LEPTON FAMILY NUMBER VIOLATION

Peter Herczeg
Theoretical Division
Los Alamos National Laboratory
Los Alamos, NM 87545

1 Introduction

In 1955 Lokanathan and Steinberger [1] searched for the decay $\mu \to e\gamma$, and set the limit

$$B(\mu \to e\gamma) \equiv \Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\nu\overline{\nu}) \; < \; 2 \times 10^{-5} \tag{1}$$

on the branching ratio. This result began the experimental efforts [2], that are still continuing today, which use $\mu \to e\gamma$ and similar processes to probe the symmetry properties of the fundamental interactions and to search for new interactions.

In the domain of the theory a major development after the discovery of parity violation was the recognition of the $V - A$ structure of the weak interaction [3]. Shortly afterwards, Feynman and Gell-Mann pointed out [4] that if the current x current interaction [5] is due to the exchange of a heavy charged spin-one boson (the "intermediate vector boson"), the decay $\mu \to e\gamma$ should occur in the first order in the weak interaction. Subsequently, Feinberg [6] calculated $B(\mu \to e\gamma)$, finding

$$B(\mu \to e\gamma) \simeq 10^{-4} \quad . \tag{2}$$

The comparison with the experimental result (1) led to the conclusions that one has to either abandon the idea of the intermediate vector boson, or find a way to forbid or suppress $\mu \to e\gamma$. The latter line of thought led to the introduction of "muon number", which is $L_\mu = 1$ for μ^-, $L_\mu = -1$ for μ^+, and $L_\mu = 0$ for all other particles [7]. L_μ conservation forbids $\mu \to e\gamma$, as well as other similar processes ($\mu^- \to e^-$ conversion in nuclei [8], $\mu \to 3e$, etc.). Further, L_μ conservation predicted the existence of the muon neutrino, since it can allow the decay $\mu^- \to e^- + \overline{\nu}_1 + \nu_2$ only if $\nu_1 \neq \nu_2$. The neutrinos $\nu_2 (\equiv \nu_\mu)$ and $\nu_1 (\equiv \nu_e)$ have $L_\mu = 1$ and $L_\mu = 0$, respectively. These developments culminated in the discovery of ν_μ in 1962 [9].

Today we know of the existence of three lepton families:

$$\begin{pmatrix} \nu'_e \\ e' \end{pmatrix}_L, (e')_R; \quad \begin{pmatrix} \nu'_\mu \\ \mu' \end{pmatrix}_L, (\mu')_R; \quad \begin{pmatrix} \nu'_\tau \\ \tau' \end{pmatrix}_L, (\tau')_R \tag{3}$$

Their weak and electromagnetic interactions are described by the successful Standard Model (SM) [10]. The fields in Eq. (3) are the guage group eigenstates; the primes are included to distinguish them from the (unprimed) mass eigenstates. With each family one can associate a lepton family number (LFN) $L_\ell (\ell = e, \mu, \tau)$. These are assigned to the mass eigenstates and are identical with the electron number, muon number, and τ number. In the SM the

LFN are (ignoring small instanton effects) exactly conserved. The underlying reasons for this are the following features of the SM:

- masslesssness of the neutrinos (implying that the leptonic mixing matrix in the charged current weak interaction is a unit matrix);
- the absence of Z-lepton couplings nondiagonal in the lepton mass eignestates (such as $\overline{e}\Gamma_\lambda \mu Z^\lambda$, $\overline{\nu}_e \Gamma_\lambda \nu_\mu Z^\lambda$, etc.),
- the absence of H-lepton couplings nondiagonal in the lepton mass eigenstates.

The above features imply global symmetries, which guarantee the conservation of LFN.

At present there is evidence from neutrino oscillation searches that the neutrinos are in fact massive particles and that they mix [11]. If confirmed, this would imply that the conservation of LFN is not exact. Lepton family number violation (LFNV) has been searched for with impressive sensitivities in many processes involving charged leptons. The present experimental limits on some of them (those which we shall consider here) are shown in Table I [12]. These stringent limits are not inconsistent with the neutrino oscillation results since, given the experimental bounds on the masses of the known neutrinos and the neutrino mass squared differences required by the oscillation results, the effects of LFNV from neutrino mixing would be too small to be seen elsewhere (see Section 2). The purpose of experiments searching for LFNV involving the charged leptons is to probe the existence of other sources of LFNV. Such sources are present in many extensions of the SM. In this lecture we shall discuss some of the possibilities, focusing on processes that require muon beams. Other LFNV processes, such as the decays of the kaons and of the τ, provide complementary information. In the next Section we shall consider some sources of LFNV that do not require an extension of the gauge group of the SM (the added leptons or Higgs bosons may of course originate from models with extended gauge groups). In Section 3 we discuss LFNV in left-right symmetric models. In Section 4 we consider LFNV in supersymmetric models, first in R-parity conserving supersymmetric grand unified models, and then in the minimal supersymmetric standard model with R-parity violation. The last section is a brief summary of our conclusions.

2 LFNV in $SU(2)_L \times U(1)$ Electroweak Models

2.1 Neutrino Mixing

One of the possible sources of LFNV is the mixing of neutrinos. This requires that the neutrinos have mass. The neutrinos of the SM can acquire Dirac masses through the same mechanism as the other fermions if the fermion sector is extended by right-handed neutrinos, assigned to singlet representations of $SU(2)$. Majorana masses can be generated only if the Higgs sector is enlarged. Majorana mass terms of right-handed (left-handed) neutrinos require a singlet (triplet) Higgs boson [23].

For massive (and nondegenerate) neutrinos the gauge group eigenstates of the neutrinos are in general not identical with the mass eigenstates. One has (assuming for simplicity Dirac neutrinos)

$$\nu'_{\ell L} = \sum_{i=1}^{3} U_{\ell i}\ \nu_{iL} \qquad\qquad (\ell = e, \mu, \tau), \tag{4}$$

$$\nu'_{\ell R} = \sum_{i=1}^{3} V_{\ell i}\ \nu_{iR} \qquad\qquad (\ell = e, \mu, \tau), \tag{5}$$

where the ν_i's are the mass eigenstates, $\nu_{iL} = \frac{1}{2}(1-\gamma_5)\nu_i$, and $\nu_{iR} = \frac{1}{2}(1-\gamma_5)\nu_i$; U and V are unitary matrices.

The mixing of neutrinos breaks the conservation of the LFN. We shall illustrate this on the example of $\mu \to e\gamma$ decay. In lowest order this decay originates from the sum of one-loop diagrams, in which the loop consists of the neutrino mass eigenstates and the W, and the photon is attached to any of the charged particles. The general form of the amplitude is

$$M(\mu \to e\gamma) = \overline{e}\sigma_{\lambda\nu}q_\nu(a + b\gamma_5)\epsilon^\lambda, \tag{6}$$

where q_ν and ϵ^λ are the photon momentum and polarization vector, respectively, and a, b are constants. The decay rate is given by

$$\Gamma(\mu \to e\gamma) = \frac{m_\mu^3}{8\pi}(|a|^2 + |b|^2). \tag{7}$$

In our case the contributions of ν_i to a and b are [24]

$$a_i = -b_i = \frac{g^2}{8m_W^2}\ \frac{em_\mu}{32\pi^2}\ U_{\mu i}\ U_{ei}^*\ (\frac{10}{3} - \frac{m_i^2}{m_W^2} + \cdots), \tag{8}$$

where an expansion in powers of the small parameter m_i/m_W was made. The terms in (8) independent of the neutrino masses cancel in the total amplitudes due to the unitarity relation $\Sigma_i U_{\mu i}U_{ei}^* = 0$. One obtains

$$B(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\ \left|\sum_i U_{\mu i}U_{ei}^*\frac{m_i^2}{m_W^2}\right|^2. \tag{9}$$

The branching ratio (9) is unobservably small. The contribution which potentially can be the largest is the one from ν_τ. Using $m_{\nu_\tau} < 18.2$ MeV [25] one would have, even with $|U_{e\tau}| = |U_{\mu\tau}| = 1/\sqrt{2}$, $B(\mu \to e\gamma) \lesssim 10^{-19}$. Much larger, possibly observable branching ratios for $\mu \to e\gamma$ are possible through the above mechanism if heavy neutrinos exist and if they mix with the usual ones [26]. $B(\mu \to e\gamma)$ is governed then by a function of M_N^2/m_W^2, where M_N is the mass of the heavy neutrino. LFNV through light-heavy neutrino mixing can be present even if the light neutrinos are massless [26]. A numerical analysis [27] shows that for $M_N \gtrsim 45$ GeV the present experimental limit on $B(\mu \to e\gamma)$ (see Table I) implies

$$|U_{eN}U_{\mu N}| < 2 \times 10^{-3}. \tag{10}$$

For $M_N \gtrsim M_Z$, where there is no information from heavy neutrino production experiments at LEP, the limit (10) is the best one on $|U_{eN}U_{\mu N}|$ from all sources, except possibly from $\mu^- \to e^-$ conversion in Ti (see the text further on) [29].

With the known neutrinos only, the branching ratios of coherent $\mu^- \to e^-$ conversion in nuclei [30] and of $\mu \to 3e$ are also outside of the observable range. These processes must also occur if $\mu \to e\gamma$ does, since the photon can convert into Dalitz pairs (e^-e^+ or $q\overline{q}$). Additional contributions, which in fact dominate these processes, come from the off-shell $\mu \to e\gamma$ form factors, Z-exchange (due to the induced $Z\mu e$ vertex) and W^+W^--exchange (box diagrams) [31].

For heavy neutrinos the branching ratios for $\mu^- \to e^-$ conversion (in Cu), and $\mu \to 3e$ were calculated in Ref. [28]. The ratio $B(\mu^- \to e^-)/B(\mu \to e\gamma)$ is a function of M_N; around

$M_N = 45$ GeV it is of the order of 10, and then increases to unity around $M_N = 100$. The decay $\mu \to 3e$ is less sensitive to this type of LVNV than $\mu \to e\gamma$. Around $M_N \simeq 100$ GeV one has $B(\mu \to 3e)/B(\mu \to e\gamma) \simeq 0.04$.

2.2 Z-Mediated LFNV

The necessary and sufficient condition for the current coupled to the Z to be diagonal in the charged lepton mass eigenstates is that the gauge group eigenstates of the charged leptons of a given charge (which can mix) have the same values of T_3 ($\equiv$ third component of the weak isospin) [32]. This condition is satisfied in the SM, but would be violated if we add, for example, a charged heavy lepton with both left-handed and right-handed components belonging to singlet representations of $SU(2)$, and allow it to mix with the known charged leptons [33]. Let us consider such a scenario. The neutral current in the neutral current interaction

$$\mathcal{L}_Z = -\frac{g^2}{8m_W^2}\, J_Z^\lambda J_{Z\lambda} \tag{11}$$

will be of the form

$$J_Z^\lambda = 2\overline{\ell}'_L\gamma^\lambda T_3 \ell'_L - 2sin^2\theta_W J_{em}^\lambda, \tag{12}$$

where $\overline{\ell}' = (\overline{e}', \overline{\mu}', \overline{\tau}')$. Note that E'_L, being a singlet, does not contribute to the T_3-part of J_Z^λ. The mixing of the charged leptons is described by

$$\begin{pmatrix} \ell'_L \\ E'_L \end{pmatrix} = \begin{pmatrix} a_L, \, b_L \\ c_L, \, d_L \end{pmatrix} \begin{pmatrix} \ell_L \\ E_L \end{pmatrix}, \tag{13}$$

where a_L is a 3 x 3 matrix, and b_L, c_L, d_L are numbers. It now straightforward to show that J_Z^λ contains terms of the form $\overline{e}\gamma^\lambda(1-\gamma_5)\mu$, $\overline{\mu}\gamma^\lambda(1-\gamma_5)\tau$, etc. The underlying reason for this is that the mixing of the known left-handed charged leptons is no longer governed by a unitary matrix.

The LFNV terms in the current of the known leptons are contained in $\overline{\ell}_L\gamma^\lambda a_L^+ a_L \ell$. The experimental result on $B(\mu^- \to \overline{e})_{Ti}$ yields [34]

$$|\lambda_{e\mu}| < 4 \times 10^{-7} \tag{14}$$

on the matrix element $\lambda_{e\mu} \equiv (a_L^+ a_L)_{e\mu}$. The bound on $\lambda_{e\mu}$ from the present experimental limit on $B(\mu \to 3e)$ is $|\lambda_{e\mu}| < 2 \times 10^{-6}$ [35] (the ratio $B(\mu \to 3e)/B(\mu^- \to e^-)_{Ti}$ is about 1/4 [34]). The decay $\mu \to e\gamma$ arises from one-loop diagrams with $Z - e$, $Z - \mu$ and $Z - E$ loops, and depends therefore also on the parameters of E.

LFNV could arise in an analogous way also from a new $U(1)$ gauge boson [34]. Another possibility in models with extended gauge groups is LFNV from horizontal gauge bosons (gauge bosons associated with horizontal gauge symmetries, that are introduced to relate or distinguish the families) [36].

2.3 H-Mediated LFNV

In models with extended Higgs sectors LFNV involving the charged leptons can be present at the tree level from the exchange of Higgs bosons, if the charged leptons receive their masses from more than one neutral Higgs boson [32].

Let us consider the standard electroweak model with a Higgs sector extended by a second doublet, and allow both Higgs doublets to couple to the leptons. The Yukawa couplings of the leptons are given by

$$\mathcal{L}_Y = \sum_{a,b}[h_{ab}\overline{L}^a_L\phi_1 b_R \ + \ h'_{ab}\overline{L}^a_L\phi_2 b_R] + H.c. \tag{15}$$

where $\overline{L}^a_L = (\overline{\nu}_{aL}, \overline{a}_L)$; $a, b = e, \mu, \tau$ and $(\phi_i)^T = (\phi_i^+, \phi_i^o)$ $(i = 1, 2)$. In the following, for simplicity, we shall consider in the model only the first two families. The couplings of the neutral components of ϕ_i to the charged leptons are

$$\mathcal{L}_Y^{e,\mu} = (h_{ee}\phi_1^0 + h'_{ee}\phi_2^0)\overline{e}_L e_R + (h_{e\mu}\phi_1^o + h'_{e\mu}\phi_2^o)\overline{e}_L\mu_R + (e \leftrightarrow \mu) + H.c. \tag{16}$$

In the basis where the mass matrix is diagonal we have

$$h_{ee}v_1 + h'_{ee}v_2 = m_e, \tag{17}$$

$$h_{\mu\mu}v_1 + h'_{\mu\mu}v_2 = m_\mu, \tag{18}$$

$$h_{e\mu}v_1 + h'_{e\mu}v_2 = 0, \tag{19}$$

$$h_{\mu e}v_1 + h'_{\mu e}v_2 = 0, \tag{20}$$

where the v_i are the vacuum expectation values of ϕ_i^0. Note that if the h'_{ab} vanish, we must have $h_{e\mu} = h_{e\mu} = 0$, implying LFN conservation. This is what takes place in the SM. However, if both $f_{ab} \neq 0$ and $f'_{ab} \neq 0$, LFN conservation no longer follows [37]. In this model there are five physical Higgs bosons, a pair of charged ones and three neutral ones. Both the charged and the neutral physical Higgs bosons have couplings nondiagonal in the lepton mass eigenstates.

Contributions to $\mu \to e\gamma$ come already from one-loop diagrams [37], but two-loop diagrams need to be also considered [38].

If the neutral Higgs-lepton couplings are proportional to lepton masses the two-loop diagrams (e.g., diagrams involving $H \to \gamma\gamma$ via a top quark or W loop, with one of the photons and the Higgs boson attached to the lepton line) may dominate, since they involve only one Higgs-lepton vertex and no helicity flip is required from the propagator. This is what was found in Ref. [39], where $\mu \to e\gamma$ was studied in a model where the couplings at $\overline{f}_1 f_2 H$ are proportional to $\sqrt{m_1 m_2}$. The conclusion of the authors is that the present experimental limit on $B(\mu \to e\gamma)$ implies that the LFNV neutral Higgs bosons should be heavier than about 200 GeV. In such a model $\mu \to 3e$ is less sensitive, due to the small Higgs-lepton couplings. The same is true for $M \to \overline{M}$ and $\mu^+ \to e^+\overline{\nu}_e\nu_\mu$. In $\mu^- \to e^-$ conversion the effective nucleon-Higgs coupling is enhanced due to the heavy quark contributions. The experimental limits on $B(\mu^- - e^-)$ provide model independent bounds on the LFNV Higgs lepton couplings [40].

We conclude here our discussion of possible sources of LFNV that can be considered in the framework of the $SU(2)_L \times U(1)$ electroweak models. A further posibility is the presence of spin-zero leptoquarks (in models with extended gauge groups also spin-one leptoquarks) which can also mediate LFNV. We refer the reader for this subject to Ref. [41].

3 LFNV in Left-Right Symmetric Models

Left-right symmetric models [42, 43] are attractive extensions of the SM, which provide a framework for the understanding of parity violation in the weak interaction. The simplest such models are based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with a discrete left-right symmetry. We shall consider here the class of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ models which also provides a framework for the understanding of the smallness of the masses of the neutrinos [43].

In $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ electroweak models [44] the fermions are assigned to representations of the gauge group in a left-right symmetric manner: the left- (right-) handed fermions are doublets of $SU(2)_L$ ($SU(2)_R$) and singlets of $SU(2)_R$ ($SU(2)_L$). The models contain a new charged gauge boson W_R, and a new neutral gauge boson, Z_R. The Higgs sector of the minimal model consists of the bidoublet field ϕ with $(T_L, T_R, Y) = (2, 2, 0)$, and the triplet fields $\Delta_L(3, 1, 2)$ and $\Delta_R(1, 3, 2)$. $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is broken to $SU(2)_L \times U(1)_Y$ ($Y = 2I_{2R} + B - L$) by $\langle \Delta_R \rangle_0 = v_R \neq 0$; $\langle \phi \rangle_0 \neq 0$ completes the symmetry breaking to $U(1)_{em}$. The Higgs potential allows the phenomenologically needed pattern $v_R \gg \langle \phi \rangle_0$. The vacuum expectation values of Δ_L^0 and Δ_R^0 generate Majorana mass terms for the left-handed and right-handed neutrinos, respectively. The masses of the right-handed gauge bosons and of the right-handed neutrinos are governed by v_R. The masses of the light neutrinos vanish in the limit $v_R \to \infty$.

LFNV in the above model comes from both the gauge and the Higgs sector. In the gauge sector there are now contributions from the right-handed neutrinos. The pattern of these contributions to $\mu \to e\gamma$, $\mu \to 3e$ and $\mu^- N \to e^- N$ is similar to the pattern of heavy neutrino contributions in the $SU(2)_L \times U(1)$ model. The sizes of the corresponding branching ratios depend on the mass of the W_R and on the masses and mixings of the right-handed neutrinos. For a review of these contributions see Ref. [44].

Further LFNV comes from the triplet Higgs bosons. These give contributions to $\mu \to e\gamma$, $\mu \to 3e$, and also to $M \to \overline{M}$ and $\mu^+ \to e^+ \overline{\nu}_e \nu_\mu$. The Δ's do not couple directly to the quarks, and therefore their contributions to semileptonic LFNV processes are small.

$M \to \overline{M}$ [45] can occur at the tree level via Δ_L^{++}- and Δ_R^{++}-exchange [46]. We shall consider here the contribution from Δ_L^{++}. The Δ_R^{++}- exchange contribution is similar.

The $M \to \overline{M}$ Hamiltonian resulting from Δ_L^{++}- exchange is given by

$$H_\Delta^{M\overline{M}} = -\frac{fee f_{\mu\mu}^*}{4m_{++}^2}\, \overline{e}^c(1-\gamma_5)e\; \overline{\mu}(1+\gamma_5)\mu^c + \text{H.c.} \tag{21}$$

where f_{ee} are $f_{\mu\mu}$ and $\vec{\Delta}_L$-lepton coupling constants, and m_{++} is the mass of the Δ_L^{++}. It is convenient to write (21), applying a Fierz transformation, in the form

$$H_\Delta^{M\overline{M}} = \frac{G_\Delta^{M\overline{M}}}{\sqrt{2}}\; \overline{\mu}\gamma_\lambda(1-\gamma_5)e\overline{\mu}\gamma^\lambda(1-\gamma_5)e + \text{H.c.}, \tag{22}$$

where

$$G_\Delta^{M\overline{M}} = \sqrt{2}\; \frac{f_{ee} f_{\mu\mu}^*}{8m_{++}^2}. \tag{23}$$

The coupling constant $G_\Delta^{M\overline{M}}$ is related to the conversion probability $P_{M\overline{M}}(0)$ in the absence of a magnetic field as $G_\Delta^{M\overline{M}} = G_F[P_{M\overline{M}}(0)/2.56 \times 10^{-5}]^{1/2}$ [47, 19]. Further, for an interaction of the type (22) $P_{M\overline{M}}(0.1T) = (35/100)P_{M\overline{M}}(0)$ [48]. The experimental limit on $P_{M\overline{M}}(0.1T)$ (see Table I) and the above relations imply [19]

$$\left| G_{\Delta}^{M\overline{M}} \right|_{expt} < 3 \times 10^{-3}. \tag{24}$$

The exchange of the Δ_L^+ givesSupersymmetric rise to the decay $\mu^+ \to e^+ \overline{\nu}_e \nu_\mu$, and to other LFNV two neutrino muon decays [49]. We shall assume in the following that the mixing of leptons can be neglected. Then the decay $\mu^+ \to e^+ \overline{\nu}_e \nu_\mu$ will be the dominant one, since it is the only one that is not forbidden in the absence of family mixing. The interaction responsible for $\mu^+ \to e^+ \overline{\nu}_e \nu_\mu$ is given by

$$H_\Delta = \frac{G_\Delta}{\sqrt{2}} \, \overline{\mu}(1+\gamma_5)\nu_\mu^c \overline{\nu}_e^c (1-\gamma_5) e + \text{H.c.}, \tag{25}$$

where

$$G_\Delta = -\sqrt{2} \, \frac{f_{ee} f_{\mu\mu}^*}{2 m_+^2} = -4 \frac{m_{++}^2}{m_+^2} \, G_\Delta^{M\overline{M}}. \tag{26}$$

The branching ratio $B(\mu^+ \to e^+ \overline{\nu}_e \nu_\mu) = \Gamma(\mu^+ \to e^+ \overline{\nu}_e \nu_\mu)/\Gamma(\mu^+ \to all)$ is given by

$$B(\mu^+ \to e^+ \overline{\nu}_e \nu_\mu) \simeq \frac{1}{4} \left| \frac{G_\Delta}{G_F} \right|^2 . \tag{27}$$

The decay $\mu^+ \to e^+ \overline{\nu}_e \nu_\mu$ can be identified by detecting the $\overline{\nu}_e$'s through the inverse beta decay reaction $\overline{\nu}_e p \to e^+ n$. The present experimental limit is shown in Table I. A more stringent bound can be derived from the experimental limit on $P_{M\overline{M}}(0.1T)$ [50]. Assuming that the mixing of the $\vec{\Delta}_L$ with other Higgs fields can be neglected, the masses of Δ_L^+, Δ_L^0 and Δ_L^{++} are related as [51]

$$m_+^2 = \frac{1}{2}(m_0^2 + m_{++}^2). \tag{28}$$

The relations (26), (28), and the limit (24) yield

$$|G_\Delta| = 8 \left| G_\Delta^{M\overline{M}} \right| \frac{1}{[1 + (m_0^2/m_{++}^2)]} < 2.4 \times 10^{-2}, \tag{29}$$

and therefore

$$B(\mu^+ \to e^+ \overline{\nu}_e \nu_\mu) < 1.5 \times 10^{-4}. \tag{30}$$

It can be shown [49] that in the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model we are discussing the coupling constants $G_\Delta^{M\overline{M}}$ and G_Δ have a lower bound of about 10^{-4} and 10^{-3}, respectively, if the mass of the muon neutrino is in the range 40 keV $\lesssim m_{\nu_\mu} <$ 170 keV ($\equiv$ present experimental limit). This is the range for which the ν_μ can decay fast enough to evade the requirement that the energy density of the neutrinos in the present Universe does not exceed the upper limit on the present total energy density of the Universe (the other allowed range for m_{ν_μ} is $m_{\nu_\mu} \lesssim 35$ eV where ν_μ can be stable.

The doubly charged Higgs bosons $\Delta_{L,R}^{++}$ contribute to $\mu \to 3e$ at the tree level [52]. Unlike the contributions to $M \to \overline{M}$, the ones to $\mu \to 3e$ require (in a basis where the Δ lepton couplings are diagonal) the presence of charged lepton mixing. The exchange of $\Delta_{L,R}^{++}$ and of $\Delta_{L,R}^{+}$ contributes to $\mu \to e\gamma$ at the one loop level [52].

4 LFNV in Supersymmetric Models

4.1 Supersymmetric Grand Unified Models

In supersymmetric models [53] there are new sources of LFNV. A supersymmetric interaction, which plays an important role in this regard, is the coupling of the form

$$\mathcal{L}_g = -\sqrt{2}g\left[(\phi^* T\psi)\lambda + \lambda^+(\psi^+ T\phi)\right], \tag{31}$$

where g is a gauge coupling constant, ϕ and ψ are the bosonic and the fermionic parts of a chiral superfield, λ is a gaugino, and T is an appropriate representation matrix. Our interest here is in the part of $\mathcal{L}_g$ involving the charged leptons and the sleptons. We shall consider the minimal supersymmetric standard model (MSSM), defined to be R-parity invariant, and based on supergravity with supersymmetry broken in a hidden sector.

The Lagrangian (31) can be expressed in terms of the mass eigenstate fields, applying the matrices U_ψ, U_ϕ, which diagonalize the lepton and slepton mass matrices, respectively. At the Planck scale the matrices U_ϕ and U_ψ are identical [54], and therefore the matrix $U_\phi^+ U_\psi$ involved in (31) is a unit matrix. However, at the electroweak scale (reached by evolving the parameters using the renormalization group equations) this is no longer so, due to induced contributions to the slepton mass matrices and the trilinear scalar terms. Thus the photino (or more generally, the neutralinos) will have nondiagonal couplings to the lepton and slepton mass eigenstates [55,56]. These couplings generate at one-leep level a contribution to $\mu \to e\gamma$. The constraint implied by the experimental limit on $B(\mu \to e\gamma)$ is that the sleptons are required to be nearly degenerate $(\Delta m_{s\ell}^2/m_{s\ell}^2 \lesssim 10^{-3})$ [55]. One approach to deal with this problem is to assume that the slepton mass matrices are proportional to the unit matrix, and that the matrix A_E in the trilinear scalar interactions $LA_E EH_d$ is proportional to the lepton Yukawa matrix. In such a case the theory conserves the LFN (see Ref. [57]). LFN conservation will be broken however if the theory is grand unified above a scale M_G. This is caused by the presence of LFNV interactions in a grand unified theory. LFNV manifests itself through non-diagonal contributions to the slepton mass matrices and to the trilinear scalar terms [58]. They are generated by evolving the parameters from the Planck scale to M_G. These contributions lead to a mismatch of U_ϕ and U_ψ, an thus to LFNV processes, as discussed earlier. In Ref. [57] it was noted that these LFNV effects can be calculated reliably, and that they are large because the top coupling is large. The authors find that in $SU(5)$ $B(\mu \to e\gamma)$ is $1-2$ orders of magnitude below the present experimental limit, and in $SO(10)$ even nearer. Subsequently it was found [59] that in $SU(5)$ a correlation takes place between different diagrams and as a result $B(\mu \to e\gamma)$ is below 10^{-13} for most of the parameter space. In $SO\ (10)\mu^- \to e^-$ conversion is suppressed since only the contribution of the on-shell $\mu \to e\gamma$ form factors is enhanced [57].

4.2 The MSSM with R-Parity Violation

Unlike in the SM, in the MSSM the conservation of lepton (L) and baryon (B) numbers is not automatic: the superpotential can contain L- and B-violating gauge invariant renormalizable supersymmetric terms. The general form of these is given by [60]

$$W_R = \frac{1}{2}\lambda_{ijk}L_iL_jE_k^c \ + \ \lambda'_{ijk}L_iQ_jD_k^c \ + \ \frac{1}{2}\lambda''_{ijk}U_i^cD_j^cD_k^c \ + \ \mu_iL_iH_\mu, \tag{32}$$

where L_i, Q_i, E_i^c, U_i^c and D_i^c are the chiral superfields containin, respectively, the left-handed lepton doublet, the left-handed quark doublet, etc; H_u is the superfield containing the Higgs doublet which generates the masses of charge 2/3 quarks; the subscripts on the coupling constants λ_{ijk}, λ'_{ijk} and λ''_{ijk} are family indices.

The L-violating terms in the interaction (32) violate also LFN conservation. The λ''_{ijk} term if present would have to be extremely small, to prevent too rapid proton decay. One way to deal with this problem is to postulate R-parity symmetry. R-parity is a multiplicatively conserved quantum number, defined for each particle as $R = (-1)^{3(B-L)+2s}$ where s is the spin of the particle. Thus $R = +1$ for the SM particles and $R = -1$ for their superpartners. The requirement of R-parity conservation eliminates all the terms in Eq. (32). Alternatively, it is possible to introduce a different discrete symmetry, known as "baryon parity" symmetry, which eliminates in (32) the B-violating term, while allowing the L-violating ones.

The interaction (32) gives rise to all the LFNV processes we discuss here, and also to many others.

$M \to \overline{M}$ conversion is governed by the product $\lambda_{132}\lambda^*_{231}$ [61]. The corresponding interaction, which is generated by $\tilde{\nu}_\tau$-exchange is of the form

$$H^{M\overline{M}}_{\tilde{\nu}_\tau} = \frac{\lambda_{132}\lambda^*_{231}}{4m^2_{\tilde{\nu}_\tau}}\, \overline{\mu}(1-\gamma_5)e\, \overline{\mu}(1+\gamma_5)e + \text{H.c.} \tag{33}$$

The Hamiltonian (33) can be rewritten using a Fierz transformation as

$$H^{M\overline{M}}_{\tilde{\nu}_\tau} = \frac{G^{M\overline{M}}_{\tilde{\nu}_\tau}}{\sqrt{2}}\, \overline{\mu}\gamma^\lambda(1+\gamma_5)e\, \overline{\mu}\gamma_\lambda(1-\gamma_5)e, \tag{34}$$

where

$$G^{M\overline{M}}_{\tilde{\nu}_\tau} = -\sqrt{2}\, \frac{\lambda_{132}\lambda^*_{231}}{8m^2_{\tilde{\nu}_\tau}}. \tag{35}$$

For this interaction one has $G^{M\overline{M}}_{\tilde{\nu}_\tau} = (\frac{4}{3}P_{M\overline{M}}(0)/2.56 \times 10^{-5})^{1/2}$ [62] and $P_{M\overline{M}}(0.1T) = (77.6/100)P_{M\overline{M}}(0)$ [48]. The experimental limit on $P_{M\overline{M}}(0.1T)$ implies

$$|G^{M\overline{M}}_{\tilde{\nu}_\tau}| < 2.3 \times 10^{-3} G_F, \tag{36}$$

or, equivalently,

$$|\lambda_{132}\lambda^*_{231}| < 1.5 \times 10^{-3} \left(\frac{m_{\tilde{\nu}_\tau}}{100\ \text{GeV}}\right)^2. \tag{37}$$

The decay $\mu^+ \to e^+\overline{\nu}_e\nu_\mu$ is governed by the same product of the coupling constants as $M \to \overline{M}$ [61]. The corresponding interaction is due to $\tilde{\tau}_L$ exchange, and is of the form

$$H_{\tilde{\tau}} = \frac{G_{\tilde{\tau}}}{\sqrt{2}}\, \overline{\mu}(1-\gamma_5)\nu_e\overline{\nu}_\mu(1+\gamma_5)e + \text{H.c.}, \tag{38}$$

where

$$G_{\tilde{\tau}} = \sqrt{2}\, \frac{\lambda_{132}\lambda^*_{231}}{4m^2_{\tilde{\tau}}} = -2\, G^{M\overline{M}}_{\tilde{\nu}_\tau} \left(\frac{m^2_{\tilde{\nu}_\tau}}{m^2_{\tilde{\tau}_L}}\right). \tag{39}$$

Using $m^2_{\tilde{\nu}_\tau}/m^2_{\tilde{\tau}_L} \lesssim 4$ and the limit (36), we obtain

$$B(\mu^+ \to e^+\overline{\nu}_e\nu_\mu) < 10^{-4}. \tag{40}$$

This limit is better than the direct one by about a factor of 5.

The decay $\mu^+ \to e^+\overline{\nu}_e\nu_\mu$ is only one of many other LFNV two-neutrino muon decays given rise by the first term in the interaction (32). An investigation of these is under way [63].

Stringent limits on various $\lambda_{ijk}\lambda_{k\ell m}$, $\lambda_{ijk}\lambda'_{k\ell m}$, and $\lambda'_{ijk}\lambda'_{k\ell m}$ products follow also from $\mu \to 3e$, $\mu \to e\gamma$, and $\mu^- \to e^-$ conversion [64].

Conclusions

In the Standard Model LFNV is unobservably small, even if it is extended to allow the neutrinos to have mass. Experiments searching for LFNV processes are therefore important probes of other types of LFNV. In many extensions of the SM $\mu \to e\gamma$, $\mu \to 3e$, $\mu^- \to e^-$ conversion and $M \to \overline{M}$ can occur near the present experimental limits. The continuing efforts to improve the present limits are therefore of great importance.

Acknowledgements

I would like to thank Dirk Graudenz and Milan Locher for the interesting and stimulating School, and to both them and Christine Kunz for making our stay most enjoyable.

This work was supported by the United State Department of Energy.

Table I. The experimental situation for some LFNV processes

Process		Present limit (90% c.l.)	Expected sensitivity of the ongoing experiment	Proposals
$\mu \to e\gamma$		3.8×10^{-11} [13]	$3 - 5 \times 10^{-12}$ [13]	$\sim 10^{-14}$ [14]
$\mu \to 3e$		1×10^{-12} [15]		
$(\mu^- N \to e^- N)_{coh}$	Ti	6.1×10^{-13} [16]	$\sim 2 \times 10^{-14}$ [16]	
	Pb	4.6×10^{-11} [17]		
	Au		$\sim 2 \times 10^{-14}$ [16]	
	Al			19^{-16} [18]
$M \to \overline{M}$		8.2×10^{-11} [19] [a]		
$\mu^+ \to e^+ \overline{\nu}_e \nu_\mu$		1.75×10^{-13} (Bayesian) [20] 1.17×10^{-13} (unified) [20]		

[a] The limit on muonium to antimuonium conversion ($M \to \overline{M}$) is on the conversion probability $P_{M\overline{M}}(0.1T)$ in a $0.1T$ magnetic field.

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