Non-minimal flavor violation in the MSSM
Outline:

- The SUSY flavor and CP problem
- Self-energies in the MSSM
- Resummation of chirally enhanced corrections
- Effective Higgs vertices
- Chirally enhanced corrections to FCNC processes
- Flavor from SUSY
- Right-handed $W$-coupling and the determination of $V_{ub}$ and $V_{cb}$.
- Constraints on the squark mass splitting from Kaon and D mixing
Introduction

Sources of flavor violation in the MSSM
Quark masses

- Top quark is very heavy: \( m_t \approx v \)
- Bottom quark rather light, but \( Y^b \) can be big at large \( \tan(\beta) \)
- All other quark masses are very small
  - sensitive to radiative corrections

\( u \, c \, t \, d \, s \, b \)
CKM matrix

- CKM matrix is the only source of flavor and CP violation in the SM.
- No tree-level FCNCs: $V_{CKM} = \begin{vmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{vmatrix}$
- Off-diagonal CKM elements are small
- Flavor-violation is suppressed in the Standard Model.
SUSY flavor (CP) problem

- The squark mass matrices are not necessarily diagonal (and real) in the same basis as the quark mass matrices.
- Especially the trilinear A-terms can induce dangerously large flavor-mixing (and complex phases) since they don’t necessarily respect hierarchy of the SM.
- The MSSM possesses two Higgs-doublets: Flavor-changing charged and (loop-induced) neutral Higgs interactions.

Why is the observed flavor violation so small?

Possible solutions:
- MFV D’Ambrosio, Giudice, Isidori, Strumia hep-ph/0207036
- Flavor-symmetries
- effective SUSY Barbieri et al hep-ph/10110730
- Radiative flavor violation
**Squark mass matrix**

\[
M_{\tilde{q}}^2 = \begin{pmatrix}
M_{\tilde{q}}^{\tilde{q} \tilde{q}}^{LL} & \Delta \tilde{q}^{\tilde{q} LR} \\
\Delta \tilde{q}^{\tilde{q} LR \dagger} & M_{\tilde{q}}^{\tilde{q} \tilde{q}}^{RR}
\end{pmatrix}
\]

**hermitian:** \[ W^{\tilde{q} \dagger} M_{\tilde{q}}^2 W^{\tilde{q}} = M_{\tilde{q}}^{2(D)} \]

\[ M_{\tilde{q}}^{\tilde{q} \tilde{q}}^{LL,RR} \] involves only bilinear terms (in the decoupling limit)

The chirality-changing elements are proportional to a vev

\[
\Delta_{ij}^{d LR} = -v_d \left( \mu \tan(\beta) Y_i^{d(0)} \delta_{ij} + A_{ij}^d \right) \\
\Delta_{ij}^{u LR} = -v_u \left( \mu \cot(\beta) Y_i^{u(0)} \delta_{ij} + A_{ij}^u \right)
\]

\[\tan(\beta) = \frac{v_u}{v_d}\]
Mass insertion approximation


- Useful to visualize flavor-changes in the squark sector
  \[ \Delta_{ij}^{q^{AB}} \] off-diagonal element of the squark mass matrix
- \( q = u, d \)
- \( i, j \) flavor indices 1,2,3
- \( A, B \) chiralitys L,R

\[ \tilde{q}_j^B \quad \times \quad \tilde{q}_f^A \]

\[ -i\Delta_{\tilde{f}\tilde{i}}^{q^{AB}} \]
Self-energies

in the MSSM
SQCD self-energy:

\[-i\Sigma(0)_{fi}^{q_{LR}} = \]

\[\Sigma_{fi}^{q_{LR}} = \alpha_s \frac{2}{3\pi} m_g W_{fs} W_{i+3,s}^* B_0 \left( m_g^2, m_{q_s}^2 \right) \]

Finite and proportional to at least one power of \( \Delta_{fi}^{q_{LR}} \)

\[\Sigma_{fi}^{q_{LR}} = \alpha_s \frac{2}{3\pi} m_g W_{fs}^q W_{js}^{q*} \Delta_{jl}^{q_{LR}} W_{i+3,t}^q W_{i+3,t}^{q*} C_0 \left( m_g^2, m_{q_s}^2, m_{q_l}^2 \right) \]
Decomposition of the self-energy

Decompose the self-energy

$$\Sigma_{ii}^{d LR} = \Sigma_{ii}^{d LR}_A + \Sigma_{ii}^{d LR}_Y$$

into a holomorphic part proportional to an A-term

$$\Sigma_{fi A}^{d LR} = -v_d \alpha_s \frac{2}{3\pi} m_g W_{fs}^d W_{js}^{d*} A_j^q W_{lt}^d W_{it}^{d*} C_0 \left( m_{\tilde{g}}^2, m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2 \right)$$

non-holomorphic part proportional to a Yukawa

$$\Sigma_{fi Y}^{d LR} = -v_u \alpha_s \frac{2}{3\pi} m_g W_{fs}^d W_{js}^{d*} Y_j^d W_{jt}^d W_{it}^{d*} C_0 \left( m_{\tilde{g}}^2, m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2 \right)$$

Define dimensionless quantity

$$\varepsilon_{i}^{d} = \frac{\Sigma_{ii}^{d LR}}{v_u Y_i^d}$$

which is independent of a Yukawa coupling
Chargino self-energy:

\[ -i \Sigma (0)_{\text{LR}}^{d} = \]

\[ \sum_{d_{i}d_{j}}^{\pm} \frac{-1}{16\pi^{2}} \mu Y_{d_{j}}^{d} \left[ V_{3f}^{\text{CKM(0)}} \ Y_{u_{3}}^{*} \Delta_{33}^{u_{RL}} \sum_{s,t=1}^{6} V_{s_{33}}^{u_{RL}} V_{t_{33}}^{d_{LL}} C_{0} \left( |\mu|^{2}, m_{u_{s}}^{2}, m_{u_{t}}^{2} \right) \right] \]

\[ -\sqrt{2} g_{2} \sin(\beta) M_{W} M_{2} \sum_{s=1}^{6} V_{s_{33}}^{d_{LL}} C_{0} \left( m_{u_{s}}^{2}, |\mu|^{2}, |M_{2}|^{2} \right) \]
Finite Renormalization

and resummation of chirally enhanced corrections

AC, Ulrich Nierste, arXiv:0810.1613
AC, Ulrich Nierste, arXiv:0908.4404
AC, arXiv:1012.4840
AC, Lars Hofer, Janusz Rosiek arXiv:1103.4272
Renormalization I

- All corrections are finite; counter-term not necessary.
- Minimal renormalization scheme is simplest.

**Mass renormalization**

\[ m_{d_i} = v_d Y^{d_i(0)} + \sum_{ii}^{d \text{LR}} \]

\[ = v_d Y^{d_i(0)} + \sum_{ii \ A}^{q \text{LR}} + v_d \tan(\beta) Y^{d_i(0)} \varepsilon_{d_i} \]

\[ Y^{d_i(0)} = \frac{m_{d_i} - \sum_{ii \ A}^{q \text{LR}}}{v_d (1 + \tan(\beta) \varepsilon_{d_i})} \]

- \( \tan(\beta) \) is automatically resummed to all orders
Renormalization II

- Flavour-changing corrections

\[
U^q_L = \left( 1 - \frac{\left| \Sigma_{12}^q \right|^2}{2m_{q_2}^2} \right) \left( -\frac{1}{m_{q_2}} \sum_{21}^q \right) \left( \frac{1}{m_{q_2}} \sum_{12}^q \right) \left( \frac{1}{m_{q_2}} \sum_{13}^q \right)
\]

\[
+ \left( -\frac{1}{m_{q_3}} \sum_{31}^q \right) \left( \frac{1}{m_{q_2}} \sum_{32}^q \sum_{21}^q \right) \left( -\frac{1}{m_{q_3}} \sum_{32}^q \right)
\]
Renormalization III

- Renormalization of the CKM matrix:
  \[ V^{(0)} = U^u_L V U^d_L \dagger \]

- Decomposition of the rotation matrices
  \[ U^q_L = U^q_L^{\text{CKM}} U^q_L \]

- Corrections independent of the CKM matrix
  \[ \tilde{V} = U^{u L \dagger}_{\text{CKM}} V^{(0)} U^{d L}_{\text{CKM}} \]

- CKM dependent corrections
  \[ U^{u L \dagger}_{\text{CKM}} \tilde{V} U^{d L}_{\text{CKM}} \]

\[ V_{13,23}^{(0)} = \frac{\tilde{V}_{13,23}}{1 + \varepsilon_{FC}} \]
Effective gaugino and higgsino vertices

- No enhanced genuine vertex corrections.

- Calculate $\varepsilon_{d_i}, \varepsilon_{FC}, \sum_{ii}^q LR, \sum_{ii}^q LR_{CKM}$

- Determine the bare Yukawas and bare CKM matrix

- Insert the bare quantities for the vertices.

- Apply rotations $U_{f_i}^{q L,R}$ to the external quark fields.

- Similar procedure for leptons (up-quarks)
Chiral enhancement

\[ \Sigma_{\text{fi}}^{d\ \text{LR}} \approx \frac{1}{50} \frac{\Delta_{\text{fi}}^{q\ \text{LR}}}{M_{\text{SUSY}}} = \frac{-V_d}{50} \left( \tan(\beta) Y_{i}^{d(0)} \delta_{ij} + \frac{A_{ij}^{d}}{M_{\text{SUSY}}} \right) \]

- For the bottom quark only, the term proportional to \(\tan(\beta)\) is important.
  - \(\tan(\beta)\) enhancement

- For the light quarks also, the part proportional to the A-term is relevant.

\[ \Sigma_{33\ Y}^{d\ \text{LR}} = \frac{-1}{100} V_d \tan(\beta) Y_{b}^{b(0)} \sim m_b \]

\[ O\left(\frac{\tan(\beta)}{100}\right) \]

\[ \Sigma_{22\ A}^{d\ \text{LR}} = O(1) \overset{\wedge}{=} A_{22}^{d} \approx M_{\text{SUSY}} \]

\[ \Sigma_{11\ A}^{d\ \text{LR}} = O(1) \overset{\wedge}{=} A_{11}^{d} \approx \frac{1}{50} M_{\text{SUSY}} \]

Blazek, Raby, Pokorski, hep-ph/9504364
Flavor-changing corrections

\[
\sum_{fi}^{qL} \frac{q_{LR}}{m_{q_{\text{max}(f,i)}}} \sim V_{fi}^{\text{CKM}}
\]

\[
V_{cb}^{\text{CKM}} : \quad A_{23}^q \approx M_{\text{SUSY}}
\]

\[
V_{ub}^{\text{CKM}} : \quad A_{13}^q \approx M_{\text{SUSY}} \times 10^{-1}
\]

\[
V_{us}^{\text{CKM}} : \quad A_{12}^q \approx M_{\text{SUSY}} \times 10^{-1}
\]

- Flavor-changing A-term can easily lead to order one correction.
Chirally enhanced Corrections to FCNC processes

AC, Ulrich Nierste, arXiv:0908.4404
Improvement of FCNC analysis necessary if A-terms are big:

Self energies can be of $O(1)$ in the flavor conserving case, and have to be resummed.


They are still of $O(1)$ in the flavor violating case, when the mixing angle is divided out.

Two- or even three-loop processes can be of the same order as the LO process!
Inclusion of the self-energies

- We treat all diagrams in which no flavor appears twice on an external leg as one particle irreducible.

- Use of the MS-bar scheme allows for a direct relation between the parameters in the squark mass matrices and observables.

- Computations are easiest if one includes the chirally enhanced self-energies into a renormalized quark-squark-gluino vertex:

\[
W_{s,i}^\tilde{q}^* \rightarrow W_{s,j}^\tilde{q}^* \left(1 + \Delta U_L^q\right)_{ji}, \quad W_{s,i+3}^\tilde{q}^* \rightarrow W_{s,j}^\tilde{q}^* \left(1 + \Delta U_R^q\right)_{ji}
\]
Two-loop effects enter only if also $m_{b\mu} \tan(\beta)$ is large.

Behavior of the branching ratio for $\delta^{d\ LR}_{23}$
Constraints on $\delta_{23}$ from $b \to s\gamma$

$\tan(\beta) = 50$

- $m_{\tilde{g}} = 2000\text{GeV}$
- $m_{\tilde{g}} = 1500\text{GeV}$
- $m_{\tilde{g}} = 1000\text{GeV}$
- $m_{\tilde{g}} = 500\text{GeV}$
Effective Higgs vertices

AC, arXiv:1012.4840
Higgs vertices in the EFT I

Flavour-diagonal case
M. Spira et al arXiv:0305101
Higgs vertices in the EFT II

\[ L_Y^{\text{eff}} = \bar{Q}_f^a L \left( \left( Y_i^d \delta_{fi} + E_{fi}^d \right) \epsilon_{ba} H_d^b + E_{fi}'^d H_u^{a*} \right) d_i R \]

- Non-holomorphic corrections: \( E_{fi}'^d = \sum_{fi}^{d \text{LR}} / \nu_u \)

- Holomorphic corrections: \( E_{fi}^d = \sum_{fi}^{d \text{LR}} / \nu_d \)

- The quark mass matrix \( m_{fi}^d = \nu_d \left( Y_i^d \delta_{fi} + E_{fi}^d \right) + \nu_u E_{fi}'^d \) is no longer diagonal in the same basis as the Yukawa coupling

Flavor-changing neutral Higgs couplings
Effective Yukawa couplings

- Final result: \( Y_{ij}^{\text{eff}} = \frac{1}{V_d} \left( m_{d_i} \delta_{ij} - \tilde{\Sigma}_{ij}^{dLR} \right) \) with

\[ \tilde{\Sigma}_{jk}^{dLR} = U_{jf}^d \sum_{jk}^{dLR} U_{ki}^d \]

\[ \approx \sum_{fi}^{dLR} - \begin{pmatrix}
0 & \sum_{22}^{dLR} \sum_{12}^{dLR} & \sum_{33}^{dLR} \sum_{13}^{dLR} \\
\sum_{22}^{dLR} m_{d_2} & 0 & \sum_{33}^{dLR} m_{d_3} \\
\sum_{33}^{dLR} m_{d_3} & \sum_{33}^{dLR} m_{q_3} & 0 
\end{pmatrix} \]

Diagrammatic explanation in the full theory:
Higgs vertices in the full theory

Cancellation incomplete since $v_d Y^{d_3} \neq m_{d_3}$
Part proportional to $\sum_{33}^{d LR} Y$ is left over.

A-terms generate flavor-changing Higgs couplings
Radiative generation of light quark masses and mixing angels

AC, Ulrich Nierste, arXiv:0908.4404
AC, Jennifer Girrbach, Ulrich Nierste, arXiv:1010.4485
SU(2)$^3$ flavor-symmetry in the MSSM superpotential:

- CKM matrix is the unit matrix.
- Only the third generation Yukawa coupling is different from zero.

\[
V_{\text{CKM}}^{(0)} = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\]

\[
Y^q = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & Y^{q_3} \\
\end{pmatrix}
\]

All other elements are generated radiatively using the trilinear A-terms!
Features of the model

- Additional flavor symmetries in the superpotential.
- Explains small masses and mixing angles via a loop-suppression.
- Deviations from MFV if the third generation is involved.
- Solves the SUSY CP problem via a mandatory phase alignment. (Phase of $\mu$ enters only at two loops)
- The SUSY flavor problem reduces to the elements $\delta_{32}^{q\, LR}, \delta_{31}^{q\, LR}$
- Can explain the $B_s$ mixing phase
CKM generation in the down-sector:

\[ \Sigma_{13}^{d\,LR} = m_b \, V_{ub} \]
\[ \Sigma_{23}^{d\,LR} = m_b \, V_{cb} \]

- **Allowed regions from** \(b \to s\gamma\).
  Chirally enhanced corrections must be taken into account.
  A.C., Ulrich Nierste 2009

- \(m_b \mu \tan (\beta) = 0.12 \text{ TeV}^2\)
- \(m_b \mu \tan (\beta) = 0 \text{ TeV}^2\)
- \(m_b \mu \tan (\beta) = 0.12 \text{ TeV}^2\)
Non-decoupling effects

- Non-holomorphic self-energies induce flavour-changing neutral Higgs couplings.
- Effect proportional to $\varepsilon_b$

\[
\varepsilon_b = \frac{\sum_{33}^{dLR} Y}{v_u Y^b} \approx \frac{\alpha_s}{3\pi} \frac{m_g \mu}{\max(m_q^2, m_g^2)}
\]
Higgs effects: $B_s \rightarrow \mu^+ \mu^-$

- Constructive contribution due to

$$\sum_{23}^d L R = m_b V_{cb}$$

- $\epsilon_b = 0.005$
- $\epsilon_b = 0.01$
- $\epsilon_b = -0.005$
- $\epsilon_b = -0.01$
Higgs effects: $B_s$ mixing

- Contribution only if

$$V_{23}^R = \frac{\sum d_{RL}^{23}}{m_b} \neq 0$$

due to Peccei-Quinn symmetry

- $\tan(\beta) = 11$
- $\tan(\beta) = 14$
- $\tan(\beta) = 17$
- $\tan(\beta) = 20$
Correlations between $B_s$ mixing and $B_s \rightarrow \mu^+\mu^-$

- $\text{Br}[B_s \rightarrow \mu^+\mu^-] \times 10^{-9}$
  - exluded by $B_s \rightarrow \mu^+\mu^-$
  - allowed from $B_s$ mixing for $\tan(\beta) = 11$
CKM generation in the up-sector:

\[ \Sigma_{13}^{u LR} = m_t V_{td}^* \]
\[ \Sigma_{23}^{u LR} = m_t V_{cb}^* \]

- Constraints from Kaon mixing.
- \( \delta_{u LR}^{31}, \delta_{u LR}^{32} \) unconstrained from FCNC processes.
- \( \delta_{u LR}^{31} \) can induce a sizable right-handed W coupling.

\[ m_\tilde{g}, M_2 = 200\text{GeV} \]
\[ M_2 = 400\text{GeV} \]
\[ M_2 = 800\text{GeV} \]
- Effects in $K \rightarrow \pi \nu \nu$
- Verifiable predictions for NA62
A right-handed W coupling
in the MSSM

Effects on the determination of $V_{ub}$ and $V_{cb}$

AC, arXiv:0907.2461
Motivation for a right-handed $W$ coupling

- $2.2 \sigma$ discrepancy between the inclusive and exclusive determination of $V_{cb}$
- $2.5-2.7 \sigma$ deviation from the SM expectation in $B \to \tau \nu$
- Tree-level processes. Commonly believed to be free of NP. (Charged Higgs contribution to $B \to \tau \nu$ is destructive.)

Notoriously difficult to explain the deviations from the SM
Effective field theory

\[ L = L_{\text{SM}} + \frac{1}{\Lambda} \sum_i C_i Q_i^{(5)} + \frac{1}{\Lambda^2} \sum_i C_i Q_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right) \]


- \( O^{(5)} \) gives rise to neutrino masses
- Focus on the dimension 6 operator

\[ Q_{RR} = \bar{u}_f \gamma^\mu P_R d_i \left( \tilde{\phi}^+ iD_{\mu} \phi \right) \]

which generates the anomalous W couplings

\[ -i \frac{g_W}{\sqrt{2}} \gamma^\mu \left( P_L V_{\tilde{f}i}^L + P_R V_{\tilde{f}i}^R \right) \]
Possible size of $V^R$

- $V^R_{tb}$ strongly constrained from $b\rightarrow s\gamma$
  Misiak et. al. 0802.1413

- $V^R_{ts} \left( V^R_{td} \right)$ also constrained from $b\rightarrow s\gamma (b\rightarrow d\gamma)$
  A.C. Lorenzo Mercolli arXiv:1106.5499

- No large effect for the first two generations possible because the CKM elements are big and the chirality violation is small.

- Sizable effects possible in $V^R_{ub}$ and $V^R_{cb}$
Genuine vertex-correction

- \( \Lambda_{\mu}^{\tilde{W}_g} = \frac{g_2}{\sqrt{2}} \frac{\alpha_s}{3\pi} \sum_{s,t=1}^{6} \sum_{j,k=1}^{3} \left( W_{ls}^{\tilde{u}t} W_{ks}^{\tilde{d}s} V_{kj}^{CKM} W_{jt}^{\tilde{d}s} W_{lt}^{\tilde{d}s} P_L \right.

- \left. + W_{f+3,s}^{\tilde{u}t} W_{ks}^{\tilde{d}s} V_{kj}^{CKM} W_{jt}^{\tilde{d}s} W_{i+3,t}^{\tilde{d}s} P_R \right) C_2 \left( m_{\tilde{u}}, m_{\tilde{d}}, m_{\tilde{g}} \right) \)

- Corrections to the left-handed coupling suppressed because the hermitian part of the WFR cancels with the genuine vertex correction.

- Right-handed coupling not suppressed!
Where are SUSY effects possible?

- $\delta_{fi}^{dLR/RL}$ strongly constrained from FCNC processes.
- $\delta_{13,23}^{uLR}$ less constrained from FCNC but constrained from the CKM renormalization.
- $\delta_{12,21}^{uLR,LL,RR}$ constrained from D mixing
- $\delta_{13,23}^{uRL}$ nearly unconstrained from FCNCs and not involved in the CKM renormalization.
- Large $\delta_{33}^{dLR}$ possible if $A^b$ or $\tan(\beta)$ is large.
- $V_{ud}, V_{us}, V_{cd}, V_{cs}$ are too large for observable effects
  
  Only $V_{ub}, V_{cb}$ can be affected by SUSY effects.
Biggest SUSY effect in $V_{ub}$.

Effect in $V_{cb}$ $\approx$ 10%
Right-handed W coupling in exclusive and inclusive B decays

V = measured CKM element

- Exclusive leptonic B decays: $\sim|\gamma^\mu\gamma^5|^2$
  $$V^L = V^L + V^R$$

- Exclusive semi-leptonic B decays to pseudo-scalar mesons $\sim|\gamma^\mu|^2$
  $$V^L = V^L - V^R$$

- Exclusive semi-leptonic B decays to vector mesons $\sim|\gamma^\mu\gamma^5|^2$ at $\omega = 1$

- Inclusive B→u decay $\sim|1+\gamma^\mu\gamma^5|^2+|1-\gamma^\mu\gamma^5|^2$
  $$V^L \approx V$$

- Inclusive B→c decay receive correction proportional to $m_c/m_b$

  Dassinger, Feger, Mannel: Complete Michel Parameter Analysis of inclusive semileptonic b→c transition

  $$V^L = V + 0.56V^R$$
Effects of a right-handed W-coupling on $V_{ub}$

- $B \rightarrow \tau \nu$
- $B \rightarrow \pi l \nu$

Graph showing inclusive CKM unitarity with $|V_{ub}^L| \times 10^3$ and $\text{Re}[V_{ub}^R/V_{ub}^L]$.
Connection to Single Top Production

Feynman diagrams contributing to Single Top production

Integrated luminosity necessary to discover Single Tops

Plehn, Rauch, Spannowski: 0906.1803
Constraints on the mass splitting between left-handed squarks from D and K mixing

AC, Momchil Davidkov, arXiv:1002.2653
K and D mixing

- Mass difference is small:
  \[ \Delta m_K / m_K = (7.01 \pm 0.01) \times 10^{-15} \]
  \[ \Delta m_D / m_D = (8.6 \pm 2.1) \times 10^{-15} \]

- CP violation is tiny
  \[ \varepsilon_K = (2.23 \pm 0.01) \times 10^{-3} \]
  \[ A_\Gamma = (1.2 \pm 2.5) \times 10^{-3} \]

Constrains FCNCs between the first two generations in a stringent way.
SU(2)_L relation: \[ m_{\tilde{u}^2}^{LL} = V^\dagger m_{\tilde{d}^2}^{LL} V \]

SUSY contributions to D and Kaon mixing can only be simultaneously avoided if \( m_{\tilde{u}^2}^{LL} \) is proportional to the unit matrix.

\[ \delta_{12}^{q LL} \] elements are induced if the left-handed squarks of the first two generations are not degenerate.
Electroweak contributions are important for

In non-minimal flavor violation the main focus is on the gluino contributions, however:

- The gluino contributions suffer from cancellations between the crossed and uncrossed boxes for \( m_{\tilde{q}} = 1.5 m_{\tilde{g}} \).
- Chargino diagrams do not suffer from such a cancellation.
- Winos couple directly to left-handed squarks with \( g_2 \).
- The wino mass is often assumed to be much lighter than the gluino mass. In most GUT scenarios: \( M_2 \approx m_\tilde{g} \frac{\alpha_2}{\alpha_s} \).
Relative importance of the contributions to $C_1$

$m_q = 1000$ GeV, $M_2 = 400$ GeV

normalized to the chargino contribution:

16.05.11
Allowed mass splitting

- Non-degenerate squark masses are allowed.
- More space for models with abelian flavor symmetries.
- Interesting for LHC benchmark scenarios.

Maximally allowed mass-splitting

Alignment to $Y_d$

Alignment to $Y_u$

Minimally allowed mass-splitting

$M_2=400$ GeV, $m_{\tilde{q}_{2,3}}=1000$ GeV
Conclusions

- Self-energies in the MSSM can be of order one.
- Chirally enhanced corrections must be taken into account in FCNC processes.
- $A$-terms generate flavor-changing neutral Higgs couplings.
- Radiative generations of light fermion masses and mixing angles solves the SUSY flavor and the SUSY CP problem. It can explain $B_s$ mixing and enhance $B_s \to \mu^+\mu^-$. 
- The first two generations of left-handed squarks can be non-degenerate.
- The MSSM can generate a sizeable right-handed $W$-coupling.

**Tree-level processes are not necessarily free of NP!**
Finite renormalization

(General formalism)

Decomposition of the self-energies:

\[
\Sigma_i^q(p) = \Sigma_i^{LR}(p^2)P_R + \Sigma_i^{RL}(p^2)P_L + p\left(\Sigma_i^{RR}(p^2)P_R + \Sigma_i^{LL}(p^2)P_L\right)
\]

Corrections to the Mass:

\[
m_{q_i}^{(0)} \rightarrow m_{q_i}^{(0)} + \Sigma_{ii}^{LR}(m_{q_i}^2) + \frac{1}{2}m_{q_i}\left(\Sigma_{ii}^{LL}(m_{q_i}^2) + \Sigma_{ii}^{RR}(m_{q_i}^2)\right) + \delta_{q_i}^m = m_{q_i}^{\text{phys}}
\]

Flavor valued wave-function corrections:

\[
U_{fi}^{L(0)} \rightarrow U_{fi}^{L(0)} + \sum_{j=1}^{3}U_{fi}^{L(0)}\Delta U_{ji}^{q L}
\]

with

\[
\Delta U_{ji}^{q L} = \frac{1}{m_{q_i}^2 - m_{q_j}^2}\left(m_{q_i}^2\Sigma_{ji}^{LL}(m_{q_j}^2) + m_{q_j}m_{q_i}\Sigma_{ji}^{LR}(m_{q_j}^2) + m_{q_i}^2\Sigma_{ji}^{RR}(m_{q_j}^2)\right), \quad f \neq i
\]

\[
\Delta U_{ii}^{q L} = \frac{1}{2}\text{Re}\left[\Sigma_{ii}^{LL}(m_{q_i}^2) + 2m_{q_i}\Sigma_{ii}^{LR}(m_{q_i}^2) + m_{q_i}^2\left(\Sigma_{ii}^{LL'}(m_{q_i}^2) + \Sigma_{ii}^{RR'}(m_{q_i}^2)\right)\right]
\]
Fine-tuning constraints

‘t Hooft’s naturalness argument:

- A small parameter is natural if a symmetry is gained if parameter is put to zero
- Large accidental cancellations, not enforced by a symmetry are unnatural

The SUSY corrections to the masses and mixing angels should not exceed the measured values.
Constrains from fermion masses I:

\( m_\ell \) in GeV

\( \Delta \rho_{ij} \times 10^{-4} \)

\( \Delta \rho_{11} \times 10^{-4} \)

\( \Delta \rho_{22} \times 10^{-2} \)

\( \Delta \rho_{33} \times 10^{-3} \)
Constrains from the CKM matrix:

Example: $V_{ub}$
Constraints on $\delta_{13}^{dLL}$ from $V_{ub}$

For large helicity flipping elements, for example $m_{b\mu}\tan(\beta)$, also $\delta_{ij}^{qLL}$ can be constrained. Strongest for $\delta_{13}^{qLL}$:

\[ |(\delta_{LL})_{13}| \]

\[ m_{G} \]

\[ \frac{\mu \tan(\beta)}{1 + \Delta} = 10 \text{TeV} \]

\[ \frac{\mu \tan(\beta)}{1 + \Delta} = 20 \text{TeV} \]

\[ \frac{\mu \tan(\beta)}{1 + \Delta} = 30 \text{TeV} \]
Constrains from fermion masses II:

- $m_{\text{SUSY}} = 500\text{GeV}$
- $m_{\text{SUSY}} = 1000\text{GeV}$
- $m_{\text{SUSY}} = 1500\text{GeV}$
- $m_{\text{SUSY}} = 2000\text{GeV}$
Existing results:

FCNC bounds (decoupling):
• M. Ciuchini et al. [arXiv:hep-ph/9808328]
• D. Becirevic et al. [arXiv:hep-ph/0112303]
• M. Ciuchini et al. [arXiv:hep-ph/0703204]
• ...

Vacuum stability bounds (non-decoupling):
J.A. Casas and S. Dimopoulos,
Stability bounds on flavor-violating trilinear soft terms in the MSSM,
## Results and comparison

<table>
<thead>
<tr>
<th>quantity</th>
<th>our bound</th>
<th>bound from FCNC</th>
<th>bound from vacuum stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta^d_{12}$</td>
<td>0.0011</td>
<td>0.006, K mixing</td>
<td>0.00015</td>
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<tr>
<td>$\delta^d_{13}$</td>
<td>0.001</td>
<td>0.15, $B_d$ mixing</td>
<td>0.005</td>
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<tr>
<td>$\delta^d_{23}$</td>
<td>0.01</td>
<td>0.06, $b \rightarrow s \gamma$</td>
<td>0.05</td>
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<tr>
<td>$\delta^u_{13}$</td>
<td>0.032</td>
<td>0.5, $B_d$ mixing</td>
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<tr>
<td>$\delta^u_{12}$</td>
<td>0.0047</td>
<td>0.016, D mixing</td>
<td>0.0012</td>
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<tr>
<td>$\delta^u_{13}$</td>
<td>0.027</td>
<td>--</td>
<td>0.22</td>
</tr>
<tr>
<td>$\delta^u_{23}$</td>
<td>0.27</td>
<td>--</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Bounds calculated with $m_{\text{squark}} = m_{\text{gluino}} = 1000 \text{GeV}$
Allowed range for NP in $\Delta F=2$ processes

Results taken for: UTfit Collaboration: www.utfit.org

Example: $B_s$ mixing

$$C_{B_s}e^{2i\phi_{B_s}} = \frac{\langle B_s | H_{\text{full}}^{\text{eff}} | \bar{B}_s \rangle}{\langle B_s | H_{\text{SM}}^{\text{eff}} | \bar{B}_s \rangle}$$
Effect of including the self-energies in $\Delta F=2$ processes

$$\frac{\Delta M_{B_{\text{Bren}}}}{\Delta M_B} \quad \text{for} \quad m_g = 1000 \text{GeV}$$
Constraints on \((\delta_{LR})_{23}\) from \(B_s\) mixing

\[ M_{1,2L/R}^{\tilde{q}} = 2M_{3L/R}^{\tilde{q}} = 1\text{TeV} \]

- \(m_{\tilde{g}} = 2000\text{GeV} \)
- \(m_{\tilde{g}} = 1500\text{GeV} \)
- \(m_{\tilde{g}} = 1000\text{GeV} \)
- \(m_{\tilde{g}} = 500\text{GeV} \)
Constraints on $\delta_{LR}$ from D, B, and K mixing
Constraints on $\delta_{23}$ from $b \to s \gamma$

$$\tan(\beta) = 50$$

- $m_{\tilde{g}} = 2000\text{GeV}$
- $m_{\tilde{g}} = 1500\text{GeV}$
- $m_{\tilde{g}} = 1000\text{GeV}$
- $m_{\tilde{g}} = 500\text{GeV}$
experimentally allowed range

$m_b \mu \tan(\beta) = 0 \text{TeV}$

$m_b \mu \tan(\beta) = -30 \text{TeV}$

$m_b \mu \tan(\beta) = 30 \text{TeV}$
Effects of a right-handed $W$-coupling on $V_{cb}$

inclusive

$B \rightarrow D\ell\nu$

$B \rightarrow D^{*}\ell\nu$