QCD_2 in the axial gauge

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Abstract

The 't Hooft model for the two-dimensional QCD in the limit of infinite number of colours is considered in the axial gauge and the Hamiltonian approach is developed. The Hamiltonian of the model is diagonalized in the mesonic sector by means of the generalized Bogoliubov transformation. Massless solution of the bound state equation, chiral pion, which is exact in the chiral limit, is found explicitly. The chiral properties of the model are considered using the developed formalism. Strong hadronic decay are discussed and the "Adler zero" condition is derived which states that the amplitude of the decay $A \to B + C$ vanishes identically if at least one of the final mesons is the pion.

The model for QCD in two dimensions in the limit of infinite number of colours suggested by 't Hooft in 1974 [1] is given by the Lagrangian

$$L(x) = -\frac{1}{4}TrF^2_{\mu\nu}(x) + \bar{q}(i\hat{\partial} - g\hat{A} - m)q, \quad \gamma = \frac{g^2N_C}{4\pi} \underset{N_C\to\infty}{\to} \mathrm{const} \qquad (1)$$

where we fix the axial gauge $A_1^a = 0$ for the gluonic field and use the following convention for γ-matrices: $\gamma_0 = \sigma_3$, $\gamma_1 = i\sigma_2$, $\gamma_5 = \gamma_0\gamma_1$. Principal value prescription is used to regularize the infrared divergences [2].

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This toy model attracts considerable attention as it possesses many features similar to those of four-dimensional QCD, like confinement and chiral symmetry breaking.

Introducing the dressed quark field defined by means of the Bogoliubov-Valatin angle θ, arranging the normal ordering of the Hamiltonian of the theory in this basis and minimizing the vacuum energy one can diagonalize the model in the quark sector that leads to the gap equation for θ [2]. As a next step creation and annihilation operators for bound $q\bar{q}$ states — mesons can be introduces. Performing the second, generalized, Bogoliubov transformation [3] one can arrive at the bound state equation [2] which provides diagonalization of the Hamiltonian in the mesonic sector. Each mesonic state is defined via two w.f. $\varphi_\pm$ describing forward and backward in time motion of the $q\bar{q}$ pair inside meson. The φ_- component is suppressed for heavy quarks and highly excited states, but it is of the same order of magnitude as φ_+ in case of pion — the massless (in the chiral limit) solution of the bound state equation. The explicit form of this solution is found in [3, 4] and the chiral properties of the model are reproduced in the given approach [3].

Strong decays with pions in the final state are of most interest in the hadronic spectroscopy and they may shed light on the chiral nature of the pion. One can study the amplitude of the decay $A \to \pi + C$ in the 't Hooft model using the Hamiltonian approach developed above or in the alternative matrix approach [2, 4]. In both cases the pion-quark-antiquark dressed vertex appears as a superposition $\Gamma_\pi(p,P) = -iP_\mu v_\mu(p,P) - iP_\mu a_\mu(p,P)$ where vector and axial-vector currents v_μ and a_μ are conserved in the weak sense in the chiral limit so that any amplitude of the type $M(A \to \pi + C)$ vanishes identically for any pion momentum [4]. The latter statement is the Adler selfconsistency condition ("Adler zero") [5] for the 't Hooft model.

References

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