Positronium Decay and Dispersion Relations

S. Trine *

Institut de Physique Théorique, Université Catholique de Louvain,
Chemin du Cyclotron 2, B-1348 Louvain-la-Neuve, Belgium

Positronium is a bound state of electron and positron. The binding is due to pure electromagnetic interactions; that's why it is considered as an ideal test ground for bound state treatment in Quantum Field Theory. We argue that dispersion relations can give us more insight into this bound state decay, showing in particular that some $O(\alpha^2)$ corrections to the decay rate are missing in the standard approach.

The standard amplitude [1] for parapositronium (spin 0) or orthopositronium (spin 1) decay into an even or an odd number of photons respectively is given by the convolution of the on-shell scattering amplitude for e^+e^- into photons with the bound state wavefunction. The integral is carried over all e^+e^- relative momenta and the intermediate electron and positron are constrained to the required spin state (0 or 1) through a spin projector. However, this standard formula seems to be in conflict with energy conservation as the intermediate electron and positron are taken on-shell, being thus heavier than the initial bound state. Besides, the spin projector commonly used is the spin projector for particles at rest, while electron and positron are moving particles. The behaviour of the standard amplitude in the soft photon limit (in the case where this limit is kinematically allowed) is also problematic, as it is in conflict with Low's theorem [2].

In order to solve these problems, we turn to a loop model, where positronium decays into *virtual* electron and positron, which subsequently annihilate into photons. The coupling of positronium to its constituents is described by an effective form factor, proportional to the bound state wavefunction. In such a model, the intermediate electron and positron are always off-shell

* *Electronic mail address:* trine@fyma.ucl.ac.be

and the energy is manifestly conserved throughout. We then use an unsubstracted dispersion relation [3] in order to express the loop model amplitude as the integral of the imaginary part over the bound state mass.

In the case of parapositronium decay into two photons, the loop model amplitude corresponding to the vertical cuts just gives back the factorized standard amplitude, except for the spin projector. Indeed, we find that the projector for particles at rest must be replaced by the projector for moving electron and positron. This brings new $O(\alpha^2)$ corrections to the usual prediction for the decay rate [4].

The standard amplitude for positronium decay must thus be seen as a dispersion integral corresponding to the vertical cuts, and not as a simple convolution integral. That's why the integral is carried over values of the total energy larger than two times the electron mass, where the imaginary part is nonzero. There is no conflict with energy conservation.

In the case of orthopositronium decay into three photons, we find that new, non-factorizable, contributions arise from dispersion relations. These correspond to the oblique cuts, which were not present in the parapositronium case. They are essential to restore the correct analytical properties of the amplitude in the soft photon limit and bring still new $O(\alpha^2)$ corrections to the decay rate [5].

To conclude, I would say that there is still a lot of work before knowing exactly the $O(\alpha^2)$ corrections to positronium decay.

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References

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[3] See for example B. Kniehl, Acta Phys. Polon. B27, 3631 (1996) (hep-ph/9607255).

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