Reduze 2 - Distributed Feynman Integral Reduction

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PSI: Particle Theory Seminar
March 1, 2012

Outline

1. Introduction

2. Job system (load balancing)

3. Topological analyis

4. Distributed reductions

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Overwiew

Reduze 2

- computer program written in C++ to perform reductions of scalar Feynman integrals to master integrals
- arXiv:1201.4330, A.v.Manteuffel, CSt
- successor and major rewrite of `Reduze 1` by CSt
- dependencies
  - requires: `GiNaC` by Bauer, Frink, Kreckel
  - optional: `Open MPI`
  - optional: `Berkeley DB`
  - optional: `Fermat CAS` by Lewis (closed source, non-free)

Main features:

- topological analysis of **graphs** of integrals
- fully **parallelized** reductions
- resume aborted reductions
- computation of QCD diagram interferences up to masters
- generation of differential equations for masters
- ...
- `QGRAF` input and `FORM`, `Mathematica`, `Maple` output

Typical steps in calculations in pertubative Quantum Field Theories

- generate Feynman diagrams: e.g. QGRAF by Nogueira, FeynArts by Hahn
- apply Feynman rules
- build scalar interference terms: multiply diagrams or use projectors
- scalar Feynman integrals: loop/external momenta k_i/p_j

$$\int \mathrm{d}^d k_1 \ldots \int \mathrm{d}^d k_L \, \frac{(q_i q_j)^{\alpha_{ij}}}{D_1^{r_1} \ldots D_t^{r_t}}, \qquad D_i = q_{comb_i}^2 - m_i^2, \qquad q_n \in \{k_i, p_j\}$$

- use integration-by-parts (IBP) identities to reduce the integrals to master integrals: e.g. `AIR` by Anastasiou, `FIRE` by Smirnov, `Reduze`
- calculate the master integrals
- ...

need: standardized representation of integrals

- define **integral family** ("auxiliary topology"): set of propagators $\{1/D_1, \ldots, 1/D_N\}$ such that: all scalar products are linear combinations of D_i and kinematic invariants
- counting propagator exponents **indexes integrals**:

$$\text{Feynman integrals of some topologies} \quad \rightarrow \quad \mathbb{Z}^N$$

$$\int d^d k_1 \cdots d^d k_L \frac{1}{D_1^{n_1} \cdots D_N^{n_N}} \mapsto \{n_1, \ldots, n_N\} \quad \text{with } n_i \in \mathbb{Z}$$

- integrals belong to a **sector** of an integral family

```
I[FAM       T ID   R S  n_1     ...      n_9]
I[planarbox 5 182  6 1  -1 1 1 0 1 1 0 2 0]
```

- define an ordering for integrals (e.g. fewer denominators means simpler)

Overwiev of the main jobs in Reduze 2

input: a list of user-defined integral families

job: setup_sector_mappings

- construct graphs for the sectors (identify physical sectors)
- find zero sectors
- identify isomorphic graphs
- derive shifts to relate isomorphic sectors (sector relations)
- find shifts from sector to itself (sector symmetries)

job: reduce_sectors

- reduce integrals of a collection of sectors to master integrals

need other jobs:

- generate indexed integrals (seed integrals)
- generate IBP identities from the seed integrals
- use reduction results from sub-sectors $\rightarrow$ reduce them first

need a job system to handle the dependencies

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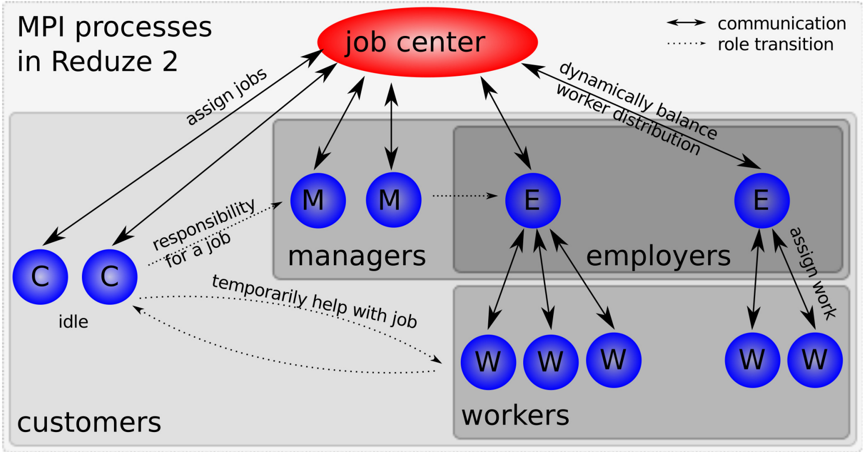
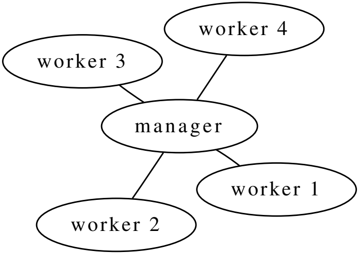
Available jobs in Reduze 2

```
$ reduze -h jobs

List of available job types:

apply_crossings:                  Generates reduction results for crossed sectors.
cat_files:                        Concatenates files.
collect_integrals:                Collects all integrals appearing in the input file.
compute_diagram_interferences:    Computes interferences of diagrams.
compute_differential_equations:   Computes derivatives of integrals wrt invariants.
export:                           Exports to FORM, Mathematica or Maple format.
extract_database_contents:        Extracts intermediate results from aborted reduction.
find_diagram_shifts:              Matches diagrams to sectors via graphs.
find_diagram_shifts_alt:          Matches diagrams to sectors via combinatorics.
generate_identities:              Generates identities like IBPs for given seeds.
generate_seeds:                   Generates integrals from a sector.
insert_reductions:                Inserts reductions in expressions.
normalize:                        Simplifies linear combinations and equations.
print_reduction_info_file:        Analyzes reductions in a file.
print_reduction_info_sectors:     Analyzes reductions available for sectors.
print_sector_info:                Prints diagrams and other information for sectors.
reduce_files:                     Reduces identities in given files.
reduce_sectors:                   Reduces integrals from a selection of sectors.
run_reduction:                    Low-level job to run a reduction.
select_reductions:                Selects reductions for integrals.
setup_sector_mappings:            Finds shifts between sectors via graphs.
setup_sector_mappings_alt:        Finds shifts between sectors via combinatorics.
sum_terms:                        Sums terms.
test:                             Performs some tests.
verify_same_terms:                Verifies two files contain the same terms.
```

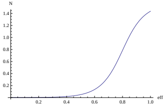
MPI processes in Reduze 2



Dynamical load balancing



- high efficiency: manager idle, workers busy
- dynamically reassing workers to managers
  - minimal worker requirement
  - worker distribution



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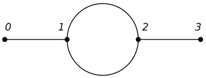
Graph Isomorphism

- **Graph** $G = (V, E)$, vertices V and edges E (pairs of vertices)
- Representation of graphs: eg. **adjacency matrix**



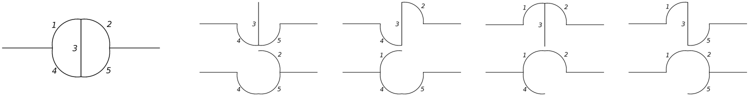
$$\begin{matrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{matrix}$$

- Two graphs $G_1 = (V, E_1)$, $G_2 = (V, E_2)$ are **isomorphic**, $G_1 \sim G_2$, if there is a permutation $\sigma : V \to V$ such that $\sigma(E_1) = E_2$
  eg. if (v_1, v_2) has k edges then also $(\sigma(v_1), \sigma(v_2))$
- **Canonical labeling** (unique representation):
  Permute the n vertices such that the adjacency matrix is minimal w.r.t. lexicographic ordering: $n!$ cases to check (if allowed, if minimal)
- **Refinement procedure**: Brendan D. McKay, Practical Graph Isomorphism, 1981
  define a degree for vertices (#adjacent edges) and partition vertices into a sequence of subsets with equal degree **(equitable partition)**

$$(\{0,1,2,3\}) \to (\{0,3\},\ \{1,2\}) \quad \Rightarrow \quad (\{0\},\{3\},\{1\},\{2\}) \to \begin{matrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{matrix}$$

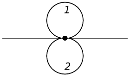
Graphs of a sector

- **Spanning tree** of a graph are all connected tree graphs which contain all vertices of the original graph (Obtained by deleting #loop edges).



- **U-polynomial**
  Sum over product of edges not in the spanning tree:
  $U = x_1x_2 + x_1x_5 + x_4x_5 + x_4x_2 + x_3x_1 + x_3x_2 + x_3x_5 + x_3x_4$
  Can also be calculated by given (inverse) propagators $P_i = q^2 - m_i^2$:
  $U = \det(M), \quad k_i M_{ij} k_j = x_l P_l, \quad$ loop momenta k_i

- Select propagators of one term, eg. x_1, x_2, attach them to a node, split vertex and try to insert the rest of the propagators (momentum conservation)



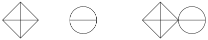
- Given a Feynman integral (sector) ⇒ corresponding graph is not unique in general.

Matroids

- Given a Feynman integral (sector) ⇒ corresponding graph is not unique in general.

- Both graphs have the same (cyclic) **matroid**
  - matroids introduced as a generalization of the concept of "linear independence"
  - graphs with the same cyclic matroid have the same U-polynomial

- **Theorem**: Hassler Whitney, 2-isomorphic graphs, AJM 55:245-254, 1933

  Two cyclic matroids are isomorphic if their graphs can be transformed into each other by a sequence of the operations:
  - vertex cleaving and identification: 
  - twisting: 

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k_i/p_j loop/(independent)external momenta
$q_n \in \{k_i, p_j\}$
$\mathbf{I}'(p_1, \ldots, p_N, k_1, \ldots, k_L)$ is integrand of a Feynman integral

$$\int \mathrm{d}^d k_i \, \frac{\partial}{\partial k_i^\mu} \left[q^\mu \, \mathbf{I}'(p_1, \ldots, p_N, k_1, \ldots, k_L) \right] = 0$$

sum over μ (no sum over i)
$L(N+L)$ equations per seed integral

Laporta algorithm:

- define **ordering** for integrals
- generate IBPs: **sparse system** of equations
- **solve linear system** of equations

- generate the system of equations
- sort equations in blocks with the **same leading integral**
- send blocks to workers

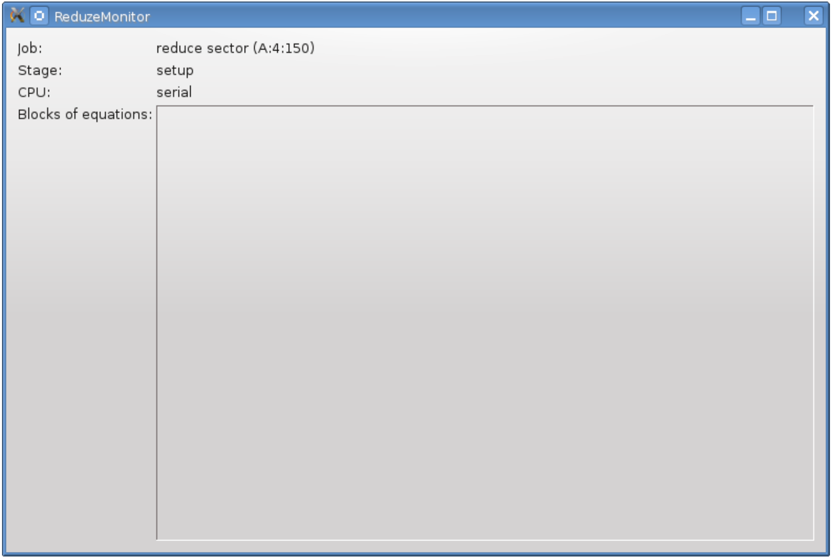
$$\begin{aligned}
\mathbf{I_5} + c_{14}\mathbf{I_4} + c_{13}\mathbf{I_3} &= 0 \\
\mathbf{I_5} + c_{24}\mathbf{I_4} + c_{22}\mathbf{I_2} &= 0 \\
\mathbf{I_5} + c_{33}\mathbf{I_3} + c_{32}\mathbf{I_2} &= 0
\end{aligned}$$

$$\begin{aligned}
\mathbf{I_3} + c_{42}\mathbf{I_2} &= 0 \\
\mathbf{I_3} + c_{51}\mathbf{I_1} &= 0
\end{aligned}$$

$$\mathbf{I_2} + c_{61}\mathbf{I_1} = 0$$

Example: distributed reduction of one sector

visualisation of reduction (subtopology of 2-loop massive double box):



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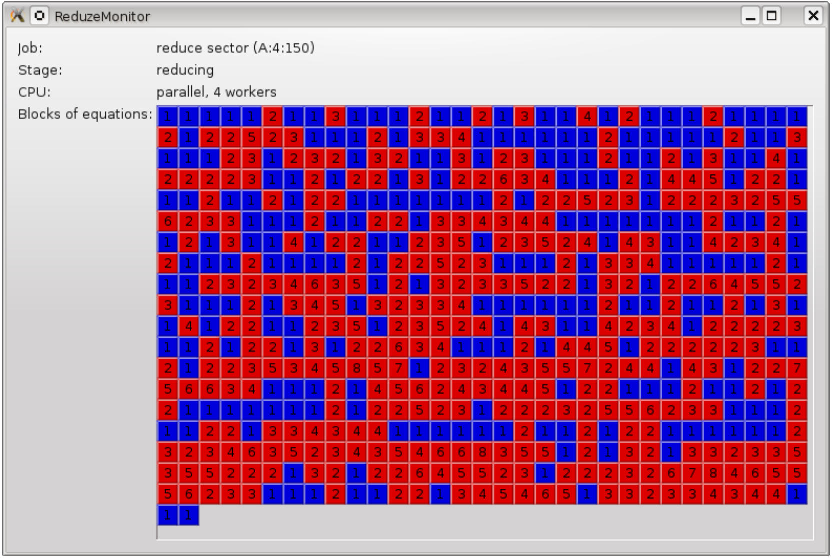

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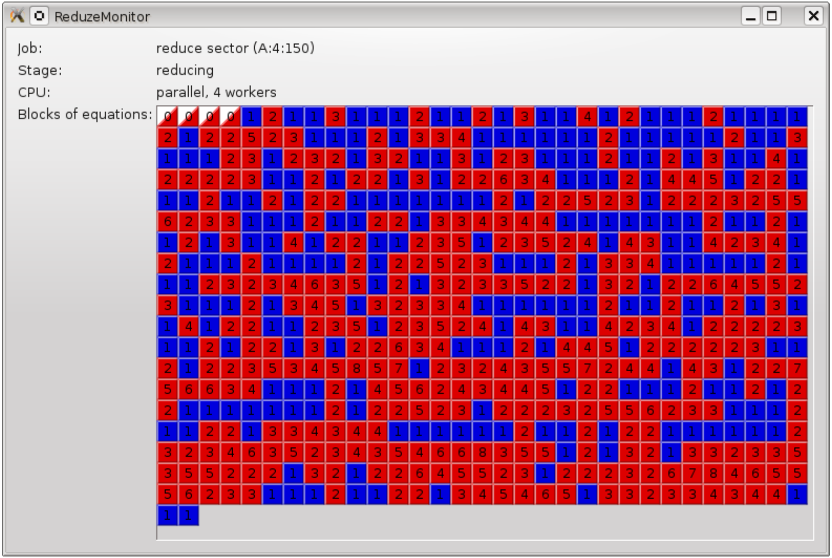

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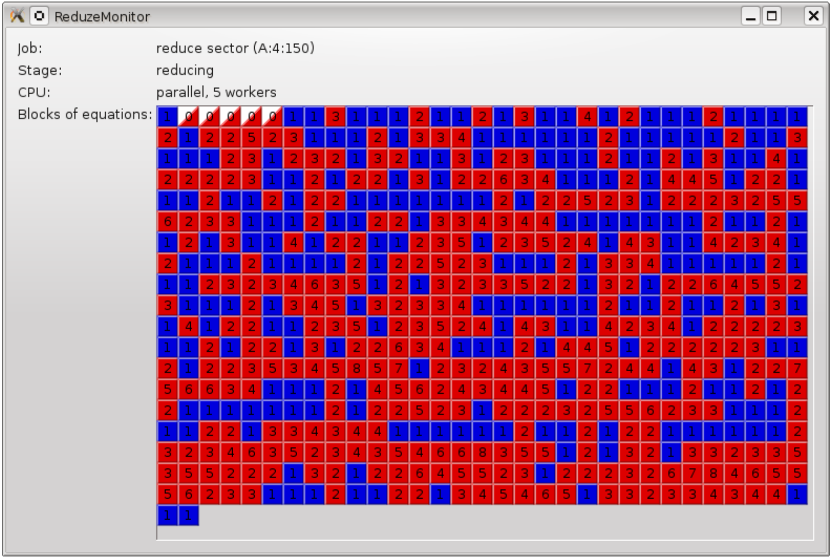

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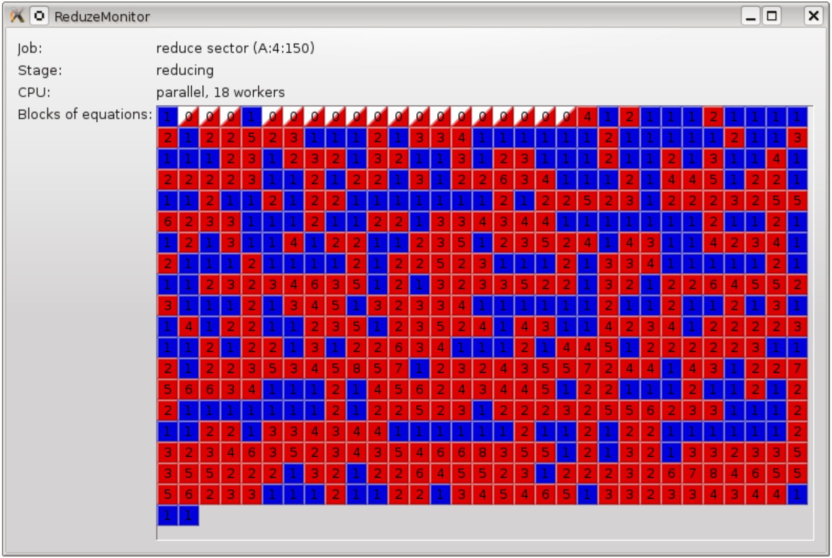

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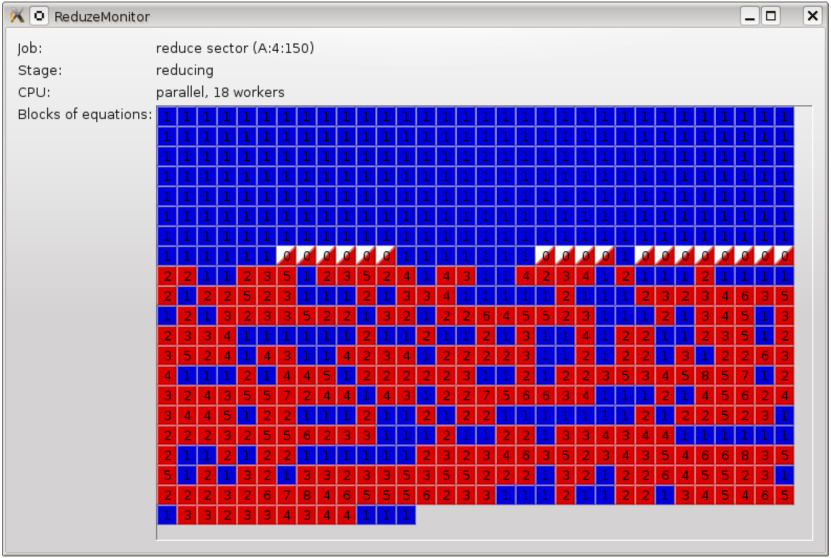

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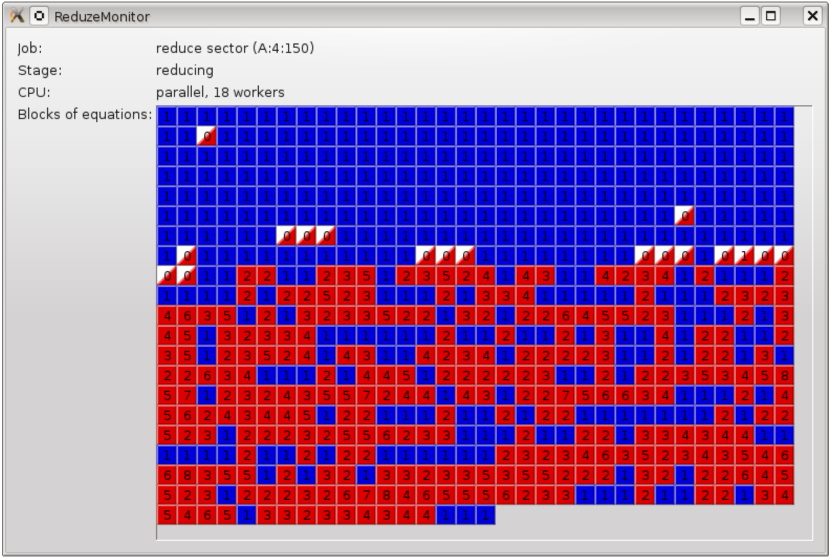
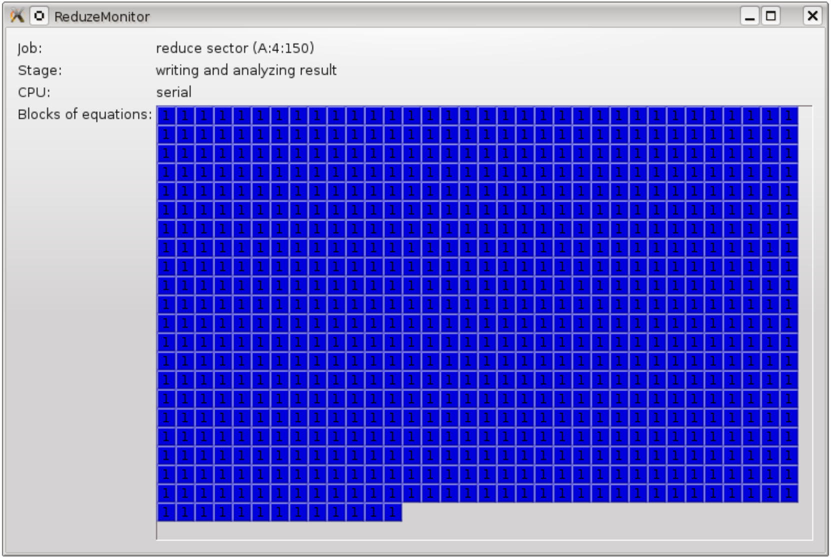


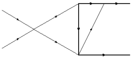
EXAMPLE: DISTRIBUTED REDUCTION OF ONE SECTOR

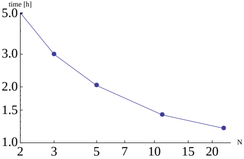
visualisation of reduction (subtopology of 2-loop massive double box):

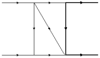


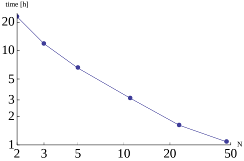
Performance: single sectors

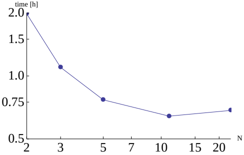
GiNaC | **Fermat**

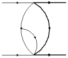


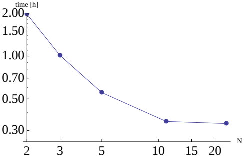




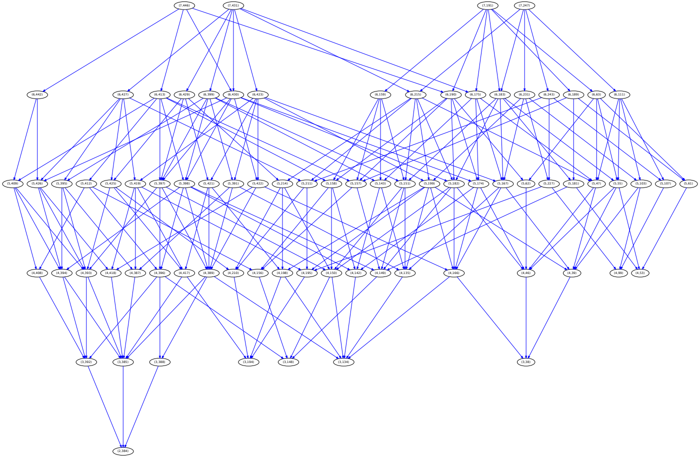


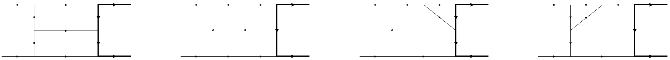




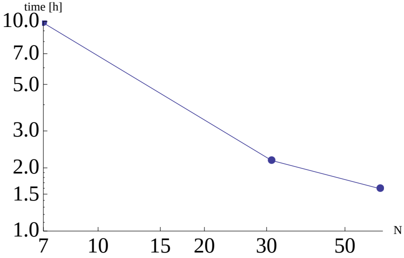


Performance: sector tree

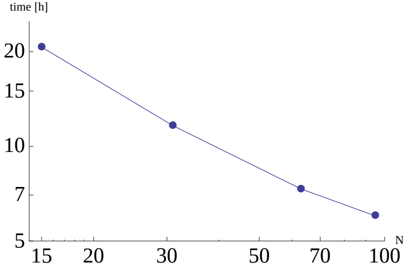




$s \in \{0, 1, 2, 3\}$



$s \in \{0, 1, 2, 3, 4\}$



Usage

Required input files in directory "config"

```
# kinematics.yaml

kinematics:
  incoming_momenta: [ p1, p2 ]
  outgoing_momenta: [ p3, p4 ]
  momentum_conservation: [p4, p1 + p2 - p3]
  kinematic_invariants:
    - [mt, 1]
    - [s,  2]
    - [t,  2]
  scalarproduct_rules:
    - [[p1,p1],  0]
    - [[p2,p2],  0]
    - [[p3,p3],  mt^2]
    - [[p1+p2, p1+p2], s]
    - [[p1-p3, p1-p3], t]
    - [[p2-p3, p2-p3], -s-t+2*mt^2] # == u
  symbol_to_replace_by_one: mt
```

```
# integralfamilies.yaml

integralfamilies:
  - name: planarbox
    loop_momenta: [k1, k2]
    propagators:
      - [ k1, 0 ]
      - [ k2, 0 ]
      - [ k1-k2, 0 ]
      - [ k1-p1, 0 ]
      - [ k2-p1, 0 ]
      - [ k1-p1-p2, 0 ]
      - [ k2-p1-p2, 0 ]
      - [ k1-p3, mt ]
      - [ k2-p3, mt ]
    permutation_symmetries:
      - [ [ 1, 6 ], [ 2, 7 ] ]
      - [ [ 1, 2 ], [ 4, 5 ], [ 6, 7 ], [ 8, 9] ]
```

Optional input files in directory "config"

```
# global.yaml

paths:
  fermat: /path/to/fermat/executable
```

```
# feynmanrules.yaml

feynmanrules: {} # see example 2 of the source package
```

Usage

Job file:

```
# myjobs.yaml

jobs:
  - setup_sector_mappings: {}
  - reduce_sectors:
      sector_selection:
        select_recursively:
          - [planarbox, 182]
      identities:
        ibp:
          - { r: [t, 5], s: [0, 1] }
        sector_symmetries:
          - { r: [t, t], s: [0, 1] }
      reduzer_options:
        use_transactions: true
  - select_reductions:
      input_file: "myintegrals"
      output_file: "myintegrals.sol"
  - export:
      input_file: "myintegrals.sol"
      output_file: "myintegrals.sol.inc"
      output_format: "form"
```

and start the program with:

```
$ reduze myjobs.yaml
```

or in parallel mode:

```
$ mpirun -np 32 reduze myjobs.yaml
```

Output: default directories: `graphs`, `sectormappings`, `reductions`, `log`, `tmp`

Thank you!