SUSY GUTs & Extra Dims.

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Plan

- symm. structure of SM
- basic SU(5) GUT
- basic 5d SU(5) orbifold GUT
- SO(10) and its subgroups
- basic 6d SO(10) orbifold GUT
- larger groups — lecture 1
- spinors in $d > 4$ — lecture 2
- SUSY in $d > 4$
- proton decay in 4d & 5d SUSY GUTs
- power law unification
- relation to heterotic string theory

Symmetry structure of the SM

SM:
- gauge group $SU_3 \times SU_2 \times U_1$
- charged fermions
- Higgs field

crucial: write all fermions in terms of Weyl spinors (= only l.h. fields)

e.g. $u_{Dirac} = \begin{pmatrix} u_L \\ \bar{u}_R \end{pmatrix}$

or

$u_{Dirac,L} = \begin{pmatrix} u_L \\ 0 \end{pmatrix}$; $u_{Dirac,R} = \begin{pmatrix} 0 \\ \bar{u}_R \end{pmatrix}$

$\boxed{\text{consider } u_L, u_R \text{ as fund. fields}}$

charged means that a field transforms in a repres. of the gauge group

notation:	fund. repr. of SU_2	→	2
	fund. repr. of SU_3	→	3
	adj. repr. of SU_3	→	8
	U_1-charge	→	$(\cdots)_Y$

SM fermions: 3 generations of

$$\underbrace{(3,2)_{1/3}}_{Q_L} + \underbrace{(\bar{3},1)_{-4/3}}_{u_R} + \underbrace{(\bar{3},1)_{2/3}}_{d_R} + \underbrace{(1,2)_{-1}}_{L_L} + \underbrace{(1,1)_{2}}_{e_R}$$

Higgs doublet: $(1,2)_{-1}$

$$\boxed{Q = T_3 + \frac{Y}{2}}$$

Further data:

- size of g_1, g_2, g_3 at m_Z
- Yukawa couplings: coefficients of

  $\bar{H}Qu \; ; \; HQd \; ; \; HLe$

  (3 matrices 3×3)

  (with $H \to H_d$; $\bar{H} \to H_u$ this could be superfield notation for the MSSM)
- neutrino masses: induced by operator

  $$\frac{1}{M}(\bar{H}L)^2$$

SU_5 unification (Georgi, Glashow, '74)

$$SU_3 \times SU_2 \times U_1 \subset SU_5$$

at Lie-algebra level ($U = \exp(T)$), consider 5×5 antihermitian, traceless matrices

$$\left.\left(\begin{array}{c:c} SU_3 & X,Y \\ \hdashline X,Y & SU_2 \end{array}\right)\right\} \text{ } SU_5 \text{ gauge bosons } (24 \text{ of } SU_5)$$

$$\nwarrow U_1$$

- $SU_3 \times SU_2 \subset SU_5$ obvious from matrix above
- U_1-generator T_Y :

$$T_1 = \frac{i}{\sqrt{60}} \left(\begin{array}{ccc:cc} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ \hdashline & & & -3 & \\ & & & & -3 \end{array}\right)$$

$$\boxed{\text{convention: } \operatorname{tr} T^a T^b = -\tfrac{1}{2}\delta^{ab}}$$

crucial observation:

$10 + \bar{5}$ of SU_5 give rise to precisely the field content of a SM generation

$\uparrow$

antisymm. part of 5×5

Outline of a proof

$\bar{5} = \left(\frac{d}{L}\right) = \bar{3} + \bar{2}$; however: $\bar{2} \cong 2$ for SU_2

equivalence of repres.:

$\exists C$ so that

$$\begin{tikzcd} \psi \arrow[r, "U"] \arrow[d, "C"'] & U\psi \\ \psi' \arrow[r, "U^*"] & U^*\psi' \arrow[u, "C^{-1}"'] \end{tikzcd}$$

here: $C_{ij} = \varepsilon_{ij}$

$$10 = \left(\begin{array}{c|c} u & Q \\ \hline -Q & e \end{array}\right) = (\bar{3},1) + (3,2) + (1,1)$$

trf. rule: $M_{ij} \Rightarrow U_{ik} U_{jl} M_{kl} = (UMU^T)_{ij}$

$Q = (3,2)$ - obvious

$e = (1,1)$: $e = \begin{pmatrix} 0 & e \\ -e & 0 \end{pmatrix}$ - only 1 component

$u = (\bar{3},1)$:

$$u = \begin{pmatrix} 0 & u_3 & -u_2 \\ -u_3 & 0 & u_1 \\ u_2 & -u_1 & 0 \end{pmatrix}$$

antisymm. part of $3 \times 3 \cong \bar{3}$

(needs to be checked)

Derivation:

ψ - antisymm. 3×3 matrix ; $\psi_{ij} \to U_{ik} U_{jl} \psi_{kl}$

write $\underline{\psi_{ij} = \varepsilon_{ijk} \psi_k}$

$$\psi'_{ij} = U_{ik} U_{jl} \varepsilon_{klm} \psi_m$$

$$1 = \det U = \frac{1}{3!} \varepsilon_{ijk} U_{ii'} U_{jj'} U_{kk'} \varepsilon_{i'j'k'}$$

$$U_{ii'} U_{jj'} U_{kk'} \varepsilon_{i'j'k'} = \varepsilon_{ijk}$$

$$U_{ii'} U_{jj'} \varepsilon_{i'j'k'} = \varepsilon_{ijk} U^{+}_{k'k}$$

$$\Rightarrow \psi'_{ij} = \varepsilon_{ijk} (U^{+})_{kk'} \psi_{k'} = \varepsilon_{ijk} (U^{*}\psi)_k$$

$$\Rightarrow \boxed{\psi_i \to U^{*}_{ij} \psi_j}$$

- given the successful identification of the $SU_3 \times SU_2$-multiplets, checking the U_1-charges is straightforward

$\boxed{\text{All Y-charge ratios are reproduced!}}$

Gauge coupling unification

let $D_\mu = \partial_\mu + iA_\mu$, so that $\mathcal{L} \supset \frac{1}{g^2} F^2$

natural quantity: $\underline{\alpha_i^{-1}(\mu)}$ $\quad i \in \{1,2,3\}$

experimental:

$$\alpha_i^{-1}(m_Z) = (\underline{59.0,\ 29.6,\ 8.4})$$

running:

$$\alpha_i^{-1}(m_Z) = \alpha^{-1}(M_{GUT}) + \sum_{r_i} d_{r_i} T_{r_i} \underline{\ln \frac{M_{GUT}}{m_Z}}$$

where $\mathrm{tr}[T^a T^b] = \delta^{ab} T_{r_i}$

$$d_{r_i} \sim \begin{array}{rl} +1 & \text{for scalar} \\ +8 & \text{spinor} \\ -22 & \text{vector} \end{array}$$

let $\alpha_{ij} = \alpha_i^{-1} - \alpha_j^{-1}$

the only truly fund. quantity is

$$\frac{\alpha_{12}}{\alpha_{23}} = 1.39_{exp.} = \frac{\underline{22\, T^A_{12}} - 2\, T^H_{12}}{\underline{22\, T^A_{23}} - 2\, T^H_{23}}$$

↓

pure gauge gives $\underline{2.0}$

$\underline{\text{SUSY}}$: more weight for Higgs relative to gauge $\longrightarrow$ $\underline{1.4}$

So far we have:

- $SU_3 \times SU_2 \times U_1$ "fits naturally" in SU_5
  (is a maximal subgroup of)
- SM fermions fill out $10 + \bar{5}$ of SU_5
- with SUSY, the measured SM gauge couplings are reproduced if

$$\alpha_{GUT}^{-1} \simeq 25 \quad \text{at} \quad M_{GUT} \simeq 10^{16}\,\text{GeV}$$

- the measured neutrino mass scale is reproduced if the relevant higher-dim. operator is suppressed by

$$M \simeq 10^{15}\,\text{GeV}$$

(• given $M_P \simeq 10^{19}$ GeV and $\alpha_{GUT}^{-1} \simeq 25$, the simplest string-GUTs predict

$$M_{GUT} \simeq 10^{17}\,\text{GeV}$$
)

Arguably, GUTs are our best guess for going beyond the SM.

SU_5 breaking

(conventional Higgs breaking)

1) $\phi \in 5$ (fund.) ; VEV $H = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}$

$$\Downarrow$$

$$SU_5 \supset SU_4$$

2) $\phi \in 24$ (adjoint) ; in general

VEV $\phi = \begin{pmatrix} \alpha_1 & & 0 \\ & \ddots & \\ 0 & & \alpha_5 \end{pmatrix}$

recall: $\phi \to u \phi u^+$

$\sum_i \alpha_i = 0$ (tracelessness)

$$\Downarrow$$

$$SU_5 \supset (U_1)^4$$

however: for $\phi \sim \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$,

the breaking is to

$$SU_5 \supset \underline{SU_3 \times SU_2 \times U_1}$$

serious problems of GUT-Higgs-breaking

we need:

GUT Higgs ϕ (24 of SU_5)

SM Higgs H (5 of SU_5) $\quad (5 = 3 + \underline{\underline{2}})$

- already the minimal potential is complicatea

$$V \supset m_\phi^2 \, tr\,\phi^2 \; ; \; (tr\,\phi^2)^2 \; ; \; tr\,\phi^4 \; ;$$
$$\underline{m_H^2 \, H^+ H} \; ; \; (H^+ H)^2 \; ;$$
$$(tr\,\phi^2)(H^+ H) \; ; \; H^+ \phi^2 H$$

- need to arrange

$$\phi_{min} = v_\phi \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}$$

- resulting masses for doublet (d) & triplet (t):

$$m_t^2 = \underline{m_H^2} + \lambda v_\phi^2 \longrightarrow \text{need} \sim m_{GUT}^2 \quad (10^{16}\,GeV)$$
$$m_d^2 = \underline{m_H^2} + \lambda' v_\phi^2 \longrightarrow \text{need} \sim m_{EW}^2 \quad (10^2\,GeV)$$

(→ fine tuning!) <u>"doublet-triplet-splitting"</u>

- still: proton decay constraints (via H_t) (<u>almost</u>) kill the model

Yukawa unification (very brief)

SM Higgs H: 5_H of SU_5

Matter: $3\times(10_M + \bar{5}_M)$ of SU_5

possible Yukawa couplings:

$$\underline{5_H 10_M 10_M} \quad ; \quad \underline{\bar{5}_H 10_M \bar{5}_M}$$

invariance by contraction with ε_{ijklm} ; invariance obvious

$$\downarrow \qquad\qquad \downarrow$$

3rd generation: λ_t ; λ_b, λ_τ

$\Longrightarrow$ $\lambda_b = \lambda_\tau$ at M_{GUT}

(after running consistent with measured low-energy values m_b and m_τ)

<u>but</u>: obvious problems for light generations

<u>possible solution:</u> <u>higher-dim. operators</u>

$$\text{e.g.} \quad \mathcal{L} \supset \frac{1}{M_P} \cdot \bar{5}_H 24_H 10_M \bar{5}_M$$

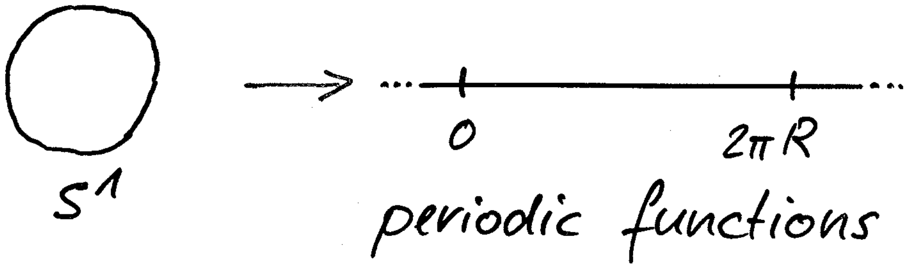
(recall $24_H \longrightarrow U \cdot 24_H \cdot U^{-1}$)

try new ingredient: **Extra Dimensions**

(*here:* just on a simple, field-theoretic level, as used by Kaluza & Klein; *however:* well-motivated in string theory, especially at $M_{GUT} \sim 10^{16}\,\text{GeV}$)

recall:

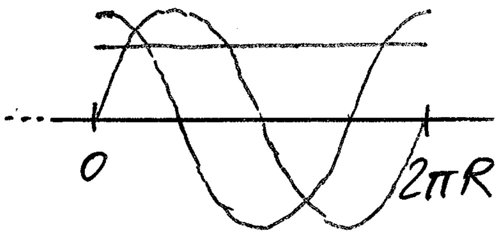
space $\mathbb{R}^4 \times S^1$



periodic functions

field theory on that space:

$$\varphi = \sum_{n=0}^{\infty} \varphi^{c}_{(n)}(x) \cdot \cos(ny/R) + \sum_{n=1}^{\infty} \varphi^{s}_{(n)}(x) \cdot \sin(ny/R)$$



low energy physics:

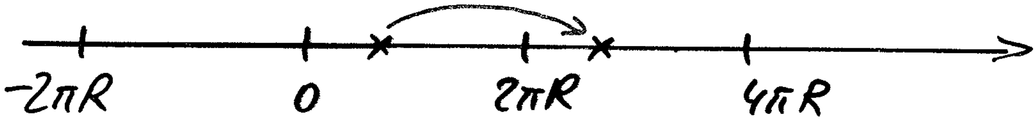
- described by 0-modes $(n=0)$ of field φ
- Kaluza-Klein (KK) excitations start at mass $\frac{1}{R} \sim 10^{16}\,\text{GeV}$

Old & new idea: Gauge symm. breaking through extra-dim. structure

1) consider $\mathbb{R}^4 \times S^1$ with radius small enough to be unobservable (KK-idea)

2) consider S^1 as $\underline{S^1 = \mathbb{R}/\mathbb{Z}}$

(where the action of $\mathbb{Z}$ on $\mathbb{R}$ is defined by $n: x \to x + 2\pi R \cdot n$)



3) more generally:

$\underline{M}$ - extra-dim. manifold

$\underline{K}$ - discrete symm. group of M

($k \in K$; $k : x \to k \cdot x$)

$\underline{M/K}$ - manifold of equivalence classes, where $x \sim x'$ iff $\exists\, k$ with $x' = k \cdot x$

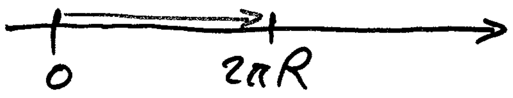
(M/K is manifold only if K acts freely, i.e., $k x = x$ for some $x \Rightarrow k = \mathbb{1}$)

4) consider gauge-theory on $\mathbb{R}^4 \times M$;
let $K \to G$ be group homomorphism;
K acts on field space:

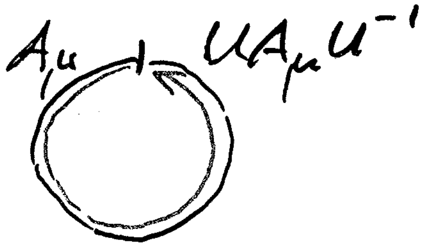
$$k: \; A_\mu(x) \longrightarrow U(k) A_\mu(k^{-1}x) U(k)^{-1}$$

- in field theory on M/K, only fields are allowed that are invariant under K.

<u>example:</u> $S^1 = \mathbb{R}/\mathbb{Z}$



$\Longrightarrow$



5) A_μ = const. is forbidden
$\Rightarrow$ <u>gauge symm. breaking in eff. 4d theory</u>

6) more abstract, geometrical language:
gauge connection is such that parallel transport around S^1 is non-trivial
(Wilson loop $\neq 1$)

<u>obvious problem:</u> precise value of W-loop is <u>modulus</u>

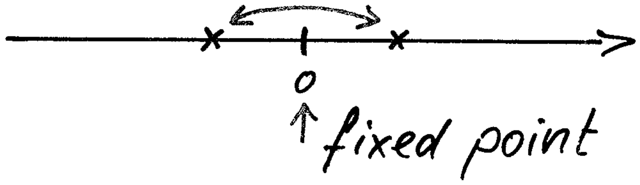
(→ Hosotani '83)

Orbifolding

M/K (exactly as before), but with a <u>non-free</u> action of K on M

simplest example: $\mathbb{R}/\mathbb{Z}_2$

($\mathbb{Z}_2 = \{1, -1\}$, $-1 \cdot x = -x$)



$\mathbb{R}/\mathbb{Z}_2$ for scalar field φ: two possibilities

① $\varphi(x) \longrightarrow \varphi(-x)$

② $\varphi(x) \longrightarrow -\varphi(-x)$ ← in this case, φ will have no zero-mode

$\mathbb{R}/\mathbb{Z}_2$ for gauge field A_μ:

certain components A_μ^a can be forced to vanish at fixed point

$\Rightarrow$ in contrast to Wilson line breaking, orbifold breaking leads to <u>specific points</u> where the gauge symm. is broken

(cf. Dixon, Harvey, Vafa, Witten '85 for orbifolding in string theory)

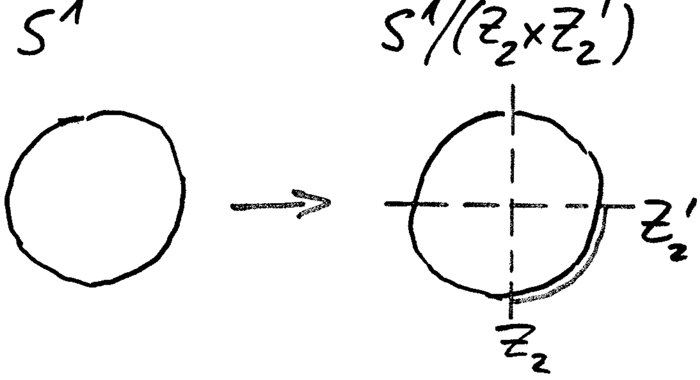
Simplest realistic orbifold GUT

Kawamura 02/99 ... 12/00

$d=5$; $S^1/(Z_2\times Z_2')$

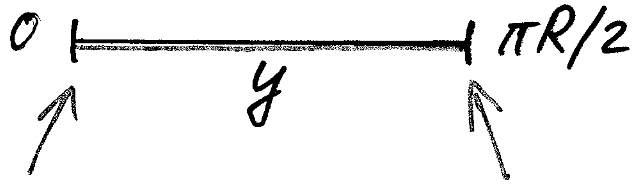
also:

Altarelli, Feruglio
Hall, Nomura
....



phys. space:

$\mathbb{R}^4 \times [0, \frac{\pi R}{2}]$



$A_\mu(y) = P A_\mu(-y) P^{-1}$

$A_\mu(y') = P' A_\mu(-y') P'^{-1}$

$(y' = y + \pi R/2)$

choose: $P = \mathbb{1}$

$$P' = \begin{pmatrix} 1 &&&& \\ & 1 &&& \\ && 1 && \\ &&& -1 & \\ &&&& -1 \end{pmatrix}$$

removes zero-modes of all A_5-fields

breaks gauge symm. from SU_5 to SM

furthermore:

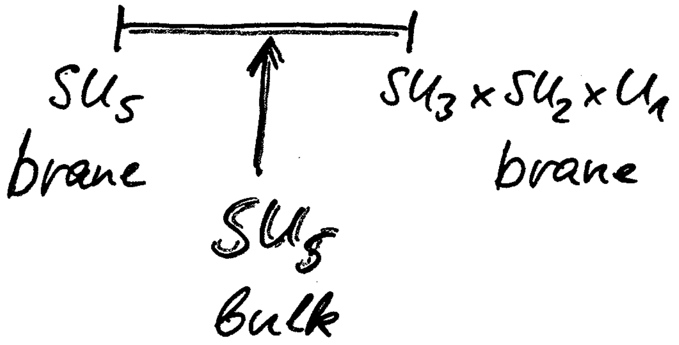
let Higgs = 5 be in the bulk

$H(y) = P H(-y)$

$H(y') = -P' H(-y')$

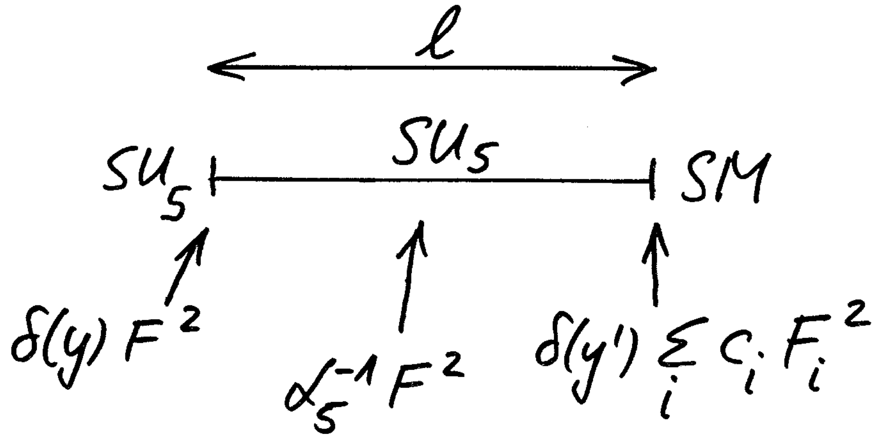
$\Rightarrow$ only doublet survives!

in summary:



- $\frac{1}{R} \sim 10^{15}$ GeV ; proton decay can be completely avoided
- unification prediction remains as good as usual due to dominance of bulk

in more detail:



$$\alpha_5^{-1} \sim M \gg M_c \sim 1/\ell$$

$$\boxed{\alpha_{4,i}^{-1} = \ell \cdot \alpha_5^{-1} + c_i}$$

$\sim 25 \quad + \; O(1)$ corrections

(4d) $\underline{\underline{SO_{10}}}$ Georgi Fritzsch, Minkowski '74

Can one do better than $10 + \bar{5} + 1$ per generation?

↑ r.h. neutrino

Yes!

$$SO_{10} \supset SU_5 \supset SU_3 \times SU_2 \times U_1$$

↑ can be understood by writing

$$\begin{pmatrix} \xi_1 \\ \vdots \\ \xi_{10} \end{pmatrix} \hat{=} \begin{pmatrix} \xi_1 + i\xi_2 \\ \vdots \\ \xi_9 + i\xi_{10} \end{pmatrix}$$

obvious SO_{10} action

obvious SU_5 action (smaller symm.!)

Need <u>branching rules</u>

e.g. $SU_5 \supset SU_3 \times SU_2$

$$5 = (3,1) + (1,2)$$

here we need, e.g., $SO_{10} \supset SU_5$

$$10 = 5 + \bar{5} \quad \text{(Higgses!)}$$

↑ note: 10 <u>complex</u> numbers

$10: x_1, \cdots, x_{10}$ $\quad$ $5: x_1 + ix_2, \cdots, x_9 + ix_{10}$

$\bar{5}: x_1 - ix_2, \cdots, x_9 - ix_{10}$

crucial point for SM matter:

SO_{10} has a spinor repres. : 16

(just like the spinor 2 of SO_3 or $SO_{1,3}$)

branching rules:

$$\begin{array}{ccc} SO_{10} & \supset & SU_5 \\ 10 & = & 5 + \bar{5} \\ \hline 16 & = & 10 + \bar{5} + 1 \\ \hline 45 & = & 24 + 10 + \overline{10} + 1 \end{array}$$

r.h. neutrino ↙

Thus: SO_{10} with 10 & 3×16
give SM with 2 Higgses !

even better:

$$\phi = v \cdot \left(\begin{array}{ccc|cc} \begin{smallmatrix}0&-1\\1&0\end{smallmatrix} & & & & \\ & \begin{smallmatrix}0&-1\\1&0\end{smallmatrix} & & & \\ & & \begin{smallmatrix}0&-1\\1&0\end{smallmatrix} & & \\ \hline & & & 0 & \\ & & & & 0 \end{array}\right)$$

$(H^1_{10})^T \phi_{45} H^2_{10} \implies$ masses for triplets, but not for doublets

(→ Dimopoulos-Wilczek mechanism for doublet-triplet-splitting ; '81)

different possibilities to get from SO_{10} to SM

$$\begin{tikzcd}
& SO_{10} \arrow[dl] \arrow[d] \arrow[dr] & \\
SU_5\times U_1 \arrow[dr] & SU_5'\times U_1' \arrow[d, "\text{``flipped'' } SU_5"] & SU_4\times SU_2\times SU_2 \arrow[dl, "\text{Pati-Salam '73}"'] \\
& \underbrace{SU_3\times SU_2\times U_{1,Y}}_{SM}\times U_{1,X} &
\end{tikzcd}$$

note:
- $SU_5' \supset SU_3\times SU_2$ of SM, but not the $U_{1,Y}$
- flipped SU_5 and Pati-Salam are not "real" GUTs since they don't predict coupling unification
- if no SM subgroup is specified $SU_5\times U_1$ & $SU_5'\times U_1'$ are equivalent
- $SO_{10} \to SU_4\times SU_2\times SU_2 \to$ SM is a possibility of non-SUSY unification*

* a different proposal for non-SUSY unification: "Split supersymmetry"

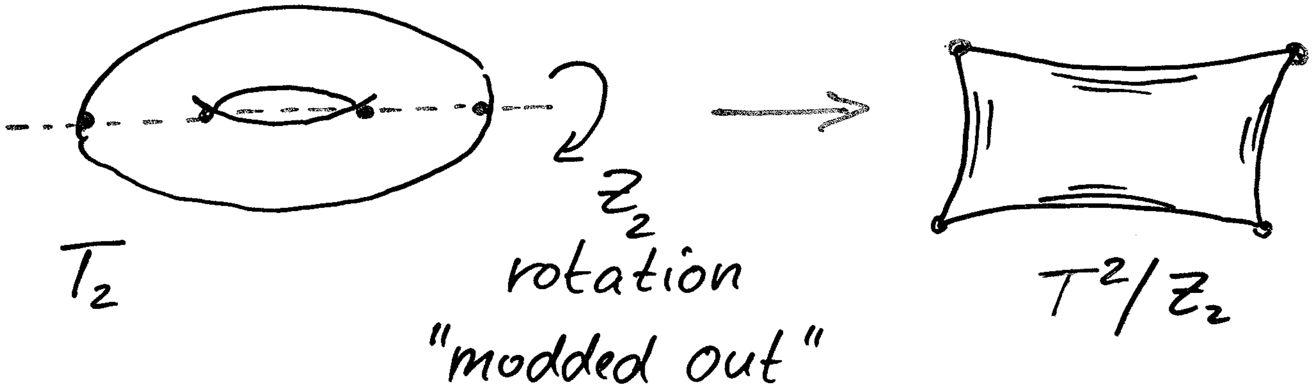
→ Arkani-Hamed, Dimopoulos
Giudice, Romanino

- accept fine-tuning of Higgs mass and break SUSY at high scale
- squarks are heavy ($\gtrsim 10^{10}$ GeV)
- gauginos & Higgsinos are light (this is technically natural since they are fermions)
- unification still works since the squarks come in full SU_5 multiplets and don't contribute to the differential running of couplings

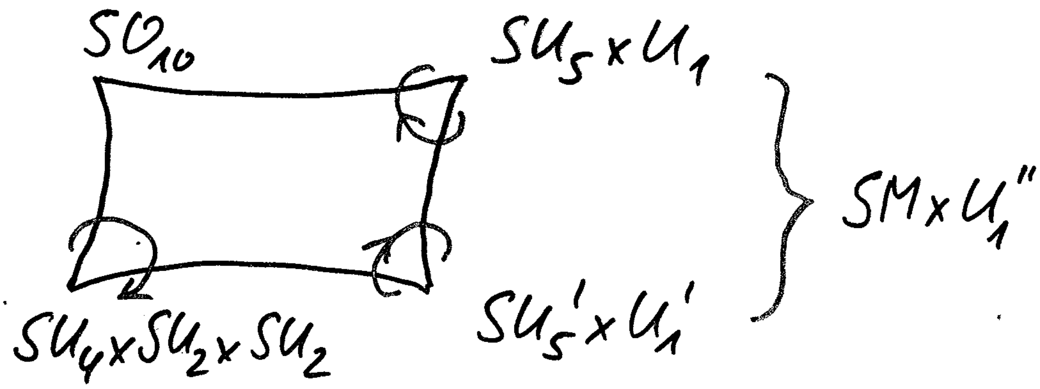
back to SO_{10}

- the 5d SU_5 model does not directly generalize to SO_{10} since there is no appropriate $\mathbb{Z}_2$-automorphism of SO_{10}
- an interesting possibility exists in $d=6$

→ Asaka, Buchmüller, Covi, '01
Hall, Nomura, Okui, Smith, '01



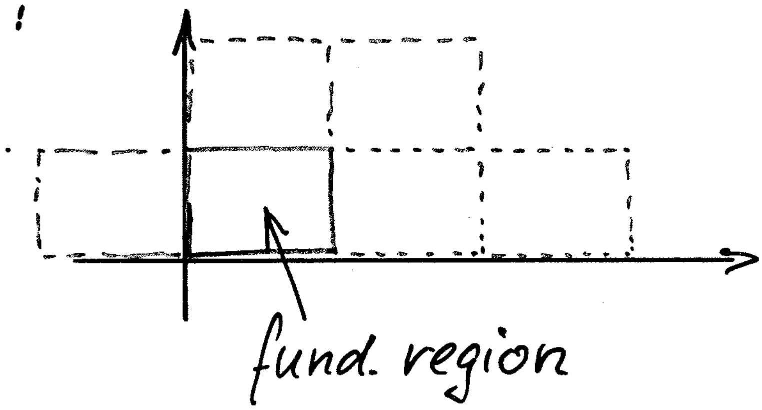
gauge-breaking by orbifolding corresponds to Wilson lines around "corners" of the "pillow":



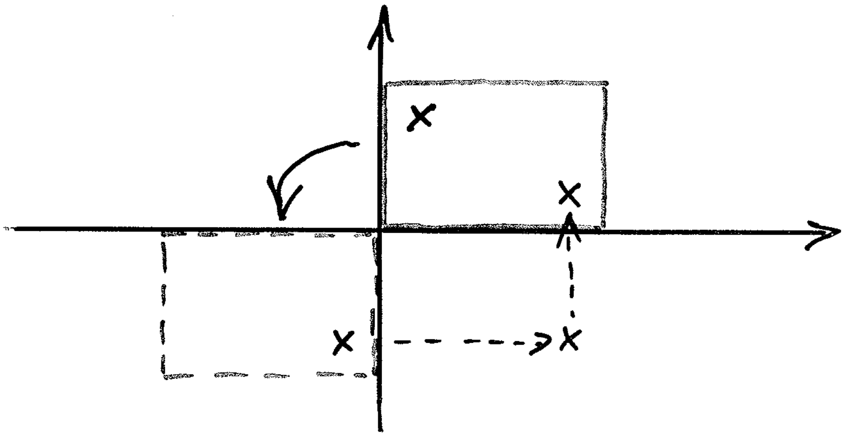
(in fact, one needs $T^2/(\mathbb{Z}_2\times \mathbb{Z}_2'\times \mathbb{Z}_2''))$

Technical description:

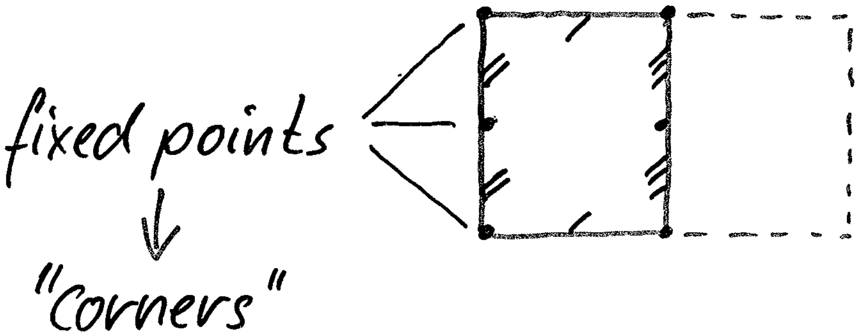
- T_2 is obtained by modding out translation lattice from $\mathbb{R}^2$:



- this lattice has a $\mathbb{Z}_2$ rotation symmetry, which can be used to further reduce the fund. space:



- new fund. space is the "pillow":



for large groups, more powerful tools needed:

- let $\mathfrak{g}$ be the Lie-alg. of G
- consider its action on itself:

$$X : Y \longrightarrow [X,Y] \quad \text{(adjoint repr.)}$$

- diagonalize max. set of operators:

$$\{H_i\} \quad \text{(Cartan subalg.)}$$

$$\dim \{H_i\} \equiv \text{rank}\,(G)$$

- normalize them

$$\text{tr}\,(H_i H_j) = \lambda\,\delta_{ij} \quad \text{(Killing metric)}$$

- the other generators (vectors of repr. space) are completely characterized by their eigenvalues

$$[H_i, E_\alpha] = \alpha_i E_\alpha$$

(α_i - vectors in r-dim. "root space")

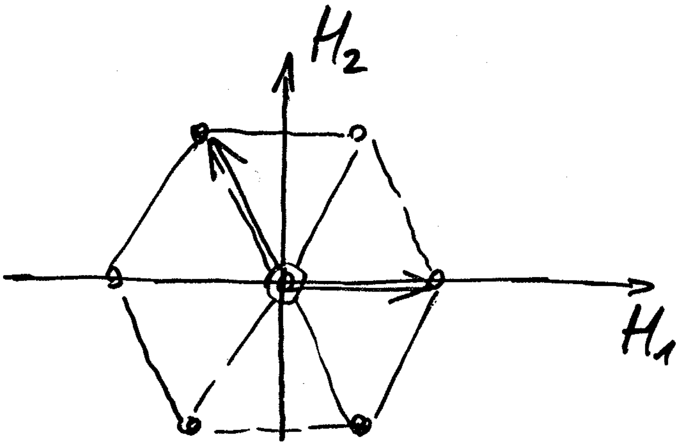
- order the α's:

$$\alpha - \beta > 0 \iff \left\{ \text{first non-zero component of } (\alpha-\beta) \right\} > 0$$

- the smallest r positive roots form a basis!

famous example:

octet of SU_3
(flavour)



- the above vectors are called roots
- the two distinguished vectors suffice to build the whole diagram (by adding & flipping sign) ⇒ simple roots

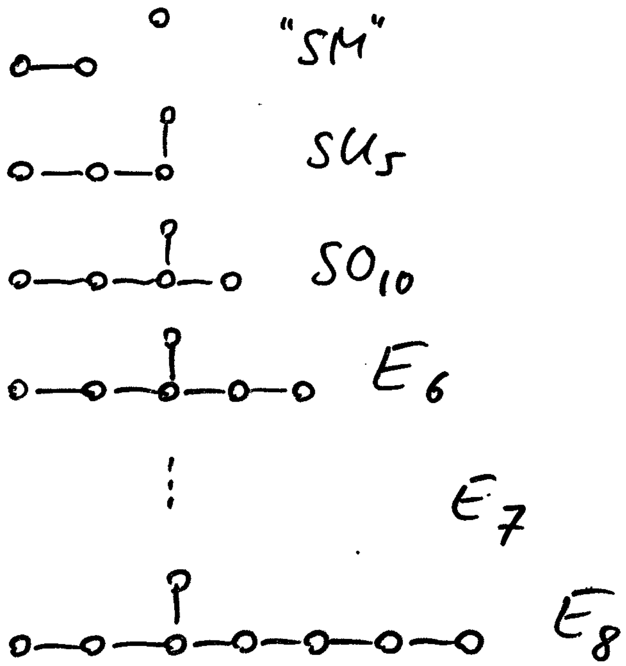
→ Dynkin diagrams ; e.g.

for SU_3

famous series:

o – simple root

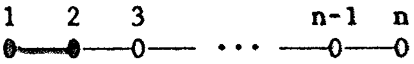
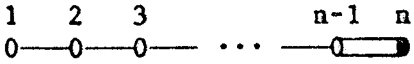
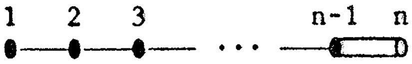
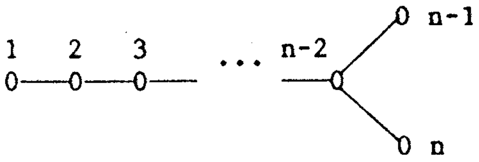
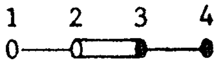
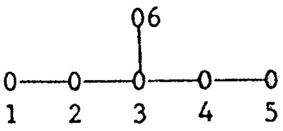
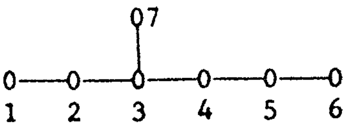
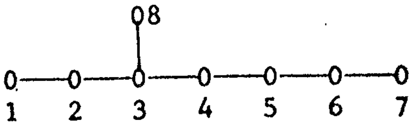
"—" – angle 120° between them

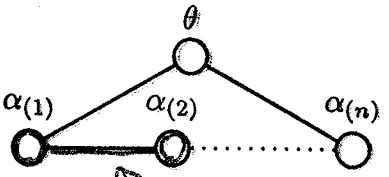
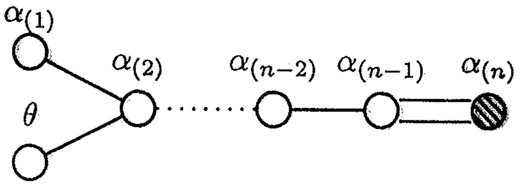
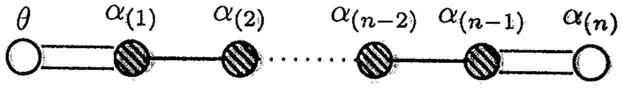
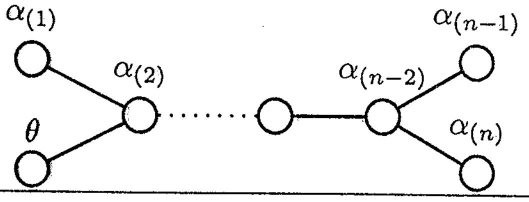


(cf. $E_8 \times E_8$ of heterotic string)

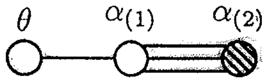
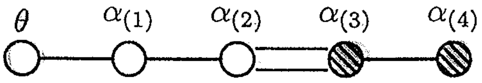
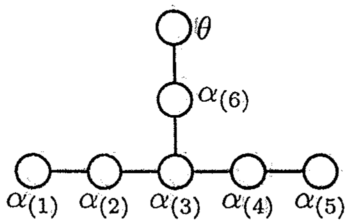
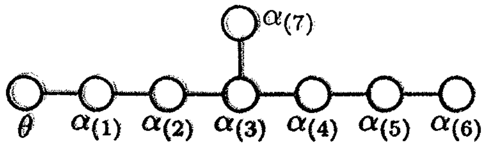
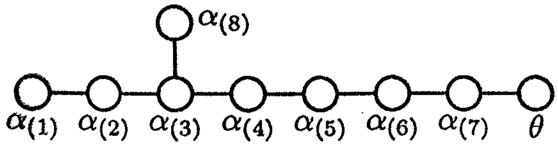
from Slansky
(Phys. Rept.)

Table 5
Dynkin diagrams for simple Lie algebras. (Black dots represent shorter roots.)

A_n		SU_{n+1}	SU_{n+1}
B_n			SO_{2n+1}
C_n			Sp_{2n}
D_n		SO_{2n}	SO_{2n}
G_2			
F_4			
E_6		E_6	
E_7		E_7	
E_8		E_8	

Name	Real algebra	Extended Dynkin diagram
A_n	$\mathfrak{su}(n+1)$	
B_n	$\mathfrak{so}(2n+1)$	
C_n	$\mathfrak{sp}(2n)$	
D_n	$\mathfrak{so}(2n)$	

su_3

Name	Extended Dynkin diagram
G_2	
F_4	
E_6	
E_7	
E_8	

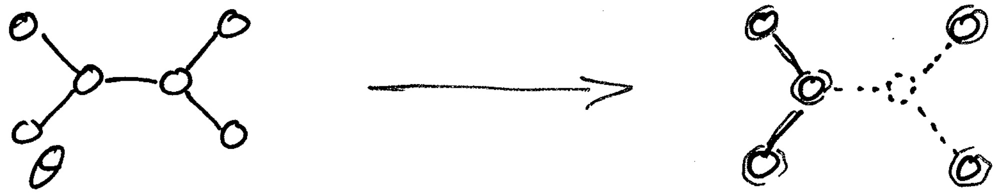
$\underline{\theta}$
most negat.
root

Orbifolding to any max. reg. subgroup

Dynkin's prescription ('57):

"remove any node from ext. Dynkin diagram"

e.g.



$$SO_{10} \longrightarrow SU_4 \times SU_2 \times SU_2$$

By orbifolding:

$$P = \exp(2\pi i \, V \cdot H)$$

$$P E_\alpha P^{-1} = \exp(2\pi i \, V \cdot \alpha) \, E_\alpha$$

$$P H_i P^{-1} = H_i$$

- to remove $\alpha_{(i)}$, choose $V \sim \underset{\uparrow}{\mu^{(i)}}$

dual basis vector

$$\mu^{(i)} \cdot \alpha_{(j)} \sim \delta^i_j$$

- to check for survival of θ, look at <u>Coxeter labels</u>:

$$\theta = - \sum_k \underset{\uparrow}{c_k} \, \alpha_{(k)}$$

<u>many further interesting possibilities,</u>

e.g.

$$E_6 \supset SO_{10}$$

$$\underset{(\text{adj.})}{78} = 45 + 16 + \overline{16} + 1$$

(just the gauge fields contain the right quantum numbers for a SM generation)

$$E_8 \supset SU_3 \times E_6$$

$$248 = (8,1) + (1,78) + (3,27) + (\overline{3},\overline{27})$$

and

$$E_6 \supset SO_{10}$$

$$27 = 1 + 10 + 16$$

(everything could come from gauge! <u>in higher dimensions</u>!

<u>note</u>: E_7, E_8 have only real repr., so getting the chiral SM is difficult in $d=4$ (<u>but easy by orbifolding in $d>4$</u>)

higher-dim. spinor representations

(needed for SO_{10} GUT & for FT in $d > 4$)

consider $SO(2n)$

introduce Γ_α $(\alpha = 1 \cdots 2n)$ with

$$\{\Gamma_\alpha, \Gamma_\beta\} = 2\delta_{\alpha\beta} \cdot \mathbb{1}$$

(Clifford alg.)

let $\gamma_a = \frac{1}{2}(\Gamma_{2a} - i\Gamma_{2a-1})$; $\gamma_a^+ = \frac{1}{2}(\Gamma_{2a} + i\Gamma_{2a-1})$

$(a = 1 \cdots n)$

$$\Rightarrow \{\gamma_a, \gamma_b^+\} = \delta_{ab}$$

define: $|0\rangle$ with $\gamma_a|0\rangle = 0$

$\Rightarrow$ the γ's create Hilbert space of
n fermionic oscillators
(2^n states)

e.g. $|\psi\rangle = |{+}{+}{-}{-}{+}\rangle$ for $n=5$

$\Gamma_{\alpha\beta} = \frac{1}{2}[\Gamma_\alpha, \Gamma_\beta]$ form SO_{10}-Lie-Alg.

$\Rightarrow$ SO_{10} acts on spinors

introduce $\Gamma = i^n \Gamma_1 \Gamma_2 \cdots \Gamma_{2n}$

$(\Gamma^2 = 1 \; , \; \{\Gamma, \Gamma_\alpha\} = 0)$

$P_\pm = \frac{1 \pm \Gamma}{2}$ – projectors on two invariant subspaces of 2^n-spinor

<u>for SO_{10}</u>: $2^5 = 32$ "=" $16 + \overline{16}$

(with, say, <u>even</u> & <u>odd</u> number of "–" signs)

- <u>SO_{10} generators:</u> flip two signs in $|++--+\rangle$ etc.
- <u>SU_5 generators:</u> interchange two signs

Thus, under $SO_{10} \supset SU_5$

$$16 = 10 + \bar{5} + 1$$

16 ↑ even number of "–" signs

10 ↑ two "–" signs

$\bar{5}$ ↑ four "–" signs

1 ↑ no "–" sign

$$\underbrace{10 + \bar{5} + 1}_{\text{one SM generation + r.h. neutrino (total singlet)}}$$

SUSY in $d>4$ (very brief)

- the crucial part of the Super-Poinc.-Alg.,

$$\{Q, \bar{Q}\} \sim P \qquad \text{(schematic)}$$

generalizes to $d \neq 4$, but not in a trivial (universal) way

- essential difficulty:

varying features of minimal spinor

$d=4$: $2^2 = 4$ (Dirac) — $2 \cdot 2$ (Weyl)
— $2 \cdot 4_{real}$ (Majorana)

$d=5$: $2^2 = 4$ (Dirac) — no further reduction, since $\Gamma = \Gamma_5$ is already in $SO(1,4)$-alg.

$d=6$: $2^3 = 8$ (Dirac) — $2 \cdot 4$ (Weyl)
(no Majorana)

⋮

$d=11$: $2^5 = 32$ (Dirac) — $2 \cdot 32_{real}$ (Majorana)

no higher SUSY FTs known

as an illustration:

- minimal 4d SUSY can be written as

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^m_{\alpha\dot\beta} P_m$$

↑ Weyl

$$\{Q_a, Q_b\} = 2(\gamma_m C)_{ab} P^m$$

↑ Majorana

- either one or the other form will be relevant in specific cases in $d \neq 4$

for example, in $d=10$:

$$\{Q_a, Q_b\} = \left[\frac{1+\Gamma}{2}(\Gamma_m C)\right]_{ab} P^m$$

↑ Weyl and Majorana

→ J. Strathdee, J. of Mod. Phys. A 2 ('87) 273

Appendix of Polchinski, vol. II

The "amount" of SUSY

$d=4$, min. SUSY - $N=1$ (by def.)

$d=5,6$ $N=2$

(no Yuk. couplings , gauge-couplings run only at 1-loop)

$d=7,8,9,10$ $N=4$

(just pure gauge theory & gravity ; gauge th. finite!)

$d=11$ $N=8$

(just gravity)

The 5d SUSY model in more detail

(→ Mirabelli/Peskin 1997 and refs. therein)

$M,N : 0,1,2,3,5$; $m,n : 0,1,2,3$

$$\gamma^M = \left(\begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix}, \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \right)$$

4-spinor ψ irred. under $SO(1,4)$

$\psi = \begin{pmatrix} \psi_L \\ \bar{\psi}_R \end{pmatrix}$; ψ_L & ψ_R are mixed by $SO(1,4)$

5d gauge multiplet:
- A^M vector
- Σ real scalar
- λ gaugino
- X^a $(a=1,2,3)$ – real auxil. fields

$$\mathcal{L} = \frac{1}{g^2}\left\{ -\frac{1}{2} F_{MN}^2 - (D_M \Sigma)^2 + \bar{\lambda} i \gamma^M D_M \lambda + (X^a)^2 - \bar{\lambda}[\Sigma,\lambda] \right\}$$

SUSY-trf.: $\delta_\xi A^M = i \bar{\xi} \gamma^M \lambda + h.c.$

$\vdots$

$\xi = \begin{pmatrix} \xi_L \\ \bar{\xi}_R \end{pmatrix}$ – corresponds to $N=2$ SUSY from 4d perspective already the 'trivial' Z_2 (with $P = \mathbb{1}$) breaks $N=2$ to $N=1$ SUSY:

- lagrangian contains terms like

$$\lambda_R \underline{\underline{\partial_5}} \lambda_L$$

$\Rightarrow$ either λ_R or λ_L must be odd Z_2

- A_μ even $\Rightarrow$ A_5 odd

- vector multiplet: $\{ A_\mu , \lambda_L , \underbrace{X^3 - \partial_5 \Sigma}_{\text{"D"}} \}$

  $\hookrightarrow$ real superfield V

- scalar multiplet: $\{ \Sigma + i A_5 , \lambda_R , X^1 + i X^3 \}$

  $\hookrightarrow$ chiral superfield Φ

$\rightarrow$ Marcus, Sagnotti, Siegel, '83

Arkani-Hamed, Gregoire, Wacker, '01

....

- define covar. deriv. $\qquad \nabla_5 = \partial_5 + \Phi$

5d lagrangian – 4d superfield language

- combine the lowest-dim. invariant operators that can be built from V & ∇_5 :

$$\mathcal{L} = \frac{1}{2g^2} \operatorname{tr} \left\{ \int_{\theta^2} W^\alpha W_\alpha + \int_{\bar{\theta}^2} \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} + \int_{\theta^2\bar{\theta}^2} \underbrace{\left(e^{-2V} \nabla_5 e^{2V} \right)}_{\text{superfield } Z}{}^2 \right\}$$

$$\boxed{\mathcal{L} = \frac{1}{2g^2} \operatorname{tr} \left\{ \int_{\theta^2} W^2 + h.c. + \int_{\theta^2\bar{\theta}^2} Z^2 \right\}}$$

- 4d SUSY manifest
- gauge invariance manifest
- relat. normaliz. of W^2 & Z^2 terms fixed by demanding 5d Lorentz inv.

$\Downarrow$

full 5d SUSY automatic

The theory can be <u>constructed</u> by starting from V, introducing $\nabla_5 = \partial_5 + \Phi$, and requiring 5d Lorentz inv. (<u>also for higher dim. operators!</u>)

The matter multiplet (hypermultiplet)

in components: H^i , ψ , F^i

scalars / Dirac spinor / auxil. fields

4d superfields:

$$H = H^1 + \sqrt{2}\theta\psi_L + \theta^2(F_1 + D_5 H^2 - \Sigma H^2)$$

$$H^c = \bar{H}_2 + \sqrt{2}\theta\psi_R + \theta^2(-\bar{F}^2 - D_5\bar{H}_1 - \bar{H}_1\Sigma)$$

gauge trf.: $H \to e^{-\Lambda}H$; $H^c \to H^c e^{\Lambda}$

lagrangian:

$$\mathcal{L} = \int_{\theta^2\bar{\theta}^2}\left(\bar{H}e^{2V}H + H^c e^{-2V}\bar{H}^c\right) + \int_{\theta^2} H^c \nabla_5 H + h.c.$$

---

summary:

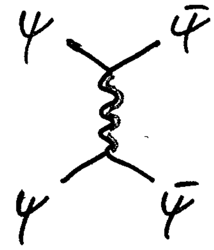
gauge: V, ϕ matter: H, H^c

$\mathbb{Z}_2$-orbifolding: $+ \; -$ or $- \; +$ $+ \; -$ or $- \; +$

(1 SUSY survives)

Proton decay: 4d GUTs

SU_5 gauge fields: $\left(\begin{array}{c|c} SU_3 & X,Y \\ \hline X,Y & SU_2 \end{array}\right)$



$\longrightarrow$ operators $\sim \frac{g^2}{M_{X,Y}^2} \cdot (\bar\psi\bar\psi)(\psi\cdot\psi)$

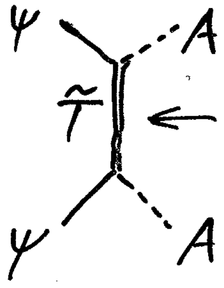
"dimension 6"

$\Gamma(\text{e.g. } p \to \pi^0 e^+) \sim \frac{g^4}{M_{X,Y}^4} \cdot \Lambda^5$ $\Leftarrow$ hadronic scale

(below present limit of $1/\Gamma \sim 5 \cdot 10^{33} y$)

<u>however</u>: in SUSY-GUTs, the <u>triplet-Higgs</u> dominates:

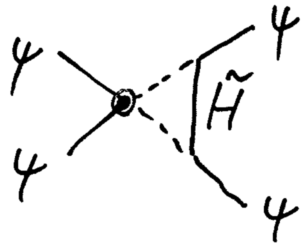
e.g. $\overset{(-)}{5}_{Higgs} = \begin{pmatrix} T \\ \hdashline H \end{pmatrix} \leftarrow$ coloured Higgs



$\leftarrow$ fermionic partner of T $\Rightarrow \frac{\lambda^2}{M_T} \cdot (\psi\psi)(AA)$

"dimension 5"

although the required "dressing" by SUSY partners leads to a suppression,



the resulting rate still dominates (e.g. $p \to K^+\nu$)

and has been claimed to <u>exclude</u> the minimal SUSY GUT (Murayama, Pierce '02)

<u>however:</u>

- the minimal GUT is, in fact, excluded by Yukawa unification;
- the minimal <u>consistent</u> GUT (including higher-dim. operators) can avoid the bound (by fine-tuning)

→ Emmanuel-Costa, Wiesenfeld '03

(Bajc, Perez, Senjanovic '02)

Crucial technical comment:

- Why doesn't the X,Y-gaugino induce dim.-5 proton decay?

  $\lambda = \lambda_{X,Y}$ gets its mass from

  $$M\,\lambda\cdot\chi \qquad (\text{not } M\cdot\lambda\cdot\lambda)$$

  ↑ superpartner of GUT-Higgs

  χ doesn't couple to matter

  ⇒ no dim.-5 diagram

- This can be repeated in SM Higgs sector:

  $$H_{u,d} \longrightarrow T_{u,d}$$

  let $T_{u,d}$ be heavy because of

  $$\begin{matrix} M\,T_u T_u' \\ M\,T_d T_d' \end{matrix} \qquad (\text{not } M\,T_u T_d)$$

  ("missing partner mechanism")

  This is automatically realized in 5d models!

Proton decay in 5d orbifold GUTs

- Triplet-Higgsino-mediated decay avoided because mass comes from

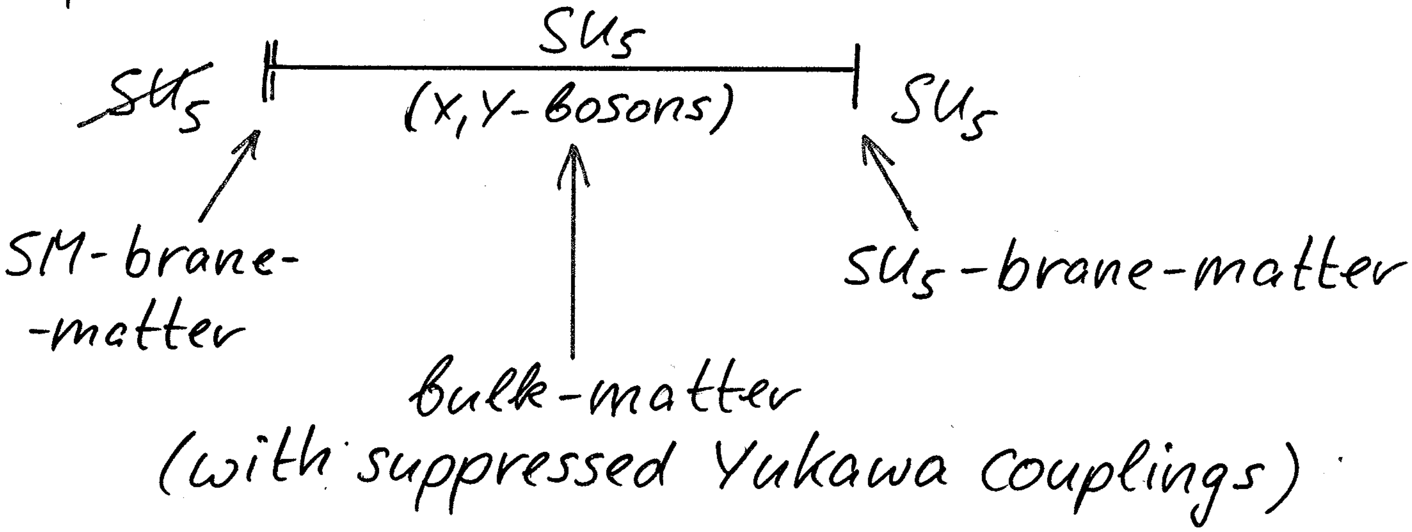
$$T_u \partial_5 T_u^c \quad \& \quad T_d \partial_5 T_d^c$$

"partners" in 5d-spinor

("primed" fields do not couple to SM matter)

Altarelli, Feruglio
Hall, Nomura, '01

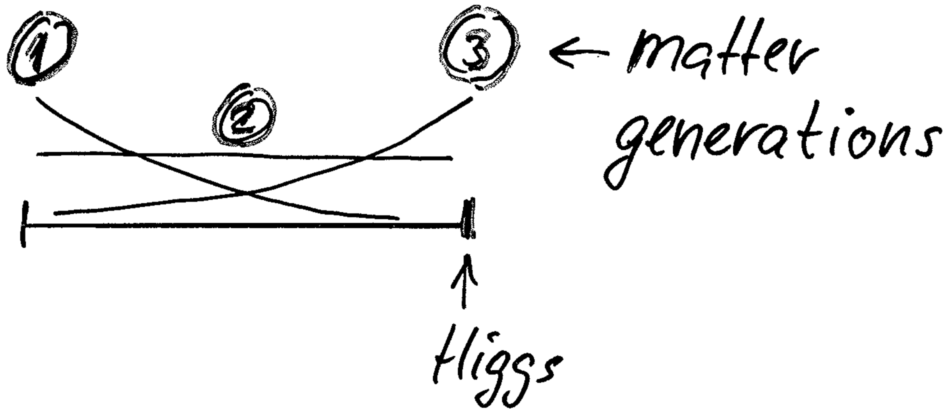
- X,Y-gauge-boson-mediated decay can become observable ;
various signals depend on localization of matter:



Comment:

partial (or complete) localization of bulk fields is induced by bulk mass terms and can be used (as an alternative to Froggat-Nielsen) for flavour model building:

e.g.



Power-law-unification

Logarithmic corrections

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\Lambda) + b_i \ln(\Lambda/\mu)$$

$(i = 1,2,3$ for U_1, SU_2, $SU_3)$

$$\alpha_i^{-1}(\Lambda) = \alpha_{GUT}^{-1}(\Lambda) + \underbrace{O(1)}_{\text{unknown threshold corrections}}$$

Power-corrections in higher dimensions

Taylor, Veneziano, '88
Roberts, Ross, '92
Dienes, Dudas, Gherghetta, '98

$$\alpha_i^{-1}(0) = \alpha_i^{-1}(\Lambda) + \underbrace{b_i \cdot \Lambda}_{\substack{\text{linear divergence of} \\ \text{1-loop integral in } d=5}}$$

$$\alpha_i^{-1}(\Lambda) = \alpha_{GUT}^{-1}(\Lambda) + \underbrace{O(\Lambda)}_{\text{unknown threshold corrections}}$$

(same order as
1-loop "running" effect).

Soft breaking in d=5 dimensions

5d VEV $\langle \phi \rangle$ breaks, e.g.,

$$SU_5 \rightarrow U_1 \times SU_2 \times SU_3$$

inducing vector boson mass $M_V \sim \langle \phi \rangle$

$$\alpha_i^{-1}(0) = \alpha_{GUT}^{-1}(\Lambda) + \underbrace{b_i \cdot M_V}_{\substack{\text{finite}\\\text{and}\\\text{calculable}}} + \underbrace{b \cdot \Lambda}_{\substack{\text{irrelevant}\\\text{for}\\\text{unification}}}$$

problem: Higher-Dimension Operators:

in general: $\mathcal{L} \supset \frac{1}{\Lambda^n} \operatorname{tr}(\phi^n F^2)$

after breaking: $\mathcal{L} \supset \left(\frac{M_V}{\Lambda}\right)^n \sum_i c_i F_i^2$

can be comparable to calculable 1-loop effect!

Much better situation in

5d Super-Yang-Mills Theory

1) $N=2$ SUSY ; no corrections beyond 1 loop

2) no operators beyond mass-dimension 6
   - F^2 (gauge-kinetic)
   - AF^2 (Chern-Simons)

multiplet: A, ψ, ϕ (adjoint scalar)

$$\underbrace{\mathcal{L}_{CS} \supset \mathrm{tr}(\phi F^2) + \underbrace{\sum_i |C^i_m \phi^m| F_i^2}_{\text{1-loop terms, non-analytic in } \phi}}_{\text{parts of Quantum Exact Prepotential}}$$

parts of Quantum Exact Prepotential

Seiberg, '96

Intriligator, Morrison, Seiberg, '97

Check: resulting $\Delta\alpha_i^{-1}$ agree with "power-law running" calculation

---

full calculability! even TeV-scale unification conceivable!

Connection with string theory

most direct contact: heterotic string

$\downarrow$

10d SYM + gravity

(6 dims. to be compactified)

(one could even say that orbifold GUTs are just a "poor man's" version of this)

[another option: Dp-brane models for $p>3$

(gauge group on brane)

could give rise to effective orbifold GUTs in the last steps of compactification]

here: focuss on het. string

allowed groups in 10d: $SO(32)$ and $E_8 \times E_8$

(first found in field theory on the basis of anomaly considerations in SUGRA → Green, Schwarz, '84) historically, most attention has always been paid to $E_8 \times E_8$ (for aesthetic and model-building reasons)

- $E_8 \rightarrow SU_3 \times E_6$; $E_6 \rightarrow SO_{10} \rightarrow SU_5 \rightarrow \cdots$

  $27 = \underline{\underline{16}} + 10 + 1$

  $\uparrow$ by simplest $\mathbb{Z}_3$ orbifold

- E_8' useful for hidden-sector ~~SUSY~~.

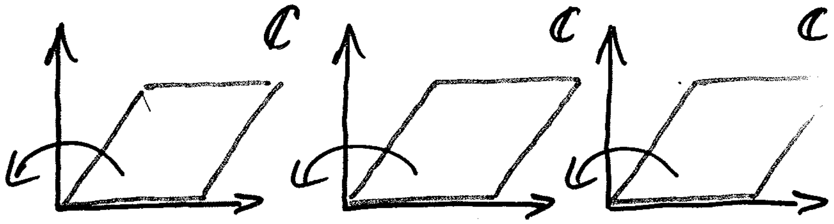
Focus on 10d SYM

- gauge field – 10d vector → 8 d.o.f. on shell
- gaugino – 10d spinor – 16 (Majoran-Weyl) → 8 d.o.f. on shell

this is all!

simplest orbifold models:

$\underline{\underline{T^6/\mathbb{Z}_n}}$, e.g.



choose some action: (e.g. $\mathbb{Z}_3$) $\times e^{2\pi i/3}$ $\times e^{2\pi i/3}$ $\times e^{2\pi i/3}$

($\underline{\text{many}}$ other options)

<u>crucial constraint:</u>

(both fundamental & phenomenological)

<u>preserve $N=1$ SUSY in orbifolding</u>

10 Lorentz:

$$SO(1,9) \supset \underbrace{SO(6)}_{\substack{\text{relevant for}\\ \text{orbifolding}}} \times SO(1,3) = \underbrace{SU(4)} \times SO(1,3)$$

$$16 = \underbrace{(4,2)}_{\substack{4 \text{ 2-comp. Weyl spinor.}\\ \text{"mixed" by } SU(4)}} + (\bar{4},2)$$

- to preserve SUSY, 1 spinor should survive orbifolding, i.e., <u>not be rotated</u>

$$\Rightarrow \text{use only } \underset{\underset{\mathbb{Z}_3}{\cup}}{SU(3)} \subset SU(4) = SO(6)$$

The action of SU(3) on gauge fields is given by:

$$\begin{array}{lcl}
\text{inert} & \text{---}\; V & \supset A_0 \cdots A_3 \\
\text{rotated as } SU_3\text{-triplet} & \left\{\begin{array}{l} \phi_1 \\ \phi_2 \\ \phi_3 \end{array}\right. & \begin{array}{l} \supset A_5 + i A_6 \\ \supset A_7 + i A_8 \\ \supset A_9 + i A_{10} \end{array}
\end{array}$$

in addition:

string theory <u>requires</u> an accompanying rotation in group space, i.e.

$$Z_n \subset SU_3$$

<u>and</u> $\quad Z_n \subset E_8 \times E_8$ (in a specific way)

---

in this languge, 5d & 6d orbifold GUTs are a (maybe phenom. imortant) intermediate step (for 1 or 2 radii large)

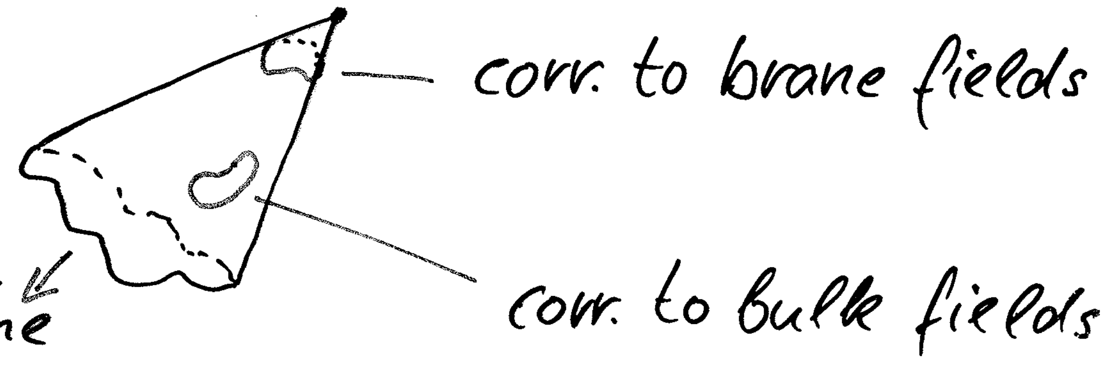
→ Kobayashi, Raby, Zhang, '04

Förste, Nilles, Vaudrevange, Wingerter, '04 important advantage of a string-theory embedding:

brane-localized fields and their couplings are predicted rather than introduced by hand

they correspond to the "twisted sector"

consider one of the orbifold fixed points:



also: orbifold fixed points (singularities) can be "blown up" ⇒ Calabi-Yau spaces

(very naive)

Conclusions

- GUTs have a very strong position as candidates for beyond-the-SM physics
- discovery of SUSY and/or proton decay will further strengthen this option
- orbifold GUTs are the (maybe) simplest explicit models
- they fit well into the string-theory framework and (probably) have to rely on string theory for better predictivity
- surprises (low-scale GUTs, warped orbifold GUTs, ...) are possible
- the ultimate challenge remains a quanitative theory of flavour