A test of the Kugo-Ojima confinement criterion by lattice QCD

Sadataka Furui[a] and Hideo Nakajima[b] *

[a] School of Science and Engineering, Teikyo University

[b] Department of Information Science, Utsunomiya University

Abstract

The Kugo-Ojima confinement criterion is tested in the lattice Landau gauge QCD simulation. The dependence of the confinement signal on the definition of the gauge field $A_\mu(x)$ is studied and the comparison to the Dyson-Schwinger approach is made.

About two decades ago, Kugo and Ojima proposed a criterion for the absence of coloured massless asymptoptic states in Landau gauge QCD using the BRST symmetry[1]. On a lattice we measure

$$\frac{1}{V}\sum_{x,y} e^{-ip(x-y)}\langle \mathrm{tr}\left(\lambda^{a\dagger} D_\mu \frac{1}{\partial D}[-A_\nu \lambda^b]\right)_{xy}\rangle = (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) u^{ab}(p^2), \qquad (1)$$

where D is the lattice covariant derivative[2], λ^a is a normalized ($\mathrm{tr}\lambda^{a\dagger}\lambda^b = \delta^{ab}$) antihermitian basis of Lie algebra, V a lattice volume.

Usually, the gauge field on lattice $A_\mu(x)$ is defined from the link variable $U_\mu(x)$ as $A_{x,\mu} = \frac{1}{2}(U_{x,\mu} - U^\dagger_{x,\mu})|_{traceless\ part}$, which we call U-linear version. A more natural definition[2] is $U_{x,\mu} = \exp A_{x,\mu}, \quad A^\dagger_{x,\mu} = -A_{x,\mu}$, which we call $\log U$ version.

We observe that the absolute value of the Kugo-Ojima parameter $u^{ab}(0)$ at $\beta = 5.5$ is about -0.7 in the $\log U$ and about -0.6 in the U-linear version. Deviation from -1 is expected from the absence of strict BRST symmetry on the lattice. The ghost propagator is infrared divergent, while the gluon propagator is infrared finite[2].

*furui@liberal.umb.teikyo-u.ac.jp, nakajima@is.utsunomiya-u.ac.jp

The QCD running coupling $\alpha(\mu) = g^2/4\pi$ can be measured from the three gluon vertices[5] as, $g(\mu^2) = \frac{G_A{}^{(3)}(p_i{}^2,p_f{}^2,p_c{}^2)Z_3^{3/2}(\mu^2)}{G_A{}^{(2)}(p_i{}^2)G_A{}^{(2)}(p_f{}^2)G_A{}^{(2)}(p_c{}^2)}$. Since the lattice data of $G_A{}^{(2)}$ is infrared finite, in contrast to the conjecture that the gluon propagator is infrared vanishing[3], $g(\mu^2)$ decreases as μ decreases to $0.78/a$. This behaviour does not agree with the results of Dyson-Schwinger approach[4]. The preliminary results of the ghost-gluon-ghost coupling $\tilde{g}^2/4\pi$, where $\tilde{g}(\mu^2) = \frac{G_g{}^{(3)}(p_i{}^2,p_f{}^2,p_c{}^2)Z_3^{1/2}(\mu^2)\tilde{Z}_3(\mu^2)}{G_g{}^{(2)}(p_i{}^2)G_g{}^{(2)}(p_f{}^2)G_A{}^{(2)}(p_c{}^2)}$, at $\sum_\mu n_\mu^2 = 4$ and 8 shown in Fig2 yields Λ_{LATT} consistent to that obtained from $g^2/4\pi$ at $\mu > 1.2/a$, when $\Lambda_{\widetilde{MOM}} = 83.2\Lambda_{LATT}$ is assumed[5].

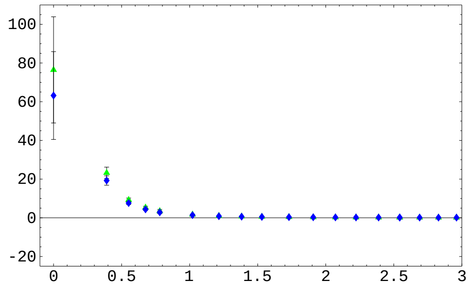

Figure 1: The gluon propagator as a function of the lattice momentum. $\beta = 6.0$, 16^4. Triangles are $\log U$ version and diamonds are U-linear version. 50 samples.

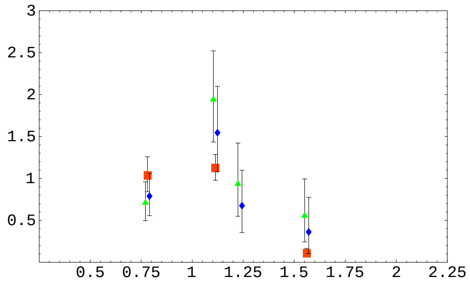

Figure 2: The QCD running coupling constant as a function of momentum μ. Triangles and diamonds are $g^2/4\pi$, the same as Fig1, and boxes are $\tilde{g}^2/4\pi$, $\log U$ version.

References

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