SARGE: for generating QCD-antennas

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Abstract

SARGE [1] is an algorithm to generate random massless momenta in phase space, with a distribution that contains the kinematical pole structure that is typically found in multi-parton QCD-processes. The reason for its need is explained, and it is compared with RAMBO in the calculation of cross-sections of QCD-processes.

Considering that many multi-jet processes will occur in future hadron colliders, such as the LHC, it is necessary to calculate their cross-sections. A part of the amplitude of these processes consists of a multi-parton QCD-amplitude, and it is well known [2] that the leading kinematic singularity structure of the squared matrix elements is given by the so-called *antenna pole structure* (APS). In particular, for n gluons it is given by all permutations in the momenta of

$$\frac{1}{(p_1p_2)(p_2p_3)(p_3p_4)\cdots(p_{n-1}p_n)(p_np_1)} \quad , \tag{1}$$

where (p_ip_j) denotes the Lorentz invariant scalar product of the gluon momenta p_i and p_j. Actually, it is this kinematical structure that is implemented in algorithms based on the so called SPHEL approximation to calculate the amplitudes [2]. But it is expected, and observed, that the same structure occurs in the exact matrix elements (cf. [3]).

For the integration of the differential cross-sections of the processes under consideration, the Monte Carlo method is the only option, and a phase space

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(PS) generator is needed. `RAMBO` [4] is a robust and efficient algorithm to generate any number of random massless momenta in their center-of-mass frame (CMF) with a given energy, but generates the momenta distributed uniformly in PS, so that a large number of events is needed to integrate integrands with the APS to acceptable precision. When the evaluation of the integrand is time-consuming, which is the case for the exact matrix elements, this is highly inconvenient. The table below givens the cpu-times in seconds to evaluate the matrix element with n final gluons in *one* phase space point.

n	4	5	6	7
`SPHEL`	5.40×10^{-5}	2.70×10^{-4}	1.80×10^{-3}	1.41×10^{-2}
exact	3.07×10^{-1}	1.08	3.35	10.92

A solution to this problem is importance sampling, and `SARGE` is an explicit implementation that generates any number of random massless momenta in their CMF with a given energy, distributed with a density that contains the APS of (1). It takes account for a substantial reduction in cpu-time in the calculation of cross-sections of multi-parton processes, because it needs far less PS points than `RAMBO` to reach the same precision. As an illustration, we present in the table below the cpu-times in hours and the number of PS points needed to integrate the squared `SPHEL`-amplitudes to a given precision, both with `RAMBO` and `SARGE`. The cpu-times for the exact amplitudes are extrapolated from the previous table and the `SPHEL`-results.

	preci-	SPHEL: cpu-time		SPHEL: # PS points		exact: cpu-time	
n	sion	`RAMBO`	`SARGE`	`RAMBO`	`SARGE`	`RAMBO`	`SARGE`
4	1%	0.198	0.0172	3,065,227	40,910	262	3.51
5	1%	0.286	0.0758	1,712,518	36,385	514	11.7
6	1%	0.685	0.471	700,482	34,095	653	32.2
7	3%	0.224	0.452	49,915	3,256	152	10.3

The cuts used were a minimal transverse momentum of 0.04 times the CM-energy, and a minimal angle between the partons of 30°.

References

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