Precision Tests of the Standard Model and of the MSSM

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- Electroweak precision observables
- Standard Model and precision data
- The MSSM and precision data
- A light Higgs boson in the MSSM
- Conclusions

The Standard Model

- the symmetry group $SU(2)_L \times U(1) \times SU(3)_C$

- the principle of local gauge invariance
  $\longrightarrow$ fermion – vector boson interaction
  $\longrightarrow$ vector boson – vector boson interaction

- Higgs mechanism and Yukawa interactions
  $\longrightarrow$ masses $M_W,\ M_Z,\ m_{\text{fermion}}$

renormalizable quantum field theory
accurate theoretical predictions

The (minimal) supersymmetric Standard Model (MSSM)

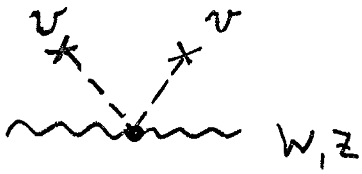
- same gauge symmetry

- Higgs mechanism with two scalar doublets,

- modified Yukawa interactions

- SUSY partners + SUSY interactions

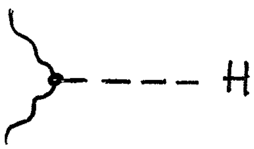
- Gauge interaction $\rightarrow W,\ Z$ mass

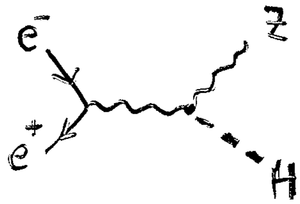


$$M_W^2 = \frac{1}{2} g_2^2 v^2$$
$$M_Z^2 = \frac{1}{2} (g_1^2 + g_2^2)^2 v^2$$

$$\frac{M_W}{M_Z} = \cos\theta_W$$

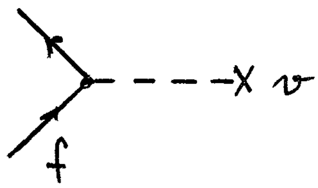
residual interaction $\sim M_{W,Z}$



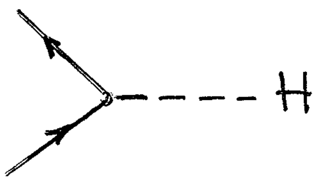


$M_H > 114$ GeV (LEP)

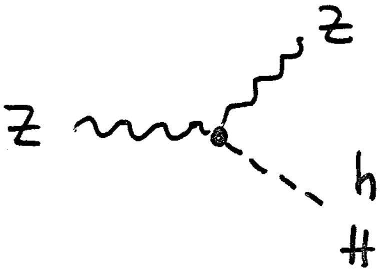
- Yukawa interaction $\rightarrow$ fermion masses



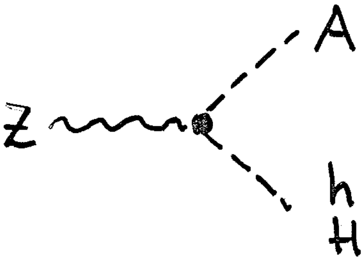
residual interaction $\sim m_f$



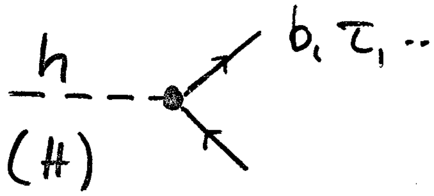
$\alpha, \beta \longrightarrow$ all couplings determined:



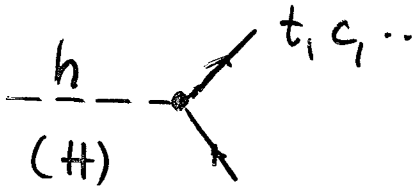
$$\frac{g_2}{c_W} M_Z \begin{cases} \sin(\beta-\alpha) \\ \cos(\beta-\alpha) \end{cases}$$



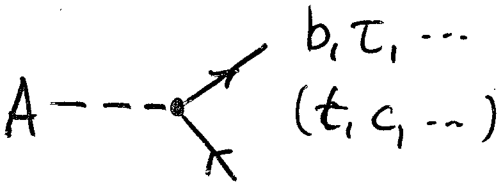
$$\frac{g_2}{c_W} \begin{cases} \cos(\beta-\alpha) \\ \sin(\beta-\alpha) \end{cases}$$



$$g_2 \frac{m_f}{M_W} \cdot \frac{\sin\alpha}{\cos\beta} \qquad \left(\frac{\cos\alpha}{\cos\beta}\right)$$



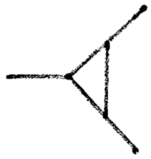
$$g_2 \frac{m_f}{M_W} \cdot \frac{\cos\alpha}{\sin\beta} \qquad \left(\frac{\sin\alpha}{\sin\beta}\right)$$



$$g_2 \frac{m_f}{M_W} \cdot \tan\beta \qquad (\cot\beta)$$

Quantum Effects and Precision Tests

quantum effects: beyond Born approximation

loop diagrams   . . .

Theory $\overset{?}{\longleftrightarrow}$ Experiments

precise predictions $\longleftrightarrow$ measurements with high accuracy

Z resonance M_Z, Γ_Z, ...

M_W

m_{top}

G_F

⋮

- LEP1/SLC: $e^+e^- \to Z \to f\bar{f}$
  LEP1: $\sim 4 \times 10^6$ events/experiment
  4 experiments (1989 – 1995)

- LEP2: $e^+e^- \to W^+W^-$
  $\mathcal{O}(10^4)$ W pairs (1996 – 2000)

- Tevatron: $q\bar{q}' \to W \to l\nu, q\bar{q}'$
  ($p\bar{p}$) $q\bar{q}' \to t\bar{t},\ t \to W^+b \to \ldots$

- low-energy experiments (μ decay, νN scattering, νe scattering, atomic parity violation, ...)

exp. results

M_Z [GeV]	$= 91.1875 \pm 0.0021$	0.002%
Γ_Z[GeV]	$= 2.4952 \pm 0.0023$	0.09%
$\sin^2\theta^{\text{lept}}_{\text{eff}}$	$= 0.23148 \pm 0.00017$	0.07%
M_W [GeV]	$= 80.426 \pm 0.034$	0.04%
m_t [GeV]	$= 178.0 \pm 4.3$	2.4%
G_F [GeV^{-2}]	$= 1.16637(1)10^{-5}$	0.001%

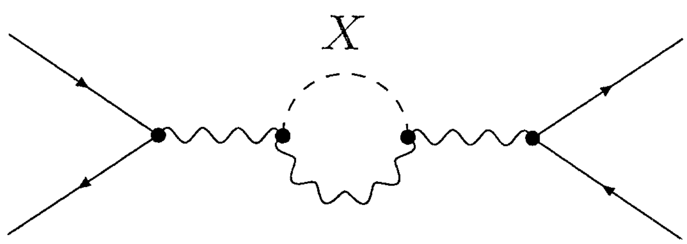
Comparison of electro-weak precision observables with theory:

EW Precision data: $\Delta\rho, \Delta r, M_W, \sin^2\theta_{\text{eff}}$	$\leftrightarrow$	Theory: SM, MSSM , ...

$\Downarrow$

Test of theory at quantum level:

Sensitivity to loop corrections



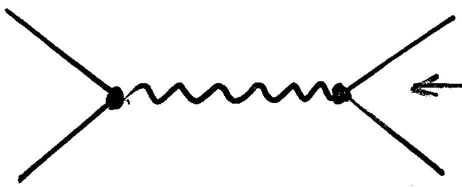
$\Downarrow$

sensitivity to internal particles (X)

Standard Model is renormalizable

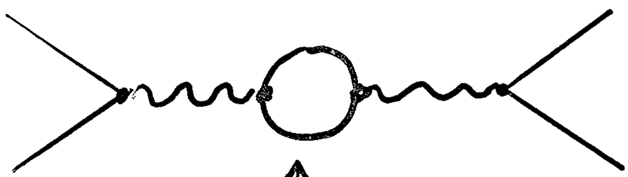
→ quantum effects are calculable

- lowest order (Born approximation)



← $g = \mathcal{O}(e)$

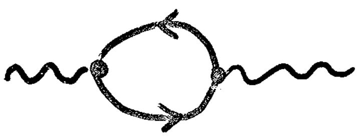
- next order (1-loop):



↑

all particles of the SM
(virtual states)

top: $\sim m_t^2$, Higgs: $\sim \log M_H$



top

⇓

determination of top mass

Higgs

⇓

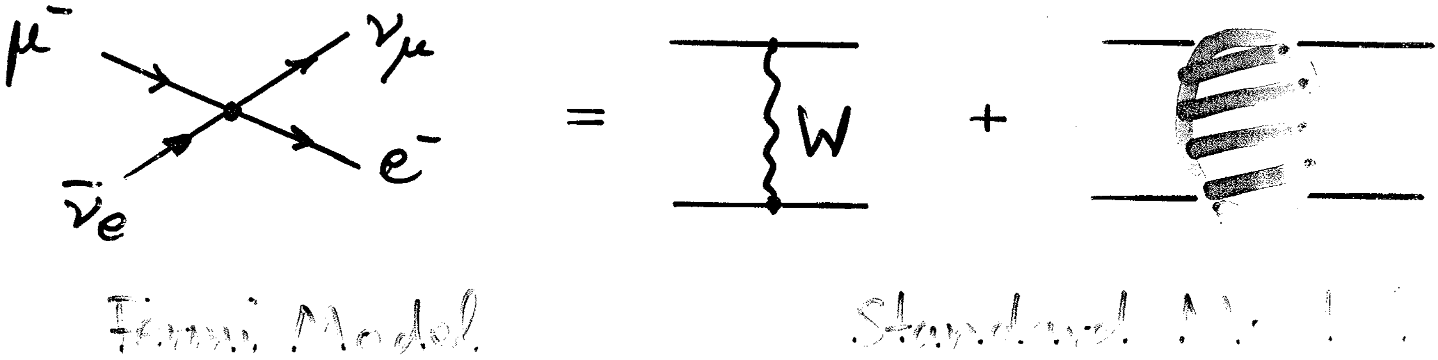
constraints on Higgs-boson mass

Masses of W and Z bosons

correlated via muon lifetime $\leftrightarrow$ Fermi constant G_μ

$$\frac{1}{\tau_\mu} = \frac{G_\mu^{\,2}\, m_\mu^5}{192\pi^3}\left(1 - \frac{8m_e^2}{m_\mu^2}\right)\cdot(1+\delta_{\text{QED}})$$

$$G_\mu = 1.16637(1)\cdot 10^{-5}\ \text{GeV}^{-2}$$

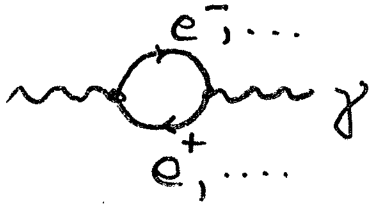


$$G_\mu = \frac{\pi}{\sqrt{2}}\cdot\frac{\alpha}{M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}\cdot\frac{1}{1-\Delta r}$$

$$\Delta r = \Delta\alpha + \Delta r_W\,(m_t, M_H \ldots)$$

6%

[QED]



mass renormalization

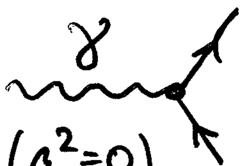
$$M_Z^2 \longrightarrow M_Z^2 + \delta M_Z^2$$

$$M_W^2 \longrightarrow M_W^2 + \delta M_W^2$$

$$\sin^2\theta_W \longrightarrow 1 - \frac{M_W^2 + \delta M_W^2}{M_Z^2 + \delta M_Z^2}$$

$$1 - \frac{M_W^2}{M_Z^2} + \frac{M_W^2}{M_Z^2}\underbrace{\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right)}_{\sim m_t^2,\ \sim \log M_H}$$

charge renormalization



$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137.036\ldots}$$

$$e \longrightarrow e + \delta e$$

$$\alpha \longrightarrow \alpha + \delta\alpha$$

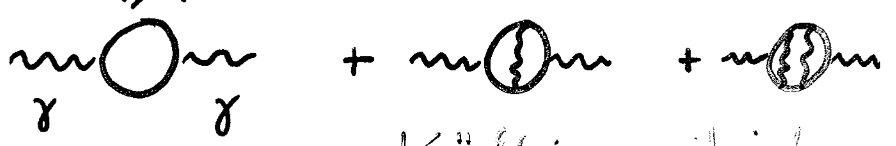
for electroweak processes:

$$\Delta\alpha = \Pi^\gamma(0) - \Pi^\gamma(M_Z^2) \simeq 0.06$$

$$\alpha \longrightarrow \frac{\alpha}{1-\Delta\alpha} = \alpha(M_Z)$$ effective e.m. α

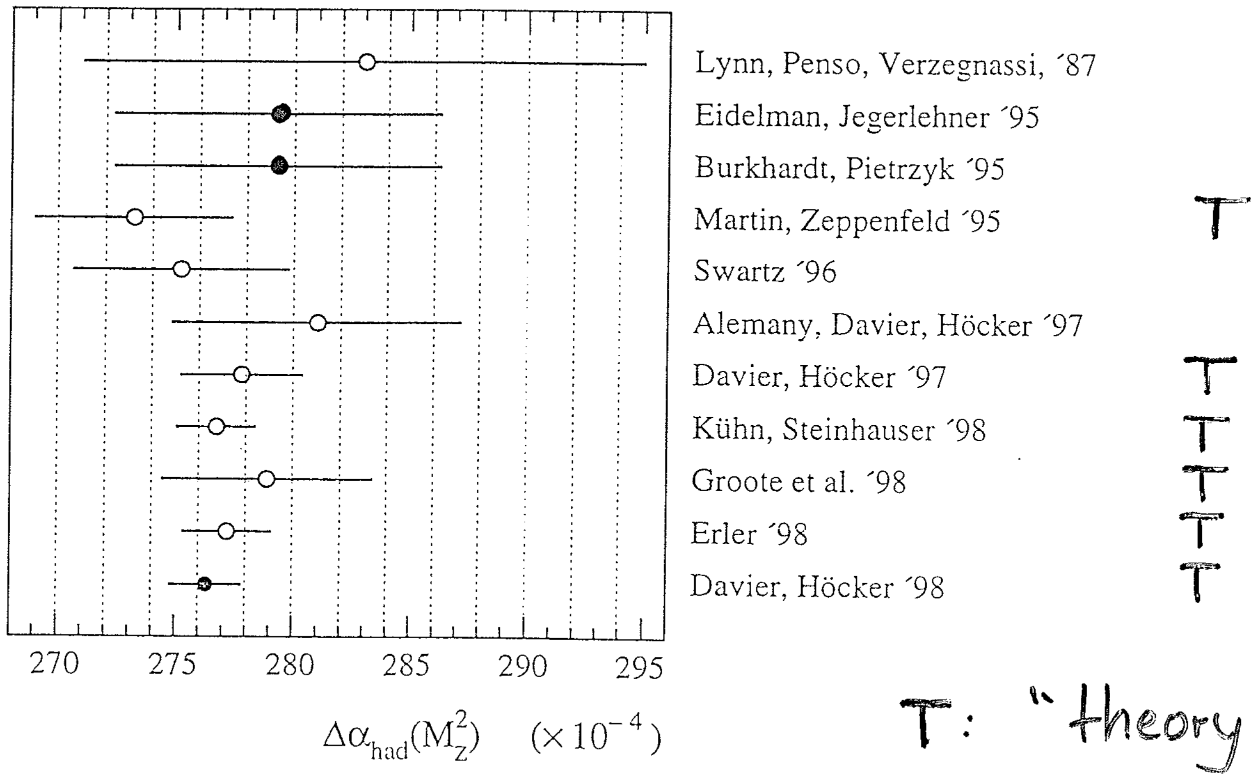
$$\Delta\alpha = (\Delta\alpha)_{lep} + (\Delta\alpha)^{(5)}_{had} + (\Delta\alpha)_{top}$$

$$(\Delta\alpha)_{lept} = \sum_{e,\mu,\tau}$$

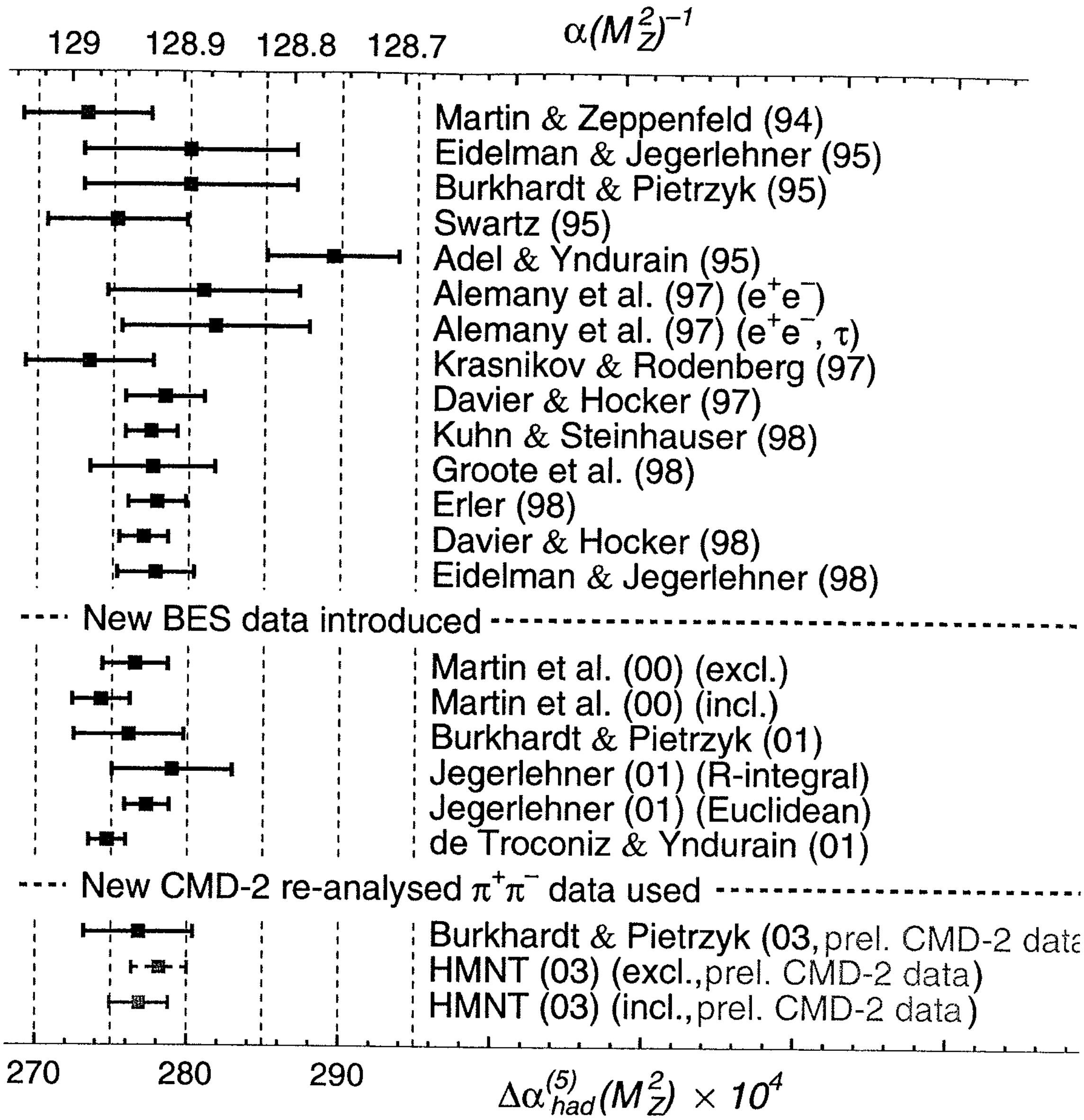


$$(\Delta\alpha)^{(5)}_{had} = -\frac{M_Z^2}{4\pi^2\alpha}\,\mathrm{Re}\int_{4m_\pi^2}^{\infty} ds\, \frac{\sigma(e^+e^- \to had)}{s - M_Z^2 - i\varepsilon}$$

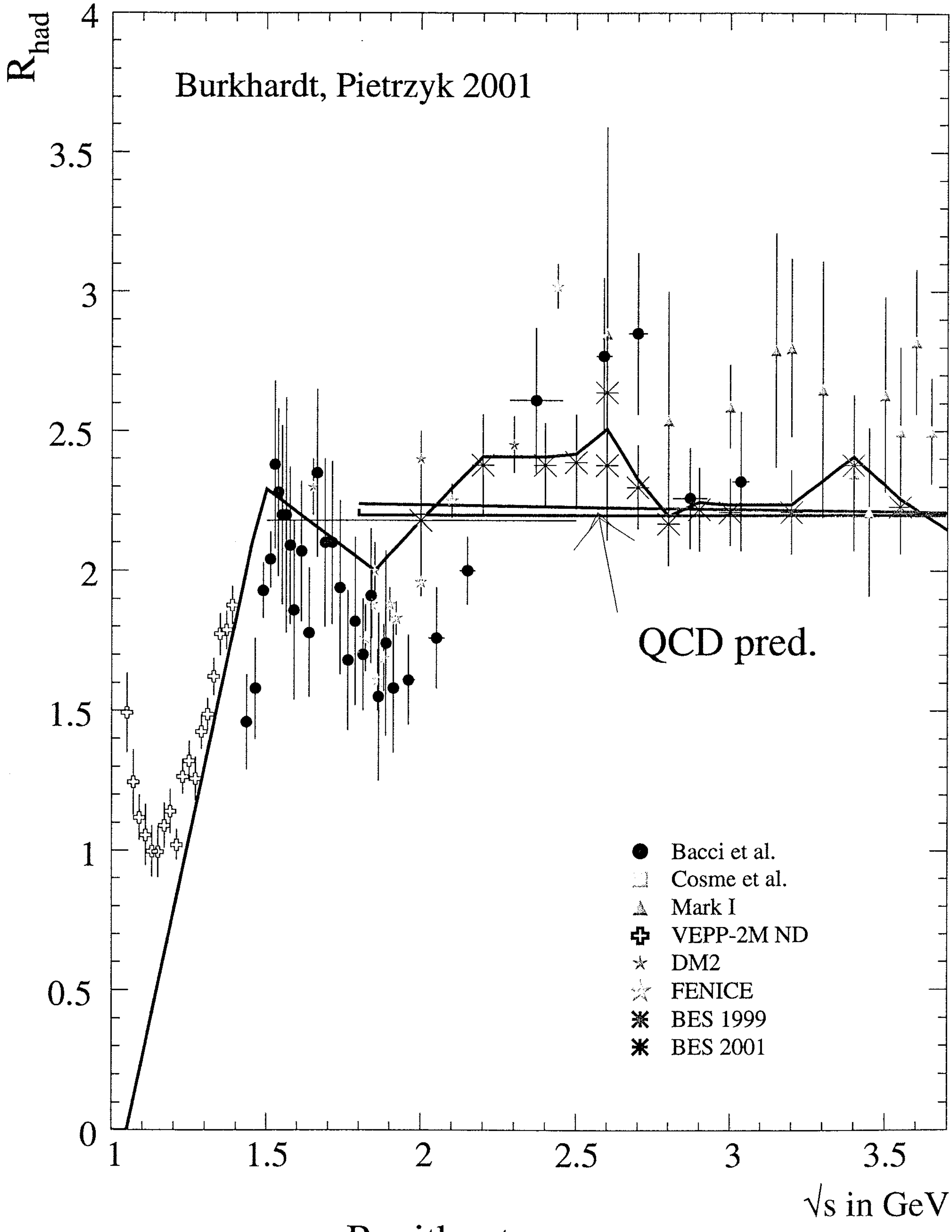
$$\alpha(M_Z) = \frac{\alpha}{1-\Delta\alpha} = \alpha\left[1 + \Delta\alpha + \Delta\alpha^2 + \cdots\right]$$



T: "theory driven"

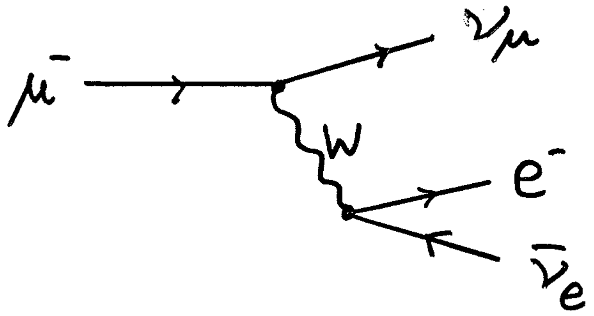


129
128.9
128.8
128.7
$\alpha(M_Z^2)^{-1}$
Martin & Zeppenfeld (94)
Eidelman & Jegerlehner (95)
Burkhardt & Pietrzyk (95)
Swartz (95)
Adel & Yndurain (95)
Alemany et al. (97) (e^+e^-)
Alemany et al. (97) (e^+e^-, τ)
Krasnikov & Rodenberg (97)
Davier & Hocker (97)
Kuhn & Steinhauser (98)
Groote et al. (98)
Erler (98)
Davier & Hocker (98)
Eidelman & Jegerlehner (98)
New BES data introduced
Martin et al. (00) (excl.)
Martin et al. (00) (incl.)
Burkhardt & Pietrzyk (01)
Jegerlehner (01) (R-integral)
Jegerlehner (01) (Euclidean)
de Troconiz & Yndurain (01)
New CMD-2 re-analysed $\pi^+\pi^-$ data used
Burkhardt & Pietrzyk (03,prel. CMD-2 data
HMNT (03) (excl.,prel. CMD-2 data)
HMNT (03) (incl.,prel. CMD-2 data)
270
280
290
$\Delta\alpha^{(5)}_{had}(M_Z^2) \times 10^4$

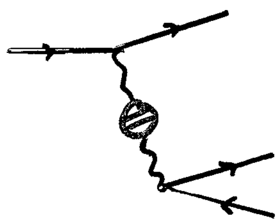


R without resonances

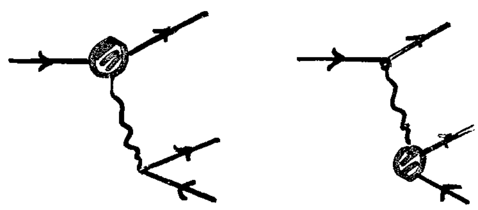
μ decay



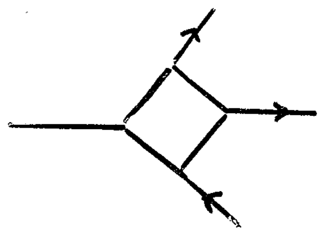
Born



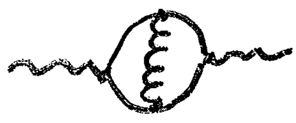
self energy



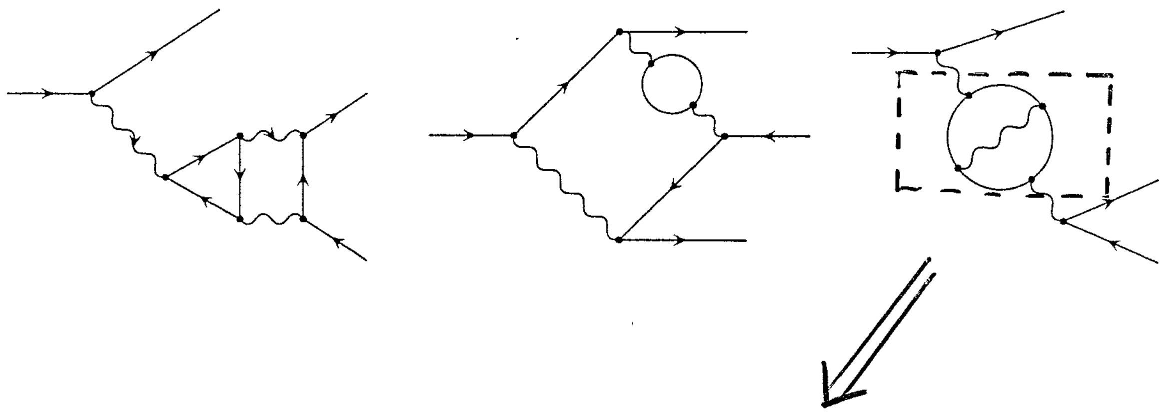
vertex corrections

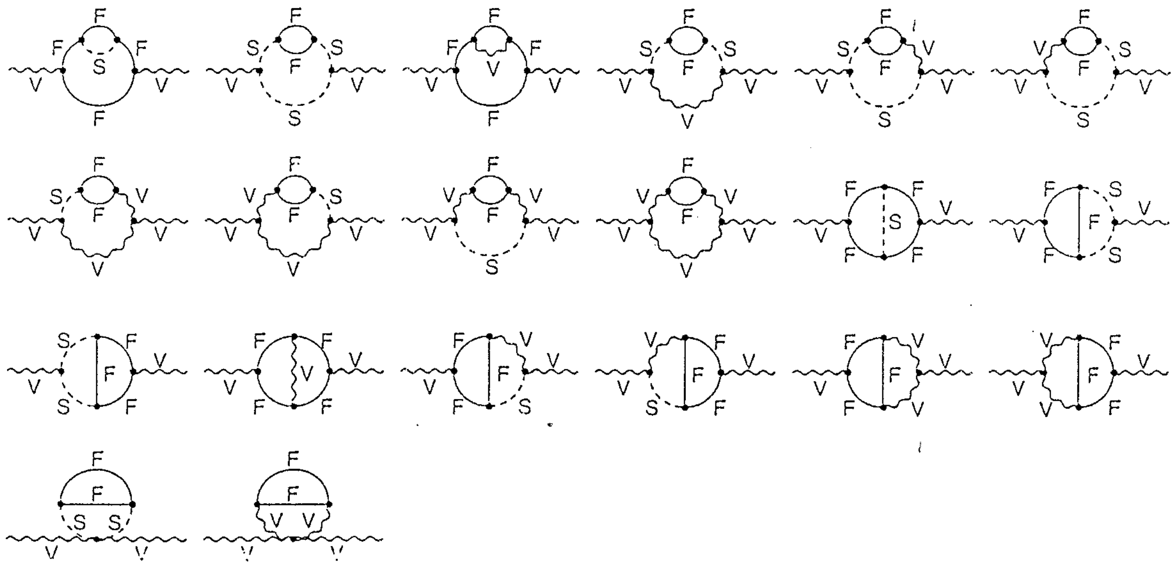


box graphs

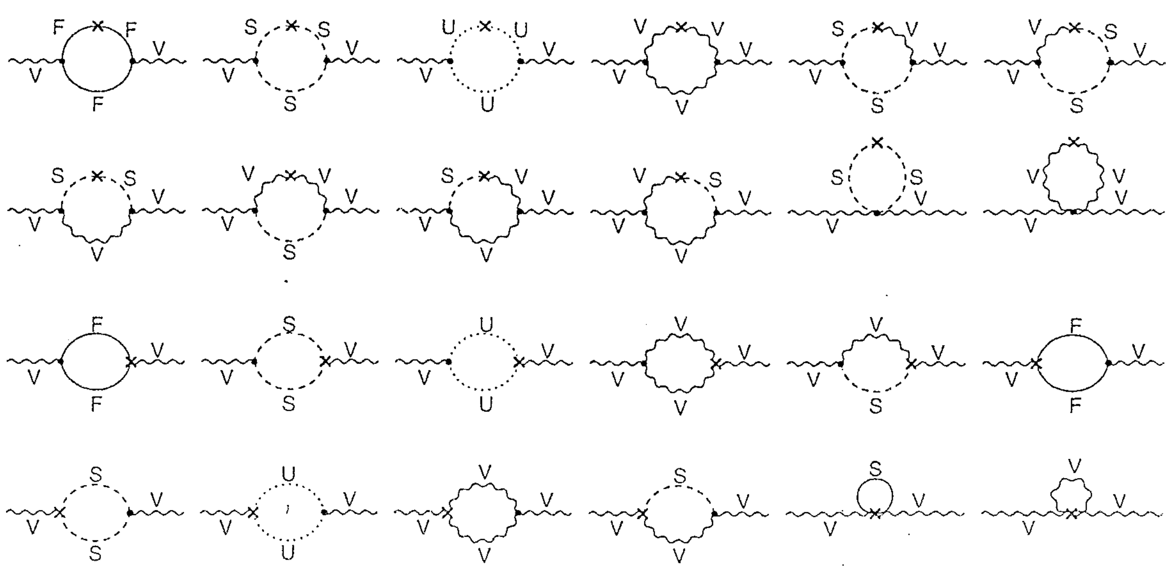
- 1-loop: complete
- 2-loop: QCD 
  EW fermionic loops
  EW bosonic loops
- 3-loop: leading QCD + EW
  (ρ-parameter)

Exact two-loop corrections to Δr with fermionic loops



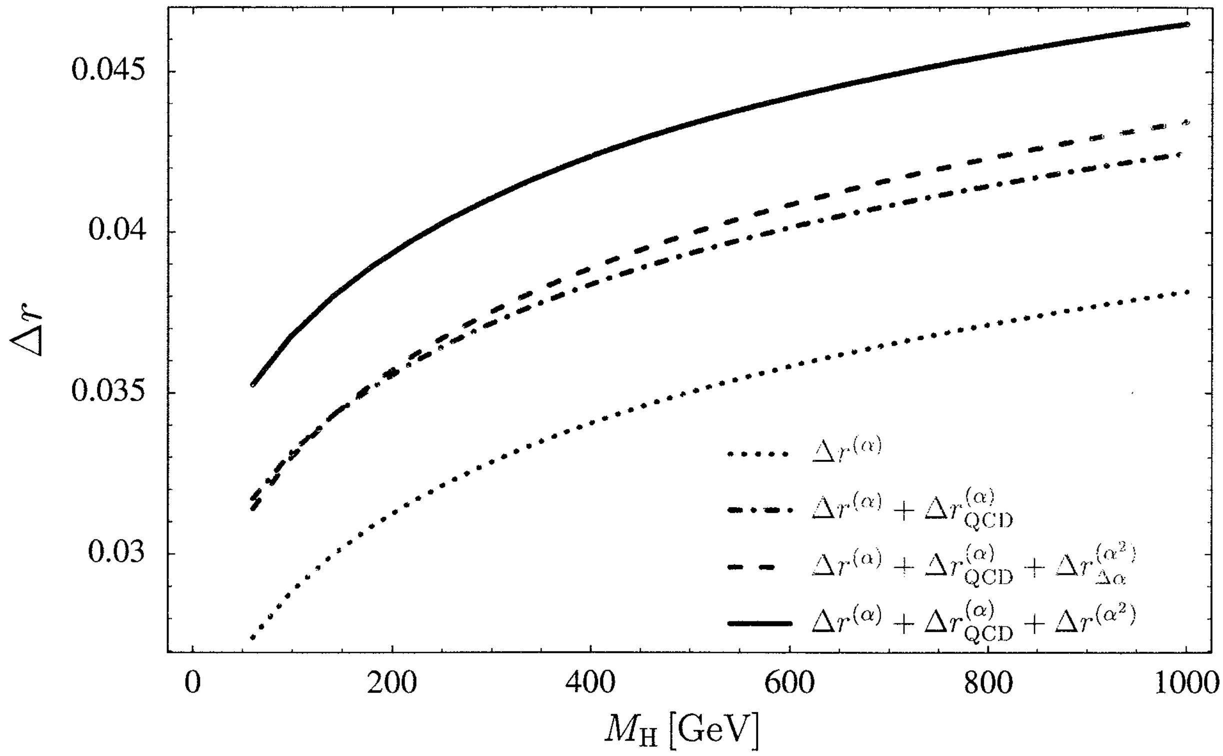


Counter term insertions (1-loop)



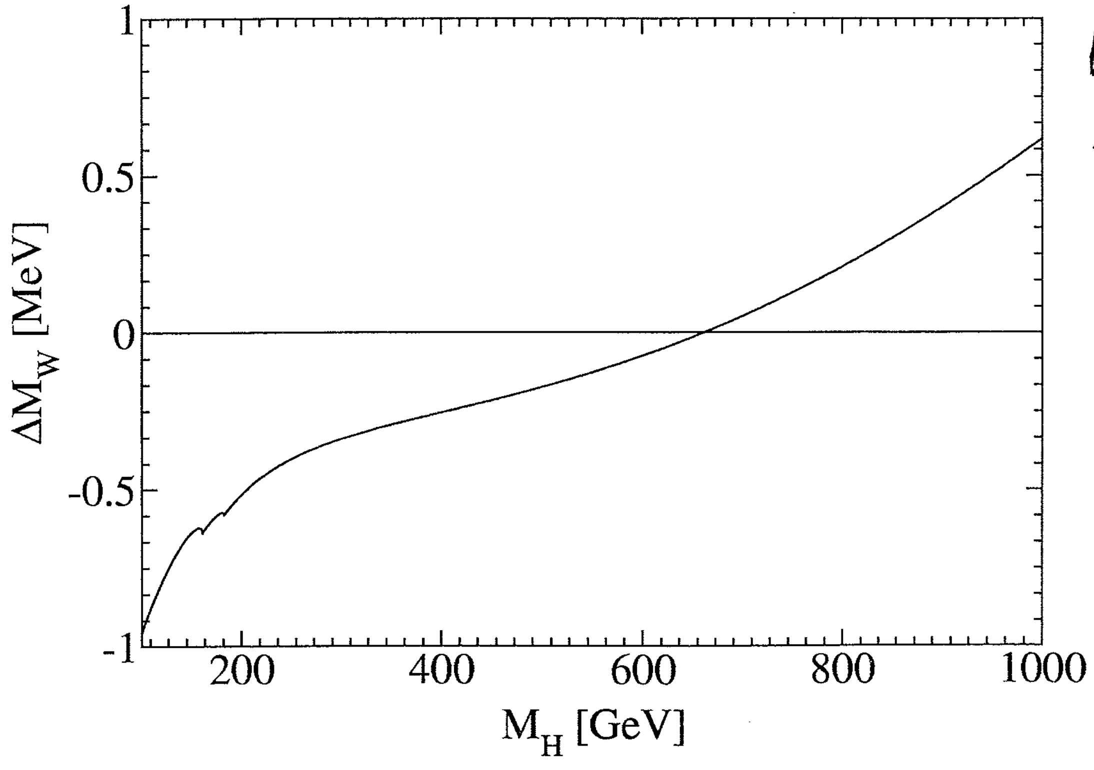
+  2-loop counter-terms

[Freitas, WH, Walter, Weiglein]



W Boson Mass Shift

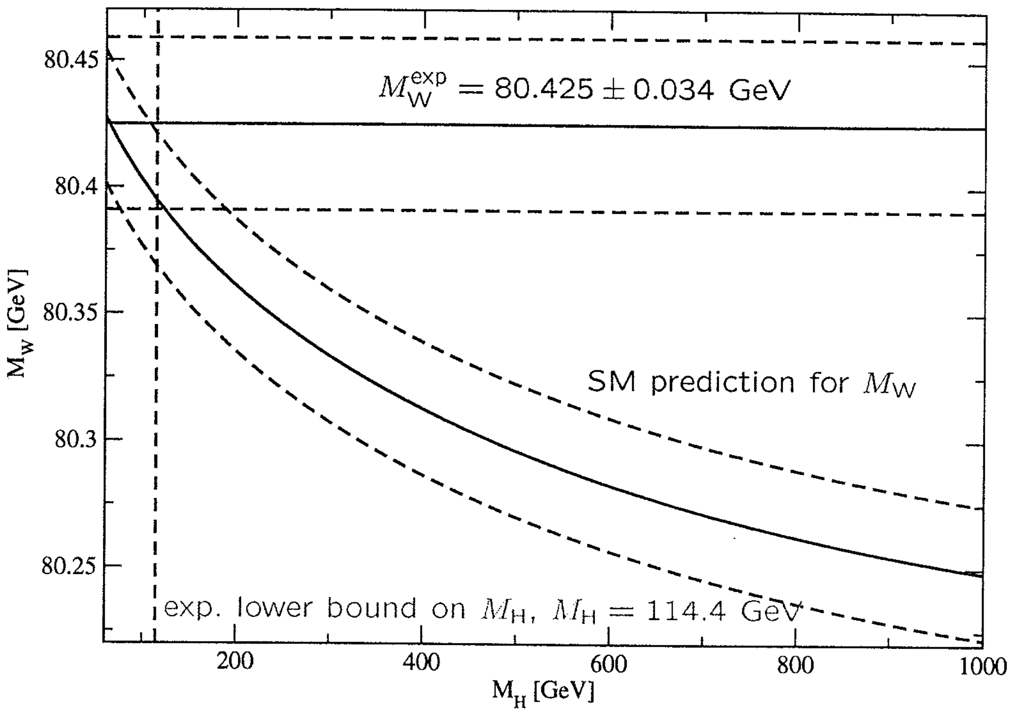
$$M_W = M_Z \sqrt{\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha}{\sqrt{2} G_F M_Z^2}(1 + \Delta r)}}$$



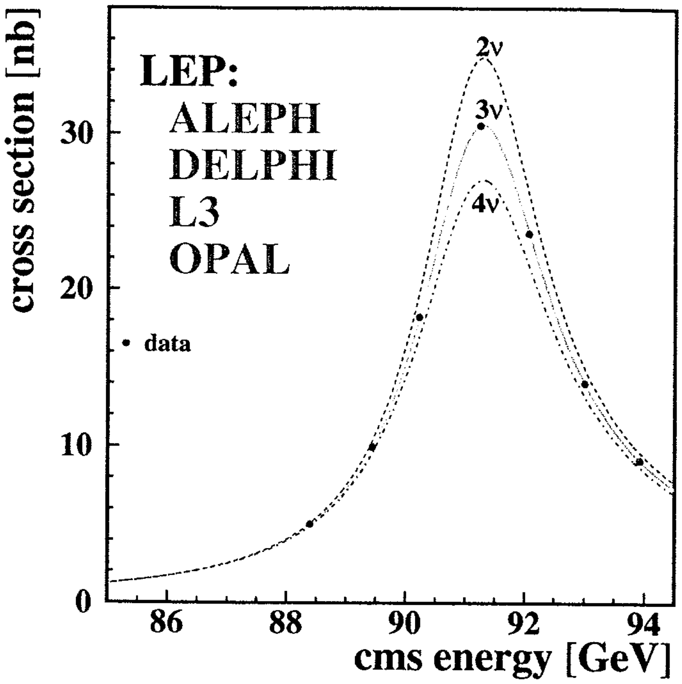
bosonic EW 2-loop

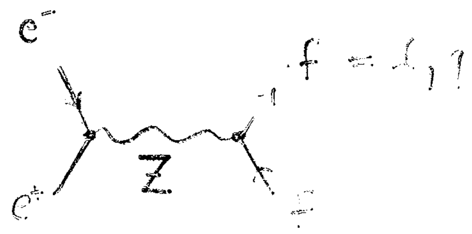
[Awramik, Czakon] [Onishchenko, Veretin]

[Awramik, Czakon, Freitas, Weiglein '04]



Z Resonance





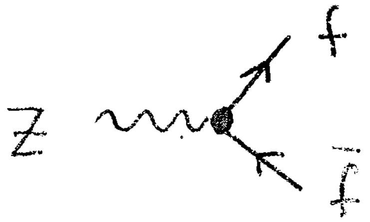
$$\sigma(\mathrm{s}) \;=\; \frac{\Gamma(Z\to e^+e^-)\cdot\Gamma(Z\to f\bar{f})}{(s-M_Z^2)^2 \;+\; M_Z^2\,\Gamma_Z^2}$$

$$\Gamma_Z \;=\; \Gamma(e,\mu,\tau)+\sum_q \Gamma(q\bar{q}) + N_\nu\,\Gamma(\nu\bar{\nu})$$

→ mass M_Z , width Γ_Z , partial widths $\Gamma_e, \ldots$

→ $\sin^2\theta_W$ from angular distribution, asymmetries

Z boson observables



g_V^f, g_A^f and $(G_\mu M_Z^2)$ normalizatio

$$\Gamma_{Z\to f\bar{f}} = N_c^f \frac{\sqrt{2}\, G_\mu M_Z^3}{12\pi}\left[(g_V^f)^2 + (g_A^f)^2\right]\left(1+\frac{3\alpha}{4\pi}Q_f^2\right)\cdot K_{QCD}$$

$$A_{FB}^f = \frac{3}{4}\cdot\frac{2g_V^e g_A^e}{g_V^{e\,2}+g_A^{e\,2}}\cdot\underbrace{\frac{2g_V^f g_A^f}{g_V^{f\,2}+g_A^{f\,2}}}_{=A_f}$$

$$P_\tau = \frac{2g_V^\tau g_A^\tau}{g_V^{\tau\,2}+g_A^{\tau\,2}}$$

$$A_{LR} = \frac{2g_V^e g_A^e}{g_V^{e\,2}+g_A^{e\,2}}$$

depend on ratio $\frac{g_V^f}{g_A^f} \leftrightarrow \sin^2\theta_f$

b-quark asymmetry:

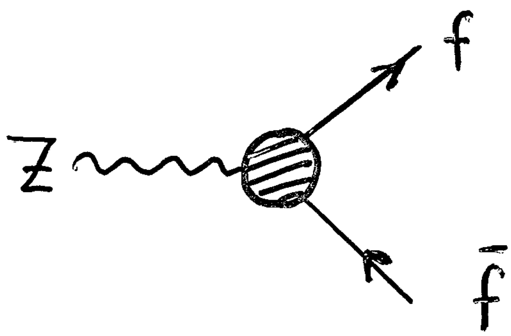
$$A_{FB}^b = \frac{3}{4}A_e\cdot A_b$$

$A_e \Rightarrow \sin^2\theta_e$

$A_b \simeq$ const in SM

0.935 ± 0.001

Effective Z couplings



$$g_A = \sqrt{\rho_f}\, I_3^f$$
$$g_V = \sqrt{\rho_f}\, (I_3^f - 2 Q_f \sin^2\Theta_f)$$

$$\rho = \rho(m_t, M_H, \dots)$$

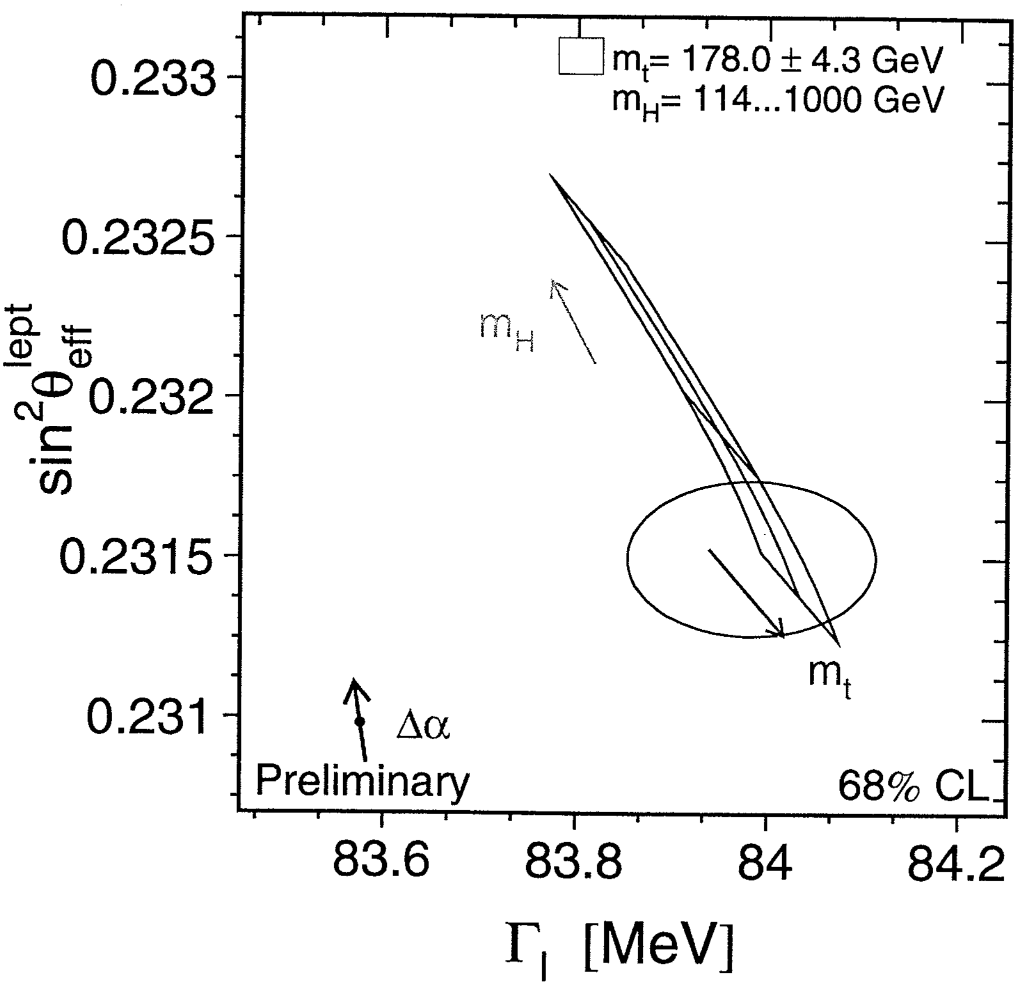
$$\sin^2\Theta_f = \sin^2\Theta_f(m_t, M_H, \dots)$$

$f =$ lepton: $\sin^2\Theta_{eff}$

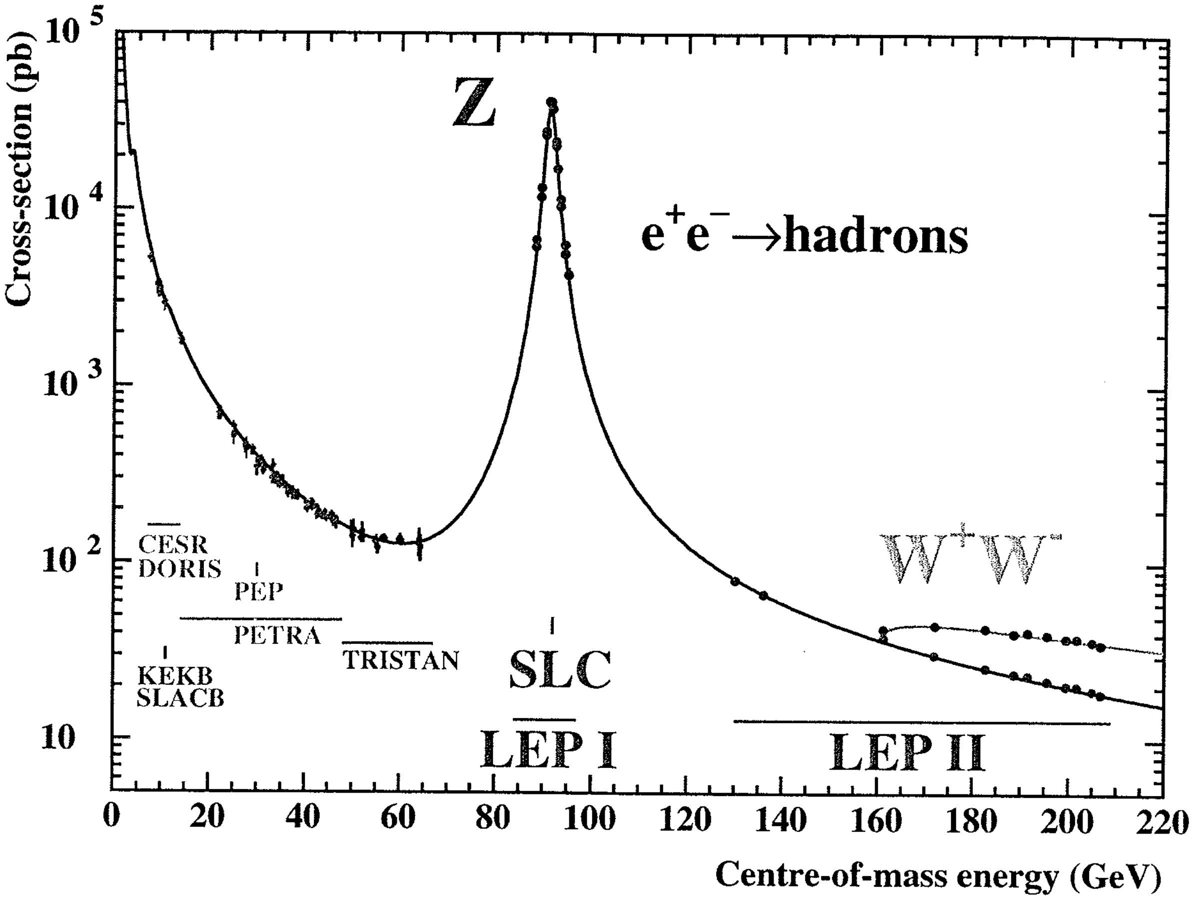
complete at 1-loop

+ 2-loop m_t^4, m_t^2 terms

↑ Degrassi, Gambino, Sirlin

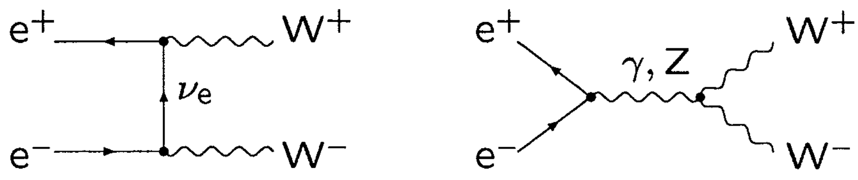


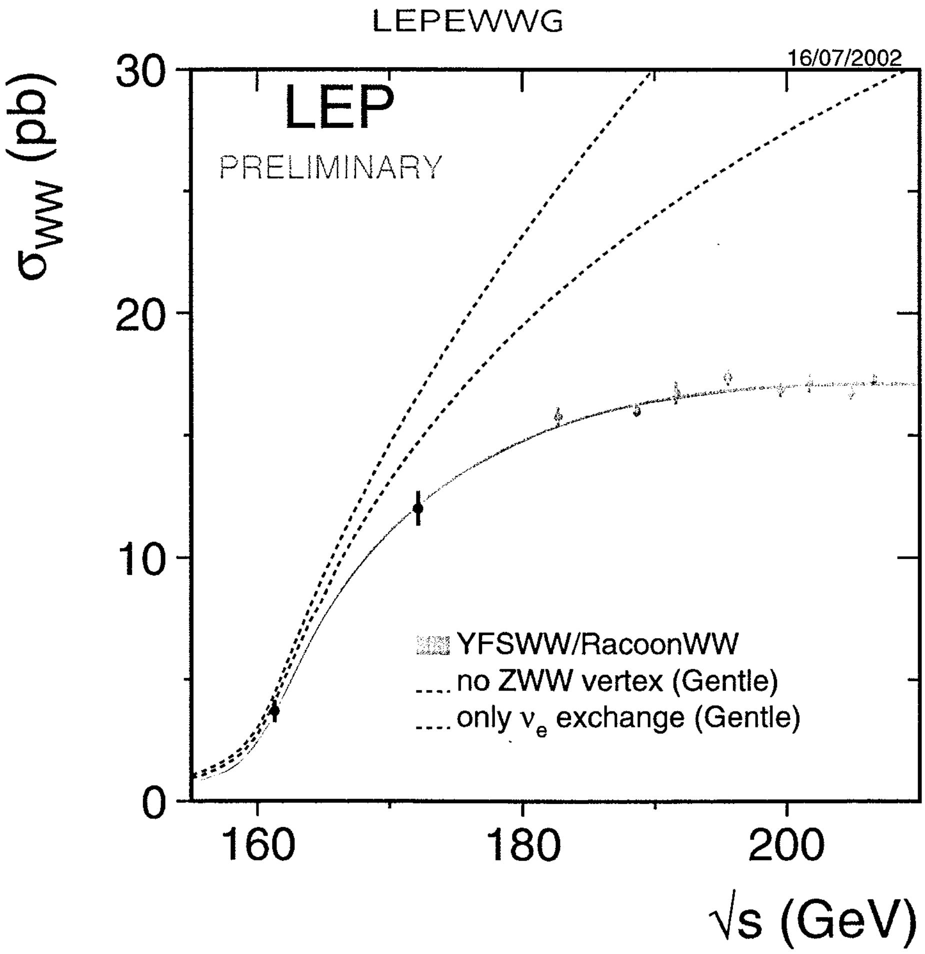
0.233
0.2325
0.232
0.2315
0.231
$\sin^2\theta_{eff}^{lept}$
m_t= 178.0 ± 4.3 GeV
m_H= 114...1000 GeV
m_H
m_t
Δα
Preliminary
68% CL
83.6
83.8
84
84.2
Γ_l [MeV]



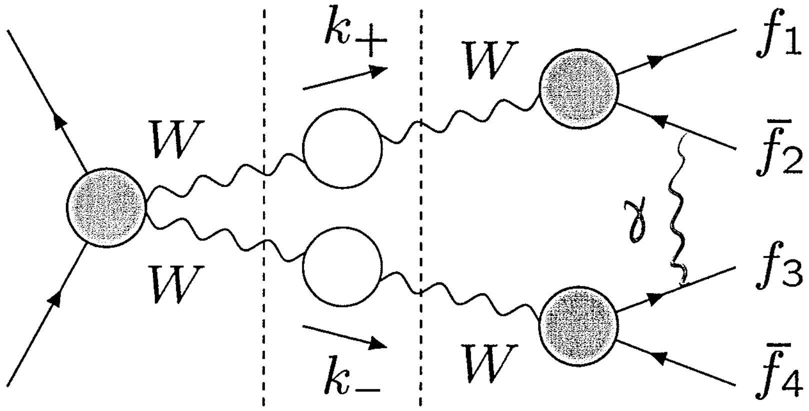
Cross-section (pb)
10 5
10 4
10 3
10 2
10
Z
e+e−→hadrons
W+W-
CESR
DORIS
PEP
PETRA
TRISTAN
KEKB
SLACB
SLC
LEP I
LEP II
0
20
40
60
80
100
120
140
160
180
200
220
Centre-of-mass energy (GeV)

W-pair-production cross-section





$e^+e^- \to W^+W^-$



non-factorizable corrections

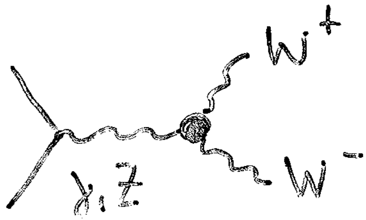
+ radiation of photons

+ background processes

[LEP: $\Delta M_W = 42$ MeV]

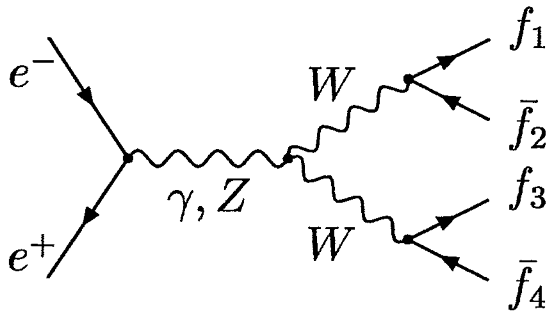
* determination of M_W
  - from reconstruction (continuum)
  - from threshold

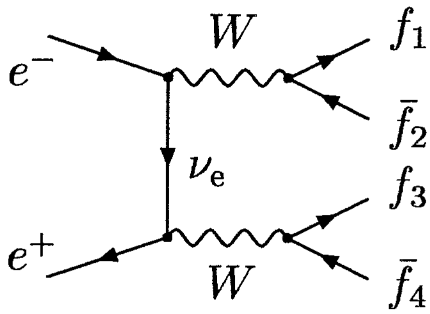
* measurement of triple-gauge couplings



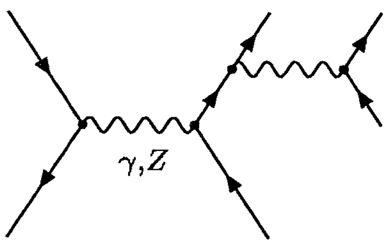
Lowest-order four-fermion cross section

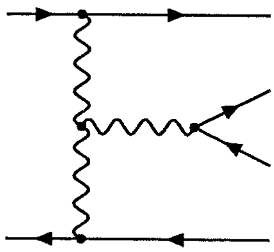
Signal diagrams: two resonant W bosons

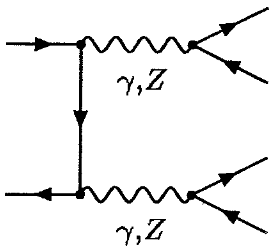




Background diagrams: at most one resonant W







etc.

Typical size $\approx \frac{\Gamma_W}{M_W} \approx 2.5\%$

Monte Carlo generator RacoonWW

$\mathcal{O}(\alpha)$ corrections with RacoonWW

Denner, Dittmaier, Roth, Wackeroth '99, '00

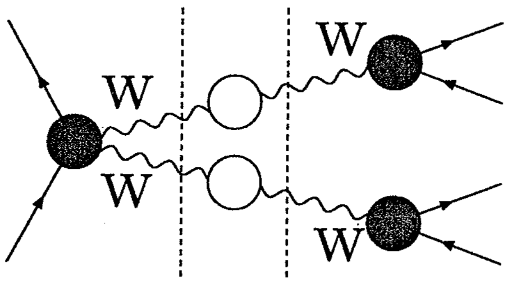
virtual

real

$e^+e^- \to 4f + \gamma$ (tree level):
Full matrix element

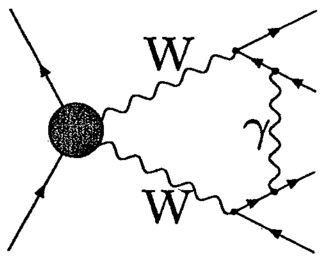
$e^+e^- \to WW \to 4f$ (1 loop):
Double-pole approximation

Factorizable corrections



building blocks:
- W-pair production
- W decay

Non-factorizable corrections

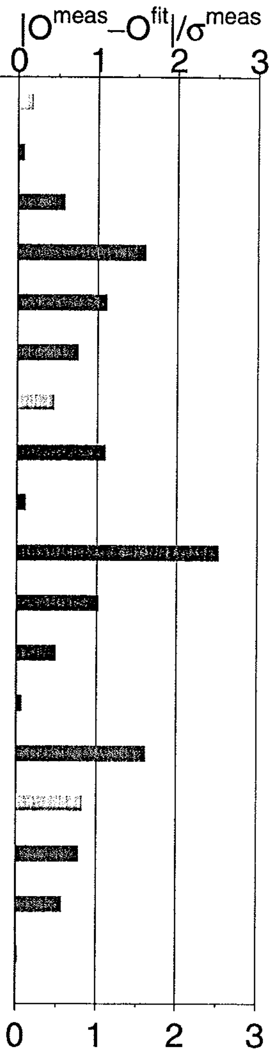


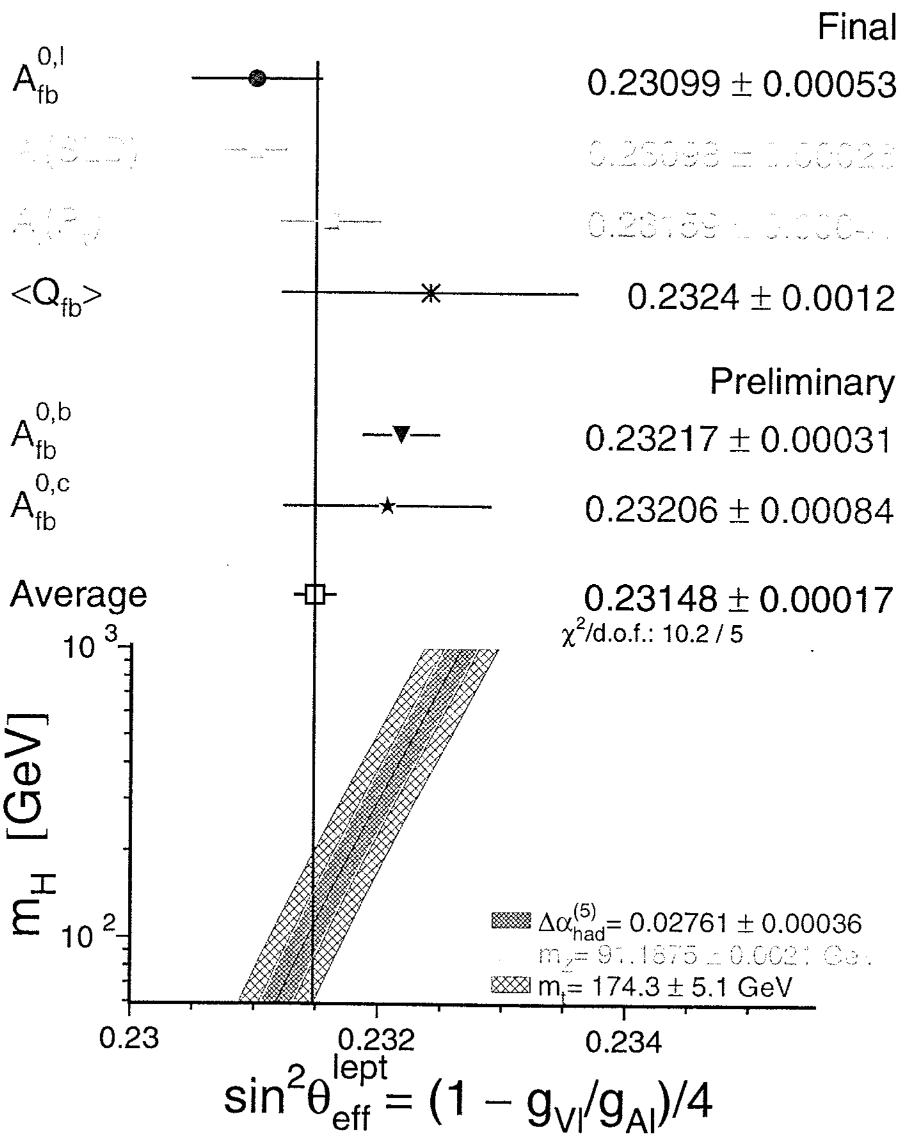
etc.

γ-exchange between production- and decay-subprocesses with $E_\gamma \lesssim \Gamma_W$

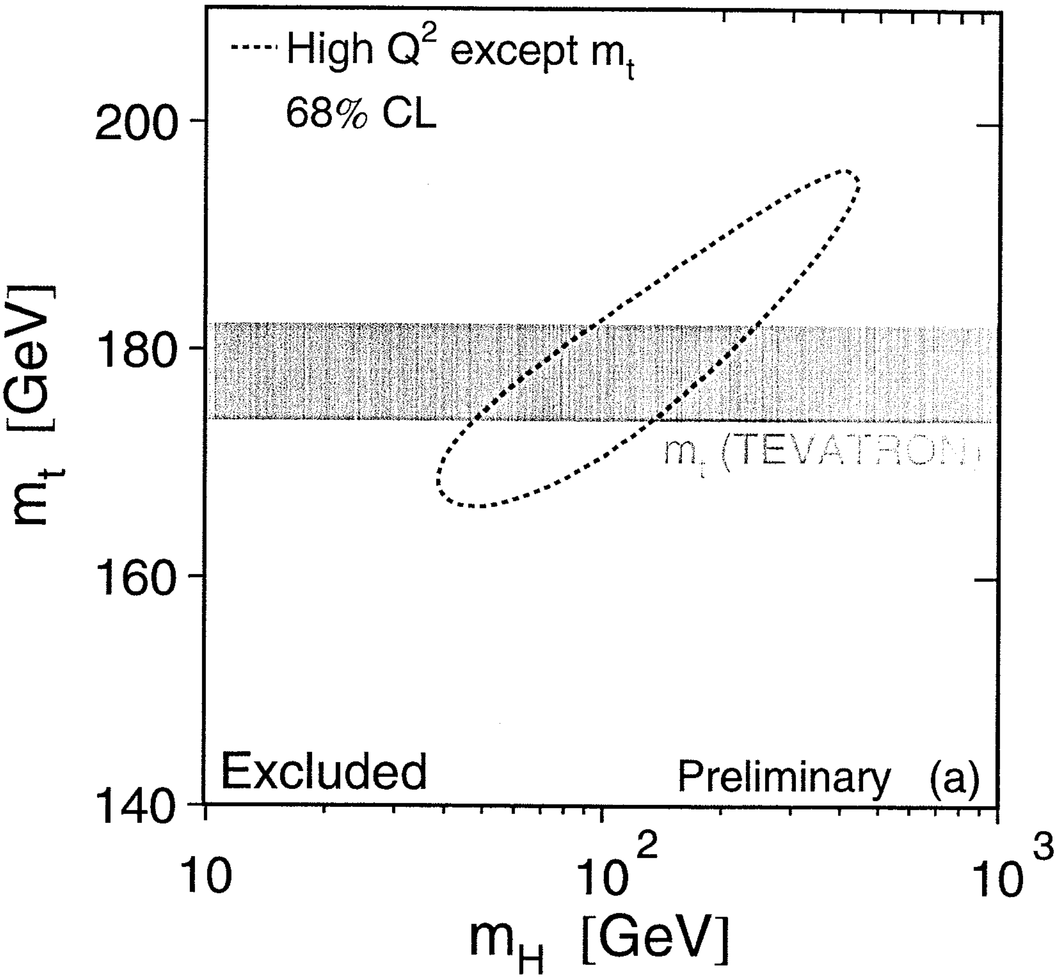
Winter 2004

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1873
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ^0_{had} [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21638 ± 0.00066	0.21566
R_c	0.1720 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0997 ± 0.0016	0.1037
$A^{0,c}_{fb}$	0.0706 ± 0.0035	0.0742
A_b	0.925 ± 0.020	0.935
A_c	0.670 ± 0.026	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.398
Γ_W [GeV]	2.133 ± 0.069	2.094
m_t [GeV]	178.0 ± 4.3	178.1

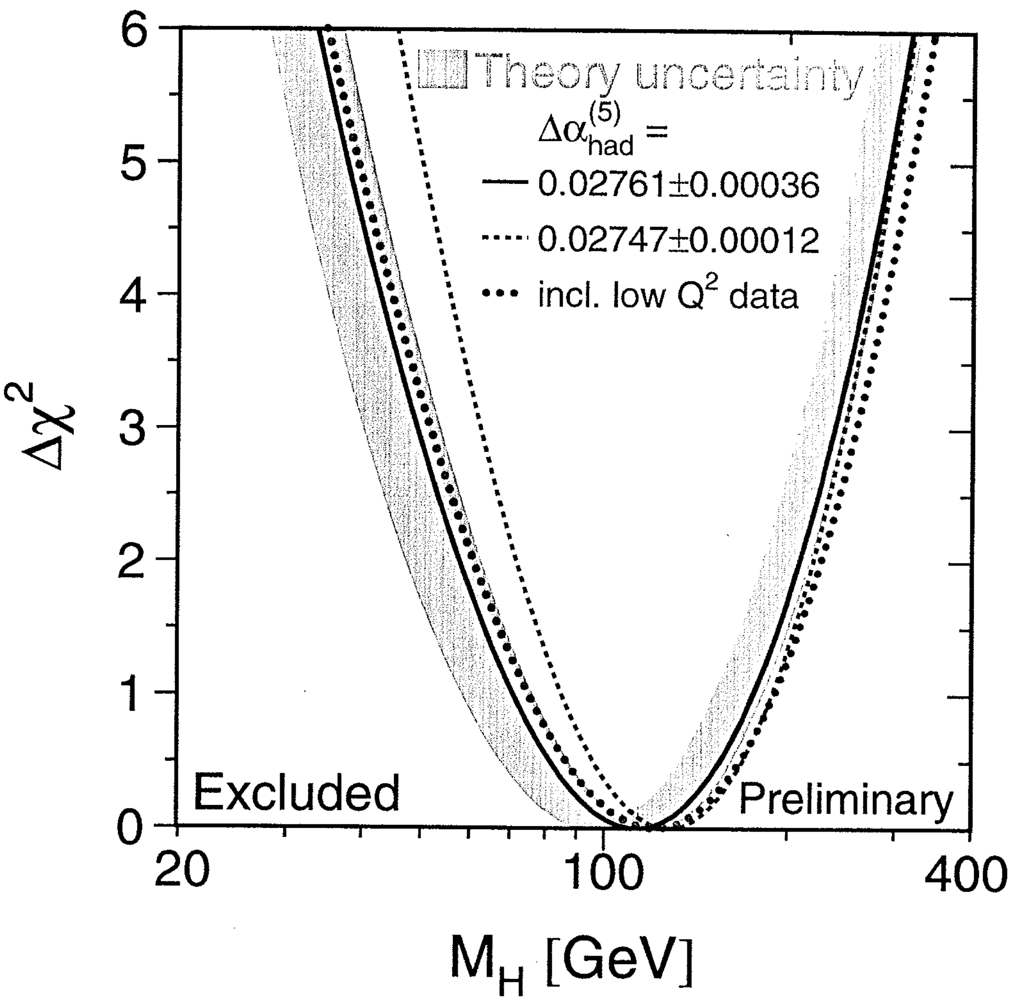




Final
A_fb^0,l
0.23099 ± 0.00053
<Q_fb>
0.2324 ± 0.0012
Preliminary
A_fb^0,b
0.23217 ± 0.00031
A_fb^0,c
0.23206 ± 0.00084
Average
0.23148 ± 0.00017
χ²/d.o.f.: 10.2 / 5
10^3
10^2
m_H [GeV]
Δα_had^(5) = 0.02761 ± 0.00036
m_t = 174.3 ± 5.1 GeV
0.23
0.232
0.234
sin²θ_eff^lept = (1 − g_Vl/g_Al)/4

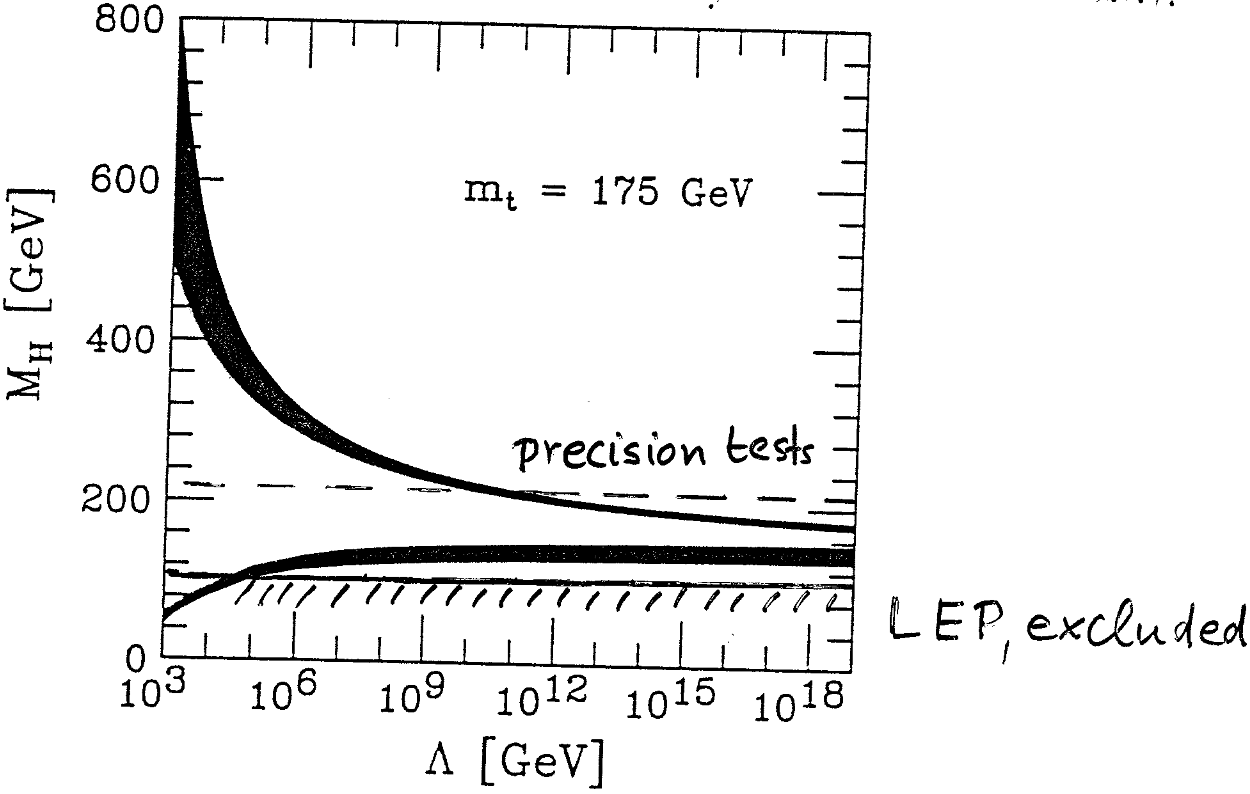


High Q^2 except m_t
68% CL
200
180
160
140
m_t [GeV]
m_t (TEVATRON)
Excluded
Preliminary (a)
10
10^2
10^3
m_H [GeV]

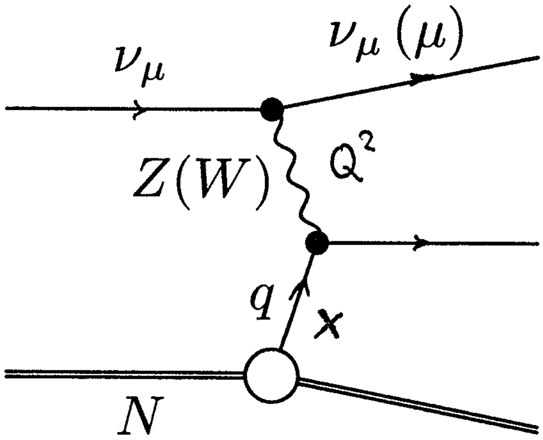


$M_H < 251$ GeV at 95% C.L.

Hambye, Riesselmann



Deep-inelastic neutrino scattering



$$x = \frac{q}{P_N}$$

$$y = \frac{Q^2}{xS}, \qquad S = (P_N + P_\nu)^2$$

$$\sigma^{\nu}_{\mathrm{NC,CC}} = \sum_q \int \mathrm{d}y \int \mathrm{d}x \; f_q(x, Q^2) \left(\frac{\mathrm{d}\sigma}{\mathrm{d}y}\right)^{\mathrm{NC,CC}}_{\nu q}$$

$$R^{\nu} = \frac{\sigma^{\nu}_{\mathrm{NC}}(\nu_\mu N \to \nu_\mu X)}{\sigma^{\nu}_{\mathrm{CC}}(\nu_\mu N \to \mu^- X)}$$

$$R^{\bar{\nu}} = \frac{\sigma^{\bar{\nu}}_{\mathrm{NC}}(\bar{\nu}_\mu N \to \bar{\nu}_\mu X)}{\sigma^{\bar{\nu}}_{\mathrm{CC}}(\bar{\nu}_\mu N \to \mu^+ X)}$$

isoscalar target (# protons = # neutrons):

$$R^{\nu} \simeq \frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W$$

The NuTeV Result

from 1.62×10^6 ν_μ and 0.35×10^6 $\bar{\nu}_\mu$ events:

$\sin^2\theta_W = 0.2277 \pm 0.0013(\text{stat.}) \pm 0.0009(\text{sys.})$

(G.P. Zeller et al., **PRL 88** (2002) 091802)

as compared to world average

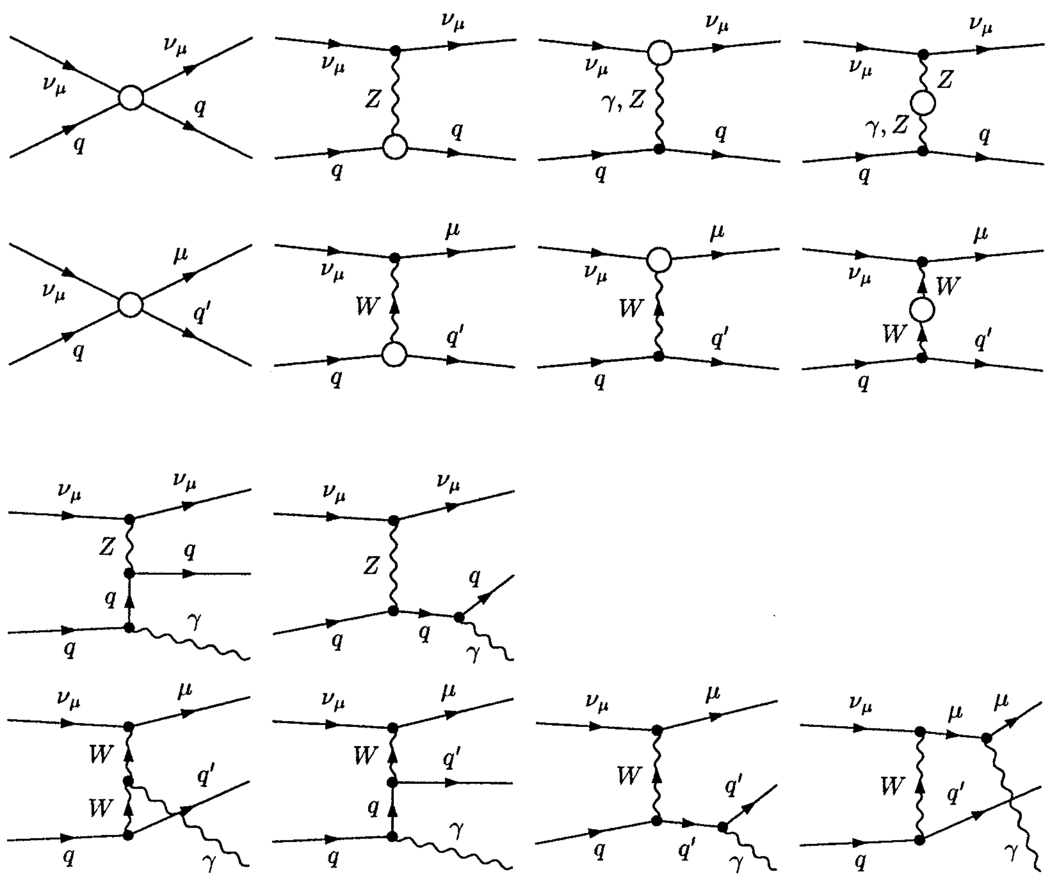
$$\sin^2\theta_W = 0.2227 \pm 0.00037$$

(LEPEWWG Homepage)

$\mathcal{O}(\alpha)$ corrections implemented in NuTeV MC as in D.Yu. Bardin, V.A. Dokuchaeva, JINR-E2-86-260, (1986) **(BD)**

new calculation of EW $O(\alpha)$ contributions

[K. Diener, S. Dittmaier, W.H.]



$$\delta R^\nu = R^\nu \left(\frac{\delta\sigma^\nu_{\rm NC}}{\sigma^\nu_{\rm NC}} - \frac{\delta\sigma^\nu_{\rm CC}}{\sigma^\nu_{\rm CC}} \right) = R^\nu \left(\delta R^\nu_{\rm NC} + \delta R^\nu_{\rm CC} \right)$$

$$\Delta \sin^2\theta_W = \frac{\frac{1}{2} - \sin^2\theta_W + \frac{20}{27}\sin^4\theta_W}{1 - \frac{40}{27}\sin^2\theta_W} \left(\delta R^\nu_{\rm NC} + \delta R^\nu_{\rm CC} \right)$$

Hadronic energy cut: $E^{\rm LAB}_{\rm had} > 10{\rm GeV}$

IPS	R^ν_0	$\delta R^\nu_{\rm NC}$	$\delta R^\nu_{\rm CC}$	$\Delta\sin^2\theta_W$
$\alpha(0)$	0.31766(2)	0.0582(1)	−0.0758(4)	−0.0082(2)
$\alpha(M_Z)$	0.31766(2)	−0.0639(1)	0.0452(4)	−0.0088(2)
G_F	0.31766(2)	0.0003(1)	−0.0185(4)	−0.0085(2)

Hadronic+photonic energy cut: $E^{\rm LAB}_{\rm had+phot} > 10{\rm GeV}$

IPS	R^ν_0	$\delta R^\nu_{\rm NC}$	$\delta R^\nu_{\rm CC}$	$\Delta\sin^2\theta_W$
$\alpha(0)$	0.31766(2)	0.0589(1)	−0.0842(4)	−0.0118(2)
$\alpha(M_Z)$	0.31766(2)	−0.0632(1)	0.0363(4)	−0.0126(2)
G_F	0.31766(2)	0.0011(1)	−0.0272(4)	−0.0122(2)

A successful decade of precision tests

Standard Model as a Quantum Field Theory

quantum effects are established

indirect versus direct determination of m_{top}

constraints on $M_{\mathrm{Higgs}} \rightarrow$ light Higgs-boson

gauge-boson self-interactions

not yet *directly* **tested:**

existence of the Higgs boson

Higgs-boson self interaction $\rightarrow$ Higgs potential

Yukawa interaction

$\longrightarrow$ **future experiments**

(expected) experimental precision

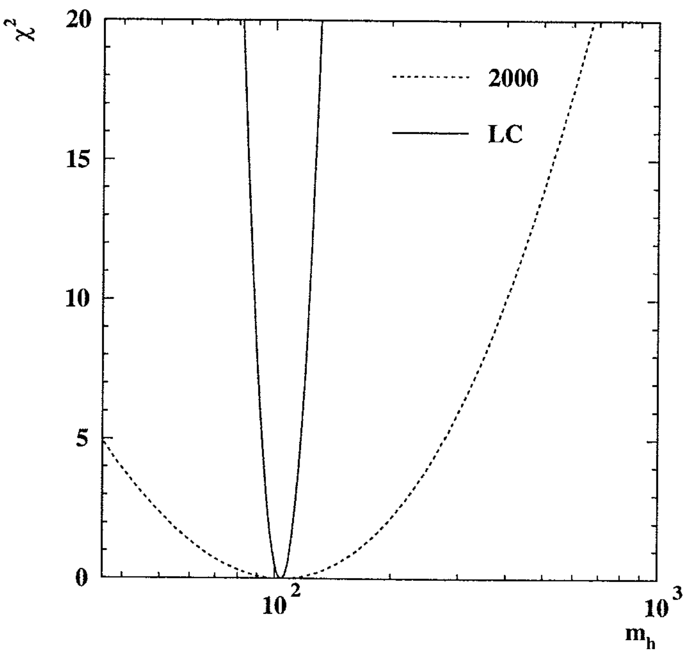
error for	LEP/Tev	Tev/LHC	LC	GigaZ
M_W [MeV]	33	15	15	6
$\sin^2\theta_{\rm eff}$	0.00017 LEP + SLC	0.00021		0.000013
$m_{\rm top}$ [GeV]	5.1	2	0.2	0.13
$M_{\rm Higgs}$ [GeV]	–	0.1	0.05	0.05

together with

$\delta M_Z = 2.1$ MeV (LEP)

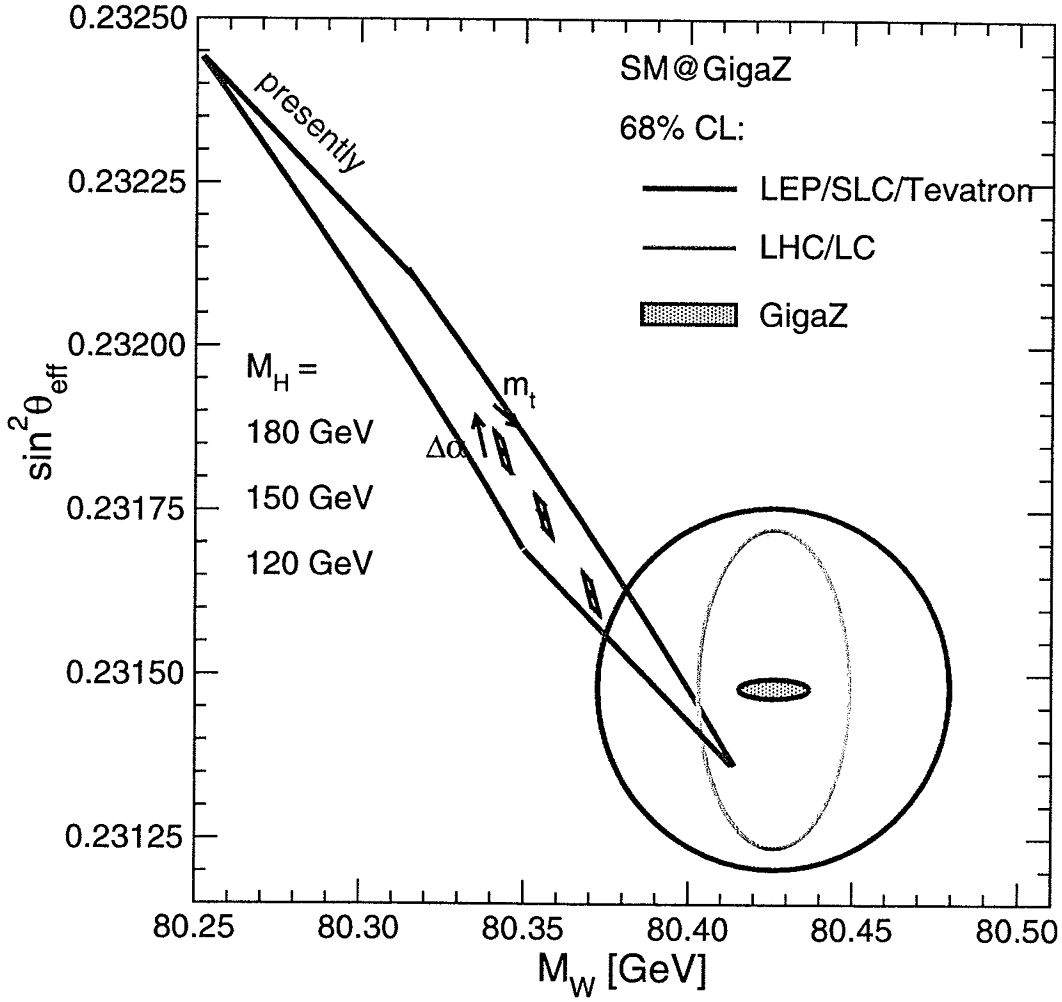
$\delta G_{\rm F}/G_{\rm F} = 1 \cdot 10^{-5}$ (μ lifetime)

	LEP/SLC/Tev [[illegible]]	TESLA
$\sin^2\theta^{\ell}_{\rm eff}$	0.23146 ± 0.00017	± 0.000013
lineshape observables:		
M_Z	$91.1875 \pm 0.0021\,\mathrm{GeV}$	$\pm 0.0021\,\mathrm{GeV}$
$\alpha_s(M_Z^2)$	0.1183 ± 0.0027	± 0.0009
$\Delta\rho_\ell$	$(0.55 \pm 0.10) \cdot 10^{-2}$	$\pm 0.05 \cdot 10^{-2}$
N_ν	2.984 ± 0.008	± 0.004
heavy flavours:		
$\mathcal{A}_{\rm b}$	0.898 ± 0.015	± 0.001
$R^0_{\rm b}$	0.21653 ± 0.00069	± 0.00014
M_W	$80.436 \pm 0.036\,\mathrm{GeV}$	$\pm 0.006\,\mathrm{GeV}$



$$\frac{\delta M_H}{M_H} \sim 7\%$$

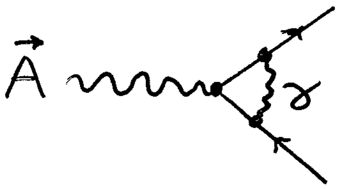
[Erler, Heinemeyer, Hollik, Weiglein, Zerwas]



Anomalous g-factor of the Muon

- Dirac Theory: $g = 2$

- QED, 1-loop-order: $g = 2 + \frac{\alpha}{\pi}$



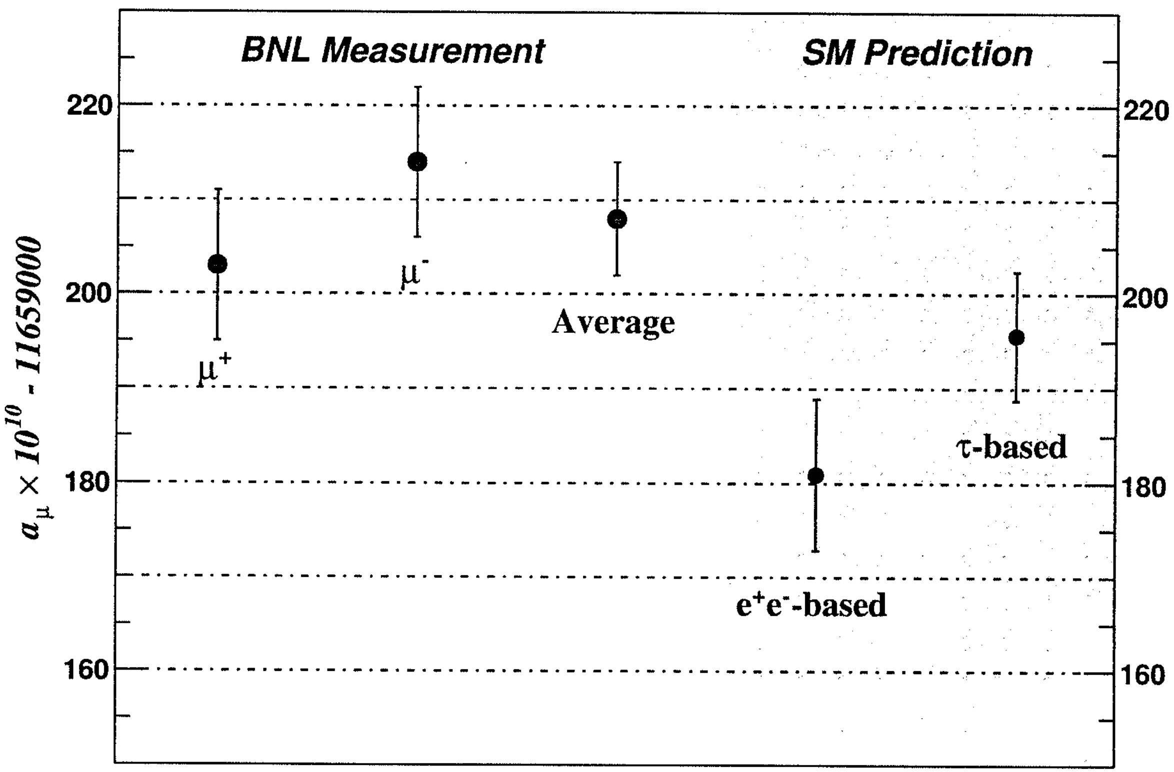
- Theory prediction 2001:

  Electroweak: 2-loop
  QED part: 4-loop (5-loop estimate)

- **Experiment 2004:**
  Brookhaven E821

$$a_\mu = \frac{g-1}{2} = 11659208(6) \cdot 10^{-10}$$

Theory vs Experiment



The e^+e^- and τ based predictions are below the experimental value by 2.7σ (2.1σ) and 0.7σ, respectively!

Beyond the Standard Model

further substructure	elementary fundamental fields
effects from new strong interaction	interactions remain weak
Grand Unified Theories	
new strong dynamics at high enery scale	new symmetry supersymmetry

Minimal Supersymmetric Standard Model (MSSM)

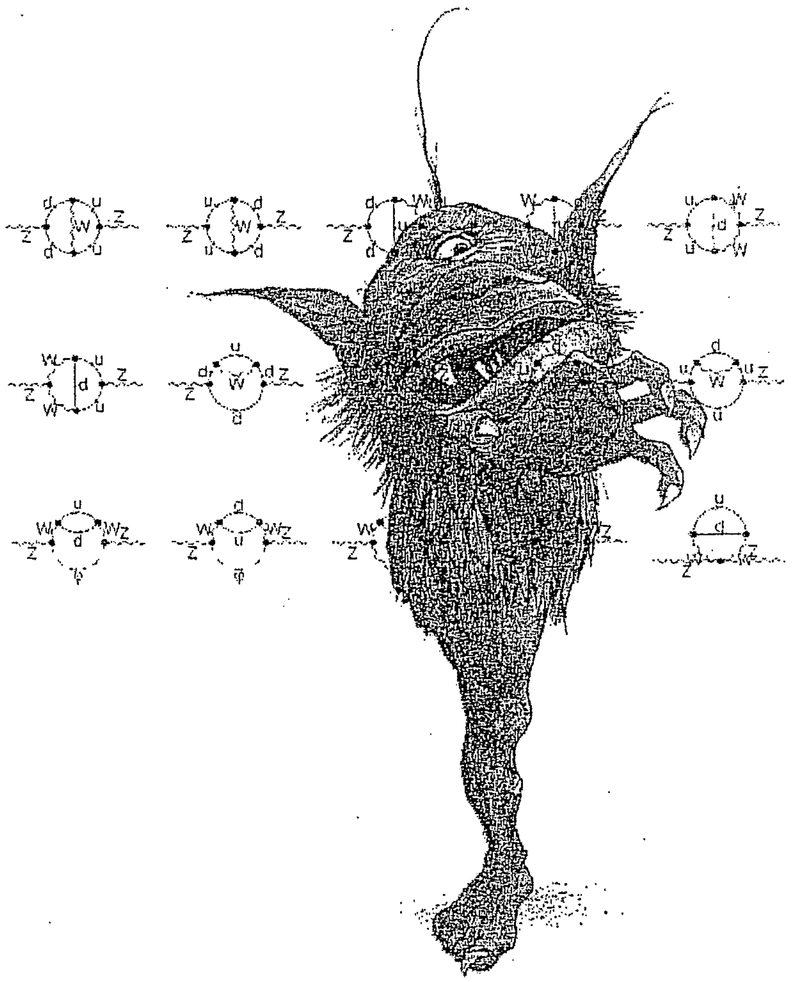
SM		Spin	SUSY		Spin
leptons	$\ell,\ \nu_\ell$	$\frac{1}{2}$	sleptons	$\tilde{\ell},\ \tilde{\nu}_\ell$	0
quarks	q	$\frac{1}{2}$	squarks	$\tilde{q}$	0
gluons	g	1	gluinos	$\tilde{g}$	$\frac{1}{2}$
EW bosons	γ, Z, W	1	charginos	$\tilde{\chi}^{\pm}_{1,2}$	$\frac{1}{2}$
Higgs	$h, H, A, H^{\pm}$	0	neutralinos	$\tilde{\chi}^{0}_{1,2,3,4}$	$\frac{1}{2}$

lightest SUSY particle stable LSP $= \tilde{\chi}^0_1$

- masses of SUSY partners $> 100\,\mathrm{GeV}$ (experimentally)
- lightest Higgs boson $< 135\,\mathrm{GeV}$ (theoretically)

Minimal Supersymmetric Standard Model (MSSM)

- renormalizable quantum field theory, precision calculations possible $\longrightarrow$ FeynArts
- predictions $\longleftrightarrow$ experiments



FeynArts 3 User's Guide

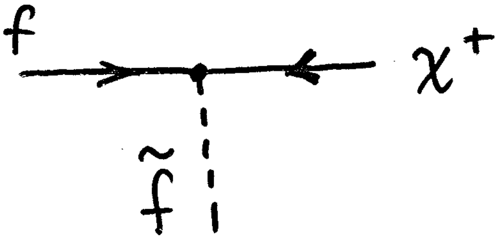
September 23, 2001 Thomas Hahn

http://www.feynarts.de

MSSM Feynman rules

Contents

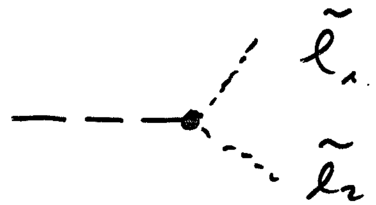
New types of vertices /propagators :



Algorithm:

Denner, Eck, Hahn, Küblbeck.

Higgs – slepton – slepton



214 $$C(A^0, \tilde{e}^1_{j1}, -\tilde{e}^2_{j2}) = \frac{-\left(e\left(-\mu - t_\beta A^e_{j1}\right) m_{e_{j1}} \delta_{j1,j2}\right)}{2 M_W s_W}$$

215 $$C(A^0, \tilde{e}^2_{j1}, -\tilde{e}^1_{j2}) = \frac{e\left(-\mu - t_\beta A^e_{j1}\right) m_{e_{j1}} \delta_{j1,j2}}{2 M_W s_W}$$

216 $$C(h^0, \tilde{e}^1_{j1}, -\tilde{e}^1_{j2}) = i\, e\, \delta_{j1,j2} \left(-\frac{M_Z s_{\alpha+\beta} s_W}{c_W} + \frac{s_\alpha {m_{e_{j1}}}^2}{c_\beta M_W s_W} + \frac{M_Z s_{\alpha+\beta} \left(-1 + 4 {s_W}^2\right) U^{e_{j1}*}_{11} U^{e_{j1}}_{11}}{2 c_W s_W} - \frac{\left(-\frac{c_\alpha \mu t_\beta}{s_\beta} - \frac{s_\alpha A^e_{j1}}{c_\beta}\right) m_{e_{j1}} U^{e_{j1}*}_{12} U^{e_{j1}}_{11}}{2 M_W s_W} - \frac{\left(-\frac{c_\alpha \mu t_\beta}{s_\beta} - \frac{s_\alpha A^e_{j1}}{c_\beta}\right) m_{e_{j1}} U^{e_{j1}*}_{11} U^{e_{j1}}_{12}}{2 M_W s_W} \right)$$

217 $$C(h^0, \tilde{e}^1_{j1}, -\tilde{e}^2_{j2}) = i\, e\, \delta_{j1,j2} \left(\frac{M_Z s_{\alpha+\beta} \left(-1 + 4 {s_W}^2\right) U^{e_{j1}*}_{21} U^{e_{j1}}_{11}}{2 c_W s_W} - \frac{\left(-\frac{c_\alpha \mu t_\beta}{s_\beta} - \frac{s_\alpha A^e_{j1}}{c_\beta}\right) m_{e_{j1}} U^{e_{j1}*}_{22} U^{e_{j1}}_{11}}{2 M_W s_W} - \frac{\left(-\frac{c_\alpha \mu t_\beta}{s_\beta} - \frac{s_\alpha A^e_{j1}}{c_\beta}\right) m_{e_{j1}} U^{e_{j1}*}_{21} U^{e_{j1}}_{12}}{2 M_W s_W} \right)$$

218 $$C(h^0, \tilde{e}^2_{j1}, -\tilde{e}^1_{j2}) = i\, e\, \delta_{j1,j2} \left(\frac{-\left(\left(-\frac{c_\alpha \mu t_\beta}{s_\beta} - \frac{s_\alpha A^e_{j1}}{c_\beta}\right) m_{e_{j1}} U^{e_{j1}*}_{12} U^{e_{j1}}_{21}\right)}{2 M_W s_W} - \frac{\left(-\frac{c_\alpha \mu t_\beta}{s_\beta} - \frac{s_\alpha A^e_{j1}}{c_\beta}\right) m_{e_{j1}} U^{e_{j1}*}_{11} U^{e_{j1}}_{22}}{2 M_W s_W} + \frac{M_Z s_{\alpha+\beta} \left(1 - 4 {s_W}^2\right) U^{e_{j1}*}_{12} U^{e_{j1}}_{22}}{2 c_W s_W} \right)$$

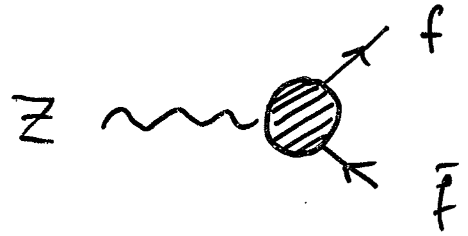
219 $$C(h^0, \tilde{e}^2_{j1}, -\tilde{e}^2_{j2}) = i\, e\, \delta_{j1,j2} \left(\frac{M_Z s_{\alpha+\beta} \left(-\frac{1}{2} + {s_W}^2\right)}{c_W s_W} + \frac{s_\alpha {m_{e_{j1}}}^2}{c_\beta M_W s_W} - \frac{\left(-\frac{c_\alpha \mu t_\beta}{s_\beta} - \frac{s_\alpha A^e_{j1}}{c_\beta}\right) m_{e_{j1}} U^{e_{j1}*}_{22} U^{e_{j1}}_{21}}{2 M_W s_W} - \frac{\left(-\frac{c_\alpha \mu t_\beta}{s_\beta} - \frac{s_\alpha A^e_{j1}}{c_\beta}\right) m_{e_{j1}} U^{e_{j1}*}_{21} U^{e_{j1}}_{22}}{2 M_W s_W} + \frac{M_Z s_{\alpha+\beta} \left(1 - 4 {s_W}^2\right) U^{e_{j1}*}_{22} U^{e_{j1}}_{22}}{2 c_W s_W} \right)$$

220 $$C(G^0, \tilde{e}^1_{j1}, -\tilde{e}^2_{j2}) = \frac{-\left(e\left(-\left(\mu t_\beta\right) + A^e_{j1}\right) m_{e_{j1}} \delta_{j1,j2}\right)}{2 M_W s_W}$$

221 $$C(G^0, \tilde{e}^2_{j1}, -\tilde{e}^1_{j2}) = \frac{e\left(-\left(\mu t_\beta\right) + A^e_{j1}\right) m_{e_{j1}} \delta_{j1,j2}}{2 M_W s_W}$$

222 $$C(H^0, \tilde{e}^1_{j1}, -\tilde{e}^1_{j2}) = i\, e\, \delta_{j1,j2} \left(\frac{c_{\alpha+\beta} M_Z s_W}{c_W} - \frac{c_\alpha {m_{e_{j1}}}^2}{c_\beta M_W s_W} + \frac{c_{\alpha+\beta} M_Z \left(1 - 4 {s_W}^2\right) U^{e_{j1}*}_{11} U^{e_{j1}}_{11}}{2 c_W s_W} - \frac{\left(-\frac{\mu s_\alpha t_\beta}{s_\beta} + \frac{c_\alpha A^e_{j1}}{c_\beta}\right) m_{e_{j1}} U^{e_{j1}*}_{12} U^{e_{j1}}_{11}}{2 M_W s_W} - \frac{\left(-\frac{\mu s_\alpha t_\beta}{s_\beta} + \frac{c_\alpha A^e_{j1}}{c_\beta}\right) m_{e_{j1}} U^{e_{j1}*}_{11} U^{e_{j1}}_{12}}{2 M_W s_W} \right)$$

- Effective Z boson couplings:



$$g_A^f = \sqrt{\rho_f}\; I_3^f$$

$$g_V^f = \sqrt{\rho_f}\,\left(I_3^f - 2\, Q_f \sin^2\theta_f \right)$$

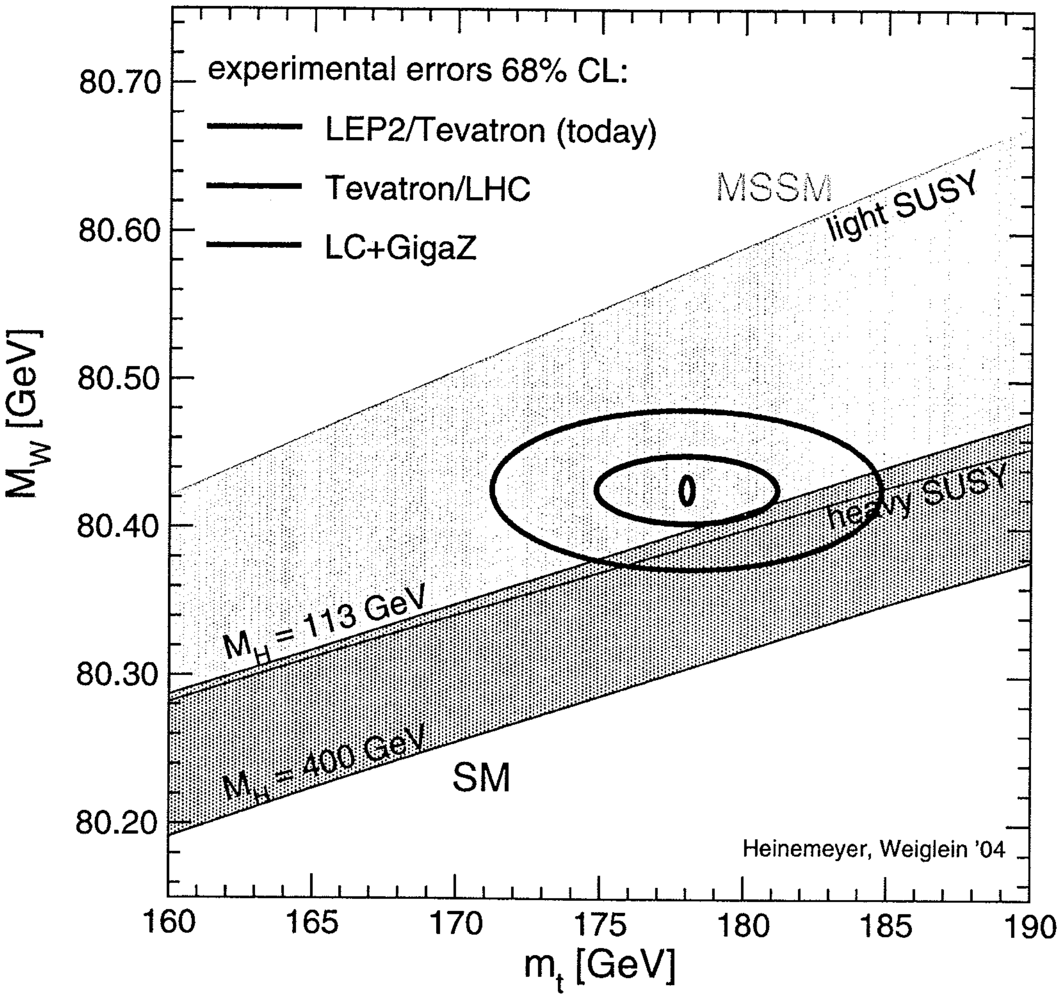
ρ_f (m_t, α_s, M_A, $\tan\beta$, SUSY-parameters)

$\sin^2\theta_f$ (m_t, α_s, M_A, $\tan\beta$, SUSY-parameters)

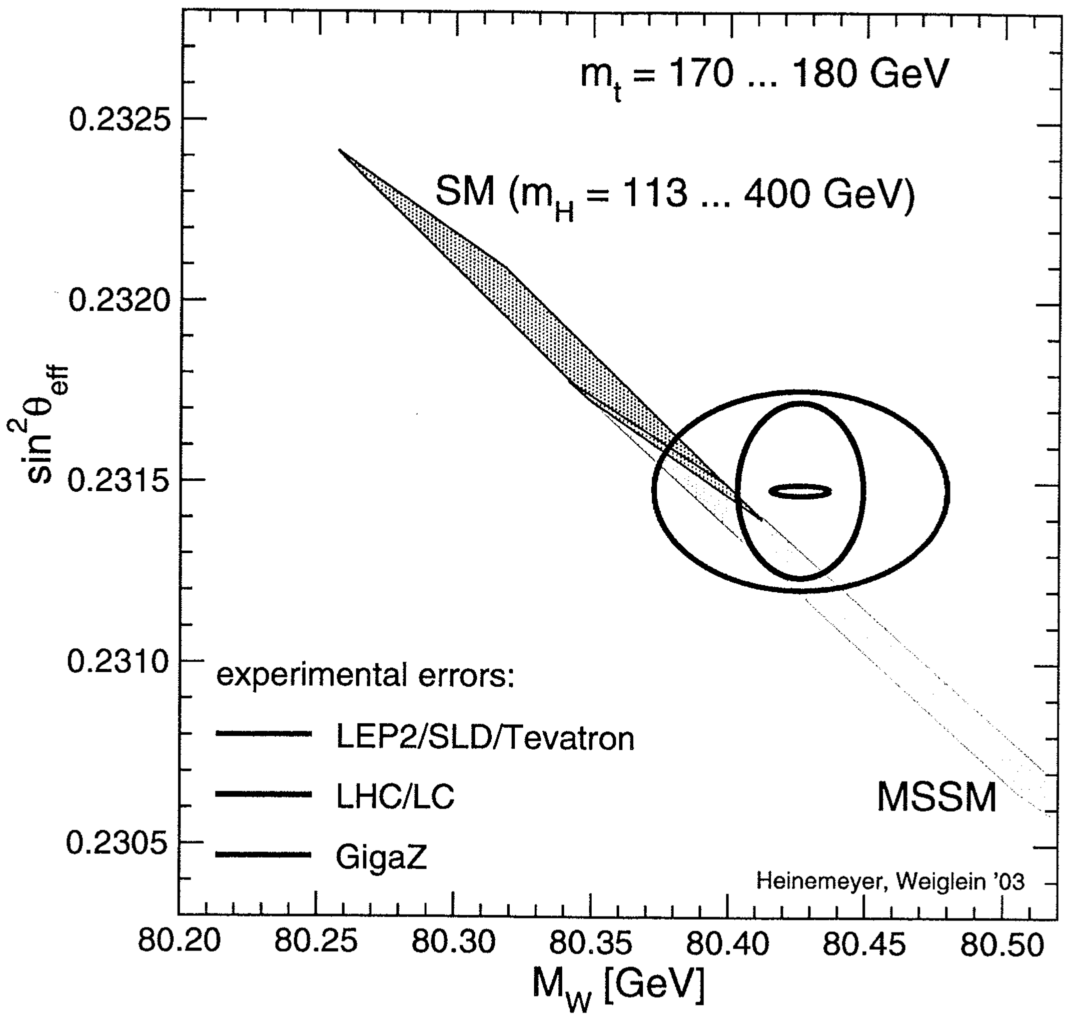
- M_W - M_Z correlation:

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}\, G_\mu} \cdot \frac{1}{1 - \Delta r}$$

$\Delta r = \Delta r$ (m_t, α_s, M_A, $\tan\beta$, SUSY-par.)



experimental errors 68% CL:
LEP2/Tevatron (today)
Tevatron/LHC
LC+GigaZ
MSSM
light SUSY
heavy SUSY
M_H = 113 GeV
M_H = 400 GeV
SM
Heinemeyer, Weiglein '04
80.70
80.60
80.50
80.40
80.30
80.20
M_W [GeV]
160
165
170
175
180
185
190
m_t [GeV]



m_t = 170 ... 180 GeV
SM (m_H = 113 ... 400 GeV)
0.2325
0.2320
0.2315
0.2310
0.2305
$\sin^2\theta_{eff}$
experimental errors:
LEP2/SLD/Tevatron
LHC/LC
GigaZ
MSSM
Heinemeyer, Weiglein '03
80.20
80.25
80.30
80.35
80.40
80.45
80.50
M_W [GeV]

Squark Generation Mixing via Soft Breaking

$$M^2_{\tilde{u}} = \left(\begin{array}{cc|cc} M^2_{\tilde{L}_c} & \Delta^t_{LL} & m_c X_c & \Delta^t_{LR} \\ \Delta^t_{LL} & M^2_{\tilde{L}_t} & \Delta^t_{RL} & m_t X_t \\ \hline m_c X_c & \Delta^t_{RL} & M^2_{\tilde{R}_c} & \Delta^t_{RR} \\ \Delta^t_{LR} & m_t X_t & \Delta^t_{RR} & M^2_{\tilde{R}_t} \end{array}\right)$$

$$M^2_{\tilde{d}} = \left(\begin{array}{cc|cc} M^2_{\tilde{L}_s} & \Delta^b_{LL} & m_s X_s & \Delta^b_{LR} \\ \Delta^b_{LL} & M^2_{\tilde{L}_b} & \Delta^b_{RL} & m_b X_b \\ \hline m_s X_s & \Delta^b_{RL} & M^2_{\tilde{R}_s} & \Delta^b_{RR} \\ \Delta^b_{LR} & m_b X_b & \Delta^b_{RR} & M^2_{\tilde{R}_b} \end{array}\right)$$

with

$$\begin{aligned} M^2_{\tilde{L}_q} &= M^2_{\tilde{Q}_q} + m_q^2 + \cos 2\beta \, M_Z^2 (T_3^q - Q_q s_W^2) \\ M^2_{\tilde{R}_q} &= M^2_{\tilde{U}_q} + m_q^2 + \cos 2\beta \, M_Z^2 Q_q s_W^2 \ (q = t, c) \\ M^2_{\tilde{R}_q} &= M^2_{\tilde{D}_q} + m_q^2 + \cos 2\beta \, M_Z^2 Q_q s_W^2 \ (q = b, s) \\ X_q &= A_q - \mu (\tan\beta)^{-2T_3^q} \end{aligned}$$

Mass eigenstates:

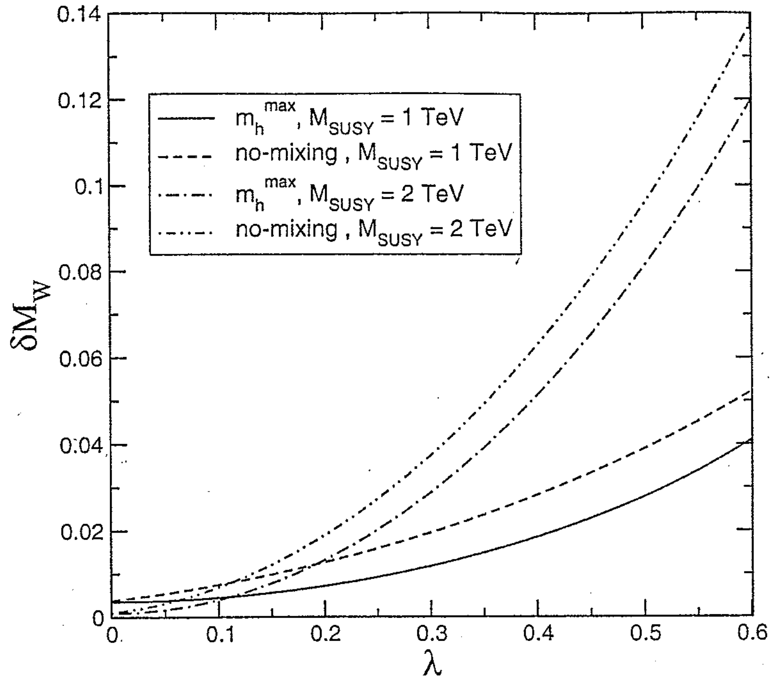
$$\tilde{u}_\alpha = R^{\alpha,j}_{\tilde{u}} \begin{pmatrix} \tilde{c}_L \\ \tilde{t}_L \\ \tilde{c}_R \\ \tilde{t}_R \end{pmatrix}_j , \quad \tilde{d}_\alpha = R^{\alpha,j}_{\tilde{d}} \begin{pmatrix} \tilde{s}_L \\ \tilde{b}_L \\ \tilde{s}_R \\ \tilde{b}_R \end{pmatrix}_j ,$$

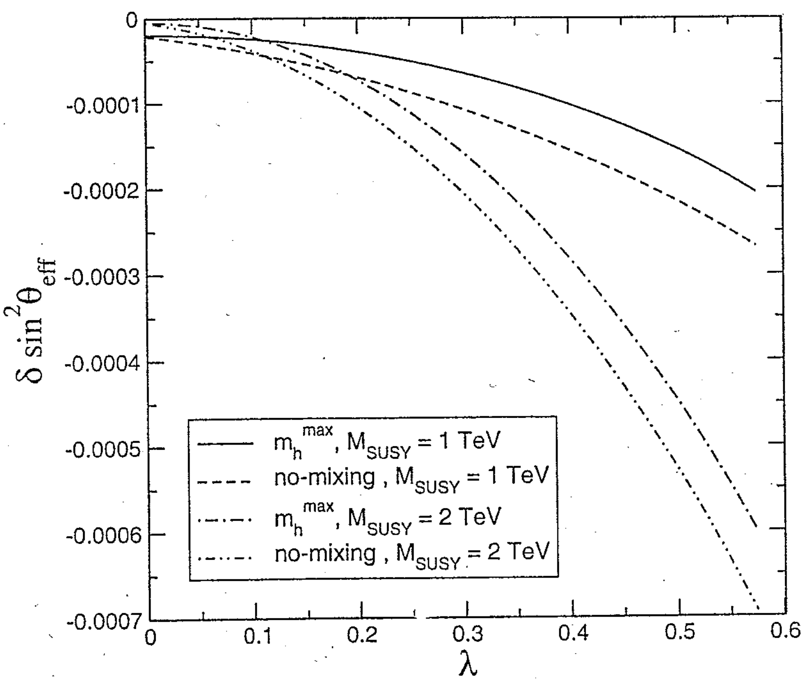
Simplest scenario:

$$\Delta^t_{LL} = \lambda^t M_{\tilde{L}_t} M_{\tilde{L}_c} , \qquad \Delta^t_{LR} = \Delta^t_{RL} = \Delta^t_{RR} = 0 ,$$

$$\Delta^b_{LL} = \lambda^b M_{\tilde{L}_b} M_{\tilde{L}_s} , \qquad \Delta^b_{LR} = \Delta^b_{RL} = \Delta^b_{RR} = 0 ,$$

[Heinemeyer, Will, Merz, Peñaranda]



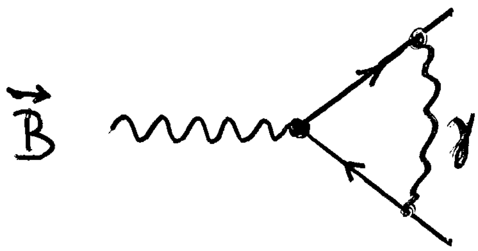


$\lambda_t = \lambda_b = \lambda$

Anomalous g-factor of the Muon

- Dirac Theory: $g = 2$

- QED, 1-loop-order: $g = 2 + \frac{\alpha}{\pi}$



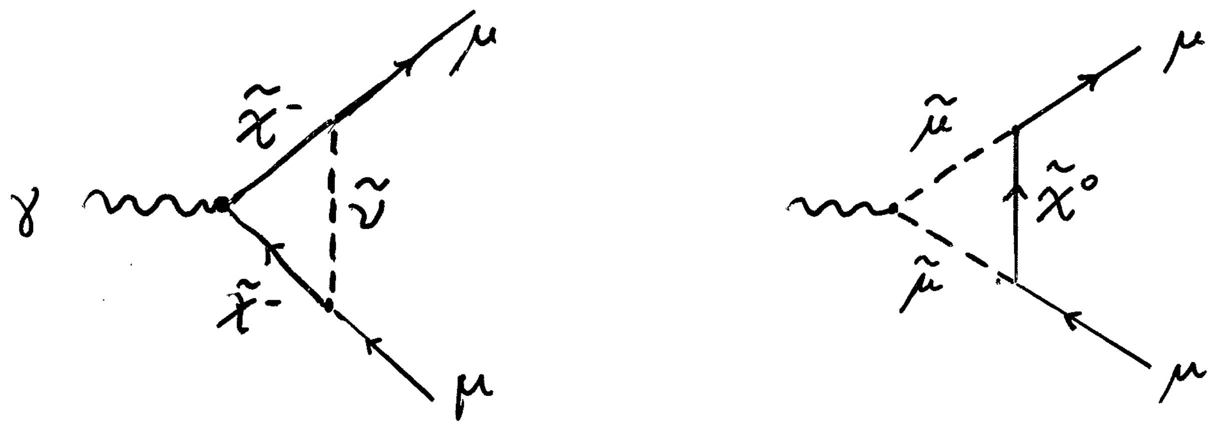
- **Theory prediction 2003:**

  Electroweak: 2-loop
  QED part: 4-loop (5-loop estimate)

- **Experiment 2003:**
  Brookhaven E821
  Phys. Rev. Lett. 86 (2001) 2227
  Phys. Rev. Lett. 89 (2002) 101804

SUSY :

positive contribution to a_μ for large $\tan\beta$, "light" $\tilde{\mu}$, $\tilde{\chi}^{\pm,0}$



- simple model : $m_{\tilde{\chi}} \simeq m_{\tilde{\mu}} = \tilde{m}$

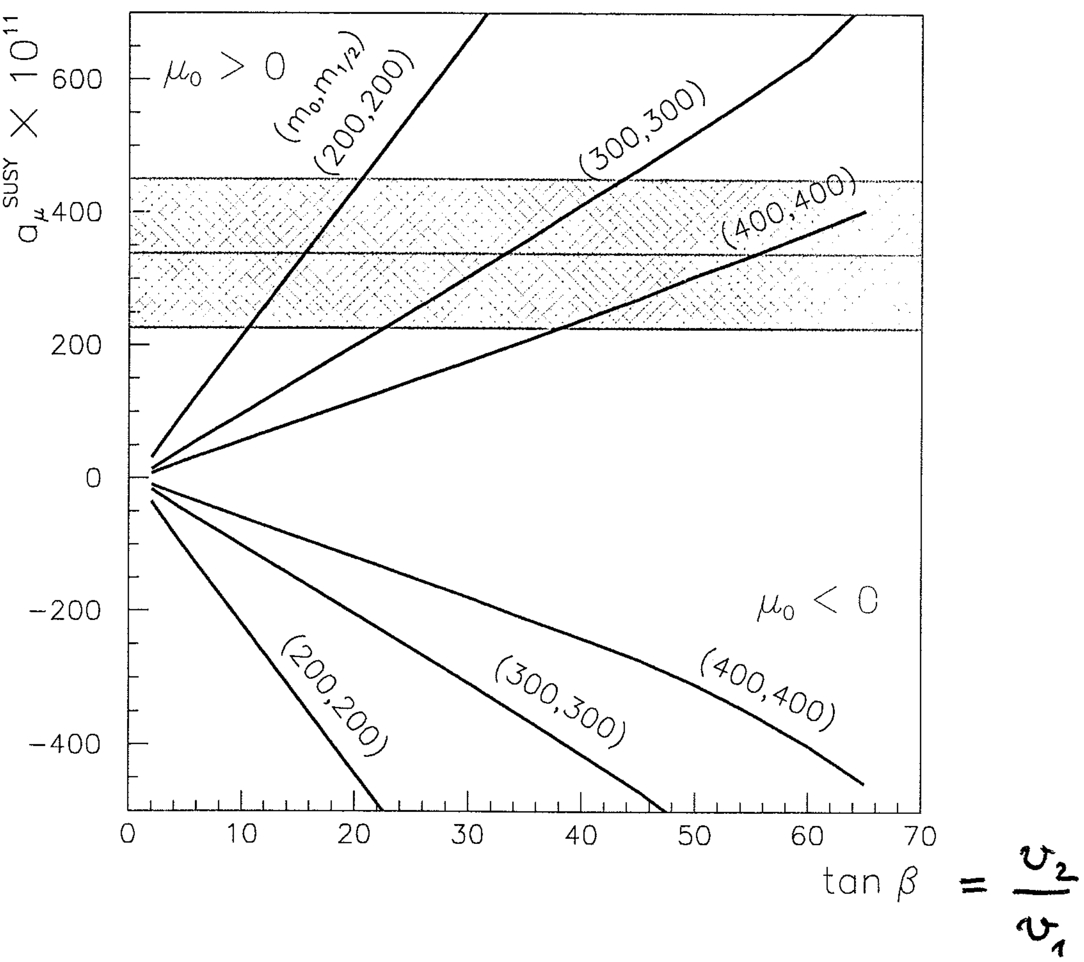
$$(\Delta a_\mu)^{SUSY} \simeq 130 \cdot 10^{-11} \tan\beta \left(\frac{100 \text{ GeV}}{\tilde{m}} \right)$$

[Czarnecki, Marciano]

$200 \text{ GeV} < \tilde{m} < 500 \text{ GeV}$ if $\langle a_\mu^{exp} \rangle - a_\mu^{SM}$ entirely SUSY

- More detailed study:

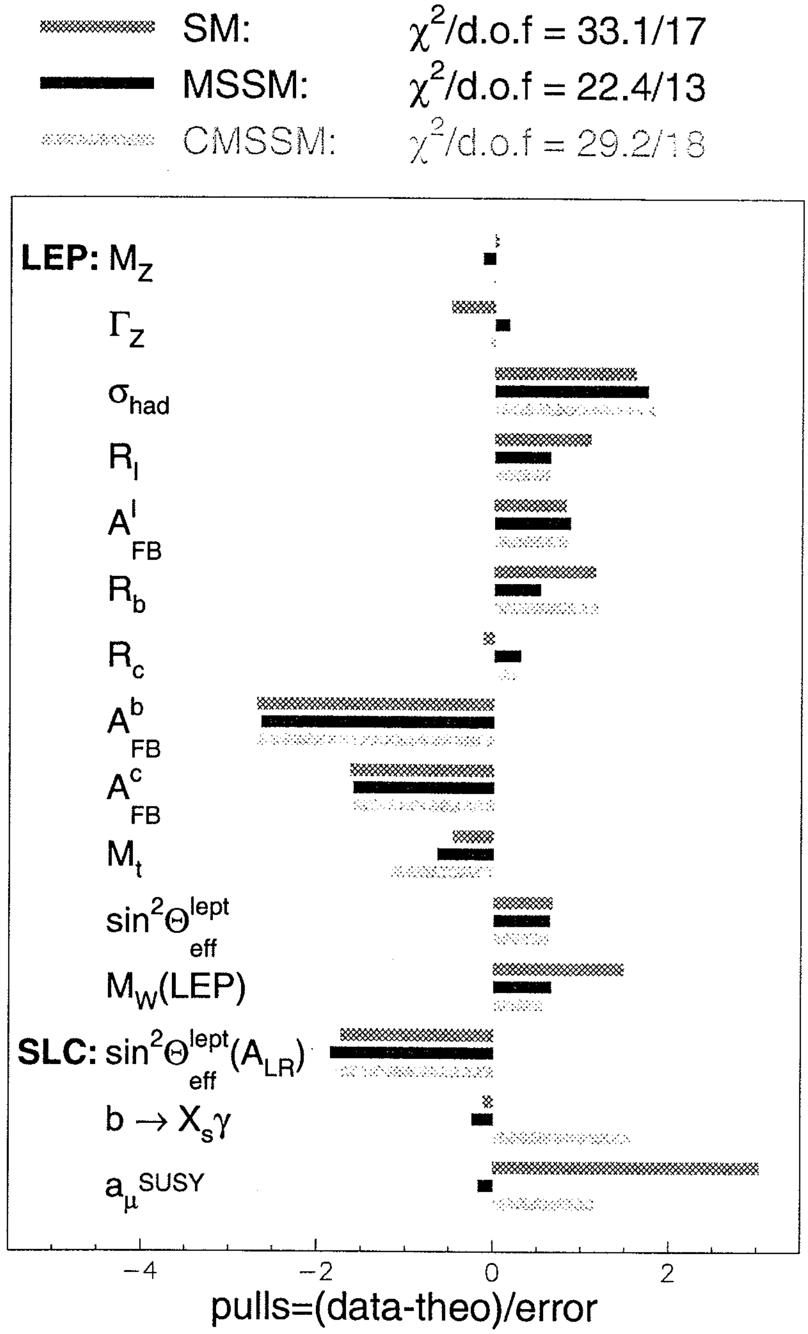
Baer et al, hep-ph/0103280


$a_\mu^{SUSY} \times 10^{11}$
600
400
200
0
−200
−400
0
10
20
30
40
50
60
70
$\tan\beta$
$= \frac{v_2}{v_1}$
$\mu_0 > 0$
$(m_0, m_{1/2})$
(200,200)
(300,300)
(400,400)
$\mu_0 < 0$
(200,200)
(300,300)
(400,400)

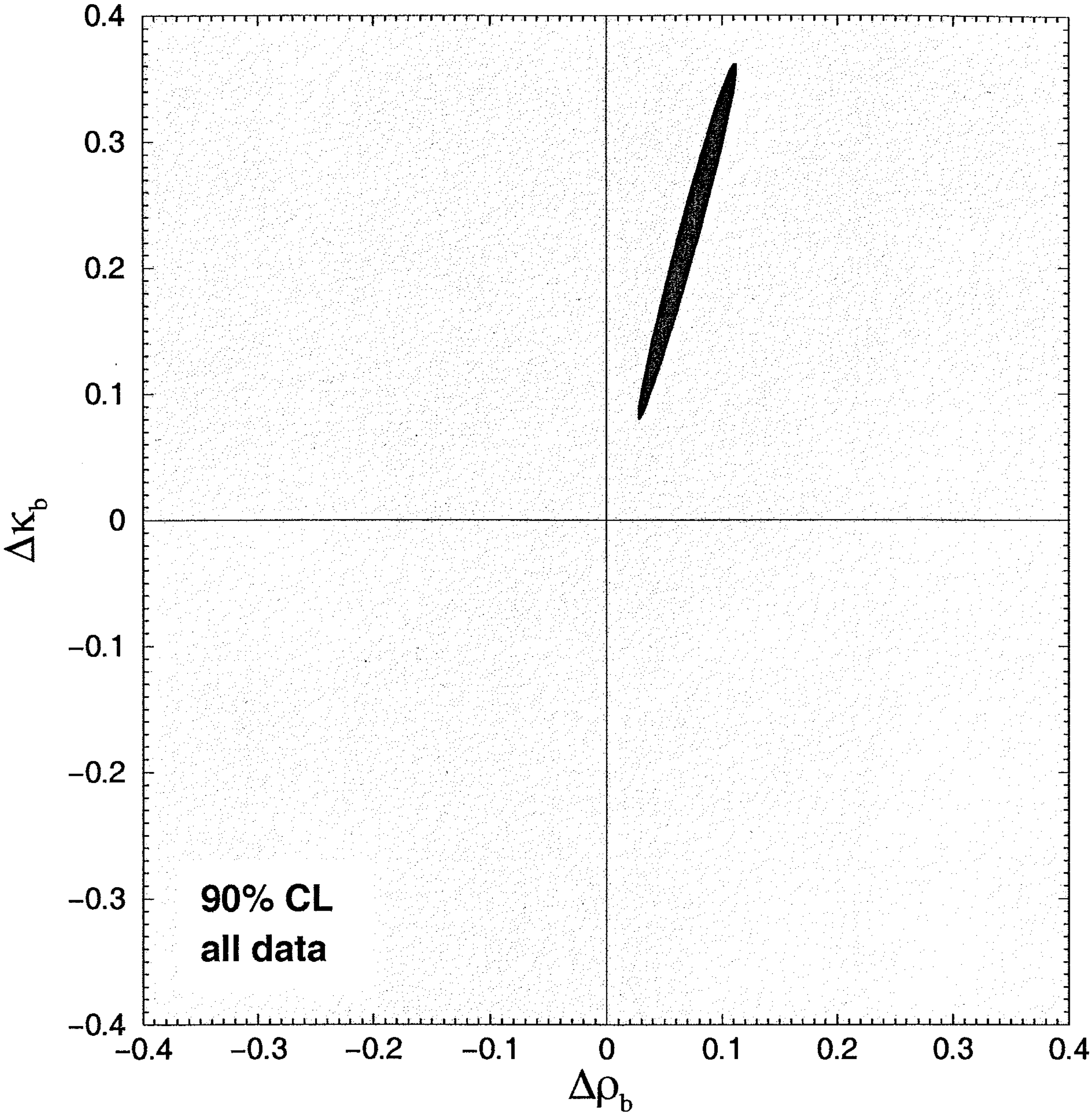
Global Fits in the MSSM

[de Boer, Dabelstein, WH, Mösle, Schwickerath]

[de Boer, Sander]



SM: χ²/d.o.f = 33.1/17
MSSM: χ²/d.o.f = 22.4/13
CMSSM: χ²/d.o.f = 29.2/18
LEP: M_Z
Γ_Z
σ_had
R_l
A^l_FB
R_b
R_c
A^b_FB
A^c_FB
M_t
sin²Θ^lept_eff
M_W(LEP)
SLC: sin²Θ^lept_eff(A_LR)
b → X_s γ
a_μ^SUSY
−4
−2
0
2
pulls=(data-theo)/error



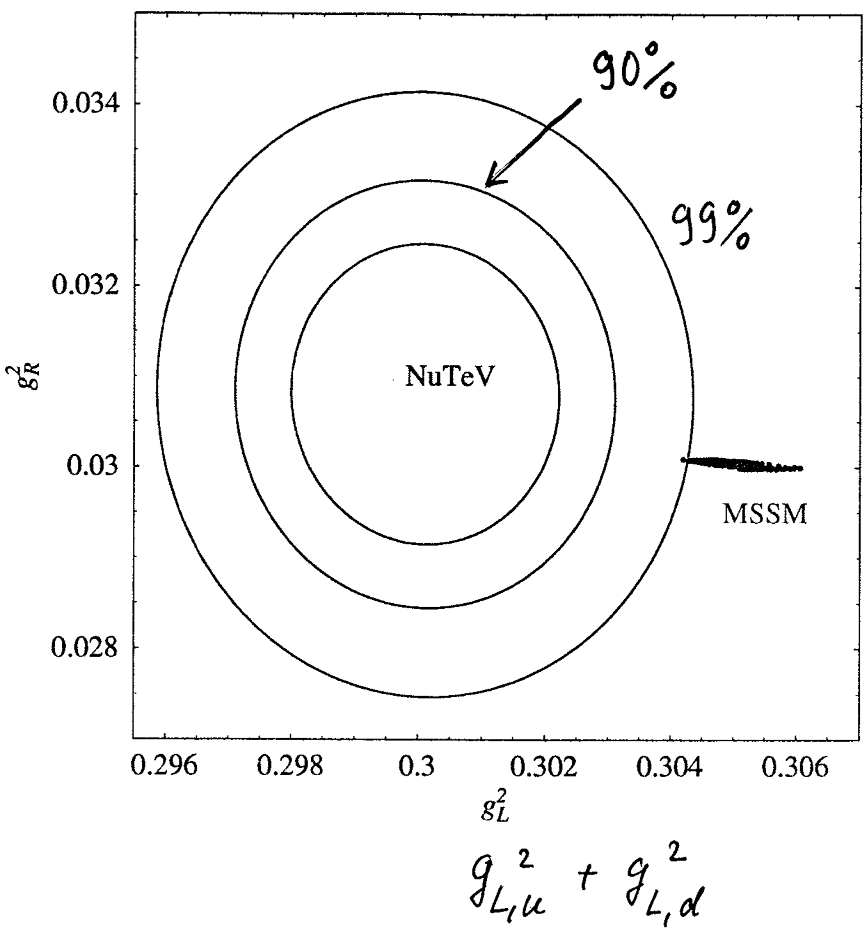
$$g_A^b = \sqrt{\rho_b} \cdot I_3^b$$

$$\sin^2\theta_b = \kappa_b \, s_W^2$$

$$\uparrow$$

$$1 - \frac{M_W^2}{M_Z^2}$$

[Davidson, Forte, Gambino, Rius, Strumia]



Higgs bosons in the MSSM

MSSM parameters $\longrightarrow$ m_{h^0}

precise prediction

higher orders are needed

exp. constraints on m_{h^0} $\longrightarrow$ constraints on MSSM parameters

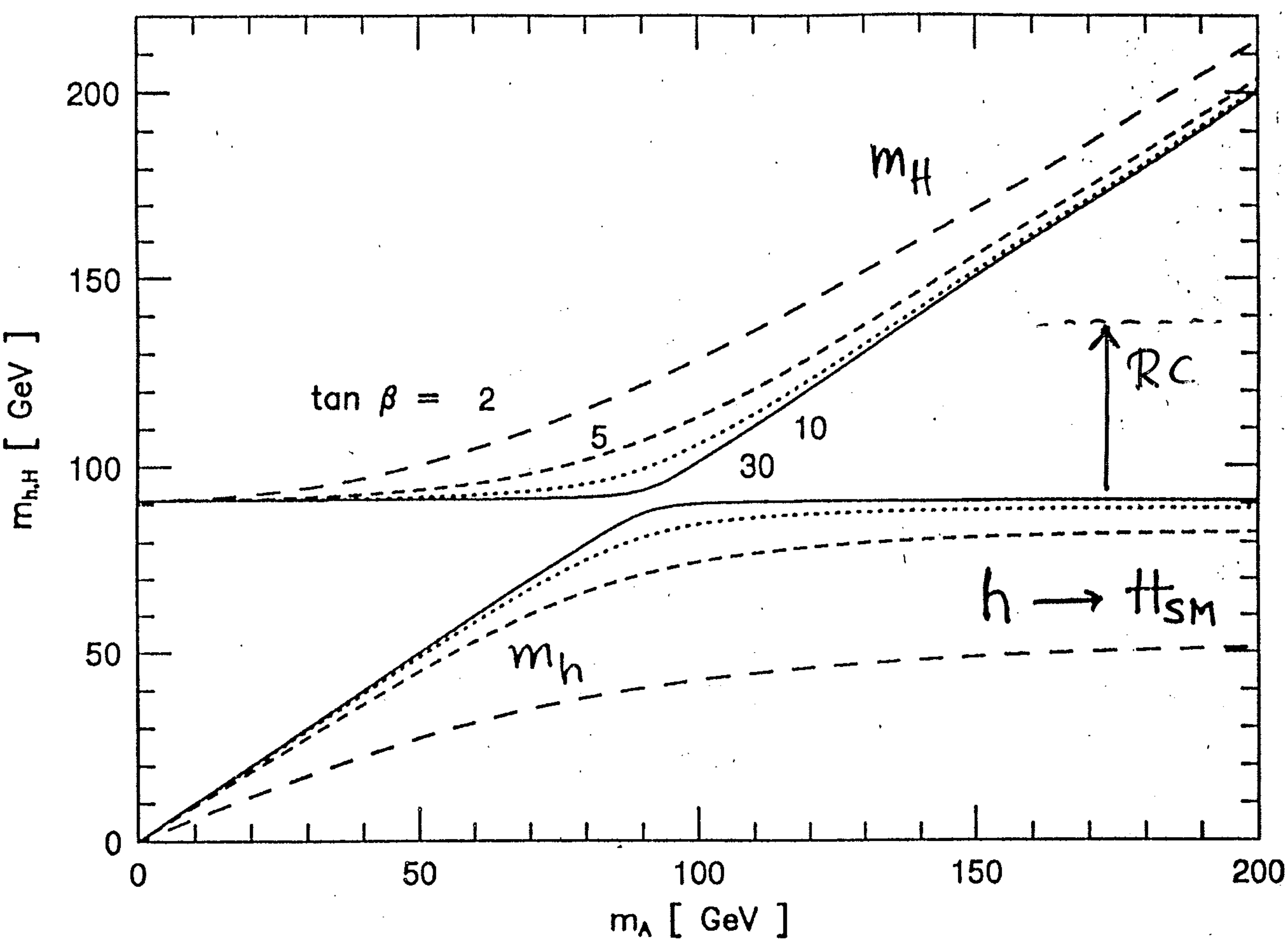
EW Precision Observables $\quad\quad\quad\quad$ m_{h^0}

$\updownarrow$ $\quad\quad\quad\quad$ $\updownarrow$

MSSM parameters

complementary in special regions

light and heavy scalar Higgs mass



<u>lowest order</u>

$$m_h < m_Z$$

$$m_h < m_A$$

- Maximum value of m_h:

  $$m_A \to \infty$$

  $$\tan\beta \to \infty$$

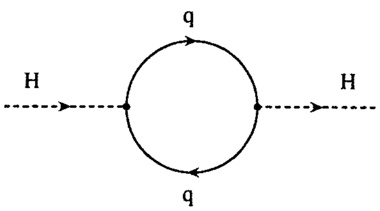
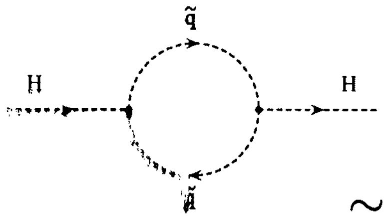
- large radiative corrections

• MSSM Higgs Mass Predictions

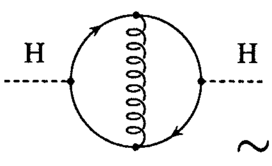
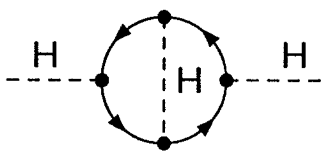
The tree-level prediction of the MSSM is already ruled out by the present LEP data.

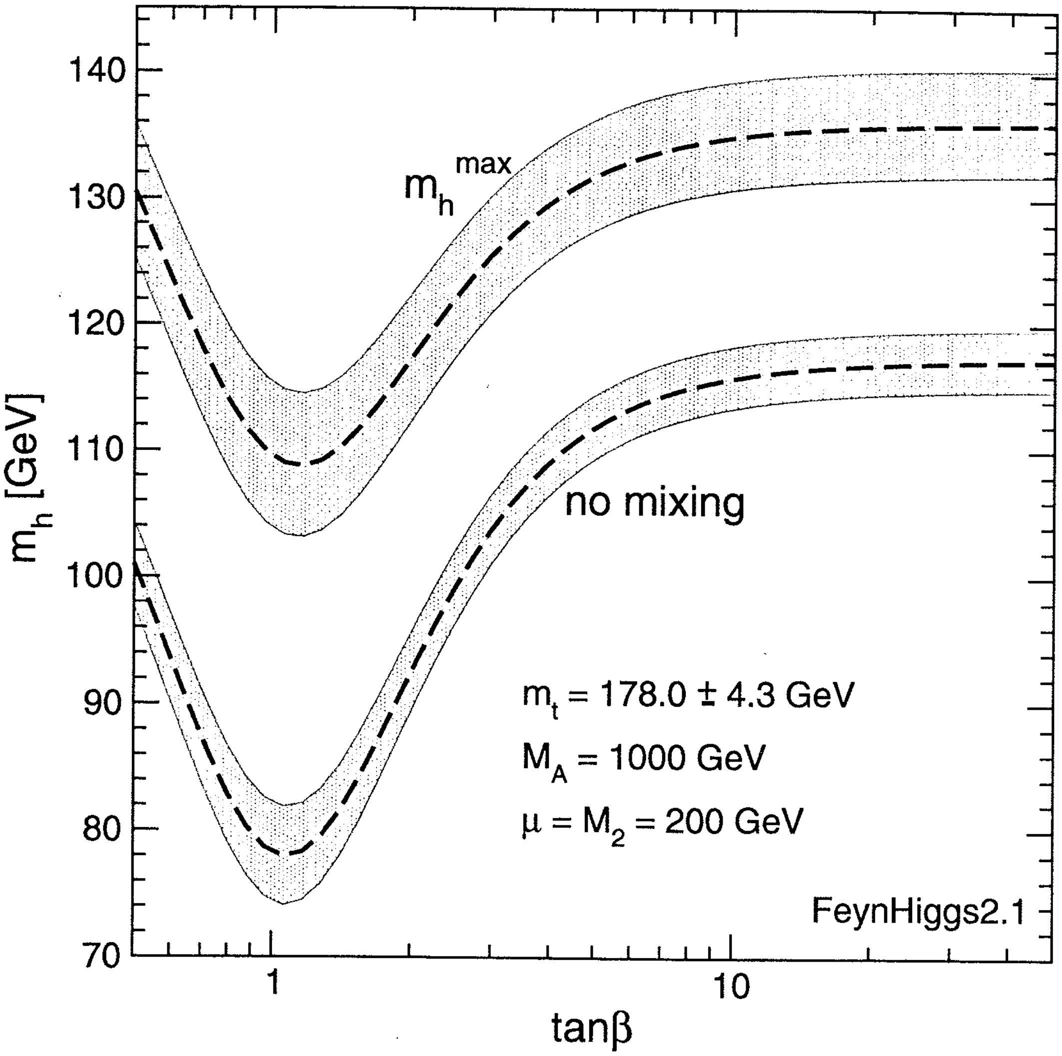
But:

- There are large quantum corrections to m_{h^0}

e.g.   $\sim \alpha_t$

- Even 2-loop corr. to m_{h^0} are significant.

e.g.  $\sim \alpha_t \alpha_s$  $\sim \alpha_t^2$

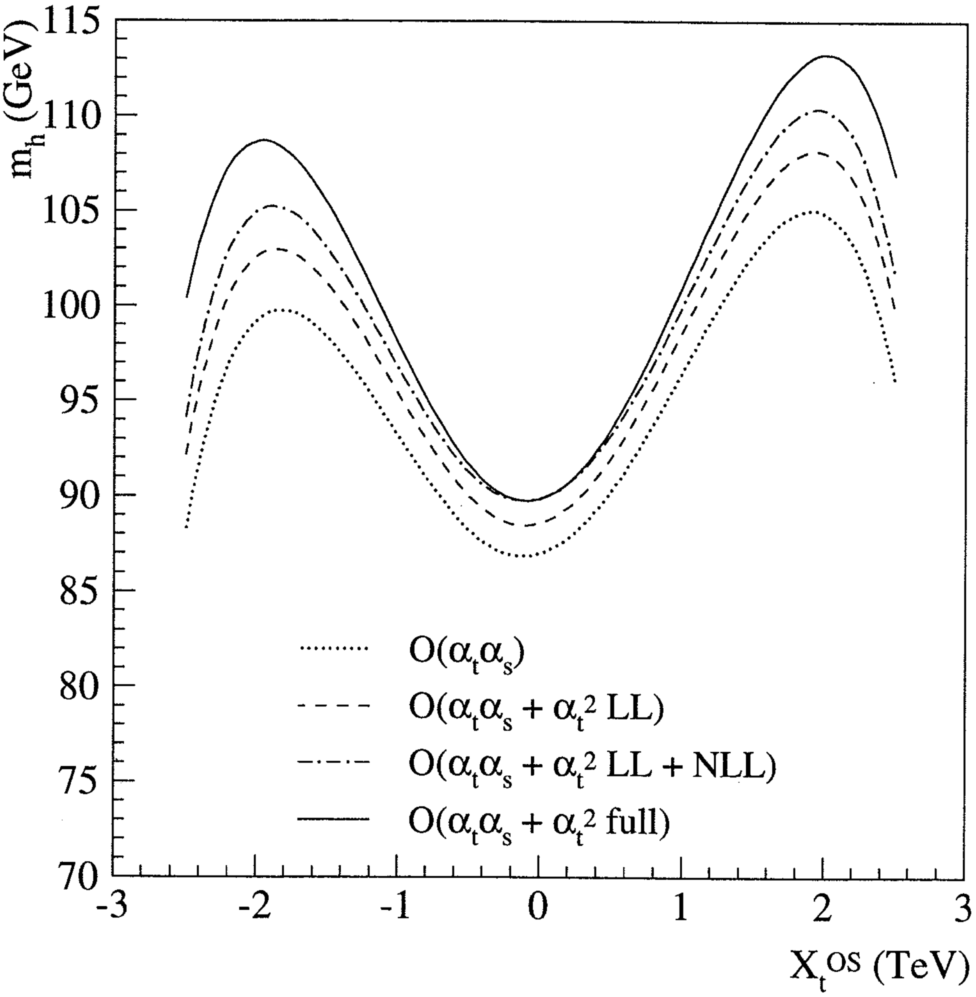


140
130
120
110
100
90
80
70
m_h [GeV]
m_h^max
no mixing
m_t = 178.0 ± 4.3 GeV
M_A = 1000 GeV
μ = M_2 = 200 GeV
FeynHiggs2.1
1
10
tanβ

The h mass at the two-loop level

combined strong and top-Yukawa terms

[Degrassi, Heinemeyer, WH, Slavich, Weiglein]

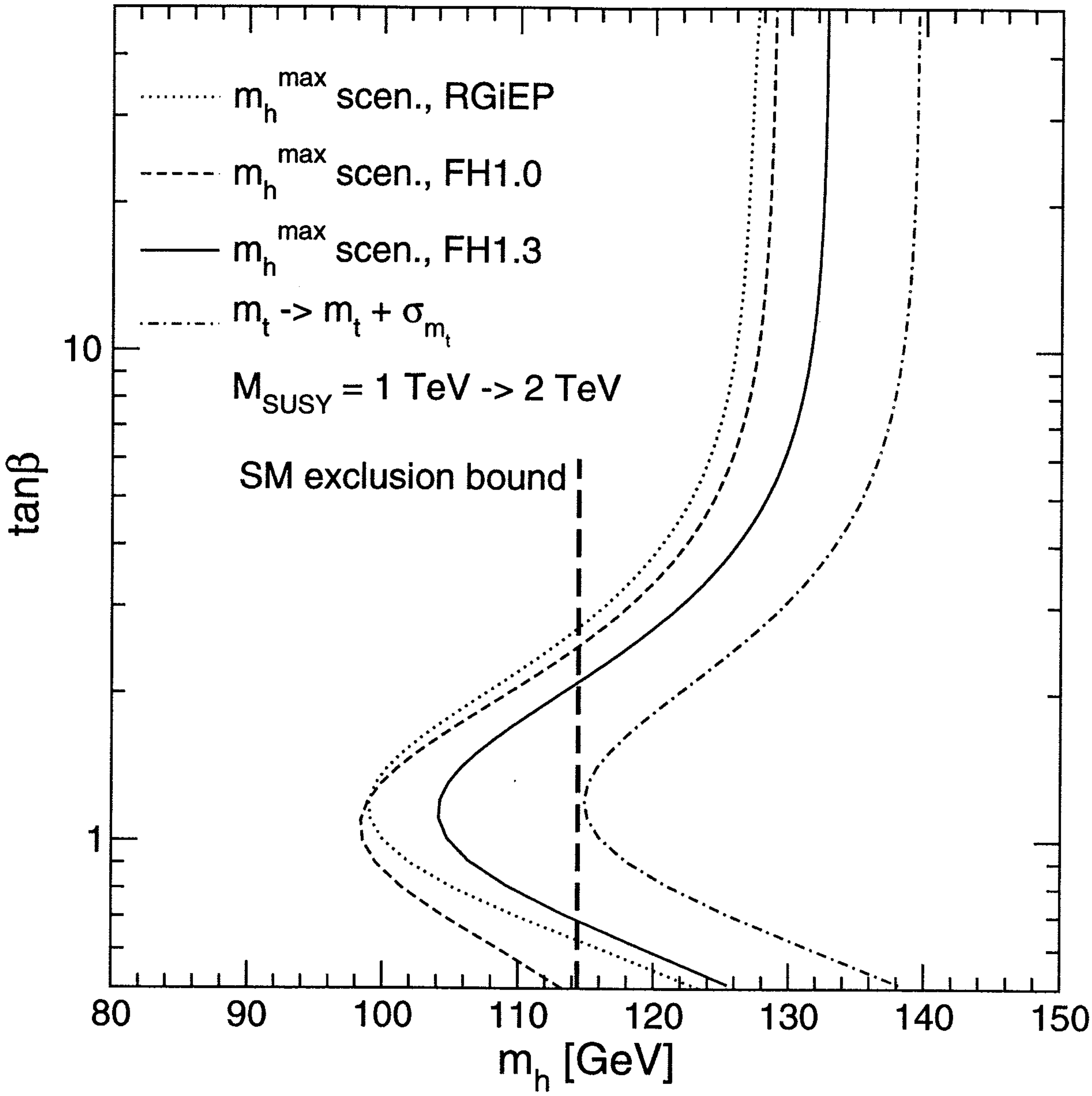


$= A_t - \mu \cot\beta$

$\tan\beta = 3, \quad M_{\tilde{Q}} = M_A = 1 \text{ TeV}, \quad m_{\tilde{g}} = 800 \text{ GeV}$

$$A \log^2 \frac{M_{\tilde{Q}}}{m_t} + B \log \frac{M_{\tilde{Q}}}{m_t} + C$$

A ↑ RG 1-loop; B ↑ RG 2-loop; C ↑ only diagram.



m_h^{max} scen., RGiEP
m_h^{max} scen., FH1.0
m_h^{max} scen., FH1.3
m_t -> $m_t + \sigma_{m_t}$
M_{SUSY} = 1 TeV -> 2 TeV
SM exclusion bound
tanβ
10
1
80
90
100
110
120
130
140
150
m_h [GeV]

Possible scenarios

- a single light Higgs boson
  - SM Higgs boson?
  - SUSY light Higgs boson?
    with H, A, $H^{\pm}$ heavy (decoupling scenario)
    $h \sim H_{\text{SM}}$
- a light Higgs boson + more $(H, A, H^{\pm})$
  - SUSY Higgs?
  - non-SUSY 2-Higgs-Doublet model?
- a single heavy Higgs boson ($\gg 200$ GeV)
  - SUSY ruled out
  - SM + (?)   strong interaction?
- no Higgs boson
  - strongly interacting weak interaction
    new strong force $\sim$ TeV scale