Hadron Structure in Lattice QCD

C. Alexandrou
University of Cyprus and Cyprus Institute

THE CYPRUS INSTITUTE

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Outline

Motivation

Ab initio **calculation of hadron properties**

- Post-diction of hadron properties:
  - Masses of low-lying, magnetic moments and radii of low-lying hadrons
  - Masses of excited states
  - Width of unstable particles - just starting
  - Form factors e.g. the electromagnetic form factors of the nucleon are precisely measured; the transition form factors in $N\gamma^* \to \Delta$
  - Generalized parton distributions (GPDs)

  $\to$ can we reproduce these from lattice QCD?

- Prediction of hadron properties:
  - Hybrids and exotics
  - Form factors and coupling constants of unstable particles e.g. hyperons, resonances etc
  - Hadronic contributions to weak matrix elements, electric dipole moment of the neutron, the muon-magnetic moment $\leftarrow$ awarded the *First K. Wilson prize*, X. Feng, K. Jansen, M. Petschlies and D. Renner (ETMC)
  - Phase diagram of QCD
  - New Physics?

Recent Work:

- As part of the European Twisted Mass Collaboration (ETMC) we have been studying nucleon structure
- With MIT we have been looking at N to Δ transition and Δ form factors using domain wall fermions (DWF).

$\Longrightarrow$ Calculate within lattice QCD the form factors of the nucleon/Δ system $\to$ global fit

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QCD – Gauge theory of the strong interaction

- Lagrangian: formulated in terms of quarks and gluons

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} + \sum_f \overline{\psi}_f\,(i\gamma^\mu D_\mu - m_f)\,\psi_f, \quad f = u, d, s, c, b, t$$

$$D_\mu = \partial_\mu - ig(\tfrac{1}{2}\lambda^a)A^a_\mu$$

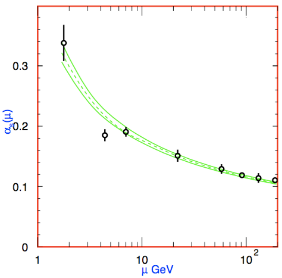

Harald Fritzsch


Murray Gell-Mann


Heinrich Leutwyler

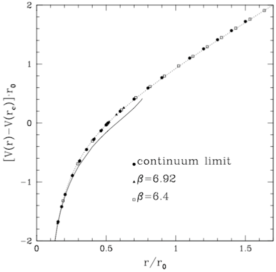
Properties of QCD

Asymptotic freedom: $g(\mu)$



[Yao et al., PDG 2006]

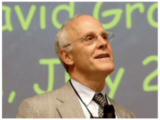
Confinement



[Necco & Sommer, Nucl Phys B622 (2002) 328]

Nobel Prize in Physics 2004

"...for the discovery of asymptotic freedom in the theory of the strong interaction"



David Gross

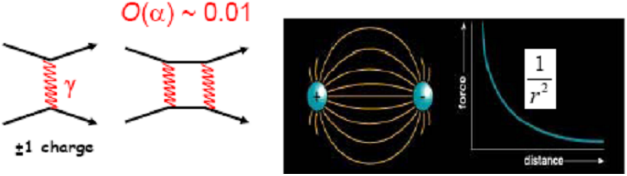
Frank Wilczek

David Politzer

QCD versus QED

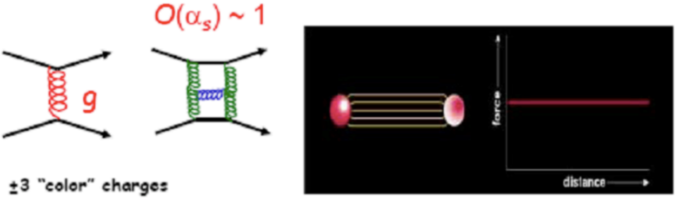
Quantum Electrodynamics (QED): The interaction is due to the exchange of photons. Every time there is an exchange of a photon there is a correction in the interaction of the order of 0.01.

→ we can apply perturbation theory reaching whatever accuracy we like



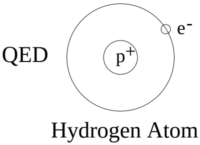
QCD: Interaction due to exchange of gluons. In the energy range of ~ 1GeV the coupling constant is ~1

→ We can no longer use perturbation theory

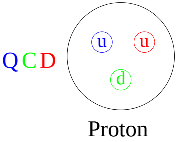


QCD versus QED

- Conventional perturbative approach cannot be applied for hadronic process at scales $\lesssim$ 1 GeV
$\Longrightarrow$ we cannot calculate the masses of mesons and baryons from QCD even if we are given α_s and the masses of quarks.
- Bound state in QCD very different from QED e.g. the binding energy of a hydrogen atom is to a good approximation the sum of it constituent masses. Similarly for nuclei the binding energy is $\mathcal{O}(\mathrm{MeV})$. For the proton almost all the mass is attributed to the strong non-linear interactions of the gluons.



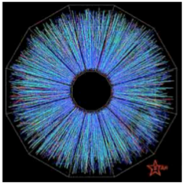
$M_e = 0.5$ MeV
$M_p = 938$ MeV
$E_{binding} = 13.6$ eV
(EM force)



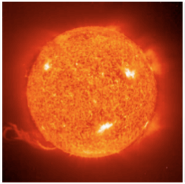
$M_u \sim 3$ MeV
$M_d \sim 6$ MeV
$M_p = 938$ MeV
(Strong force)

Strong Interaction Phenomena

The Strong Interactions describe the evolution from the big-bag to the present Universe and beyond.



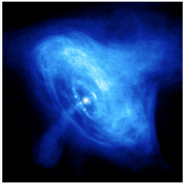
Birth



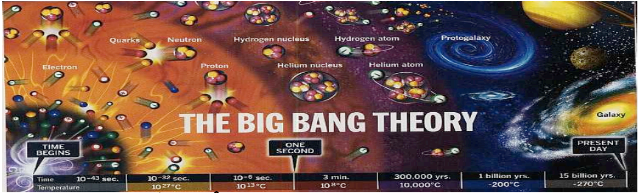
Fusion

Metals

Supernova



Collapse

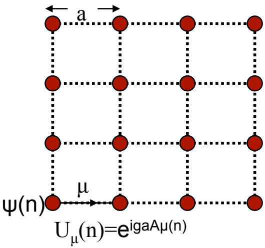


Numerical simulation of QCD already provides essential input for a wide class of physical phenomena

- QCD phase diagram relevant for Quark-Gluon Plasma: $t \sim 10^{-32}$s and $T \sim 10^{27}$, studied in heavy ion collisions at RHIC and LHC
- Hadron structure: $t \sim 10^{-6}$ s, experimental program at JLab, Mainz.
  - Momentum distribution of quarks and gluons in the nucleon
  - Hadron form factors e.g. the nucleon axial charge g_A
- Nuclear forces: $t \sim 10^9$ years, affect the large scale structure of the Universe

Exa-scale machines are required to go beyond hadrons to nuclei

QCD on the lattice



- Discretization of space-time in 4 Euclidean dimensions $\rightarrow$ $\Longrightarrow$ Rotation into imaginary time is the most drastic modification
  Lattice acts as a non-perturbative regularization scheme with the lattice spacing a providing an ultraviolet cutoff at $\pi/a \rightarrow$ no infinities
- Gauge fields are links and fermions are anticommuting Grassmann variables defined at each site of the lattice. They belong to the fundamental representation of SU(3)
- Construction of an appropriate action such that when $a \rightarrow 0$ (and Volume$\rightarrow \infty$) it gives the continuum theory
- Construction of the appropriate operators with their renormalization to extract physical quantities
- Can be simulated on the computer using methods analogous to those used for Statistical Mechanics systems $\rightarrow$ Allows calculations of correlation functions of hadronic operators and matrix elements of any operator between hadronic states in terms of the fundamental quark and gluon degrees of freedom with only input parameters the coupling constant a_s and the quarks masses.

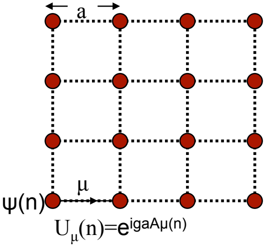
$\Longrightarrow$ Lattice QCD provides a well-defined approach to calculate observables non-perturbative starting directly from the QCD Langragian.

Consider simplest isotropic hypercubic grid: $a = a_S = a_T$ and size $N_S \times N_S \times N_S \times N_T$, $N_T > N_S$.

Lattice artifacts

- **Finite Volume:**
  1. Only discrete values of momentum in units of $2\pi/N_S$ are allowed.
  2. Finite volume effects need to be studied $\rightarrow$ Take box sizes such that $L_S m_\pi \gtrsim 3.5$.
- **Finite lattice spacing:** Need at least three values of the lattice spacing in order to extrapolate to the continuum limit.
- **q^2-values:** Fourier transform of lattice results in coordinate space taken numerically $\rightarrow$ for large values of momentum transfer results are too noisy $\Longrightarrow$ Limited to $Q^2 = -q^2 \sim 2\ \text{GeV}^2$.

QCD on the lattice



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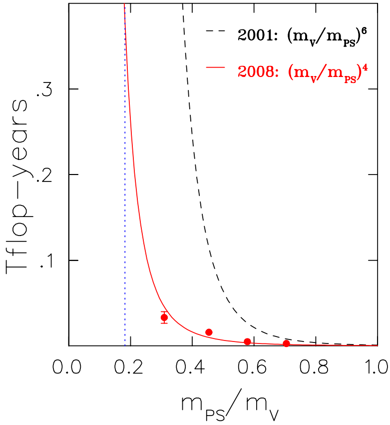
Computational cost

Simulation cost: $C_{\rm sim} \propto \left(\frac{300\text{MeV}}{m_\pi}\right)^{c_m} \left(\frac{L}{2\text{fm}}\right)^{c_L} \left(\frac{0.1\text{fm}}{a}\right)^{c_a}$



L=2.1 fm, a=0.089 fm, K. Jansen and C. Urbach, arXiv:0905.3331

Coefficients c_m, c_L and c_a depend on the discretized action used for the fermions.
State-of-the-art simulations use improved algorithms:

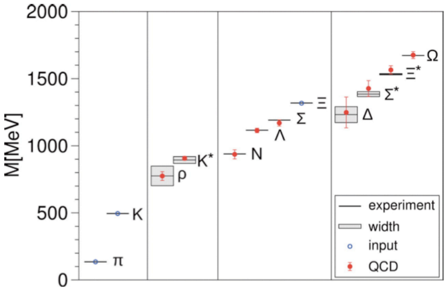
- Mass preconditioner, M. Hasenbusch, Phys. Lett. B519 (2001) 177
- Multiple time scales in the molecular dynamics updates

$\Longrightarrow$ for twisted mass fermions: $c_m \sim 4$, $c_L \sim 5$ and $c_a \sim 6$.

- Precise results at physical quark masses, $a \sim 0.1$ fm and $L \sim 5$ fm would require $\mathcal{O}(1)$ Pflop.Years.
- After post-diction of well measured quantities the goal is to predict quantities that are difficult or impossible to measure experimentally.

Mass of low-lying hadrons

$N_F = 2+1$ smeared Clover fermions, BMW Collaboration, S. Dürr et al. Science 322 (2008)
$N_F = 2$ twisted mass fermions, ETM Collaboration, C. Alexandrou et al. PRD (2008)



- BMW with $N_F = 2+1$:
  - 3 lattice spacings: $a \sim 0.125, 0.085, 0.065$ fm set by m_Ξ
  - Pion masses: $m_\pi \gtrsim 190$ MeV
  - Volumes:$m_\pi^{\rm min} L \gtrsim 4$
- ETMC with $N_F = 2$:
  - 3 lattice spacings: $a = 0.089, 0.070, a = 0.056$ fm, set my m_N
  - $m_\pi \gtrsim 260$ MeV
  - Volumes:$m_\pi^{\rm min} L \gtrsim 3.3$

Good agreement between different discretization schemes $\Longrightarrow$ Significant progress in understanding the masses of low-lying mesons and baryons

$\rightarrow$ For Δ to N and Δ form factors we will use domain wall fermions (DWF)

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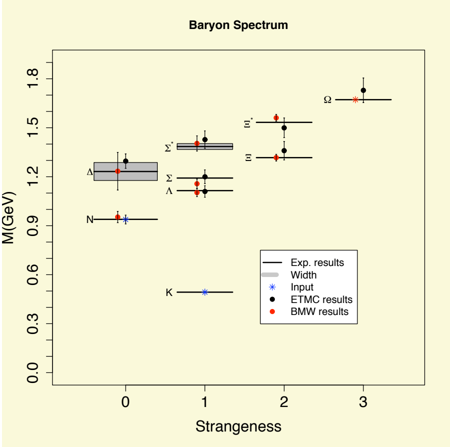

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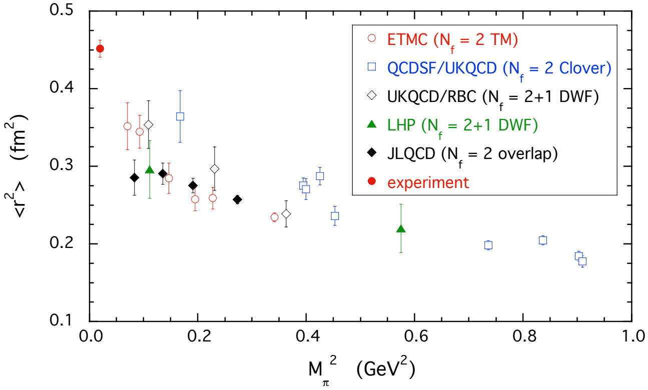
Pion form factor

Several Collaborations using dynamical quarks with pion masses down to about 300 MeV

ETMC, $N_F = 2$, R. Frezzotti, V. Lubicz and S. Simula, PRD 79, 074506 (2009)

- Examine volume and cut-off effects $\Rightarrow$ estimate continuum and infinite volume values
- Twisted boundary conditions to probe small $Q^2 = -q^2$
- All-to-all propagators and 'one-end trick' to obtain accurate results
- Chiral extrapolation using NNLO $\rightarrow$ $\langle r^2 \rangle$ and $F_\pi(Q^2) = \left(1 + \langle r^2 \rangle Q^2/6\right)^{-1}$



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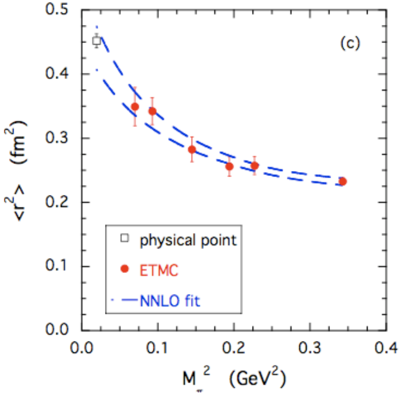

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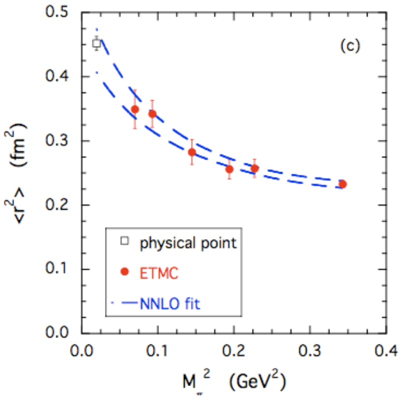
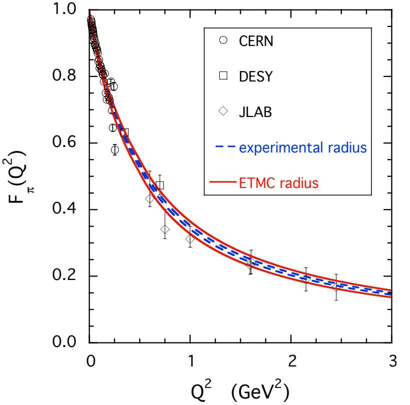
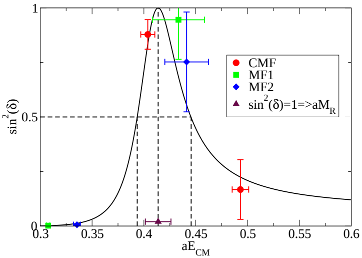




ρ-meson width

- Consider $\pi^+\pi^-$ in the $I = 1$-channel
- Estimate P-wave scattering phase shift $\delta_{11}(k)$ using finite size methods
- Use Lüscher's relation between energy in a finite box and the phase in infinite volume
- Use Center of Mass frame and Moving frame
- Use effective range formula: $tan\delta_{11}(k) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{E\left(m_R^2 - E^2\right)}$, $k = \sqrt{E^2/4 - m_\pi^2}$ $\rightarrow$ determine M_R and $g_{\rho\pi\pi}$ and then extract $\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k_R^3}{m_R^2}$, $k_R = \sqrt{m_R^2/4 - m_\pi^2}$

$m_\pi = 309$ MeV, $L = 2.8$ fm



$N_F = 2$ twisted mass fermions, Xu Feng, K. Jansen and D. Renner, Phys. Rev. D83 (2011) 094505

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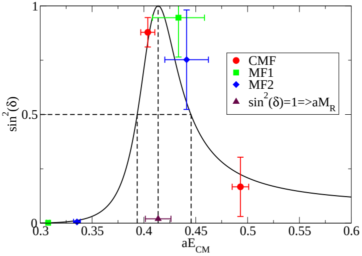

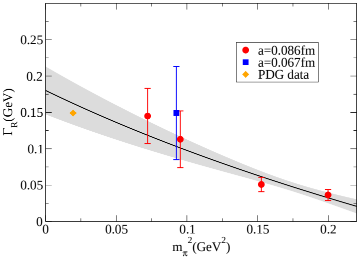

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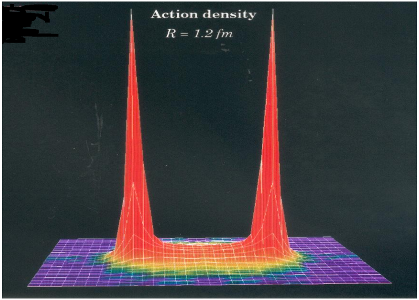
Nuclear forces

From the $q\bar{q}$ potential to the determination of nuclear forces



K. Schilling, G. Bali and C. Schlichter, 1995

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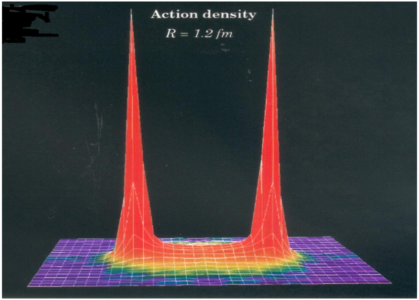

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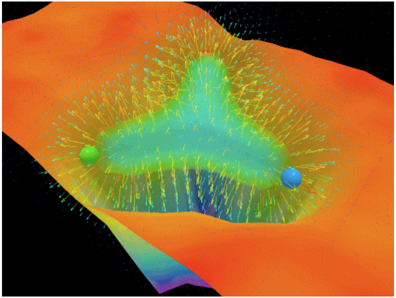
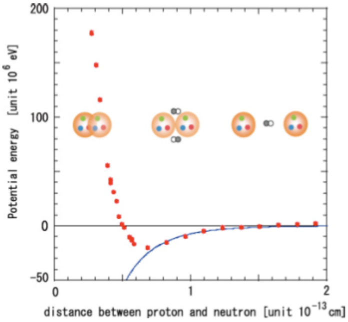


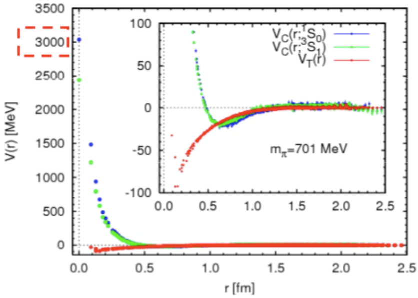
A.I. Signal, F.R.P. Bissey and D. Leinweber, arXiv:0806.0644

Nuclear forces

From the $q\bar{q}$ potential to the determination of nuclear forces

Determination of the nuclear force is essential for understanding the binding and stability of atomic nuclei, the structure of neutron stars and supernova explosions

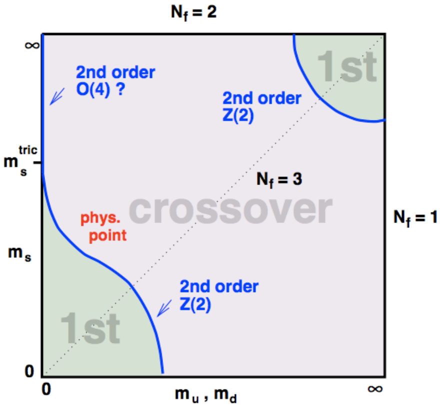




Calculate Bethe-Salteper wave-function and define from that a potential in a finite box $\Longrightarrow$ extrapolate to $L \to \infty$, S. Aoki, HAL QCD Collaboration, arXvi:1107.1284

QCD phase diagram

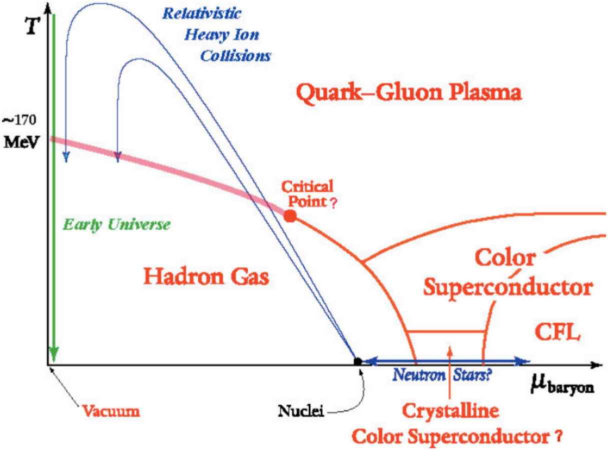
Zero baryon density, phase transition extensively studied



- 1^{st} order transition for large quark masses
- 1^{st} order transition for small quark masses
- No transition for physical u-, d- and s- quarks

QCD phase diagram

Non-zero density action becomes complex → need new techniques

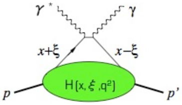


Definition of Generalized Parton Distributions (GPDs)

High energy scattering: Formulate in terms of light-cone correlation functions, M. Diehl, Phys. Rep. 388 (2003)
Consider one-particle states p' and $p \to$ GPDs, X. Ji, J. Phys. G24 (1998) 1181

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar\psi(-\lambda n/2)\Gamma \mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(n\alpha)} \psi(\lambda n/2)|p\rangle$$

"Handbag" diagram



where $q = p' - p$, $\bar P = (p'+p)/2$, n is a light-cone vector with and $\bar P.n = 1$

$$\Gamma = \not n \quad \to \frac{1}{2}\bar u(p')\left[\not n H(x,\xi,q^2) + i\frac{n_\mu q_\nu \sigma^{\mu\nu}}{2m} E(x,\xi,q^2)\right] u(p)$$

$$\Gamma = \not n\gamma_5 \quad \to \frac{1}{2}\bar u(p')\left[\not n\gamma_5 \tilde H(x,\xi,q^2) + \frac{n.q\gamma_5}{2m}\tilde E(x,\xi,q^2)\right] u(p)$$

$$\Gamma = n_\mu\sigma^{\mu\nu} \quad \to \text{tensor GPDs}$$

Expansion of the light cone operator leads to a tower of local twist-2 operators $\mathcal{O}^{\mu_1\cdots\mu_n}$, related to moments:

- Diagonal matrix element $\langle P|\mathcal{O}(x)|P\rangle$ (DIS) $\to$ parton distributions: $q(x)$, $\Delta q(x)$, $\delta q(x)$

$$\mathcal{O}^{\mu_1\cdots\mu_n} = \bar q\gamma^{\{\mu_1} iD^{\mu_2}\dots iD^{\mu_n\}} q \quad \overset{unpolarized}{\to} \quad \langle x^n\rangle_q = \int_0^1 dx\, x^n\left[q(x) - (-1)^n\bar q(x)\right]$$

$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n} = \bar q\gamma_5\gamma^{\{\mu_1} iD^{\mu_2}\dots iD^{\mu_n\}} q \quad \overset{helicity}{\to} \quad \langle x^n\rangle_{\Delta q} = \int_0^1 dx\, x^n\left[\Delta q(x) + (-1)^n\Delta\bar q(x)\right]$$

$$\mathcal{O}_T^{\rho\mu_1\cdots\mu_n} = \bar q\sigma^{\rho\{\mu_1} iD^{\mu_2}\dots iD^{\mu_n\}} q \quad \overset{transversity}{\to} \quad \langle x^n\rangle_{\delta q} = \int_0^1 dx\, x^n\left[\delta q(x) - (-1)^n\delta\bar q(x)\right]$$

where $q = q_\downarrow + q_\uparrow$, $\Delta q = q_\downarrow - q_\uparrow$, $\delta q = q_T + q_\perp$

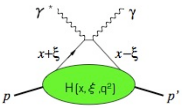
- Off-diagonal matrix elements (DVCS) $\to$ generalized form factors

Definition of Generalized Parton Distributions (GPDs)

High energy scattering: Formulate in terms of light-cone correlation functions, M. Diehl, Phys. Rep. 388 (2003)
Consider one-particle states p' and $p \to$ GPDs, X. Ji, J. Phys. G24 (1998) 1181

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar\psi(-\lambda n/2)\Gamma\mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(n\alpha)} \psi(\lambda n/2)|p\rangle$$

"Handbag" diagram



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- Off-diagonal matrix elements (DVCS) $\to$ generalized form factors

Nucleon generalized form factors

Decomposition of matrix elements into generalized form factors - contain both form factors and parton distributions:

$$\langle N(p')|\mathcal{O}^{\mu_1\cdots\mu_n}_{\not n}|N(p)\rangle = \bar{u}(p')\Big[\sum_{\substack{i=0\\ \text{even}}}^{n-1}\Big(A_{ni}(q^2)\gamma^{\{\mu_1} + B_{ni}(q^2)\frac{i\sigma^{\{\mu_1\alpha}q_\alpha}{2m}\Big)q^{\mu_2}\dots q^{\mu_{i+1}}\bar{P}^{\mu_{i+2}}\dots\bar{P}^{\mu_n\}}$$

$$+\delta^n_{\text{even}}C_{n0}(q^2)\frac{1}{m}q^{\{\mu_1}\dots q^{\mu_n\}}\Big]u(p)$$

And similarly for $\mathcal{O}_{\not n\gamma_5}$ in terms of $\tilde{A}_{ni}(q^2)$, $\tilde{B}_{ni}(q^2)$ and $\mathcal{O}_T$ in terms of A^T_{ni}, B^T_{ni}, C^T_{ni} and D^T_{ni}

Special cases:

- $n=1$: ordinary nucleon form factors

$$A_{10}(q^2)=F_1(q^2)=\int_{-1}^1 dxH(x,\xi,q^2),\quad B_{10}(q^2)=F_2(q^2)=\int_{-1}^1 dxE(x,\xi,q^2)$$
$$\tilde{A}_{10}(q^2)=G_A(q^2)=\int_{-1}^1 dx\tilde{H}(x,\xi,q^2),\quad \tilde{B}_{10}(q^2)=G_p(q^2)=\int_{-1}^1 dx\tilde{E}(x,\xi,q^2)$$

  where

  - $j_\mu=\bar\psi\gamma_\mu\psi \Longrightarrow \gamma_\mu F_1(q^2)+\frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2)$
    The Dirac F_1 and Pauli F_2 are related to the electric and magnetic Sachs form factors:
    $G_E(q^2)=F_1(q^2)-\frac{q^2}{(2m)^2}F_2(q^2),\qquad G_M(q^2)=F_1(q^2)+F_2(q^2)$
  - $j_\mu=\bar\psi\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)\Longrightarrow i\Big[\gamma_\mu\gamma_5 G_A(q^2)+\frac{q^\mu\gamma_5}{2m}G_p(q^2)\Big]\frac{\tau^a}{2}$

- $A_{n0}(0)$, $\tilde{A}_{n0}(0)$, $A^T_{n0}(0)$ are moments of parton distributions, e.g. $\langle x\rangle_q=A_{20}(0)$ and $\langle x\rangle_{\Delta q}=\tilde{A}_{20}(0)$ are the spin independent and helicity distributions
  $\to$ can evaluate quark spin, $J_q=\frac{1}{2}[A_{20}(0)+B_{20}(0)]=\frac{1}{2}\Delta\Sigma_q+L_q$
  $\to$ nucleon spin sum rule: $\frac{1}{2}=\frac{1}{2}\Delta\Sigma_q+L_q+J_g$, momentum sum rule: $\langle x\rangle_g=1-A_{20}(0)$

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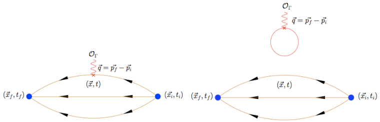
Lattice evaluation

Evaluation of two-point and three-point functions

$$G(\vec{q},t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\,\Gamma^4_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\bar{J}_\beta(0)\rangle$$

$$G^{\mu\nu}(\Gamma,\vec{q},t) = \sum_{\vec{x}_f,\vec{x}} e^{i\vec{x}\cdot\vec{q}}\,\Gamma_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\mathcal{O}^{\mu\nu}(\vec{x},t)\bar{J}_\beta(0)\rangle$$



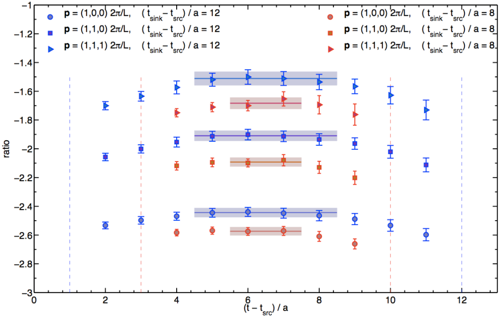
Sequential inversion "through the sink" $\rightarrow$ fix sink-source separation $t_f - t_i$, final momentum $\vec{p}_f = 0$, Γ

Apply smearing techniques to improve ground state dominance in three-point correlators

Ratios: Leading time dependence cancels

$$aE_{\rm eff}(\vec{q},t) = \ln\left[G(\vec{q},t)/G(\vec{q},t+a)\right] \rightarrow aE(\vec{q})$$

$$R^{\mu\nu}(\Gamma,\vec{q},t) = \frac{G^{\mu}(\Gamma,\vec{q},t)}{G(\vec{0},t_f)}\sqrt{\frac{G(\vec{p}_i,t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(\vec{p}_i,t)G(\vec{p}_i,t_f)}} \rightarrow \Pi^{\mu\nu}(\vec{q},\Gamma)$$



Variational approach, great improvement on plateaux: B. Blossier *et at.*, (Alpha Collaboration), JHEP 0904 (2009)

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Evaluation of two-point and three-point functions

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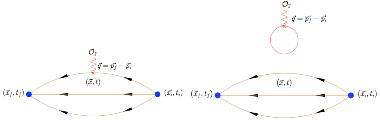

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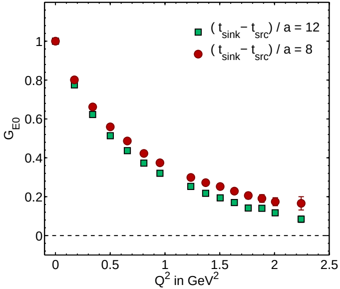
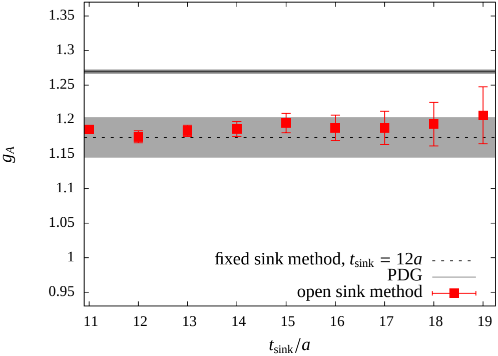


Electric form factor $\rightarrow t_f - t_i > 1$ fm

However, this might be operator dependent

Study of excited state contributions

Vary source- sink separation:



S. Dinter, C.A. M. Constantinou, V. Drach, K. Jansen and D. Renner, arXiv: 1108.1076

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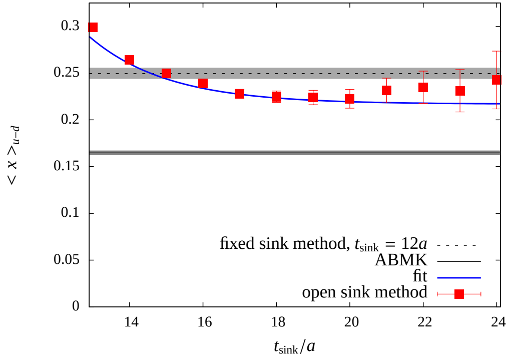


$\Longrightarrow$ Excited contribution are operator dependent

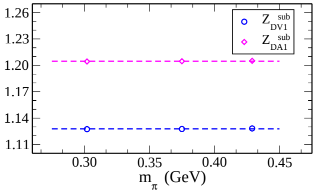
g_A unaffected, $\langle x \rangle_{u-d}$ 10% lower

S. Dinter, C.A. M. Constantinou, V. Drach, K. Jansen and D. Renner, arXiv: 1108.1076

Non-perturbative renormalization

Most collaborations use non-perturbative renormalization.

ETMC: RI'-MOM renormalization scheme as in e.g. M. Göckeler *et al.*, Nucl. Phys. B544,699

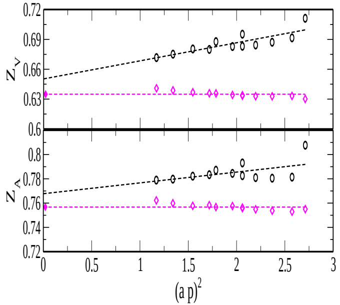


- Fix configurations to Landau gauge.

  $S^u(p) = \frac{a^8}{V} \sum_{x,y} e^{-ip(x-y)} \langle u(x)\bar{u}(y)\rangle$

  $G(p) = \frac{a^{12}}{V} \sum_{x,y,z,z'} e^{-ip(x-y)} \langle u(x)\bar{u}(z)\mathcal{J}(z,z')d(z')\bar{d}(y)\rangle$

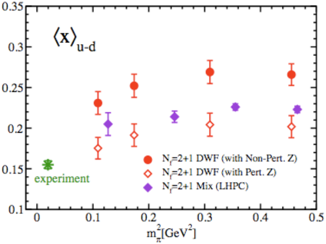
  $\rightarrow$ Amputated vertex functions $\Gamma(p) = (S^u(p))^{-1}\, G(p)\, (S^d(p))^{-1}$
- Renormalization functions: Z_q and $Z_\mathcal{O}$
- Mass independent renormalization scheme $\rightarrow$ need chiral extrapolations
- Subtract $\mathcal{O}(a^2)$ perturbatively.



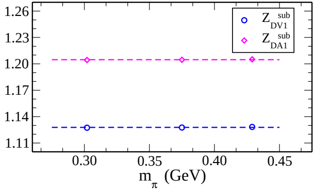
Non-perturbative renormalization

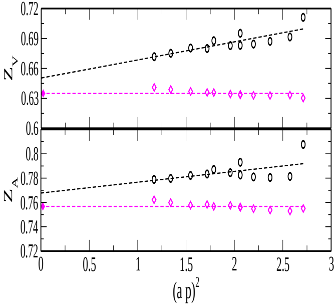
Most collaborations use non-perturbative renormalization.

- RBC: Also uses a RI'-MOM renormalization scheme but with momentum independent source, Y. Aoki *al.* arXiv:1003.3387



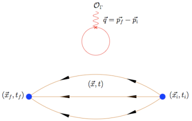
Similarly for $< x >_{\Delta u - \Delta d} \rightarrow$ non-perturbative renormalization may explain the lower values observed by LHPC

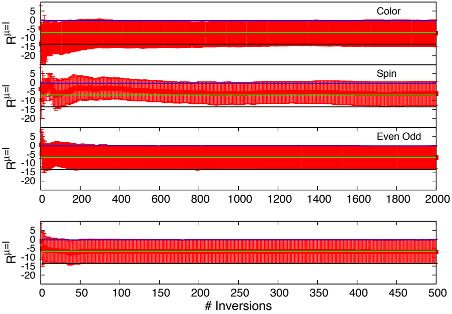


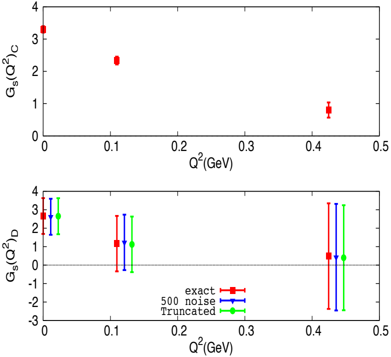


Disconnected contributions

- Approximate using stochastic techniques
- Compare with exact evaluation, enabled using GPUs
- Nucleon σ-term evaluated exactly and compared to various dilution schemes as well as the truncated solver method, G. Bali, S. Collins, A. Schafer Comput.Phys.Commun. 181 (2010) 1570



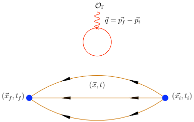


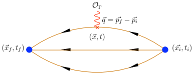


C.A., K. Hadjiyiannakou, G. Koutsou, A. 'O Cais, A. Strelchenko, arXiv:1108.2473

Nucleon form factors

Experimental measurements since the 50's but still open questions $\rightarrow$ high-precision experiments at JLab.





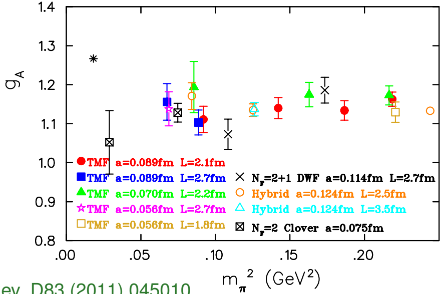
- The electric and magnetic Sachs form factors:

  $G_E(q^2) = F_1(q^2) - \frac{q^2}{(2m)^2} F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$

  - Many lattice studies down to lowest pion mass of $m_\pi \sim 300$ MeV $\Longrightarrow$ Lattice data in general agreement, but still slower q^2-slope
  - Disconnected diagrams neglected so far

- Axial-vector FFs: $A^a_\mu = \bar{\psi}\gamma_\mu\gamma_5 \frac{\tau^a}{2}\psi(x)$

  $\Longrightarrow \frac{1}{2}\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m} G_p(q^2)\right]$

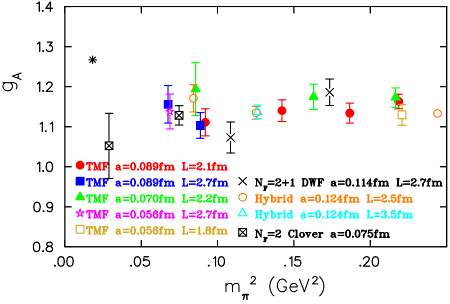


C. A. *et al.* (ETMC), Phys. Rev. D83 (2011) 045010

Similar discrepancy also for the momentum fraction, C. A. *et al.* (ETMC), PRD 83 (2011) 114513

Physical results on nucleon form factors

Axial charge is well known experimentally



- Agreement among recent lattice results - all use non-perturbative Z_A
- Weak light quark mass dependence
- What can we say about the physical value of g_A?
- Use ETMC results taking continuum limit and estimate volume corrections, A. Ali Khan, *et al.*, PRD **74**, 094508 (2006)
- Use one-loop chiral perturbation theory in the small scale expansion (SSE), T. R. Hemmert, M. Procura and W. Weise, PRD **68**, 075009 (2003).
- 3 fit parameters, $g_A^0 = 1.10(8)$, $g_{\Delta\Delta}$=2.1(1.3), $C^{SSE}(1\ GeV) = -0.7(1.7)$, axial N$\Delta$ coupling fixed to 1.5: $\Rightarrow g_A = 1.14(6)$
- Fitting lattice results directly leads to $g_A = 1.12(7)$

Results shown are from:

- $N_F = 2$ twisted mass fermions, ETMC, C.A. *et al.* PRD 83 (2011) 045010.
- $N_F = 2 + 1$ Domain wall fermions, RBC-UKQCD, T. Yamazaki *et al.*, PRD 79 (2009) 14505.
- $N_F = 2 + 1$ hybrid action, LHPC, J. D. Bratt *et al.*,PRD 82 (2010) 094502.
- $N_F = 2$ Clover, QCDSF, D. Pleiter *et al.*, arXiv:1101.2326; CLS, S.Capitano, B.Knippschild, M. Della Morte, H. Wittig arXiv:1011.1358; B. B. Brandt *et al.*, arXiv:1106.1554.
- $N_F = 2 = 1$ DWF, RBC-UKQCD, S. Ohta, arXiv:1011.1388.

Δ axial charge can be extracted from lattice

$\Longrightarrow$ Study N-Δ system to determine parameters that can help with chiral expansions

In a similar spirit, the determination of the axial charges for other octet baryons can also provide input for χPT, H.- W. Lin and K. Orginos, PRD 79, 034507 (2009); M. Gockeler *et l.*, arXiv:1102.3407

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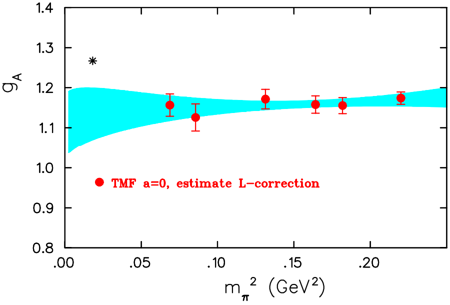

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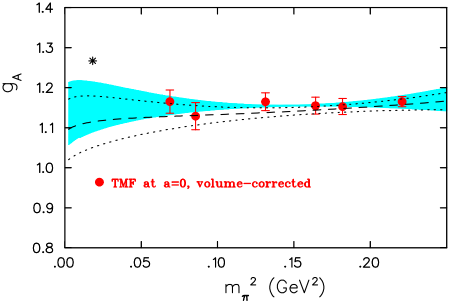
- $N_F = 2 + 1$ hybrid action, LHPC, J. D. Bratt *et al.*,PRD 82 (2010) 094502.
- $N_F = 2$ Clover, QCDSF, D. Pleiter *et al.*, arXiv:1101.2326; CLS, S.Capitano, B.Knippschild, M. Della Morte, H. Wittig arXiv:1011.1358; B. B. Brandt *et al.*, arXiv:1106.1554.
- $N_F = 2 = 1$ DWF, RBC-UKQCD, S. Ohta, arXiv:1011.1388.

Δ axial charge can be extracted from lattice
$\Longrightarrow$ Study N-Δ system to determine parameters that can help with chiral expansions
In a similar spirit, the determination of the axial charges for other octet baryons can also provide input for χPT, H.- W. Lin and K. Orginos, PRD 79, 034507 (2009); M. Gockeler *et l.*, arXiv:1102.3407

Physical results on nucleon form factors

Axial charge is well known experimentally



- Agreement among recent lattice results - all use non-perturbative Z_A
- Weak light quark mass dependence
- What can we say about the physical value of g_A?
- Use ETMC results taking continuum limit and estimate volume corrections, A. Ali Khan, *et al.*, PRD **74**, 094508 (2006)
- Use one-loop chiral perturbation theory in the small scale expansion (SSE), T. R. Hemmert, M. Procura and W. Weise, PRD **68**, 075009 (2003).
- 3 fit parameters, $g_A^0 = 1.10(8)$, $g_{\Delta\Delta}$=2.1(1.3), $C^{SSE}(1\ GeV) = -0.7(1.7)$, axial N$\Delta$ coupling fixed to 1.5: $\Rightarrow g_A = 1.14(6)$
- Fitting lattice results directly leads to $g_A = 1.12(7)$

Results shown are from:

- $N_F = 2$ twisted mass fermions, ETMC, C.A. *et al.* PRD 83 (2011) 045010.
- $N_F = 2 + 1$ Domain wall fermions, RBC-UKQCD, T. Yamazaki *et al.*, PRD 79 (2009) 14505.
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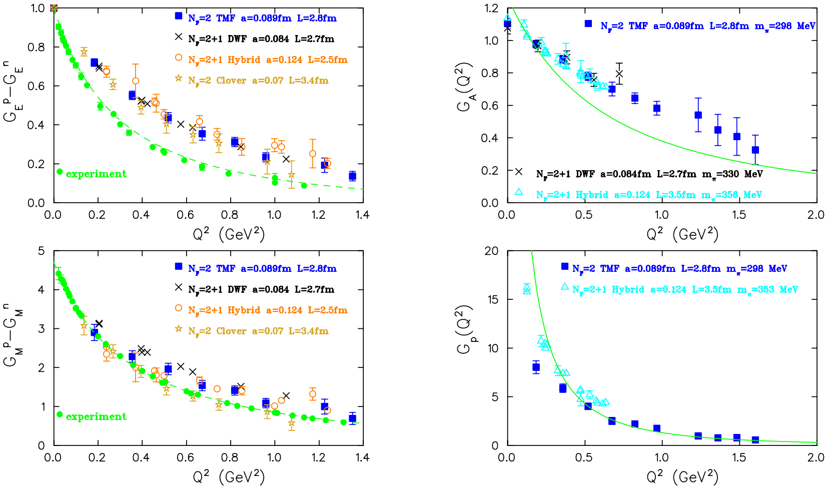
Results on nucleon form factors

Nucleon electromagnetic and axial form factors

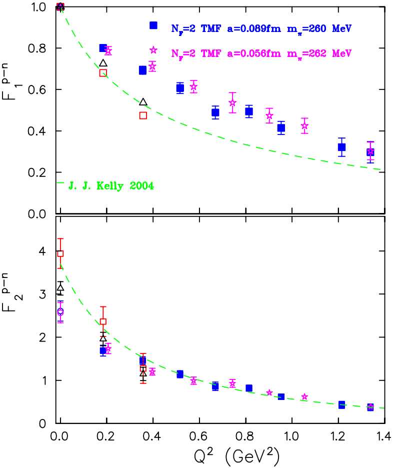


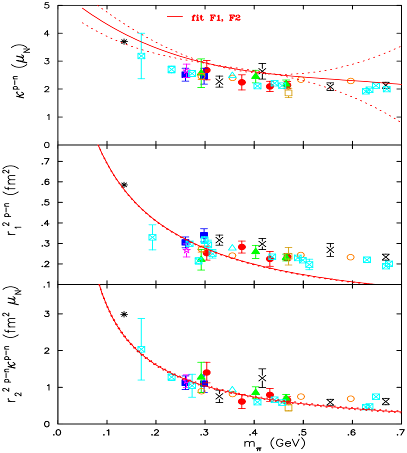
Results from ETMC (arXiv:0910.3309), LHPC using DWF (S. N. Syritsyn, PRD 81, 034507 (2010)) and a hybrid action (J. D. Bratt *et al.*, arXiv:1001:3620), and from CLS using Clover, (H. Wittig)

Can we get results at physical point?

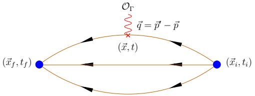
Chiral extrapolation of electromagnetic form factors

Baryon chiral perturbation theory to one-loop, with Δ d.o.f.(SSE) and iso-vector $N\Delta$ coupling included in LO, T. R. Hemmert and W. Weise, Eur. Phys. J. A **15**,487 (2002); M. Gockeler *et al.*, PRD **71**, 034508 (2005).
Fit $F_1(m_\pi, Q^2)$ and $F_2(m_\pi, Q^2)$ with 5 parameters: κ_v, the isovector (c_v) and axial N to Δ ($g_{\pi N\Delta}$ or c_A) couplings and two counter-terms

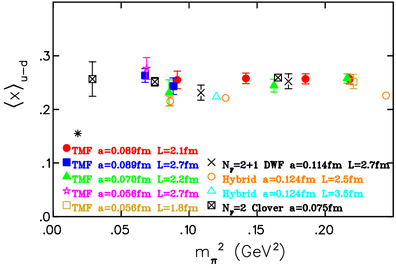


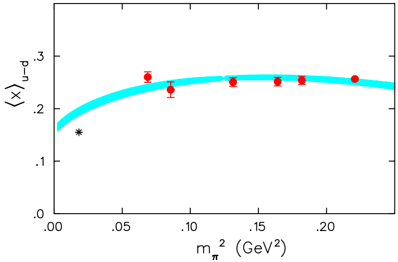


Nucleon momentum fraction



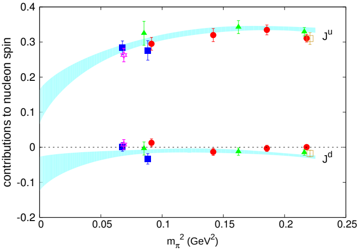
Matrix of the one derivative vector current
$\Longrightarrow$ probes the momentum fraction carried by quarks

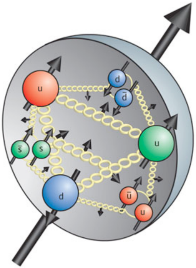




Origin of the spin of the Nucleon

Results using $N_F = 2$ TMF for 270 MeV $< m_\pi < 500$ MeV, C. Alexandrou *et al.* (ETMC), arXiv:1104.1600
Apply HBχPT, W. Detmold, W. Melnitchouk, A. W. Thomas, PRD66 (2002) 054501





- Total spin for u-quarks $J^u \sim 0.25$ and for d-quark $J^d \sim 0 \Longrightarrow$ Where is the other half?
- In qualitative agreement with J. D. Bratt *et al.* (LHPC), PRD82 (2010) 094502

Origin of the spin of the Nucleon

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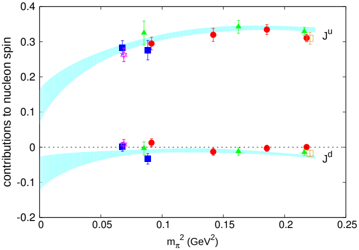
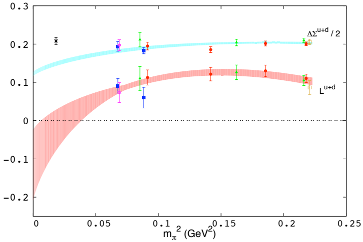
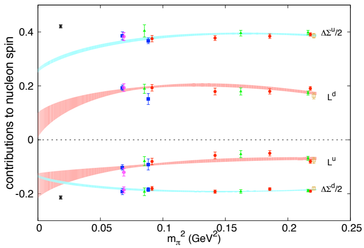
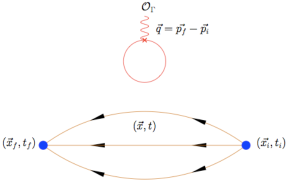
Apply HBχPT, W. Detmold, W. Melnitchouk, A. W. Thomas, PRD66 (2002) 054501





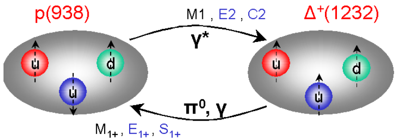


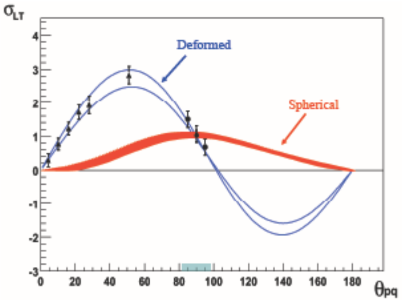


Disconnected contributions neglected

$N\gamma^* \to \Delta$ form factors

- A dominant magnetic dipole, M1
- An electric quadrupole, E2 and a Coulomb, C2 signal a deformation in the nucleon/Δ
- 1/2-spin particles have vanishing quadrupole moment in the lab-frame
- Probe nucleon shape by studying transitions to its excited Δ-state
- Difficult to measure/calculate since quadrupole amplitudes are sub-dominant





- $R_{EM}(\text{EMR}) = -\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)}$,

  $R_{SM}(\text{CMR}) = -\frac{|\vec{q}|}{2m_\Delta}\frac{G_{C2}(Q^2)}{G_{M1}(Q^2)}$,

  in lab frame of the Δ.
- Precise data strongly "suggesting" deformation in the Nucleon/Δ
  At $Q^2 = 0.126$ GeV^2:
  EMR=$(-2.00 \pm 0.40_{\text{stat+sys}} \pm 0.27_{\text{mod}})$%,
  CMR=$(-6.27 \pm 0.32_{\text{stat+sys}} \pm 0.10_{\text{mod}})$%

C. N. Papanicolas, Eur. Phys. J. A18 (2003); N. Sparveris *et al.*, PRL **94**, 022003 (2005)

$N\gamma^* \to \Delta$ form factors

- A dominant magnetic dipole, M1
- An electric quadrupole, E2 and a Coulomb, C2 signal a deformation in the nucleon/Δ
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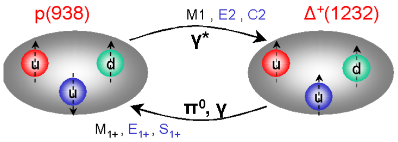
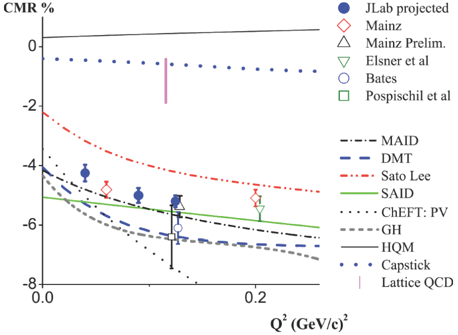




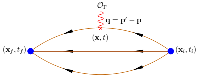
- I. Aznauryan *et al.*, CLAS, Phys. Rev. C 80 (2009) 055203
- New measurement of the Coulomb quadrupole amplitude in the low momentum transfer region (E08-010) , N. Sparveris *et al.*, Hall A

Thanks to N. Sparveris.

Lattice evaluation

$$\langle \Delta(p', s')|j_\mu|N(p,s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta\ m_N}{E_\Delta(\mathbf{p}')\ E_N(\mathbf{p})}\right)^{1/2} \bar{u}_\sigma(p',s')\left[G^*_{M1}(q^2)K^{M1}_{\sigma\mu} + G^*_{E2}(q^2)K^{E2}_{\sigma\mu} + G^*_{C2}K^{C2}_{\sigma\mu}\right]u(p,s)$$

- Evaluation of two-point and three-point functions



$$G(\vec{q}, t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\,\Gamma^4_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f, t_f)\bar{J}_\beta(0)\rangle$$

$$G^{\mu\nu}(\Gamma, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x}\cdot\vec{q}}\,\Gamma_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f, t_f)\mathcal{O}^\mu(\vec{x}, t)\bar{J}_\beta(0)\rangle$$

$$R^J_\sigma(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma_\tau; \mu) = \frac{\langle G^{\Delta J_\mu N}_\sigma(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma_\tau)\rangle}{\langle G^{\Delta\Delta}_{ii}(t_2, \mathbf{p}'; \Gamma_4)\rangle}\left[\frac{\langle G^{\Delta\Delta}_{ii}(t_2, \mathbf{p}'; \Gamma_4)\rangle}{\langle G^{NN}(t_2, \mathbf{p}; \Gamma_4)\rangle}\ \frac{\langle G^{NN}(t_2 - t_1, \mathbf{p}; \Gamma_4)\rangle\ \langle G^{\Delta\Delta}_{ii}(t_1, \mathbf{p}'; \Gamma_4)\rangle}{\langle G^{\Delta\Delta}_{ii}(t_2 - t_1, \mathbf{p}'; \Gamma_4)\rangle\ \langle G^{NN}(t_1, \mathbf{p}; \Gamma_4)\rangle}\right]^{1/2}$$

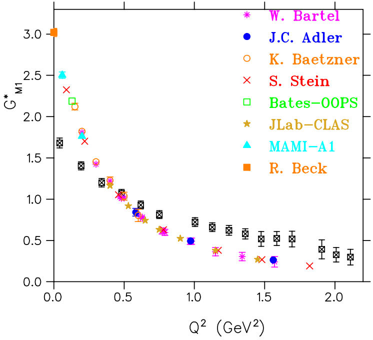
- Construct optimized sources to isolate quadrupoles → three-sequential inversions needed

$$S^J_1(\mathbf{q}; J) = \sum_{\sigma=1}^{3} \Pi^J_\sigma(\mathbf{0}, -\mathbf{q}; \Gamma_4; J) \quad , \quad S^J_2(\mathbf{q}; J) = \sum_{\sigma\neq k=1}^{3} \Pi^J_\sigma(\mathbf{0}, -\mathbf{q}; \Gamma_k; J)$$

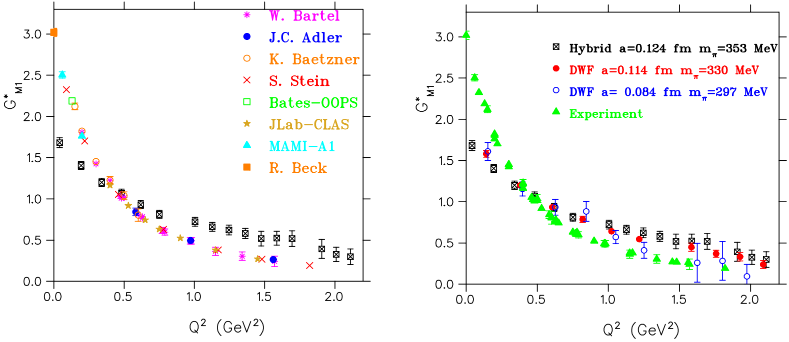
$$S^J_3(\mathbf{q}; J) = \Pi^J_3(\mathbf{0}, -\mathbf{q}; \Gamma_3; J) - \frac{1}{2}\left[\Pi^J_1(\mathbf{0}, -\mathbf{q}; \Gamma_1; J) + \Pi^J_2(\mathbf{0}, -\mathbf{q}; \Gamma_2; J)\right]$$

- Use the coherent sink technique: create four sets of forward propagators for each configuration at source positions separated in time by one-quarter of the total temporal size, Syritsyn et al. (LHPC), Phys. Rev. D81 (2009) 034507.

Results on magnetic dipole



Results on magnetic dipole



Slope smaller than experiment, underestimate G^*_{M1} at low Q^2 → pion cloud effects?

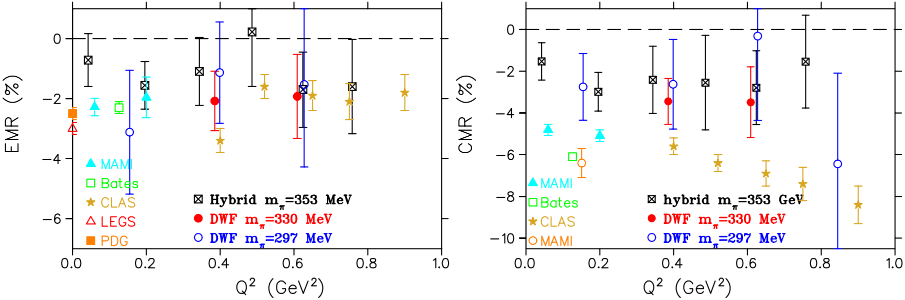
New results using $N_f = 2 + 1$ dynamical Domain Wall Fermions, simulated by RBC-UKQCD Collaborations $\Longrightarrow$ No visible improvement.
C. A., G.Koutsou, J.W. Negele, Y. Proestos, A. Tsapalis, Phys. Rev. D83 (2011)

Situation like for nucleon form factors, independent of lattice discretization

$\Longrightarrow$ nucleon FFs under study by a number of lattice groups.

Results on EMR and CMR

Systematic errors may cancel in rations: G_{E2} and G_{C2} are suppressed at low Q^2 like G^*_{M1}
$\Longrightarrow$ look at EMR and CMR



New results using $N_f = 2 + 1$ dynamical domain wall fermions by RBC-UKQCD Collaborations
Need large statistics to reduce the errors $\Longrightarrow$ as $m_\pi \to 140$ MeV $\mathcal{O}(10^3)$ need to be analyzed.

N - Δ axial-vector form factors

$$\langle \Delta(p',s')|A^3_\mu|N(p,s)\rangle = \mathcal{A}\bar{u}^\lambda(p',s')\left[\left(\frac{C^A_3(q^2)}{m_N}\gamma^\nu + \frac{C^A_4(q^2)}{m_N^2}p'^\nu\right)\left(g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu}\right)q^\rho + C^A_5(q^2)g_{\lambda\mu} + \frac{C^A_6(q^2)}{m_N^2}q_\lambda q_\mu\right]u(p,s)$$

$$\mathcal{A} = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta m_N}{E_\Delta(\mathbf{p}')E_N(\mathbf{p})}\right)^{1/2}$$

- $C^A_5(q^2)$ analogous to the nucleon $G_A(q^2)$
- $C^A_6(q^2)$, analogous to the nucleon $G_p(q^2) \longrightarrow$ pion pole behaviour
- $C^A_3(q^2)$ and $C^A_4(q^2)$ are suppressed (transverse part of the axial-vector)

- Study also the pseudo-scalar transition form factor $G_{\pi N\Delta}(q^2)$
  $\Longrightarrow$ Non-diagonal Goldberger-Treiman relation:

$$C^A_5(q^2) + \frac{q^2}{m_N^2}C^A_6(q^2) = \frac{1}{2m_N}\frac{G_{\pi N\Delta}(q^2)f_\pi m_\pi^2}{m_\pi^2 - q^2} \quad .$$

Pion pole dominance relates C^A_6 to $G_{\pi N\Delta}$ through:

$$\frac{1}{m_N}C^A_6(q^2) \sim \frac{1}{2}\frac{G_{\pi N\Delta}(q^2)f_\pi}{m_\pi^2 - q^2}$$

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$$G_{\pi N\Delta}(q^2)\,f_\pi = 2m_N C^A_5(q^2)$$

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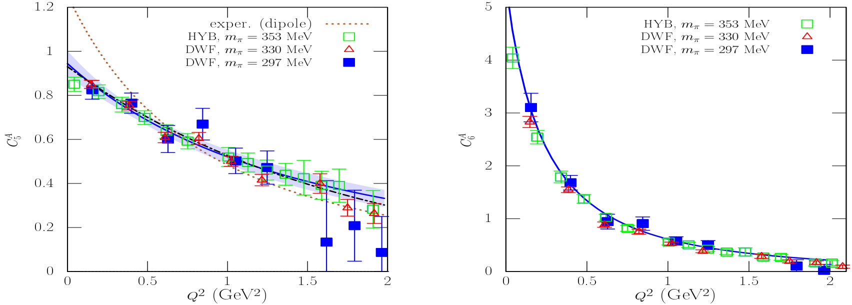
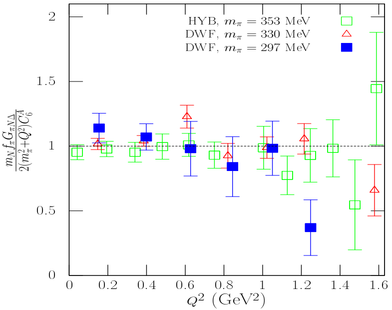
Results on Δ to N axial-vector form factors

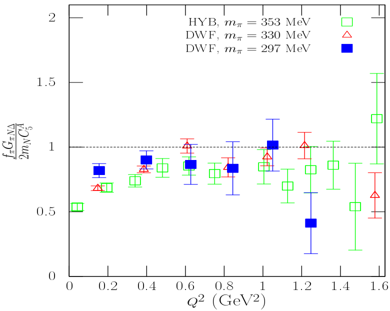


Similar behavior as in the nucleon system, i.e. between $G_{\pi NN}$ and G_p, and $G_{\pi NN}$ and G_A.

Ratio: $G_{\pi N\Delta}/G_{\pi NN} \sim 1.6$, independent of Q^2.

Results on Δ to N axial-vector form factors





Pion-pole dominance: $\frac{1}{m_N} C_6^A(Q^2) \sim \frac{1}{2} \frac{G_{\pi N\Delta}(Q^2) f_\pi}{m_\pi^2 + Q^2}$ Goldberger-Treiman rel.: $G_{\pi N\Delta}(Q^2) f_\pi = 2 m_N C_5^A(Q^2)$

Similar behavior as in the nucleon system, i.e. between $G_{\pi NN}$ and G_p, and $G_{\pi NN}$ and G_A.

Ratio: $G_{\pi N\Delta}/G_{\pi NN} \sim 1.6$, independent of Q^2.

Δ electromagnetic form factors

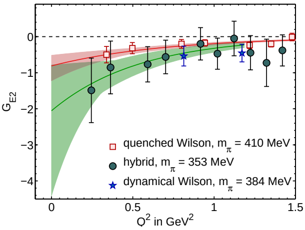
$$\langle\Delta(p',s')|j^\mu(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\left\{\left[F_1^*(Q^2)g^{\alpha\beta} + F_3^*(Q^2)\frac{q^\alpha q^\beta}{(2M_\Delta)^2}\right]\gamma^\mu + \left[F_2^*(Q^2)g^{\alpha\beta} + F_4^*(Q^2)\frac{q^\alpha q^\beta}{(2M_\Delta)^2}\right]\frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}\right\}u_\beta(p,s)$$

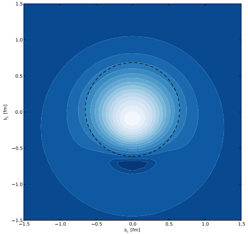
with e.g. the quadrupole form factor given by: $G_{E2} = \left(F_1^* - \tau F_2^*\right) - \frac{1}{2}(1+\tau)\left(F_3^* - \tau F_4^*\right)$, where $\tau \equiv Q^2/(4M_\Delta^2)$

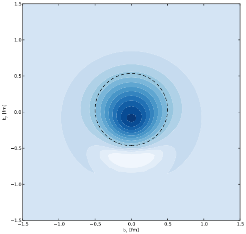
Construct an optimized source to isolate G_{E2} $\rightarrow$ additional sequential propagators needed.
Neglect disconnected contributions in this evaluation.

Transverse charge density of a Δ polarized along the x-axis can be defined in the infinite momentum frame $\rightarrow$ $\rho^\Delta_{T\,\frac{3}{2}}(\vec{b})$ and $\rho^\Delta_{T\,\frac{1}{2}}(\vec{b})$.

Using G_{E2} we can predict 'shape' of Δ.







Δ with spin 3/2 projection elongated along spin axis compared to the Ω^-

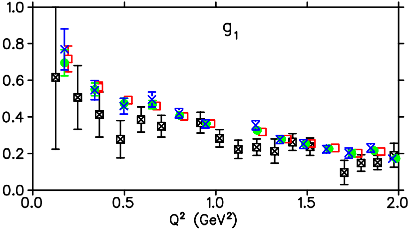
C. A., T. Korzec, G. Koutsou, C. Lorcé, J. W. Negele, V. Pascalutsa, A. Tsapalis, M. Vanderhaeghen, NPA825 ,115 (2009).

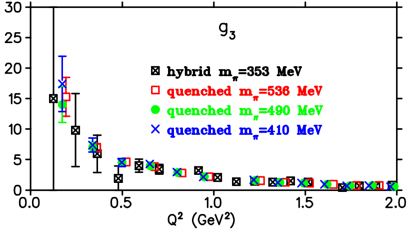
Δ axial-vector form factors

Axial-vector current: $A^a_\mu(x) = \overline{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$

$$\langle\Delta(p',s')|A^3_\mu(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\frac{1}{2}\left[-g^{\alpha\beta}\left(g_1(q^2)\gamma^\mu\gamma^5 + g_3(q^2)\frac{q^\mu}{2M_\Delta}\gamma^5\right) + \frac{q^\alpha q^\beta}{4M_\Delta^2}\left(h_1(q^2)\gamma^\mu\gamma^5 + h_3(q^2)\frac{q^\mu}{2M_\Delta}\gamma^5\right)\right]u_\beta(p,s)$$

i.e. 4 axial form-factors, g_1, g_3, h_1 and h_3 $\longrightarrow$ at $q^2 = 0$ we can extract the Δ axial charge





$\Longrightarrow$ Using a consistent chiral perturbation theory framework extract the chiral Lagrangian couplings g_A, c_A, g_Δ from a combined chiral fit to the lattice results on the nucleon and Δ axial charge and the axial N-to-Δ form factor $C_5(0)$.

C. A., E. Gregory, T. Korzec, G. Koutsou, J. W. Negele, T. Sato, A. Tsapalis, arXiv:1011.0411

Δ axial-vector form factors

Axial-vector current: $A^a_\mu(x) = \overline{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$

$$\langle\Delta(p',s')|A^3_\mu(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\frac{1}{2}\left[-g^{\alpha\beta}\left(g_1(q^2)\gamma^\mu\gamma^5 + g_3(q^2)\frac{q^\mu}{2M_\Delta}\gamma^5\right) + \frac{q^\alpha q^\beta}{4M^2_\Delta}\left(h_1(q^2)\gamma^\mu\gamma^5 + h_3(q^2)\frac{q^\mu}{2M_\Delta}\gamma^5\right)\right]u_\beta(p,s)$$

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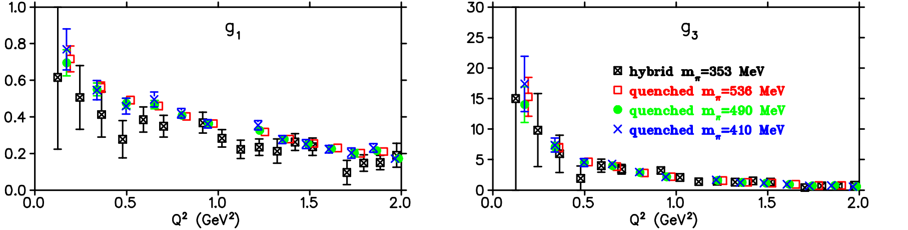

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Δ pseudoscalar couplings

Pseudoscalar current: $P^a(x) = \overline{\psi}(x)\gamma_5 \frac{\tau^a}{2}\psi(x)$

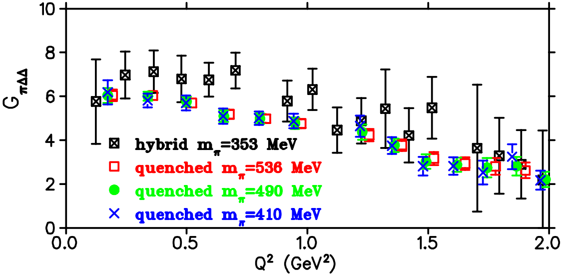
- $\Delta - \Delta$ matrix element:

$$\langle\Delta(p',s')|P^3(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\frac{1}{2}\left[-g^{\alpha\beta}\tilde{g}(q^2)\gamma^5 + \frac{q^\alpha q^\beta}{4M_\Delta^2}\tilde{h}(q^2)\gamma^5\right]u_\beta(p,s)$$

  i.e. two $\pi\Delta\Delta$ couplings $\Longrightarrow$ two Goldberger-Treiman relations.

- $G_{\pi\Delta\Delta}$ is given by: $m_q\tilde{g}(Q^2) \equiv \frac{f_\pi m_\pi^2 G_{\pi\Delta\Delta}(Q^2)}{(m_\pi^2+Q^2)}$ and $H_{\pi\Delta\Delta}$ is given by: $m_q\tilde{h}(Q^2) \equiv \frac{f_\pi m_\pi^2 H_{\pi\Delta\Delta}(Q^2)}{(m_\pi^2+Q^2)}$

- Goldberger-Treiman relations: $f_\pi G_{\pi\Delta\Delta}(Q^2) = m_\Delta g_1(Q^2)$, $\quad f_\pi H_{\pi\Delta\Delta}(Q^2) = m_\Delta h_1(Q^2)$

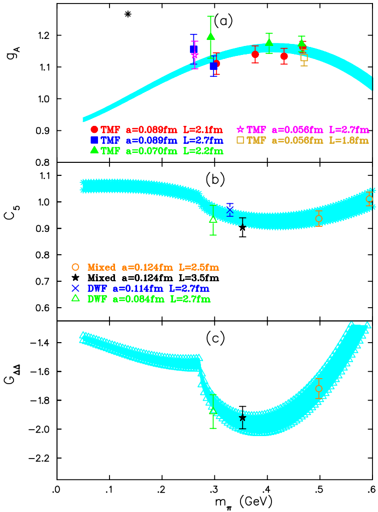


C. A., E. Gregory, T. Korzec, G. Koutsou, J. W. Negele, T. Sato, A. Tsapalis, arXiv:1011.0411

Global chiral fit to the axial couplings

Use heavy baryon χPT to describe the pion mass dependence of the axial nucleon and Δ charge g_A and g_Δ as well as the axial N to Δ coupling $C_5(0)$.

T. R. Hemmert, M. Procura, W. Weise, PRD68, 075009 (2003); F. J. Jiang and B. C. Tiburzi, PRD 78, 017504 (2008); M. Procura, PRD 78, 094021 (2008).



Conclusions

- Large scale simulations using the underlying theory of the Strong Interactions have made spectacular progress
  $\Longrightarrow$ we now have simulations of the full theory at near physical parameters
- The low-lying hadron spectrum is reproduced
- Nucleon form factors are being computed by a number of collaborations in order to understand the discrepancy with the experimental values
- N to Δ transition form factors can be extracted in a similar way to the nucleon
  Ratios of form factors expected to be less affected by lattice artifacts $\to$ EMR and CMR allow comparison to experiment
- Δ form factors are predicted
- Resonance width can be computed within Euclidean Lattice QCD as illustrated for the ρ-meson $\to$ similar techniques can be applied to Δ
  $\Longrightarrow$ We expect many physical results using $N_F = 2+1$ simulations at the physical pion mass in the next few years

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Thank you for your attention