Recent progress in ab initio few (*and not-so-few*) - body theories for nuclear physics

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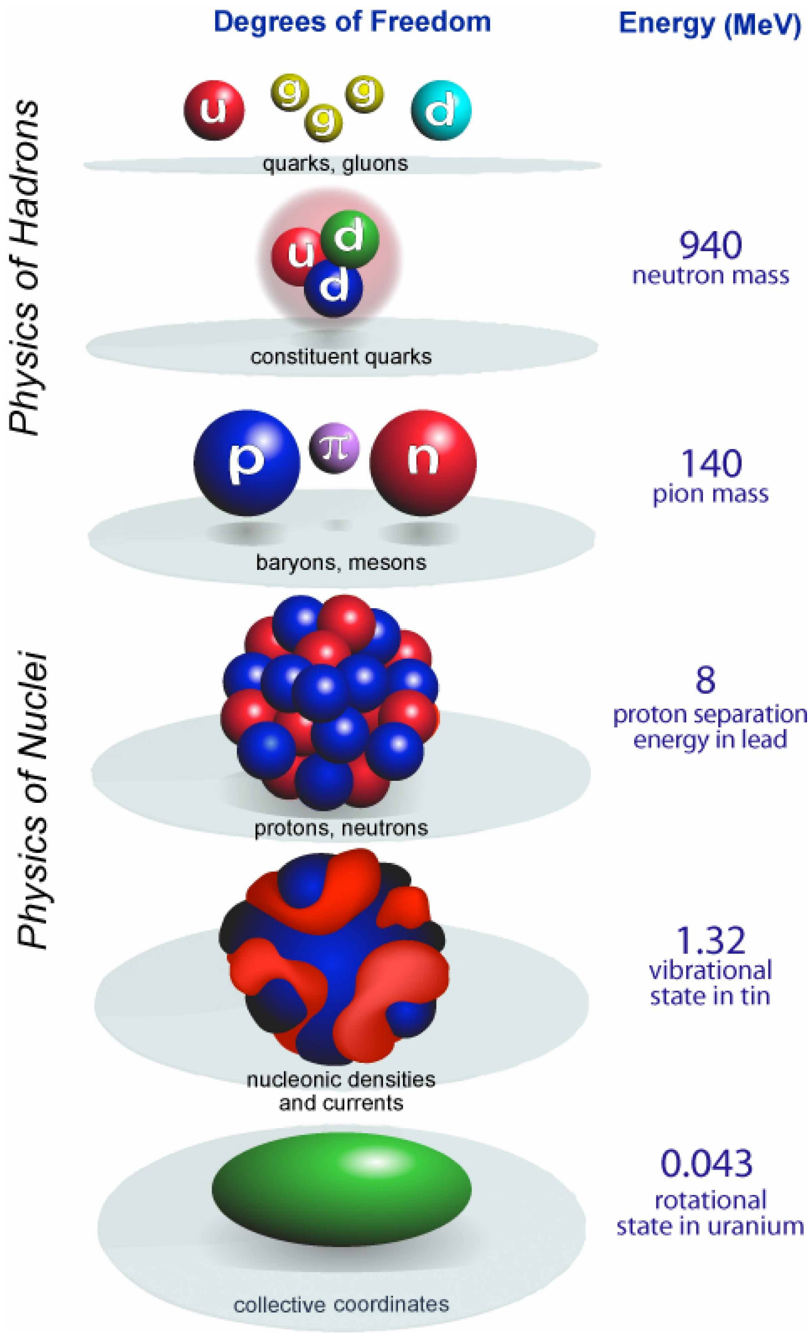
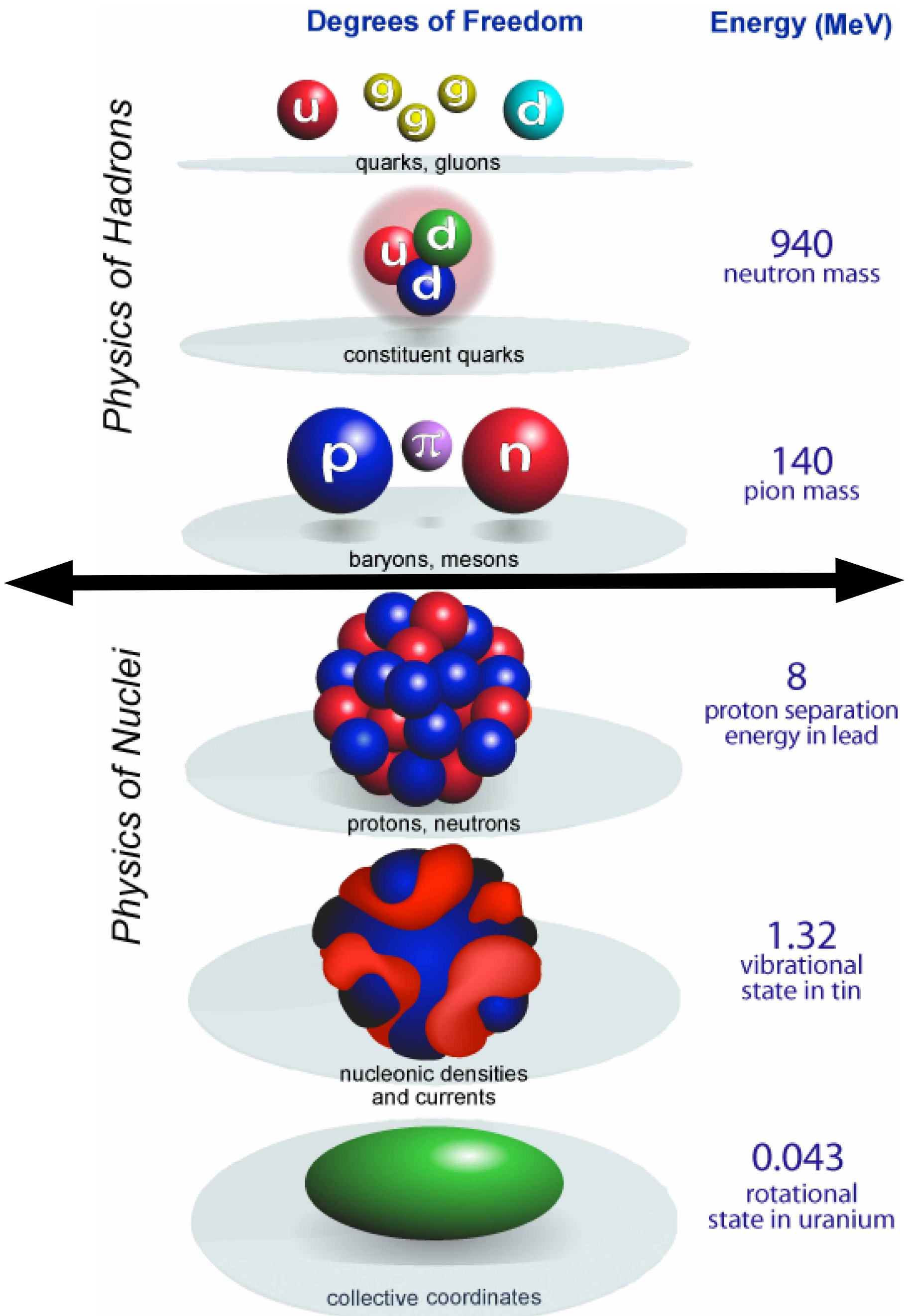
Summary:

- General remarks
- Short review of ab initio methods for bound states (**structure**) and scattering states (**reactions**)
- The LIT method for reactions
- Selected results



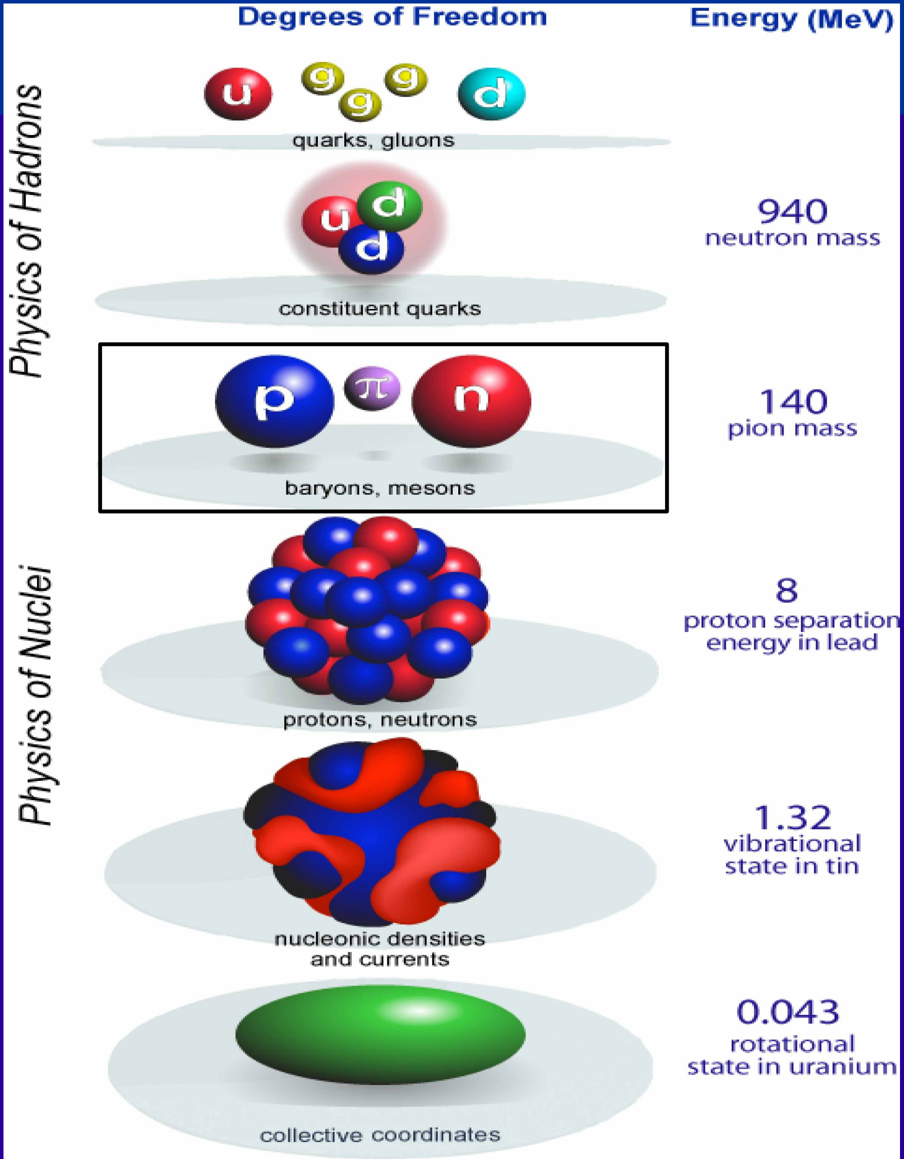
Degrees of Freedom
Energy (MeV)
Physics of Hadrons
Physics of Nuclei
u
g
g
g
d
quarks, gluons
u
d
d
constituent quarks
940
neutron mass
p
π
n
baryons, mesons
140
pion mass
protons, neutrons
8
proton separation
energy in lead
nucleonic densities
and currents
1.32
vibrational
state in tin
collective coordinates
0.043
rotational
state in uranium



FEW-BODY PHYSICS

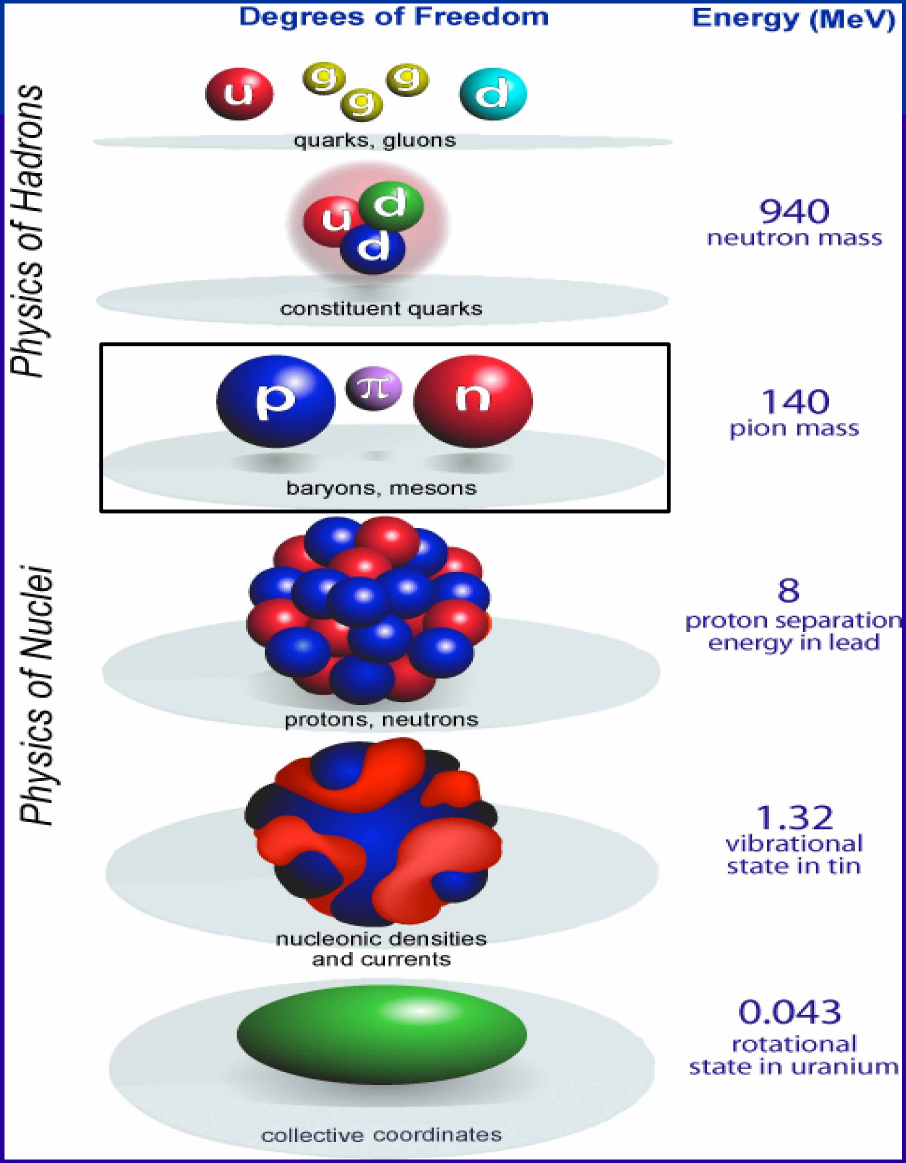
***ab initio* methods**

fundamental issues in nuclear physics:



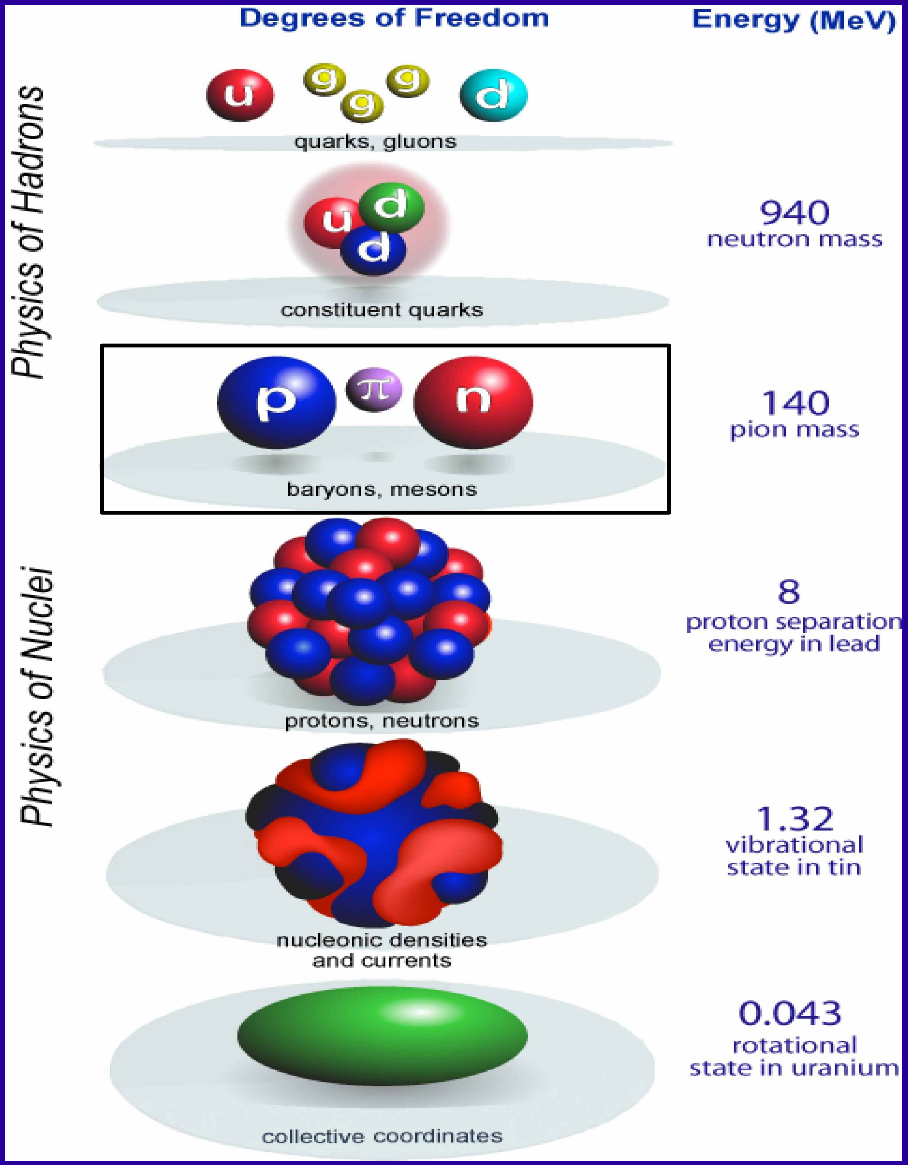
relevant degrees of freedom

fundamental issues in nuclear physics:



What is the Nuclear Interaction?

fundamental issues in nuclear physics:



QCD

What is the
Nuclear Interaction?

typical MB properties
(clusterization, collectivity,
mean field properties
etc.)

“**Few(?)**-body” theories are ***ab initio*** in the following sense:

d.o.f.: **A nucleons**

Potential: **“realistic” interaction**

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Nucleons are not elementary -------> N-N, N-N-N, N-N-N-N... potential

State of the art of ab initio theories:

bound states:

A= 2 – 4 many different methods (see PRC 64 (2001) 044001)

ab initio A=4 *bound state* *calculations*

E_b = binding energy of ^{4}He

TABLES

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	-128.33(10)	-25.94(5)	1.485(3)
CRCGV	102.30	-128.20	-25.90	1.482
SVM	102.35	-128.27	-25.92	1.486
HH	102.44	-128.34	-25.90(1)	1.483
GFMC	102.3(1.0)	-128.25(1.0)	-25.93(2)	1.490(5)
NCSM	103.35	-129.45	-25.80(20)	1.485
EIHH	100.8(9)	-126.7(9)	-25.944(10)	1.486

from H.Kamada et al. (18 auhors 7 groups) PRC 64 (2001) 044001

ab initio A=4 bound state calculations

Excellent accuracy !

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Bound State ***Methods can be grouped in 3 classes***

A

Monte Carlo methods:

GFMC = **G**reen **F**unction **M**onte **C**arlo
AFMC = **A**uxiliary **F**ield **M**onte **C**arlo
......

B

Expansion methods + "**effective interaction**" (to accelerate convergence):

NCSM = **N**o **C**ore **S**hell **M**odel
EIHH = **E**ffective **I**nteraction in **Hyperspherical** **Harmonics** expansion
....

C

correlation operator (e^S) methods:

CC = Coupled **C**luster
UCOM = **U**nitary **C**orrelation **O**perator **M**ethod

State of the art of ab initio theories:

bound states:

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A > 4	**EIHH,**	**GFMC,**	**NCSM,**	AFMC, CC, UCOM
upper limit in A:	**6 - 8?**	**10-12?**	**12-14??**	**40-56 ???**

^{8}Be: From **ab initio Monte Carlo** to **two alpha – cluster** !

From

S.C. Pieper & R.B. Wiringa
Ann. Rev. Nucl. Part. Sci
51 (2001) 53

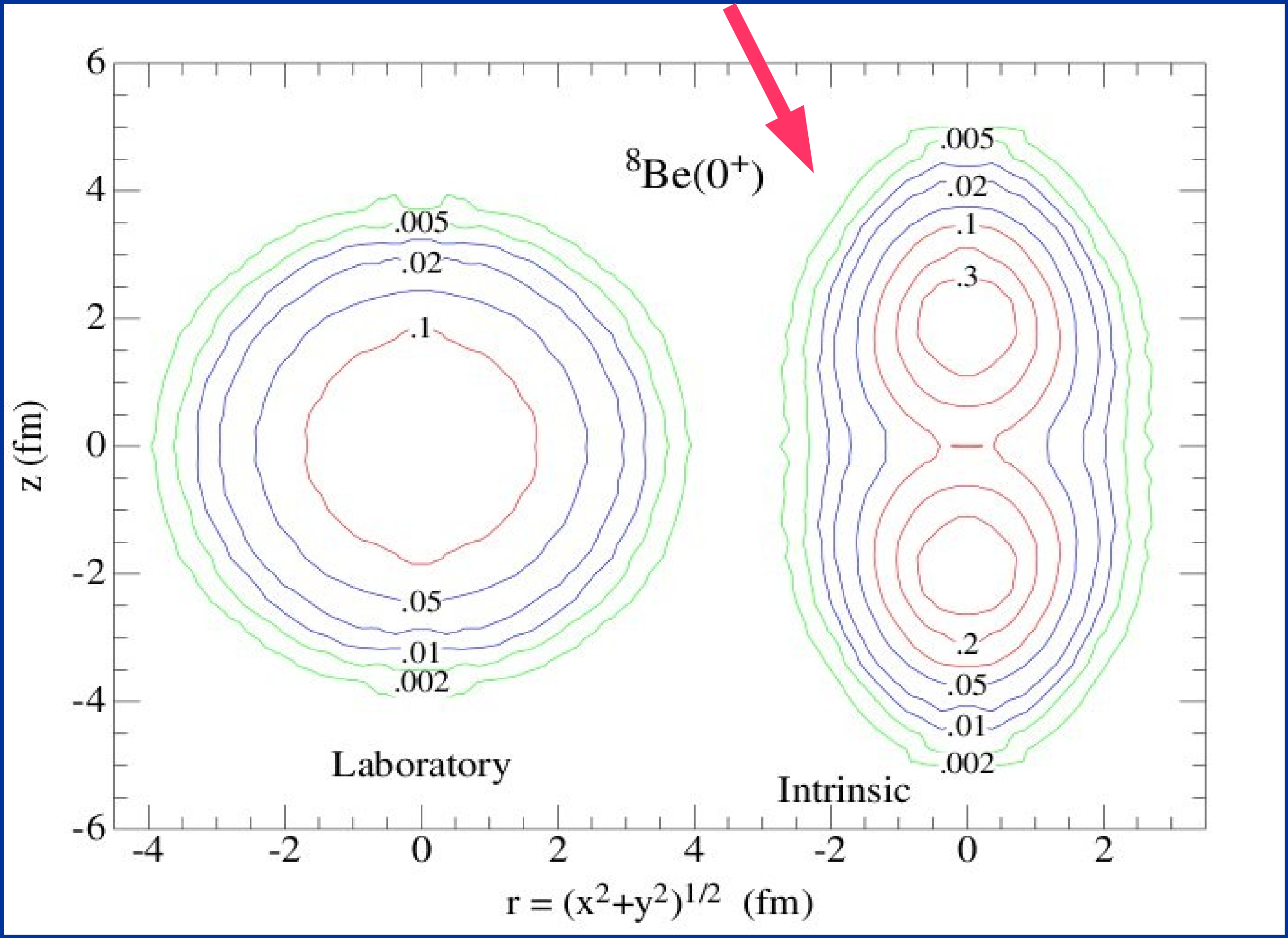

Fig.11 - Calculated density contours of ^{8}Be in the lab frame (left) and the intrinsic frame (right), labeled with densities in fm^{-3}

State of the art of ab initio theories:

bound states:

continuum (scattering) states ---------> REACTIONS

conventional approach:
A=3: Faddeev equations (2-body and 3-body break-up) , HH
A=4: Yakubowski (AGS) only 2-body break up , HH

A > 4 ???

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A > 4 ???

unconventional approach: **LIT method**
reduces the continuum problem to bound state like problem

The Lorentz Integral Transform (LIT) method

First proposed in:

V. D. Efros, W. Leidemann and G. Orlandini,
Phys. Lett. B338, 130 (1994)

Recent Topical Review:

V. D. Efros, W. Leidemann, G. Orlandini and N. Barnea
J. Phys. G: Nucl. Part. Phys. 34 (2007) R459-R528

Integral transform approaches

There are many examples in physics where one uses
“integral transform approaches”

$$T = K\, F$$

accessible object

object of interest

There are many classes of problems that are difficult to solve in their original representations. An integral transform "maps" an equation from its **original "domain"** into **another domain**. Manipulating and solving the equation in the **target domain** is sometimes much easier than manipulation and solution in the **original domain**. The solution is then **mapped back** to the original domain with the inverse of the integral transform.

In theoretical physics:

Laplace Kernel

$$\Phi(\tau) = \int \langle\, |\Theta^{\dagger}(\tau, \mathbf{x})\, \Theta(0, 0)|\, \rangle \; d^3\mathbf{x} \;\; \text{--->} \;\; \int e^{-\tau\omega}\, S(\omega)\, d\omega$$

$$\tau = it$$

In Condensed Matter Physics:

Θ = Density Operator

$S(\omega)$ = Dynamical Structure Function

$\Phi(\tau)$ is obtained with Monte Carlo Methods

In Nuclear Physics:

Θ= Charge or current density operator

$S(\omega)$ = **R**(ω) "Response" Function (to external perturbative probe)

$\Phi(\tau)$ is obtained with Monte Carlo Methods

In QCD

Θ = quark or gluon creation operator

$S(\omega)$ = Hadronic Spectral Function

$\Phi(\tau)$ is obtained by OPE - QCD sum rules or Lattice

$$\Phi(\tau) = \int d\omega \; K(\omega,\sigma)\, S(\omega)$$

One is able to calculate $\Phi(\tau)$ but wants $S(\omega)$, which is the quantity of direct physical meaning.

Problem:

The "inversion" of $\Phi(\tau)$ may be problematic ("**ill posed problem**")

It is well known that the numerical inversion of the **Laplace** Transform is a terribly **ill-posed** problem

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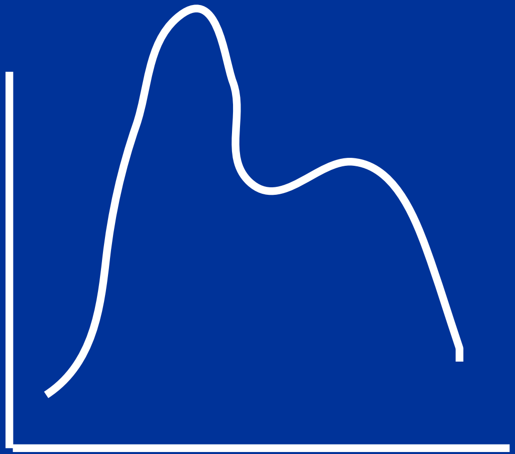
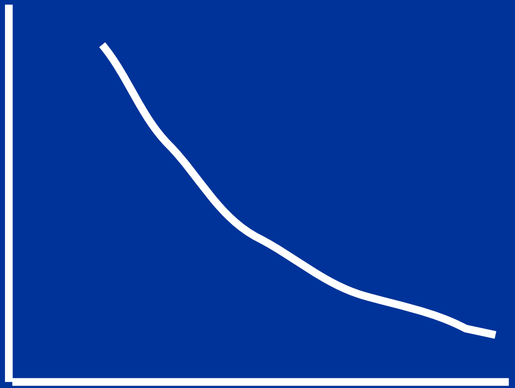
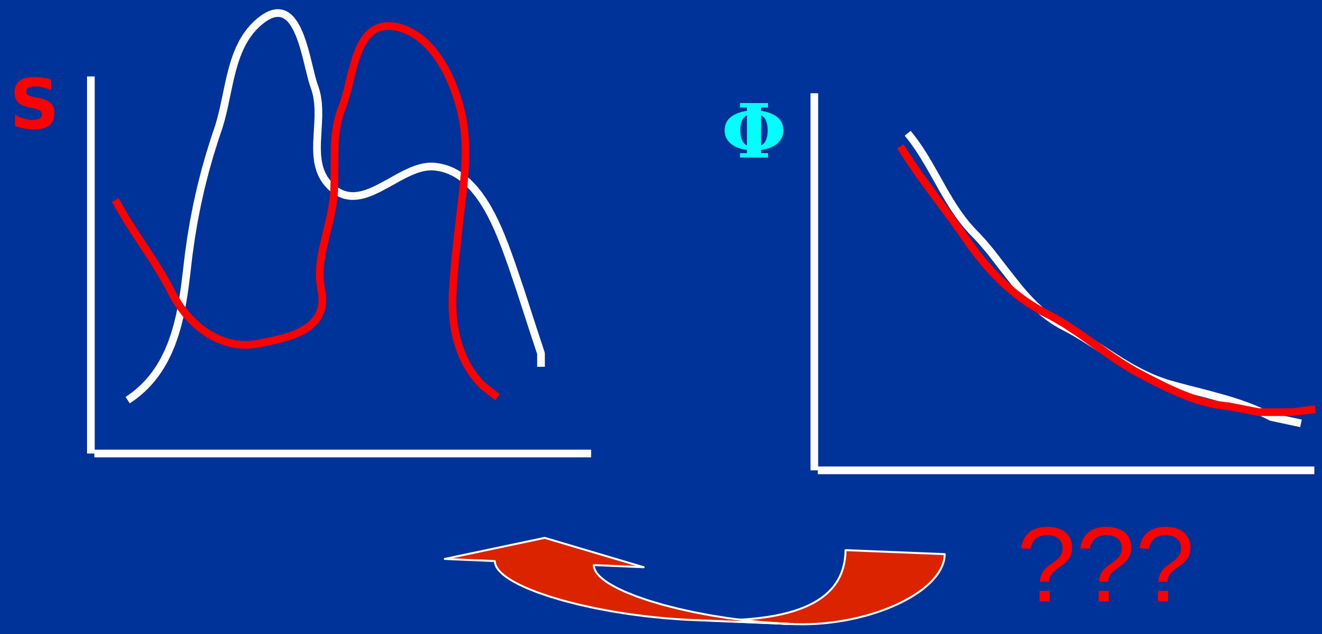
S



Φ



It is well known that the numerical inversion of the **Laplace** Transform is a terribly **ill-posed** problem



a “good” Kernel has to satisfy two requirements

1) one must be able to calculate the integral transform

2) one must be able to invert the transform minimizing instabilities

What is the perfect Kernel?

What is the perfect Kernel?

the delta-function!

What would be the “perfect” Kernel?

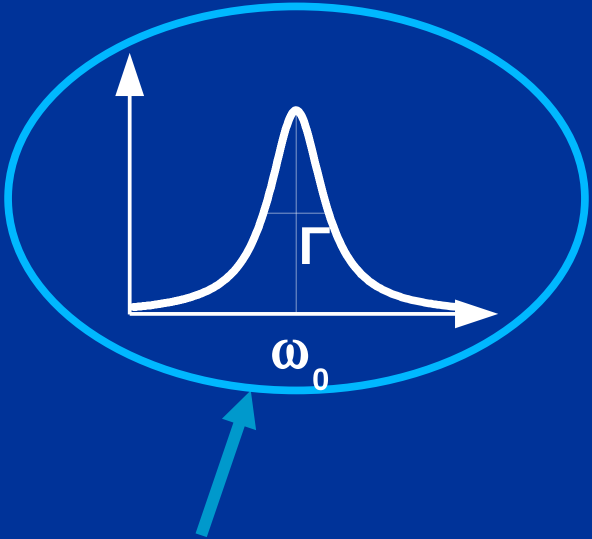
the delta-function!

in fact

$$\Phi(\tau) = S(\tau) = \int \delta(\omega - \tau)\, S(\omega)\, d\omega$$

the **LIT method** is based on the idea to use one of the so-called "representations of the delta-function":

it turns out that a very good Kernel is the Lorentzian function



$$\Phi(\omega, \Gamma) = \int [(\omega - \omega_0)^2 + \Gamma^2]^{-1} \, S(\omega) \, d\omega$$

The Lorentz Kernel satisfies the two requirements !

N.1. one can calculate the integral transform

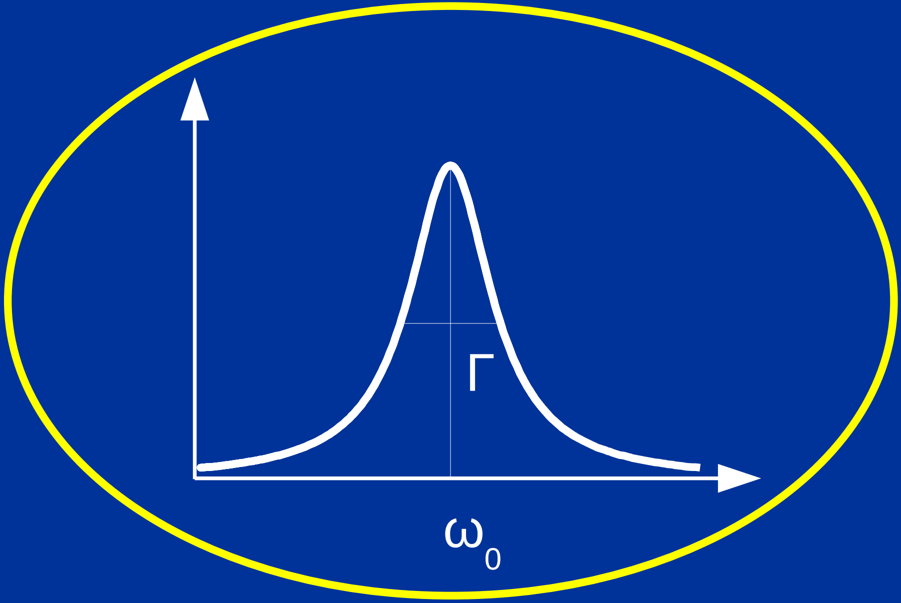
N.2 one is able to invert the transform, minimizing instabilities with controlled resolution

Illustration of requirement **N.1:** one can calculate the integral transform

Suppose we want an **R(ω)** defined as

(for example for perturbation induced inclusive reactions)

$$R(\omega) = \sum\!\!\!\!\!\!\!\int_n \left|\langle n|\Theta|0\rangle\right|^2 \delta(\omega - E_n + E_0)$$



THEOREM:

$$\Phi(\omega_0,\Gamma) = \left\langle \tilde{\Psi} | \tilde{\Psi} \right\rangle = \int R(\omega) L(\omega, \omega_0, \Gamma) d\omega$$

where $$|\tilde{\Psi} > = \frac{1}{(H - E_0 - \omega_0 + i\Gamma)} \Theta |0 >$$

Proof of the theorem:

$$\Phi(\omega_0,\Gamma) = \int_{E_{th}^-}^{\infty} d\omega \frac{R(\omega)}{(\omega-\omega_0)^2+\Gamma^2}$$

$$= \int_{E_{th}^-}^{\infty} d\omega \frac{\int\!\!\!\!\sum dn \, |\langle n|\Theta|0\rangle|^2 \, \delta(\omega - E_n + E_0)}{(\omega-\omega_0-i\Gamma)(\omega-\omega_0+i\Gamma)}$$

$$= \int\!\!\!\!\!\sum dn < 0|\Theta^\dagger \frac{1}{(E_n - E_0 - \omega_0 - i\Gamma)} |n> \quad \text{x}$$

$$\text{x} \quad < n| \frac{1}{(E_n - E_0 - \omega_0 + i\Gamma)} \Theta |0>$$

Closure = 1

$$= \int\!\!\!\!\!\sum dn < 0|\Theta^\dagger \frac{1}{(H - E_0 - \omega_0 - i\Gamma)} |n>< n| \frac{1}{(H - E_0 - \omega_0 + i\Gamma)} \Theta |0>$$

$$= < 0|\Theta^\dagger \frac{1}{(H - E_0 - \omega_0 - i\Gamma)} \frac{1}{(H - E_0 - \omega_0 + i\Gamma)} \Theta |0>$$

$$= < \tilde{\Psi} | \tilde{\Psi} >$$

$$|\tilde{\Psi}> = \frac{1}{(H - E_0 - \omega_0 + i\Gamma)} \Theta |0>$$

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$$= < 0|\Theta^\dagger \frac{1}{(H - E_0 - \omega_0 - i\Gamma)} \frac{1}{(H - E_0 - \omega_0 + i\Gamma)} \Theta |0>$$

$$= < \tilde\Psi | \tilde\Psi >$$

$$|\tilde\Psi> = \frac{1}{(H - E_0 - \omega_0 + i\Gamma)} \Theta |0>$$

The LIT in practice:

1.

$$|\tilde{\Psi}> = \frac{1}{(H - E_0 - \omega_0 + i\Gamma)} \Theta |0>$$

is found solving for fixed Γ and many ω_0

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = \Theta\,|0\rangle$$

2. the overlap $\left\langle \tilde{\Psi} | \tilde{\Psi} \right\rangle$ is calculated

3. the transform is inverted

$$\left\langle \tilde{\Psi} | \tilde{\Psi} \right\rangle = \int R(\omega) L(\omega, \omega_0, \Gamma) d\omega$$

main point of the LIT :

Schrödinger-like equation with a source

$$\left(H - E_0 - \omega_0 + i\Gamma\right)\tilde{\Psi} = S$$

S = Θ |0>

main point of the LIT :

Schrödinger-like equation with a source

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = S$$

Theorem:

The $\tilde{\Psi}$ solution is unique and has ***bound state*** asymptotic behavior

main point of the LIT :

Schrödinger-like equation with a source

$$(H - E_0 - \omega_0 + i\Gamma)\,\tilde{\Psi} = S$$

Theorem:

The $\tilde{\Psi}$ solution is unique and has ***bound state*** asymptotic behavior

→ one can apply ***bound state methods***

The LIT method

- reduces the continuum problem to a bound state-like problem
- needs only a “good” method for bound state calculations (FY, HH, NCSM, ...???)
- has been benchmarked in “directly solvable” systems (A=2,3)

Illustration of requirement

N.2: one can invert the integral transform minimizing instabilities

Inversion of the LIT: the **regularization** method

$$R(\omega) = \sum_{n=1}^{N_{max}} c_n \chi_n(\omega, \alpha_i)$$

The χ_n are given functions with nonlinear parameters α_i.

Substituting such an expansion in the integral equation

$$\Phi(\omega_0, \Gamma) = \sum_{n=1}^{N_{max}} c_n \tilde{\chi}_n(\omega_0, \alpha_i) \; ,$$

where

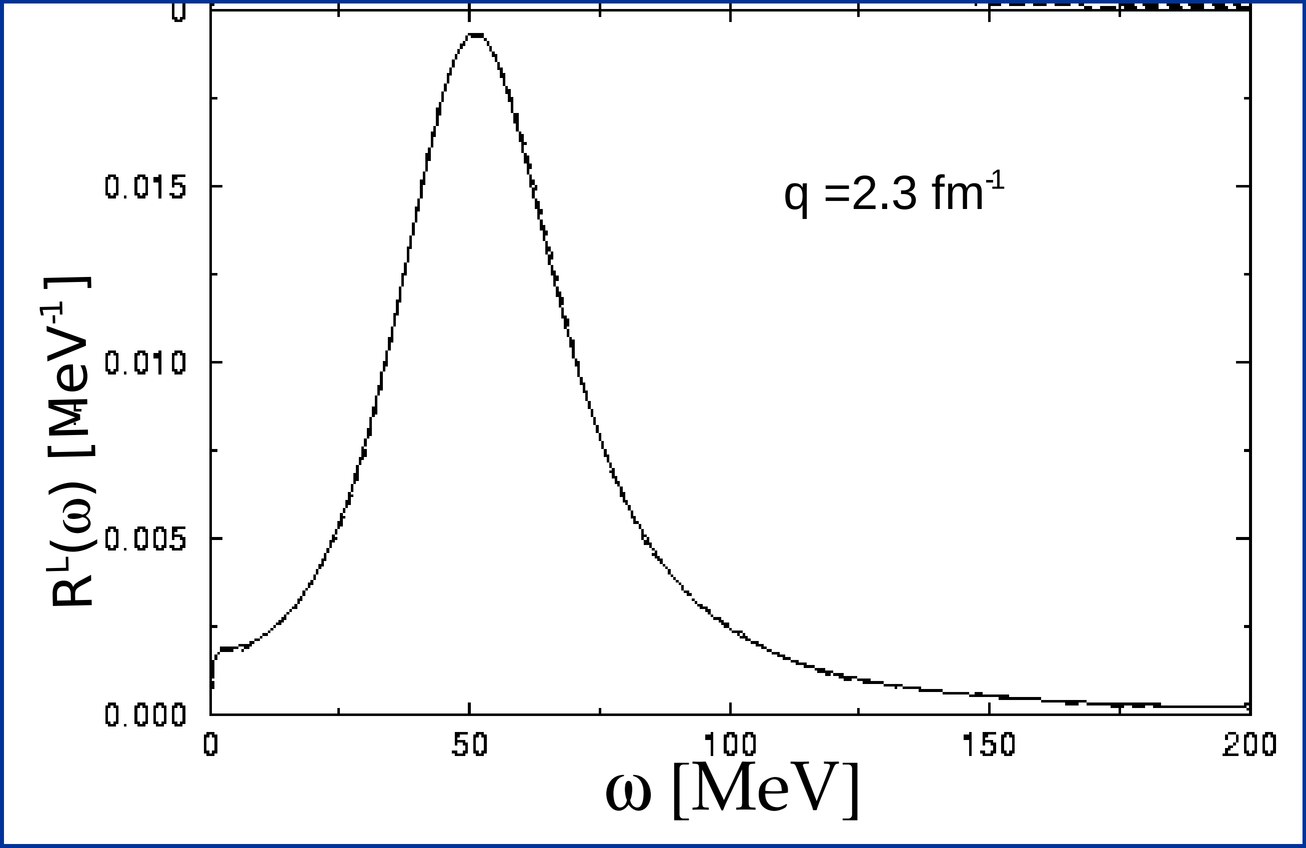
$$\tilde{\chi}_n(\omega_0, \alpha_i) = \int_0^{\infty} d\omega \, \frac{\chi_n(\omega, \alpha_i)}{(\omega - \omega_0)^2 + \Gamma^2} \; .$$

For given α_i the linear parameters c_n are determined from a least–square best fit to the calculated $\Phi(\omega_0, \Gamma)$ for a number of ω_0 points much larger than N_{max}.

Works very well with "bell shaped" kernels

test on the Deuteron:

$R(\omega)$ is the longitudinal (e,e') response function

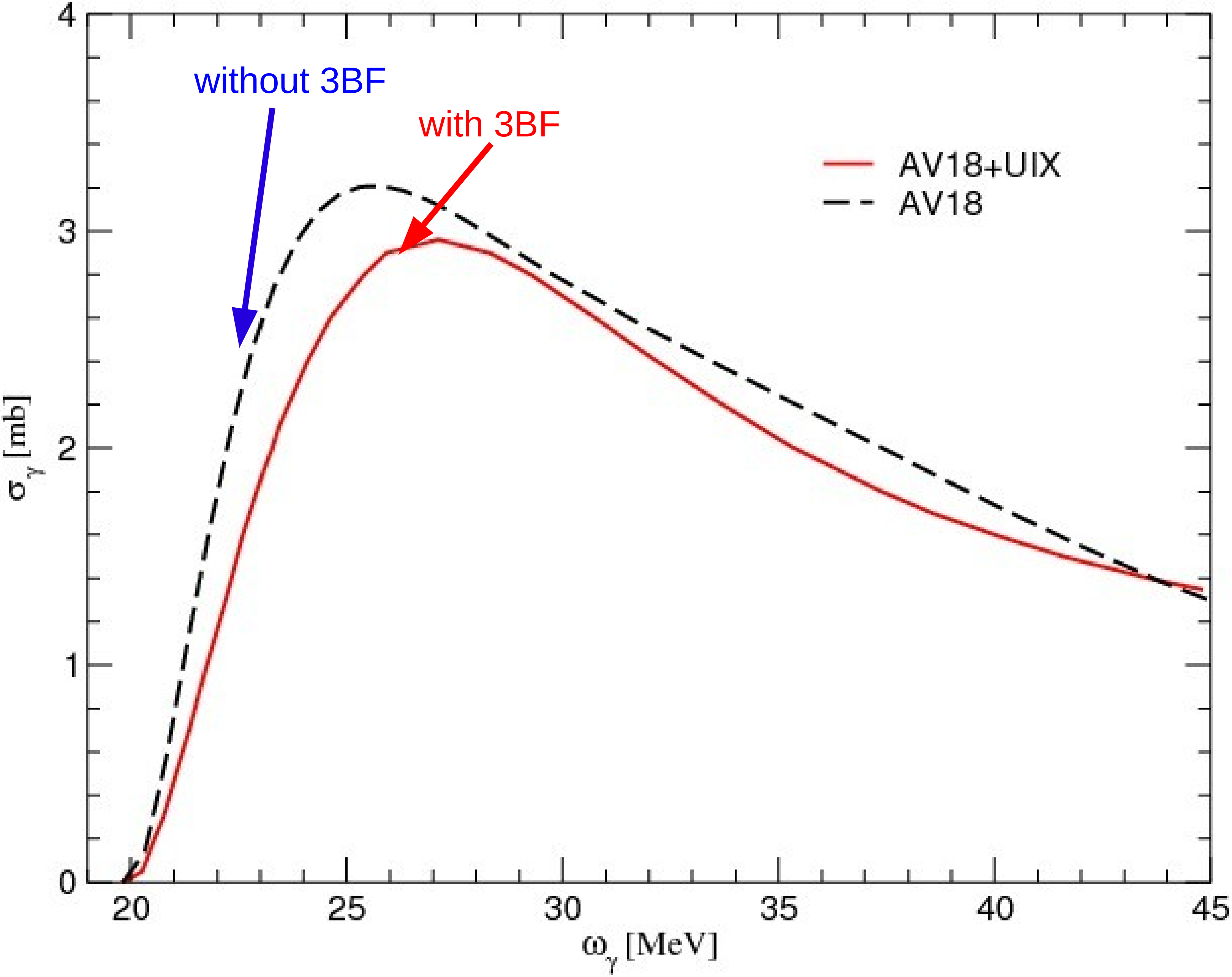


Phys Lett. B338 (1994) 130

Electromagnetic probes (photons, electrons) are a very "clean" source of information

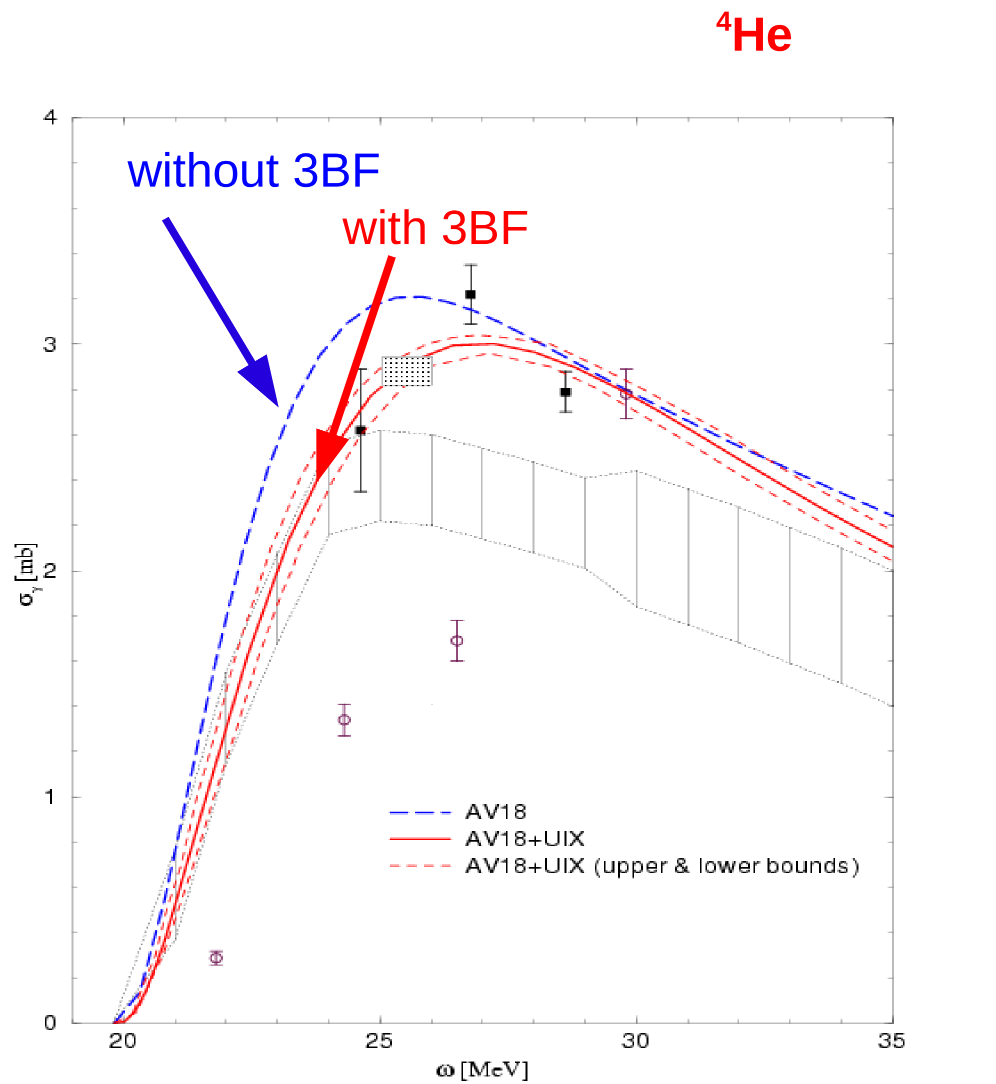
Total Photodisintegration of 4He

AV18 + UIX



Total Photoabsorption Cross Section of ^{4}He

Theory: LIT + EIHH



D.Gazit, S. Bacca et al. PRL 96 (2006) 112301

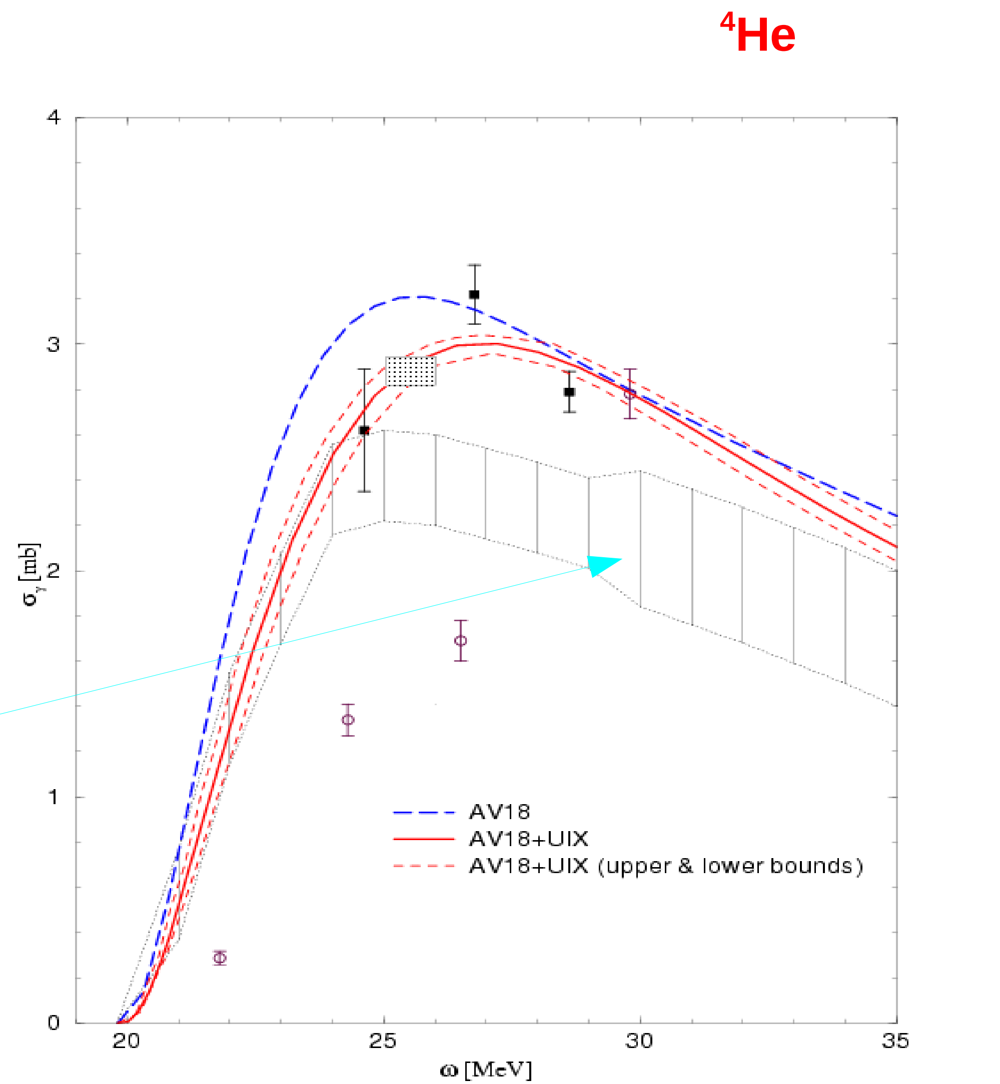
Total Photoabsorption Cross Section of ^{4}He

OLD data:

(γ,n) Berman et al. '80

+

(γ,p) Feldman et al. '90



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Total Photoabsorption Cross Section of ^4He

NEW data:

Nilsson et al. MAXLAB Lund (2005)

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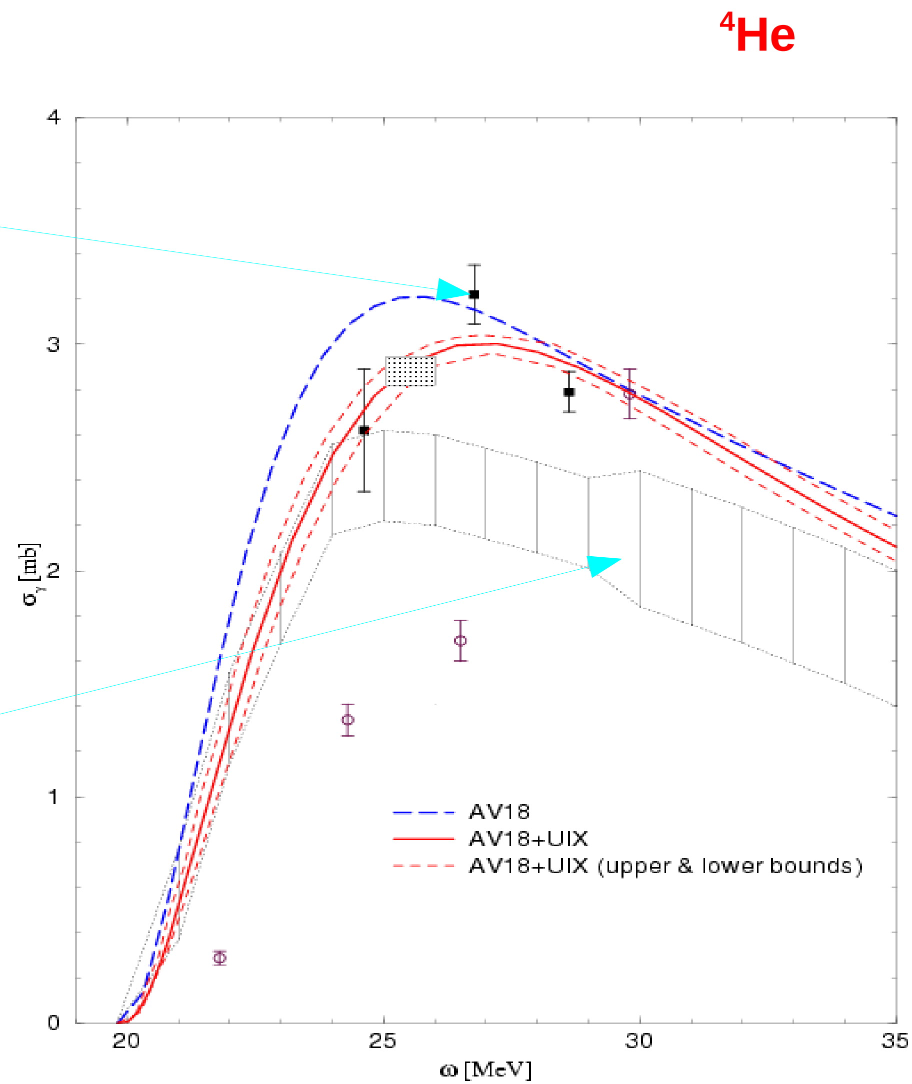

D.Gazit, S. Bacca et al. PRL 96 (2006) 112301

Total Photoabsorption Cross Section of ^{4}He

NEW data:

Nilsson et al. MAXLAB Lund (2005)

Shima et al. Osaka (2005)

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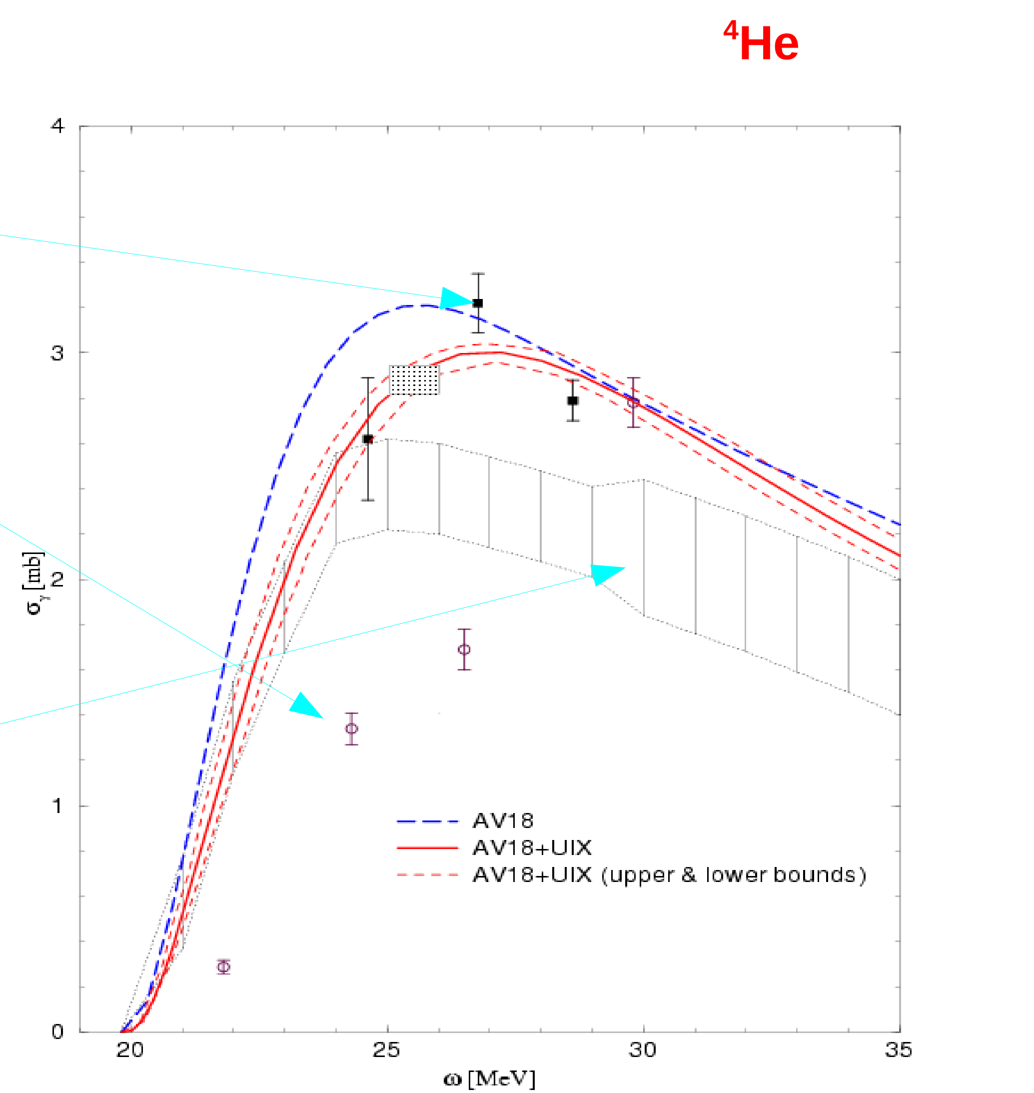
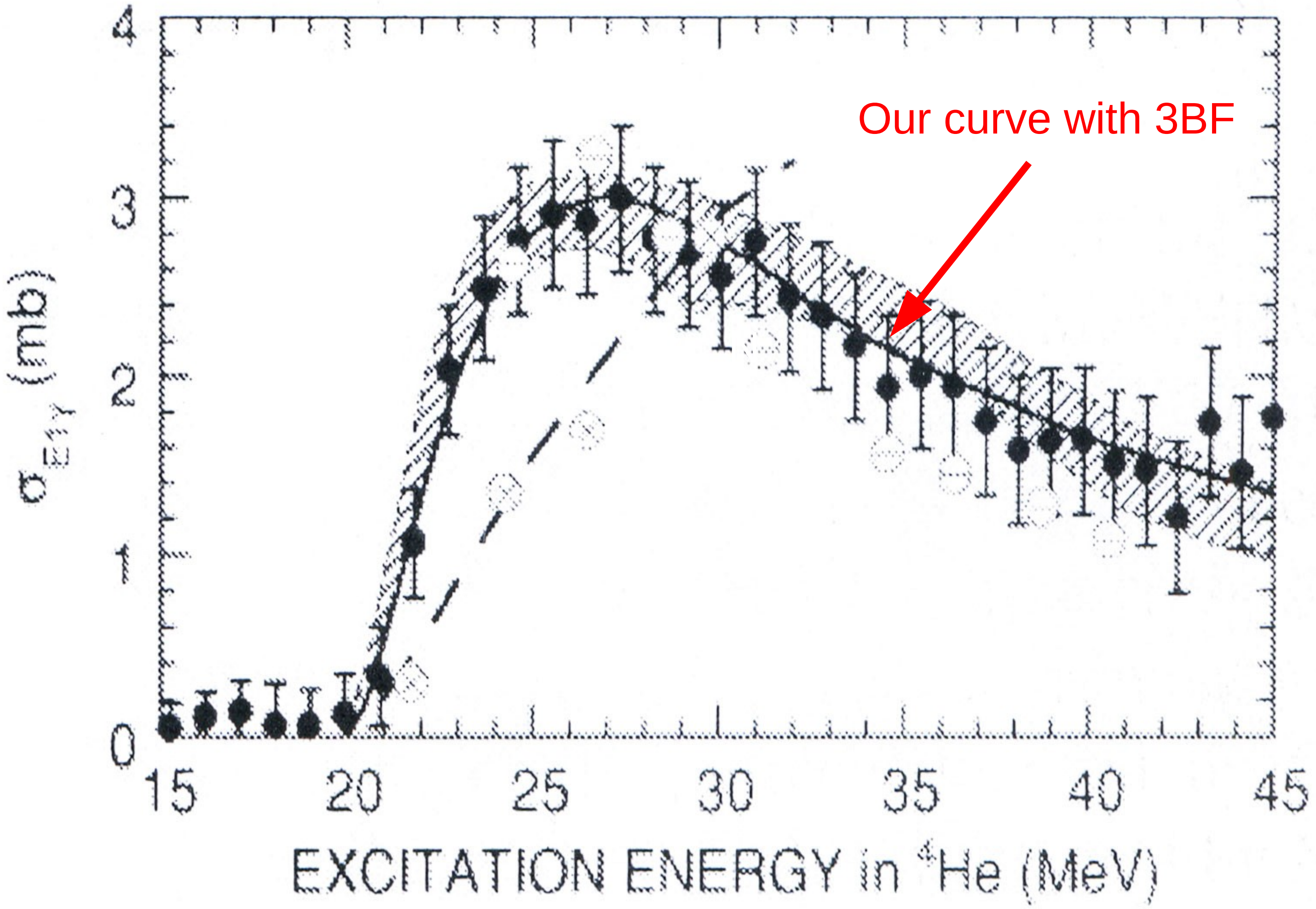


D.Gazit, S. Bacca et al. PRL 96 (2006) 112301

Nakayama et al. PHYSICAL REVIEW C 76, 021305(R) (2007)

Analog of GDR by $^4He(^7Li,^7Be)$



Electromagnetic probes (electrons, photons) are a very "clean" source of information

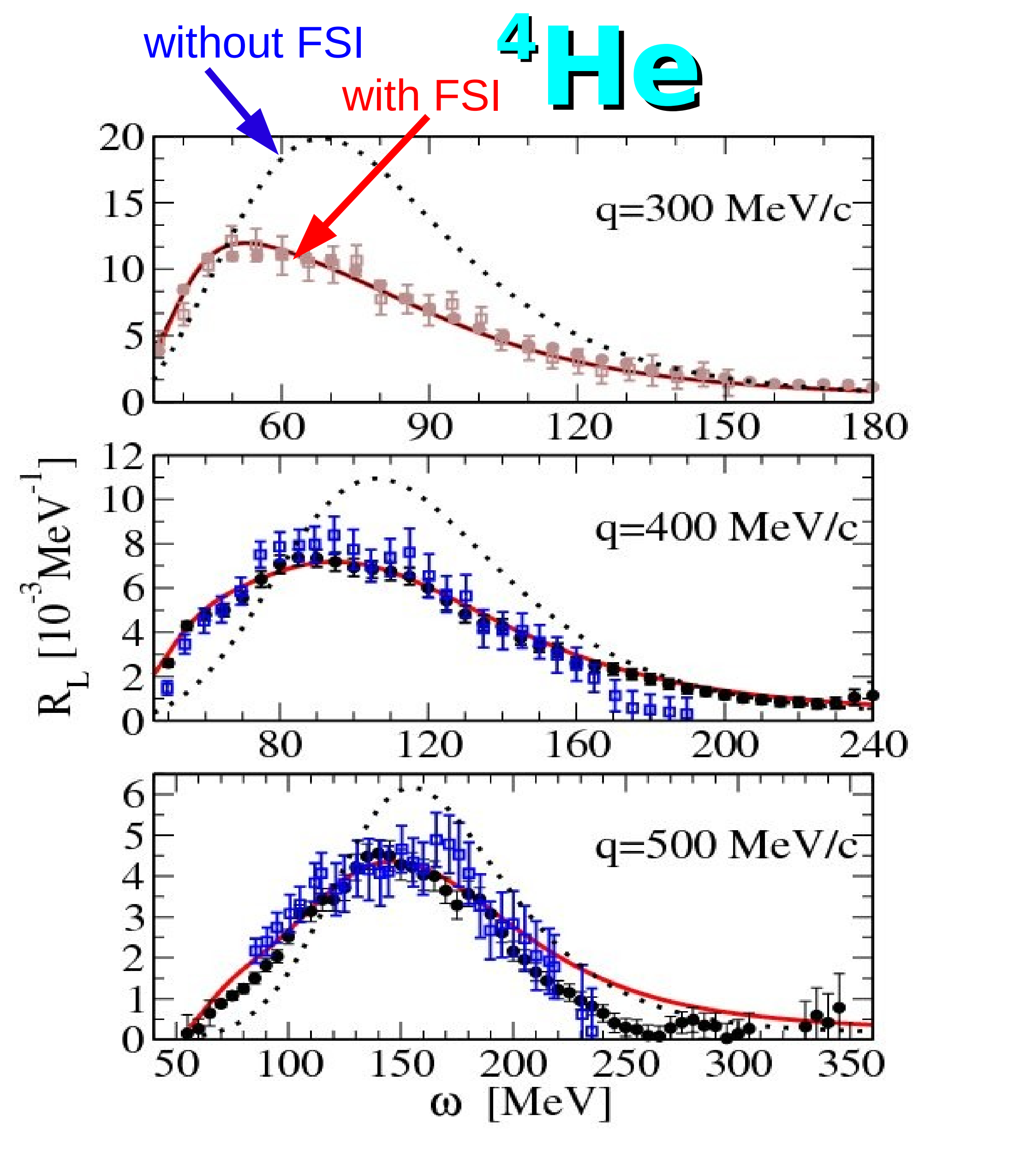
Electrons can explore the entire nucleus
at **different scales** (varying momentum transfer **q**)
and **different excitation energies** (varying energy transfer $\boldsymbol{\omega}$).
e.g. low ω , high q

“quasi elastic” electron scattering

Role of
Final State
Interaction:

dotted: PWIA

full: AV18+UIX



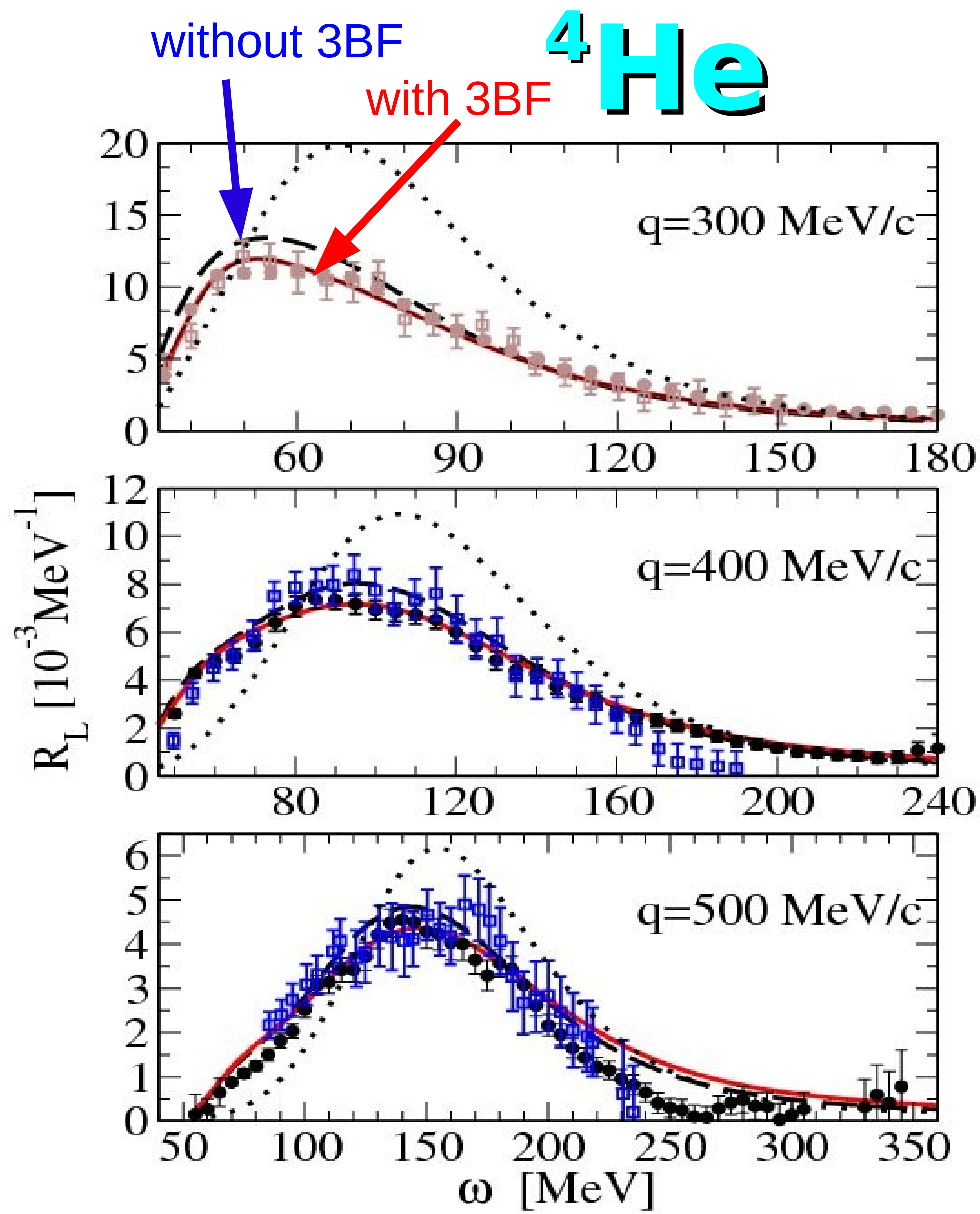
S.Bacca et al.,
PRL 102 (2009) 162501

“quasi elastic” electron scattering

Role of 3-body force

dashed: AV18

full: AV18+UIX



S.Bacca et al.,
PRL 102 (2009) 162501

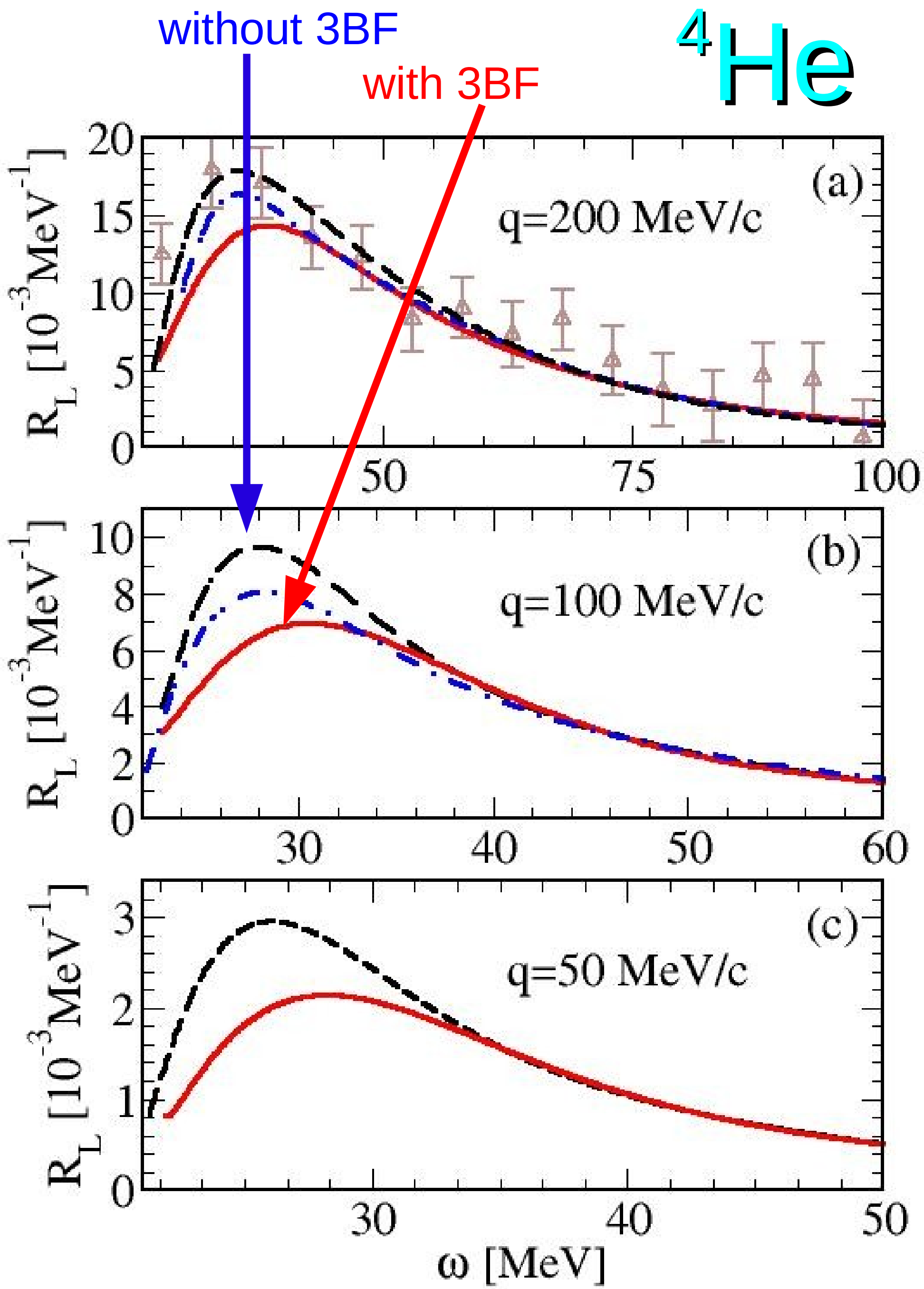
LOW q:

SURPRISE !

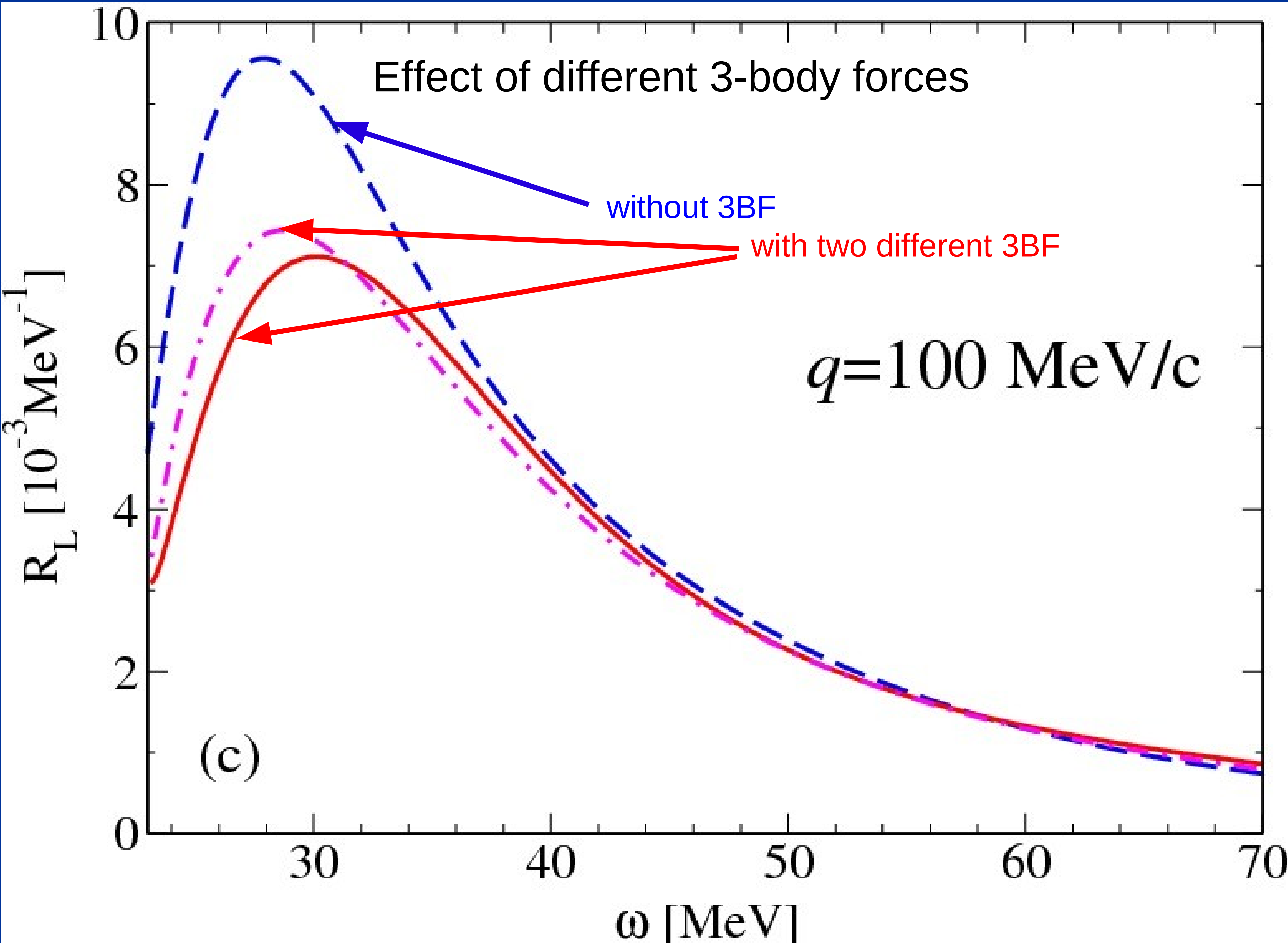
LARGE EFFECT OF
3-BODY FORCE

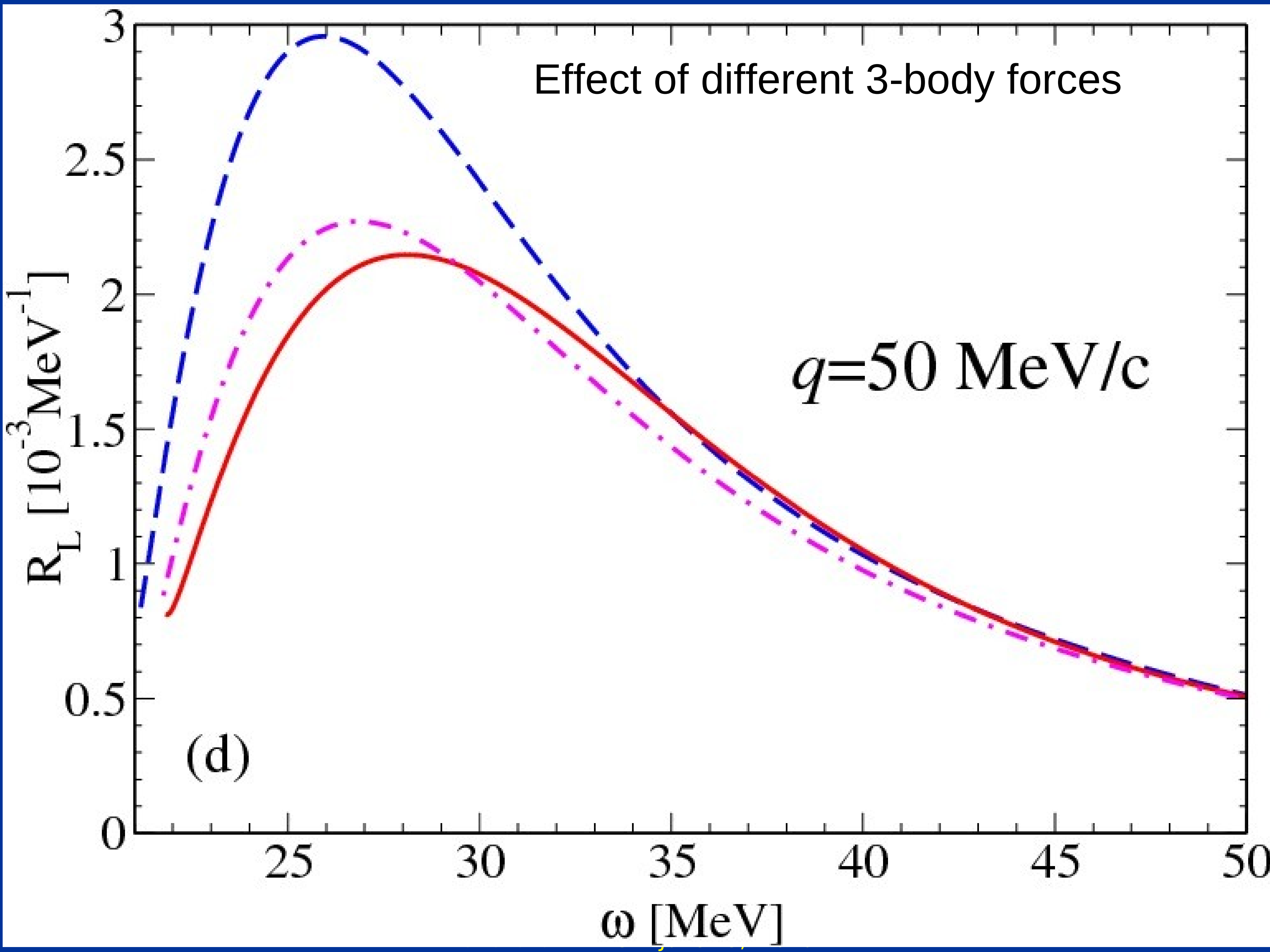
NO MEASUREMENTS
AT LOW q !!!

S.Bacca et al., PRL 102 (2009) 162501



G.

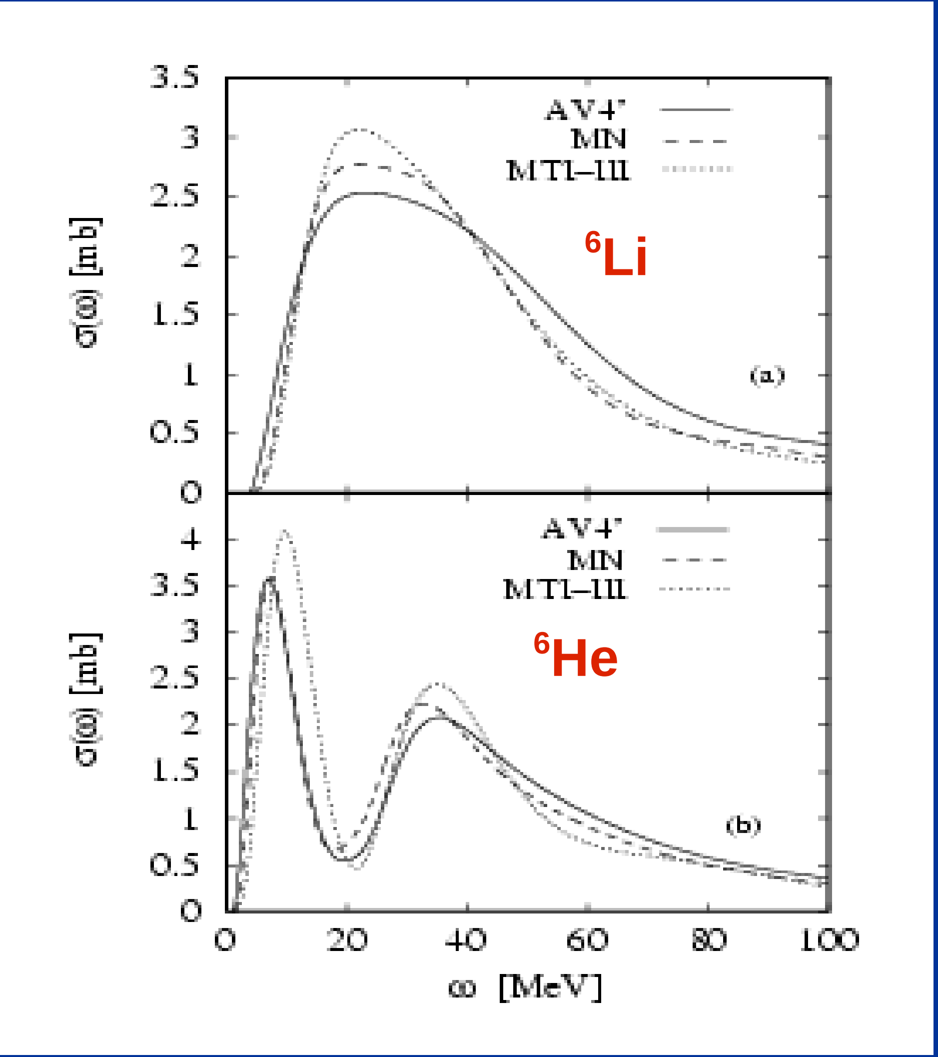

Effect of different 3-body forces
without 3BF
with two different 3BF
q=100 MeV/c
(c)
R_L [10^{-3}MeV^{-1}]
ω [MeV]
0
2
4
6
8
10
30
40
50
60
70



Effect of different 3-body forces
q=50 MeV/c
(d)
R_L [10^-3 MeV^-1]
ω [MeV]
0
0.5
1
1.5
2
2.5
3
25
30
35
40
45
50

6-Body E1 excitation

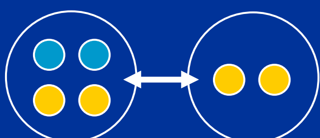
S. Bacca et al.PRL89(2002)052502



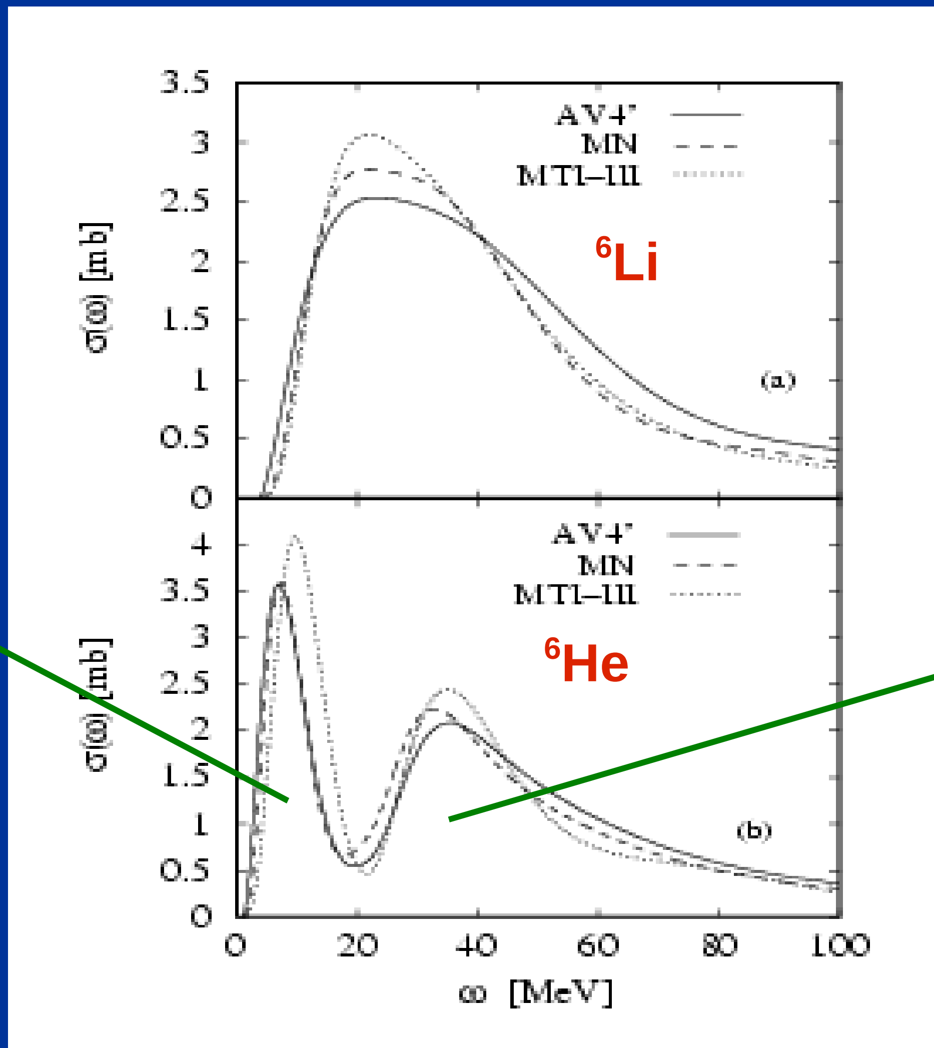
LIT + EIHH methods

6-Body E1 excitation

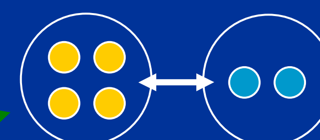
S. Bacca et al.PRL89(2002)052502



soft mode



LIT + EIHH methods



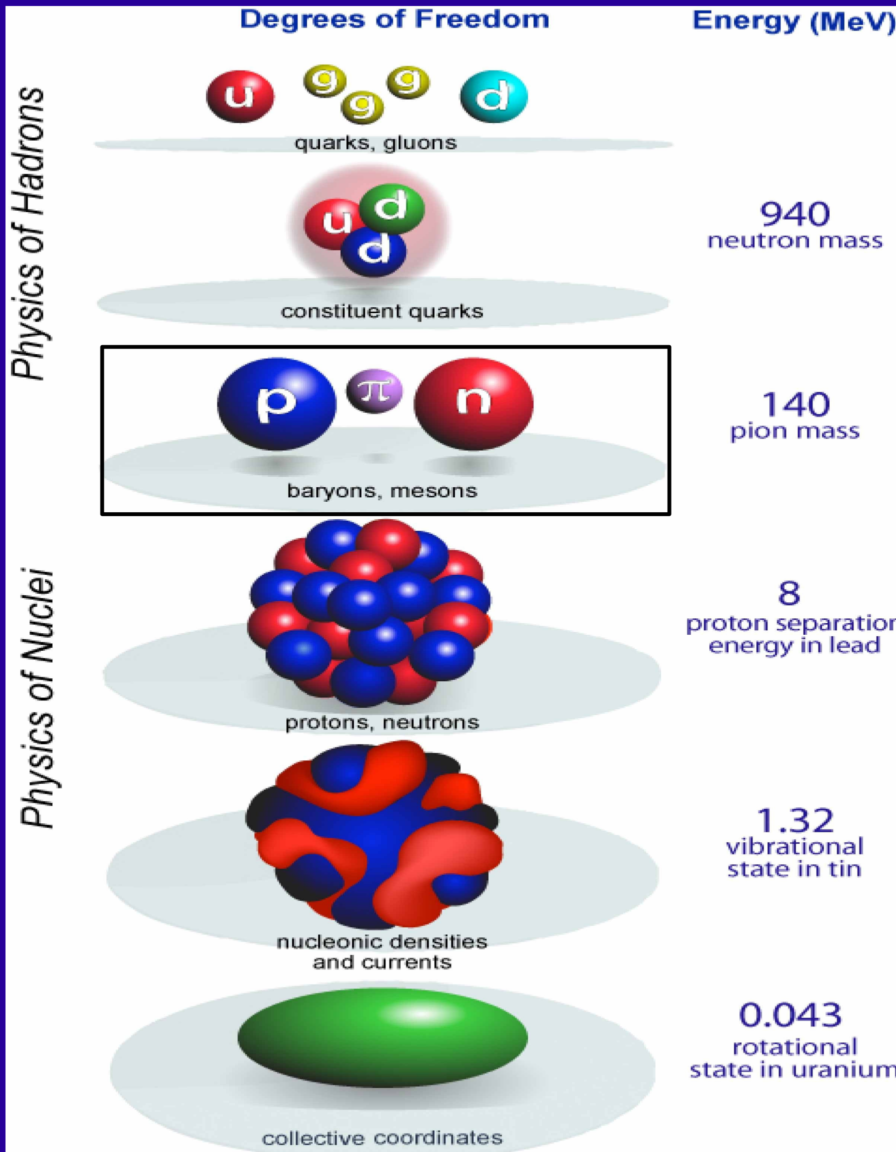
classical GT mode

Conclusions

- Few body nuclear physics has made important progress both in studying the nuclear interaction and in accounting for typical many – body phenomena

- The **LIT** method has opened up new perspectives to study reactions with ab initio methods, avoiding the many-body scattering problem

fundamental issues in nuclear physics:



Connects to QCD

nuclear interaction

Connects to typical MB properties (clusterization, collectivity, mean field properties etc.)

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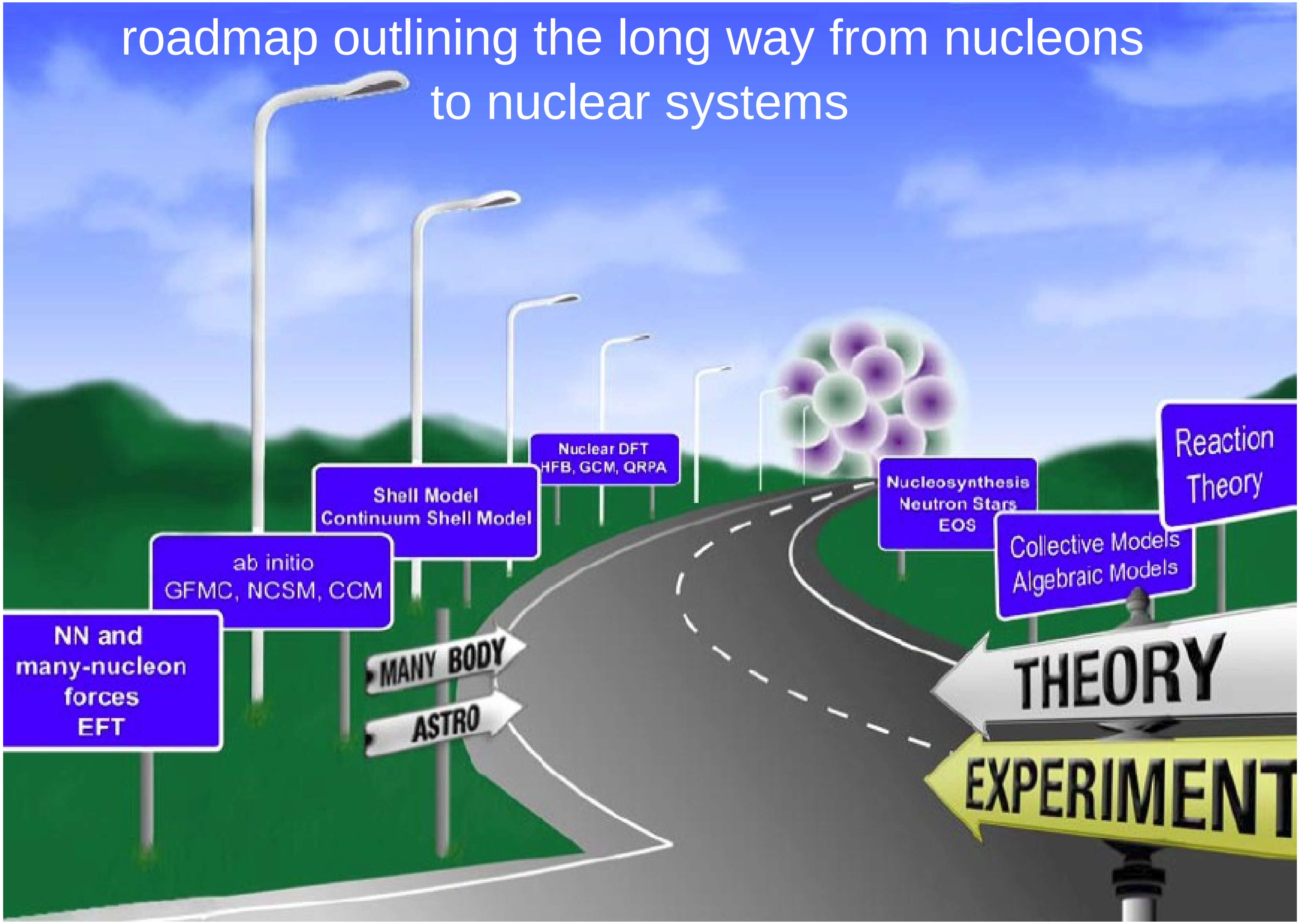
roadmap outlining the long way from nucleons to nuclear systems



roadmap outlining the long way from nucleons to nuclear systems