Angular distribution of thrust axis with power-suppressed contribution in e^+e^- annihilation

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Outline

Outline

$e^+e^- \to Z \to hadrons$

We restrict our attention to the case when Z-boson is the intermediate state in the processes $e^+e^- \to hadrons$.

$$\frac{d\sigma(T)}{d\cos\theta} \sim L^{\mu\nu}(\mathbf{n}_e)\, H_{\mu\nu}(T, \mathbf{n}_T),$$

Leptonic tensor:

$$L^{\mu\nu} = \frac{1}{4}\sum_{\lambda_{e_-},\lambda_{e_+}} \langle 0|j^\nu|e^+e^-\rangle^\star \langle 0|j^\mu|e^+e^-\rangle, \quad j^\mu = \bar\psi_e\gamma^\mu(g_{ve} - g_{ae}\gamma_5)\psi_e.$$

Hadronic tensor:

$$H^{\mu\nu} = \sum_X \langle X|J^\nu|0\rangle^\star \langle X|J^\mu|0\rangle\, \Theta\left(TQ - \sum_{i\in X}|\mathbf{p}_i\cdot\mathbf{n}_T|\right), \quad J^\mu = \bar\psi_q\gamma^\mu(g_{vq} - g_{aq}\gamma_5)\psi_q,$$

$$\sum_{i\in X}|\mathbf{p}_i\cdot\mathbf{n}_T| = \max_{\mathbf{n}}\sum_{i\in X}|\mathbf{p}_i\cdot\mathbf{n}|, \qquad \text{where} \qquad \mathbf{n}^2 = \mathbf{n}_T^2 = 1$$

Tensor structures

Leptonic tensor:

$$L^{\mu\nu}(\mathbf{n}_e) = \left(g_{al}^2 + g_{vl}^2\right)\left[-g_\perp^{\mu\nu}(\mathbf{n}_e)\right] - 2g_{al}g_{vl}a^{\mu\nu}(\mathbf{n}_e),$$

Hadronic tensor can be parameterized as follows

$$H^{\mu\nu} = (g_{vq}^2 + g_{aq}^2)\left\{F(\tau)\left[-g_\perp^{\mu\nu}(\mathbf{n}_T)\right] + G(\tau)\,g_\parallel{}^{\mu\nu}(\mathbf{n}_T)\right\} - 2g_{vq}g_{aq}K(\tau)\left[a^{\mu\nu}(\mathbf{n}_T)\right]^\star,$$

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{n^\mu n_+^\nu + n^\nu n_+^\mu}{n\cdot n_+},$$

$$g_\parallel^{\mu\nu} = \frac{1}{4}\left(n^\mu - n_+^\mu\right)\left(n^\nu - n_+^\nu\right),$$

$$a^{\mu\nu} = \frac{i}{n\cdot n_+}\varepsilon^{\mu\nu\alpha\beta}n^\alpha n_+^\beta,$$

$$\mathbf{u}^2 = 1, \qquad n = (1, -\mathbf{u}), \qquad n_+ = (1, \mathbf{u}).$$

$$H^{\mu\nu}(n+n_+)_\nu = 0$$

Angular distribution

Let us assume, for the sake of simplicity, that $g_{vl} = g_{al} = 1$.

$$j^{\mu} = \bar{\psi}_e \gamma^{\mu} (1 - \gamma_5) \psi_e .$$

Therefore Z-boson is produced in the state $|J, J_z\rangle = |1, -1\rangle$, so that $\mathbf{n}_z = \mathbf{n}_e$.

Thus we obtain the following angular distribution

$$\frac{d\sigma(T)}{d\cos\theta} \sim (g_v^2 + g_a^2) \left[F(\tau) \left(1 + \cos^2\theta_T \right) + G(\tau) \sin^2\theta_T \right] + 2 g_v g_a K(\tau) 2\cos\theta_T ,$$

where $\cos\theta_T = \mathbf{n}_T \cdot \mathbf{n}_e$.

Large energy symmetry

$$\frac{d\sigma}{d\cos\theta} \sim (g_v+g_a)^2 \left|d^1_{-1,-1}\right|^2 + (g_v-g_a)^2 \left|d^1_{1,-1}\right|^2$$
$$= \frac{1}{4}\left[(g_v+g_a)^2 \left(1+\cos\theta_q\right)^2 + (g_v-g_a)^2 \left(1-\cos\theta_q\right)^2\right]$$
$$= \frac{1}{2}\left[\left(g_v^2+g_a^2\right)\left(1+\cos^2\theta_q\right) + (2g_vg_a)\, 2\cos\theta_q\right].$$

$$\frac{d\sigma(T)}{d\cos\theta} \sim (g_v^2+g_a^2)\left[F(\tau)\left(1+\cos^2\theta_T\right) + G(\tau)\sin^2\theta_T\right] + 2g_vg_aK(\tau)\, 2\cos\theta_T,$$

If $\tau << 1$, then

$$F(\tau) = K(\tau)$$
$$G(\tau) = 0.$$

Why symmetry?

Operators

$$\hat{g} = g_{\perp}^{\mu\nu} = g^{\mu\nu} - \frac{n^{\mu}n_{+}^{\nu} + n^{\nu}n_{+}^{\mu}}{n \cdot n_{+}},$$
$$\hat{a} = a^{\mu\nu} = \frac{1}{n \cdot n_{+}} \varepsilon^{\mu\nu\alpha\beta} n_{\alpha} n_{+\beta}.$$

Algebra

$$\hat{g}^2 = \hat{g},$$
$$\hat{a}\hat{g} = \hat{g}\hat{a} = \hat{a},$$
$$\hat{a}^2 = -\hat{g}.$$

Rotation $U(1)$

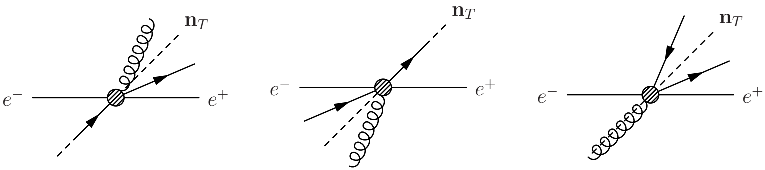
$$\hat{U} = \hat{g}\cos\phi + \hat{a}\sin\phi = \hat{g}\exp(\hat{a}\phi)$$
$$\hat{U}^{\dagger} = \hat{g}\cos\phi - \hat{a}\sin\phi = \hat{g}\exp(-\hat{a}\phi)$$

Hadronic tensor

$$\hat{H} = A\hat{g} + B\hat{a} = \hat{U}(\phi)\hat{H}\hat{U}^{\dagger}(\phi)$$
$$\hat{H}(n, n_{+}) = \hat{H}(\alpha n, \beta n_{+})$$

Outline

Gluon radiation

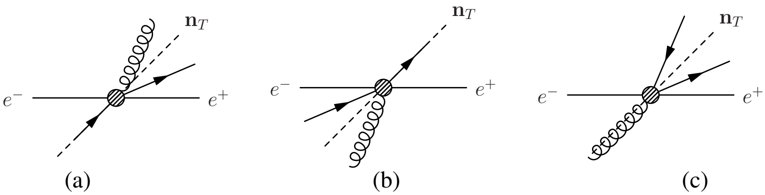


(a) (b) (c)

(a), (b): mainly contribute to $G(\tau)$ and slightly violates the relation $F(\tau) = G(\tau)$

(c): can be, in principle, excluded from the analysis. To do this requires a simultaneous tagging of B and $\bar{B}$ mesons. If not, it does not contribute to $K(\tau)$ and gives a leading contribution to $F(\tau)$. The contribution to $G(\tau)$ is negligible.

Gluon radiation



This talk

We will restrict our attention to the topologies (a), (b) such that $\left|\theta_q - \theta_{\bar{q}}\right| \approx \pi$ in the $\tau \to 0$ limit.

Therefore, we will consider the corrections to $G(\tau)$.

Mechanisms of symmetry breaking

Old-fashioned perturbative theory provides a clear physical explanation.

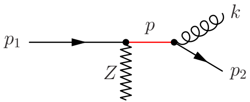
We can (to some extent arbitrary) single out two mechanisms.

1st Additional partons are radiated off the primary $q\bar{q}$-pair *after* the Z-boson decay:

Breaking is of kinematic nature: $\theta_q - \theta_T$ misfit

2nd A virtual hadronic state appears *before* the Z-boson decay.

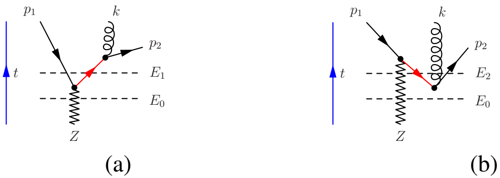
Retarded and advanced propagations



The sum of the quark and gluon momenta:

$$p^\mu = p_2^\mu + k^\mu = (p \cdot n)\,\frac{n_+^\mu}{2} + (p \cdot n_+)\,\frac{n^\mu}{2},$$

Retarded and advanced propagations



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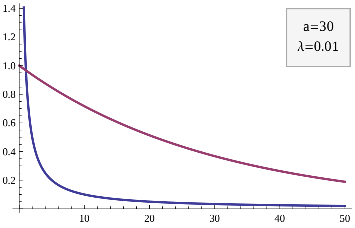
Splitting of the propagator into two parts:

$$\frac{\not p}{p^2 + i0} = \frac{1}{E_0 - E_1 + i0}\,\frac{\not n_+}{2} + \frac{-1}{E_0 - E_2 - i0}\,\frac{\not n}{2}$$

Z-boson disappears being absorbed by the intermediate antiquark from $d^1_{0,-1}(\theta_T)$

Outline

A very simple integral



The exponential integral function

$$\int_0^\infty \frac{e^{-x/a}}{x+\lambda}\mathrm{d}x = e^{\lambda/a}\int_{\lambda/a}^\infty \frac{e^{-x}}{x}\mathrm{d}x$$
$$= -e^{\lambda/a}\,\mathrm{Ei}\left(-\lambda/a\right)$$

Expansion in the $\lambda \ll a$ limit:

$$\int_0^\infty \frac{e^{-x/a}}{x+\lambda}\mathrm{d}x = \int_0^\infty \frac{e^{-x}}{x+\lambda/a}\mathrm{d}x \neq \sum_{n=0}^\infty \left(-\frac{\lambda}{a}\right)^n \int_0^\infty \frac{e^{-x}}{x^{1+n}}\mathrm{d}x$$

One can not ignore the region $x \sim \lambda$.

Separation of the regions

- Dimensional regularization $\mathrm{d}x \longrightarrow \left(\frac{x}{\mu}\right)^{\varepsilon} \mathrm{d}x$

Separation of the regions

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- Integrand expansion in the soft region:

$$I(x) = \frac{e^{-x/a}}{x+\lambda} \to I_{\text{soft}}(x) = \frac{1}{x+\lambda} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{-x}{a}\right)^{n},$$

Soft region

$x \sim \lambda$

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Soft region

$x \sim \lambda$

- Subtraction of the soft region:

$$\int_0^{\infty} I(x) \left(\frac{x}{\mu}\right)^{\varepsilon} \mathrm{d}x = \int_0^{\infty} [I(x) - I_{\text{soft}}(x)] \left(\frac{x}{\mu}\right)^{\varepsilon} \mathrm{d}x + \int_0^{\infty} I_{\text{soft}}(x) \left(\frac{x}{\mu}\right)^{\varepsilon} \mathrm{d}x$$

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$$\int_0^{\infty} I(x) \left(\frac{x}{\mu}\right)^{\varepsilon} \mathrm{d}x = \int_0^{\infty} \left[I(x) - I_{\text{soft}}(x)\right] \left(\frac{x}{\mu}\right)^{\varepsilon} \mathrm{d}x + \int_0^{\infty} I_{\text{soft}}(x) \left(\frac{x}{\mu}\right)^{\varepsilon} \mathrm{d}x$$

- Integrand expansion in the hard region:

$$I_{\text{hard}}(x) = I(x) - I_{\text{soft}}(x) = \frac{e^{-x/a}}{x+\lambda} - \frac{1}{x+\lambda} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{-x}{a}\right)^n$$

$$\to \sum_{n=0}^{\infty} \left(-\frac{\lambda}{x}\right)^n \frac{e^{-x/a}}{x}$$

Hard region

$x \sim a$

Integration over regions

- In fact,the contributions are separated out

$$\int_0^\infty I(x)\left(\frac{x}{\mu}\right)^\varepsilon \mathrm{d}x = \sum_{n=0}^\infty \left(-\frac{\lambda}{a}\right)^n \times \left\{\left(\frac{a}{\mu}\right)^\varepsilon \int_0^\infty I_{\text{hard}}^{(n)}(x)\, x^\varepsilon \mathrm{d}x + \left(\frac{\lambda}{\mu}\right)^\varepsilon \int_0^\infty I_{\text{soft}}^{(n)}(x)\, x^\varepsilon \mathrm{d}x\right\}$$

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- Each of which can be easily evaluated

$$\int_0^\infty I_{\mathrm{hard}}^{(n)}(x)\,x^\varepsilon \mathrm{d}x = \int_0^\infty \frac{e^{-x}}{x^{1+n}}x^\varepsilon \mathrm{d}x = \Gamma(\varepsilon-n)$$

$$\int_0^\infty I_{\mathrm{soft}}^{(n)}(x)\,x^\varepsilon \mathrm{d}x = \frac{1}{n!}\int_0^\infty \frac{x^n}{x+1}x^\varepsilon \mathrm{d}x = -\frac{\pi}{n!\sin\pi\varepsilon}$$

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- The singularities with respect to ε and the μ-dependence drop out of the sum of the all contributions

$$\int_0^\infty \frac{e^{-x/a}}{x+\lambda}\mathrm{d}x = \sum_{n=0}^\infty \left(\frac{\lambda}{a}\right)^n \frac{\pi}{\sin(\pi\varepsilon)}\left[\left(\frac{a}{\mu}\right)^\varepsilon \frac{1}{\Gamma(1-\varepsilon+n)} - \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^\varepsilon\right] = \sum_{n=0}^\infty \left(\frac{\lambda}{a}\right)^n \frac{1}{n!}\left[\ln\frac{a}{\lambda} + \psi^{(0)}(n+1)\right].$$

Quantum field theory

- It is (usually) difficult to calculate multiloop Feynman integrals as exact function of external kinematic parameters.
- Expansion is sometimes sufficient (smooth fields).
- In general case, one can not expand the integrand before the integration.
- In general case, there is no regular multiple Taylor series (nonanalytic behaviour: $\ln q$, $\sqrt{q^2}$).

The strategy of expanding by regions

- One has to analyze all scales in a problem and single out the field modes with momentum components is of order of one of the scales (power counting rules).

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The most nontrivial step. The scales can be hidden:

$$a \ll b \ll c \Rightarrow a \left(\frac{b}{c} \right)^n \ll a$$

Usually the region appears near poles of the propagators (potential, soft, collinear etc.).

The strategy of expanding by regions

- One has to analyze all scales in a problem and single out the field modes with momentum components is of order of one of the scales (power counting rules).
- Separation of the regions. It implies regularization with the help of intermediate scales ($a \ll \mu_1 \ll b \ll \mu_2 \ll c$) such that the integration convergent in the definite region (can be implicit) and expansion of the integrand in the regions.

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  Dimensional regularization is not always enough (sometimes with analytic regularization). Pauli-Villards is OK, but additional intermediate regions appear.

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Some artifacts appear: $\log \frac{a}{\mu}$, $\frac{1}{\varepsilon^n}$, *etc.*

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- Integration of every expansion over the whole integration domain.
- Sum all the contributions.

  All artifacts should disappear

Outline

Thrust distribution

Let us demonstrate how to apply this method to the perturbative calculation of the thrust distribution

$$F(\tau) = \frac{1}{\sigma_0} \int \mathrm{d}\sigma_{e^+e^- \to h} \Theta \left(TQ - \sum_{i \in h} |\mathbf{p}_i \cdot \mathbf{n}_T| \right),$$

in the region where $\tau = 1 - T \ll 1$.

We introduce a small parameter λ such that $\tau \sim \lambda^2$.

Power counting rules

Region	Scale	Power counting $Q^{-1}(k\cdot n, k_\perp, k\cdot n_+)$
Hard	Q^2	$(1,1,1)$
Right collinear	τQ^2	$(1,\lambda,\lambda^2)$
Left collinear	τQ^2	$(\lambda^2,\lambda,1)$
Soft	$(\tau Q)^2$	$(\lambda^2,\lambda^2,\lambda^2)$

Integrations

Hard regions gives the usual on-shell QCD Sudakov form factor:

$$F^{\text{hard}}(\tau) = \frac{\alpha_S C_{\text{F}}}{4\pi}\left(\frac{Q^2}{\mu^2}\right)^{-\varepsilon}\left[-\frac{4}{\varepsilon^2} - \frac{6}{\varepsilon} - 16 + \frac{7\pi^2}{3} + O(\varepsilon)\right].$$

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Expansion in the collinear region gives the DGLAP kernel:

$$F^{\text{col.R}}(\tau) = \frac{g^2 C_{\text{F}}}{(4\pi)^2}\int_0^{Q^2\tau}\frac{\mathrm{d}p_{\text{L}}^2}{p_{\text{L}}^2}\left(\frac{p_{\text{L}}^2}{\mu^2}\right)^{\mathscr{D}/2-2}\int_0^1 \mathrm{d}z\,\frac{2\,(z\bar{z})^{\mathscr{D}/2-2}}{\Gamma(\mathscr{D}/2-1)}P_{qq}(z)$$

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Soft regions corresponds the soft radiation off two-parton antenna:

$$F^{\text{soft}}(\tau) = 2\int \mathrm{d}\rho_{\text{soft}}\,\Theta(k\cdot n - k\cdot n_+)\,\Theta(\tau Q - k\cdot n_+)\,\frac{\alpha_S}{\pi}C_{\text{F}}\left(\frac{n}{n\cdot k}-\frac{n_+}{n_+\cdot k}\right)^2$$

Integrations

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The singularities with respect to ε and the μ^2-dependence drop out of the sum of the all contributions, that occurs always if one correctly uses the method of expanding by regions.

$$F = 1 + \frac{\alpha_S}{4\pi} C_\mathrm{F}\left(-4\ln^2\frac{1}{\tau} + 6\ln\frac{1}{\tau} - 2 + \frac{2\pi^2}{3}\right).$$

I can see them

Quarks. Neutrinos. Mesons. All those damn particles you can't see. That's what drove me to drink. But now I can see them!

Outline

Soft collinear effective theory

Effective theory amplitude can be understood as an expansion of the full QCD amplitude in a certain region

Soft collinear effective theory

$$\psi = \xi + \eta, \qquad \xi = \frac{\not{n}\,\not{n}_+}{4}\psi, \qquad \eta = \frac{\not{n}_+\,\not{n}}{4}\psi,$$

$$\mathscr{L} = \bar{\psi}\left(i\not{D} + i\varepsilon\right)\psi,$$

$$\mathscr{L}' = \bar{\xi}(x)\, in\cdot D\frac{\not{n}_+}{2}\xi(x) + i\int_{-\infty}^{0} \mathrm{d}s\left[\bar{\xi}\, i\overleftarrow{\not{D}}_\perp W\right](x)\left[W^\dagger i\not{D}_\perp\frac{\not{n}_+}{2}\xi\right](x + sn_+),$$

$\mathscr{L}'$ is equivalent to $\mathscr{L}$ and identical to that of QCD in the infinite momentum frame (*Kogut, Soper*...1970) or light-cone quantization (*Brodsky, Pauli, Pinsky*... 1998).

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$$\langle 0\left|\mathrm{T}\xi\left(x\right)\bar{\xi}\left(y\right)\right|0\rangle = \frac{\not{n}\,\not{n}_+}{4}\langle 0\left|\mathrm{T}\psi\left(x\right)\bar{\psi}\left(y\right)\right|0\rangle\frac{\not{n}_+\,\not{n}}{4} = \int\frac{\mathrm{d}^4 p}{(2\pi)^2}\frac{\not{n}}{2}\frac{in_+\cdot p}{p^2+i0}e^{-ip(x-y)}.$$

Soft collinear effective theory

$$\psi = \xi + \eta, \qquad \xi = \frac{\not{n}\,\not{n}_+}{4}\psi, \qquad \eta = \frac{\not{n}_+\,\not{n}}{4}\psi,$$

$$\mathscr{L} = \bar{\psi}\,(i\not{D} + i\varepsilon)\,\psi,$$

$$\mathscr{L}' = \bar{\xi}\,(x)\,in\cdot D\frac{\not{n}_+}{2}\xi\,(x) + i\int_{-\infty}^{0} \mathrm{d}s\left[\bar{\xi}\,i\overleftarrow{\not{D}}_\perp W\right](x)\left[W^\dagger i\not{D}_\perp\frac{\not{n}_+}{2}\xi\right](x+sn_+),$$

If one counts as “collinear field” the modes with $(n_+\cdot p, p_\perp, n\cdot p) \sim \left(1,\lambda,\lambda^2\right)$, so that $\mathrm{d}^4p \sim \lambda^4$ then

$$\xi \sim \lambda, \qquad \eta \sim \lambda^2.$$

Soft collinear effective theory

$$\psi = \xi + \eta, \qquad \xi = \frac{\not{n}\,\not{n}_+}{4}\psi, \qquad \eta = \frac{\not{n}_+\,\not{n}}{4}\psi,$$

$$\mathscr{L} = \bar{\psi}\,(i\not{D} + i\varepsilon)\,\psi,$$

$$\mathscr{L}' = \bar{\xi}\,(x)\,in\cdot D\frac{\not{n}_+}{2}\xi\,(x) + i\int_{-\infty}^{0} \mathrm{d}s\left[\bar{\xi}\,i\overleftarrow{\not{D}}_{\perp}W\right](x)\left[W^{\dagger}i\not{D}_{\perp}\frac{\not{n}_+}{2}\xi\right](x+sn_+),$$

Hierarchy of modes

$$\xi \sim \lambda, \qquad \eta \sim \lambda^2.$$

$$n_+A_c \sim 1, \qquad A_{c\perp} \sim \lambda, \qquad n\cdot A_c \sim \lambda^2,$$

$$q_s \sim \lambda^3, \qquad A_s \sim \lambda^2.$$

$$\mathscr{L}'_{\mathrm{SCET}} = \mathscr{L}_{\xi}^{(0)} + \mathscr{L}_{\xi}^{(1)} + \mathscr{L}_{\xi}^{(2)} + \mathscr{L}_{\xi q}^{(1)} + \mathscr{L}_{\xi q}^{(2)} + \mathscr{L}_{qq}^{(2)} + \ldots$$

Factorization formula for thrust

Integration over the hard region is the matching of the QCD operator (weak current) on the SCET one

$$\hat{O}_2 = \bar{\xi}_{n_+} W_{n_+} Y^\dagger_{n_+} \Gamma Y_n W_n \xi_n .$$

Bauer, Fleming, Lee, and Sterman (2008) – factorization formula for *angularities* ($a = 0 \longrightarrow$ thrust, $a = 1 \longrightarrow$ broadening)

$$e(X) = \frac{1}{Q} \sum_{i \in X} e^{-|\eta|(1-a)} |\mathbf{p}_\perp^{(i)}|$$

$$F(\tau) = H\left(Q^2, \mu^2\right) \int \mathrm{d}p_\mathrm{L}^2 \mathrm{d}p_\mathrm{R}^2 \mathrm{d}k \, J\left(p_\mathrm{L}^2, \mu^2\right) J\left(p_\mathrm{R}^2, \mu^2\right) S_T\left(k, \mu^2\right) \Theta\left(Q^2 \tau - p_\mathrm{L}^2 - p_\mathrm{R}^2 - Qk\right).$$

It is claimed that

1. factorization of the complete set of hadronic final states is not needed.
2. any logarithmic accuracy can be achieved (LL, NLL, NNLL...)

Objects

- $H\left(Q^2,\mu^2\right)$ is the hard function, that is the square of the usual on-shell QCD Sudakov form factor

- $J\left(p^2,\mu^2\right)$ is the jet function

$$J\left(p^2,\mu^2\right) = \frac{1}{(p\cdot n_+)\,N_c}\,\frac{1}{2\pi}\,\mathrm{Im}\left[i\int \mathrm{d}^4x e^{-ipx}\left\langle 0\left|\mathrm{T}\left\{\bar{\xi}_n'(x)\,W_n(x)\,\frac{\hat{n}_+}{2}W_n^\dagger(0)\,\xi_n'(0)\right\}\right|0\right\rangle\right],$$

that is, up to a factor, the imaginary part of the QCD quark propagator in the light-cone gauge

- The soft factor $S_T\left(k,\mu^2\right)$ is defined as follows:

$$S_T\left(k,\mu^2\right) = \sum_X \left|\left\langle X\left|Y_n^\dagger Y_{n_+}\right|0\right\rangle\right|^2 \delta\left(k - n\cdot p_{X_\mathrm{L}} - n_+\cdot p_{X_\mathrm{R}}\right).$$

Evolution equations

It is convenient to take the Laplace transform

$$j\left(sQ^2,\mu^2\right) \equiv \int_0^\infty \mathrm{d}p^2 e^{-\nu p^2} J\left(p^2,\mu^2\right), \qquad s_T\left(sQ,\mu^2\right) \equiv \int_0^\infty \mathrm{d}k\, e^{-\nu Qk} S_T\left(k,\mu^2\right),$$

where we use the notation $s = 1/\left(\nu Q^2 e^{\gamma_E}\right) \sim \tau$, so that the thrust distribution takes the form:

$$F(\tau) = \frac{1}{2\pi i}\int_C \frac{\mathrm{d}\nu}{\nu} H\left(Q^2,\mu^2\right) j^2\left(sQ^2,\mu^2\right) s_T\left(sQ,\mu^2\right).$$

Evolution equations

$$\frac{dH\left(Q^2,\mu^2\right)}{d\ln\mu^2} = \left\{\Gamma_{\text{cusp}}\left[\alpha_S\left(\mu^2\right)\right]\ln\frac{Q^2}{\mu^2} + \gamma^{\text{H}}\left[\alpha_S\left(\mu^2\right)\right]\right\} H\left(Q^2,\mu^2\right),$$

$$\frac{dj\left(sQ^2,\mu^2\right)}{d\ln\mu^2} = \left\{-\Gamma_{\text{cusp}}\left[\alpha_S\left(\mu^2\right)\right]\ln\frac{sQ^2}{\mu^2} - \gamma^{\text{J}}\left[\alpha_S\left(\mu^2\right)\right]\right\} j\left(sQ^2,\mu^2\right),$$

$$\frac{ds_T\left(sQ,\mu^2\right)}{d\ln\mu^2} = \left\{\Gamma_{\text{cusp}}\left[\alpha_S\left(\mu^2\right)\right]\ln\frac{s^2Q^2}{\mu^2} - \gamma^{\text{S}}\left[\alpha_S\left(\mu^2\right)\right]\right\} s_T\left(sQ^2,\mu^2\right).$$

$$\Gamma_{\text{cusp}}\left(\alpha_S\right) = \frac{\alpha_S}{4\pi}\Gamma_{(0)} + \left(\frac{\alpha_S}{4\pi}\right)^2\Gamma_{(1)} + \ldots,$$

$$\gamma^i\left(\alpha_S\right) = \frac{\alpha_S}{4\pi}\gamma^i_{(0)} + \left(\frac{\alpha_S}{4\pi}\right)^2\gamma^i_{(1)} + \ldots.$$

NLL accuracy: 2-loop Γ_{cusp}, 2-loop $\alpha_S\left(\rho^2\right)$, 1-loop γ^i, initial conditions up to $O\left(\alpha_S\right)$.

CTTW approach

- Correct description of soft radiation implies *color coherence*, which results in the angular ordering constraints (*Ermolaev and Fadin, Müller 1981*)
- Ordering + two-loop DGLAP splitting kernels + proper normalization of coupling constant = branching algorithm
- *Catani, Trentadue, Turnock, Webber (1993)*

$$F(\tau) = \int \mathrm{d}P_{\mathrm{L}}^2 \mathrm{d}P_{\mathrm{R}}^2 J^{\mathrm{CTTW}}\left(P_{\mathrm{L}}^2, Q^2\right) J^{\mathrm{CTTW}}\left(P_{\mathrm{R}}^2, Q^2\right) \Theta\left(Q^2\tau - P_{\mathrm{L}}^2 - P_{\mathrm{R}}^2\right),$$

where the Laplace transform of $J^{\mathrm{CTTW}}\left(P^2\right)$ is found to be

$$\ln \tilde{J}^{\mathrm{CTTW}}\left(\nu, Q^2\right) = \ln \int_0^\infty \mathrm{d}\nu\, e^{-P^2\nu} J^{\mathrm{CTTW}}\left(P^2, Q^2\right)$$
$$= -\int_0^1 \frac{\mathrm{d}u}{u} \Theta\left(u - \frac{1}{e^{\gamma_E}\nu Q^2}\right)\left\{\int_{u^2Q^2}^{uQ^2} \frac{\mathrm{d}\rho^2}{\rho^2} A\left[\alpha_S\left(\rho^2\right)\right] + \frac{1}{2} B\left[\alpha_S\left(uQ^2\right)\right]\right\},$$

Antenna pattern

Ignoring the angular ordering does not necessarily lead to an incorrect result for an inclusive quantity.

If event shape is not sensitive to the structure of gluon subjets, then one can assemble final partons into the gluon subjets radiated off the primary $q\bar{q}$ line and consider this radiation as a sequence of QED-type soft independent emissions (*Parisi, Petronzio, 1979*)

Antenna pattern

$$R(\tau) = \sum_n \frac{1}{n!} \int \mathrm{d}\Phi_n \prod_i^n W(k_i)\, \Theta\left(Q\tau - \sum_{i\in \mathrm{R}} k_i \cdot n - \sum_{i\in \mathrm{L}} k_i \cdot n_+\right),$$

$$W(k_i) = \frac{\alpha_S C_{\mathrm{F}}}{\pi} \left(\frac{n}{n\cdot k_i} - \frac{n_+}{n_+\cdot k_i}\right)^2$$

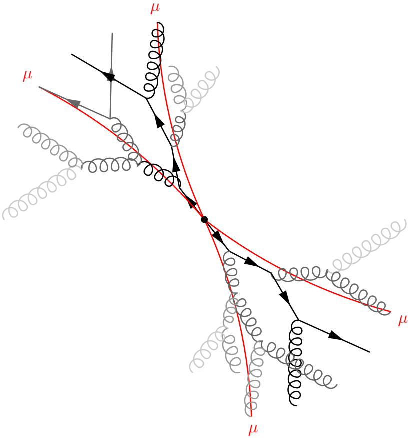
$$R(\tau) = \int_C \frac{\mathrm{d}\nu}{2\pi i\nu} \exp\left[\nu Q\tau + 2\mathscr{R}(\nu, Q)\right],$$

- α_S should be used in the so-called Monte-Carlo scheme
- some part of collinear radiation has to be taken into account *a posteriori* (rescaling with $r = \exp(3/4)$)

$$\mathscr{R}(\nu, Q) = -C_{\mathrm{F}} \int_{1/(e^{\gamma_E}\tilde{\nu})}^{rQ} \frac{d\beta}{\beta} \int_{\beta}^{rQ} \frac{d\alpha}{\alpha} \frac{\alpha_S^{\mathrm{MC}}(\alpha\beta)}{\pi}.$$

This approach is used in calculations of such event shape variables as the three-jet aplanarity or the D-parameter (*Banfi, Dokshitzer, Marchesini, Zanderighi, 2001*)

Inter- and intra-jet radiation



Antenna approach is not without merit because, in contrast to the angular-ordered branchings of partons, which can be referred as an *intra*-jet radiation, it claims to be the correct description of a coherent *inter*-jet radiation, which plays a crucial role for the aplanarity or D-parameter distributions

Interpretation of SCET

In fact, SCET carefully separates *intra*- and *inter*-jet radiation

$$F(\tau) = H\left(Q^2,\mu^2\right)\int \mathrm{d}p_{\mathrm{L}}^2\mathrm{d}p_{\mathrm{R}}^2\mathrm{d}k\, J\left(p_{\mathrm{L}}^2,\mu^2\right)J\left(p_{\mathrm{R}}^2,\mu^2\right)S_T\left(k,\mu^2\right)\Theta\left(Q^2\tau - p_{\mathrm{L}}^2 - p_{\mathrm{R}}^2 - Qk\right).$$

$$P_{\mathrm{L}}^2 = (p_{\mathrm{L}}+k_{\mathrm{L}})^2 = p_{\mathrm{L}}^2 + Qn\cdot k_{\mathrm{L}} + O\left(\lambda^3\right),$$

therefore

$$P_{\mathrm{L}}^2 + P_{\mathrm{R}}^2 = p_{\mathrm{L}}^2 + p_{\mathrm{R}}^2 + kQ + O\left(\lambda^3\right), \quad \text{where} \quad k = n\cdot k_{\mathrm{L}} + n_+\cdot k_{\mathrm{R}}.$$

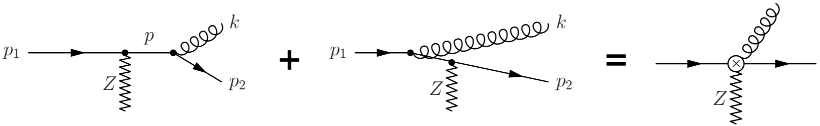
- If $\mu^2 = s^2Q^2 \sim \tau^2Q^2 \sim kQ$, we exclude soft region

$$\tilde{J}^{\mathrm{CTTW}}\left(\nu,Q^2\right) = \tilde{H}^{1/2}\left(Q^2,s^2Q^2\right)\tilde{j}\left(sQ^2,s^2Q^2\right)$$

- If $\mu^2 = sQ^2 \sim \tau Q^2 \sim p_{\mathrm{L}}^2 \sim p_{\mathrm{R}}^2$, we exclude collinear region

$$\mathscr{R}(\nu Q,Q) = \frac{1}{2}\ln H\left(Q^2,sQ^2\right)s_T\left(sQ^2,sQ^2\right)$$

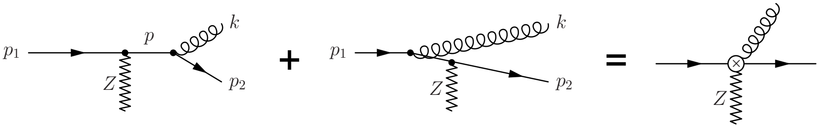
Tree-level local operator



$$(\not{p} - \not{k}) \left[\hat{V}^{\mu}_{(1)} + \hat{V}^{\mu}_{(2)} \right] q_n \Big|_{\lambda^0 + \lambda^1} = (\not{p} - \not{k}) \frac{2 g_S t^a}{Q} \left[\left(g^{\mu\nu} - \frac{k^{\mu} n^{\nu} + k^{\nu} n^{\mu}}{k \cdot n + i0} \right) \gamma_{\perp\nu} \right] q_n + \dots$$

$$\int \mathrm{d}^{\mathscr{D}} x \, e^{-ikx} \left\langle \mathrm{T} A^{(a)\mu}(x) A^{(b)\mu}(0) \right\rangle = -i \frac{\delta^{ab}}{k^2 + i0} \left(g^{\mu\nu} - \frac{k^{\mu} n^{\nu} + k^{\nu} n^{\mu}}{k \cdot n + i0} \right),$$

Tree-level local operator

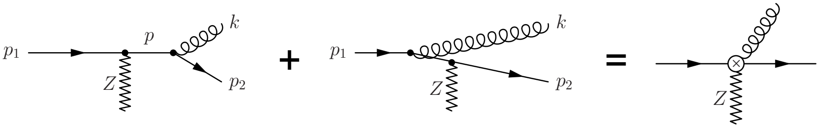


$$(\not{p}-\not{k})\left[\hat{V}^{\mu}_{(1)}+\hat{V}^{\mu}_{(2)}\right]q_n\Big|_{\lambda^0+\lambda^1}=(\not{p}-\not{k})\frac{2g_S t^a}{Q}\left[\left(g^{\mu\nu}-\frac{k^{\mu}n^{\nu}+k^{\nu}n^{\mu}}{k\cdot n+i0}\right)\gamma_{\perp\nu}\right]q_n+\dots$$

In LC gauge the effective vertex takes a particular simple form:

$$\hat{V}^{\mu}_{\text{eff}}=\hat{V}^{\mu}_{(1)}+\hat{V}^{\mu}_{(2)}=\frac{2g_S}{Q}t^a\gamma^{\mu}_{\perp},$$

Tree-level local operator



$$(\not{p} - \not{k})\left[\hat{V}^{\mu}_{(1)} + \hat{V}^{\mu}_{(2)}\right] q_n\Big|_{\lambda^0+\lambda^1} = (\not{p} - \not{k})\,\frac{2g_S t^a}{Q}\left[\left(g^{\mu\nu} - \frac{k^{\mu}n^{\nu} + k^{\nu}n^{\mu}}{k\cdot n + i0}\right)\gamma_{\perp\nu}\right] q_n + \dots$$

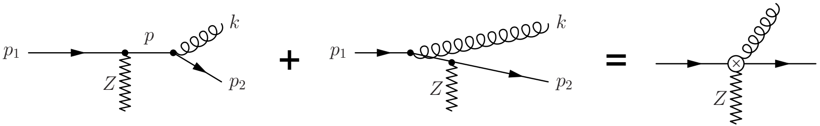
In LC gauge the effective vertex takes a particular simple form:

$$\hat{V}^{\mu}_{\text{eff}} = \hat{V}^{\mu}_{(1)} + \hat{V}^{\mu}_{(2)} = \frac{2g_S}{Q} t^a \gamma^{\mu}_{\perp},$$

Contributions to the thrust distribution:

$$G^{(0)}(\tau) = \frac{1}{2N_c}\int_0^{\tau Q^2} \mathrm{d}p_{\mathrm{R}}^2 \int \left|\bar{q}_{n_+}\hat{A}_{\perp} q_n\right|^2 \frac{\mathrm{d}\rho_3}{\mathrm{d}p_R^2} = C_{\mathrm{F}}\,\frac{\alpha_S}{\pi}\,\tau.$$

Tree-level local operator



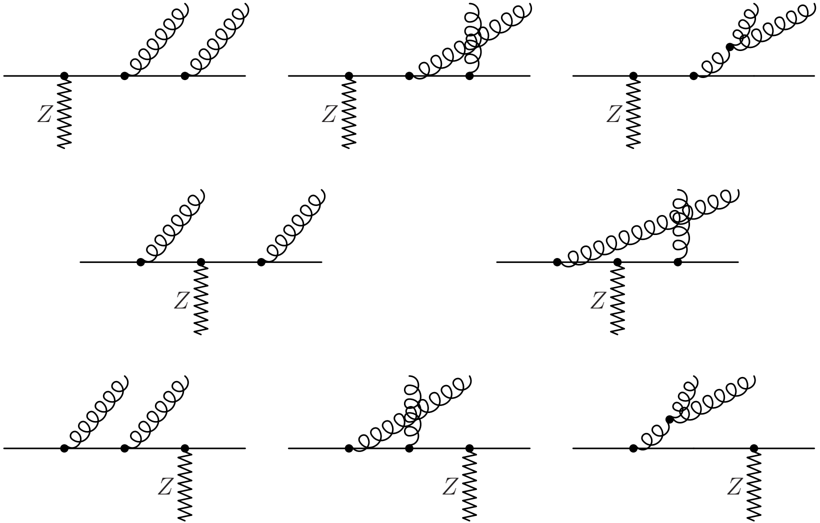
$$(\not p - \not k)\left[\hat{V}^{\mu}_{(1)} + \hat{V}^{\mu}_{(2)}\right] q_n\Big|_{\lambda^0+\lambda^1} = (\not p - \not k)\,\frac{2g_S t^a}{Q}\left[\left(g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n + i0}\right)\gamma_{\perp\nu}\right] q_n + \dots$$

In LC gauge the effective vertex takes a particular simple form:

$$\hat{V}^{\mu}_{\text{eff}} = \hat{V}^{\mu}_{(1)} + \hat{V}^{\mu}_{(2)} = \frac{2g_S}{Q} t^a \gamma^{\mu}_{\perp},$$

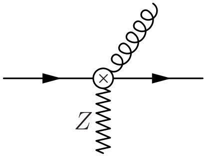
The main feature of the effective vertex is that it is *local* and therefore the expanded amplitude can be considered as a matrix element of a local operator with two r-collinear and one l-collinear particles in the final state.

Beyond tree level



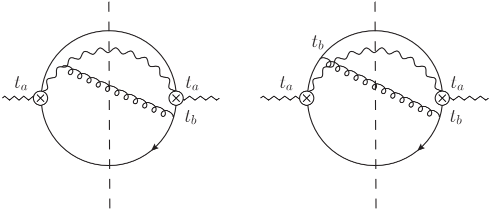
Beyond tree level

1. r-collinear–r-collinear or l-collinear–l-collinear
2. r-collinear–l-collinear



Beyond tree level

1. r-collinear–r-collinear or l-collinear–l-collinear
2. r-collinear–l-collinear
3. collinear-soft

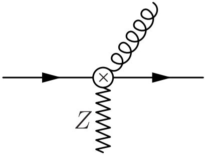


$$if^{abc}t^c \frac{e(k_{\text{soft}})\cdot n_+}{k_{\text{soft}}\cdot n_+} + t^b t^a \frac{e(k_{\text{soft}})\cdot n_+}{k_{\text{soft}}\cdot n_+} = t^a t^b \frac{e(k_{\text{soft}})\cdot n_+}{k_{\text{soft}}\cdot n_+}$$

$$|\mathcal{M}|^2 = |\mathcal{M}_0|^2 \times \frac{\alpha_S C_F}{\pi}\left(\frac{n}{n\cdot k} - \frac{n_+}{n_+\cdot k}\right)^2$$

Local operator in SCET

$$\mathcal{O}_3 = \mathcal{O}_{3R} + \mathcal{O}_{3L},$$
$$\mathcal{O}_{3R} = 2g_S\bar{\xi}_{n_+}\hat{A}_{\perp,n_+}\xi_n, \qquad \mathcal{O}_{3L} = 2g_S\bar{\xi}_{n_+}\hat{A}_{\perp,n}\xi_n,$$



Arbitrary gauge (*Beneke, Feldman (2003), Bauer et al. (2002)*):

$$\xi = YW^\dagger\xi', \qquad g_S A_\perp = Y\left(W^\dagger iD'_{\perp c}W - i\partial_\perp\right)Y^\dagger,$$

Wilson lines:

$$W_{n_+}(x) = \mathrm{P}\exp\left[ig_S\int_0^\infty \mathrm{d}s\, n\cdot A'_c(x+sn)\right], \qquad Y_{n_+}(x) = \mathrm{P}\exp\left[ig_S\int_0^\infty \mathrm{d}s\, n\cdot A'_s(x+sn)\right]$$

SCET operator:

$$\mathcal{O}_3 = 2g_S\bar{\xi}'_{n_+}\widetilde{A}_{\perp,n_+}W_{n_+}Y^\dagger_{n_+}Y_nW^\dagger_n\xi'_n + 2g_S\bar{\xi}'_{n_+}W_{n_+}Y^\dagger_{n_+}Y_nW_n\tilde{A}_{\perp,n}\xi'_n,$$

where

$$\tilde{A}_\perp = A'_\perp - \frac{i}{g_S}W\left[\partial_\perp, W^\dagger\right].$$

Local operator in SCET

We are under the conditions of: *Bauer, Fleming, Lee, and Sterman (2008)*

$$G(\tau) = 2H_3\left(Q^2,\mu^2\right)\int \mathrm{d}p_{\mathrm{L}}^2\mathrm{d}p_{\mathrm{R}}^2\mathrm{d}k \times \Sigma_\perp\left(p_{\mathrm{R}}^2,\mu^2\right)J\left(p_{\mathrm{L}}^2,\mu^2\right)S_T\left(k,\mu^2\right)\Theta\left(Q^2\tau - p_{\mathrm{L}}^2 - p_{\mathrm{R}}^2 - Qk\right)$$

Local operator in SCET

We are under the conditions of: *Bauer, Fleming, Lee, and Sterman (2008)*

$$G(\tau) = 2H_3\left(Q^2,\mu^2\right)\int \mathrm{d}p_\mathrm{L}^2\mathrm{d}p_\mathrm{R}^2\mathrm{d}k$$
$$\times\,\Sigma_\perp\left(p_\mathrm{R}^2,\mu^2\right)J\left(p_\mathrm{L}^2,\mu^2\right)S_T\left(k,\mu^2\right)\Theta\left(Q^2\tau - p_\mathrm{L}^2 - p_\mathrm{R}^2 - Qk\right)$$

- where $S_T\left(k,\mu^2\right)$ is the same soft factor
- $J\left(p_\mathrm{L}^2,\mu^2\right)$ is the jet function
- H_3 is the square of the hard matching coefficient of the QCD operator $(n-n_+)^\mu j_\mu/2$ onto SCET operator $\mathscr{O}_3$.

Local operator in SCET

We are under the conditions of: *Bauer, Fleming, Lee, and Sterman (2008)*

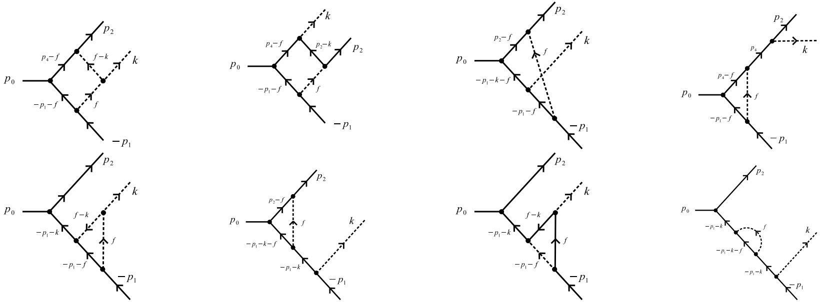
$$G(\tau) = 2H_3\left(Q^2,\mu^2\right)\int \mathrm{d}p_\mathrm{L}^2\mathrm{d}p_\mathrm{R}^2\mathrm{d}k \times \Sigma_\perp\left(p_\mathrm{R}^2,\mu^2\right)J\left(p_\mathrm{L}^2,\mu^2\right)S_T\left(k,\mu^2\right)\Theta\left(Q^2\tau - p_\mathrm{L}^2 - p_\mathrm{R}^2 - Qk\right)$$

New object:

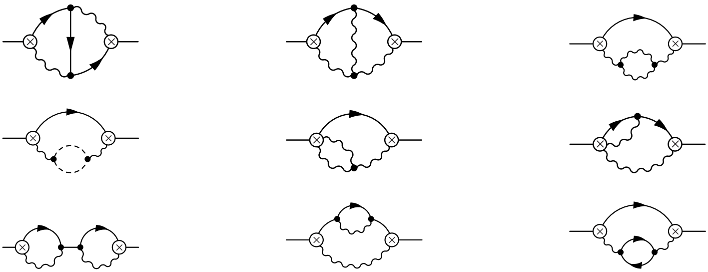
$$\Sigma_\perp\left(p^2,\mu^2\right) = \frac{g_S^2}{(p\cdot n)\,Q^2 N_\mathrm{c}} \times \frac{1}{\pi}\,\mathrm{Im}\left[i\int \mathrm{d}^{\mathscr{D}}x e^{-ipx}\left\langle 0\middle|\mathrm{T}\left\{\left(\bar{\xi}'_{n_+}\widetilde{A}_{\perp,n_+}W_{n_+}\right)(x)\,\frac{\hat{n}}{2}\left(W^\dagger_{n_+}\widetilde{A}_{\perp,n_+}\xi'_{n_+}\right)(0)\right\}\middle|0\right\rangle\right],$$

$$\Sigma_\perp^{(0)}\left(p_\mathrm{R}^2,\mu^2\right) = \frac{\alpha_S\left(\mu^2\right)C_\mathrm{F}}{4\pi Q^2}\left(\frac{p_\mathrm{R}^2}{4\pi\mu^2}\right)^{\mathscr{D}/2-2}\frac{(\mathscr{D}-2)\|\mathscr{D}/2-1\|}{\|\mathscr{D}-2\|}.$$

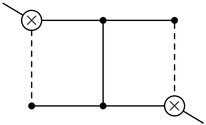
Fixed order: Hard coefficient



Fixed order: Transverse self energy



Master in main topology



$$J(1,1,1,1,1,1,1) = \frac{8\pi}{\sin \pi d}\left[\frac{\|2-d/2\|\|d/2-1\|^3}{\|d-1\|^2}\cos\frac{\pi d}{2}\,{}_3F_2\left(\begin{array}{c} 1,1,d/2-1 \\ d-1,d/2 \end{array}\middle|\, 1\right)\right.$$
$$-\frac{\|d/2\|\|d/2-1\|}{\|d-1\|\,(d-2)}\sum_{n=0}^{\infty}\frac{\|n+1\|}{\|d+n-1\|}\left[{}_3F_2\left(\begin{array}{c} d-2,d-2,d/2-1 \\ 2d-4,d+n-1 \end{array}\middle|\, 1\right)\right.$$
$$\left.\left. + \frac{d/2-1}{d+n-1}\,{}_3F_2\left(\begin{array}{c} d-2,d-2,d/2 \\ 2d-4,d+n \end{array}\middle|\, 1\right)\right]\right],$$

where $d = \mathscr{D} - 2$ and $\|x\| = \Gamma(x)$.

Perturbative corrections

$$H_3\left(Q^2,\mu^2\right) = 1 + \frac{\alpha_S}{4\pi} H_3^{(1)}\left(Q^2,\mu^2\right),$$

$$\int_0^{p^2} \mathrm{d}p_{\mathrm{L}}^2\, J\left(p_{\mathrm{L}}^2,\mu^2\right) = 1 + \frac{\alpha_S}{4\pi} J_{\mathrm{int}}^{(1)}\left(p^2,\mu^2\right),$$

$$\Sigma_\perp\left(p_{\mathrm{R}}^2,\mu^2\right) = \Sigma_\perp^{(0)}\left(p_{\mathrm{R}}^2,\mu^2\right)\left[1 + \frac{\alpha_S}{4\pi}\Sigma_\perp^{(1)}\left(p_{\mathrm{R}}^2,\mu^2\right)\right],$$

$$\int_0^{p} \mathrm{d}k\, S\left(k,\mu^2\right) = 1 + \frac{\alpha_S}{4\pi} S_{\mathrm{int}}^{(1)}\left(p^2,\mu^2\right),$$

Perturbative corrections

$$H_3^{(1)}\left(Q^2,\mu^2\right)=\left(\frac{Q^2}{\mu^2}\right)^{-\varepsilon}\left\{2C_{\mathrm{F}}\left[-\frac{2}{\varepsilon^2}+\frac{1}{\varepsilon}\left(3-\frac{2\pi^2}{3}\right)+\frac{\pi^2}{2}+17-16\zeta\left(3\right)\right]\right.$$
$$\left.+\,C_{\mathrm{A}}\left[\frac{1}{\varepsilon}\left(\frac{2\pi^2}{3}-4\right)-16+\frac{2\pi^2}{3}+16\zeta\left(3\right)\right]+O\left(\varepsilon\right)\right\}.$$

$$\Sigma_{\perp}^{(1)}\left(p_{\mathrm{R}}^2,\mu^2\right)=\left(\frac{p_{\mathrm{R}}^2}{\mu^2}\right)^{-\varepsilon}\left\{2C_{\mathrm{F}}\left[\frac{2}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\frac{2\pi^2}{3}-\frac{9}{2}\right)-\frac{5\pi^2}{6}-\frac{85}{4}+22\zeta(3)\right]\right.$$
$$\left.+C_{\mathrm{A}}\left[\frac{1}{\varepsilon}\left(\frac{23}{3}-\frac{2\pi^2}{3}\right)+\frac{503}{18}-22\zeta(3)\right]-2T_{\mathrm{F}}N_{\mathrm{f}}\left(\frac{2}{3\varepsilon}+\frac{19}{9}\right)+O\left(\varepsilon\right)\right\}.$$

$$J_{\mathrm{int}}^{(1)}\left(p^2,\mu^2\right)=\frac{\alpha_S}{4\pi}C_{\mathrm{F}}\left(\frac{p^2}{\mu^2}\right)^{-\varepsilon}\left(\frac{4}{\varepsilon^2}+\frac{3}{\varepsilon}+7-\pi^2+O\left(\varepsilon\right)\right),$$
$$S_{\mathrm{int}}^{(1)}\left(p^2,\mu^2\right)=\frac{\alpha_S}{4\pi}C_{\mathrm{F}}\left(\frac{p^2}{\mu^2}\right)^{-\varepsilon}\left(-\frac{4}{\varepsilon^2}+\frac{\pi^2}{3}+O\left(\varepsilon\right)\right).$$

Correction to distribution

$$G(\tau) = G^{(0)}(\tau)\left(1 + \frac{\alpha_S}{4\pi} G^{(1)}(\tau)\right),$$

where $G^{(1)}(\tau)$ can be presented as follows:

$$G^{(1)}(\tau) = -\frac{\beta_0}{\varepsilon} + H_3^{(1)}\left(Q^2, \mu^2\right) + \frac{1}{\tau Q^2}\left[\int_0^{\tau Q^2} \Sigma_\perp^{(1)}\left(p_R^2, \mu^2\right) dp_R^2 + \int_0^{\tau Q^2} J_{\text{int}}^{(1)}\left(p^2, \mu^2\right) dp^2 + \int_0^{(\tau Q)^2} S_{\text{int}}^{(1)}\left(p^2, \mu^2\right) dp^2\right]$$

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$$+ C_{\mathrm{F}}\left[-\frac{31}{2} + 12\zeta(3)\right] + C_{\mathrm{A}}\left[\frac{353}{18} - 6\zeta(3)\right] - \frac{50}{9} T_{\mathrm{F}} N_{\mathrm{f}},$$

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Resummation of large logs

$$G(\tau) = 2H_3\left(Q^2,\mu^2\right)\frac{1}{2\pi i}\int_C \frac{\mathrm{d}\nu}{\nu}\,\tilde{\Sigma}_\perp\left(sQ^2,\mu^2\right)j\left(sQ^2,\mu^2\right)s_T\left(sQ,\mu^2\right).$$

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We put $\mu^2 = \tau Q^2$:

$$\begin{aligned}\tilde{\Sigma}_\perp\left(sQ^2,\tau Q^2\right) &\equiv \int_0^\infty \mathrm{d}p_\mathrm{R}^2 e^{-\nu p_\mathrm{R}^2}\Sigma_\perp^{(0)}\left(p_\mathrm{R}^2,\tau Q^2\right) \\ &= \frac{\alpha_S\left(\tau Q^2\right)C_\mathrm{F}}{2\pi}\frac{1}{\nu Q^2}\left[1+\mathscr{C}_\Sigma\alpha_S\left(Q^2\right)\right].\end{aligned}$$

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$$\begin{aligned}G(\tau) = {} & [1+(\mathscr{C}_\Sigma+\mathscr{C}_J)\alpha_S]\frac{\alpha_S\left(\tau Q^2\right)C_\mathrm{F}}{\pi} \\ & \times H_3\left(Q^2,\tau Q^2\right)\frac{1}{2\pi i}\int_C \frac{\mathrm{d}\nu}{\nu^2 Q^2}s_T\left(sQ,\tau Q^2\right).\end{aligned}$$

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Ratio of the distributions

$$\frac{G(\tau)}{F(\tau)} = G^{(0)}(\tau)\, e^{\omega(\tau)}, \qquad G^{(0)} = \frac{\alpha_S}{\pi} C_{\mathrm{F}}\, \tau$$

where

$$\omega(\tau) = \frac{\gamma_0^{H_3} - \gamma_0^{H_2} - \beta_0}{\beta_0} \ln(1-\lambda) - \ln[1-\gamma(\lambda)] + \alpha_S\left(Q^2\right)(\mathscr{C}_3 - \mathscr{C}_2),$$

$$\gamma(\lambda) = \frac{\Gamma_0}{\beta_0}\left[\ln(1-2\lambda) - \ln(1-\lambda)\right], \qquad \lambda = \frac{\beta_0 \alpha_S(Q^2)}{4\pi} \ln\frac{1}{\tau}$$

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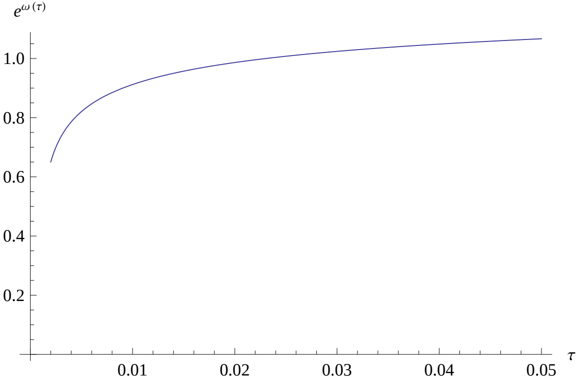
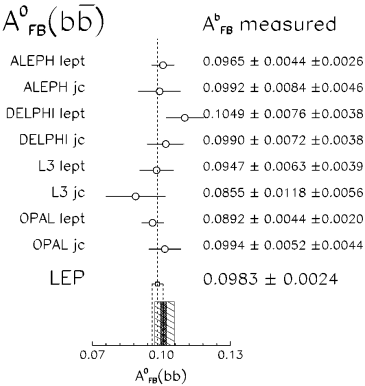


Experimental result

A^b_{FB} is measured by LEP2

Martinez et al. (1999)

$$A^{(0)}_{FB} = \frac{\int_0^{\pi/2} \mathrm{d}\theta\, w(\theta) - \int_{\pi/2}^{\pi} \mathrm{d}\theta\, w(\theta)}{\int_0^{\pi} \mathrm{d}\theta\, w(\theta)} = \frac{3}{4} \frac{2 g_{al} g_{vl}}{(g_{al}^2 + g_{vl}^2)} \frac{2 g_{aq} g_{vq}}{(g_{aq}^2 + g_{vq}^2)}.$$



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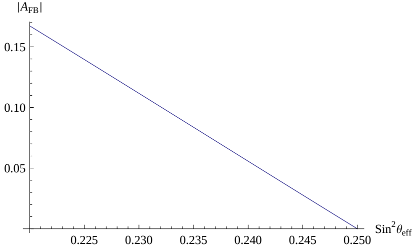
$$\left| \frac{\mathrm{d} A^{(0)}_{FB}}{\mathrm{d} \sin^2 \theta_{\mathrm{eff}}} \right| \approx 5.6$$



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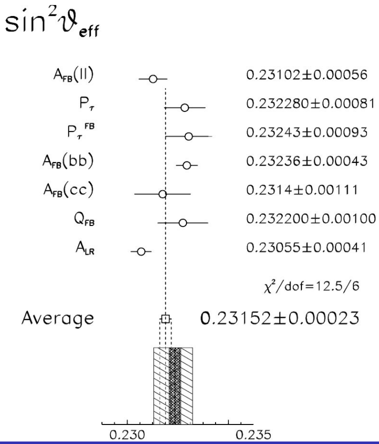
$$\frac{\sigma\left(\sin^2\theta_{\mathrm{eff}}\right)}{\sin^2\theta_{\mathrm{eff}}} = 1.8\times 10^{-3}$$

Martinez et al. (1999)



Tree level

The experimental cuts bias the theoretical corrections (*Abbaneo et al. (1997)* e.g. momentum cut in lepton tagging). The event shape can also been used to select the events (*Djouadi et al. (1990)*)

$$A(\tau) = A^{(0)} \frac{K(\tau)}{F(\tau) + G(\tau)},$$

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$$F_{\text{tree}}(\tau) - K_{\text{tree}}(\tau) = \frac{\alpha_S}{4\pi} C_{\text{F}} \left\{ -\frac{2\pi^2}{3} + \frac{\tau\left(12\tau^2 + 17\tau - 45\right)}{\tau - 1} + \left(\frac{5}{2} - 8\ln 2 - 4\tau - 2\tau^2\right) \right.$$
$$\left. \times \ln(1-2\tau) + 2\tau(\tau+2)\ln\tau + 8\ln(1-\tau)\left[\ln\tau - \ln(1-2\tau) + 6\right] + 8\left[\text{Li}_2(\tau) - \text{Li}_2(2\tau - 1)\right] \right\}$$

$$G_{\text{tree}}(\tau) = \frac{\alpha_S}{\pi} C_{\text{F}} \left\{ \tau - 4\left[\frac{\tau(2-\tau)}{1-\tau} + 2\ln(1-\tau)\right] \right\}$$

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$$G_{\text{tree}}(\tau) = G^{(0)}(\tau) + O\left(\tau^2\right) = \frac{\alpha_S}{\pi} C_{\text{F}} \left[\tau + O\left(\tau^2\right)\right].$$

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$$F\left(\frac{1}{3}\right) - K\left(\frac{1}{3}\right) + G\left(\frac{1}{3}\right)$$
$$= \frac{\alpha_S}{\pi} C_{\mathrm{F}} \left[\frac{\pi^2}{6} - \frac{9}{4} - \ln^2\frac{3}{2} + (2+\ln 2)^2 - \frac{37}{8}\ln 3 - 2\,\mathrm{Li}_2\left(\frac{1}{3}\right)\right] \approx \frac{\alpha_S}{\pi} 0.89$$

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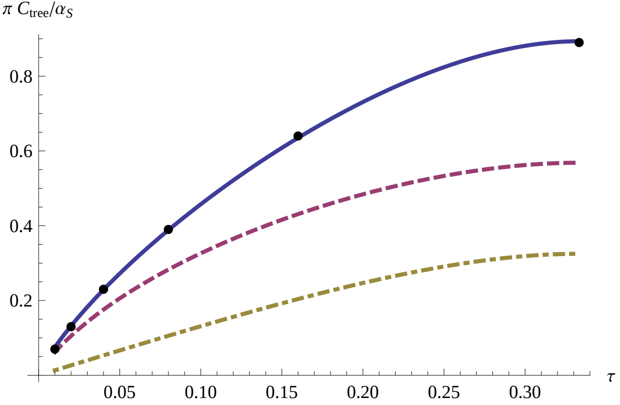

Tree level

In the region $0.03 \lesssim \tau \lesssim 0.07$, $0.2 \lesssim \frac{\pi}{\alpha_s} C_{\rm tree}(\tau) \lesssim 0.4$, thereby simulating the real experimental cuts.

$$C(\tau) = \frac{F(\tau) - K(\tau) + G(\tau)}{F(\tau) + G(\tau)} \approx \frac{F(\tau) - K(\tau)}{F(\tau)} + \frac{G(\tau)}{F(\tau)}.$$

$$A^{(\rm obs)} = A^{(\rm corr)} \left[1 - C(\tau)\right].$$

$$A^{(\rm corr)}_{\rm imp}(\tau) = A^{(\rm corr)}_{\rm tree}(\tau) \frac{1 - C_{\rm tree}(\tau)}{1 - C_{\rm imp}(\tau)} \approx A^{(\rm corr)}_{\rm tree}(\tau) \left\{1 + G^{(0)}(\tau) \left[e^{\omega(\tau)} - 1\right]\right\}.$$

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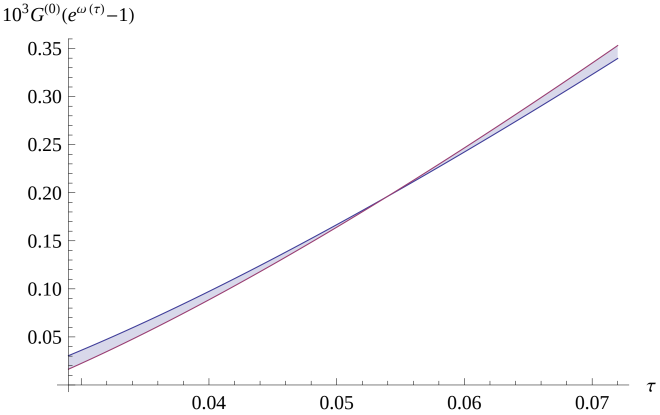

Conclusion

- If the angular distributions $1+\cos^2\theta$, $\sin^2\theta$ and $\cos\theta$ are measured independently, one finds "jets" with different internal structure.
- SCET is the relevant framework to establish factorization formulae and perform resummation
- If one takes into account tree-level distributions correctly, the effect of resummation is absolutely negligible to the present level of experimental accuracy.