VII. Supersymmetric extension of the Standard Model (finally!)

Steps to construct a supersymmetric extension of the Standard Model:

1. Add a superpartner to all Standard Model particles such that:

$$\mathrm{Str}\, 1 = n_B - n_F = 0$$

2. In step 1, new fermions are added which can generate a gauge anomaly. This will require a slight extension of the original Standard Model.

The standard model contains a Higgs boson with quantum numbers $(1, 2, 1)$ under $SU(3) \times SU(2) \times U(1)$.

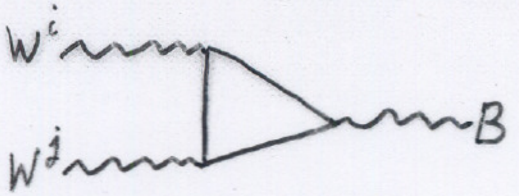
$$(\underset{\substack{\text{color}\\\text{singlet}}}{1},\ \underset{\substack{\text{weak}\\\text{doublet}}}{2},\ \underset{\substack{\text{hypercharge}\\ Y=1.}}{1})$$

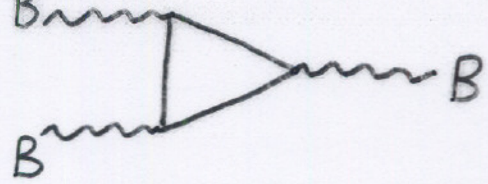
Its higgsino partner will generate an anomaly.
To avoid this problem, add a second Higgs doublet with opposite hypercharge: $(1, 2, -1)$. Now, the supersymmetric extended model contains two pairs of higgsinos

$$(1, 2, 1) \oplus (1, 2, -1)$$

which is now vector-like rather than chiral; the total contribution to the gauge anomalies cancel.

Cancellation of anomalies:





$$\mathrm{Tr}\, T_3^2 Y = 0 \qquad\qquad \mathrm{Tr}\, Y^3 = 0$$

(Remember the color factor when performing the trace!)

For example, for the Standard Model fermions,

$$\left(\mathrm{Tr}\, Y^3\right)_{SM} = 3\left(\frac{1}{27} + \frac{1}{27} - \frac{64}{27} + \frac{8}{27}\right) - 1 - 1 + 8 = 0$$

If we just had one Higgs doublet, then the (left-handed) Higgsino superpartners would be $(\tilde{H}^+, \tilde{H}^0)$ with $T_3 = \pm\frac{1}{2}$ and $Y = 1$.
Then, $\mathrm{Tr}\, Y^3 = \left(\mathrm{Tr}\, Y^3\right)_{SM} + 2$.

(Note: the only other supersymmetric fermions – the gauginos – have $Y = 0$ and so do not contribute to the anomaly or its cancellation.)

By including Higgs multiplets in pairs, with opposite hypercharge, the Higgsinos' contributions to the anomaly will cancel.

Finally, it is worth pointing out that if we require the cancellation of pure gravitation and mixed gravitational/gauge anomalies (by substituting some or all gauge bosons above with gravitons), we deduce one more new requirement: $\mathrm{Tr}\, Y = 0$. Again this implies that we must include Higgs doublets in pairs with opposite hypercharge.

3. Include supersymmetric interactions

As we shall see, the most general set of allowed interactions contain some terms that violate baryon number B or lepton number L. These can be removed if a particular discrete symmetry is imposed.

4. Break the supersymmetry by adding the most general set of soft-supersymmetry breaking terms.

The result of steps 1 – 4:

The minimal supersymmetric extension of the Standard Model (MSSM).
The MSSM is defined such that all B and L violating interactions are forbidden by a discrete symmetry.

To go beyond the MSSM, either:

(i) allow for some B or L violation at step 3

(ii) extend the matter sector of the MSSM and repeat steps 1 – 4

(iii) extend the gauge sector of the MSSM and repeat steps 1 – 4.

A summary of supersymmetric interactions

① Self-interaction of the gauge supermultiplet

origin of terms: $\frac{1}{2}\int d^2\theta \, \mathrm{tr}\, W^\alpha W_\alpha$ + h.c.

- self-coupling of gauge fields [dictated by non-abelian gauge theory]
- coupling of the gauge field to the gauginos

(also fixed by the non-abelian gauge theory)

② Interaction of the gauge and matter supermultiplet

origin of terms: $\int d^4\theta \, \bar{\Phi} e^{2gV} \Phi$

- coupling of the gauge field to spin-0 matter
- coupling of the gauge field to spin-½ matter

(these two are fixed by gauge invariance)

- coupling of the gaugino to spin-½ matter and its superpartner

$$\mathcal{L} = -i\sqrt{2}\, g \left(\bar{\lambda}^a \bar{\psi}_i T^a_{ij} A_j - A^*_i T^a_{ij} \psi_j \lambda^a \right)$$

this is a consequence of the supersymmetry

③ Self-interaction of the matter supermultiplet

- scalar potential

$$V_{scalar} = \sum_i F^*_i F_i + \frac{1}{2}\left[D^a D^a + (D')^2 \right]$$

if the gauge group contains a U(1) factor

- Yukawa interactions

$$\mathcal{L} = -\frac{1}{2}\left[\frac{d^2 W}{dA_i dA_j} \psi_i \psi_j + \left(\frac{d^2 W}{dA_i dA_j} \right)^* \bar{\psi}_i \bar{\psi}_j \right]$$

origin of these two terms: $\int d^2\theta \, W(\phi)$ + h.c.

The spectrum of the MSSM

$Q = T_3 + \frac{Y}{2}$

Gauge Supermultiplets:

BOSON FIELDS	FERMIONIC SUPER-PARTNERS	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	Q
g	$\tilde{g}$	8	0	0	0
W^a	$\tilde{W}^a$	1	3	0	$+1, 0, -1$
B	$\tilde{B}$	1	1	0	0

Matter Supermultiplets:

BOSON FIELDS	FERMION FIELDS	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	Q
leptons:					
$\tilde{L}^j = (\tilde{\nu}, \tilde{e}^-_L)$	$(\nu, e^-)_L$	1	2	-1	$0, -1$
$\tilde{E} = \tilde{e}^+_R$	e^c_L	1	1	2	$+1$
quarks:					
$\tilde{Q}^j = (\tilde{u}_L, \tilde{d}_L)$	$(u, d)_L$	3	2	$1/3$	$2/3, -1/3$
$\tilde{U} = \tilde{u}^*_R$	u^c_L	3^*	1	$-4/3$	$-2/3$
$\tilde{D} = \tilde{d}^*_R$	d^c_L	3^*	1	$2/3$	$1/3$
Higgs:					
$H_1^i = (H_1^0, H_1^-)$	$(\tilde{H}_1^0, \tilde{H}_1^-)_L$	1	2	-1	$0, -1$
$H_2^i = (H_2^+, H_2^0)$	$(\tilde{H}_2^+, \tilde{H}_2^0)_L$	1	2	$+1$	$+1, 0$

The matter multiplets of the MSSM originate from chiral superfields:

$$\hat{L},\ \hat{E},\ \hat{Q},\ \hat{U},\ \hat{D},\ \hat{H}_1 \text{ and } \hat{H}_2$$

where hats will be used to indicate superfields.

For example,

$$\hat{E} = A_E + \sqrt{2}\,\theta\psi_E - \theta\theta F_E$$

with $A_E = \tilde{e}_R^+$ and $\begin{pmatrix}\psi_E \\ 0\end{pmatrix} = P_L e^+ = e_L^c$.

Supersymmetric interactions

The SUSY interactions are known once we specify the superpotential. The most general gauge invariant superpotential has the following form:

WARNING: two sign conventions exist in the literature

$$W = W_R + W_{NR}$$

where

$$W_R = \epsilon_{ij}\left[h_\tau \hat{H}_1^i \hat{L}^j \hat{E} + h_b \hat{H}_1^i \hat{Q}^j \hat{D} + h_t \hat{H}_2^j \hat{Q}^i \hat{U} - \mu \hat{H}_1^i \hat{H}_2^j\right]$$

i and j are weak $SU(2)$ indices. Here $\epsilon_{12} = -\epsilon_{21} = +1$.

μ = supersymmetric Higgs mass parameter.

Generation labels have been suppressed; h_τ, h_b and h_t are actually 3×3 matrices. In a one-generation model,

$$h_\tau = \frac{\sqrt{2}\,m_\tau}{v_1}, \quad h_b = \frac{\sqrt{2}\,m_b}{v_1}, \quad h_t = \frac{\sqrt{2}\,m_t}{v_2}$$

where $\langle H_1^0\rangle = \frac{v_1}{\sqrt{2}}$, $\langle H_2^0\rangle = \frac{v_2}{\sqrt{2}}$

$$W_{NR} = \varepsilon_{ij}\left[\lambda_L \hat{L}^i \hat{L}^j \hat{E} + \lambda_L' \hat{L}^i \hat{Q}^j \hat{D} - \mu' \hat{L}^i \hat{H}_2^j\right] + \lambda_B \hat{U}\hat{D}\hat{D}$$

where generation labels are again suppressed. Note that λ_L must be antisymmetric upon interchange of $\hat{L}^i$ and $\hat{L}^j$.

One quickly observes that the terms in W_{NR} violate either baryon number (B) or lepton number (L):

$$\hat{L}\hat{L}\hat{E},\ \hat{L}\hat{Q}\hat{D},\ \hat{L}\hat{H} \qquad \Delta L \neq 0$$

$$\hat{U}\hat{D}\hat{D} \qquad \Delta B \neq 0$$

$$\boxed{\text{In the MSSM, set } W_{NR} = 0.}$$

In this regard, the MSSM is not as elegant as the SM. Recall that if one imposes $SU(3)\times SU(2)\times U(1)$ on all possible SM interactions, one finds that all terms of dimension ≤ 4 preserve B and L. Not so in supersymmetry!

How does one impose $W_{NR}=0$?

Here, we introduce a discrete symmetry to do our dirty work. There are two equivalent descriptions:

1. Matter parity

The MSSM does not distinguish Higgs and Quark/Lepton Superfields. Define matter parity such that all Quark/Lepton superfields change sign but the Higgs superfields do not.

2. R-parity

In superspace, a chiral superfield is

$$\hat{\Phi} = A(x) + \sqrt{2}\,\theta\Psi(x) + \theta\theta F(x)$$

Under a continuous $U(1)_R$ symmetry, $\theta \to e^{i\alpha}\theta$ and $\hat{\Phi} \to e^{in\alpha}\hat{\Phi}$. The superfield has $R=n$. This means that the component fields have $R(A)=n$, $R(\Psi)=n-1$, $R(F)=n-2$. The superpotential W must have $R(W)=2$ in order that the theory conserve $U(1)_R$, since $[W]_F + h.c.$ appears in the supersymmetric Lagrangian.

Thus, to set $W_{NR}=0$, choose

$$R=1 \quad \text{for } \hat{H}_1, \hat{H}_2$$
$$R=1/2 \quad \text{for } \hat{L}, \hat{E}, \hat{Q}, \hat{U}, \hat{D}$$

In fact, $U(1)_R$ is too restrictive. Consider the gauge super-multiplet, $\hat{V}$. Since $\hat{V}$ is real, we must have $R(\hat{V})=0$ which means that $R(V_\mu)=0$, $R(\lambda)=1$. That is, $U(1)_R$ forbids Majorana masses for the gaugino.

In the MSSM, we will add Majorana mass terms for the gaugino via the soft-SUSY-breaking terms. Since these terms are quadratic in λ, we see that $U(1)_R$ is broken to a discrete Z_2 symmetry called R-parity. It is easy to check that:

$$R = (-1)^{3(B-L)+2S}$$

for particles of spin S. By imposing R-parity invariance, $W_{NR}=0$.

At this point, our model consists of a supersymmetric gauge field theory based on $SU(3)\times SU(2)\times U(1)$.

But, it is not yet realistic for two reasons:

- supersymmetry is an exact symmetry
- $SU(2)\times U(1)$ is unbroken

To illustrate the second point, examine the scalar potential:

$$V_{scalar} = \sum_i F_i^* F_i + \tfrac{1}{2}\left[D^a D^a + (D')^2\right]$$

The result:

$$V_{Higgs} = |\mu|^2\left[|H_1|^2 + |H_2|^2\right] \longleftarrow \text{from the F-terms}$$

$$+\tfrac{1}{8}(g^2+g'^2)\left[|H_1|^2 - |H_2|^2\right]^2 + \tfrac{1}{2} g^2 |H_1^* H_2|^2$$

(exercise: derive this!)

from the D-terms

Clearly, $V_{Higgs} \geq 0$ and $H_1 = H_2 = 0$ minimizes the Higgs potential (giving $\langle V_{Higgs}\rangle = 0$ as expected for a supersymmetric vacuum). Hence, no $SU(2)\times U(1)$ breaking.

We shall see that the addition of soft-SUSY-breaking terms will also permit the breaking of the electroweak symmetry.

Soft-SUSY-breaking terms

Assume R-parity invariance

notation: $\tan\beta \equiv \frac{v_2}{v_1}$ $\qquad v^2 \equiv v_1^2 + v_2^2 = (246\text{ GeV})^2$

$$m_W = \tfrac{1}{2} g v$$

$$\begin{aligned}
V_{soft} = {} & m_{H_1}^2 |H_1|^2 + m_{H_2}^2 |H_2|^2 - m_{12}^2 (\epsilon_{ij} H_1^i H_2^j + h.c.) \\
& + M_Q^2 (\tilde{t}_L^* \tilde{t}_L + \tilde{b}_L^* \tilde{b}_L) + M_U^2 \tilde{t}_R^* \tilde{t}_R + M_D^2 \tilde{b}_R^* \tilde{b}_R \\
& + M_L^2 (\tilde{\nu}^* \tilde{\nu} + \tilde{\tau}_L^* \tilde{\tau}_L) + M_E^2 \tilde{\tau}_R^* \tilde{\tau}_R \\
& + \frac{g}{\sqrt{2}\, m_W} \epsilon_{ij} \Big[\frac{m_\tau A_\tau}{\cos\beta} H_1^i \tilde{\ell}_L^j \tilde{\tau}_R^* + \frac{m_b A_b}{\cos\beta} H_1^i \tilde{q}_L^j \tilde{b}_R^* \\
& \qquad + \frac{m_t A_t}{\sin\beta} H_2^i \tilde{q}^i \tilde{t}_R^* \Big] \\
& + \tfrac{1}{2} \left[M_3 \tilde{g}\tilde{g} + M_2 \widetilde{W}^a \widetilde{W}^a + M_1 \widetilde{B}\widetilde{B} + h.c. \right]
\end{aligned}$$

where $\tilde{\ell}_L \equiv \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$ and $\tilde{q}_L \equiv \begin{pmatrix} \tilde{t}_L \\ \tilde{b}_L \end{pmatrix}$. I have written this for the case of one generation. For three generations, $M_Q^2, M_U^2, M_D^2, M_L^2, M_E^2$ are 3×3 matrices.

<u>color code</u>

—— scalar soft squared-mass terms

—— B-term $\quad m_{12}^2 \equiv b = B\mu$

—— A-terms

—— gaugino Majorana mass terms

Mass Eigenstates of the MSSM

By examing the contributions of $\mathcal{L}_{SUSY}$ and V_{soft} to the quadratic terms, we discover that any set of particles of a given spin, B and L and $SU(3)_c \times U(1)_{EM}$ can mix. Thus, we must diagonalize mass matrices to obtain the mass eigenstates and the corresponding mass eigenvalues.

(i) scalar-quark sector

In principle, I must diagonalize 6×6 matrices corresponding to the basis $(\tilde{q}_{iL}; \tilde{q}_{iR})$ $i=1,2,3 \leftarrow$ generation labels.

To make these transparencies readable, I shall only display the one-generation case.

$$M^2_{\tilde{t}} = \begin{bmatrix} M_Q^2 + m_t^2 + m_Z^2 \cos 2\beta\left(\tfrac{1}{2} - e_u s_W^2\right) & m_t(A_t - \mu\cot\beta) \\ m_t(A_t - \mu\cot\beta) & M_U^2 + m_t^2 + m_Z^2\cos 2\beta\, e_u s_W^2 \end{bmatrix}$$

$$M^2_{\tilde{b}} = \begin{bmatrix} M_Q^2 + m_b^2 - m_Z^2 \cos 2\beta\left(\tfrac{1}{2} + e_d s_W^2\right) & m_b(A_b - \mu\tan\beta) \\ m_b(A_b - \mu\tan\beta) & M_D^2 + m_b^2 + m_Z^2\cos 2\beta\, e_d s_W^2 \end{bmatrix}$$

where $e_u = 2/3$, $e_d = -1/3$, $s_W^2 \equiv \sin^2\theta_W$ and $\tan\beta \equiv \langle H_2^0\rangle / \langle H_1^0\rangle$.

(ii) scalar-lepton sector

$$M_{\tilde{\nu}}^2 = M_L^2 + \tfrac{1}{2} m_Z^2 \cos 2\beta$$

$$M_{\tilde{\tau}}^2 = \begin{bmatrix} M_L^2 + m_\tau^2 - m_Z^2 \cos 2\beta \left(\tfrac{1}{2} - s_W^2\right) & m_\tau (A_\tau - \mu \tan\beta) \\ m_\tau (A_\tau - \mu \tan\beta) & M_E^2 + m_\tau^2 - m_Z^2 \cos 2\beta \, s_W^2 \end{bmatrix}$$

These results suggest that $\tilde{f}_L - \tilde{f}_R$ mixing is unimportant in the first two generations. In the third generation, top-squark ("stop") mixing is likely to be the most significant, while bottom-squark ("sbottom") mixing and tau-slepton ("stau") mixing may also be relevant if $\tan\beta \gg 1$.

<u>Remark</u>: expectations for $\tan\beta$

The Higgs-fermion Yukawa coupling are:

$$h_b = \frac{\sqrt{2}\, m_b}{v_1} = \frac{\sqrt{2}\, m_b}{v \cos\beta}, \qquad h_t = \frac{\sqrt{2}\, m_t}{v_2} = \frac{\sqrt{2}\, m_t}{v \sin\beta}$$

Perturbativity of couplings suggest that h_b and h_t should not be too large. This would imply (crudely) that:

$$1 \lesssim \tan\beta \lesssim \frac{m_t}{m_b}$$

(More convincing arguments exist, but will not be given here.)

(iii) charged SUSY fermions ($\tilde{\chi}_i^\pm$, $i=1,2$) <u>CHARGINOS</u>

Charginos are mixtures of gauginos and higgsinos. They arise from the SUSY-interaction

$$-\mathcal{L} = i\sqrt{2}\, g^a \left(\bar{\lambda}^a \bar{\psi}_i T^a_{ij} A_j - A_i^* T^a_{ij} \psi_j \lambda^a \right)$$

when the Higgs bosons acquire their vacuum expectation values. Two other sources for mass terms are:

(i) $-\mathcal{L} = \frac{1}{2} \left\langle \frac{d^2 W}{dA_i\, dA_j} \right\rangle \psi_i \psi_j$ due to the μ-term

(ii) soft-SUSY-breaking Majorana mass term for the gaugino.

Writing

$$-\mathcal{L}_{mass} = \frac{1}{2} (\psi^+ \ \psi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + h.c.$$

with:

$$\psi^+ = (-i\lambda^+ \ \psi^+_{H_2})$$
$$\psi^- = (-i\lambda^- \ \psi^-_{H_1})$$

we find:

$$X = \begin{pmatrix} M_2 & g v_2 \\ g v_1 & \mu \end{pmatrix} = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}$$

Mass diagonalization works as follows. Let:

$$\chi_i^+ = V_{ij} \psi_j^+$$
$$\chi_i^- = U_{ij} \psi_j^-$$

V, U unitary

Then,

$$-\mathcal{L}_{mass} = \chi_i^- (M_D)_{ij} \chi_j^+ + h.c.$$

where M_D is a diagonal matrix with positive entries and:

$$U^T X V^{-1} = M_D \qquad X = \text{complex matrix}$$

To determine U and V, note that:

$$M_D^\dagger M_D = V X^\dagger X V^{-1}$$
$$M_D M_D^\dagger = U^* X X^\dagger U^{*-1}$$

so all we have to do is to diagonalize $X^\dagger X$ and $XX^\dagger$ and adjust the relative phases of U^* and V such that the entries of M_D are positive.

The chargino masses are the positive square roots of the eigenvalues of $X^\dagger X$ (or $XX^\dagger$):

$$m^2_{\chi_i^\pm} = \frac{1}{2}\Big\{|\mu|^2+|M_2|^2+2m_W^2 \mp \Big[(|\mu|^2+|M_2|^2+2m_W^2)^2 - 4|\mu|^2|M_2|^2 - 4m_W^4 \sin 2\beta + 8m_W^2 \sin 2\beta \, Re(\mu M_2)\Big]^{1/2}$$

__eigenstates__: $\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$ with $m_{\tilde{\chi}_1} < m_{\tilde{\chi}_2}$.

(iv) neutral SUSY fermions ($\tilde{\chi}_i^0$, $i=1,\ldots,4$) <u>NEUTRALINOS</u>

Neutralinos are mixtures of neutral gauginos and higgsinos. Following the previous analysis, we obtain:

$$-\mathcal{L}_{mass} = \frac{1}{2}\psi_i^0 Y_{ij} \psi_j^0 + h.c.$$

with

$$\psi^0 = (-i\lambda' \;\; -i\lambda^3 \;\; \psi_{H_1}^0 \;\; \psi_{H_2}^0)$$

λ' is the hypercharge gaugino ("bino")
λ^3 is the W^3-gaugino ("wino")

and

$$Y = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

Mass diagonalization:

$s_W = \sin\theta_W$
$c_W = \cos\theta_W$
$s_\beta = \sin\beta$
$c_\beta = \cos\beta$

$$\chi_i^0 = N_{ij}\psi_j^0$$

$$-\mathcal{L}_{mass} = \chi_i^0 (M_D)_{ij} \chi_j^0 + h.c.$$

Then:

$$N^T Y N^{-1} = M_D$$

N = unitary
Y = complex symmetric

The phases of N can be adjusted such that all entries of the diagonal matrix M_D are positive.

Limiting cases for the neutralino mass matrix:

(i) $M_1 = M_2 = \mu = 0$

$$\tilde{\chi}_1^0 = \tilde{\gamma} \qquad m = 0$$

$$\tilde{\chi}_2^0 = \tilde{H}_1^0 \sin\beta + H_2^0 \cos\beta \qquad m = 0$$

$$\tilde{\chi}_3^0 = \sqrt{\tfrac{1}{2}}\left[\tilde{Z} + \tilde{H}_1^0 \cos\beta - \tilde{H}_2^0 \sin\beta\right] \qquad m = m_Z$$

$$\tilde{\chi}_4^0 = \sqrt{\tfrac{1}{2}}\left[-\tilde{Z} + \tilde{H}_1^0 \cos\beta - \tilde{H}_2^0 \sin\beta\right] \qquad m = m_Z$$

where

$$\tilde{\gamma} = c_W \tilde{B} + s_W \tilde{W}^3 \qquad \text{"photino"}$$

$$\tilde{Z} = -s_W \tilde{B} + c_W \tilde{W}^3 \qquad \text{"zino"}$$

Clearly, nature is not very close to this limit.

(ii) $M_1, M_2, |\mu| >> m_Z$

$$\tilde{\chi}_i^0 = \left\{ \tilde{B}, \tilde{W}_3, \tfrac{1}{\sqrt{2}}(\tilde{H}_1^0 - \tilde{H}_2^0), \tfrac{1}{\sqrt{2}}(\tilde{H}_1^0 + \tilde{H}_2^0) \right\}$$

with masses $|M_1|$, $|M_2|$, $|\mu|$ and $|\mu|$ respectively

Standard notation: $m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_3^0} < m_{\tilde{\chi}_4^0}$.

(v) The MSSM Higgs sector

Including the soft-SUSY-breaking terms, the Higgs potential is given by:

SUSY-breaking (B-term)

$$V_{Higgs} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - \left(m_{12}^2 \, \varepsilon_{ij} H_1^i H_2^j + h.c. \right)$$
$$+ \frac{1}{8}(g^2+g'^2)\left(|H_1|^2 - |H_2|^2\right)^2 + \frac{1}{2} g^2 |H_1^* H_2|^2$$

D-term contributions

where:

$$m_1^2 \equiv |\mu|^2 + m_{H_1}^2$$
$$m_2^2 \equiv |\mu|^2 + m_{H_2}^2$$

SUSY-breaking

F-term contributions

Let us search for a minimum where

$$\langle H_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad \langle H_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

where the vacuum expectation values (v.e.v.'s) appear in the charge-neutral components.

$$\langle V_{Higgs} \rangle = \frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_2^2 v_2^2 - m_{12}^2 v_1 v_2 + \frac{1}{32}(g^2+g'^2)(v_1^2 - v_2^2)^2$$

Minimize this:

$$\frac{d}{dv_1} \langle V_{Higgs} \rangle = 0 \qquad m_1^2 = m_{12}^2 \frac{v_2}{v_1} + \frac{1}{8}(g^2+g'^2)(v_2^2 - v_1^2)$$

$$\frac{d}{dv_2} \langle V_{Higgs} \rangle = 0 \qquad m_2^2 = m_{12}^2 \frac{v_1}{v_2} - \frac{1}{8}(g^2+g'^2)(v_2^2 - v_1^2)$$

We can write:

$$\langle V_{Higgs}\rangle = \frac{1}{32}(g^2+g'^2)(v_1^2-v_2^2)^2 + \frac{1}{2}(v_1 \ v_2)\begin{pmatrix} m_1^2 & -m_{12}^2 \\ -m_{12}^2 & m_2^2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

Then, for the vev's to correspond to a potential minimum,

$$\det\begin{pmatrix} m_1^2 & -m_{12}^2 \\ -m_{12}^2 & m_2^2 \end{pmatrix} < 0$$

or $\boxed{m_1^2 m_2^2 < m_{12}^4}$.

A second condition arises, since for $v_1 = v_2$, $\langle V_{Higgs}\rangle = \frac{v^2}{2}(m_1^2+m_2^2-2m_{12}^2)$, which would be unbounded from below unless:

$$\boxed{m_1^2+m_2^2 \geq 2m_{12}^2}$$

<u>Technicality:</u> In obtaining the above two conditions, I assumed that m_{12}^2, v_1 and v_2 are real. However, I can always absorb the phase of m_{12}^2 into the definition of one of the Higgs fields and without loss of generality take $v_1 > 0$ (by an appropriate hypercharge rotation). Then, the minimum conditions imply that v_2 is real. Finally, I can choose $v_2 > 0$ by absorbing a potential sign into the definition of H_2.

To compute the Higgs mass spectrum, we write:

$$H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 + h_1^0 + i a_1^0 \\ h_1^- \end{pmatrix} \qquad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} h_2^+ \\ v_2 + h_2^0 + i a_2^0 \end{pmatrix} .$$

From the potential, we compute the quadratic terms in the scalar fields to derive squared-mass matrices. Diagonalizing these matrices yield the physical states:

$$(h_1^0, h_2^0) \longrightarrow h^0, H^0$$

CP-even Higgs bosons with $m_h < m_H$

$$(a_1^0, a_2^0) \longrightarrow A^0, G^0$$

CP-odd Higgs boson

Goldstone boson that gives mass to the Z^0

$$(h_1^\pm, h_2^\pm) \longrightarrow H^\pm, G^\pm$$

charged Higgs bosons

Goldstone bosons that give mass to the $W^\pm$

Physical Higgs degrees of freedom = 5

$$h^0, H^0, A^0, H^\pm$$

+ 3 Goldstone bosons = 8 degrees of freedom of the two complex Higgs doublets

CP-even Higgs mass matrix:

$$\mathcal{M}_{ij}^2 = \frac{\partial^2 \langle V_{\rm Higgs}\rangle}{\partial v_i \partial v_j} = \frac{1}{v_1^2+v_2^2}\begin{pmatrix} m_A^2 v_2^2 + m_Z^2 v_1^2 & -(m_A^2+m_Z^2)v_1v_2 \\ -(m_A^2+m_Z^2)v_1v_2 & m_A^2 v_1^2 + m_Z^2 v_2^2 \end{pmatrix}$$

where:

$$m_A^2 \equiv \frac{m_{12}^2}{v_1v_2}(v_1^2+v_2^2)$$

m_A is the mass of A (as we will soon see)

$$m_Z^2 = \tfrac{1}{4}(g^2+g'^2)(v_1^2+v_2^2)$$

eigenvalues

$$m_{H,h}^2 = \tfrac{1}{2}\left[m_A^2+m_Z^2 \pm \sqrt{(m_A^2+m_Z^2)^2 - 4m_A^2m_Z^2\cos^2 2\beta}\right]$$

where

$$\boxed{\tan\beta \equiv \frac{v_2}{v_1}}$$

Note: $v_1^2+v_2^2 = (246\ \text{GeV})^2$

eigenstates

$$h = -h_1^0\sin\alpha + h_2^0\cos\alpha$$
$$H = h_1^0\cos\alpha + h_2^0\sin\alpha$$

where the CP-even Higgs mixing angle is given by:

$$\cos 2\alpha = -\cos 2\beta\left(\frac{m_A^2-m_Z^2}{m_H^2-m_h^2}\right)$$

$$\sin 2\alpha = -\sin 2\beta\left(\frac{m_H^2+m_h^2}{m_H^2-m_h^2}\right)$$

In our convention, $v_1>0$, $v_2>0$ implies that $\sin 2\beta > 0$.

Hence,

$$0<\beta<\frac{\pi}{2}$$

$$-\frac{\pi}{2}<\alpha<0$$

CP-odd Higgs mass matrix

$$M^2_{odd} = m_{12}^2 \begin{pmatrix} v_2/v_1 & 1 \\ 1 & v_1/v_2 \end{pmatrix}$$

eigenvalues: $m_G = 0$

$$m_A^2 = \operatorname{Tr} M^2_{odd} = \frac{m_{12}^2}{v_1 v_2}(v_1^2 + v_2^2)$$

Charged Higgs mass matrix

$$M^2_{charged} = \left(\frac{m_{12}^2}{v_1 v_2} + \frac{1}{4}g^2\right) \begin{pmatrix} v_2^2 & v_1 v_2 \\ v_1 v_2 & v_1^2 \end{pmatrix}$$

eigenvalues: $m_{G^\pm} = 0$

$$m^2_{H^\pm} = \operatorname{Tr} M^2_{charged} = \left(\frac{m_{12}^2}{v_1 v_2} + \frac{1}{4}g^2\right)(v_1^2 + v_2^2)$$

note: $m^2_{H^\pm} = m_W^2 + m_A^2$ since $m_W^2 = \frac{1}{4}g^2(v_1^2 + v_2^2)$.

A possible phenomenological disaster ?

The mass formula for m_h implies that:

$$m_h^2 \leq m_Z^2 \cos^2 2\beta \leq m_Z^2$$

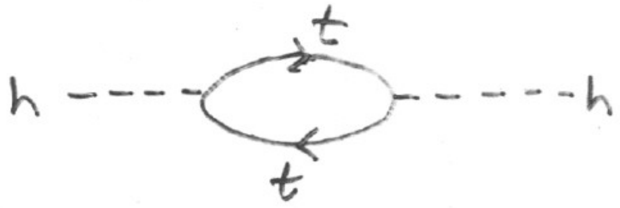
But the LEP Higgs search found no significant evidence for a Higgs boson. Their MSSM Higgs analysis concluded that:

$$m_h > 91 \text{ GeV} \quad (\text{for } \tan\beta > 10)$$

$$m_h > 114.6 \text{ GeV} \quad (\text{for } 2 \lesssim \tan\beta \lesssim 8)$$

The radiatively-corrected Higgs mass

The formula for m_h previously given is a tree-level formula. At one-loop, there are important corrections. The dominant corrections are due to:



In the SUSY limit, these two diagrams above cancel. But, due to supersymmetry breaking, there is an incomplete cancellation. A quick and dirty computation will expose the largest effect.

Recall the formula for the effective potential. Setting $\operatorname{Str} M^2(\phi) = 0$,

$$V_{eff}(\phi) = V_{tree}(\phi) + \frac{1}{64\pi^2} \operatorname{Str}\left\{ M_i^4(\phi)\left[\ln \frac{M_i^2(\phi)}{\Lambda^2} - \frac{1}{2}\right]\right\}$$

Examine the contribution of t and $\tilde{t}_L, \tilde{t}_R$ to Str. For simplicity, let us ignore stop mixing and put $M_Q = M_U \equiv M_S$. Then,

$$m_{\tilde{t}}^2 = m_{\tilde{t}_L}^2 = m_{\tilde{t}_R}^2 \simeq M_S^2 + \tfrac{1}{2} h_t^2 v_2^2 \qquad\qquad m_t = \frac{1}{\sqrt{2}} h_t v_2 .$$

Then, the one-loop contribution to V_{eff} is:

$$V^{(1)}(v_1, v_2) = \frac{\overset{\text{color}}{3}}{64\pi^2}\cdot \underbrace{2}_{\text{complex}} \cdot \underbrace{2}_{\tilde{t}_L, \tilde{t}_R \text{ or } \underbrace{2}_{\text{spin-}1/2 \text{ degrees of freedom}} \times t} \left[\left(M_S^2 + \tfrac{1}{2} h_t^2 v_2^2\right)^2 \left[\ln\left(\frac{M_S^2 + \frac{1}{2} h_t^2 v_2^2}{\Lambda^2}\right) - \frac{1}{2}\right] - \frac{1}{4} h_t^4 v_2^4 \left[\ln\left(\frac{\frac{1}{2} h_t^2 v_2^2}{\Lambda^2}\right) - \frac{1}{2}\right]\right]$$

Then,

$$V = \frac{1}{32}(g^2+g'^2)(v_1^2-v_2^2)^2 + \frac{1}{2}(v_1\ v_2)\begin{pmatrix} m_1^2 & -m_{12}^2 \\ -m_{12}^2 & m_2^2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + V^{(1)}(v_1, v_2)$$

$$\frac{\partial V}{\partial v_2} = 0 \quad \Rightarrow \quad m_2^2 = m_{12}^2\frac{v_1}{v_2} - \frac{1}{8}(g^2+g'^2)(v_2^2-v_1^2) - \frac{1}{v_2}\frac{\partial V^{(1)}}{\partial v_2}$$

Using the minimum condition to eliminate m_2^2,

$$\frac{\partial^2 V}{\partial v_2^2} = m_{12}^2\frac{v_1}{v_2} + \frac{1}{4}(g^2+g'^2)v_2^2 - \frac{1}{v_2}\frac{\partial V^{(1)}}{\partial v_2} + \frac{\partial^2 V^{(1)}}{\partial v_2^2}$$

Inserting our expression for $V^{(1)}$,

$$\frac{\partial^2 V^{(1)}}{\partial v_2^2} - \frac{1}{v_2}\frac{\partial V^{(1)}}{\partial v_2} = \frac{3}{8\pi^2} h_t^4 v_2^2 \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

Note that Λ has cancelled out!

The effect of the t and $\tilde{t}_L, \tilde{t}_R$ loops is to modify M_{22}^2 of the CP-even squared mass matrix:

$$\delta M_{22}^2 = \frac{3g^2 m_t^4}{8\pi^2 m_W^2 \sin^2\beta} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

Diagonalizing the CP-even Higgs mass matrix in the limit of $m_A \gg m_Z$ yields the new upper limit for m_h:

$$\boxed{m_h^2 \le m_Z^2\cos^2 2\beta + \frac{3g^2 m_t^4}{8\pi^2 m_W^2} \ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)}$$

One can repeat the previous computation to include the effects of stop mixing. Recall the stop mass matrix:

$$M^2_{stop} = \begin{pmatrix} M_Q^2 + m_t^2 + D_L & m_t X_t \\ m_t X_t & M_U^2 + m_t^2 + D_R \end{pmatrix}$$

where D_L and D_R are the $O(m_Z^2)$ D-term contributions and

$$X_t \equiv A_t - \mu \cot\beta$$

The new upper bound, which is saturated when $\tan\beta \gg 1$, is given by:

$$m_h^2 \le m_Z^2 + \frac{3g^2 m_t^4}{8\pi^2 m_W^2}\left[\ln\left(\frac{M_S^2}{m_t^2}\right) + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right]$$

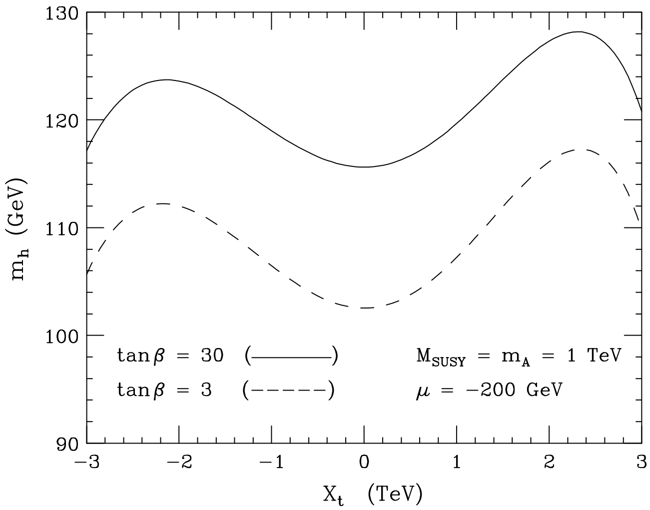
where $M_S^2 \equiv \frac{1}{2}(m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})$.

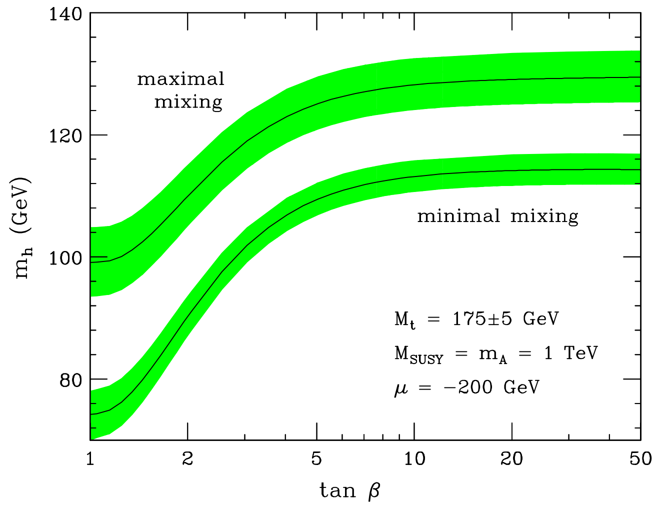
The upper bound is maximal when $X_t = \sqrt{6}\, M_S$ (which is called the "maximal mixing" point).

This result was first obtained in 1991. Since then, computations have become much more sophisticated. Complete one-loop results and nearly complete two-loop results (plus leading log resummations) lead to the conclusion that:

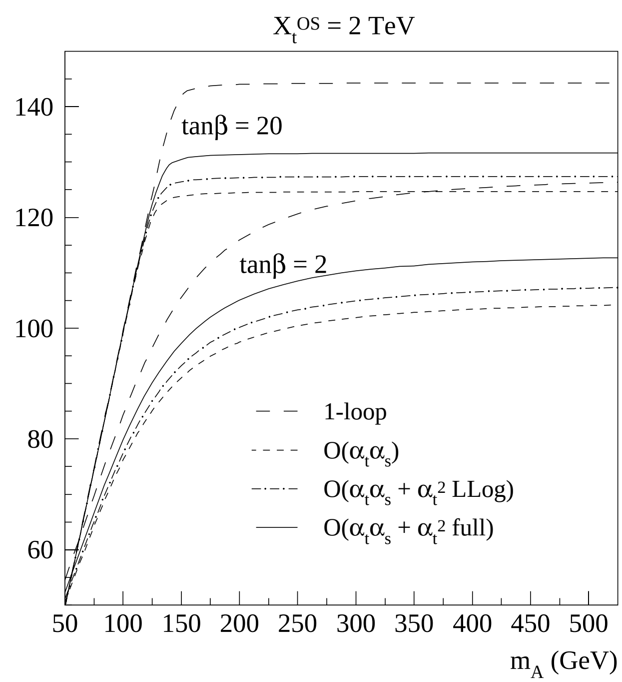
$$\boxed{m_h \lesssim 135 \text{ GeV}} \qquad \text{assuming } M_S \lesssim 2 \text{ TeV}$$

For further details on MSSM Higgs theory and phenomenology and references, see M. Carena and H.E. Haber, Prog. Nucl. Part. Phys.

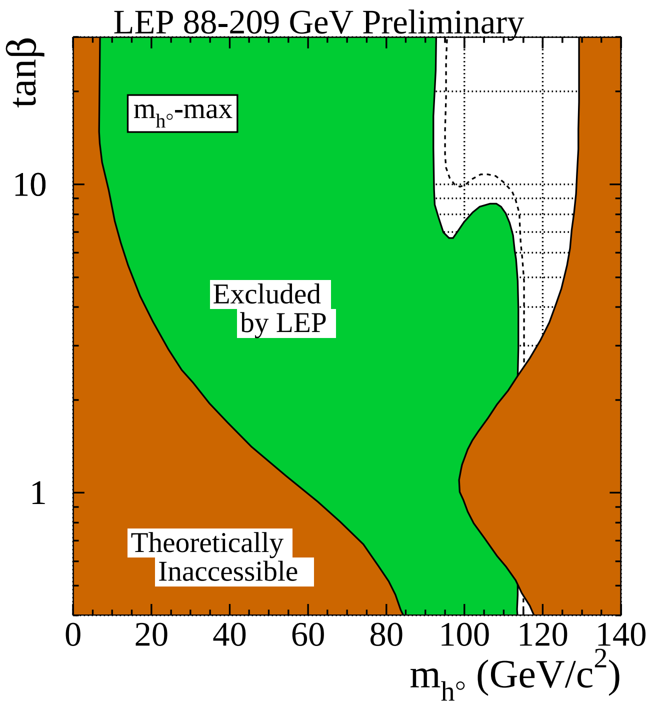


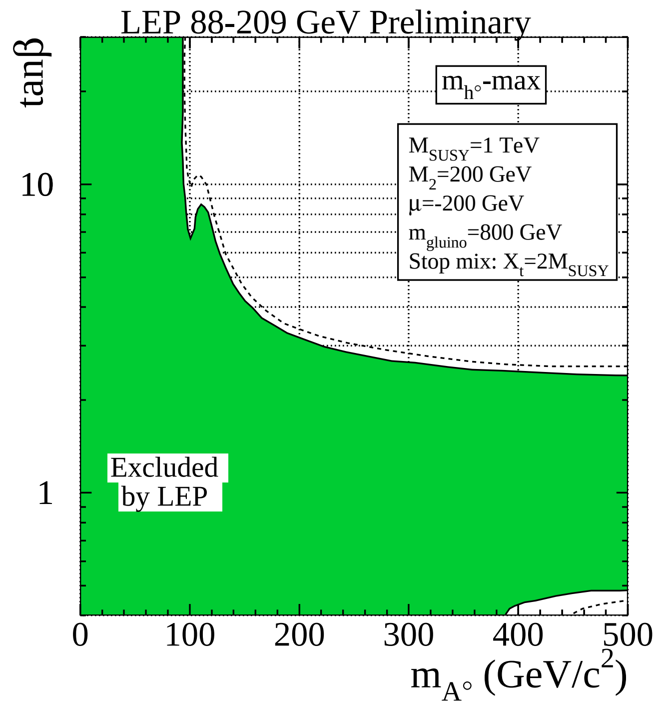


Maximal mixing corresponds to choosing the MSSM Higgs parameters in such a way that m_h is maximized (for a fixed $\tan\beta$). This occurs for $X_t/M_S \sim 2$. As $\tan\beta$ varies, m_h reaches is maximal value, $(m_h)_{\rm max} \simeq 130$ GeV, for $\tan\beta \gg 1$ and $m_A \gg m_Z$.



recent calculation including new 2-loop contributions (Brignole et al.)





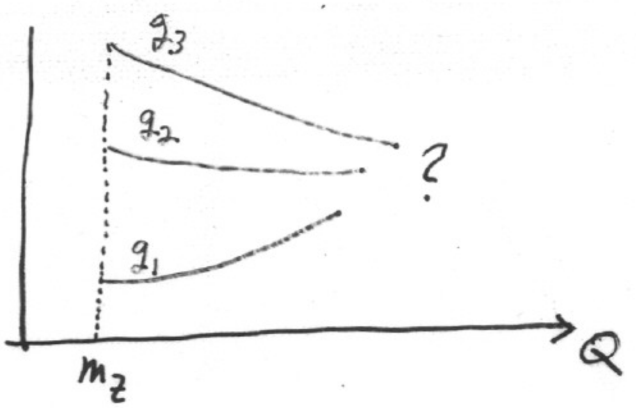
Present status of the LEP Higgs Search [95% CL limits]

- Standard Model Higgs boson: $m_h > 114.6$ GeV
- Charged Higgs boson: $m_{H^\pm} > 78.6$ GeV
- MSSM Higgs: $m_h > 91.0$ GeV; $m_A > 91.9$ GeV

At large $\tan\beta$, supersymmetric radiative corrections can also have a significant impact on the Higgs branching ratios. Example: the dominant decay mode $h \to b\bar{b}$ is suppressed in some regions of MSSM Higgs parameter space.

UNIFICATION OF COUPLINGS

Grand unification predicts the unification of gauge couplings. Since the running of the coupling constants below the grand unified scale is dictated by the Standard Model particle spectrum, one can test the hypothesis of coupling constant unification.



A subtlety: normalization of the $U(1)_Y$ coupling

In $SU(2)_L \times U(1)_Y$ theory, the overall normalization of the $U(1)_Y$ coupling was a matter of convention. If g' is unified in a non-abelian group, then the relative normalization of g and g' is fixed. To work out the proper normalization, consider the covariant derivative:

$$D_\mu = \partial_\mu + i g_a T^a A^a_\mu$$

At scales above unification, we have complete unification, so $g_a = g_U$ and $\mathrm{Tr}(T^a T^b) = T_R \delta^{ab}$.

Below unification,

$$g_a T^a A^a_\mu = g T^3 W^3_\mu + g' \frac{Y}{2} B + \ldots$$

Coupling constant evolution

$$\frac{dg_i^2}{dt} = \frac{b_i g_i^4}{16\pi^2}$$

Solution:

$$\frac{1}{g_3^2(m_Z)} = \frac{1}{g_U^2} - \frac{b_3}{16\pi} \ln\left(\frac{m_Z^2}{M_X^2}\right)$$

$$\frac{1}{g_2^2(m_Z)} = \frac{1}{g_U^2} - \frac{b_2}{16\pi} \ln\left(\frac{m_Z^2}{M_X^2}\right)$$

$$\frac{1}{g_1^2(m_Z)} = \frac{1}{g_U^2} - \frac{b_1}{16\pi} \ln\left(\frac{m_Z^2}{M_X^2}\right)$$

Define:

$$\sin^2\theta_W(m_Z) = \frac{g'^2(m_Z)}{g^2(m_Z)+g'^2(m_Z)} = \frac{\frac{3}{5}g_1^2(m_Z)}{\frac{3}{5}g_1^2(m_Z)+g_2^2(m_Z)}$$

$$= \frac{3}{8} - \frac{5}{32\pi}\alpha(m_Z)(b_1-b_2)\ln\left(\frac{M_X^2}{m_Z^2}\right)$$

$$\ln\frac{M_X^2}{m_Z^2} = \frac{32\pi}{5b_1+3b_2-8b_3}\left(\frac{3}{8\alpha(m_Z)} - \frac{1}{\alpha_s(m_Z)}\right)$$

Thus, at the unification point,

$$g_U(W^3T^3 + BT^0) = g\,W^3T^3 + g'B\frac{Y}{2}$$

where T^0 is the properly normalized hypercharge generator when embedded in the grand unified group.

Thus, $g_U = g_3 = g_2 = g_1 = g$ at the unification point

$$T^0 = \frac{g'}{g_1}\frac{Y}{2}$$

Using

$$\mathrm{Tr}(T^3)^2 = \mathrm{Tr}(T^0)^2$$

$$= \frac{1}{4}\frac{g'^2}{g_1^2}\,\mathrm{Tr}\,Y^2$$

we conclude that

$$\boxed{g_1^2 = g'^2\,\frac{\mathrm{Tr}\,Y^2}{4\,\mathrm{Tr}(T^3)^2}} \implies \boxed{g_1^2 = \frac{5}{3}g'^2}$$

two component fields	T_3	Y	$\mathrm{Tr}\,T_3^2$	$\mathrm{Tr}\,Y^2$
ψ_{Q_1}	$1/2$	$1/3$	$3(1/4)$	$3(1/9)$
ψ_{Q_2}	$-1/2$	$1/3$	$3(1/4)$	$3(1/9)$
ψ_U	0	$-4/3$	$3(0)$	$3(16/9)$
ψ_D	0	$+2/3$	$3(0)$	$3(4/9)$
ψ_{L_1}	$1/2$	-1	$1/4$	1
ψ_{L_2}	$-1/2$	-1	$1/4$	1
ψ_E	0	$+2$	0	4
			2	$40/3$

don't forget the color factor of 3 for the quark fields!

[assuming one generation of the Standard Model fills up complete representations of the grand unified group.]

Introduce $\boxed{x \equiv \frac{1}{5}\left(\frac{b_2-b_3}{b_1-b_2}\right)}$

Then, we find:

$$\boxed{\sin^2\theta_W(m_Z) = \frac{1}{1+8x}\left[3x + \frac{\alpha(m_Z)}{\alpha_s(m_Z)}\right]}$$

To compute the b_i, we employ:

$$\boxed{b_i = \frac{2}{3} T_f(R_k) \prod_{k \neq j} d_f(R_j) + \frac{1}{6} T_s(R_k) \prod_{k \neq j} d_s(R_j) - \frac{11}{3} C_2(G_i)}$$

f = fermion
s = scalar

$$\prod_{k \neq j} d(R_j) = \text{multiplicity factors}$$

$d(R) = \dim(R)$

$$\mathrm{Tr}\, T^a T^b = T(R)\,\delta^{ab}$$

$$(T^a T^a)_{ij} = C_2(G)\,\delta_{ij}$$

(<u>Note</u>: for the adjoint representation ($R=A$), we have: $T(A) = C_2(G)$.)

For $G = SU(N)$, $C_2(G) = N$
For $G = U(1)$, $C_2(G) = 0$

<u>Note</u>: using the properly normalized hypercharge generator, $\sqrt{\frac{3}{5}}\,\frac{Y}{2}$, it follows that:

$$T(R_1) = \left[\sqrt{\tfrac{3}{5}}\,\tfrac{1}{2} Y\right]^2 = \frac{3}{20} Y^2$$

fields	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$T(R_3)$	$\prod_{i\neq 3} d(R_i)$	$T(R_2)$	$\prod_{i\neq 2} d(R_i)$	$T(R_1)$ $\left(\frac{3}{20}Y^2\right)$	$\prod_{i\neq 1} d(R_i)$
$\begin{pmatrix}\psi_{Q_1}\\ \psi_{Q_2}\end{pmatrix}$	3	2	$\frac{1}{3}$	$\frac{1}{2}$	2	$\frac{1}{2}$	3	$\frac{1}{60}$	6
ψ_U	3^*	1	$-\frac{4}{3}$	$\frac{1}{2}$	1	0	3	$\frac{4}{15}$	3
ψ_D	3^*	1	$\frac{2}{3}$	$\frac{1}{2}$	1	0	3	$\frac{1}{15}$	3
$\begin{pmatrix}\psi_{L_1}\\ \psi_{L_2}\end{pmatrix}$	1	2	-1	0	2	$\frac{1}{2}$	1	$\frac{3}{20}$	2
ψ_E	1	1	2	0	1	0	1	$\frac{3}{5}$	1
$\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$	1	2	1	0	2	$\frac{1}{2}$	1	$\frac{3}{20}$	2
$\begin{pmatrix}\tilde{\phi}^0\\ \tilde{\phi}^-\end{pmatrix}$	1	2	-1	0	2	$\frac{1}{2}$	1	$\frac{3}{20}$	2

<u>example</u>:

$$b_3 = \frac{2}{3}\left[\left(\tfrac{1}{2}\right)(2) + \left(\tfrac{1}{2}\right)(1) + \left(\tfrac{1}{2}\right)(1)\right] N_G - \frac{11}{3}(3)$$

$$= \frac{4}{3} N_G - 11$$

N_G = number of fermion generations $= 3$

Final result

$$b_3 = \frac{4}{3} N_G - 11$$

$$b_2 = \frac{1}{6} N_H + \frac{4}{3} N_G - \frac{22}{3}$$

$$b_1 = \frac{1}{10} N_H + \frac{4}{3} N_G$$

where I have allowed for N_H copies of the Standard Model Higgs boson.

For the Standard Model, $N_G = 3$ and $N_H = 1$.

$$b_3 = -7$$

$$b_2 = \frac{-19}{6}$$

$$b_1 = \frac{41}{10}$$

$$x = \frac{1}{5}\left(\frac{b_2 - b_3}{b_1 - b_2}\right) = \frac{23}{218} = 0.1055$$

Remark: Notice that N_G drops out completely in the expression for x (this is a special feature of the one-loop calculation). Thus, in this approximation, the success (or failure) of unification does not depend on the number of fermion generations.

Check the prediction of $\alpha_s(m_Z)$.

$$\alpha_s(m_Z) = \frac{\alpha(m_Z)}{(1+8x)\sin^2\theta_W(m_Z) - 3x}$$

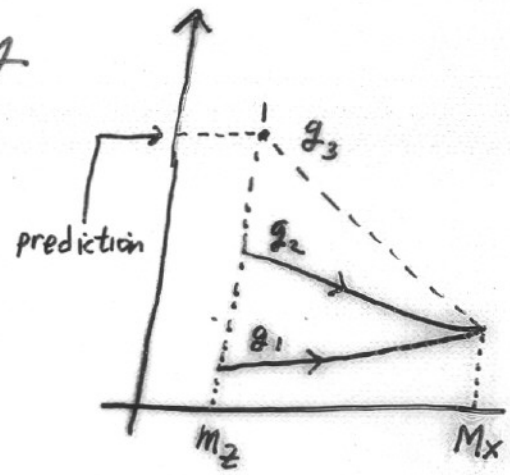
These are $\overline{MS}$-couplings.

$$\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.2315 \pm 0.0004$$

$$\alpha^{-1}(m_Z)_{\overline{MS}} = 127.90 \pm 0.09$$

$$x = 0.1055$$

$$\Longrightarrow \quad \alpha_s(m_Z) = 0.071$$



to be compared with the world average:

$$\alpha_s(m_Z) = 0.118 \pm 0.003$$

Is this a hint for the minimal supersymmetric standard model (MSSM)?

<u>exercise</u>: Show that $x = \frac{1}{7}$ in the MSSM

$$\Longrightarrow \quad \alpha_s(m_Z) = 0.116$$

<u>Note</u>: two-loop corrections to unification are not negligible. The prediction for $\alpha_s(m_Z)$ increases by roughly 0.01, not nearly enough to save Standard Model unification.

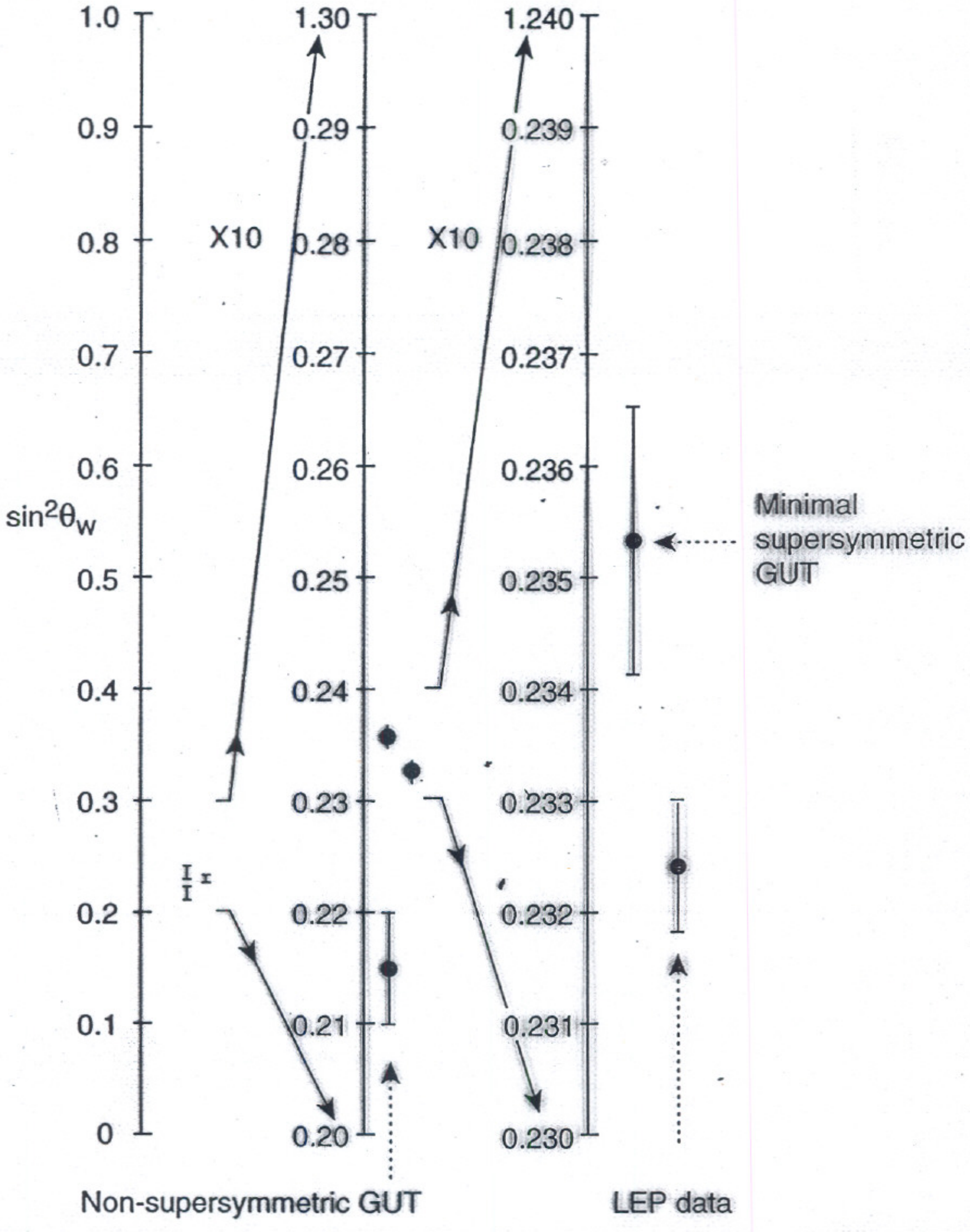


Fig. 8. Gee-whizz plot showing how well GUT predictions of $\sin^2\theta_W$ agree with the experimental data.

from J. Ellis (Lepton-Photon Conference
Beijing 1995)

Beyond the MSSM

A priori, there is nothing to stop us from considering extensions to the Standard Model involving new gauge or matter fields and then constructing its supersymmetric extension. The success of the SUSY-GUT prediction limits somewhat which extensions to consider.

<u>example</u>: the supersymmetric extension of the four-Higgs-doublet Standard Model would <u>not</u> exhibit a successful unification of couplings.

Unification is not affected by the introduction of $SU(3)\times SU(2)\times U(1)$ - singlet fields. Let us promote this to a superfield $\hat{N}$.

motivations for $\hat{N}$

(i) The $\mu \hat{H}_1 \hat{H}_2$ term of the MSSM is puzzling. It is supersymmetric, and requires μ to be of order the supersymmetry breaking scale. How can this happen?

There are a number of proposals, but we shall not consider them here.

NMSSM

Remove the $\mu \hat{H}_1 \hat{H}_2$ term from the superpotential. Replace it with:

$$\lambda_1 \hat{H}_1 \hat{H}_2 \hat{N} + \lambda_2 \hat{N}^3$$

where λ_i is dimensionless. One obtains an effective μ, namely $\mu = \lambda_1 \langle \hat{N} \rangle$.

In this model, the number of neutralinos and neutral Higgs fields are increased.

(ii) The see-saw model of neutrino masses introduces a heavy right-handed neutrino. One can construct a supersymmetric extension of the seesaw model by introducing $\hat{N}$ as above, but with a different superpotential.

<u>remark</u>: in the MSSM, R-parity conservation is imposed to guarantee L conservation in the low-energy theory. The supersymmetric see-saw model is R-parity conserving since L is violated by two units in the neutrino mass term: $-\mathcal{L}_m = m_\nu \nu\nu + h.c.$

The supersymmetric see-saw

In the R-Parity-Conserving (RPC) MSSM:

- $R = (-1)^{3(B-L)+2S}$
- The LSP is stable
- Neutrinos are massless

To obtain neutrino masses consistent with RPC, one must violate L by two units. The simplest model is the supersymmetric extension of the seesaw.

One-generation model

$$W \ni \epsilon_{ij}\left[\lambda \hat{H}_U^i \hat{L}^j \hat{N} - \mu \hat{H}_D^i \hat{H}_U^j\right] + \tfrac{1}{2} M \hat{N}\hat{N}$$

$$V_{\rm soft} \ni m_L^2 \tilde{\nu}^* \tilde{\nu} + m_{\widetilde{N}}^2 \widetilde{N}^* \widetilde{N} + \left[\lambda A_\nu H_U^0 \tilde{\nu} \widetilde{N}^* + M B_N \widetilde{N}\widetilde{N} + \text{h.c.}\right]$$

After EWSB, $\langle H_i^0 \rangle = v_i/\sqrt{2}$, with $\tan\beta \equiv v_u/v_d$, one obtains the usual seesaw result:

$$\mathcal{M}_N = \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix}$$

where $m_D \equiv \lambda v_u$. Thus, taking $m_D \ll M$, $m_\nu \simeq m_D^2/M$.

The sneutrino masses are obtained by diagonalizing a 4×4 squared-mass matrix. Here, it is convenient to define:
$\tilde\nu=(\tilde\nu_1+i\tilde\nu_2)/\sqrt{2}$ and $\tilde N=(\tilde N_1+i\tilde N_2)/\sqrt{2}$.
Then, the squared-sneutrino mass matrix ($\mathcal{M}^2$) separates into CP-even and CP-odd blocks:

$$\mathcal{M}^2=\tfrac{1}{2}\begin{pmatrix}\phi_1 & \phi_2\end{pmatrix}\begin{pmatrix}\mathcal{M}^2_+ & 0\\ 0 & \mathcal{M}^2_-\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix},$$

where $\phi_i\equiv\begin{pmatrix}\tilde\nu_i & \tilde N_i\end{pmatrix}$ and $\mathcal{M}^2_\pm$ consist of the following 2×2 blocks:

$$\begin{pmatrix} m^2_{\tilde L}+\tfrac{1}{2}m^2_Z\cos 2\beta+m^2_D & m_D[A_\nu-\mu\cot\beta\pm M]\\ m_D[A_\nu-\mu\cot\beta\pm M] & M^2+m^2_D+m^2_{\tilde N}\pm 2B_N M\end{pmatrix}.$$

To first order in $1/M$, the two light sneutrino eigenstates are $\tilde\nu_1$ and $\tilde\nu_2$, with corresponding squared masses:

$$m^2_{\tilde\nu_{1,2}}=m^2_{\tilde L}+\tfrac{1}{2}m^2_Z\cos 2\beta\mp\tfrac{1}{2}\Delta m^2_{\tilde\nu}\,,$$

where $\Delta m^2_{\tilde\nu}\equiv m^2_{\tilde\nu_2}-m^2_{\tilde\nu_1}$. Writing $\Delta m^2_{\tilde\nu}=2m_{\tilde\nu}\Delta m_{\tilde\nu}$,

$$r_\nu\equiv\frac{\Delta m_{\tilde\nu}}{m_\nu}\simeq\frac{2(A_\nu-\mu\cot\beta-B_N)}{m_{\tilde\nu}}$$

Three-generation model

In the three-generation model, one can choose various alternatives depending on the number of singlet superfields $\hat{N}$. Suppose that there are n_g SM generations.

- If there is only one $\hat{N}$ superfield, then

$$\mathcal{M}_N = \begin{pmatrix} 0 & (m_D)_j \\ (m_D)_i & M \end{pmatrix} ,$$

  where $(m_D)_i \equiv \lambda_i v_u$ [i labels generation], which yields (at tree-level) $n_g - 1$ massless neutrinos and one light neutrino with mass $m_\nu = \sum_i [(m_D)_i]^2$.

- If there are n_g singlets $\hat{N}_i$, then

$$\mathcal{M}_N = \begin{pmatrix} 0 & (m_D)_{\ell k} \\ (m_D)_{ij} & M_{ik} \end{pmatrix} ,$$

  where $(m_D)_{ij} \equiv \lambda_{ij} v_u$, which yields (at tree-level) n_g light neutrinos with nonzero mass. In this case, *all* light neutrinos have mass, since $\det\,\mathcal{M}_N = (\det\, m_D)^2 \neq 0$.

In all cases, if the unperturbed ($M \to \infty$) sneutrino masses are non-degenerate, then $(\Delta m_{\tilde{\nu}})_k \neq 0$ for *all* $k = 1, \ldots, n_g$.

R Parity Violating Models

- In a general R-Parity-Violating (RPV) model, both L and B are violated. The corresponding superpotential is

$$W = \epsilon_{ij}\Big[-\mu_\alpha \widehat{L}^i_\alpha \widehat{H}^j_u + \tfrac{1}{2}\lambda_{\alpha\beta m}\widehat{L}^i_\alpha \widehat{L}^j_\beta \widehat{E}_m + \lambda'_{\alpha n m}\widehat{L}^i_\alpha \widehat{Q}^j_n \widehat{D}_m$$
$$-h_{nm}\widehat{H}^i_u \widehat{Q}^j_n \hat{U}_m\Big] + (\lambda_B)_{pnm}\widehat{U}_p \widehat{D}_n \widehat{D}_m\,,$$

where α, $\beta = 0,\ldots,3$; m, n, $p = 1,2,3$ and $\hat{L}_0 \equiv \hat{H}_D$.

The RPC model is equivalent to introducing a $\mathbf{Z_2}$ matter parity. To avoid fast proton decay in the RPV model, one may introduce a $\mathbf{Z_3}$ triality, which conserves B. This is the unique choice for a (generation independent) discrete symmetry with no discrete gauge anomalies in a model consisting only of the MSSM superfields. [Ibanez, Ross]

Matter discrete symmetries

symmetry	$\widehat{Q}_n$	$\widehat{U}_n$	$\widehat{D}_n$	$\widehat{L}_n$	$\widehat{E}_n$	$\widehat{H}_U$	$\widehat{H}_D$
$\mathbf{Z_2}$	-1	-1	-1	-1	-1	$+1$	$+1$
$\mathbf{Z_3}$	ω	ω^{-1}	ω^{-1}	$+1$	$+1$	$+1$	$+1$

Note: $\omega \equiv e^{i\pi/3}$

The B-conserving RPV model

$\hat{H}_D$ and $\hat{L}_i$ are indistinguishable $Y=-1$ weak doublets

- Neutrinos mix with neutralinos $\Longrightarrow \quad m_\nu \neq 0$
- Sneutrinos mix with Higgs bosons $\Longrightarrow \quad \Delta m_{\tilde{\nu}} \neq 0$

$\Delta m_{\tilde{\nu}}$: sneutrino–antisneutrino mass-splitting

Denote $\hat{H}_D$ by $\hat{L}_0$ ($\hat{L}_i \to \hat{L}_\alpha \quad \alpha = 0,1,2,3$)

$(\text{MSSM})_{\text{R}}$	$(\text{MSSM})_{\text{B}}$
$\mu \hat{H}_D \hat{H}_U$	$\mu_\alpha \hat{L}_\alpha \hat{H}_U$
$h^\ell_{jk} \hat{H}_D \hat{L}_j \hat{E}_k$	$\lambda_{\alpha\beta k} \hat{L}_\alpha \hat{L}_\beta \hat{E}_k$
$h^D_{jk} \hat{H}_D \hat{Q}_j \hat{D}_k$	$\lambda'_{\alpha jk} \hat{L}_\alpha \hat{Q}_j \hat{D}_k$
$b H_D H_U$	$b_\alpha \tilde{L}_\alpha H_U$
$a^\ell_{jk} H_D \tilde{L}_j \tilde{E}_k$	$a_{\alpha\beta k} \tilde{L}_\alpha \tilde{L}_\beta \tilde{E}_k$
$a^D_{jk} H_D \tilde{Q}_j \tilde{D}_k$	$a'_{\alpha jk} \tilde{L}_\alpha \tilde{Q}_j \tilde{D}_k$
$M^2_D H^\dagger_D H_D + (M^2_{\tilde{L}})_{ij} \tilde{L}^\dagger_i \tilde{L}_j$	$(M^2_{\tilde{L}})_{\alpha\beta} \tilde{L}^\dagger_\alpha \tilde{L}_\beta$
v_d	v_α

We define: $\quad v_d^2 = \sum v_\alpha^2\,, \quad \mu^2 = \sum \mu_\alpha^2\,, \quad b^2 = \sum b_\alpha^2$

and $v^2 \equiv v_u^2 + v_d^2 = (246~\text{GeV})^2\,, \qquad \tan\beta = v_u/v_d$

$$W = \epsilon_{ij} \;\; \left[-\mu_\alpha \hat{L}^i_\alpha \hat{H}^j_U + \tfrac{1}{2}\lambda_{\alpha\beta m} \hat{L}^i_\alpha \hat{L}^j_\beta \hat{E}_m + \lambda'_{\alpha n m} \hat{L}^i_\alpha \hat{Q}^j_n \hat{D}_m \right.$$
$$\left. - h_{nm} \hat{H}^i_U \hat{Q}^j_n \hat{U}_m \right]$$

$$\begin{aligned}
V_{\rm soft} = {} & (M^2_{\widetilde{Q}})_{mn}\, \widetilde{Q}^{i*}_m \widetilde{Q}^i_n + (M^2_{\widetilde{U}})_{mn}\, \widetilde{U}^*_m \widetilde{U}_n + (M^2_{\widetilde{D}})_{mn}\, \widetilde{D}^*_m \widetilde{D}_n \\
& + (M^2_{\widetilde{L}})_{\alpha\beta}\, \widetilde{L}^{i*}_\alpha \widetilde{L}^i_\beta + (M^2_{\widetilde{E}})_{mn}\, \widetilde{E}^*_m \widetilde{E}_n + m^2_U |H_U|^2 \\
& - (\epsilon_{ij} b_\alpha \widetilde{L}^i_\alpha H^j_U + {\rm h.c.}) + \epsilon_{ij} [\tfrac{1}{2} a_{\alpha\beta m} \widetilde{L}^i_\alpha \widetilde{L}^j_\beta \widetilde{E}_m \\
& + a'_{\alpha n m} \widetilde{L}^i_\alpha \widetilde{Q}^j_n \widetilde{D}_m - (a_U)_{nm} H^i_U \widetilde{Q}^j_n \widetilde{U}_m + {\rm h.c.}] \\
& + \tfrac{1}{2} \left[M_3\, \widetilde{g}\, \widetilde{g} + M_2 \widetilde{W}^a \widetilde{W}^a + M_1 \widetilde{B} \widetilde{B} + {\rm h.c.} \right]
\end{aligned}$$

$$\begin{aligned}
V_D = {} & \tfrac{1}{8} g^2 \Big\{ \Big(|H_U|^2 - \sum_\alpha |\widetilde{L}_\alpha|^2 - \sum_m |\widetilde{Q}_m|^2 \Big)^2 - 2 \sum_{\alpha \neq \beta} |\epsilon_{ij} \widetilde{L}^i_\alpha \widetilde{L}^j_\beta|^2 \\
& + 4 \sum_\alpha |H^{i*}_U \widetilde{L}^i_\alpha|^2 - 2 \sum_{m \neq n} |\epsilon_{ij} \widetilde{Q}^i_m \widetilde{Q}^j_n|^2 \\
& + 4 \sum_m |H^{i*}_U \widetilde{Q}^i_m|^2 - 4 \sum_{\alpha m} |\epsilon_{ij} \widetilde{L}^i_\alpha \widetilde{Q}^i_m|^2 \Big\} \\
& + \tfrac{1}{8} g'^2 \Big[|H_U|^2 - \sum_\alpha |\widetilde{L}_\alpha|^2 + 2 \sum_m |\widetilde{E}_m|^2 + \tfrac{1}{3} \sum_m |\widetilde{Q}_m|^2 \\
& - \tfrac{4}{3} \sum_m |\widetilde{U}_m|^2 + \tfrac{2}{3} \sum_m |\widetilde{D}_m|^2 \Big]^2 .
\end{aligned}$$

Neutrino masses: Tree level

In the $\{\widetilde{B}, \widetilde{W}^3, \widetilde{h}_U, \nu_\alpha\}$ basis the 7×7 mass matrix, $M^{(\mathrm{n})}$ is

$$\begin{pmatrix} M_1 & 0 & m_Z s_W v_u/v & -m_Z s_W v_\beta/v \\ 0 & M_2 & -m_Z c_W v_u/v & m_Z c_W v_\beta/v \\ m_Z s_W v_u/v & -m_Z c_W v_u/v & 0 & \mu_\beta \\ -m_Z s_W v_\alpha/v & m_Z c_W v_\alpha/v & \mu_\alpha & 0_{\alpha\beta} \end{pmatrix}$$

Two zero eigenvalues: two massless neutrinos

Five non-zero eigenvalues: four $\tilde{\chi}^0$ and one ν

$$\det{}' M^{(\mathrm{n})} = m_Z^2 \mu^2 M_{\tilde{\gamma}} \cos^2\beta\, |\hat{v} \times \hat{\mu}|^2$$

$\sin^2\xi \equiv |\hat{v} \times \hat{\mu}|^2 \equiv 1 - (\hat{v} \cdot \hat{\mu})^2$ measures the alignment of v_α and μ_α

$$m_\nu = \frac{\det{}' M^{(\mathrm{n})}}{\det M_0^{(\mathrm{n})}} = \frac{m_Z^2 \mu M_{\tilde{\gamma}} \cos^2\beta \sin^2\xi}{m_Z^2 M_{\tilde{\gamma}} \sin 2\beta - M_1 M_2 \mu} \sim m_Z \sin^2\xi$$

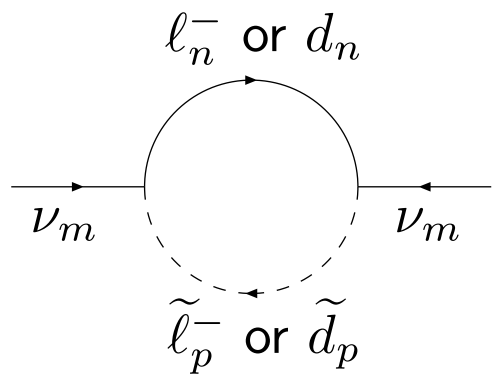
where $M_{\tilde{\gamma}} \equiv M_1 \cos^2\theta_W + M_2 \sin^2\theta_W$.

At tree level, $m_\nu \neq 0 \iff \sin\xi \neq 0$

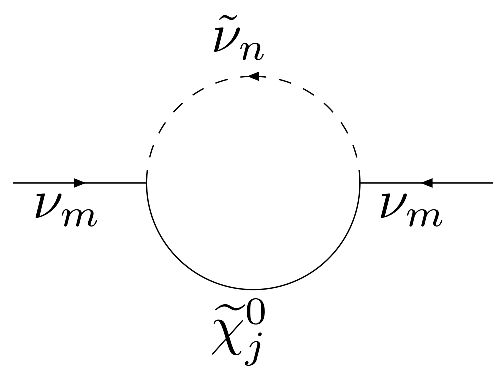
Neutrino masses: Loop effects

Contributions at one loop:

- Lepton–slepton loops and down type quark–squark loops. Proportional to trilinear lepton number violating interactions



- Sneutrino and neutralinos loops. Proportional to sneutrino–antisneutrino mass splitting. Exist in any model with lepton number violation



Fermion–sfermion loops

The lepton-slepton loop contribution is:

$$(m_\nu)_{qm}^{(\ell)} = \frac{1}{32\pi^2} \sum_{\ell,p} \lambda_{q\ell p}\lambda_{mp\ell} m_\ell \sin 2\phi_\ell \ln\left(\frac{M_{p_1}^2}{M_{p_2}^2}\right)$$

$$\sim \frac{1}{16\pi^2} \lambda_{q\ell p}\lambda_{mp\ell} \frac{m_\ell^2}{\widetilde{M}}$$

since $\sin 2\phi_\ell \ln(M_{p1}^2/M_{p2}^2) \propto m_\ell$.

The down-type quark-squark loops are similar

$$(m_\nu)_{qm}^{(q)} \sim \frac{3}{16\pi^2} \lambda'_{qdr}\lambda'_{mrd} \frac{m_q^2}{\widetilde{M}}$$

The dominant loops are with τ leptons and b quarks.

General structure of the one-loop mass:

$$(m_\nu)^{(f)} \simeq (\text{loop factor}) \times (\text{RPV Yukawas}) \times \lambda_{\mathrm{f}}^2$$

Sneutrino–neutralinos loops

$$m_\nu^{(1)} = \frac{g^2 \Delta m_{\tilde\nu}}{32\pi^2 \cos^2\theta_W} \sum_j f(y_j) |Z_{jZ}|^2 \sim 10^{-3} \Delta m_{\tilde\nu}$$

where $f(y_j) = \sqrt{y_j}\,[y_j - 1 - \ln(y_j)]\,/(1-y_j)^2$, Z_{jZ} projects out the $\widetilde{Z}$ eigenstate from $\tilde\chi_j^0$, and $y_j \equiv M_{\tilde\nu}^2 / M_{\tilde\chi_j^0}^2$

This contribution exists in any model. [Y. Grossman and H.E. Haber] The general structure of the one-loop mass:

$$(m_\nu)^{(\tilde\nu)} \simeq (\text{loop factor}) \times (\text{RPV parameters})$$

If the sizes of the RPV parameters that enter here are roughly the same as the RPV Yukawas that contribute to $(m_\nu)^{(f)}$, then we would expect $(m_\nu)^{(\tilde\nu)}$ to be the dominant one-loop contribution to the neutrino mass

$$\frac{(m_\nu)^{(\tilde\nu)}}{(m_\nu)^{(f)}} \sim \frac{1}{\lambda_f^2} \gg 1$$

where λ_f is a down-type Higgs-fermion Yukawa coupling.

Sneutrino–antisneutrino mass splittings

In L-violating RPV models, $\Delta L = 1$ interactions (acting twice) yield both $\Delta L = 2$ neutrino masses and sneutrino–antisneutrino mass splitting. The latter arises as a consequence of a squared-mass term: $m^2_{\Delta L=2}\tilde{\nu}\tilde{\nu}$ +h.c.

One expects

- Large ($\sim m_Z$) $\Delta L = 0$ SUSY breaking mass
- Small ($\sim m_\nu$) $\Delta L = 2$ “Majorana” mass

The sneutrino squared-mass matrix is schematically

$$\begin{pmatrix} m^2_{\tilde{\nu}} & m^2_{\Delta L=2} \\ m^2_{\Delta L=2} & m^2_{\tilde{\nu}} \end{pmatrix}$$

This results in sneutrino–antisneutrino mixing and small mass splitting of order $\Delta m_{\tilde{\nu}} \sim m_\nu$.

VIII. CONSTRAINING THE MSSM

The MSSM parameter count

Sutter, Dimopoulos

In the previous lecture, we constructed the MSSM. But to simplify the presentation, flavor degrees of freedom were suppressed.

Now, it is time to review the full set of parameters of the MSSM, given three generations of quarks and leptons. [Generation labels: $i, j, k = 1, 2, 3$]

slight change of notation

$$\hat{H}_1 \longrightarrow \hat{H}_D$$
$$\hat{H}_2 \longrightarrow \hat{H}_U$$

the subscript indicates which right-handed quark superfield couples to $\hat{H}_1$ and $\hat{H}_2$.

$m_{12}^2 = B\mu$ the "B-term"

Remark on the Fayet-Iliopoulos term

Since the MSSM gauge group contains a $U(1)$ factor, I could introduce an associated Fayet-Iliopoulos term (and parameter ξ). I choose to omit this term. Presumably, it does not arise if $SU(3)\times SU(2)\times U(1)$ is the broken subgroup of some non-abelian grand unified group.

There exists a non-renormalization theorem that states that if $\mathrm{Tr}\, T^a = 0$, then by setting $\xi = 0$ at tree level, it remains zero to all orders in perturbation theory. (Conversely, if $\mathrm{Tr}\, T^a \neq 0$, there is only one renormalization at one-loop order. But it is quadratically divergent!)

Parameters of the MSSM

SUSY-conserving sector

$g_1, g_2, g_3, \theta_{QCD}$

$\mu \, \hat{H}_D \hat{H}_U$

$h^\ell_{jk} \, \hat{H}_D \hat{L}_j \hat{E}_k$

$h^D_{jk} \, \hat{H}_D \hat{Q}_j \hat{D}_k$

$h^U_{jk} \, \hat{H}_U \hat{Q}_j \hat{U}_k$

SUSY-breaking sector

$B\mu \, H_D H_U$

$(h^\ell A^\ell)_{jk} \, H_D \tilde{L}_j \tilde{E}_k$

$(h^D A^D)_{jk} \, H_D \tilde{Q}_j \tilde{D}_k$

$(h^U A^U)_{jk} \, H_U \tilde{Q}_j \tilde{U}_k$

$M^2_D \, H^\dagger_D H_D + M^2_U \, H^\dagger_U H_U$

$(M^2_{\tilde{Q}})_{ij} \, \tilde{Q}^\dagger_i \tilde{Q}_j + (M^2_{\tilde{L}})_{ij} \, \tilde{L}^\dagger_i \tilde{L}_j$

$(M^2_{\tilde{D}})_{ij} \, \tilde{D}^\dagger_i \tilde{D}_j + (M^2_{\tilde{U}})_{ij} \, \tilde{U}^\dagger_i \tilde{U}_j + (M^2_{\tilde{E}})_{ij} \, \tilde{E}^\dagger_i \tilde{E}_j$

$M_1 \, \widetilde{B}\widetilde{B} + M_2 \, \widetilde{W}^a \widetilde{W}^2 + M_3 \, \widetilde{g}\widetilde{g}$

To see how to count parameters, let us first consider the Standard Model. Its parameters are:

$g_1, g_2, g_3, \theta_{QCD}$	$3+1$	θ_{QCD} can be regarded as the imaginary part of the strong coupling constant.
Higgs sector: v^2, λ	2	
$V_{Higgs} = \lambda(\phi^2 - v^2)^2$		
Yukawas: h_u, h_d, h_e	$\underset{\text{real}}{27} + \underset{\text{imaginary}}{27}$	

Here we have used the fact that h is a 3×3 complex matrix with no special properties.

But, most of these degrees of freedom are unphysical. In the limit of $h_u = h_d = h_e = 0$, the Standard Model possesses an exact $U(3)^5$ global symmetry corresponding to three generations of the five $SU(3)\times SU(2)\times U(1)$ multiplets:

$$(\nu, e^-)_L,\ e^+_L,\ (u,d)_L,\ u^c_L,\ d^c_L$$

Note that by gauge invariance, I must rotate each multiplet by a unique global symmetry rotation.

So, $U(3)^5$ rotations leave the total $\mathcal{L}$ invariant if $h_u = h_d = h_e = 0$. If h_u, h_d and h_e are non-zero, the $U(3)^5$ rotation does not leave $\mathcal{L}$ invariant. In particular, the Yukawa terms of the Lagrangian would shift. But I can also view the $U(3)^5$ rotation as a field-redefinition, which does not alter the physical predictions of the theory.

Thus, I can use these rotations to remove unphysical degrees of freedom from the parameters.

How many degrees of freedom can be removed?

<u>answer</u>: the number of parameters that define the $U(3)^5$ rotation minus the number of parameters of any subgroup of $U(3)^5$ that does leave $\mathcal{L}$ invariant (since the latter has no effect on the parameters).

$U(3)$ is parameterized by 3 angles and 6 phases

$U(3)^5$ is parameterized by 15 angles and 30 phases

Four global symmetries that live inside $U(3)^5$ leave $\mathcal{L}$ invariant. These are B and the three separate lepton numbers L_e, L_μ, L_τ (remember that ν is massless in the Standard Model). These are $U(1)$-phase rotations.

Thus, we started with

$$32 \text{ real} + 28 \text{ phases}$$

Using $U(1)^5 - B - L_e - L_\mu - L_\tau$, we can remove

$$15 \text{ real} + 26 \text{ phases}$$

What remains are 17 real parameters and 2 phases*
for a total of 19 Standard Model parameters.

* one of which is θ_{QCD}.

In fact, we can explicitly identify them:

$g_1, g_2, g_3, \theta_{QCD}, m_H, m_Z$, 6 quark masses, 3 lepton masses, 3 CKM mixing angles and 1 CKM phase,

for a total of 19 parameters. ✓

The MSSM count

		real + imaginary	
	$g_1, g_2, g_3, \theta_{QCD}$	$3 + 1$	
gaugino masses	M_1, M_2, M_3	$3 + 3$	
	$m_{H_U}^2, m_{H_D}^2$	2	← equivalently: v_U and v_D OR v and $\tan\beta$
	B, μ	$2 + 2$	
	h_U, h_D, h_E	$27 + 27$	
	A_U, A_D, A_E	$27 + 27$	note:
	$M_{\tilde{Q}}^2, M_{\tilde{U}}^2, M_{\tilde{D}}^2, M_{\tilde{L}}^2, M_{\tilde{E}}^2$	$30 + 15$	← a 3×3 hermitian matrix contains 6 real parameters and 3 imaginary parameters.
		$94 + 75$	

Removing unphysical degrees of freedom

This time, I apply the $U(3)^5$-rotation to the five $SU(3)\times SU(2)\times U(1)$ supermultiplets:

$$\hat{L}, \hat{E}, \hat{Q}, \hat{U}, \hat{D}$$

In this way, I protect supersymmetric interactions such as $A^* \psi \lambda + h.c.$, since I am rotating simultaneously the partners and superpartners.

Among the $U(3)^5$-rotations, only B and L leave $\mathcal{L}_{MSSM}$ invariant. Note that L_e, L_μ, and L_τ are not separately conserved, assuming that sneutrinos are not mass-degenerate. We can introduce CKM-like rotations in the slepton sector which need not align with the corresponding definitions of L_e, L_μ, L_τ.

Using $U(3)^5 - B - L$, we can remove

15 real + 28 phases.

There are two other global symmetries that we can use to remove degrees of freedom. They correspond, respectively, to global chiral symmetries that protect gaugino and higgsino masses while leaving $\lambda\Psi A^* + h.c.$ interactions and $\mu \hat{H}_U \hat{H}_D$ invariant. Consider these $U(1)$ transformations:

PQ = Peccei-Quinn

	$U(1)_R$	$U(1)_{PQ}$
$\tilde{Q}, \tilde{U}, \tilde{D}, \tilde{L}, \tilde{E}$	$1/2$	1
H_D, H_U	1	-2
h_u, h_d, h_e	0	0
A_U, A_D, A_E	-2	0
M_1, M_2, M_3	-2	0
μ	0	4
$m_{12}^2 = B\mu$	-2	4
gauge bosons	0	0
gauginos	1	0

Here, I pretended that the parameters also rotate under the $U(1)$ transformation, and chose the corresponding $U(1)$ quantum numbers that make $\mathcal{L}$ invariant. Of course, the parameters do <u>not</u> rotate, so if they have a non-zero entry above, this means that the parameter shifts under the $U(1)$-rotation, and the latter can be used to remove unphysical degrees of freedom.

We therefore use $U(1)_R$ to remove a phase from M_3, and we use $U(1)_{PQ}$ to remove a phase from $m_{12}^2 = B\mu$.

In fact, we have implicitly performed this last step, when we studied the Higgs sector and noticed that we could redefine the phases of the Higgs fields such that m_{12}^2 was real and $v_1, v_2 > 0$.

<u>Note:</u> As a result, the tree-level MSSM Higgs sector is automatically CP-conserving.

The final count

	real	imaginary
original count	94	75
remove with $U(3)^5 - B - L$	-15	-28
remove with $U(1)_R \times U(1)_{PQ}$		-2
	79	45

TOTAL 124

I call this theory MSSM-124.

The Breakdown

18 Standard Model parameters (include v^2 but not λ)
2 Higgs-sector parameters (m_A, $\tan\beta$)
104 SUSY-parameters

124

real parameters		phases	
6	quark masses	1	CKM phase
3	lepton masses	40	super-CKM phases
12	squark masses ($\tilde{q}_L, \tilde{q}_R \times 6$ flavors)	1	θ_{QCD}
9	slepton masses (no $\tilde{\nu}_R$ here)	3	$\arg M_1$, $\arg M_2$, $\arg \mu$
3	CKM angles	45	
36	super-CKM angles		
7	$\lvert M_1\rvert, \lvert M_2\rvert, M_3, B\mu, m^2_{H_U}, m^2_{H_D}, \lvert\mu\rvert$		
3	g_1, g_2, g_3		
79			

Note in particular 36 super-CKM angles and 40 super-CKM phases. These arise since squarks and sleptons need not be diagonal in the basis in which quarks and leptons are diagonal.

Unconstrained Low-Energy SUSY is not Viable

- No conservation of lepton numbers L_e, L_μ and L_τ
- Unsuppressed flavor-changing neutral-currents
- New sources of CP-violation in conflict with experimental constraints

The MSSM is a phenomenologically viable theory only in tiny "exceptional" regions of the full parameter space. That is, there needs to be many *a priori* unexplained small (soft-SUSY-breaking) parameters in the model.

In the bottom-up approach, one attempts to assess the viable parameter regimes and deduce implications for the fundamental theory of SUSY-breaking.

In the top-down approach, one looks for simple theories of SUSY-breaking that yield acceptable low-energy SUSY parameters.

Examples of the bottom-up approach

Place constraints directly on the ("low-energy") MSSM parameters. Two alternatives are:

1. Horizontal universality

Take $M^2_{\tilde{Q}}, M^2_{\tilde{U}}, M^2_{\tilde{D}}, M^2_{\tilde{L}}, M^2_{\tilde{E}}$ and the associated matrix A-parameters to be proportional to $I_{3\times 3}$.

2. Flavor alignment

Take $M^2_{\tilde{Q}}, M^2_{\tilde{U}}, M^2_{\tilde{D}}, M^2_{\tilde{L}}, M^2_{\tilde{E}}$ are the associated matrix A-parameters to be flavor diagonal in a basis where the quark and lepton mass matrices are flavor diagonal

These alternatives are phenomenologically viable, but rather ad-hoc from a theoretical perspective.

The top-down approach

The MSSM parameters evolve with energy scale according to renormalization group equations (RGE's) - Impose a particular (simple) structure on the soft-SUSY-breaking terms at a common high energy scale (e.g. M_{PL}).
The initial conditions depend on the theory of supersymmetry breaking. Then, using RG-evolution, one can compute the low-energy SUSY spectrum.

possible bonus: radiative electroweak symmetry breaking

Evolution of SUSY parameters – the SUSY RGE's

notation:

$$a_t = h_t A_t$$

$$a_b = h_b A_b$$

$$a_\tau = h_\tau A_\tau$$

$$b = B\mu$$

$$Y_t = 2h_t^2(m_{H_u}^2 + m_{Q_3}^2 + m_{U_3}^2) + 2a_t^2$$

$$Y_b = 2h_b^2(m_{H_d}^2 + m_{Q_3}^2 + m_{D_3}^2) + 2a_b^2$$

$$Y_\tau = 2h_\tau^2(m_{H_d}^2 + m_{L_3}^2 + m_{E_3}^2) + 2a_\tau^2$$

$$\frac{d}{dt} = \mu \frac{d}{d\mu}. \qquad \mu = \text{evolution scale}$$

Keeping just the 3rd generation Yukawas, and assuming all parameters real, $g_1^2 = \frac{5}{3}g'^2$

$$\frac{dh_t}{dt} = \frac{h_t}{16\pi^2}\left[6h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right]$$

$$\frac{dh_b}{dt} = \frac{h_b}{16\pi^2}\left[6h_b^2 + h_t^2 + h_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right]$$

$$\frac{dh_\tau}{dt} = \frac{h_\tau}{16\pi^2}\left[4h_\tau^2 + 3h_b^2 - 3g_2^2 - \frac{9}{5}g_1^2\right]$$

$$\frac{d\mu}{dt} = \frac{\mu}{16\pi^2}\left[3h_t^2 + 3h_b^2 + h_\tau^2 - 3g_2^2 - \frac{3}{5}g_1^2\right]$$

μ = supersymmetric Higgs mass parameter.

$$\frac{d}{dt} M_a = \frac{b_a g_a^2 M_a}{8\pi^2}$$

$b_a = \left(\frac{33}{5}, 1, -3\right)$

M_a = gaugino mass $a = 1, 2, 3$

$$\frac{d}{dt} g_a = \frac{b_a g_a^3}{16\pi^2}$$

Thus, $\frac{d}{dt}\left(\frac{M_a}{g_a^2}\right) = 0$ [at one-loop only]

In grand unified models, both g_a and M_a unify at the grand unification scale, M_X. That is,

$$g_a(M_X) = g_U$$
$$M_a(M_X) = M_U$$

Then,

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} \quad \text{at any scale.}$$

e.g.

$M_3 = \frac{g_3^2}{g_2^2} M_2 \simeq 3.5 M_2$ gluino mass

$M_1 = \frac{5}{3}\frac{g'^2}{g_2^2} M_2 \simeq 0.5 M_2$ bino mass (often the lightest SUSY particle)

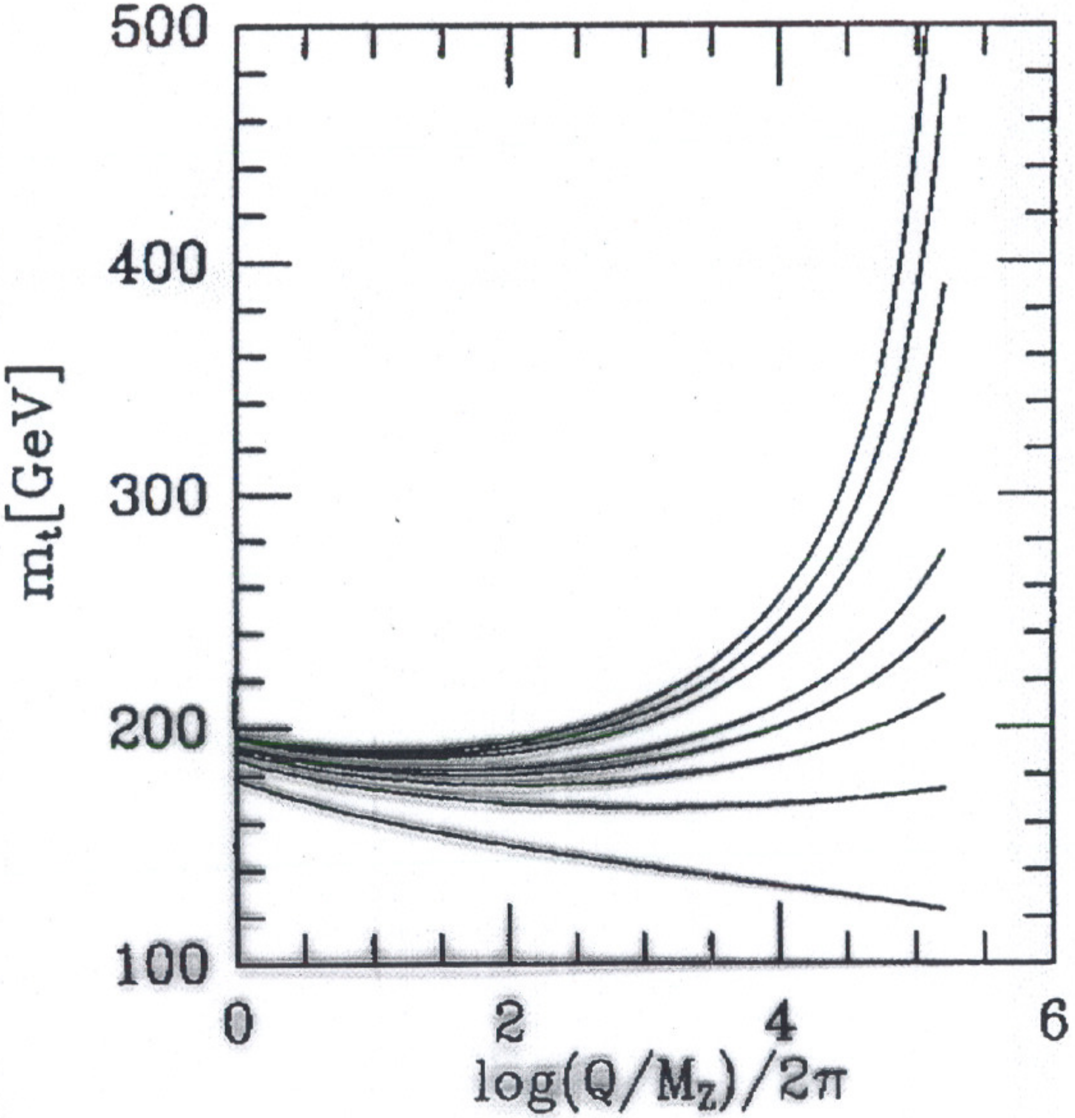


Fig. 1

$$16\pi^2 \frac{da_t}{dt} = a_t\left[18h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right] + 2a_b h_b h_t + h_t\left[\frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15}g_1^2 M_1\right]$$

$$16\pi^2 \frac{da_b}{dt} = a_b\left[18h_b^2 + h_t^2 + h_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right] + 2a_t h_b h_t + 2a_\tau h_\tau h_b + h_b\left[\frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15}g_1^2 M_1\right]$$

$$16\pi^2 \frac{da_\tau}{dt} = a_\tau\left[12h_\tau^2 + 3h_b^2 - 3g_2^2 - \frac{9}{5}g_1^2\right] + 6a_b h_b h_\tau + h_\tau\left[6g_2^2 M_2 + \frac{18}{5}g_1^2 M_1\right]$$

$$16\pi^2 \frac{db}{dt} = b\left[3h_t^2 + 3h_b^2 + h_\tau^2 - 3g_2^2 - \frac{3}{5}g_1^2\right] + \mu\left[6a_t h_t + 6a_b h_b + 2a_\tau h_\tau + 6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1\right]$$

$$16\pi^2 \frac{dm_{Q_3}^2}{dt} = Y_t + Y_b - \frac{32}{3}g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{15}g_1^2 M_1^2$$

$$16\pi^2 \frac{dm_{U_3}^2}{dt} = 2Y_t - \frac{32}{3}g_3^2 M_3^2 - \frac{32}{15}g_1^2 M_1^2$$

$$16\pi^2 \frac{dm_{D_3}^2}{dt} = 2Y_b - \frac{32}{3}g_3^2 M_3^2 - \frac{8}{15}g_1^2 M_1^2$$

$$16\pi^2 \frac{dm_{L_3}^2}{dt} = Y_\tau - 6g_2^2 M_2^2 - \frac{6}{5}g_1^2 M_1^2$$

$$16\pi^2 \frac{dm_{E_3}^2}{dt} = 2Y_t - \frac{24}{5}g_1^2 M_1^2$$

For the first two generations, the soft-SUSY-breaking squared-masses obey

$$16\pi^2 \frac{d}{dt} m_\phi^2 = -\sum_a 8 g_a^2 C_a^\phi M_a^2$$

where:

$$C_3^\phi = \begin{cases} 4/3 & \text{for } \phi = \tilde{Q}, \tilde{U}, \tilde{D} \\ 0 & \text{for } \phi = \tilde{L}, \tilde{E}, H_U, H_D \end{cases}$$

$$C_2^\phi = \begin{cases} 3/4 & \text{for } \phi = \tilde{Q}, \tilde{L}, H_U, H_D \\ 0 & \text{for } \phi = \tilde{U}, \tilde{D}, \tilde{E} \end{cases}$$

$$C_1^\phi = \frac{3}{20} Y_\phi^2$$ where Y_ϕ is the hypercharge of ϕ.

Finally, the soft-SUSY-breaking squared-masses for H_U and H_D satisfy:

$$16\pi^2 \frac{dm_{H_U}^2}{dt} = 3X_t - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2$$

$$16\pi^2 \frac{dm_{H_D}^2}{dt} = 3X_b + X_t - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2$$

Since $X_t = 2h_t^2 (m_{H_U}^2 + m_{Q_3}^2 + m_{U_3}^2) + 2a_t^2$

and $h_t \simeq 1$, we see that $\frac{dm_{H_U}^2}{dt} > 0$. That is, $m_{H_U}^2$ decreases as one evolves from the high energy scale to the low-energy scale. Thus, $m_{H_U}^2$ will be the first squared-mass to be driven negative, thus triggering electroweak symmetry breaking. This is a feasable mechanism because h_t is of $O(1)$, or equivalently because m_t is so large.

Constraining SUSY—Top-down

Models of SUSY-Breaking

Gravity-mediated SUSY-breaking

- SUSY-breaking effects mediated by Planck-scale physics

- The minimal model (mSUGRA) assumes a universal scalar mass, m_0, a universal gaugino mass, $m_{1/2}$, and a universal A-parameter, A_0 at the Planck scale. In addition, the parameters μ and B can be traded in for the Higgs vevs, v_d and v_u, with a two-fold ambiguity in $\mathrm{sign}(\mu)$. The W mass fixes $v_d^2 + v_u^2 = (246\ \mathrm{GeV})^2$, while $\tan\beta \equiv v_u/v_d$ remains a free parameter.

- Use RGEs to predict the MSSM spectrum. In particular,

$$M_3 = (g_3^2/g_2^2)M_2\ , \qquad M_1 = (5g_1^2/3g_2^2)M_2 \simeq 0.5 M_2$$

- $m_{3/2} \sim 1$ TeV; $\widetilde{g}_{3/2}$ is irrelevant for phenomenology.

Anomaly-mediated SUSY-breaking (AMSB)

Randall and Sundrum
Giudice, Luty, Murayama and Rattazzi

- A model-independent contribution to super-partner masses (and A-terms) arises from the super-conformal anomaly of supergravity.

- If tree-level gaugino masses are absent, then

$$M_i \simeq \frac{b_i g_i^2}{16\pi^2} m_{3/2} \,,$$

where b_i are the coefficients of the MSSM gauge beta-functions: $(b_1, b_2, b_3) = (33/5, 1, -3)$.

Gauge-mediated SUSY-breaking (GMSB)

Dine, Nelson and Shirman

- SUSY-breaking effects mediated by gauge forces generated at intermediate-scales (between m_Z and $M_{\rm PL}$)

- $m_{3/2} \sim 1$ eV–1 keV with phenomenological consequences