Higher order Wess-Zumino consistency conditions in gauge theories

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Abstract

Higher order Wess-Zumino (WZ) consistency conditions in gauge theories are analyzed. Assuming that the Slavnov-Taylor (ST) identity has not been recovered at the first order in perturbation theory, second order WZ consistency conditions are used to discuss non-local contributions to the second order ST breaking term, in the example of the massive Abelian Higgs-Kibble model.

In gauge theories, nilpotency of the BRST transformations and the Quantum Action Principle (QAP) allow for a fully algebraic characterization of the first order Slavnov-Taylor (ST) breaking term. This algebraic characterization is based upon a set of constraints known as the first order Wess-Zumino (WZ) consistency conditions [1]. The WZ consistency conditions transform the problem of computing the first order ST breaking term into a cohomological problem: one just needs to find out the representatives of all non-trivial cohomology classes of the classical linearized ST operator in the space of local functionals with FP charge +1.

The restriction to the space of local functionals is an essential ingredient in this procedure. This restriction is a consequence of the QAP and crucially depends on the assumption that the ST identity is fulfilled at lower orders in the loop-wise expansion. Moreover, locality of the first order ST breaking

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term ensures that, if there are no anomalies, the ST breaking term can be compensated by a suitable choice of finite first order counterterms, so that the ST identity is fulfilled at the first order in perturbation theory.

In some cases, for example when the γ_5 matrix is involved, dimensional regularization could not be able to respect from the beginning the ST identity of the model. Thus one must carefully construct the needed finite first order counterterms to recover the ST identity for a non-anomalous theory.

One might however imagine to sidestep this corrective procedure and to go on renormalizing the theory at higher orders in perturbation theory. What happens to the next order ST breaking terms?

One can derive a set of higher order WZ consistency conditions starting from the algebraic identity

$$\mathcal{S}_\Gamma\left(\mathcal{S}(\Gamma)\right) = 0\,. \tag{1}$$

$\mathcal{S}_\Gamma$ is the linearized ST operator with respect to the full quantum effective action Γ.

We study eq.(1) in the example of the massive Abelian Higgs-Kibble model [2]. Suppose that first order normalization conditions have been chosen in such a way that $\mathcal{S}(\Gamma)^{(1)} \neq 0$. Then eq.(1) gives

$$\mathcal{S}_{\Gamma^{(0)}}(\Delta^{(2)}) + \mathcal{S}_{\Gamma^{(1)}}(\mathcal{S}(\Gamma)^{(1)}) = 0\,, \qquad \Delta^{(2)} \equiv \mathcal{S}(\Gamma)^{(2)}\,. \tag{2}$$

Notice that the QAP cannot be applied in this case to ensure that $\Delta^{(2)}$ is local. Indeed it turns out [3] that, because of the second term in the l.h.s. of eq.(2), $\Delta^{(2)}$ is actually a non-local functional of the fields and the external sources of the model. This is a major difference with respect to the case in which $\mathcal{S}(\Gamma)^{(1)} = 0$, where $\Delta^{(2)}$ is a local functional of the fields and the external sources. Moreover, it can be shown that $\Delta^{(2)}$ is an actual formal power series in the fields and external sources. This spoils the effectiveness of the WZ consistency conditions in constructing the ST breaking functional $\Delta^{(2)}$ and the possibility to recover at the second order the ST identity by fine-tuning the local second order finite counterterms, if the ST identity is not fulfilled at the first order in perturbation theory.

References

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