LMU-TPW 2003-..

History of Supersymmetry

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Abstract

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Supersymmetry:

Supersymmetry is based on the algebra:

$$\ldots \qquad (1)$$

Q_α is a spinorial quantity and $\bar{Q} = Q^{+}\gamma^{0}$. I shall talk about four-dimensional space, $m = 0, 1, 2, 3$. It is remarkable that the spinorial charges can be expressed in terms of local currents in a Lorentz-covariant quantum field theory.

$$\ldots \qquad (2)$$

The supersymmetry algebra is then a consequence of the canonical commutation relations of the fields.

The beginning of supersymmetry, a brief history:

It was discovered independently three times in the second half of the last century.

I. By Golfand and Likhtman in Moscow in the year: 1971
II. By Volkov and Akulov and by Volkov and Sorokov in Kharkov in the year:
III. By Zumino and Wess at CERN, Geneva, and in Karlsruhe in the year:

The way that led to supersymmetry was different in the work of the three groups, this might be the reason why each group was unaware of the work done by the other groups. The reason why all three groups ended up by a version of supersymmetry in the context of four-dimensional QFT has a deep reason which I will try to explain later.

Golfand and Likhtmann where the first to envisage the concept of supersymmetry. They tried to extend the Poincare algebra by spinorial charges. From an algebraic point of view Berezin and Katz had already pioneered this idea and had published a paper on "Lie groups with commuting and anticommuting parameters" in the year 1970. Golfand and Likhtman tried to use such algebras with spinorial charges in the framework of quantum field theory and to generalize the Coleman-Mandula theorem to such algebras. O'Raiffeartaigh, Coleman and Mandula had shown that the Lie algebras that can be realized in a local Lorentz-invariant QFT are of the type of a direct product of a compact group and the Lorentz group. Golfand and Likhtman were well on the way to supersymmetric QFT.

Volkov, Akulov and Soroka invented supersymmetry in the spirit of nonlinear σ-models. In such a model Goldstone fields transform nonlinearly and are massless. In nature fermions (neutrinos) are closest to have mass zero. Can they be Goldstone fields for a spontaneously broken symmetry? Starting from this idea, Volkov et al. constructed a σ-model type of interaction and the fermions transformed under a nonlinear

Supersymmetry

Algebra:

$$\{ Q_\alpha , \bar{Q}_\beta \}_+ = 2 \gamma^m_{\alpha\beta} P_m$$

$$\{ Q_\alpha , Q_\beta \}_+ = [P_m , Q_\alpha] = [P_m , P_n] = 0$$

Q_α : spinorial $\quad \bar{Q} = Q^\dagger \gamma^0$

$m = 0, 1, 2, 3$

Supersymmetry in four dimensions and its relation to QFT:

Local currents

$$Q_\alpha = \int d^3x \, J^0_\alpha (x)$$

$$P_m = \int d^3x \, \Theta^0{}_m$$

$$\underbrace{\partial_n J^n_\alpha (x) = 0}_{\text{spinorial currents}} , \quad \partial_n \Theta^{n\mu}(x) = 0$$

Historic background

Q.F.T, 4.dim

a) Axioms :

1) Lorentz invariance P^m, M^{mn}

2) QM : Hilbertspace

3) local fields (finite number)

$$P^m = \int d^3x\, \Theta^{om}$$

$$\partial_n \Theta^{nm} = 0$$

$$[\Theta^{nm}(x), \Theta^{rs}(x')] = 0, \quad x \sim x' \text{ spacelike}$$

4) Spectrum :

Vacuum: $|0\rangle$, $|m\rangle$ one particle states

5) Conservation of probability (Unitarity of S-Matrix)

$\Rightarrow$ Spin Statistik

CPT Theorem

Coleman Mandula theorem

b) Field theoretic Models

QED, A^4

free field theory with canonical quantization

interactions

perturbation theory

renormalization
(fight against singularities)

symmetries:

$U(1)$: ED
$SU(2)$: Isospin } $SU(3)$

$SU(3)_c \times SU(2) \times U(1)$: standard model

GUT: $SU(5)$

Wigner Hund : $SU(4) \simeq \underset{\text{Spin}}{SU(2)} \times \underset{\text{Isospin}}{SU(2)}$

$SU_3 \times SU_2 \subset SU(6)$

NO GO THEOREM

O'Raifeartaigh,
Coleman Mandula

space time || inner space

Yang Mills theories

t' Hooft and Veltman

renormalizability on the basis

of gauge invariance,

gauge models : vector particles

Anomalies

Co. Ma.

Large playground,

Spontaneous symmetry breaking

renormalization properties

survived.

conserved currents, Q does not exist

J. Cornwell

Llewellyn Smith

Steve Weinberg

renormalizability ⇒ gauge theory

History of the beginning of supersymmetry in four dimensions.

Brief and personal!

Prehistory:

anticommuting parameters, η, ξ,

Schwinger, Symanzik

Lie groups with commuting and anticommuting parameters

F. A. Berezin and G. I. Katz (1970)

Supersymmetry and supercurrents in 2-dim models

A. Neveu, J. Schwarz,

J.L. Gervais, B. Sakita

Symmetrie on massshell only

Early history :

There were three independent roads to supersymmetry : at second half of last century

1) : Golfand and Likhtmann at Moscow

1971

2) Volkov and Akulov
Volkov and Soroka
at Kharkov

1973

3) Wess and Zumino
CERN, Geneva
Karlsruhe

1973 / 1974

1) Golfand and Likhtmann
were the first to envisage the
concept of supersymmetry.

extended the Lorentz algebra
by spinorial charges
tried to realize this algebra in QFT
were restricted by Co. Ma

found supersymmetry algebra
constructed models, ~ QED

did not find supersym transf.
off massshell.

2) Volkov, Akulov, Soroka
invented supersymmetry in the spirit of non linear σ - models.

σ model: goldstons : transform nonlinear

nature : Fermions (neutrinos) are closest to have mass zero (Heisenberg)

they constructed such theories and they were supersymmetric

$$W = \frac{1}{a} \int d^4x \, \mathrm{Det}\left(\eta_{\mu\nu} + \frac{a}{2i} (\bar{\psi} \sigma_\mu \partial_\nu \psi - \partial_\nu \bar{\psi} \sigma_\mu \psi) \right)$$

unfortunately:
de Wit and Freedman:
neutrinos do not satisfy the <u>low energy theorem</u> of goldstinos

model is notrenormalizable!

did not draw much interest

3) Bruno + myself
started from spinorial currents
in two dimensions.
Supercurrents (gave the name)

Sakita gave a talk at CERN
on the Iwasaki Kikkawa model

Our starting point
spinorial currents
→ Spinorial charges
Noether → spinorial symmetries
: Supersymmetry.

Only on mass shell

Auxiliary fields
Weyl - Majorana field

$N_B = N_F$

SUSY

All susy multiplets have $N_F = N_B$ [7]

Werner Nahm:

$$(-1)^{N_f} Q_\alpha = -Q_\alpha (-1)^{N_f}$$

$$\mathrm{Tr}\,(-1)^{N_f}\{Q_\alpha \bar{Q}_\beta + \bar{Q}_\beta Q_\alpha\}$$

cyclic property of the trace

$$\mathrm{Tr}(-1)^{N_f} Q_\alpha \bar{Q}_\beta = -\mathrm{Tr}\, Q_\alpha (-1)^{N_f} \bar{Q}_\beta$$

$$= -\mathrm{Tr}\,\bar{Q}_\beta Q_\alpha (-1)^{N_f} = -\mathrm{Tr}(-1)^{N_f}\bar{Q}_\beta Q_\alpha$$

$$= 0$$

Algebra

$$\mathrm{Tr}(-1)^{N_f}\, 2\gamma_{\alpha\beta}^{m} P_m$$

$$\Rightarrow \mathrm{Tr}\,(-1)^{N_f} = 0$$

Spontaneous broken symmetry
(σ-model)
Q_α does not exist, only currents
(Reh, Schlieder)

Chiral Susy multiplet:

A, ψ, F

$$\delta_\xi A = \sqrt{2}\, \xi \psi$$

$$\delta_\xi \psi = i\sqrt{2}\, \sigma^m \bar{\xi}\, \partial_m A + \sqrt{2}\, \xi F$$

$$\delta_\xi F = -i\sqrt{2}\, \bar{\xi} \bar{\sigma}^m \partial_m \psi$$

1) ξ: anticommuting parameter:

Dim $[\xi]$:

$[A] = 1$ $\quad$ $[\xi] = -1/2$

$[\psi] = 3/2$ $\quad$ $[F] = 2$

2) $F = 0$: $\bar{\sigma}^m \partial_m \psi = 0$, $\Box A = 0$

3) Degrees of freedom

off : $A: 2$, $\psi: 4$, $F = 2$

on : $A: 2$, $\psi: 2$, $F = 0$

4) highest component transforms into a derivative!

Anticommuting parameters

$$[\xi\psi, \eta\chi]_- = -\xi\eta\{\psi, \chi\}_+$$

$$[\xi Q, \bar{\xi}\bar{Q}]_- = 2\xi\sigma^m\bar{\xi}P_m$$

Vector multiplet:

$\underset{1}{C}$ (real) $\underset{4}{\chi}$, $\underset{2}{M}$ (complex) $\underset{4}{v_m}$ (real)

$\underset{4}{\lambda}$, $\underset{1}{D}$ (real)

$N_F = 4+4$ $N_B = 1+2+4+1$

Fermions : Greek
Bosons : Latin

Lagrangians:
highest component of a multiplet

$$\mathcal{L}_0 = -\partial_m \bar{\psi} \bar{\sigma}^m \psi + A^* \Box A + F^* F$$

$$\mathcal{L}_m = m\left(AF + A^* F^* - \tfrac{1}{2}\psi\psi - \tfrac{1}{2}\bar{\psi}\bar{\psi}\right)$$

$\mathcal{L}_0 + \mathcal{L}_m$: Field equations

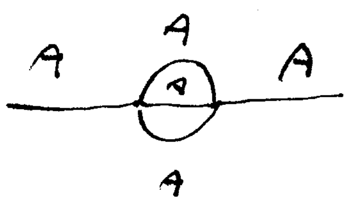
$$i\bar{\sigma}^m \partial_m \psi + m\bar{\psi} = 0$$

$$F + mA^* = 0$$

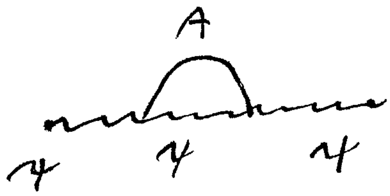
$$\Box A + mF^* = 0$$

Interaction: $g(FAA + A\psi\psi)$
$+$ c.c

leads to A^4 couplings
It is componentwise renormalizable



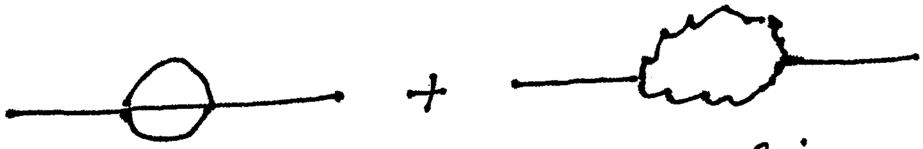
$\dfrac{p^8}{p^6} \sim p^2$



$\dfrac{p^4}{p^3} \sim p$

A and ψ are in the same multiplet!

Chancelation of divergencies:
or anomalies

 : only log.

no mass renormalization
only log. fieldstrength renorm.

Zumino: vacuum-vacuum
diagrams chancel.

no vacuum energie ⇝
cosmological constant.

A new way to supersymmetry
(Cornwell, Lewely + Smith, Weinberg)
spin 0, 1/2 fields
calculate the one loop contributions
ask them to be as smooth as possible
→ Susy model.

Fujikawa, Lang A^6 theorie
it remains nonrenormalizable

gravity?

SQED : Salam Strathdee
Ferara, Zumino, J.W

start from a vector multiplet
8 + 8 components
invariant under supergauge transf.
$\lambda(x) \longrightarrow \Lambda(x)$: chiral superfield
4 + 4 gauge parameters.

Weyl fields combine to a Dirac field
4 + 4 components on m.s
Bosons v^m : 2, A complex

W.Z gauge : transform to
physical degrees of freedom
brakes supersymm.
renormalizable.

Superspace :

Salam, Strathdee

Ferara, Zumino, J.W.

ξ, θ : Anticommuting parameters

group element:

$$G(x, \theta, \bar\theta) = e^{i(-x^m P_m + \theta Q + \bar\theta \bar Q)}$$

$$G(0, \xi, \bar\xi)\, G(x, \theta, \bar\theta) =$$

$$G(x + i\theta\sigma\bar\xi - i\xi\sigma\bar\theta,\ \theta + \xi,\ \bar\theta + \bar\xi)$$

$\{x^m, \theta, \bar\theta\} \in$ Superspace

motion: $x^m \to x^m + i\theta\sigma^m\bar\xi - i\xi\sigma^m\bar\theta$

$\theta \to \theta + \xi, \quad \bar\theta \to \bar\theta + \bar\xi$

generated by differential operator

$$\xi Q = \xi\left(\frac{\partial}{\partial\theta} - i\sigma^m\bar\theta\frac{\partial}{\partial x^m}\right)$$

$$\bar\xi\bar Q = \bar\xi\left(\frac{\partial}{\partial\bar\theta} - i\theta\sigma^m\frac{\partial}{\partial x^m}\right)$$

$(x^m, \theta^\alpha, \bar\theta_{\dot\alpha})$: Superspace

Superfied :

$$F(x, \theta, \bar\theta) = f(x) + \theta\phi + \bar\theta\bar\chi + \cdots$$
$$+ i\theta\sigma^m\bar\theta V_m + \cdots \theta^2\bar\theta^2 d(x)$$

$(f, \phi, \chi, \cdots d)$: component fields

Transformation law :

$$\delta_\xi F = (\underbrace{\xi Q + \bar\xi\bar Q}_{\text{Dif. op.}}) F$$
$$= \delta_\xi f + \theta\delta_\xi\phi + \bar\theta\delta_\xi\bar\chi + \cdots$$
$$+ i\theta\sigma^m\bar\theta\delta_\xi V_m + \theta^2\bar\theta^2\delta_\xi d$$

Superfield $\longrightarrow$ transformation law of component fields.

: Chiral multiplet : scalar s.f A
vector s.f V

component fields $\rightarrow$ superfield

lowest component field (dimension)

$A(x)$

$$\mathcal{A}(x,\theta,\bar\theta) = e^{i(\theta Q + \bar\theta \bar Q)} \times A(x)$$

$Q, \bar Q$ acting on $\mathcal{A}$ is known in terms of component fields

$$\delta_\xi \mathcal{A} = \underbrace{e^{i(\xi Q + \bar\xi \bar Q)}}_{\text{diff operator}} \mathcal{A}(x,\theta,\bar\theta)$$

Product of superfields is again a superfield

solves the Clebsch-Gordan problem

$$\mathcal{L} = \phi^+\phi + m(\phi^2 + \phi^{+2}) + g(\phi^3 + \phi^{+3})$$

take the highest component:

Lagrangian

$$W = \int d^4x \, d\theta \, d\bar\theta \, \mathcal{L}$$

Superpropagator:

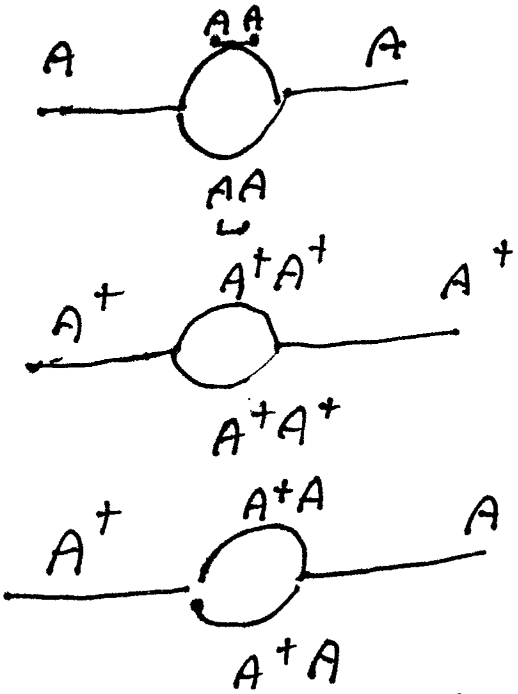
$$A(x, \theta, \bar\theta)\, A(x', \theta', \bar\theta')$$

contract component fields

$$AA, \quad A^+A^+, \quad AA^+, \quad A^+A$$

Feynman rules:

$\phi \quad A^3$



$\rightarrow A^+A, \; A^2, \; A^{+2}$ contributions

all diagrams with only A or only A' cancel to all orders.

Nonrenormalization theorem.

A^3, A^{+3}: also not renormalized

chirarchy problem:

parameters of the Higgs sector
are not renormalized.

Stability of symmetry breaking
(after finetuning)
in GUT theories.

This is the reason why we
expect susy breaking in the
order of elektroweak scale.

This is the present status

Present status:

....

Haag, Lopuszanski, Sohnius

How many supersym. in local QFT.

Coleman Mandula Theorem gives the answer for bosonic charges

P^m (spin 1), M^{mn} (spin 2) L^A (spin 0)

Extended with spinorial charges:

$A, B = 1 \ldots N$

$$\{ Q^A_\alpha , \bar{Q}_{\dot\beta B} \} = 2 \sigma^m_{\alpha\dot\beta} P_m \delta^A_B$$

$$\{ Q^A_\alpha , \bar{Q}_{\dot\beta B} \} = \varepsilon_{\alpha\beta} Z^{AB}$$

$$Z^{AB} = - Z^{BA}$$

together with internal symmetries maximal symmetry in local QFT

$\boxed{\text{local QFT likes Susy}}$

(So the reason that all found the same supersym.)

Future after more than 30000 papers

Frame of local QFT seems to be exhausted.

After more than 100 years of locality (Heinrich Hertz → Rubin)

we might have to go into a new period without locality

↓

String theory : in the moment the best candidate