ $BR \gtrsim 10^{-15}$
over all range

For a "maximal" model:

$BR \gtrsim 10^{-11} - 10^{-12}$
over all range

- Susy - GUTS

  $V_{CKM} \to V_{leptons}$ (GUT-relation)

  $\to$ lepton flavour violations too

  Models yield $\mu \to e\gamma$, $\tau \to \mu\gamma$ etc.

  (Barbieri, Hall, 1995)

  middle values $\sim 10^{-11} - 10^{-13}$

8. Other effects

- A few years ago, $g-2$ was a hot topic; large SUSY-effects (L-R transition)

- Various EDM's give good bounds on Im's; in particular on μ.

Conclusions

The smallness of flavour violation in general is not natural in SUSY and imposes signifant bounds. Dificult to account for.

A solution to the flavour problem more urgent than in the SM, where effects "automatically" small.

This might (?) lead to a breakthrough