$U_A(1)$ PROBLEMS AND GLUON TOPOLOGY

– ANOMALOUS SYMMETRY IN QCD

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Abstract

Many of the distinctive and subtle features of the dynamics in the $U_A(1)$ channel in QCD can be related to gluon topology, more precisely to the topological susceptibility $\chi(k^2) = i \int d^4x \ e^{ikx} \langle 0|T \ Q(x) \ Q(0)|0\rangle$, where $Q = \frac{\alpha_s}{8\pi} \mathrm{tr} G_{\mu\nu} \tilde{G}^{\mu\nu}$ is the gluon topological charge density. The link is the $U_A(1)$ axial (ABJ) anomaly. In this lecture, we describe the anomalous $U_A(1)$ chiral Ward identities in a functional formalism and show how two apparently unrelated '$U_A(1)$ problems' – the mass of the η' and the violation of the Ellis-Jaffe sum rule in polarised deep-inelastic scattering – can be explained in terms of the gluon topological susceptibility. They are related through a $U_A(1)$ extension of the Goldberger-Treiman formula, which is derived here for QCD with both massless and massive quarks.

1. Introduction

The central theme of this school is symmetry, in particular the realisation of 'hidden' or 'spontaneously broken' symmetries in quantum field theory. Another especially interesting realisation of symmetry in QFT is exemplified by the flavour singlet chiral $U_A(1)$ symmetry in QCD with massless quarks. This is a subtle case, because although global chiral $U_A(1)$ is a symmetry of the classical lagrangian of QCD, the corresponding current is not conserved in the quantum theory due to the well-known ABJ anomaly[1]. Nevertheless, this 'anomalous symmetry' has important implications for the phenomenology of QCD.

In this lecture, we describe how to analyse the consequences of the $U_A(1)$ symmetry in QCD, using the formalism of anomalous chiral Ward identities. This formalism is then used to relate two '$U_A(1)$ problems' – the mass of the η' and the violation of the Ellis-Jaffe sum rule[2] in polarised deep-inelastic scattering (DIS) – to gluon topology in the form of the gluon topological susceptibility

$$\chi(k^2) = i \int d^4x \; e^{ikx} \langle 0|T \; Q(x) \; Q(0)|0\rangle \tag{1.1}$$

where $Q = \frac{\alpha_s}{8\pi} \mathrm{tr} G_{\mu\nu} \tilde{G}^{\mu\nu}$ is the gluon topological charge density.

The topological susceptibility is a fundamental correlation function in pure gluodynamics (Yang-Mills theory) or QCD itself and is the key to understanding much of the distinctive dynamics in the $U_A(1)$ channel. It has been studied non-perturbatively using lattice gauge theory[3-5], spectral sum rules[6-8], instanton models of the vacuum[9], etc., with the following results:

$$\begin{array}{llll} \text{Gluodynamics} & \chi^{YM}(0) \simeq -(180 \text{ MeV})^4 & \chi'_{YM}(0) \simeq -(10 \text{ MeV})^2 & \\ \text{QCD } (m_q = 0) & \chi(0) = 0 & \chi'(0) \simeq (26 \text{ MeV})^2 & \end{array} \tag{1.2}$$

where $\chi'(0) = \frac{d}{dk^2}\chi(k^2)\big|_{k=0}$. In gluodynamics, the value $\chi^{YM}(0)$ is well-established[3,6] and can be calculated in lattice gauge theory. In QCD with massless quarks, the result $\chi(0) = 0$ is an exact identity, following (as we show below) directly from the anomalous chiral Ward identities. The quoted value for $\chi'(0)$ in full QCD was obtained using spectral sum rules[7,8]. Lattice techniques are still being refined[4,5] to produce a reliable result.

The connection with topology arises as follows. Q is a total divergence, viz.

$$Q = \partial^\mu K_\mu \tag{1.3}$$

where $K_\mu = \frac{\alpha_s}{4\pi} \epsilon_{\mu\nu\rho\sigma} \mathrm{tr}\big(A^\nu G^{\rho\sigma} - \frac{1}{3} g A^\nu [A^\rho, A^\sigma]\big)$ is the Chern-Simons current. However, the integral over (Euclidean) spacetime of Q need not vanish. In fact, for gauge field configurations which become pure gauge at infinity,

$$\int d^4x \; Q = n \in \mathbf{Z} \tag{1.4}$$

where the integer n is the topological winding number (technically, an element of the homotopy group $\pi_3(SU(N_c))$, where $SU(N_c)$ is the gauge group[10, ch. 23]) or 'instanton number'. Instantons are classical field configurations which contribute to the path integral for which eq.(1.4) gives $n \neq 0$.

The connection with the $U_A(1)$ current in QCD arises through the famous ABJ anomaly. In a sense we shall make precise shortly, the flavour singlet axial current $J^0_{\mu 5}$ is not conserved in the quantum theory, even though it is the Noether current for the classical QCD lagrangian with massless quarks, but satisfies the anomaly equation (for $m_q = 0$)

$$\partial^\mu J^0_{\mu 5} - 2n_f Q \sim 0 \tag{1.5}$$

where n_f is the number of quark flavours. This identity provides the link between the quark dynamics and gluon topology.

Our first example of a '$U_A(1)$ problem' concerns the mass of the η'. This is much larger than would be the case if it were the pseudo-Goldstone boson for spontaneously broken $U_A(1)$, as we would expect in the absence of the anomaly. Indeed, for massless QCD, the η' would be an exact, massless Goldstone boson but for the anomaly. In section 3, we explain how the Goldstone theorem is circumvented by the anomaly and derive the Witten-Veneziano mass formula[11,12] for the η'. This relates, to leading order in the $1/N_c$ expansion, the mass of the η' to the topological susceptibility in pure gluodynamics.[(1)] For QCD with massless quarks, the formula is simply

$$m_{\eta'} \simeq \frac{\sqrt{6}}{f_\pi}\sqrt{\chi^{YM}(0)} \tag{1.6}$$

The second example concerns the much-publicised 'proton spin' problem (for a review, see e.g. ref.[16]), i.e. the violation of the Ellis-Jaffe sum rule observed in measurements of the first moment of the polarised proton structure function g_1^p in deep inelastic scattering (DIS). As explained in section 4, the first moment $\int dx g_1^p(x;Q^2)$ is related, through the OPE for the product of two electromagnetic currents, to the proton matrix elements of the $SU(3)$ flavour singlet and non-singlet axial currents $\langle p|J^a_{\mu 5}|p\rangle$, for $a = 0,3,8$. The corresponding 'axial charges' of the proton are denoted by $a^0(Q^2)$, a^3 and a^8, where the singlet charge (only) has an explicit dependence on the scale Q^2 of the deep inelastic process. This is due to the non-trivial renormalisation of the singlet axial current induced by the anomaly. We have found[17,18,19] the following formula relating the ratio of the flavour singlet and non-singlet axial charges to the slope of the gluon topological susceptibility:

$$\frac{a^0(Q^2)}{a^8} \simeq \frac{\sqrt{6}}{f_\pi}\sqrt{\chi'(0)}\big|_{Q^2} \tag{1.7}$$

The similarity to eq.(1.6) is striking. This formula is the basis of our quantitative resolution of the 'proton spin' problem[7], which is explained as a consequence of topological charge screening by the QCD vacuum.

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(1) In this lecture, we derive this and other results directly from the anomalous chiral Ward identities. For the related approach of using effective chiral lagrangians in the $1/N_c$ expansion to study η' physics, see e.g. refs.[13,14,15].

Although apparently quite different, these two examples are in fact intimately related through a $U_A(1)$ extension of the familiar Goldberger-Treiman relation. The essence of the conventional GT relation is that it links the dynamics of the pseudovector and pseudoscalar channels. This is just what we need to explain the OZI violation at the heart of the 'proton spin' problem, provided an extension of the GT relation to the anomalous $U_A(1)$ can be found. This was achieved in refs.[17,18,19] and underlies our result (1.7). In section 5, we derive the $U_A(1)$ GT relation. For massless QCD, we find

$$2m_N G_A^0 = 2n_f \sqrt{\chi'(0)}\ \hat{\Gamma}_{\eta^0 NN}\big|_{k=0} \tag{1.8}$$

(Here, G_A^a is just an alternative notation for the nucleon axial charges, with a more convenient $SU(3)$ normalisation.) $\hat{\Gamma}_{\eta^0 NN}$ is a vertex describing the coupling of the nucleon to the η^0 – an unphysical state in QCD which in the OZI limit (see section 3) is identified with the exact $U_A(1)$ Goldstone boson. For QCD with massive quarks and flavour $SU(3)$ breaking, the corresponding formula mixes the flavour singlet and non-singlet sectors. In this case, we find[8]

$$2m_N G_A^a \ = \ F_{ab}\ \hat{\Gamma}_{\eta^b NN}\big|_{k=0} \tag{1.9}$$

where F is determined from

$$F_{ac}F_{cb}^T = \lim_{k=0} \frac{d}{dk^2}\ i \int dx\ e^{ikx}\ \langle 0|T\ \partial^\mu J_{\mu 5}^a(x)\ \partial^\nu J_{\nu 5}^b(0)|0\rangle \tag{1.10}$$

providing a natural generalisation of eq.(1.8).

2. Chiral Ward Identities and the Renormalisation Group

To study the phenomenology of the $U_A(1)$ anomaly, we will need to find the Green functions of the composite operators which couple to the relevant physical states. These are the currents and pseudoscalar operators $J_{\mu 5}^a$, Q, ϕ_5^a together with ϕ^a where

$$\begin{aligned} J_{\mu 5}^a &= \bar{q}\gamma_\mu\gamma_5 T^a q \qquad\qquad & Q &= \frac{\alpha_s}{8\pi}\mathrm{tr} G_{\mu\nu}\tilde{G}^{\mu\nu} \\ \phi_5^a &= \bar{q}\gamma_5 T^a q & \phi^a &= \bar{q}T^a q \end{aligned} \tag{2.1}$$

$G_{\mu\nu}$ is the field strength for the gluon field A_μ. In this notation, $T^i = \frac{1}{2}\lambda^i$ are flavour $SU(n_f)$ generators, and we include the singlet $U_A(1)$ generator $T^0 = \mathbf{1}$ and let the index $a = 0, i$. We will only need to consider fields where i corresponds to a generator in the Cartan sub-algebra, so that $a = 0, 3, 8$ for $n_f = 3$ quark flavours. We define d-symbols by $\{T^a, T^b\} = d_{abc}T^c$. Since this includes the flavour singlet $U_A(1)$ generator, they are only symmetric on the first two indices. For $n_f = 3$, the explicit values are $d_{000} = d_{033} = d_{088} = 2, d_{330} = d_{880} = 1/3, d_{338} = d_{383} = -d_{888} = 1/\sqrt{3}$. (For further notation and description of the formalism used here, see refs.[19,8].)

The Green functions, or correlation functions, are constructed from the generating functional $W[V_{\mu 5}^a, \theta, S_5^a, S^a]$, where $V_{\mu 5}^a, \theta, S_5^a, S^a$ are the sources for the composite operators $J_{\mu 5}^a, Q, \phi_5^a, \phi^a$ respectively. Functional derivatives of W yield Green functions which are '1PI' w.r.t. the designated fields (composite operators). Explicitly,

$$e^{iW} \;=\; \int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q \;\exp i\int d^4x\big(\mathcal{L}_{\rm QCD} + V_{\mu 5}^a J_{\mu 5}^a + \theta Q + S_5^a\phi_5^a + S^a\phi^a\big) \tag{2.2}$$

The chiral Ward identities are found by exploiting the invariance of W under a change of variables in the path integral corresponding to a chiral transformation $q \to e^{i\alpha T^a\gamma_5}q$. This gives

$$\int \mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q \,\Big[\partial^\mu J_{\mu 5}^a - 2n_f\delta^{a0}Q - d_{adc}m^d\phi_5^c - \delta\Big(i\int d^4x\mathcal{L}_{\rm QCD}\Big)\Big] \;\exp i\int d^4x(\ldots) \;=\; 0 \tag{2.3}$$

The terms in the square bracket are simply those arising from Noether's theorem, which defines the symmetry current through a functional derivative of the action, with the addition of the anomaly term. This can be understood as arising from the non-invariance of the path integral measure $\mathcal{D}\bar{q}\mathcal{D}q$ in a background gauge field A_μ. A careful derivation of the anomaly by this method can be found in several standard textbooks on QFT, e.g. ref.[10, ch. 22]. The chiral variation term w.r.t. the elementary fields is then simply re-expressed as a variation w.r.t. the sources, giving finally the functional form of the (anomalous) chiral Ward identities:

$$\partial_\mu\frac{\delta W}{\delta V_{\mu 5}^a} - 2n_f\delta_{a0}\frac{\delta W}{\delta\theta} - d_{adc}m^d\frac{\delta W}{\delta S_5^c} + d_{adc}S^d\frac{\delta W}{\delta S_5^c} - d_{adc}S_5^d\frac{\delta W}{\delta S^c} = 0 \tag{2.4}$$

This is the key equation which will be the basis of all the results derived in this lecture. It makes precise the anomaly equation (1.5).

It will be useful in what follows to introduce some streamlined notation. The quark mass matrix is written as m^aT^a, so that for $n_f = 3$,

$$\begin{pmatrix} m_u & 0 & 0\\ 0 & m_d & 0\\ 0 & 0 & m_s\end{pmatrix} = m^0\mathbf{1} + m^3T^3 + m^8T^8 \tag{2.5}$$

The chiral symmetry breaking condensates may be similarly written as

$$\begin{pmatrix} \langle\bar{u}u\rangle & 0 & 0\\ 0 & \langle\bar{d}d\rangle & 0\\ 0 & 0 & \langle\bar{s}s\rangle\end{pmatrix} = \frac{1}{3}\phi^0\mathbf{1} + 2\phi^3T^3 + 2\phi^8T^8 \tag{2.6}$$

where $\langle\phi^c\rangle$ is the VEV $\langle\bar{q}T^cq\rangle$. It is also convenient to introduce the still more compact notation

$$M_{ab} = d_{acb}m^c \qquad\qquad \Phi_{ab} = d_{abc}\langle\phi^c\rangle \tag{2.7}$$

for which

$$\frac{1}{8}\mathrm{det}M = m_u m_d m_s \qquad\qquad \frac{1}{6}\mathrm{det}\Phi = \langle \bar{u}u\rangle\ \langle \bar{d}d\rangle\ \langle \bar{s}s\rangle \tag{2.8}$$

Notice that the derivation of the chiral Ward identities sketched above was entirely in terms of the bare operators. Renormalised composite operators are defined as follows[20]:

$$\begin{aligned}
&J^0_{\mu 5R} = Z J^0_{\mu 5B} \qquad\qquad J^{a\neq 0}_{\mu 5R} = J^{a\neq 0}_{\mu 5B} \\
&Q_R = Q_B - \frac{1}{2n_f}(1-Z)\partial^\mu J^0_{\mu 5B} \\
&\phi^a_{5R} = Z_\phi \phi^a_{5B} \qquad\qquad \phi^a_R = Z_\phi \phi^a_B
\end{aligned} \tag{2.9}$$

where Z_ϕ is the inverse of the mass renormalisation, $Z_\phi = Z_m^{-1}$. The anomalous dimensions associated with Z and Z_ϕ are denoted γ and γ_ϕ respectively. Notice the mixing of the operator Q with $\partial^\mu J^0_{\mu 5}$ under renormalisation. Most importantly, with these definitions the combination $\partial^\mu J^0_{\mu 5} - 2n_f Q$ occurring in the $U_A(1)$ anomaly equation is RG invariant. The chiral Ward identities therefore take precisely the same form expressed in terms of the bare or renormalised operators. From now on, therefore, we use eq.(2.4) as an identity for renormalised composite operators (omitting the label 'R' for notational simplicity).

It is also convenient to use a condensed notation where a functional derivative is represented simply by a subscript, with a spacetime integral assumed where appropriate. Also transforming to momentum space, we therefore write eq.(2.4) compactly as

$$ik_\mu W_{V^a_{\mu 5}} - 2n_f\delta_{a0}W_\theta - M_{ac}W_{S^c_5} + d_{adc}S^d W_{S^c_5} - d_{adc}S^d_5 W_{S^c} = 0 \tag{2.10}$$

The Ward identities for composite operator Green functions are derived by taking functional derivatives of this basic identity. We will need the following identities for 2-point functions:

$$\begin{aligned}
&ik_\mu W_{V^a_{\mu 5}V^b_{\nu 5}} - 2n_f\delta_{a0}W_{\theta V^b_{\nu 5}} - M_{ac}W_{S^c_5 V^b_{\nu 5}} = 0 \\
&ik_\mu W_{V^a_{\mu 5}\theta} - 2n_f\delta_{a0}W_{\theta\theta} - M_{ac}W_{S^c_5\theta} = 0 \\
&ik_\mu W_{V^a_{\mu 5}S^b_5} - 2n_f\delta_{a0}W_{\theta S^b_5} - M_{ac}W_{S^c_5 S^b_5} - \Phi_{ab} = 0
\end{aligned} \tag{2.11}$$

Combining the individual equations in (2.11), we find the important identity:

$$k_\mu k_\nu W_{V^a_{\mu 5}V^b_{\nu 5}} - M_{ac}\Phi_{cb} = W_{S^a_D S^b_D} \tag{2.12}$$

where S^a_D is the source for the current divergence operator $D^a = 2n_f\delta_{a0}Q + M_{ac}\phi^c_5$. In canonical notation,

$$\begin{aligned}
W_{S^a_D S^b_D} &= i\int dx\ e^{ikx}\ \langle 0|T\ D^a(x)\ D^b(0)|0\rangle \\
&= i\int dx\ e^{ikx}\ \langle 0|T\ \partial^\mu J^a_{\mu 5}(x)\ \partial^\nu J^b_{\nu 5}(0)|0\rangle
\end{aligned} \tag{2.13}$$

The zero-momentum Ward identities play a special role. These follow immediately from eqs.(2.11) under the assumption that there are no massless particles (in particular, no exact Goldstone bosons) contributing $1/k^2$ poles in the 2-point functions. With this assumption, we find simply

$$\begin{aligned} &2n_f \delta_{a0} W_{\theta\theta} + M_{ac} W_{S_5^c \theta} = 0 \\ &2n_f \delta_{a0} W_{\theta S_5^b} + M_{ac} W_{S_5^c S_5^b} + \Phi_{ab} = 0 \end{aligned} \tag{2.14}$$

Combining these, we find that the topological susceptibility $\chi(0) \equiv W_{\theta\theta}(0)$ satisfies the identity

$$(2n_f)^2 \chi(0) = M_{0b} M_{0c} W_{S_5^b S_5^c} + M_{0b} \Phi_{0b} \tag{2.15}$$

Another key ingredient in the discussion of the 'proton spin' problem and GT relations in sections 4 and 5 is the use of proper vertices for this set of operators. These are defined as functional derivatives of a generating functional Γ, which is itself constructed from W by a partial Legendre transform in which the transform is made only on the fields Q, ϕ_5^a, ϕ^a and not on the currents. The resulting proper vertices are 1PI w.r.t. the propagators for these composite operators only. As explained fully in refs.[19,21], by separating off the particle poles in the propagators, this is the definition which gives the closest identification of these field-theoretic vertices with physical low-energy couplings such as e.g. $g_{\pi NN}$. We therefore define the generating functional $\Gamma[V_{\mu 5}^a, Q, \phi_5^a, \phi^a]$ as:

$$\Gamma[V_{\mu 5}^a, Q, \phi_5^a, \phi^a] = W[V_{\mu 5}^a, \theta, S_5^a, S^a] - \int dx \, \Big(\theta Q + S_5^a \phi_5^a + S^a \phi^a\Big) \tag{2.16}$$

The chiral Ward identities corresponding to eq.(2.10) are therefore:

$$ik_\mu \Gamma_{V_{\mu 5}^a} - 2n_f \delta_{a0} Q - M_{ac} \phi_5^c + d_{acd} \phi^d \Gamma_{\phi_5^c} - d_{acd} \phi_5^d \Gamma_{\phi^c} = 0 \tag{2.17}$$

The Ward identities for the 2-point vertices will also be important. These follow directly from eq.(2.17):

$$\begin{aligned} &ik_\mu \Gamma_{V_{\mu 5}^a V_{\nu 5}^b} + \Phi_{ac} \Gamma_{\phi_5^c V_{\nu 5}^b} = 0 \\ &ik_\mu \Gamma_{V_{\mu 5}^a Q} - 2n_f \delta_{a0} + \Phi_{ac} \Gamma_{\phi_5^c Q} = 0 \\ &ik_\mu \Gamma_{V_{\mu 5}^a \phi_5^b} + \Phi_{ac} \Gamma_{\phi_5^c \phi_5^b} - M_{ab} = 0 \end{aligned} \tag{2.18}$$

It is then straightforward to derive the following important identity, analogous to eq.(2.12):

$$k_\mu k_\nu \Gamma_{V_{\mu 5}^a V_{\nu 5}^b} + M_{ac} \Phi_{cb} = \Phi_{ac} \Gamma_{\phi_5^c \phi_5^d} \Phi_{db} \tag{2.19}$$

The renormalisation group equations for these quantities also play a key role in understanding the physics of the $U_A(1)$ channel. We therefore include here a brief and somewhat novel discussion of the RGEs for the Green functions and proper vertices of these composite operators in a functional formalism. For further details, see especially refs.[22,19].

The fundamental RGE for the generating functional W follows immediately from the definitions (2.9) of the renormalised composite operators. It is:

$$\mathcal{D}W = \gamma\Big(V_{\mu5}^0 - \frac{1}{2n_f}\partial_\mu\theta\Big)W_{V_{\mu5}^0} + \gamma_\phi\Big(S_5^a W_{S_5^a} + S^a W_{S^a}\Big) + \ldots \tag{2.20}$$

where $\mathcal{D} = \Big(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} - \gamma_m\sum_q m_q\frac{\partial}{\partial m_q}\Big)\Big|_{V,\theta,S_5,S}$. The notation $+\ldots$ refers to the additional terms which are required to produce the contact term contributions to the RGEs for n-point Green functions of composite operators. These are discussed fully in refs.[22,19], but will be omitted here for simplicity. They vanish at zero-momentum.

The RGEs for Green functions are found simply by differentiating eq.(2.20) w.r.t. the sources. Simplifying the results using the chiral Ward identities (2.11), we find a complete set of RGEs for the 2-point functions. These are:

$$\begin{aligned}
&\mathcal{D}W_{V_{\mu5}^0 V_{\nu5}^0} = 2\gamma W_{V_{\mu5}^0 V_{\nu5}^0} + \ldots \qquad \mathcal{D}W_{V_{\mu5}^0 V_{\nu5}^b} = \gamma W_{V_{\mu5}^0 V_{\nu5}^b} + \ldots \qquad \mathcal{D}W_{V_{\mu5}^a V_{\nu5}^b} = 0 + \ldots \\
&\mathcal{D}W_{V_{\mu5}^0\theta} = 2\gamma W_{V_{\mu5}^0\theta} + \gamma\frac{1}{2n_f}M_{0b}W_{V_{\mu5}^0 S_5^b} + \ldots \\
&\mathcal{D}W_{V_{\mu5}^a\theta} = \gamma W_{V_{\mu5}^a\theta} + \gamma\frac{1}{2n_f}M_{0b}W_{V_{\mu5}^0 S_5^b} + \ldots \\
&\mathcal{D}W_{V_{\mu5}^0 S_5^b} = (\gamma+\gamma_\phi)W_{V_{\mu5}^0 S_5^b} + \ldots \qquad \mathcal{D}W_{V_{\mu5}^a S_5^b} = \gamma_\phi W_{V_{\mu5}^a S_5^b} + \ldots \\
&\mathcal{D}W_{\theta\theta} = 2\gamma W_{\theta\theta} + 2\gamma\frac{1}{2n_f}M_{0b}W_{\theta S_5^b} + \ldots \\
&\mathcal{D}W_{\theta S_5^b} = (\gamma+\gamma_\phi)W_{\theta S_5^b} + \gamma\frac{1}{2n_f}\big(M_{0c}W_{S_5^c S_5^b} + \Phi_{0b}\big) + \ldots \\
&\mathcal{D}W_{S_5^a S_5^b} = 2\gamma_\phi W_{S_5^a S_5^b} + \ldots
\end{aligned} \tag{2.21}$$

It is straightforward to check the self-consistency of these RGEs with the Ward identities (2.11) and (2.14). The pattern of cancellations which ensures this is nevertheless quite intricate.

Next, we need the RGE for the generating functional of the 1PI vertices. This follows immediately from its definition in eq.(2.16) and the RGE (2.20) for W:

$$\tilde{\mathcal{D}}\Gamma = \gamma\Big(V_{\mu5}^0 - \frac{1}{2n_f}\Gamma_Q\partial_\mu\Big)\Gamma_{V_{\mu5}^0} - \gamma_\phi\Big(\phi_5^a\Gamma_{\phi_5^a} + \phi^a\Gamma_{\phi^a}\Big) + \ldots \tag{2.22}$$

where $\tilde{\mathcal{D}} = \Big(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} - \gamma_m\sum_q m_q\frac{\partial}{\partial m_q}\Big)\Big|_{V,Q,\phi_5,\phi}$.

The RGEs for the 1PI vertices are found by differentiation, and using the Ward identities (2.18) to simplify the results, we find for the pseudoscalar sector:

$$
\begin{aligned}
\mathcal{D}\Gamma_{QQ} &= -2\gamma\Gamma_{QQ} + 2\gamma\frac{1}{2n_f}\Big[\Phi_{0c}\Gamma_{QQ}\Gamma_{\phi_5^c Q}\Big] + \ldots \\
\mathcal{D}\Gamma_{Q\phi_5^b} &= -(\gamma+\gamma_\phi)\Gamma_{Q\phi_5^b} + \gamma\frac{1}{2n_f}\Big[\Phi_{0c}\big(\Gamma_{QQ}\Gamma_{\phi_5^c\phi_5^b} + \Gamma_{Q\phi_5^c}\Gamma_{Q\phi_5^b}\big) - M_{0b}\Gamma_{QQ}\Big] + \ldots \\
\mathcal{D}\Gamma_{\phi_5^a\phi_5^b} &= -2\gamma_\phi\Gamma_{\phi_5^a\phi_5^b} + \gamma\frac{1}{2n_f}\Big[\Phi_{0c}\Gamma_{\phi_5^a Q}\Gamma_{\phi_5^c\phi_5^b} - M_{0b}\Gamma_{\phi_5^a Q} + \ a\leftrightarrow b\ \Big] + \ldots
\end{aligned}
\tag{2.23}
$$

Here, $\mathcal{D} = \tilde{\mathcal{D}} + \gamma_\phi\langle\phi^a\rangle\frac{\delta}{\delta\phi^a}$. As explained in ref.[19], this is identical to the RG operator $\mathcal{D}$ defined above (acting on W) when the sources are set to zero and the fields to their VEVs.

It will also be useful to know the RGEs for the 3-point vertices coupling a pseudoscalar operator to the nucleon. In the same way, we find[19,8]

$$
\begin{aligned}
\mathcal{D}\hat{\Gamma}_{QNN} &= -\gamma\hat{\Gamma}_{QNN} + \gamma\frac{1}{2n_f}\Big[\Phi_{0b}\Big(\Gamma_{Q\phi_5^b}\hat{\Gamma}_{QNN} + \Gamma_{QQ}\hat{\Gamma}_{\phi_5^b NN}\Big)\Big] + \ldots \\
\mathcal{D}\hat{\Gamma}_{\phi_5^a NN} &= -\gamma_\phi\hat{\Gamma}_{\phi_5^a NN} + \gamma\frac{1}{2n_f}\Big[\Phi_{0b}\Big(\Gamma_{\phi_5^a\phi_5^b}\hat{\Gamma}_{QNN} + \Gamma_{\phi_5^a Q}\hat{\Gamma}_{\phi_5^b NN}\Big) - M_{0a}\hat{\Gamma}_{QNN}\Big] + \ldots
\end{aligned}
\tag{2.24}
$$

These RGEs play two roles in the discussion that follows. First, they will be used as consistency checks on the various formulae we derive. Second, and most important, they will provide the clue to identifying quantities which are likely to show violations of the OZI rule and those for which we may reasonably expect the OZI limit to be a good approximation. This is because we can identify quantities which will be particularly sensitive to the $U_A(1)$ anomaly as those which have RGEs involving the anomalous dimension γ.

3. Pseudoscalar Mesons and the Witten-Veneziano Mass Formula for η'

The anomalous chiral Ward identities have some immediate and important consequences. For simplicity, we specialise to massless QCD in this section. Then, integrating eq.(2.4) and evaluating with the sources set to their physical values (i.e. zero, apart from the source $\theta(x)$ which becomes the QCD theta angle θ), we find

$$
\frac{\partial W}{\partial\theta} \ = \ \int d^4x\ \frac{\delta W}{\delta\theta(x)} \ = \ 0 \tag{3.1}
$$

That is, massless QCD is independent of the theta angle. In fact[10, ch. 23], the same conclusion holds if any of the quark masses were to vanish. Then, the theta angle would have no effect and the strong CP problem would be automatically resolved in QCD. This is, however, an unrealistic solution since even $m_u \neq 0$.

It is equally simple to establish that the (zero-momentum) topological susceptibility vanishes for massless QCD. This can be read off immediately from the $M = 0$ limit of eq.(2.15). There is, however, one subtlety here which is worth noticing. Writing the second identity in eq.(2.11) in canonical form, we have

$$ik^\mu \langle 0|T\ J^0_{\mu 5}\ Q|0\rangle - 2n_f \langle 0|T\ Q\ Q|0\rangle = 0 \tag{3.2}$$

If there is no massless pseudoscalar meson (e.g. a $U_A(1)$ Goldstone boson) coupling to the current, then clearly the first term vanishes at zero momentum. However, in this case the same conclusion follows even if there does exist such a particle, since the anomaly equation implies that the coupling of Q to this massless boson would vanish on-shell. In either case, we deduce

$$\chi(0) = \langle 0|T\ Q\ Q|0\rangle\big|_{k=0} = 0 \tag{3.3}$$

The next question is what happens to Goldstone's theorem in the presence of the anomaly. For the non-anomalous flavour non-singlet currents, the third identity in eq.(2.11) shows as usual that (for $M = 0$) if there is a symmetry breaking VEV $\Phi \neq 0$ then there must exist, by Goldstone's theorem, a massless boson coupling derivatively to the current. What about the $U_A(1)$ current? The relevant Ward identity, in canonical form, reads

$$ik^\mu \langle 0|T\ J^0_{\mu 5}\ \phi^0_5|0\rangle - 2n_f \langle 0|T\ Q\ \phi^0_5|0\rangle = 2\langle \phi^0\rangle \tag{3.4}$$

Since the r.h.s. is non-zero, in the absence of the anomaly term the only way the identity can be satisfied at $k = 0$ is if there exists a massless Goldstone boson coupling to the current. However, the presence of the extra correlation function involving Q means that this conclusion no longer holds. The Goldstone theorem is evaded by virtue of the anomaly and there is no physical massless $U_A(1)$ Goldstone boson.

At its most basic, this is the resolution of the famous $U_A(1)$ problem. However, things are of course not so simple. Recall from eq.(1.3) that Q may be written as the divergence of the (gauge non-invariant) Chern-Simons current K_μ. We can therefore construct a conserved, but gauge non-invariant, current $\hat{J}^0_{\mu 5}$ as follows

$$\hat{J}^0_{\mu 5} = J^0_{\mu 5} - 2n_f K_\mu \tag{3.5}$$

which satisfies the chiral Ward identity

$$ik^\mu \langle 0|T\ \hat{J}^0_{\mu 5}\ \phi^0_5|0\rangle = 2\langle \phi^0\rangle \tag{3.6}$$

Applying Goldstone's theorem naively, we would then deduce that after all there must exist a massless boson in the QCD spectrum. However, this conclusion is false, although the precise reasons are still the subject of some debate. A full technical analysis of this $U_A(1)$ problem would need (at least) another lecture, so instead we simply refer to the literature[23,24]. In brief, however, there are two possible escape routes (both of which may be true in a sufficiently precise formulation of the problem):

(i) the boundary conditions at spatial infinity on the behaviour of the gauge-variant current $\hat{J}^0_{\mu 5}$ imposed by the vacuum structure of QCD allow eq.(3.6) to be satisfied without coupling to a massless boson;
(ii) a massless boson does indeed exist; however, it decouples from the positive-norm Hilbert space through a mechanism known as 'quartet decoupling'. This is essentially the same mechanism as is responsible for the decoupling from the physical spectrum of the ghosts and longitudinal and scalar components of the photon field in QED formulated in a covariant gauge. Its possible application to the $U_A(1)$ problem has been elaborated by Kugo[25], who shows how the required Goldstone quartet is constructed by acting with the BRS operator Q_B on the gauge-variant Goldstone field coupling to $\hat{J}^0_{\mu 5}$. It represents a more realistic generalisation of the popular Kogut-Susskind dipole hypothesis[26] and reduces to it in the one case where the Kogut-Susskind hypothesis has been proved to work - the 2 dim Schwinger model. In this model, the 'quartet' splits into two 'dipoles' which independently cancel from the physical spectrum.

Having convinced ourselves there is no real paradox associated with the Goldstone theorem applied to the anomalous $U_A(1)$ current, we can ask whether it is possible to do better and identify the mass of the 'would-be Goldstone boson' η' in some way with the anomaly. The answer is provided, in the context of the $1/N_c$ approximation to QCD, by the Witten-Veneziano mass formula[11,12].

We know that in nature the lightest pseudoscalar meson in the flavour singlet channel coupling to the current $J^0_{\mu 5}$ is the η'. It follows from the second Ward identity in eq.(2.11) that this must also couple to the operator Q, producing a pole at $k^2 = m^2_{\eta'}$ in the topological susceptibility $W_{\theta\theta}$. However, we have also seen that $W_{\theta\theta}$ vanishes at $k = 0$. A reasonable parametrisation of the correlation function is therefore

$$W_{\theta\theta}(k^2) = \frac{1}{(2n_f)^2} \frac{k^2}{k^2 - m^2_{\eta'}} A(k^2) \tag{3.7}$$

where $A(k^2)$ is a pole-free, and therefore relatively smooth, function below the glueball threshold. $A(k^2)$ is otherwise unconstrained by the chiral Ward identities.

The residue of the pole evidently satisfies

$$\left|\langle 0|Q|\eta'\rangle\right|^2 = \frac{1}{(2n_f)^2} m^2_{\eta'} A(m^2_{\eta'}) \tag{3.8}$$

and so defining the RG non-invariant 'decay constant' $f_{\eta'}$ by (see refs.[19,21] for a careful discussion of this point)

$$\langle 0|J^0_{\mu 5}|\eta'\rangle = ik_\mu f_{\eta'} \tag{3.9}$$

and using the anomaly equation, we find

$$f^2_{\eta'} m^2_{\eta'} = A(m^2_{\eta'}) \tag{3.10}$$

Notice that neither side is a RG invariant, each scaling with the anomalous dimension 2γ.

We now introduce the large N_c approximation. To leading order in $1/N_c$, the anomaly is absent and the η' mass vanishes. Formally, this is implemented by assuming that $m_{\eta'}$ is $O(1/N_c)$. In fact, this can be alternatively re-formulated by using the OZI limit, i.e. the approximation to QCD in which the OZI rule becomes exact. The OZI limit is precisely defined[27] as the truncation of full QCD in which non-planar and quark-loop diagrams are retained, but diagrams in which the external currents are attached to distinct quark loops, so that there are purely gluonic intermediate states, are omitted. (This last fact makes the connection with the familiar phenomenological form of the OZI, or Zweig, rule.) This is a more accurate approximation to full QCD than either the leading large $1/N_c$ limit, the quenched approximation (small n_f at fixed N_c) or the leading topological expansion ($N_c \to \infty$ at fixed n_f/N_c. In the OZI limit, the $U_A(1)$ anomaly is absent, as is meson-glueball mixing, and there is an extra $U_A(1)$ Goldstone boson.

Applying the large N_c limit to the l.h.s. of eq.(3.7), we therefore find

$$\lim_{k\to 0} \lim_{N_c\to\infty} W_{\theta\theta}(k^2) = \frac{1}{(2n_f)^2} A(0) \tag{3.11}$$

(Notice that the zero-momentum and large N_c limits do not commute.) We now put eqs.(3.10) and (3.11) together. To leading order in $1/N_c$, we may write $f_{\eta'} \simeq \sqrt{2n_f} f_\pi$ and $A(m_{\eta'}) \simeq A(0)$. On the other hand, we may identify the l.h.s. of eq.(3.11) as the topological susceptibility in pure gluodynamics, since as noted above, mixing between the meson and glueball sectors vanishes to leading order in $1/N_c$, or equivalently in the OZI limit.

Putting this together, we finally deduce the Witten-Veneziano formula for the mass of the η', valid to leading order in the $1/N_c$ expansion:

$$m^2_{\eta'} \;=\; \frac{2n_f}{f^2_\pi}\, \chi^{YM}(0) \tag{3.12}$$

As discussed in the introduction, this is our first example of an explicit relation linking quark dynamics in the $U_A(1)$ channel to gluon topology in the form of the topological susceptibility of pure gluodynamics.

4. Topological Charge Screening and the Ellis-Jaffe Sum Rule

The 'proton spin' problem, i.e. the question of why the first moment of the flavour singlet component of the polarised proton structure function g_1^p is anomalously suppressed, has inspired an impressive research effort, both theoretical and experimental, for over a decade. A recent review of this whole topic from the viewpoint adopted here can be found in ref.[16]. As is well-known from standard DIS theory, the first moment of g_1^p can be expressed in terms of the axial charges of the proton as follows:

$$\Gamma_1^p \;\equiv\; \int_0^1 dx\; g_1^p(x;Q^2) \;=\; \frac{1}{12} C_1^{\rm NS}(\alpha_s)\, \Big(a^3 + \frac{1}{3}a^8\Big) + \frac{1}{9} C_1^{\rm S}(\alpha_s)\, a^0(Q^2) \tag{4.1}$$

Here, $C_1^{\rm NS}$, C_1^S are the appropriate Wilson coefficients arising from the OPE for two electromagnetic currents, while the axial charges are defined from the forward matrix elements of the axial currents with the normalisations

$$\langle p,s|J_{\mu 5}^3|p,s\rangle = \frac{1}{2}a^3 s_\mu \qquad \langle p,s|J_{\mu 5}^8|p,s\rangle = \frac{1}{2\sqrt{3}}a^8 s_\mu \qquad \langle p,s|J_{\mu 5}^0|p,s\rangle = a^0(Q^2)s_\mu \quad (4.2)$$

where $s_\mu = \bar{u}(p,s)\gamma_\mu\gamma_5 u(p,s)$ is the proton polarisation vector. a^3 and a^8 are known in terms of the F and D coefficients from beta and hyperon decay, so that an experimental determination of the first moment of g_1^p in polarised DIS allows a determination of the singlet axial charge $a^0(Q^2)$. The 'proton spin' problem is the fact that it is found experimentally that $a^0(Q^2)$ is strongly suppressed relative to a^8, which would be its expected value if the OZI rule were exact in this channel.

DIS is normally described theoretically using the QCD parton model. In this model, the axial charges are represented (in the AB renormalisation scheme[28,29]) in terms of moments of parton distributions as follows[28]:

$$a^3 = \Delta u - \Delta d \qquad a^8 = \Delta u + \Delta d - 2\Delta s \qquad a^0(Q^2) = \Delta u + \Delta d + \Delta s - n_f \frac{\alpha_s}{2\pi}\Delta g(Q^2) \quad (4.3)$$

In the parton model, the 'proton spin' problem takes the following form. In the naive, or valence quark, parton model we would expect the strange quark and gluon distributions to vanish, i.e. $\Delta s = 0$, $\Delta g(Q^2) = 0$. In that case, $a^0 = a^8$, the OZI prediction. Inserted into eq.(4.1), this gives the Ellis-Jaffe sum rule[2]. However, the observed suppression $a^0(Q^2) < a^8$ can be accommodated in the full QCD parton model by invoking either or both a non-zero polarised strange quark distribution $\Delta s \neq 0$ or a non-zero polarised gluon distribution $\Delta g(Q^2) \neq 0$. An interesting conjecture (in line with the insights of the approach discussed here) is that the suppression is primarily due to the gluon distribution[28], although a quantitative prediction would still only follow if $\Delta g(Q^2)$ can be independently measured, either through a precise analysis of the Q^2 dependence of g_1^p [29] or directly through other less inclusive high energy processes such as open charm production. Notice, however, that even in the QCD parton model picture, it is *not* possible to identify $a^0(Q^2)$ with spin[17,16]. This identification only holds for free quarks, in which case the Q^2 scale dependence, which is related through eq.(2.9) (see also eq.(5.26)) to the $U_A(1)$ anomaly, disappears from a^0. It must be emphasised that the so-called 'proton spin' problem is not a problem of *spin* – rather, it is a question of understanding the dynamical origin of the OZI violation $a^0(Q^2) < a^8$.

In this section, we shall discuss a less conventional approach (the 'CPV' method[16]) to DIS based on the composite operator – proper vertex formalism described in section 2. The starting point, as indicated above, is the use of the OPE in the proton matrix element of two currents. This gives the standard form for a generic structure function moment:

$$\int_0^1 dx\ x^{n-1}F(x;Q^2) = \sum_i C_i^n(Q^2)\langle p|\mathcal{O}_i^n(0)|p\rangle \quad (4.4)$$

where $\mathcal{O}_i^n$ are the set of lowest twist, spin n operators in the OPE and $C_i^n(Q^2)$ the corresponding Wilson coefficients. In the CPV approach, we now factorise the matrix element into the product of composite operator propagators and vertex functions, as illustrated in Fig. 1.

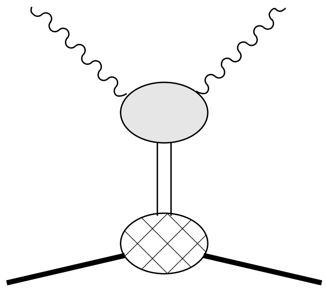

Fig.1 CPV description of DIS. The double line denotes the composite operator propogator and the lower blob the 1PI vertex.

To do this, we first select a set of composite operators $\tilde{\mathcal{O}}_i$ appropriate to the physical situation and define vertices $\Gamma_{\tilde{\mathcal{O}}_i pp}$ as 1PI with respect to this set. Technically, this is achieved as in eq.(2.16) by introducing sources for these operators in the QCD generating functional, then performing a (partial) Legendre transform to obtain a generating functional $\Gamma[\tilde{\mathcal{O}}_i]$. The 1PI vertices are the functional derivatives of $\Gamma[\tilde{\mathcal{O}}_i]$. The generic structure function sum rule (4.4) then takes the form

$$\begin{aligned}\int_0^1 dx\ x^{n-1}\ F(x,Q^2) &= \sum_i \sum_j C_j^{(n)}(Q^2)\langle 0|T\ \mathcal{O}_j^{(n)}\ \tilde{\mathcal{O}}_i|0\rangle \Gamma_{\tilde{\mathcal{O}}_i pp} \\ &= \sum_i \sum_j C_j P_{ji} V_i \end{aligned} \tag{4.5}$$

in a symbolic notation.

This decomposition splits the structure function into three pieces – first, the Wilson coefficients $C_j^{(n)}(Q^2)$ which control the Q^2 dependence and can be calculated in perturbative QCD; second, non-perturbative but *target-independent* QCD correlation functions $\langle 0|T\ \mathcal{O}_j^{(n)}\ \tilde{\mathcal{O}}_i|0\rangle$; and third, non-perturbative, target-dependent vertex functions $\Gamma_{\tilde{\mathcal{O}}_i pp}$ describing the coupling of the target proton to the composite operators of interest. The vertex functions cannot be calculated directly from first principles. They encode the information on the nature of the proton state and play an analogous role to the parton distributions in the more conventional parton picture.

It is important to recognise that this decomposition of the matrix elements into products of propagators and proper vertices is *exact*, independent of the choice of the set of operators $\tilde{\mathcal{O}}_i$. In particular, it is not necessary for $\tilde{\mathcal{O}}_i$ to be in any sense a complete set. All that happens if a different choice is made is that the vertices $\Gamma_{\tilde{\mathcal{O}}_i pp}$ themselves change, becoming 1PI with respect to a different set of composite fields. Of course, while any set of $\tilde{\mathcal{O}}_i$ may be chosen, some will be more convenient than others. Clearly, the set of operators should be as small as possible while still capturing the essential physics (i.e. they should encompass the relevant degrees of freedom) and indeed a good choice can result in vertices

$\Gamma_{\tilde{\mathcal{O}}_i pp}$ which are both RG invariant and closely related to low energy physical couplings, such as $g_{\pi NN}$. In this case, eq.(4.5) provides a rigorous relation between high Q^2 DIS and low-energy meson-nucleon scattering.

For the first moment sum rule for g_1^p [19,7,16], it is most convenient to use the $U_A(1)$ anomaly equation immediately to re-express $a^0(Q^2)$ in terms of the forward matrix element of the topological charge Q, i.e.

$$a^0(Q^2) \;=\; \frac{1}{2m_N} 2n_f \langle p|Q|p\rangle \tag{4.6}$$

where m_N is the nucleon mass.

Our set of operators $\tilde{\mathcal{O}}_i$ is then chosen to be the renormalised flavour singlet pseudoscalars Q and Φ_5, where Φ_5 is simply the operator ϕ_5^0 of eqs.(2.1) and (2.9) with a special, and crucial, normalisation. The normalisation factor is chosen such that in the absence of the anomaly, or more precisely in the OZI limit of QCD (see section 3), Φ_5 would have the correct normalisation to couple with unit decay constant to the $U_A(1)$ Goldstone boson which would exist in this limit. This also ensures that the vertex is RG scale independent, as we prove in section 5 where we relate this discussion to the $U_A(1)$ Goldberger-Treiman relation. The vertices are defined from the generating functional (2.16). We then have

$$\Gamma_{1\ singlet}^p = \frac{1}{9}\frac{1}{2m_N} 2n_f C_1^{\mathrm{S}}(\alpha_s)\Big[\langle 0|T\ Q\ Q|0\rangle \hat{\Gamma}_{Qpp} + \langle 0|T\ Q\ \Phi_5|0\rangle \hat{\Gamma}_{\Phi_5 pp}\Big] \tag{4.7}$$

where the propagators are at zero momentum and the vertices are 1PI wrt Q and Φ_5 only. For simplicity, we have also introduced the notation $i\bar{u}\Gamma_{Qpp}u = \hat{\Gamma}_{Qpp}\bar{u}\gamma_5 u$, etc. This is illustrated in Fig. 2.

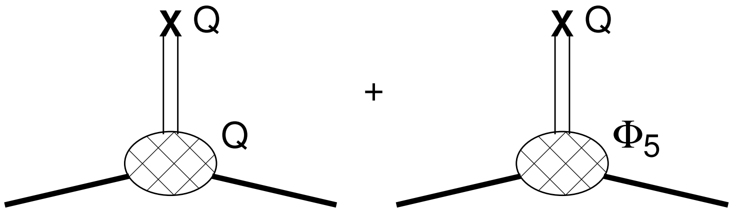

Fig.2 CPV decomposition of the matrix element $\langle p|Q|p\rangle$.

The composite operator propagator in the first term is simply the (zero-momentum) QCD topological susceptibility $\chi(0)$ which, as we have seen in sections 2 and 3, vanishes for QCD with massless quarks. Furthermore, with the normalisation specified above for Φ_5, the propagator $\langle 0|T\ Q\ \Phi_5|0\rangle$ at zero momentum is simply the square root of the slope of the topological susceptibility.

To see this, notice that by virtue of their definition in terms of the generating functional (2.16), the matrix of 2-point vertices in the pseudoscalar sector is simply the inverse of the corresponding matrix of pseudoscalar propagators, i.e.

$$\begin{pmatrix} \Gamma_{QQ} & \Gamma_{Q\phi_5^0} \\ \Gamma_{\phi_5^0 Q} & \Gamma_{\phi_5^0\phi_5^0} \end{pmatrix} \;=\; -\begin{pmatrix} W_{\theta\theta} & W_{\theta S_5^0} \\ W_{S_5^0\theta} & W_{S_5^0 S_5^0} \end{pmatrix}^{-1} \tag{4.8}$$

This implies

$$\Gamma_{\phi_5^0\phi_5^0} = -W_{\theta\theta}\big(\det\, W_{\mathcal{SS}}\big)^{-1} \tag{4.9}$$

letting $\mathcal{S}$ represent the set $\{\theta, S_5^0\}$. Differentiating w.r.t. k^2 and taking the limit $k^2 = 0$, and exploiting the fact that $W_{\theta\theta}(0)$ vanishes, we find

$$\frac{d}{dk^2}\Gamma_{\phi_5^0\phi_5^0}\big|_{k=0} = \chi'(0) W^{-2}_{\theta S_5^0}(0) \tag{4.10}$$

Finally, normalising the field Φ_5 proportional to ϕ_5^0 such that $\frac{d}{dk^2}\Gamma_{\Phi_5\Phi_5}\big|_{k=0} = 1$ (see also the discussion in section 5, following eq.(5.7)), we find the required relation

$$\langle 0|T\; Q\; \Phi_5|0\rangle\big|_{k=0} = \sqrt{\chi'(0)} \tag{4.11}$$

We therefore find:

$$\Gamma^p_{1\; singlet} \;=\; \frac{1}{9}\frac{1}{2m_N} 2n_f\; C_1^{\rm S}(\alpha_s)\; \sqrt{\chi'(0)}\; \hat{\Gamma}_{\Phi_5 pp} \tag{4.12}$$

The slope of the topological susceptibility $\chi'(0)$ is not RG invariant but, as shown in eq.(2.21), scales with the anomalous dimension 2γ, i.e.

$$\frac{d}{dt}\sqrt{\chi'(0)} = \gamma\sqrt{\chi'(0)} \tag{4.13}$$

On the other hand, the proper vertex has been chosen specifically so as to be RG invariant, as proved in section 5. The renormalisation group properties of this decomposition are crucial to our proposed resolution of the 'proton spin' problem.

Our proposal[19] is that we should expect the source of OZI violations to lie in the RG non-invariant, and therefore anomaly-sensitive, terms, i.e. in $\chi'(0)$ rather than in the RG invariant vertex. Notice that we are using RG non-invariance, i.e. dependence on the anomalous dimension γ, merely as an indicator of which quantities are sensitive to the anomaly and therefore likely to show OZI violations. Since the anomalous suppression in Γ_1^p is thus assigned to the composite operator propagator rather than the proper vertex, the suppression is a *target independent* property of QCD related to the anomaly, not a special property of the proton structure.

To convert this into a quantitative prediction we use the OZI approximation for the vertex $\hat{\Gamma}_{\Phi_5 pp}$. In terms of a similarly normalised octet field Φ_5^8, this is $\hat{\Gamma}_{\Phi_5 pp} = \sqrt{2}\hat{\Gamma}_{\Phi_5^8 pp}$. Notice that the normalisation is crucial in allowing the use of the OZI relation here. The corresponding OZI prediction for $\sqrt{\chi'(0)}$ would be $f_\pi/\sqrt{6}$. These OZI values are determined by comparing the result (4.12) (at least that part relating to the proton matrix element) with the conventional Goldberger-Treiman relation for the flavour octet axial charge in the chiral limit (see section 5). This gives our formula

$$\frac{a^0(Q^2)}{a^8} = \frac{\sqrt{6}}{f_\pi}\sqrt{\chi'(0)}\big|_{Q^2} \tag{4.14}$$

for the flavour singlet axial charge. Incorporating this into the formula for the first moment of the polarised structure function, we find

$$\Gamma^p_{1\ singlet} \;=\; \frac{1}{9}C_1^{\rm S}(\alpha_s)\; a^8\; \frac{\sqrt{6}}{f_\pi}\;\sqrt{\chi'(0)} \tag{4.15}$$

Finally, substituting this into the expression for the complete first moment and using our spectral sum rule derivation of $\chi'(0)$, which gives a suppression factor of around 0.6 (see refs.[7,8,16] for more details), we obtain our prediction in the chiral limit

$$\begin{aligned} &a^0(Q^2 = 10{\rm GeV}^2) = 0.33 \pm 0.05 \\ &\Gamma_1^p(Q^2 = 10{\rm GeV}^2) = 0.144 \pm 0.009 \end{aligned} \tag{4.16}$$

This is to be compared with the OZI (Ellis-Jaffe) prediction $a^0 = 0.58 \pm 0.03$ and the current experimental data from the SMC collaboration[30]:

$$\Gamma_1^p(Q^2 = 10{\rm GeV}^2)\big|_{(x>0.003)} \equiv \int_{0.003}^1 dx\; g_1^p(x;Q^2) = 0.141 \pm 0.012 \tag{4.17}$$

The result for the entire first moment depends on how the extrapolation to the unmeasured small x region $x < 0.003$ is performed. This is still a controversial issue. Using a simple Regge fit, SMC find $\Gamma_1^p = 0.142 \pm 0.017$ from which they deduce $a^0 = 0.34 \pm 0.17$, while using a small x fit using perturbative QCD evolution of the parton distributions[31] they find $\Gamma_1^p = 0.130 \pm 0.017$ and $a^0 = 0.22 \pm 0.17$ (all at $Q^2 = 10{\rm GeV}^2$).

More recently, SMC have published an alternative analysis[32] of their data, this time quoting a slightly lower number for the integral over the measured region of x:

$$\Gamma_1^p(Q^2 = 10{\rm GeV}^2)\big|_{(x>0.003)} \equiv \int_{0.003}^1 dx\; g_1^p(x;Q^2) = 0.133 \pm 0.009 \tag{4.18}$$

Clearly it is premature to draw too strong a conclusion given the large errors on the experimental determinations of Γ_1^p and a^0 and the uncertainty over the small x extrapolation. Nevertheless, it is extremely encouraging that our prediction is firmly in the region favoured by the data and gives us confidence that our explanation of the 'proton spin' problem in terms of the topological susceptibility is correct. (For a more extensive discussion of the phenomenological aspects of this approach to the 'proton spin' problem, and a proposal to test our 'target-independence' conjecture in semi-inclusive DIS experiments at polarised HERA, see refs.[16,33,34].)

Finally, we should draw attention to the basic dynamical mechanism responsible for the suppression of the 'proton spin'. What we have shown is that when a matrix element of the topological charge is measured, the QCD vacuum screens the topological charge through the zero or anomalously small values of the susceptibility $\chi(0)$ and its slope $\chi'(0)$ respectively (see Fig. 2). The mechanism is analogous to the screening of electric charge in QED. There, because of the gauge Ward identity, the screening is given entirely by the ('target independent') dressing of the photon propagator by vacuum polarisation diagrams, leading to the relation $e_R = e_B\sqrt{Z_3}$ (with $Z_3 < 1$) between the renormalised and bare charges, in direct analogy to eq.(4.14) above with the topological susceptibility playing the role of the photon propagator.

5. The $U_A(1)$ Goldberger-Treiman Formula[2]

In this final section, we present a unified derivation of the Goldberger-Treiman relations (1.9) for the flavour singlet and non-singlet axial charges for QCD with non-vanishing quark masses in the functional composite operator propagator – vertex formalism[8]. The zero-mass limit of the formula for the singlet $U_A(1)$ charge reduces to the expression already used in section 4 (see eq.(4.12)) in the context of the 'proton spin' problem. Notice that these new GT relations are more general than the conventional formulae. Indeed, they are *exact* field-theoretic results in QCD. The familiar PCAC forms (in the flavour non-singlet channels) are obtained by approximating the 1PI vertices by the corresponding low-energy meson-nucleon couplings and by approximating the slopes of the current correlation functions (1.10) by decay constants. Away from the chiral limit, both these approximations assume pole dominance of the matrix elements and correlation functions by the pseudo-Goldstone bosons.

The axial charges G_A^a are defined as the form factors in the forward nucleon matrix elements of the axial currents, viz.

$$\langle p,s|J_{\mu 5}^a|p,s\rangle = G_A^a s_\mu \tag{5.1}$$

(Compare with the normalisations in eq.(4.2).)

To express this matrix element in terms of composite operator propagators and the associated 1PI vertices, we first introduce an interpolating field N and source S_N for the nucleon. (Notice that this is purely a formal device – there is no dynamics implicit in this step.) The matrix element is then just the 3-point function $W_{V_{\mu 5}^a S_N S_N}$ with the external propagators amputated. This can be re-expressed in terms of the vertex functional Γ as follows (see refs.[19,8] for a derivation of the relevant formulae involving partial Legendre transforms):

$$\begin{aligned}\langle p,s|J_{\mu 5}^a|p,s\rangle &= \bar{u}(p,s)\Big[W_{S_N S_N}^{-1}\ W_{V_{\mu 5}^a S_N S_N}\ W_{S_N S_N}^{-1}\Big]u(p,s)\\ &= \bar{u}(p,s)\Big[\Gamma_{V_{\mu 5}^a NN}\ +\ W_{V_{\mu 5}^a\theta}\ \Gamma_{QNN}\ +\ W_{V_{\mu 5}^a S_5^b}\ \Gamma_{\phi_5^b NN}\Big]u(p,s)\end{aligned} \tag{5.2}$$

Since the propagators on the r.h.s. vanish at zero momentum (this requires the absence of any $1/k^2$ poles, which as we have seen in section 3 is assured by the $U_A(1)$ anomaly and quark masses), we find simply

$$2m_N G_A^a\ \bar{u}\gamma_5 u = \bar{u}\Big[k_\mu \Gamma_{V_{\mu 5}^a NN}\big|_{k=0}\Big]u \tag{5.3}$$

[2] The material in this section was not presented in the lecture in Zuoz. However, it is closely related to the topics described in sections 2-4 and should help to clarify the discussion given there.

The GT relations then follow immediately from the Ward identity (2.17) for Γ. Differentiating w.r.t. the nucleon fields, we find

$$2m_N G_A^a = \Phi_{ab}\hat{\Gamma}_{\phi_5^b NN}\big|_{k=0} \tag{5.4}$$

where as before we define $i\bar{u}\ \Gamma_{\phi_5^a NN}\ u = \hat{\Gamma}_{\phi_5^a NN}\ \bar{u}\gamma_5 u$, etc.

A non-forward version of the GT relation, which is closer to the analysis in section 4, can also be found from eq.(5.2) by using the Ward identities (2.11) for the propagators together with (2.17) for $\Gamma_{V_{\mu 5}^a NN}$. This allows us to write, for all k,

$$\begin{aligned}2m_N G_A^a(k^2) + k^2 G_P^a(k^2) = &- \Big(2n_f\delta_{a0}W_{\theta\theta} + M_{ac}W_{S_5^c\theta}\Big)\hat{\Gamma}_{QNN} \\ &- \Big(2n_f\delta_{a0}W_{\theta S_5^b} + M_{ac}W_{S_5^c S_5^b}\Big)\hat{\Gamma}_{\phi_5^b NN}\end{aligned} \tag{5.5}$$

$G_P^a(k^2)$ is the pseudoscalar form factor in the non-forward matrix element $\langle p,s|J_{\mu 5}^a|p,s\rangle$, and again has no $1/k^2$ pole for the reasons given above. This expression clearly reduces to eq.(5.4) on using the zero-momentum Ward identities (2.14) for the propagators. In the chiral limit ($M=0$), this can be compared with eq.(4.7).

The remaining step to convert eq.(5.4) into the useful form of the GT relations is to normalise the field ϕ_5^a appropriately. Clearly, eq.(5.4) is independent of the normalisation. However, with a suitable choice, the vertices can be made both RG invariant and essentially identical to the physical Goldstone boson couplings $g_{\pi NN}$ etc. To achieve this, we define normalised fields

$$\eta^a = B_{ab}\phi_5^b \tag{5.6}$$

where B is a constant matrix such that[3]

$$\frac{d}{dk^2}\Gamma_{\eta^a\eta^b}\big|_{k=0} = \delta_{ab} \tag{5.7}$$

This condition ensures that the fields η^a have unit coupling to the Goldstone bosons.

The case of the singlet η^0 is of course special, since it is only after mixing with the topological field Q (and then flavour mixing) that it becomes the physical η'. In fact, this is why it is most convenient to impose the normalisation condition as above on the matrix of 2-point vertices $\Gamma_{\eta^a\eta^b}$, which is the inverse of the pseudoscalar propagator matrix, since this most simply characterises the η^0 before mixing with Q. The corresponding state is the unphysical 'OZI Goldstone boson' introduced in ref.[17] and extensively discussed in refs.[19,21].

It can now be proved that the vertices $\hat{\Gamma}_{\eta^a NN}$ defined with the fields normalised according to eq.(5.7) are RG invariant. The proof is based on the functional form of the RGEs introduced in section 2 and is given at the end of this section.

Re-expressing eq.(5.4) in terms of the properly normalised vertices, we have

$$2m_N G_A^a = \Phi_{ac}B_{cb}^T\ \hat{\Gamma}_{\eta^b NN}\big|_{k=0} \tag{5.8}$$

[3] Applied to the flavour singlet field ϕ_5^0 in the chiral limit, this is the same condition used to normalise the field Φ_5 in section 4.

where B is to be determined from the 2-point vertex condition

$$\frac{d}{dk^2}\Gamma_{\phi_5^a\phi_5^b}\big|_{k=0} = B^T_{ac}\ \frac{d}{dk^2}\Gamma_{\eta^c\eta^d}\big|_{k=0}\ B_{db} = B^T_{ac}B_{cb} \tag{5.9}$$

The straightforward approach to finding $\Gamma_{\phi_5^a\phi_5^b}$ is to write it as one component of the inverse of the propagator matrix $W_{\mathcal{S}\mathcal{S}}$ (with $\mathcal{S} = \{\theta, S_5^a\}$). This was the approach used in section 4 to relate $\Gamma_{\phi_5^0\phi_5^0}$ to the topological susceptibility $W_{\theta\theta}$. However, inverting this matrix in the $m_q \neq 0$, multi-flavour case is cumbersome, so here we use an indirect but more elegant method. First, we use the Ward identity (2.19) to express $\Gamma_{\phi_5^a\phi_5^b}$ in terms of $\Gamma_{V_{\mu5}^a V_{\nu5}^b}$:

$$k_\mu k_\nu \Gamma_{V_{\mu5}^a V_{\nu5}^b} \;=\; \Phi_{ac}\ \Gamma_{\phi_5^c\phi_5^d}\ \Phi_{db} \;-\; M_{ac}\Phi_{cb} \tag{5.10}$$

Then, using a general identity for partial Legendre transforms[19,8], we relate $\Gamma_{V_{\mu5}^a V_{\nu5}^b}$ to the current propagator $W_{V_{\mu5}^a V_{\nu5}^b}$:

$$\Gamma_{V_{\mu5}^a V_{\nu5}^b} \;=\; W_{V_{\mu5}^a V_{\nu5}^b} \;-\; W_{V_{\mu5}^a\mathcal{S}}\ W^{-1}_{\mathcal{S}\mathcal{T}}\ W_{\mathcal{T}V_{\nu5}^b} \tag{5.11}$$

where $\mathcal{S}$ and $\mathcal{T}$ represent $\{\theta, S_5^a\}$. Combining with eqs.(2.12) and (2.19) finally yields

$$\Phi_{ac}\ \Gamma_{\phi_5^c\phi_5^d}\ \Phi_{db} \;=\; W_{S_D^a S_D^b} \;+\; k_\mu W_{V_{\mu5}^a\mathcal{S}}\ W^{-1}_{\mathcal{S}\mathcal{T}}\ W_{\mathcal{T}V_{\nu5}^b}k_\nu \tag{5.12}$$

and so, in matrix notation,

$$\Phi B^T B\Phi \;=\; \frac{d}{dk^2}W_{S_D S_D}\big|_{k=0} \;+\; \frac{d}{dk^2}\big(k_\mu W_{V_{\mu5}\mathcal{S}}\ W^{-1}_{\mathcal{S}\mathcal{T}}\ W_{\mathcal{T}V_{\nu5}}k_\nu\big)\big|_{k=0} \tag{5.13}$$

The argument is almost complete. ΦB^T is precisely the combination we need for the GT relations (5.8), and is related by eq.(5.13) directly to the first moment of the correlation function (2.13) for the divergences of the currents.

It remains to show that the final term in eq.(5.13) vanishes. The first and last factors are of $O(k^2)$, so this will contribute zero unless there is a $1/k^2$ pole in the inverse pseudoscalar propagator matrix $W^{-1}_{\mathcal{S}\mathcal{T}}$. As already mentioned, there are no $1/k^2$ poles in the propagators themselves, so all we need show is that the determinant $\Delta(k^2)$ of this propagator matrix is non-vanishing at $k = 0$. This follows from the formula

$$\Delta(0) = W_{\theta\theta}(0)\ (\mathrm{det}M)^{-1}\ \mathrm{det}\Phi \tag{5.14}$$

since $\mathrm{det}M$ and $\mathrm{det}\Phi$ are non-zero (eqs.(2.8)) and $W_{\theta\theta}$ is non-vanishing away from the chiral limit. An elegant proof of eq.(5.14), based on the zero-momentum Ward identities for $W_{\mathcal{S}\mathcal{T}}$ is as follows. Define

$$\hat{M} = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & M \end{pmatrix} \qquad\qquad W = \begin{pmatrix} W_{\theta\theta} & W_{\theta S_5^b} \\ W_{S_5^a\theta} & W_{S_5^a S_5^b} \end{pmatrix} \tag{5.15}$$

with $\Delta = \det W$. Then,

$$\det M \quad \Delta = \det\hat{M} \ \det W$$

$$= \begin{vmatrix} W_{\theta\theta} & W_{\theta S_5^b} \\ M_{ac}W_{S_5^c\theta} & M_{ac}W_{S_5^c S_5^b} \end{vmatrix} \tag{5.16}$$

Using the zero-momentum Ward identities (2.14) and, in the case of the $a = 0$ row taking a linear combination with the first (θ) row, the determinant simplifies, leaving

$$\det M \quad \Delta = \begin{vmatrix} W_{\theta\theta} & W_{\theta S_5^b} \\ 0 & -\Phi_{ab} \end{vmatrix}$$

$$= W_{\theta\theta} \ \det\Phi \tag{5.17}$$

as required.

This completes the proof of the GT relations. To summarise, we have shown that the flavour singlet and non-singlet axial charges are given by the single, unified relation

$$2m_N G_A^a \quad = \quad F_{ab} \ \hat{\Gamma}_{\eta^b NN}\big|_{k=0} \tag{5.18}$$

where $F \equiv \Phi B^T$ is determined from

$$FF^T = \frac{d}{dk^2} W_{S_D S_D}\big|_{k=0} \tag{5.19}$$

that is,

$$F_{ac}F_{cb}^T = \lim_{k=0} \frac{d}{dk^2} \ i \int dx \ e^{ikx} \ \langle 0|T \ \partial^\mu J_{\mu 5}^a(x) \ \partial^\nu J_{\nu 5}^b(0)|0\rangle \tag{5.20}$$

The current algebra approximation to eq.(5.20) is easily found. Consider just the G_A^3 relation for simplicity, since in this case isospin invariance (assume $m_u = m_d = 0$) ensures that there is no flavour mixing. Applying pole dominance to the current correlator, we find

$$F_{33}^2 = \lim_{k=0} \frac{d}{dk^2} \left(f_\pi^2 k^4 \frac{1}{k^2} \right) = f_\pi^2 \tag{5.21}$$

where in the intermediate step we have distinguished the factors of k^2 arising from the decay constant and pole terms. Under the same PCAC assumptions, the vertex can be identified with the low-energy pion-nucleon coupling constant, i.e. $\hat{\Gamma}_{\eta^3 NN}|_{k=0} = g_{\pi NN}$. Writing G_A^3 in standard notation as $g_A/2$, where g_A is the isotriplet axial-vector coupling of the nucleon measured in beta decay, we therefore recover in this approximation the standard GT formula

$$f_\pi g_{\pi NN} = m_N g_A \tag{5.22}$$

We emphasise again, however, that the relations (5.18) and (5.20) are *exact* – they are neither dependent on this interpretation nor make any PCAC assumption or approximation.

The GT relation simplifies in the chiral limit, where flavour mixing is absent. In this case, the singlet axial charge is simply given by

$$2m_N G_A^0 = 2n_f \ \sqrt{\chi'(0)} \ \hat{\Gamma}_{\eta^0 NN}\big|_{k=0} \tag{5.23}$$

where as usual $\chi'(0)$ is the slope of the topological susceptibility.

This formula can now be recognised as the basis of our resolution of the 'proton spin' problem (see eq.(4.12)). The general formula (5.18) extends this to include mixing with the flavour non-singlet sector, introducing a matrix structure which replaces the simple square root of $\chi'(0)$ in eq.(5.23) and generalising the fields in the correlation function to the entire divergence of the current including mass terms as well as the anomaly Q.

The final task is to determine the RG properties of the various quantities entering into the derivation of the GT relations, and verify the assertion that the 1PI vertices defined here are RG invariant.

The RGE for the matrix B_{ab}, which relates the ϕ_5^a fields to the canonically normalised η^a fields by $\eta^a = B_{ab}\phi_5^b$, is readily found using the RGE (2.23) for the 2-point vertex $\Gamma_{\phi_5^a\phi_5^b}$. From the definition (5.9), using eq.(2.23) and the zero-momentum limit of the Ward identities (2.18), we deduce

$$\mathcal{D}B_{ab} = -\gamma_\phi B_{ab} + \gamma B_{ac}\Phi_{c0}\Phi_{0b}^{-1} \tag{5.24}$$

The RGE for F_{ab} now follows immediately from its definition $F = \Phi B^T$ and the RGE $\mathcal{D}\Phi = \gamma_\phi \Phi$. It is simply

$$\mathcal{D}F_{ab} = \gamma\delta_{a0}F_{0b} \tag{5.25}$$

The final step in proving RG consistency of the unified GT formulae is to show that the vertices $\hat{\Gamma}_{\eta^a NN}$ (at $k=0$) are RG invariant. Eq.(5.25) then ensures the required RGE for the axial charges,

$$\mathcal{D}G_A^a = \gamma\delta_{a0}G_A^a \tag{5.26}$$

showing that only the singlet axial charge has a non-trivial RG scaling. To check this explicitly, notice that eq.(2.24) for $\hat{\Gamma}_{\phi_5^a NN}$ simplifies at $k=0$. The contact terms vanish and using the zero-momentum Ward identities (see eq.(2.18)) we find

$$\mathcal{D}\hat{\Gamma}_{\phi_5^a NN}\big|_{k=0} = -\gamma_\phi\hat{\Gamma}_{\phi_5^a NN}\big|_{k=0} + \gamma\Phi_{a0}^{-1}\Phi_{0b}\hat{\Gamma}_{\phi_5^b NN}\big|_{k=0} \tag{5.27}$$

Since $\hat{\Gamma}_{\phi_5^a NN}\big|_{k=0} = B_{ab}^T\hat{\Gamma}_{\eta^b NN}\big|_{k=0}$, and comparing eq.(5.27) with the RGE (5.24) for B, we confirm

$$\mathcal{D}\hat{\Gamma}_{\eta^a NN}\big|_{k=0} = 0 \qquad \text{for all } a \tag{5.28}$$

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