Gauge Theory and Gravitation

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Abstract

General Relativity and gauge theories have much in common, but also differ in several respects. Some aspects related to this connection will be discussed in elementary terms. In particular, a pedagogical description of a simple ungeometrical approach to General Relativity is given, which starts with a massless spin-2 theory on Minkowski spacetime. Consistency arguments will lead to the conclusion that the original normal spin-2 gauge invariance has to be extended to general invariance, and one is bound to end up with General Relativity. In a second part I shall discuss some important points of Kaluza-Klein theories, notably the connection between gauge transformations in spacetime and higher-dimensional coordinate transformations.

1 Introduction

Einstein's general theory of relativity (GR) is a non-Abelian gauge theory of a special type. The common geometrical structure of non-Abelian gauge fields and GR – best expressed in terms of connections of fiber bundles – is now widely recognized. In [1] L. O'Raifeartaigh and I have reviewed the early history of gauge theory, from GR to the appearance of non-Abelian gauge

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theories in the fifties. This complicated process was initiated by H. Weyl who in 1918 made the first attempt to extend GR in order to describe gravitation and electromagnetism within a unifying geometrical framework. Another line of development was initiated one year later by T. Kaluza, who attempted a unification of GR and electromagnetism in five-dimensional gravity theory.

In this lecture historical aspects will only be mentioned in passing. For the benefit of those who have not yet studied GR I shall spend much of my time in developing a simple ungeometrical approach to GR which follows the pattern of well understood field theories in flat spacetime, such as electrodynamics. This approach has a long tradition ([2] - [7] are some classical references). One of the interesting lessons of this will be the following: although we shall start with a theory on Minkowski spacetime, it will in the end turn out that the flat metric is actually *unobservable*, and that the physical metric is *curved* and *dynamical*, subject to field equations which agree with Einstein's equations. The physical metric is determined by the spin-2 field of the theory. In other words, the initial flat metric turns out to be a kind of unobservable ether which we will eliminate. I hope that some of you will afterwards see GR in a somewhat different light.

In a second part I shall discuss some important points of Kaluza-Klein theories, in particular the interpretation of gauge-transformations in spacetime as subgroups of coordinate transformations in higher-dimensional spaces. For the Abelian case this was discovered by O. Klein in his classical work of 1926. Kaluza-Klein theories have since been revived several times and became recently again very topical. One reason is that string theory led to the suggestion that we might be living in a brane that is embedded in a higher-dimensional world. In such a scenario it is conceivable that the string scale is much lower than the Planck scale, even close to the electroweak scale. That would provide new avenues for solving the much discussed hierarchy problem. Also the profound cosmological constant problem might appear in a different light. A considerable number of papers have recently been devoted to these issues.

This brief overview indicates only a few aspects of the connection between gravity and gauge theories. String theory is at present the most interesting attempt to unify these highly successful theories in a truly unified manner.

2 A field theoretic (pedestrian) approach to GR

A natural question shortly after 1905 was: Why not develop a field theory of gravity in close analogy to electrodynamics? Einstein's first (unpublished) attempts went in this direction.

In trying to do this, it is useful to recall the following avenue to Maxwell's equations.

As starting point we adopt the following three ingredients:

(i) **Electrostatics:**

- field equations: $\text{curl}\boldsymbol{E} = 0, \quad \text{div}\boldsymbol{E} = \rho_e$ (1)
- equation of motion for test particle: $m\ddot{\boldsymbol{x}} = e\boldsymbol{E}$. (2)

(ii) **Lorentz-invariance** of the theory. Today this is a battle-field-tested general symmetry principle. [Historically, it grew out of electrodynamics, but now we have overwhelming direct experimental evidence for its validity. I do not have to stress this here at CERN.]

(iii) **Charge conservation:**

$$\partial_\mu j^\mu = 0, \quad j^\mu = (\rho_e, \boldsymbol{J}). \tag{3}$$

[This is presumably the most important and most general conservation law of physics.]

On the basis of (i)-(iii), Maxwell's equations are more than compelling. We argue as follows:

The source of the electromagnetic field must be the current j^μ, instead of ρ_e in (1). The Lorentz-invariant generalization of the inhomogeneous equation in (1) will be of the symbolic form $(\partial \cdot F)^\mu = -j^\mu$, where $\partial \cdot F$ denotes the divergence of a tensor field. Clearly, F has to be a tensor field of second rank, and the inhomogeneous field equation is naturally expected to be of the form $\partial_\nu F^{\mu\nu} = -j^\mu$.

Now, following Maxwell, we require that current conservation is an automatic consequence of the field equations. This can only be the case if $F^{\mu\nu}$ is antisymmetric. The field tensor then transforms *irreducibly* with respect to the homogeneous Lorentz group. With similar arguments one is also led to the homogeneous Maxwell equations. Note that Lorentz-invariance implies, in particular, the existence of the magnetic field for nonstatic situations. Similarly, we shall predict in gravity theory the existence of a gravitomagnetic field.

This compelling reasoning of guessing the correct relativistic field equations is now taken as a model for gravity. We start again from three similar ingredients:

(i) **Static limit** (Newtonian theory): The gravitoelectric field $\boldsymbol{g}$ (gravitational acceleration of test bodies) satisfies the field equations:

$$\operatorname{curl} \boldsymbol{g} = 0, \quad \operatorname{div} \boldsymbol{g} = -4\pi G\rho \tag{4}$$

or in terms of the potential φ,

$$\boldsymbol{g} = -\boldsymbol{\nabla}\varphi, \quad \Delta\varphi = 4\pi G\rho. \tag{5}$$

The equation of motion for a test body is universal (weak equivalence principle):

$$\ddot{\boldsymbol{x}} = \boldsymbol{g}(\boldsymbol{x}), \quad (m_i = m_g). \tag{6}$$

(ii) **Lorentz-invariance**;

(iii) **Energy-momentum conservation:**

$$\partial_\nu T^{\mu\nu} = 0. \tag{7}$$

Since this is so similar to ED, one expects at first sight that it should not be difficult to find a satisfactory Lorentz-invariant theory of gravity. Let us try this.

First, we have to find out which type of field has to be chosen. A priori, we have the possibilities 0, 1 or 2 for the spin. Among these the spin-1 option has to be excluded, because it would lead to *repulsion*. That this is unavoidable was already known to Maxwell. Otherwise, the gravitational waves would have negative energy and the world would be unstable. (This will soon become clear.)

2.1 Scalar theory

It is instructive to consider first the simplest case of a scalar theory (Einstein, Nordström, v. Laue [8]). (I do this also because scalar-tensor theories are still under current discussion, see section 3.)

The field equation for the scalar field φ in the *weak* field case (linear field equation) is unique:

$$\Box\varphi = -4\pi G T, \quad T := T^{\mu}_{\mu}. \tag{8}$$

(For a static situation this reduces to the second equation in (5).)

We formulate the equation of motion of a test particle in terms of a Lagrangian. For weak fields this is again unique:

$$L(x^{\mu}, \dot{x}^{\mu}) = -\sqrt{\eta_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}(1+\varphi), \tag{9}$$

because only for this the Newtonian limit for weak static fields and small velocities of the test bodies comes out right:

$$L(\boldsymbol{x}, \dot{\boldsymbol{x}}) \approx \frac{1}{2}\dot{\boldsymbol{x}}^2 - \varphi + \text{const.}$$

The basic equations (8) and (9) imply a perihelion motion of the planets, but this comes out wrong, even the sign is incorrect. One finds (-1/6) times the value of GR. In spite of this failure I add some further instructive remarks.

First, I want to emphasize that the interaction is necessarily *attractive*, independent of the matter content. To show this, we start from the general form of the Lagrangian density for the scalar theory

$$\mathcal{L} = \frac{1}{2}\partial_{\mu}\phi\,\partial^{\mu}\phi - g\,T\cdot\phi + \mathcal{L}_{mat}. \tag{10}$$

(ϕ is proportional to φ; g is a coupling constant.)

Note first that only g^2 is significant: Setting $\tilde{\phi} = g\phi$, we have

$$\mathcal{L} = \frac{1}{2g^2}\partial_{\mu}\tilde{\phi}\,\partial^{\mu}\tilde{\phi} - T\cdot\tilde{\phi} + \mathcal{L}_{mat},$$

involving only g^2. Next, it has to be emphasized that it is not allowed to replace g^2 by $-g^2$, otherwise the field energy of the gravitational field would be negative. (This “solution” of the energy problem does not work.) Finally, we consider the field energy for *static* sources.

The total (canonical) energy-momentum tensor

$$T^{\mu}_{\nu} = \frac{\partial \mathcal{L}}{\partial \phi_{,\mu}} \phi_{,\nu} + \ldots - \delta^{\mu}{}_{\nu} \mathcal{L}$$

gives for the ϕ-contribution:

$$(T_\phi)_{\mu\nu} = \partial_\mu \phi \, \partial_\nu \phi - \frac{1}{2} \eta_{\mu\nu} \partial_\lambda \phi \, \partial^\lambda \phi + \eta_{\mu\nu} g \, T \phi.$$

For the corresponding total energy we find

$$\begin{aligned} E &= \int (T_\phi)_{00} d^3x = \frac{1}{2} \int \left[(\boldsymbol{\nabla} \phi)^2 + 2g \, T \phi \right] d^3x \\ &= \frac{1}{2} \int \left[\phi \, (-\Delta \phi) + 2g \, T \phi \right] d^3x = \frac{1}{2} \, g \int T \phi \, d^3x. \end{aligned}$$

Since $\Delta \phi = g \, T$, we have

$$\phi(\boldsymbol{x}) = -\frac{g}{4\pi} \int \frac{T(\boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|} \, d^3x'.$$

Inserting this above gives finally

$$E = -\frac{g^2}{4\pi} \frac{1}{2} \int \frac{T(\boldsymbol{x}) T(\boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|} d^3x \, d^3x',$$

showing that indeed the interaction is attractive.

This can also be worked out in quantum field theory by computing the effective potential corresponding to the one-particle exchange diagram with the interaction Lagrangian $\mathcal{L}_{int} = g \, \bar{\psi} \psi \, \phi_{m=0}$. One finds

$$V_{eff} = -\frac{g^2}{4\pi} \frac{1}{|\boldsymbol{x} - \boldsymbol{x}'|},$$

both for fermion-fermion and fermion-antifermion interactions. The same result is found for the exchange of massless spin-2 particles, while for spin-1 we obtain *repulsion* between particles, and attraction between particles and antiparticles.

The scalar theory predicted that there is *no light deflection*, simply because the trace of the electromagnetic energy-momentum tensor vanishes. For this reason Einstein urged in 1913 astronomers (Erwin Freundlich in Potsdam) to measure the light deflection during the solar eclipse the coming year in the Krim. Shortly before the event the first world war broke out.

Over night Freundlich and his German colleagues were captured as prisoners of war and it took another five years before the light deflection was observed.

Quite independently of the failures mentioned so far, an even more profound difficulty of the scalar theory is often claimed. The argument is based on the observation that the trace $T(\boldsymbol{x},t)$ for a moving particle is given by

$$T(\boldsymbol{x},t) = m\sqrt{1-\boldsymbol{v}^2}\,\delta^3(\boldsymbol{x}-\boldsymbol{z}(t)),$$

where $\boldsymbol{z}(t)$ is the position of the particle, $\boldsymbol{v} = \dot{\boldsymbol{z}}$. The momentum dependence shows that the moving particle generates a *weaker* gravitational field than a particle at rest.

Now, consider two boxes each containing N particles of mass m, initially at rest (gas at zero temperature). We imagine that the two containers 1 and 2 are connected by a rigid rod. The two boxes gravitationally attract each other. We can arrange things such that the forces balance each other and that the system is at rest. Suppose now that the rest mass of a single particle in box 2 is completely transformed into kinetic energy of the remaining particles. This does not change the inertial mass of container 2, but apparently the *active* gravitational mass becomes smaller, and the total system begins to accelerate. Terrible!

There is a subtle error in this argument. The moving particles are bouncing on the wall of box 2. This induces a surface tension, generating an additional gravitational field. It is easy to show that it is because of this that the total system remains at rest.

This nicely illustrates that the equivalence principle is a subtle and profound property of gravity.

Some final remarks on the scalar theory. So far we have only considered weak fields generated by T_{matter}. It is however, more than natural that the energy-momentum tensor of the gravitational field also acts as a source, so that the theory has to be nonlinear. The nonlinear generalization of Nordström's theory was set up by Einstein and Fokker in 1914 [9]. In a non-geometrical (flat-spacetime) formulation the Lagrangian is given by

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi + \mathcal{L}_{mat}\left[\psi;(1+\kappa\varphi)^2\eta_{\mu\nu}\right](1+\kappa\varphi)^4;$$

in particular, the flat metric $\eta_{\mu\nu}$ in $\mathcal{L}_{mat}$ is replaced by $(1+\kappa\varphi)^2\eta_{\mu\nu}$, $\kappa^2 = 8\pi G$.

One can eliminate the Minkowski metric and replace it by a "physical metric":

$$g_{\mu\nu} = (1+\kappa\varphi)^2\eta_{\mu\nu}\,.$$

For example, only relative to this metric the Compton wave length is constant, i.e., not spacetime dependent.

Einstein and Fokker gave a geometrical formulation of the theory. This can be summarized as follows:

(i) spacetime is conformally flat: Weyl tensor = 0;

(ii) field equation: $R = 24\pi G T$;

(iii) test particles follow geodesics.

In adapted coordinates, with $g_{\mu\nu} = \phi^2 \eta_{\mu\nu}$, one finds

$$R = -6\eta^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi/\phi^3,$$

and the field equation becomes

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\phi = -4\pi G\phi^3 T.$$

The Einstein-Fokker theory is generally covariant (as emphasized in the original paper), however, *not* generally *invariant*. I use this opportunity to point out the crucial difference of the two concepts. For a long time people (including Einstein) were not fully aware of this and this caused lots of confusion and strange controversies. (See, e.g., the preface of Fock's book on GR.)

The *invariance* group of a theory is the subgroup of the covariance group that leaves the absolute, non-dynamical elements of the theory invariant. In the Einstein-Fokker theory the conformal structure is an *absolute* element, and therefore the invariance group is the *conformal group*, whence a finite dimensional Lie group. In GR, on the other hand, the metric is entirely dynamical, and therefore the covariance group is at the same time also the invariance group. For this reason, "general relativity" is an appropriate naming, never mind Fock and others.

A very remarkable property of the Einstein-Fokker theory is, that it satisfies even the *strong* equivalence principle. Beside GR this is (to my knowledge) the only theory which does this. The Einstein-Fokker theory shows, and this is a bit puzzling, that the equivalence principle does *not* imply light deflection. (For further discussion of this, see [10].)

This was probably too much on the scalar theory, but I wanted to make some points of general significance which can more easily be explained in this context. Let us now turn to the tensor theory.

2.2 Tensor (spin-2) theory

We are led to study the spin-2 option. (There are no consistent higher spin equations with interaction.) This means that we try to describe the gravitational field by a symmetric tensor field $h_{\mu\nu}$.

Such a field has 10 components. On the other hand, we learned from Wigner that in the massless case there are only *two* degrees of freedom. How do we achieve the truncation from 10 tow 2 ?

Recall first the situation in the *massive* case. There we can require that the trace $h = h^{\mu}_{\ \mu}$ vanishes, and then the field $h_{\mu\nu}$ transforms with respect to the homogeneous Lorentz group irreducibly as $D^{(1,1)}$ (in standard notation). With respect to the subgroup of rotations this reduces to the reducible representation

$$D^1 \otimes D^1 = D^2 \oplus D^1 \oplus D^0.$$

The corresponding unwanted spin-1 and spin-0 components are then eliminated by imposing 4 subsidery conditions:

$$\partial_\mu h^{\mu}_{\ \nu} = 0.$$

The remaining 5 degrees of freedom describe (after quantisation) massive spin-2 particles (Pauli and Fierz [11]; see, e.g., the classical book of G. Wentzel [12]).

In the *massless* case we have to declare certain classes of fields as physically equivalent, by imposing - as in ED - a *gauge invariance.* The gauge transformations are

$$h_{\mu\nu} \longrightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \tag{11}$$

where ξ_μ is an arbitrary vector field.

Let us first consider the *free* spin-2 theory which is unique (Pauli and Fierz):

$$\mathcal{L} = \frac{1}{4} h_{\mu\nu,\sigma} h^{\mu\nu,\sigma} - \frac{1}{2} h_{\mu\nu,\sigma} h^{\sigma\nu,\mu} - \frac{1}{4} h_{,\sigma} h^{,\sigma} - \frac{1}{2} h_{,\sigma} h^{\nu\sigma}{}_{,\nu}\,. \tag{12}$$

Let $G_{\mu\nu}$ denote the Euler-Lagrange derivative of $\mathcal{L}$ (up to a sign),

$$\begin{aligned} G_{\mu\nu} &= \frac{1}{2}\partial^\sigma \partial_\sigma h_{\mu\nu} + \partial_\mu \partial_\nu h - \partial_\nu \partial^\sigma h_{\mu\sigma} - \partial_\mu \partial^\sigma h_{\sigma\nu} \\ &+ \eta_{\mu\nu} \left(\partial^\alpha \partial^\beta h_{\alpha\beta} - \partial^\sigma \partial_\sigma h \right). \end{aligned} \tag{13}$$

The free field equations

$$G_{\mu\nu} = 0 \tag{14}$$

are identical to the linearized Einstein equations and describe, for instance, the propagation of weak gravitational fields.

The gauge invariance of $\mathcal{L}$ (modulo a divergence) implies the identity

$$\partial_\nu G^{\mu\nu} \equiv 0 \tag{15}$$

("linearized Bianchi identity"). [This should be regarded in analogy to the identity $\partial_\mu(\Box A^\mu - \partial^\mu \partial_\nu A^\nu) \equiv 0$ for the left-hand side of Maxwell's equations.]

Let us now introduce couplings to matter. The simplest possibility is the linear coupling

$$\mathcal{L}_{int} = -\frac{1}{2}\kappa\, h_{\mu\nu} T^{\mu\nu}, \tag{16}$$

leading to the field equation

$$G^{\mu\nu} = -\frac{\kappa}{2} T^{\mu\nu}. \tag{17}$$

This can, however, not yet be the final equation, but only an approximation for weak fields. Indeed, the identity (15) implies $\partial_\nu T^{\mu\nu} = 0$ which is unacceptable (in contrast to the charge conservation of ED). For instance, the motion of a fluid would then not at all be affected by the gravitational field. Clearly, we must introduce a *back-reaction* on matter. Why not just add to $T^{\mu\nu}$ in (17) the energy-momentum tensor ${}^{(2)}t^{\mu\nu}$ which corresponds to the Pauli-Fierz Lagrangian (12)? But this modified equation cannot be derived from a Lagrangian and is still not consistent, but only the second step of an iteration process:

$$\mathcal{L}^{free} \longrightarrow^{(2)} t^{\mu\nu} \longrightarrow \mathcal{L}^{cubic} \longrightarrow^{(3)} t^{\mu\nu} \longrightarrow \ldots ?$$

The sequence of arrows has the following meaning:

A Lagrangian which gives the quadratic terms ${}^{(2)}t^{\mu\nu}$ in

$$G^{\mu\nu} = -\frac{\kappa}{2}\left(T^{\mu\nu} +^{(2)} t^{\mu\nu} +^{(3)} t^{\mu\nu} + \ldots\right) \tag{18}$$

must be cubic in $h_{\mu\nu}$, and in turn leads to cubic terms ${}^{(3)}t^{\mu\nu}$ of the gravitational energy-momentum tensor. To produce these in the field equation (18), we need quartic terms in $h_{\mu\nu}$, etc. This is an infinite process. By a clever reorganization it stops already after the second step, and one arrives at field equations which are equivalent to Einstein's equations (S. Deser, [6]). The physical metric of GR is given in terms of $\phi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} h$ by

$$\sqrt{-g}\, g^{\mu\nu} = \eta^{\mu\nu} - \phi^{\mu\nu}, \quad g := \det(g_{\mu\nu}). \tag{19}$$

At this point one can reinterprete the theory geometrically. Thereby the flat metric disappears completely and one arrives in a pedestrian way at GR.

It should, however, be pointed out that, as a result of gauge invariance, the energy arguments in this reasoning are somewhat ambiguous. In view of this we shall later (section 2.5) discuss another approach.

Let us first pursue the approximate theory, keeping only ${}^{(2)}t^{\mu\nu}$ in (18). The linearized Bianchi identity (15) implies the conservation laws

$$\partial_\nu \left(T^{\mu\nu} +^{(2)} t^{\mu\nu}\right) = 0. \tag{20}$$

This gives

$$\partial_\nu T_\mu^{\ \nu} - \frac{\kappa}{2}\,\partial_\mu h_{\alpha\beta} T^{\alpha\beta} = 0. \tag{21}$$

This is analogous to $\partial_\nu T_\mu^{\ \nu} - F_{\mu\nu} j^\nu = 0$ in ED. For a charged test particle one obtains from this the Lorentz equation of motion: $\frac{d}{d\tau}(mu^\mu) = eF^\mu_{\ \nu} u^\nu$. Similarly, from (21) one can derive the following equation of motion for a neutral test particle in a gravitational field $h_{\alpha\beta}$:

$$\frac{du_\mu}{d\tau} + \kappa\left(\partial_\beta h_{\mu\alpha} - \frac{1}{2}\partial_\mu h_{\alpha\beta}\right) u^\alpha u^\beta = 0. \tag{22}$$

Geometrically, this is just the linearization of the geodesic equation for the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \tag{23}$$

(compare this with (19)). We shall soon see that this has not only a formal meaning.

2.3 Further discussion of the linearized theory

The linearization (equations (17) and (22)) is, by the way, already contained in Einstein's Zurich note book from 1912! This is now published in Volume 3 of the *Collected Papers*.

It is convenient to introduce the fields

$$\phi_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h \tag{24}$$

and impose the Lorentz-Hilbert gauge condition

$$\partial_\nu \phi^{\mu\nu} = 0. \tag{25}$$

(This is also contained in the 1912 notes of Einstein.) Then the field equations (17), with expression (13), become simply

$$\Box \phi^{\mu\nu} = -\kappa T^{\mu\nu}. \tag{26}$$

This is again very similar to what we are used to in ED. The retarded integral of the source $T^{\mu\nu}$ describes the emission of gravitational radiation and gives, for instance, the correct damping of binary pulsars.

In the *almost Newtonian limit* we have $T_{00} \approx \rho$, and all other components are much smaller. Then only ϕ_{00} survives:

$$\phi_{00}(\boldsymbol{x}) = -\frac{\kappa}{4\pi}\int \frac{\rho(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|} d^3x'.$$

Using also the Newtonian limit of the equation of motion (22), one finds that

$$\kappa\phi_{00} = 4U, \quad U : \text{Newtonian potential}, \tag{27}$$

and

$$\kappa^2 = 16\pi G. \tag{28}$$

Therefore,

$$\kappa h_{\mu\nu} = 2U\delta_{\mu\nu}\,. \tag{29}$$

In the next order (in $1/c$) we encounter for rotating sources the gravitomagnetic field. If the spatial stress T_{ij} can be neglected, the field equations (26) reduce to

$$\Box\phi_{ij} = 0, \quad \Box\phi_{0\mu} = -\kappa T_{0\mu}\,.$$

Thus $A_\mu := -\frac{1}{4}\kappa\phi_{0\mu}$ satisfy Maxwell type equations:

$$\Box A_\mu = J_\mu, \quad \partial^\mu A_\mu = 0,$$

where $J_\mu = 4\pi G T_{0\mu}$ is proportional to the mass-energy current density. (Note that $A_0 = -U$.) It is natural to define "gravitational electric and magnetic fields" $\boldsymbol{E}$ and $\boldsymbol{B}$ by the same formulas in terms of A_μ as in ED.

Let us now assume that the time derivatives of $\phi_{\mu\nu}$ can be neglected (quasi-stationary situations). Then $\Delta\phi_{ij} = 0$ in all space, whence $\phi_{ij} = 0$, and hence A_μ describes the gravitational field. In this approximation, the equation of motion (22) reduces for non-relativistic velocities to

$$\ddot{\boldsymbol{x}} = \boldsymbol{E} + 4\dot{\boldsymbol{x}} \wedge \boldsymbol{B}. \tag{30}$$

The factor 4 in the "magnetic term" reflects the spin-2 character of the gravitational field. The potentials are given by

$$A_0 = -U, \quad A_i(\boldsymbol{x}) = G\int \frac{T_{0i}(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|} d^3x'. \tag{31}$$

On the basis of this one obtains immediately the Lense-Thirring precession of a gyroscope, by simply translating the spin precession formula in electrodynamics: Substitute in the well-known formulae

$$\dot{\boldsymbol{S}} = \boldsymbol{\mu} \wedge \boldsymbol{B}, \quad \boldsymbol{\mu} = \frac{e}{2m}\boldsymbol{S}$$

of ED e by m and $\boldsymbol{B}$ by $4\boldsymbol{B}$. This gives the Lense-Thirring precession frequency

$$\boldsymbol{\omega}_{LT} = -2\mathrm{curl}\boldsymbol{A}. \tag{32}$$

The experiment Gravity Probe - B is supposed to measure this effect directly. Its launch is scheduled for early 2001 [13].

Next, we look at the *coupling to the electromagnetic field*:

$$\mathcal{L}_{int} = -\frac{\kappa}{2} h_{\mu\nu} T^{\mu\nu}_{elm}.$$

Maxwell's equations in the presence of the gravitational field $h_{\mu\nu}$ follow from the Lagrangian

$$\begin{aligned} \mathcal{L}_{elm} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu - \frac{\kappa}{2}h_{\mu\nu}T^{\mu\nu}_{elm} \\ &= -\frac{1}{4}\left(1+\frac{\kappa}{2}h\right)\left(\eta^{\mu\nu}-\kappa h^{\mu\nu}\right)\left(\eta^{\rho\sigma}-\kappa h^{\rho\sigma}\right)F_{\mu\rho}F_{\nu\sigma} - j^\mu A_\mu \\ &= -\frac{1}{4}\sqrt{-g}\, g^{\mu\nu}g^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} - j^\mu A_\mu. \end{aligned} \tag{33}$$

In the last equality sign we used the metric (23). (All equality signs are meant to hold in lowest order of $h_{\mu\nu}$.) This gives the modified Maxwell equations:

$$\partial_\sigma\left\{\left(1+\frac{\kappa}{2}h\right)\left(\eta^{\mu\nu}-\kappa h^{\mu\nu}\right)\left(\eta^{\rho\sigma}-\kappa h^{\rho\sigma}\right)F_{\mu\rho}\right\} = -j^\nu. \tag{34}$$

Expanding this for a diagonal $h_{\mu\nu}$ (as in (29)), one finds the standard form of Maxwell's equations for macroscopic media, with a dielectric constant ϵ and a magnetic permeability μ. For an almost Newtonian situation (equation (29)) these are given by

$$\epsilon = \mu = 1 - 2U. \tag{35}$$

The corresponding refraction index is

$$n = \sqrt{\epsilon\mu} = 1 - 2U. \tag{36}$$

This result implies the correct light deflection, and more generally "all" of gravitational-lensing theory.

P. Schneider discussed in his talk some major applications of gravitational lensing. During the past ~ 15 years this field has rapidly reached an important position in present day astronomy and astrophysics. An exciting new result is the first evidence for gravitational lensing by large-scale structures [14]. This has demonstrated the technical feasibility of using weak lensing surveys to restrict the cosmological parameters. With upcoming wide field CCD cameras much progress can be expected.

Another very interesting recent result is the measurement of galaxy-galaxy weak lensing from Sloan commissioning data which show that galaxy halos are very extended, so much that the assignment to individual galaxies becomes at some distance meaningless [15].

2.4 The renormalized physical metric

Now I come to a conceptually important point: we shall see that the flat Minkowski metric is not observable.

In order to see this we study the behavior of measuring sticks and clocks in a gravitational field. To be specific, we use the hydrogen atom for defining units of length and time. Moreover, we put the H-atom into the gravitational field outside of a spherically symmetric mass distribution of total mass M at the distance R from the center, where

$$\kappa h_{\mu\nu} = -\frac{2GM}{R}\delta_{\mu\nu}\,. \tag{37}$$

In this gravitational field Maxwell's equations (34) imply the following modified Laplace-Poisson equation for the scalar potential φ for the proton:

$$\left(1 + \frac{2GM}{R}\right)\Delta\varphi = -e\,\delta^3(\boldsymbol{x}),$$

whence

$$\varphi = \frac{e}{4\pi\epsilon}\frac{1}{r}, \quad \epsilon = \frac{1}{1 - 2GM/R}. \tag{38}$$

This is the Coulomb potential for the effective charge

$$e_{eff} = \frac{e}{\epsilon}. \tag{39}$$

The equation of motion (22) or (30) reduces to the Newtonian equation

$$m_{eff}\ddot{\boldsymbol{x}} = e_{eff}\frac{\boldsymbol{x}}{4\pi r^3}, \tag{40}$$

with

$$m_{eff} = m\left(1 + 3\frac{GM}{R}\right). \tag{41}$$

These effective quantities determine the *Bohr radius*

$$a_0 = \frac{\hbar^2}{m_{eff}(e_{eff}^2/4\pi)} = \frac{\hbar^2}{me^2/4\pi}\left(1 - \frac{GM}{R}\right) \tag{42}$$

and the *Rydberg frequency*

$$\omega = \frac{1}{2\hbar}m_{eff}\left(\frac{e_{eff}^2}{4\pi\hbar}\right)^2 = \frac{1}{2\hbar}m\left(\frac{e^2}{4\pi\hbar}\right)^2\left(1 - \frac{GM}{R}\right). \tag{43}$$

Thus, using a somewhat unphysical language, we would conclude that the atoms become smaller and frequencies (times) decrease (increase).

It is, however, clearly much more physical, to express this as follows: We maintain that the Bohr radius and the Rydberg frequency define always our units of length and time. This means that we have to rescale the original "unrenormalized" length (r) and time (t) in a spacetime dependent manner:

$$\tilde{r} = r(1 + GM/R), \quad \tilde{t} = t(1 - GM/R). \tag{44}$$

In other words, the *physical metric* is

$$d\tilde{s}^2 = \eta_{\mu\nu}d\tilde{x}^\mu d\tilde{x}^\nu = g_{\mu\nu}dx^\mu dx^\nu, \tag{45}$$

with

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}\,. \tag{46}$$

Clearly, this is a dynamical field. Let me stress that in the "renormalized" description the speed of light is always 1, while it would be spacetime dependent if we would maintain the fiction of a flat Minkowski metric.

In this sense spacetime is really curved. I illustrate this in Figure 1, which demonstrates the failure of Pythagoras' theorem.

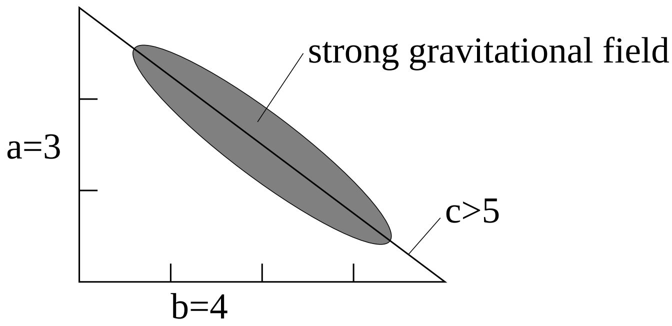

Figure 1: The failure of Pythagoras' theorem in the presence of a gravitational field.

Let me summarize this discussion:

The consequent development of the theory finally made it possible to eliminate the flat Minkowski metric, leading to a description in terms of a curved metric which has a direct physical meaning. The originally postulated Lorentz invariance turned out to be physically meaningless and plays no useful role. The flat Minkowski spacetime becomes a kind of unobservable ether. The conclusion is inevitable that spacetime is a pseudo-Riemannian (Lorentzian) manifold, whereby the metric is a dynamical field, subjected to field equations.

Once this geometrical, Einsteinian point of view is accepted, the field equations are practically unique (up to the cosmological term). For instance, the vacuum field equation of Einstein is the only one for which no derivatives of $g_{\mu\nu}$ higher than second order are allowed (Lovelock theorem). The only freedom is the cosmological term, to which we shall return later.

With this fundamental step we encounter qualitatively new phenomena. Two of the most dramatic ones are:

(i) The appearance of spacetime horizons, in particular for black hole solutions. *Direct* observational evidence for such objects would clearly be most important. The prospects for this look good. In the long run the most important results will come from gravitational wave astronomy, in particular from LISA.

(ii) Since we loose translational invariance of special relativistic field theories, energy-momentum conservation breaks down. This happens in a most dramatic way in inflationary cosmological models. Only for isolated systems we can still define total energy and momentum.

2.5 Uniqueness of nonlinearities and related issues

Before discussing some further important issues, I list the pros and cons of the field theoretic approach.

Advantages:

(i) This follows the pattern of well understood field theories (e.g., electrodynamics).

(ii) It directly predicts that nonrelativistic gravitation is attractive. [How this comes out in Einstein's geometric approach will be explained shortly.]

(iii) It leads unavoidably to a curved spacetime structure.

Note, that the equivalence principle is it not used as a cornerstone.

Disadvantages:

(i) The uniqueness of the nonlinearities is not so clear, while this is one of the most beautiful and convincing aspects of the geometric approach. I shall say more on this below.

(ii) The starting point is somewhat unphysical: Minkowski spacetime is assumed to be a good approximation. While this is the case in most applications, it is certainly not true close to black holes.

Two comments must be added to this.

1. What fixes the *sign* on the right hand side of Einstein's field equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}?$$

   Answer: Only for the correct sign do we have a positive energy theorem (PET). The latter says (in an untechnical language):

   One cannot construct an object out of "ordinary" matter, i.e., matter with positive local energy density, whose total energy (including gravitational contributions) is negative.

   This theorem is far from obvious. (A simplified proof was given by Witten [16], making use of spinors.) To emphasize its significance, I note the following:

(a) If objects with negative total energy (mass) could exist in GR, they would *repel* rather than attract nearby objects

(b) The PET implies, for example, that there are no regular interior solutions for the "anti-Schwarzschild" vacuum solution with negative mass, and hence there is attraction.

This is only true for the correct sign of Einstein's field equation. Changing the sign would lead to repulsion. Since the PET would, however, no more be true, we could presumably extract an *unlimited* amount of energy from a system with negative energy.

With the correct sign this bizarre situation cannot occur in GR. *Physically*, the reason is that as a system is compressed to take advantage of the negative gravitational binding energy, a *black hole* is inevitably formed which has *positive total energy.*

2. **Perturbation consistency and uniqueness**
The uniqueness of Einstein's vacuum equation $G_{\mu\nu}[\boldsymbol{g}] = 0$ (ignoring the Λ-term) can be translated into a perturbative consistency property, which we conversely may impose in the field theoretic approach to guarantee uniqueness. Let me explain this in some detail.

First, we decompose the Einstein tensor as

$$G_{\mu\nu} = G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)} + ..., \tag{47}$$

where $G_{\mu\nu}^{(n)}$ contains all terms of power n in the field variable $g_{\mu\nu}$. For the metric we make a perturbative (formal) expansion about the Minkowski metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon g_{\mu\nu}^{(1)} + \epsilon^2 g_{\mu\nu}^{(2)} + ... \tag{48}$$

Inserting this into the vacuum equation leads to an infinite chain of equations:

$$\begin{aligned} &G_{\mu\nu}^{(1)}[\boldsymbol{g}^{(1)}] = 0, \\ &G_{\mu\nu}^{(2)}[\boldsymbol{g}^{(1)}] + G_{\mu\nu}^{(1)}[\boldsymbol{g}^{(2)}] = 0, \\ &\ldots . \end{aligned} \tag{49}$$

Now we take the divergence of the second equation and use the linearized Bianchi identity

$$\partial^\nu G_{\mu\nu}^{(1)} \equiv 0. \tag{50}$$

This gives

$$\partial^\nu G^{(2)}_{\mu\nu}[\boldsymbol{g}^{(1)}] = 0, \tag{51}$$

and looks like an additional requirement for $g^{(1)}_{\alpha\beta}$, besides the linearized Einstein equation (first equation in (49)). That would be dangerous. However, equation (51) is *automatically* satisfied because of the Bianchi identity in second order. The same happens in higher order. Recall in this context, that the Bianchi identity can be regarded as a consequence of gauge invariance (general coordinate invariance).

The idea is now to turn the argument around. The question is: If we impose the correct linearized Einstein equations plus perturbative consistency, do the Einstein equations uniquely follow, if no derivatives higher than second order are allowed?

R. Wald [7] has analyzed this question, and came up with a qualified "yes". There are still some loose ends, but these are presumably not serious. T. Damour informed me at this meeting, that in collaboration with M. Henneaux the remaining gaps have been closed.

So general invariance is unavoidable, at least for reasonable matter couplings[1].

All this reflects once more the rigidity and beauty of GR. This theory must be contained in the low energy limit of any true unification.

Point 2. is also relevant for string-theory: The string excitations contain a massless spin-2 mode, and therefore GR has to be part of the field theoretic limit of string theory.

[1] There exist nonlinear *vacuum* theories with normal spin-2 gauge invariance: $h_{\mu\nu} \longrightarrow h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$. An example is

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(3)}, \quad \mathcal{L}^{(3)} = \left(R^{(1)}\right)^3,$$

where $R^{(1)}$ is the (gauge-invariant) linearized Ricci scalar. Explicitly,

$$\mathcal{L}^{(3)} = \left(\partial^\mu\partial^\nu h_{\mu\nu} - \Box h\right)^3.$$

3 Gauge transformations as higher-dimensional coordinate transformations

In this section a close connection between gauge transformations in spacetime and coordinate transformations in higher-dimensional spaces is established.

Already in 1919 Theodor Kaluza, a young mathematician (Privatdozent) in Königsberg, proposed a geometrical unification of gravitation and electromagnetism by extending spacetime to a five-dimensional pseudo-Riemannian manifold [17]. A few years later, shortly after the discovery of the Schrödinger equation, Oskar Klein improved and extended Kaluza's treatment, and revealed an interesting geometrical interpretation of gauge transformations [18]. This has a natural generalization to non-Abelian theories, as was discovered in 1953 – before the Yang-Mills work – by W. Pauli (see Sect. 3.3). Since then Kaluza-Klein theories experienced several revivals and have recently become again very topical.

3.1 The original Kaluza-Klein theory

In the Kaluza-Klein (KK) theory the extended spacetime is (locally) a direct product of the four-dimensional spacetime manifold and the unit circle S^1. The five-dimensional metric tensor $\hat{g}_{AB}$ $(A, B = 0, 1, 2, 3, 4, 5)$ is taken of the following form (KK ansatz):

$$(\hat{g}_{AB}) = \phi^{-1/3} \begin{pmatrix} g_{\mu\nu} - \kappa^2 \phi A_\mu A_\nu & -\kappa\phi A_\mu \\ -\kappa\phi A_\nu & -\phi \end{pmatrix}, \tag{52}$$

where $g = g_{\mu\nu} dx^\mu dx^\nu$ is the spacetime metric, and A_μ, ϕ are assumed to be independent of x^5; κ is a coupling constant to be determined. The convenience of the conformal factor $\phi^{-1/3}$ will become clear shortly.

I write this KK ansatz in a somewhat more condensed form. If we introduce the differential 1-form

$$\omega = dx^5 + \kappa A \tag{53}$$

$(A = A_\mu dx^\mu)$, we can write (52) as

$$\hat{g} = \phi^{-1/3} \left(g - \phi \, \omega \otimes \omega \right). \tag{54}$$

With Klein we consider the following subgroup of five-dimensional coordinate transformations:

$$x^\mu \mapsto x^\mu, \quad (\mu = 0, 1, 2, 3), \quad x^5 \mapsto x^5 + f(x^\mu). \tag{55}$$

Under these transformations, the pull-back of $\hat{g}$ is again of the form (54), with

$$g \to g, \quad \phi \to \phi, \quad A \to A + \frac{1}{\kappa} df. \tag{56}$$

Thus A transforms like a gauge potential under the Abelian gauge group (55), and is therefore interpreted as the electromagnetic gauge potential. (Geometrically, the 1-form ω is a connection form of a principle fiber bundle over spacetime with structure group $U(1)$.)

This interpretation is further justified by the most remarkable result derived by Kaluza and Klein, often called the Kaluza-Klein miracle. It turns out that the five-dimensional Ricci scalar $\hat{R}$ for the metric $\hat{g}$ splits as follows:

$$\hat{R} = \phi^{1/3} \left(R + \frac{1}{4}\kappa^2 \phi F_{\mu\nu}F^{\mu\nu} - \frac{1}{6\phi^2}(\nabla\phi)^2 + \frac{1}{3}\Delta \log \phi \right) \tag{57}$$

(R is the Ricci scalar of g). For $\phi \equiv 1$ this becomes the Lagrangian of the coupled Einstein-Maxwell system. In view of the gauge group (55), this split is actually no miracle, because no other gauge-invariant quantities can be formed.

With Klein we consider the $d = 5$ Einstein-Hilbert action

$$\hat{S} = -\frac{1}{\kappa^2 L} \int \hat{R}\sqrt{|\hat{g}|}\, d^5x, \tag{58}$$

assuming that the higher-dimensional space is a cylinder with $0 \leq x^5 \leq L = 2\pi R_5$. Since

$$\sqrt{|\hat{g}|}\, d^5x = \sqrt{-g}\, \phi^{-1/3} d^4x\, dx^5 \tag{59}$$

we obtain

$$\hat{S} = -\int \left[\frac{1}{\kappa^2} R + \frac{1}{4}\phi F_{\mu\nu}F^{\mu\nu} - \frac{1}{6\kappa^2\phi^2}(\nabla\phi)^2 \right] \sqrt{-g}\, d^4x. \tag{60}$$

Our choice of the conformal factor $\phi^{-1/3}$ in (54) was made so that the gravitational part in (60) is just the Einstein-Hilbert action, if we choose

$$\kappa^2 = 16\pi G. \tag{61}$$

For $\phi \equiv 1$ a beautiful geometrical unification of gravitation and electromagnetism is obtained.

We pause be noting that nobody in the early history of KK theory seems to have noticed the following inconsistency in putting $\phi \equiv 1$: The field equations for the dimensionally reduced action (60) are just the five-dimensional equations $\hat{R}_{AB} = 0$ for the KK ansatz (52). Among these, the ϕ equation, which is equivalent to $\hat{R}_{55} = 0$, becomes

$$\Box(\log\phi) = \frac{3}{4}\kappa^2 \phi F_{\mu\nu}F^{\mu\nu}. \tag{62}$$

For $\phi \equiv 1$ this implies the unphysical result $F_{\mu\nu}F^{\mu\nu} = 0$. This conclusion is avoided if one proceeds in the reverse order, i.e., by putting $\phi \equiv 1$ in the action (60) and varying afterwards. However, if the extra dimension is treated physical – a viewpoint adopted by Klein – it is clearly essential that one maintains consistency with the $d = 5$ field equations. This is an example of the crucial importance of scalar fields in KK theories (more on this below).

The KK ansatz (52) for the metric $\hat{g}_{AB}$ is, of course, not the most general five-dimensional metric. More general metrics, with nontrivial x^5-dependences, lead to very massive fields. This is most simply illustrated by considering the dimensional reduction of a massless five-dimensional scalar field $\psi(x^\mu, x^5)$, satisfying the wave equation in five dimensions. Since the fifth coordinate is periodic we can make a Fourier decomposition:

$$\psi(x, x^5) = \frac{1}{\sqrt{L}} \sum_{n\in Z} \psi_n(x) e^{inx^5/R_5}. \tag{63}$$

For the amplitudes $\psi_n(x)$ (KK modes) we obtain, for the metric (52) with $\phi \equiv 1$, the following four-dimensional wave equations

$$\left(D^\mu D_\mu - \frac{n^2}{R_5^2} \right) \psi_n = 0, \tag{64}$$

where D_μ is the doubly covariant derivative (with respect to $g_{\mu\nu}$ and A_μ) with the charge

$$q_n = n\frac{\kappa}{R_5}\,. \tag{65}$$

This shows that the mass of the nth KK mode is

$$m_n = |n|\frac{1}{R_5}\,. \tag{66}$$

Combining (65) with $q_n = ne$, we obtain

$$R_5 = \frac{2}{\sqrt{\alpha}}\,\ell_{Pl}, \tag{67}$$

where α is the fine-structure constant and ℓ_{Pl} is the Planck length.

Equations (65) and (66) show that the (bare) masses of all charged particles ($|n| \geq 1$) are of the order of the Planck mass

$$m_n = |n|\frac{\sqrt{\alpha}}{2} m_{Pl}. \tag{68}$$

This is, of course, a serious defect of the five-dimensional theory.

3.2 Remarks on the scalar field in the KK reduction

I come back to the scalar field appearing necessarily in the KK reduction. Much later ($\sim$ 1950), Jordan and Thiry tried to make use of the scalar field to obtain a theory in which the gravitational constant is replaced by a dynamical field.

Pauli wrote interesting critical letters about this theory to Jordan in 1953. In one of them (13 July) he emphasizes that, in theories containing both tensor and scalar fields, one has the freedom to make conformal transformations, and may fight about which metric is the "true" one. However, when there is matter, Pauli adds, the mass term in the Lagrangian for a Dirac field (for example) introduces a "natural choice". A natural unit of length is then provided by the Compton wave length. At this point Pauli makes the critical remark that in Jordan's theory electromagnetic standard lengths or times, like the frequencies of the Balmer lines and the radius of the hydrogen atom, then depend on the scalar field.

Three years later, Fierz [20] expanded on this, and specialized Jordan's theory (with two free constants) such that the Maxwell density, expressed in terms of the physical metric, is not multiplied with a spacetime-dependent function. Otherwise the vacuum would behave like a variable dielectric and this would have unwanted consequences, although the refraction is 1: The fine structure constant would become a function of spacetime, changing the spectra of galaxies over cosmological distances.

With these arguments Fierz arrives at a theory which was later called the Brans-Dicke theory. He did not, however, confront the theory with observations, because he did not believe in its physical relevance. [The intention of Fierz's publication was mainly pedagogical.]

3.3 Pauli's invention of non-Abelian KK theory in 1953

Let me add a few remarks on a largely unknown chapter of the early history of KK theories.

We have documents which show that W. Pauli developed in 1953 the first consistent generalization of the five-dimensional theory of Kaluza, Klein, Fock, and others to a higher dimensional internal space. He described his attempt in detailed letters to A. Paris and in two seminars in Zürich, which have been written up by one of his pupils (P. Gulmanelli). Here one finds a systematic development of non-Abelian Kaluza-Klein theory and the corresponding differential geometry. By specialization (independence of spinor fields of internal space) he got all important formulae of Yang and Mills, as he pointed out later (Feb. 1954) in a letter to Yang, after a talk of Yang in Princeton.

Pauli did not publish his study, because he was convinced that "one will always obtain vector mesons with rest mass zero" (Pauli to Paris, 6 Dec., 1953).

For a detailed discussion I refer to my contribution [21] at the Marcel Grossmann meeting in Rome (July 2000).

3.4 Are we living in a three-dimensional brane?

Recently a surprising modification of the old Kaluza-Klein picture has been discussed a lot. This is based on the idea, suggested by string theory, that ordinary matter could be confined to a three-dimensional world – our apparent Universe – while gravity could live in a higher-dimensional space. The most surprising aspect of such scenarios is that "large" extra-dimensions become conceivable, and therefore the fundamental mass (string) scale could be much lower than the three-dimensional Planck scale. Several authors have pointed out that the large extra dimension need not even be compact. Phenomenological and cosmological aspects of this scenario have been studied extensively.

For experimentalists a particularly interesting aspect of these developments is the suggestion that gravity may be modified at small distances, perhaps even at distances not much smaller than micrometers. Laboratory searches for deviations from the inverse square law at sub-millimeter scales are under way, and some new results have recently been announced by Adelberger and coworkers. The difficulty of these experiments is to control known electromagnetic and quantum mechanical (Casimir) contributions.

Time does not allow me to give an introduction to these developments. The basic papers are quoted in most recent articles of an exploding literature on these exciting but speculative new avenues (see, e.g., Ref. [22]).

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