Resonances and Unitarity in Weak Boson Scattering at the LHC

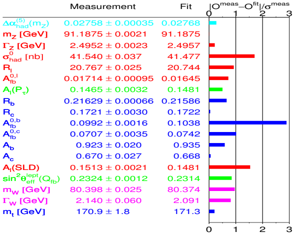
Jürgen Reuter

Albert-Ludwigs-Universität Freiburg

Alboteanu/Kilian/JR, arXiv:0806.4145 (**JHEP**); M. Mertens, 2005;
Kilian/Kobel/Mader/JR/Schumacher, work in progress;
Beyer/Kilian/Krstonošić/Mönig/JR/Schmitt/Schröder, **EPJC 48** (2006), 353 [ILC version]

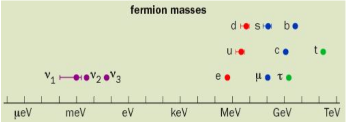
Seminar, PSI, January 22, 2009

Doubts on the Standardmodel

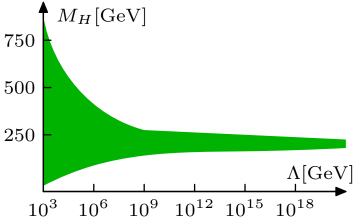


– describes microcosm (too good?)

– 28 free parameters



– Higgs ?, form of Higgs potential ?



Hierarchy Problem

chiral symmetry: $\delta m_f \propto v \ln(\Lambda^2/v^2)$

no symmetry for quantum corrections to Higgs mass

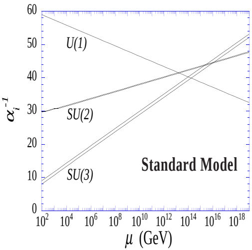
$$\delta M_H^2 \propto \Lambda^2 \sim M_{\text{Planck}}^2 = (10^{19})^2\,\text{GeV}^2$$

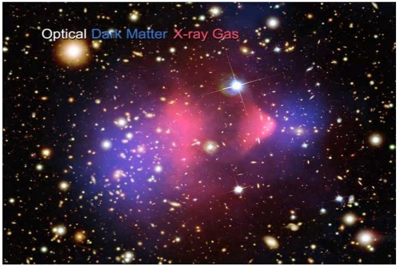
20000 GeV^2 = (1000000000000000000000000000000000020000 –

1000000000000000000000000000000000000000) GeV^2

Open Questions

- Unification of all interactions (?)
- Baryon asymmetrie $\Delta N_B - \Delta N_{\bar{B}} \sim 10^{-9}$
  missing CP violation
- Flavour: three generations
- Tiny neutrino masses: $m_\nu \sim \frac{v^2}{M}$
- Dark Matter:
  - stable
  - only weakly interacting
  - $m_{DM} \sim 100$ GeV
- Quantum theory of gravity
- Cosmic inflation
- Cosmological constant





Ideas for New Physics since 1970

Model-Independent Description of the EW sector

- Higgs boson still not observed
- Aim: describe any physics beyond the SM as generically as possible
- Implement what we know about the SM
- Implements $SU(2)_L \times U(1)_Y$ gauge invariance
- Building blocks (including longitudinal modes):

$$\psi \text{ (SM fermions)}, \quad W_\mu^a \; (a = 1,2,3), \quad B_\mu, \quad \Sigma = \exp\left[\frac{-i}{v} w^a \tau^a\right]$$

- Minimal Lagrangian including gauge interactions

$$\mathcal{L}_{\text{min}} = \sum_\psi \overline{\psi}(i\not{D})\psi - \frac{1}{2g^2}\operatorname{tr}\left[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}\right] - \frac{1}{2g'^2}\operatorname{tr}\left[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}\right] + \frac{v^2}{4}\operatorname{tr}\left[(\mathbf{D}_\mu\Sigma)(\mathbf{D}^\mu\Sigma)\right]$$

The Fundamental Building Blocks

- $\mathbf{V} = \Sigma(\mathbf{D}\Sigma)^\dagger$ (longitudinal vectors), $\mathbf{T} = \Sigma\tau^3\Sigma^\dagger$ (neutral component)

- Unitary gauge (no Goldstones): $\mathbf{w} \equiv 0$, i.e., $\Sigma \equiv 1$.

$$\mathbf{V} \longrightarrow -\frac{\mathrm{i}g}{2}\left[\sqrt{2}(W^+\tau^+ + W^-\tau^-) + \frac{1}{c_\mathrm{w}} Z\tau^3\right]$$
$$\mathbf{T} \longrightarrow \tau^3$$

- Gaugeless limit (only Goldstones) $(g, g' \to 0)$:

$$\mathbf{V} \longrightarrow \frac{\mathrm{i}}{v}\left\{\sqrt{2}\partial w^+\tau^+ + \sqrt{2}\partial w^-\tau^- + \partial z\tau^3\right\} + O(v^{-2})$$
$$\mathbf{T} \longrightarrow \tau^3 + 2\sqrt{2}\frac{\mathrm{i}}{v}\left(w^+\tau^+ - w^-\tau^-\right) + O(v^{-2})$$

So $\mathbf{T}$ projects out the neutral part:

$$\mathrm{tr}\,[\mathbf{TV}] = \frac{2\mathrm{i}}{v}\left[\partial z + \frac{\mathrm{i}}{v}\left(w^+\partial w^- - w^-\partial w^+\right)\right] + O(v^{-3})$$

Electroweak Chiral Lagrangian

Complete Lagrangian contains infinitely many parameters

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{min}} - \sum_{\psi} \overline{\psi}_L \Sigma M \psi_R + \beta_1 \mathcal{L}'_0 + \sum_i \alpha_i \mathcal{L}_i + \frac{1}{v} \sum_i \alpha_i^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^2} \sum_i \alpha_i^{(6)} \mathcal{L}^{(6)} + \ldots$$

$$\mathcal{L}'_0 = \frac{v^2}{4} \text{tr}\,[\mathbf{T}\mathbf{V}_\mu]\,\text{tr}\,[\mathbf{T}\mathbf{V}^\mu]$$

$$\mathcal{L}_1 = \text{tr}\,[\mathbf{B}_{\mu\nu}\mathbf{W}^{\mu\nu}] \qquad \mathcal{L}_6 = \text{tr}\,[\mathbf{V}_\mu\mathbf{V}_\nu]\,\text{tr}\,[\mathbf{T}\mathbf{V}^\mu]\,\text{tr}\,[\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_2 = \text{i}\text{tr}\,[\mathbf{B}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]] \qquad \mathcal{L}_7 = \text{tr}\,[\mathbf{V}_\mu\mathbf{V}^\mu]\,\text{tr}\,[\mathbf{T}\mathbf{V}_\nu]\,\text{tr}\,[\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_3 = \text{i}\text{tr}\,[\mathbf{W}_{\mu\nu}[\mathbf{V}^\mu, \mathbf{V}^\nu]] \qquad \mathcal{L}_8 = \tfrac{1}{4}\text{tr}\,[\mathbf{T}\mathbf{W}_{\mu\nu}]\,\text{tr}\,[\mathbf{T}\mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_4 = \text{tr}\,[\mathbf{V}_\mu\mathbf{V}_\nu]\,\text{tr}\,[\mathbf{V}^\mu\mathbf{V}^\nu] \qquad \mathcal{L}_9 = \tfrac{\text{i}}{2}\text{tr}\,[\mathbf{T}\mathbf{W}_{\mu\nu}]\,\text{tr}\,[\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_5 = \text{tr}\,[\mathbf{V}_\mu\mathbf{V}^\mu]\,\text{tr}\,[\mathbf{V}_\nu\mathbf{V}^\nu] \qquad \mathcal{L}_{10} = \tfrac{1}{2}\left(\text{tr}\,[\mathbf{T}\mathbf{V}_\mu]\,\text{tr}\,[\mathbf{T}\mathbf{V}^\mu]\right)^2$$

Indirect info on new physics in $\beta_1, \alpha_i, \ldots$ (Flavor physics only in M)

Electroweak precision observables (LEP I/II, SLC):

$$\Delta S = -16\pi\alpha_1 \qquad \alpha_1 = 0.0026 \pm 0.0020$$

$$\Delta T = 2\beta_1/\alpha_{\text{QED}} \qquad \beta_1 = -0.00062 \pm 0.00043$$

$$\Delta U = -16\pi\alpha_8 \qquad \alpha_8 = -0.0044 \pm 0.0026$$

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{TGC} = \mathrm{ie}\left[g_1^\gamma A_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^\gamma W_\mu^- W_\nu^+ A^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ A^{\rho\mu}\right] + \mathrm{ie}\frac{c_\mathrm{w}}{s_\mathrm{w}}\left[g_1^Z Z_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu}\right) + \kappa^Z W_\mu^- W_\nu^+ Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_\mu^{-\nu} W_{\nu\rho}^+ Z^{\rho\mu}\right]$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2\alpha_1}{c_\mathrm{w}^2 - s_\mathrm{w}^2}$ $g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_\mathrm{w}^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_\mathrm{w}^2}\alpha_3 \qquad \Delta g_1^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2\alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_\mathrm{w}^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2c_\mathrm{w}^2\Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2\left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})\right]$$

Anomalous triple and quartic gauge couplings

$$\begin{aligned}\mathcal{L}_{QGC} = e^2 &\left[g_1^{\gamma\gamma} A^\mu A^\nu W_\mu^- W_\nu^+ - g_2^{\gamma\gamma} A^\mu A_\mu W^{-\nu} W_\nu^+\right] \\ &+ e^2 \frac{c_w}{s_w}\left[g_1^{\gamma Z} A^\mu Z^\nu \left(W_\mu^- W_\nu^+ + W_\mu^+ W_\nu^-\right) - 2 g_2^{\gamma Z} A^\mu Z_\mu W^{-\nu} W_\nu^+\right] \\ &+ e^2 \frac{c_w^2}{s_w^2}\left[g_1^{ZZ} Z^\mu Z^\nu W_\mu^- W_\nu^+ - g_2^{ZZ} Z^\mu Z_\mu W^{-\nu} W_\nu^+\right] \\ &+ \frac{e^2}{2 s_w^2}\left[g_1^{WW} W^{-\mu} W^{+\nu} W_\mu^- W_\nu^+ - g_2^{WW}\left(W^{-\mu} W_\mu^+\right)^2\right] + \frac{e^2}{4 s_w^2 c_w^4} h^{ZZ} (Z^\mu Z_\mu)^2\end{aligned}$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_w^2 - s_w^2}$ $\quad g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0 \qquad \Delta\kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2}\alpha_3 \qquad \Delta\kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2\alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 \qquad \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_w^2}\alpha_3 \qquad \Delta g_1^{WW} = 2 c_w^2 \Delta g_1^{\gamma Z} + 2 g^2(\alpha_9 - \alpha_8) + g^2 \alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4}(\alpha_4 + \alpha_6) \qquad \Delta g_2^{WW} = 2 c_w^2 \Delta g_1^{\gamma Z} + 2 g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2\left[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})\right]$$

Parameters and Scales, Resonances

α_i measurable at ILC

- $\alpha_i \ll 1$ (LEP)
- $\alpha_i \gtrsim 1/16\pi^2 \approx 0.006$ (renormalize divergencies, $16\pi^2\alpha_i \gtrsim 1$)

Translation of parameters into new physics scale Λ: $\alpha_i = v^2/\Lambda^2$

- Operator normalization is arbitrary
- Power counting can be intricate

To be specific: consider resonances that couple to EWSB sector
Resonance mass gives detectable shift in the α_i

- Narrow resonances $\Rightarrow$ particles
- Wide resonances $\Rightarrow$ continuum

$\beta_1 \ll 1 \Rightarrow SU(2)_c$ custodial symmetry (weak isospin, broken by hypercharge $g' \neq 0$ and fermion masses)

	$J=0$	$J=1$	$J=2$
$I=0$	σ^0 (Higgs ?)	ω^0 $(\gamma'/Z'\ ?)$	f^0 (Graviton ?)
$I=1$	$\pi^\pm, \pi^0$ (2HDM ?)	$\rho^\pm, \rho^0$ $(W'/Z'\ ?)$	$a^\pm, a^0$
$I=2$	$\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet ?)	—	$t^{\pm\pm}, t^\pm, t^0$

accounts for weakly and strongly interacting models

Model-Independent Way – Effective Field Theories

How to clearly separate effects of heavy degrees of freedom?

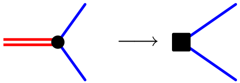
Toy model: Two interacting scalar fields φ, Φ

$$\mathcal{Z}[j,J] = \int \mathcal{D}[\Phi]\,\mathcal{D}[\varphi]\;\exp\left[i\int dx\left(\tfrac{1}{2}(\partial\varphi)^2 - \tfrac{1}{2}\Phi(\Box + M^2)\Phi - \lambda\varphi^2\Phi - \ldots + J\Phi + j\varphi\right)\right]$$

Low-energy effective theory $\Rightarrow$ integrating out heavy degrees of freedom (DOF) in path integrals, set up Power Counting

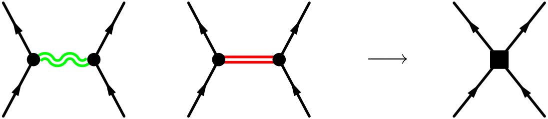
Completing the square:

$$\Phi' = \Phi + \frac{\lambda}{M^2}\left(1 + \frac{\partial^2}{M^2}\right)^{-1}\varphi^2 \Rightarrow$$

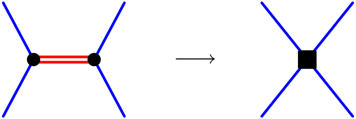


$$\frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}M^2\Phi^2 - \lambda\varphi^2\Phi = -\frac{1}{2}\Phi'(M^2 + \partial^2)\Phi' + \frac{\lambda^2}{2M^2}\varphi^2\left(1 + \frac{\partial^2}{M^2}\right)^{-1}\varphi^2.$$

Effective Dim. 6 Operators

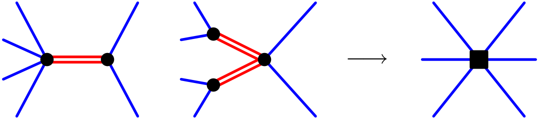


$$\mathcal{O}_{JJ}^{(I)} = \frac{1}{F^2} \mathrm{tr}\left[[]\, J^{(I)} \cdot J^{(I)}\right]$$

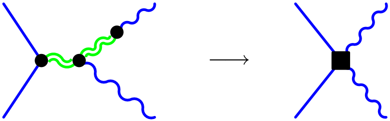


$$\mathcal{O}'_{h,1} = \frac{1}{F^2} \left((Dh)^\dagger h\right) \cdot \left(h^\dagger (D^h)\right) - \frac{v^2}{2} |Dh|^2$$

$$\mathcal{O}'_{hh} = \frac{1}{F^2} (h^\dagger h - v^2/2)\, (Dh)^\dagger \cdot (Dh)$$



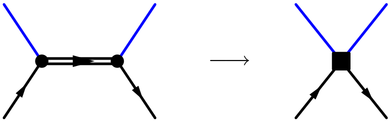
$$\mathcal{O}'_{h,3} = \frac{1}{F^2} \frac{1}{3} (h^\dagger h - v^2/2)^3$$



$$\mathcal{O}'_{WW} = -\frac{1}{F^2}\frac{1}{2}(h^\dagger h - v^2/2)\mathrm{tr}\left[W_{\mu\nu}W^{\mu\nu}\right]$$

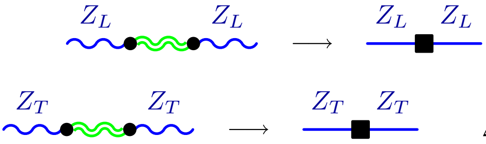
$$\mathcal{O}_B = \frac{1}{F^2}\frac{i}{2}(D_\mu h)^\dagger (D_\nu h) B^{\mu\nu}$$

$$\mathcal{O}'_{BB} = -\frac{1}{F^2}\frac{1}{4}(h^\dagger h - v^2/2)B_{\mu\nu}B^{\mu\nu}$$



$$\mathcal{O}_{Vq} = \frac{1}{F^2}\overline{q}h(\not{D}h)q$$

Oblique Corrections: S, T, U



$$\Delta T \sim \Delta\rho \sim \Delta M_Z^2 Z \cdot Z$$

$$\Delta S \sim {W^0}_{\mu\nu} B^{\mu\nu}, \Delta U \sim {W^0}_{\mu\nu} W^{0\,\mu\nu}$$

- ◇ All low-energy effects order v^2/F^2 (Wilson coefficients)
- ◇ Low-energy observables with low-energy input G_F, α, M_Z affected by **non-oblique** contributions:

$$G_F = \frac{1}{v} \longrightarrow \frac{1}{v}\left(1 - \alpha\Delta T + \delta\right),$$

$$\boxed{\delta \equiv = -\frac{v^2}{4} f_{JJ}^{(3)}}$$

$$S_{\text{eff}} = \Delta S$$
$$T_{\text{eff}} = \Delta T - \tfrac{1}{\alpha}\delta$$
$$U_{\text{eff}} = [\Delta U = 0] + \tfrac{4s_w^2}{\alpha}\delta$$

▶ non-oblique flavour-dependent corrections ⇒ enforce flavour-dependent EW fit

Integrating out resonances

Consider leading order effects of resonances on EW sector:

$$\mathcal{L}_\Phi = z\left[\Phi\left(M_\Phi^2 + DD\right)\Phi + 2\Phi J\right] \qquad \Rightarrow \qquad \mathcal{L}_\Phi^{\text{eff}} = -\frac{z}{M^2}JJ + \frac{z}{M^4}J(DD)J + \mathcal{O}(M^{-6})$$

- Simplest example: scalar singlet σ:

$$\mathcal{L}_\sigma = -\frac{1}{2}\left[\sigma(M_\sigma^2 + \partial^2)\sigma - g_\sigma v \sigma \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] - h_\sigma \mathrm{tr}\left[\mathbf{T}\mathbf{V}_\mu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\right]$$

- Effective Lagrangian

$$\mathcal{L}_\sigma^{\text{eff}} = \frac{v^2}{8M_\sigma^2}\left[g_\sigma \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] + h_\sigma \mathrm{tr}\left[\mathbf{T}\mathbf{V}_\mu\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right]\right]^2$$

- leads to anomalous quartic couplings

$$\alpha_5 = g_\sigma^2\left(\frac{v^2}{8M_\sigma^2}\right) \qquad \alpha_7 = 2g_\sigma h_\sigma\left(\frac{v^2}{8M_\sigma^2}\right) \qquad \alpha_{10} = 2h_\sigma^2\left(\frac{v^2}{8M_\sigma^2}\right)$$

- Special case: SM Higgs with $g_\sigma = 1$ and $h_\sigma = 0$

Coupl. strengths, Anomal. Couplings, Power Counting

Scalar resonance width ($M_\sigma \gg M_W, M_Z$):

$$\Gamma_\sigma = \frac{g_\sigma^2 + \frac{1}{2}(g_\sigma^2 + 2h_\sigma^2)^2}{16\pi}\left(\frac{M_\sigma^3}{v^2}\right) + \Gamma(\text{non} - WW, ZZ)$$

Largest allowed coupling for a broad continuum: $\Gamma \sim M \gg \Gamma(\text{non} - WW, ZZ) \sim 0$ translates to bounds for effective Lagrangian (e.g. scalar with no isospin violation):

$$\alpha_5 \leq \frac{4\pi}{3}\left(\frac{v^4}{M_\sigma^4}\right) \approx \frac{0.015}{(M_\sigma \text{ in TeV})^4} \qquad \Rightarrow \qquad 16\pi^2\alpha_5 \leq \frac{2.42}{(M_\sigma \text{ in TeV})^4}$$

Scalar:	$\Gamma \sim g^2 M^3,\ \alpha \sim g^2/M^2$	$\Rightarrow$	$\alpha_{\text{max}} \sim 1/M^4$
Vector:	$\Gamma \sim g^2 M,\ \alpha \sim g^2/M^2$	$\Rightarrow$	$\alpha_{\text{max}} \sim 1/M^2$
Tensor:	$\Gamma \sim g^2 M^3,\ \alpha \sim g^2/M^2$	$\Rightarrow$	$\alpha_{\text{max}} \sim 1/M^4$

Vector triplet (simplified)

$$\mathcal{L}_\rho = -\frac{1}{8}\text{tr}\left[\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu}\right] + \frac{M_\rho^2}{4}\text{tr}\left[\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu\right] + \frac{\mathrm{i}g_\rho v^2}{2}\text{tr}\left[\boldsymbol{\rho}_\mu\mathbf{V}^\mu\right]$$

$1/M^2$ term renormalizes kinetic energy (i.e. v), hence unobservable:

$$\mathcal{L}_\rho^{\text{eff}} = \frac{g_\rho^2 v^4}{4M_\rho^2}\text{tr}\left[(\mathbf{D}_\mu\Sigma)(\mathbf{D}^\mu\Sigma)\right] + \mathcal{O}(1/M_\rho^4)$$

Vector Resonances

$$
\begin{aligned}
\mathcal{L}_\rho = & -\frac{1}{8}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu}\right] + \frac{M_\rho^2}{4}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu\right] + \frac{\Delta M_\rho^2}{8}\left(\mathrm{tr}\left[\mathbf{T}\boldsymbol{\rho}_\mu\right]\right)^2 + \mathrm{i}\frac{\mu_\rho}{2}g\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{W}^{\mu\nu}\boldsymbol{\rho}_\nu\right] \\
& + \mathrm{i}\frac{\mu_\rho'}{2}g'\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{B}^{\mu\nu}\boldsymbol{\rho}_\nu\right] + \mathrm{i}\frac{g_\rho v^2}{2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{V}^\mu\right] + \mathrm{i}\frac{h_\rho v^2}{2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\mathbf{T}\right]\mathrm{tr}\left[\mathbf{T}\mathbf{V}^\mu\right] \\
& + \frac{g'v^2k_\rho}{2M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu[\mathbf{B}^{\nu\mu},\mathbf{V}_\nu]\right] + \frac{gv^2k_\rho'}{4M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu[\mathbf{T},\mathbf{V}_\nu]\right]\mathrm{tr}\left[\mathbf{T}\mathbf{W}^{\nu\mu}\right] \\
& + \frac{gv^2k_\rho''}{4M_\rho^2}\mathrm{tr}\left[\mathbf{T}\boldsymbol{\rho}_\mu\right]\mathrm{tr}\left[[\mathbf{T},\mathbf{V}_\nu]\mathbf{W}^{\nu\mu}\right] + \mathrm{i}\frac{\ell_\rho}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{W}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right] \\
& + \mathrm{i}\frac{\ell_\rho'}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{B}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right] + \mathrm{i}\frac{\ell_\rho''}{M_\rho^2}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\mathbf{T}\right]\mathrm{tr}\left[\mathbf{T}\mathbf{W}^\nu{}_\rho\mathbf{W}^{\rho\mu}\right]
\end{aligned}
$$

all $\alpha_i \sim 1/M_\rho^4$, except for $\beta_1 \sim \Delta\rho \sim T \sim h_\rho^2/M_\rho^2$

4-fermion contact interaction $j_\mu j^\mu \sim 1/M_\rho^2$ (eff. T and U parameter)

vector coupling $j_\mu V^\mu \sim 1/M_\rho^2$ (eff. S parameter)
Mismatch: measured fermionic vs. bosonic coupling g Nyffeler/Schenk, 2000; Kilian/JR, 2003

Effects on Triple Gauge Couplings

- $\mathcal{O}(1/M^2)$: Renormalization of ZWW coupling
- $\mathcal{O}(1/M^4)$: shifts in Δg_1^Z, $\Delta\kappa^\gamma$, $\Delta\kappa^Z$, λ^γ, λ^Z

Effects on Quartic Gauge Couplings

- $\mathcal{O}(1/M^4)$, orthogonal (in α_4–α_5 space) to scalar case

The Multi-Particle Generator WHIZARD

Kilian/Ohl/JR, 07

Matrix Element Generator O'Mega: Ω

Ohl, 2000/01; M.Moretti/Ohl/JR, 2001

Optimized helicity amplitudes: Avoiding all redundancies

Multi-Purpose Event Generator WHIZARD:

Ohl, 1996; Kilian, 2000; Kilian/Ohl/JR, 2007

- Adaptive Multi-Channel Monte-Carlo Integration
- Very high level of Complexity

- $e^+e^- \to t\bar{t}H \to b\bar{b}b\bar{b}jj\ell\nu$ (110,000 diagrams)
- $e^+e^- \to ZHH \to ZWWWW \to bb + 8j$ (12,000,000 diagrams)
- $pp \to \ell\ell + nj,\ n = 0, 1, 2, 3, 4, \ldots$ (2,100,000 diagrams with 4 jets + flavors)
- $pp \to \tilde{\chi}^0_1 \tilde{\chi}^0_1 bbbb$ (32,000 diagrams, 22 color flows, $\sim 10,000$ PS channels)
- $pp \to VVjj \to jj\ell\ell\nu\nu$ incl. anomalous TGC/QGC
- Test case $gg \to 9g$ (224,000,000 diagrams)

Current version:

WHIZARD 1.92 release date: 2008, April, 29th

one grand unified package (incl. VAMP, Circe, Circe 2, WHiZard, O'Mega)

New web address: `http://whizard.event-generator.org`

Standard Reference for 1.92 + new versions: Kilian/Ohl/JR, 0708.4233

- Major upgrade this fall (most code ready!!!): WHIZARD 2.0.0

Anomalous Gauge Couplings at LHC

ILC: Beyer/Kilian/Krstonošić/Mönig/JR/Schröder/Schmidt, 2006

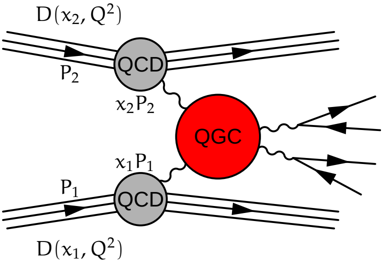
LHC: Mertens, 2006; Kilian/Kobel/Mader/JR/Schumacher

Anomalous quartic gauge couplings, by chiral EW Lagrangian:

$$\mathcal{L}_4 = \alpha_4 \frac{g^2}{2} \left\{ \left[(W^+W^+)(W^-W^-) + (W^+W^-)^2 \right] + \frac{2}{c_W^2}(W^+Z)(W^-Z) + \frac{1}{2c_W^4}(ZZ)^2 \right\}$$

$$\mathcal{L}_5 = \alpha_5 \frac{g^2}{2} \left\{ (W^+W^-)^2 + \frac{2}{c_W^2}(W^+W^-)(ZZ) + \frac{1}{2c_W^4}(ZZ)^2 \right\}$$

(all leptons, incl. τ):



$$pp \to jj(ZZ/WW) \to jj\ell^-\ell^+\nu_\ell\bar{\nu}_\ell$$

$$\sigma \approx 40\,\text{fb}$$

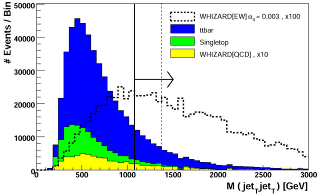
Background:

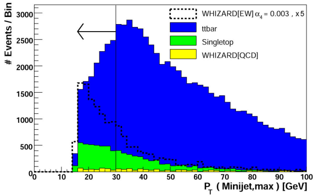
- $t\bar{t} \to WbWb$, $\sigma \approx 52$ pb
- Single t, misrec. jet: $\sigma \approx 4.8$ pb
- QCD: $\sigma \approx 0.21$ pb

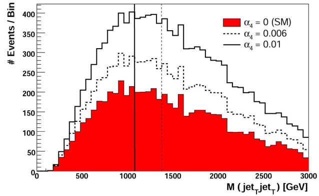
Tagging and Cuts:

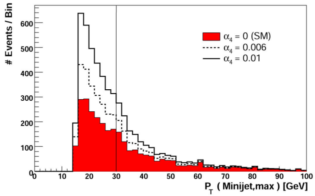
- $\ell\ell jj$-Tag, $\eta_{tag}^{min} < \eta_\ell < \eta_{tag}^{max}$, b-Veto
- $|\Delta\eta_{jj}| > 4.4$, $M_{jj} > 1080$ GeV
- Minijet-Veto: $p_{T,j} < 30$ GeV
- $E_j > 600, 400$ GeV, $p^1_{T,j} > 60, 24$ GeV

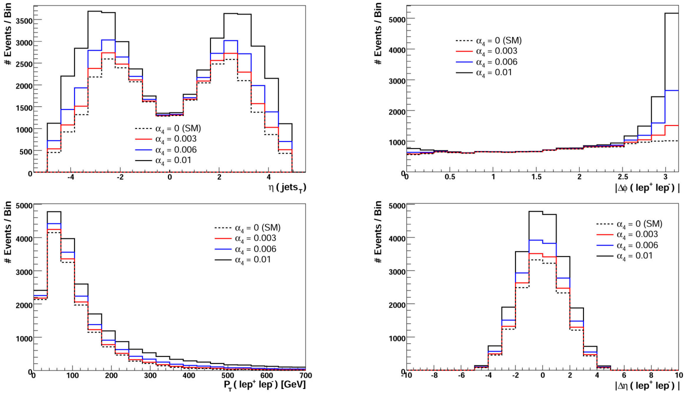
Improves $S/\sqrt{B}$ from 3.3 to 29.7











Results: (1σ Sensitivity to αs)

Coupl.	ILC ($1\,\text{ab}^{-1}$)	LHC ($100\,\text{fb}^{-1}$)
α_4	0.0088	0.00160
α_5	0.0071	0.00098

Limits for Λ [TeV]:

Spin	$I=0$	$I=1$	$I=2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

Isospin decomposition

- Lowest order chiral Lagrangian (incl. anomalous couplings)

$$\mathcal{L} = -\frac{v^2}{4}\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] + \alpha_4 \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}_\nu\right]\mathrm{tr}\left[\mathbf{V}^\mu \mathbf{V}^\nu\right] + \alpha_5 \left(\mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]\right)^2$$

- Leads to the following amplitudes: $s = (p_1 + p2)^2 \quad t = (p1 - p3)^2 \quad u = (p1 - p4)^2$

$$\boxed{\mathcal{A}(s,t,u) =:}$$

$$\begin{aligned}
\mathcal{A}(w^+ w^- \to zz) &= \frac{s}{v^2} + 8\alpha_5 \frac{s^2}{v^4} + 4\alpha_4 \frac{t^2+u^2}{v^4} \\
\mathcal{A}(w^+ z \to w^+ z) &= \frac{t}{v^2} + 8\alpha_5 \frac{t^2}{v^4} + 4\alpha_4 \frac{s^2+u^2}{v^4} \\
\mathcal{A}(w^+ w^- \to w^+ w^-) &= -\frac{u}{v^2} + (4\alpha_4 + 2\alpha_5)\frac{s^2+t^2}{v^4} + 8\alpha_4 \frac{u^2}{v^4} \\
\mathcal{A}(w^+ w^+ \to w^+ w^+) &= -\frac{s}{v^2} + 8\alpha_4 \frac{s^2}{v^4} + 4(\alpha_4 + 2\alpha_5)\frac{t^2+u^2}{v^4} \\
\mathcal{A}(zz \to zz) &= 8(\alpha_4 + \alpha_5)\frac{s^2+t^2+u^2}{v^4}
\end{aligned}$$

- (Clebsch-Gordan) Decomposition into isospin eigenamplitudes

$$\begin{aligned}
\mathcal{A}(I=0) &= 3\mathcal{A}(s,t,u) + \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t) \\
\mathcal{A}(I=1) &= \mathcal{A}(t,s,u) - \mathcal{A}(u,s,t) \\
\mathcal{A}(I=2) &= \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t)
\end{aligned}$$

Unitarity of Amplitudes

UV-incomplete theories could violate unitarity

Cross section: $\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\mathcal{M}|^2$

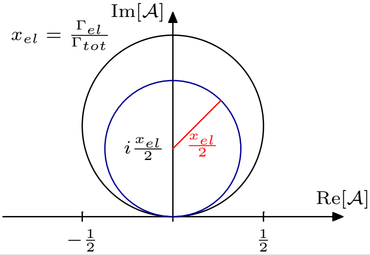
Optical Theorem (Unitarity of the S(cattering) Matrix):

$$\sigma_{\text{tot}} = \text{Im}\left[\mathcal{M}_{ii}(t=0)\right]/s \qquad t = -s(1-\cos\theta)/2$$

Partial wave amplitudes: $\mathcal{M}(s,t,u) = 32\pi \sum_\ell (2\ell+1)\mathcal{A}_\ell(s) P_\ell(\cos\theta)$

Assuming only elastic scattering:

$$\sigma_{\text{tot}} = \sum_\ell \frac{32\pi(2\ell+1)}{s}|\mathcal{A}_\ell|^2 \stackrel{!}{=} \sum_\ell \frac{32\pi(2\ell+1)}{s}\text{Im}\left[\mathcal{A}_\ell\right] \quad \Rightarrow \quad \boxed{|\mathcal{A}_\ell|^2 = \text{Im}\left[\mathcal{A}_\ell\right]}$$



Argand circle $\boxed{\left|\mathcal{A}(s) - \frac{i}{2}\right| = \frac{1}{2}}$

Resonance: $\mathcal{A}(s) = \frac{-M\Gamma_{\text{el}}}{s-M^2+iM\Gamma_{\text{tot}}}$

Counterclockwise circle, radius $\frac{x_{el}}{2}$

Pole at $s = M^2 - iM\Gamma_{\text{tot}}$

Unitarity in the EW sector: SM

- Project out isospin eigenamplitudes Lee,Quigg,Thacker, 1973

$$\mathcal{A}_\ell(s) = \frac{1}{32\pi} \int_{-s}^{0} \frac{dt}{s} \mathcal{A}(s,t,u) P_\ell(1 + 2t/s) \qquad \cos\theta = 1 + 2t/s$$

Remember Legendre polynomials: $P_0(s) = 1 \qquad P_1(s) = \cos\theta \qquad P_2(s) = (3\cos^2\theta - 1)/2$

- SM longitudinal isospin eigenamplitudes ($\mathcal{A}_{I,\text{spin}=J}$):

$$\mathcal{A}_{I=0} = 2\frac{s}{v^2} P_0(s) \qquad \mathcal{A}_{I=1} = \frac{t-u}{v^2} = \frac{s}{v^2} P_1(s) \qquad \mathcal{A}_{I=2} = -\frac{s}{v^2} P_0(s)$$

$$\boxed{\mathcal{A}_{0,0} = \frac{s}{16\pi v^2}} \qquad \boxed{\mathcal{A}_{1,1} = \frac{s}{96\pi v^2}} \qquad \boxed{\mathcal{A}_{2,0} = -\frac{s}{32\pi v^2}}$$

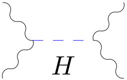
exceeds unitarity bound $|\mathcal{A}_{IJ}| \lesssim \frac{1}{2}$ at:

$I = 0: \qquad E \sim \sqrt{8\pi}v = 1.2$ TeV

$I = 1: \qquad E \sim \sqrt{48\pi}v = 3.5$ TeV

$I = 2: \qquad E \sim \sqrt{16\pi}v = 1.7$ TeV

Higgs exchange:



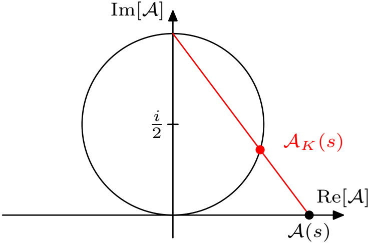
$$\mathcal{A}(s,t,u) = -\frac{M_H^2}{v^2} \frac{s}{s - M_H^2}$$

Unitarity: $M_H \lesssim \sqrt{8\pi}v \sim 1.2$ TeV

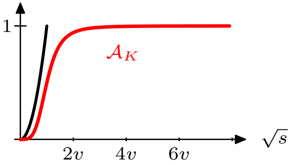
K-Matrix Unitarization and friends

K-Matrix unitarization

$$\mathcal{A}_K(s) = \frac{\mathcal{A}(s)}{1 - i\mathcal{A}(s)} = \mathcal{A}(s)\frac{1 + i\mathcal{A}(s)}{1 + \mathcal{A}(s)^2}$$



Unitarization by infinitely heavy and wide resonance



- Low-energy theorem (LET): $\frac{s}{v^2}$
- K-Matrix amplitude: $|\mathcal{A}(s)|^2 = \frac{s^2}{s^2+v^4} \overset{s\to\infty}{\longrightarrow} 1$
- Poles $\pm iv$: M_0, Γ large

Padé unitarization

separates higher chiral orders

$$\mathcal{A}_P(s) = \frac{\mathcal{A}^{(0)}(s)^2}{\mathcal{A}^{(0)}(s) - \mathcal{A}^{(1)}(s) - i\mathcal{A}^{(0)}(s)^2}$$

each partial wave dominated by single resonance

"Naive" Unitarization

Extreme case:

$$\mathcal{A}_N(s) = e^{i\mathcal{A}(s)} \sin \mathcal{A}(s)$$

Infinitely many resonances becoming denser for $s \to \infty$

BSM Unitarized Resonances: e.g. Scalar Singlet

Assumptions:

- LHC is able to detect a resonance in the EW sector
- Further resonances might exist, but out of reach or not detectable
- Describe 1st resonance by correct amplitude
- Use K-matrix unitarization to define a consistent model

Example: Scalar Singlet

- $\mathcal{L}_\sigma = -\frac{1}{2}\sigma\left(M_\sigma^2 + \partial^2\right)\sigma + \frac{g_\sigma v}{2}\sigma \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right]$
- Feynman rules: $\sigma w^+ w^- : \; -\frac{2ig_\sigma}{v}(k_+ \cdot k_-)$ $\qquad \sigma zz : \; -\frac{2ig_\sigma}{v}(k_1 \cdot k_2)$
- Amplitude (s-channel exchange): $\boxed{\mathcal{A}^\sigma(s,t,u) = \frac{g_\sigma^2}{v^2}\frac{s^2}{s - M^2}}$
- Isospin eigenamplitudes:

$$\begin{aligned}
\mathcal{A}_0^\sigma(s,t,u) &= \frac{g_\sigma^2}{v^2}\left(3\frac{s^2}{s-M^2} + \frac{t^2}{t-M^2} + \frac{u^2}{u-M^2}\right) \\
\mathcal{A}_1^\sigma(s,t,u) &= \frac{g_\sigma^2}{v^2}\left(\frac{t^2}{t-M^2} - \frac{u^2}{u-M^2}\right) \\
\mathcal{A}_2^\sigma(s,t,u) &= \frac{g_\sigma^2}{v^2}\left(\frac{t^2}{t-M^2} + \frac{u^2}{u-M^2}\right)
\end{aligned}$$

Unitarizing the scalar singlet

Alboteanu/Kilian/JR, 2008

$$\mathcal{A}^{\sigma}_{00}(s) = 3\frac{g_\sigma^2}{v^2}\frac{s^2}{s-M^2} + 2\frac{g_\sigma^2}{v^2}\mathcal{S}_0(s) \qquad \mathcal{A}^{\sigma}_{02}(s) = 2\frac{g_\sigma^2}{v^2}\mathcal{S}_2(s) \quad = \mathcal{A}^{\sigma}_{22}(s)$$

$$\mathcal{A}^{\sigma}_{11}(s) = 2\frac{g_\sigma^2}{v^2}\mathcal{S}_1(s) \qquad \mathcal{A}^{\sigma}_{13}(s) = 2\frac{g_\sigma^2}{v^2}\mathcal{S}_3(s)$$

$$\mathcal{A}^{\sigma}_{20}(s) = 2\frac{g_\sigma^2}{v^2}\mathcal{S}_0(s)$$

- S-wave coefficients no longer polynomial, e.g.:

$$\mathcal{S}_0(s) = M^2 - \frac{s}{2} + \frac{M^4}{s}\log\frac{s}{s+M^2}$$

- s-channel pole must be explicitly subtracted:

$$A_{IJ}(s) = A^{(0)}_{IJ}(s) + F_{IJ}(s) + \frac{G_{IJ}(s)}{s-M^2},$$

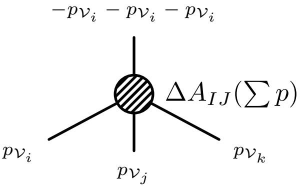
  - $F_{IJ}(s)$ is finite
  - $G_{IJ}(s) \propto s$ (vector), $\propto s^2$ (scalar, tensor)

$$\hat{A}_{IJ}(s) = \frac{A_{IJ}(s)}{1-\frac{i}{32\pi}A_{IJ}(s)} = A^{(0)}_{IJ}(s) + 32\pi i\Delta A_{IJ}(s),$$

$$\Delta A_{IJ}(s) = 32\pi i\left(1+\frac{i}{32\pi}A^{(0)}_{IJ}(s)+\frac{s-M^2}{\frac{i}{32\pi}G_{IJ}(s)-(s-M^2)\left[1-\frac{i}{32\pi}\left(A^{(0)}_{IJ}(s)+F_{IJ}(s)\right)\right]}\right)$$

Implementation and Taxonomy of Resonances

- Explicit "time arrow" in WHIZARD



  – trace back pairs of momenta at quartic vertices to external legs
  – guarantee for only s-channel insertions

- Consider the following resonances:

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma\left(M_\sigma^2 + \partial^2\right)\sigma + \sigma j_\sigma$$

$$\mathcal{L}_\phi = -\frac{1}{2}\left[\frac{1}{2}\mathrm{tr}\left[\boldsymbol{\phi}\left(M_\sigma^2 + \partial^2\right)\boldsymbol{\phi}\right] + \mathrm{tr}\left[\boldsymbol{\phi}\mathbf{j}_\phi\right]\right]$$

$$\mathcal{L}_\rho = \frac{1}{2}\left[\frac{M_\rho^2}{2}\mathrm{tr}\left[\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu\right] - \frac{1}{4}\mathrm{tr}\left[\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu}\right] + \mathrm{tr}\left[\mathbf{j}_\rho^\mu\boldsymbol{\rho}_\mu\right]\right]$$

$$\mathcal{L}_f = \mathcal{L}_{\mathrm{kin}} - \frac{M_f^2}{2} f_{\mu\nu} f^{\mu\nu} + f_{\mu\nu} j_f^{\mu\nu}$$

$$\mathcal{L}_a = \mathcal{L}_{\mathrm{kin}} - \frac{M_t^2}{4}\mathrm{tr}\left[\mathbf{t}_{\mu\nu}\mathbf{t}^{\mu\nu}\right] + \frac{1}{2}\mathrm{tr}\left[\mathbf{t}_{\mu\nu}\mathbf{j}_a^{\mu\nu}\right]$$

$$j_\sigma = \frac{g_\sigma v}{2}\mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right]$$

$$\mathbf{j}_\phi = -\frac{g_\phi v}{2}\left(\mathbf{V}_\mu \otimes \mathbf{V}^\mu - \frac{\tau^{aa}}{6}\mathrm{tr}\left[\mathbf{V}_\mu\mathbf{V}^\mu\right]\right)$$

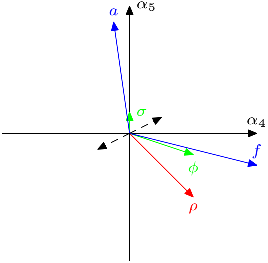
$$\mathbf{j}_\rho^\mu = \mathrm{i} g_\rho v^2 \mathbf{V}^\mu$$

$$j_f^{\mu\nu} = -\frac{g_f v}{2}\left(\mathrm{tr}\left[\mathbf{V}^\mu\mathbf{V}^\nu\right] - \frac{g^{\mu\nu}}{4}\mathrm{tr}\left[\mathbf{V}_\rho\mathbf{V}^\rho\right]\right)$$

$$\mathbf{j}_a^{\mu\nu} = -\frac{g_a v}{2}\left[\frac{1}{2}\left(\mathbf{V}^\mu \otimes \mathbf{V}^\nu + \mathbf{V}^\nu \otimes \mathbf{V}^\mu\right) - \frac{g^{\mu\nu}}{4}\mathbf{V}_\rho \otimes \mathbf{V}^\rho - \frac{\tau^{aa}}{6}\mathrm{tr}\left[\mathbf{V}^\mu\mathbf{V}^\nu\right] + \frac{g^{\mu\nu}\tau^{aa}}{24}\mathrm{tr}\left[\mathbf{V}_\rho\mathbf{V}^\rho\right]\right]$$

Taxonomy of resonances/Loops

Resonance	σ	ϕ	ρ	f	a
$\Gamma[g^2M^2/(64\pi v^2)]$	6	1	$\frac{4}{3}(\frac{v^2}{M^2})$	$\frac{1}{5}$	$\frac{1}{30}$
$\Delta\alpha_4[(16\pi\Gamma/M)(v^4/M^4)]$	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{2}$	$-\frac{5}{8}$
$\Delta\alpha_5[(16\pi\Gamma/M)(v^4/M^4)]$	$\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{3}{4}$	$-\frac{5}{8}$	$\frac{35}{8}$



- Loop corrections to LET can be switched on/off:

  (μ renormalization scale)

$$A_C^{\text{1-loop}}(s,t,u) = \frac{1}{16\pi^2}\left[\left(\frac{1}{2}\ln\frac{\mu^2}{|s|} + 8C_5\right)\frac{s^2}{v^4} + \left(\frac{t(s+2t)}{6v^4}\ln\frac{\mu^2}{|t|} + 4C_4\frac{t^2}{v^4}\right) + (t \leftrightarrow u)\right],$$

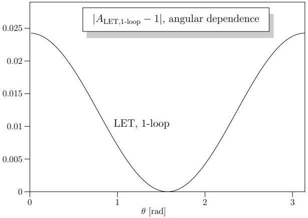
- Finite scheme-dep. matching coefficients/NLO counterterms

  (e.g. heavy Higgs regulator $\mu = M_H$ Dawson/Willenbrock, 1989)

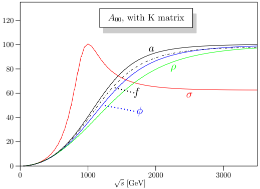
$$C_4 = -\frac{1}{18} \approx -0.056, \qquad C_5 = \frac{9\pi}{16\sqrt{3}} - \frac{37}{36} \approx -0.0075.$$

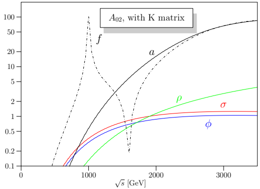
$$\alpha_4^{(1)} = \frac{1}{16\pi^2}\left(C_4 - \frac{1}{12}\ln\frac{\mu^2}{\mu_0^2}\right)$$

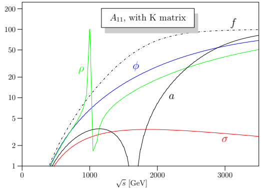
$$\alpha_5^{(1)} = \frac{1}{16\pi^2}\left(C_5 - \frac{1}{24}\ln\frac{\mu^2}{\mu_0^2}\right)$$

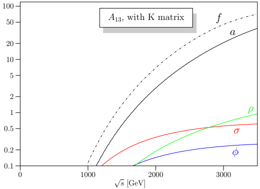


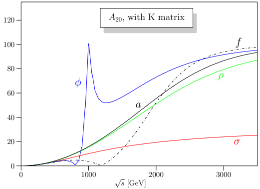
Eigenamplitudes

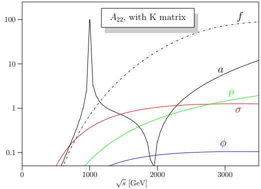


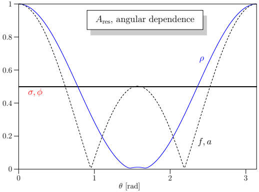


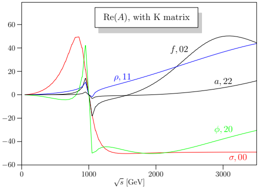




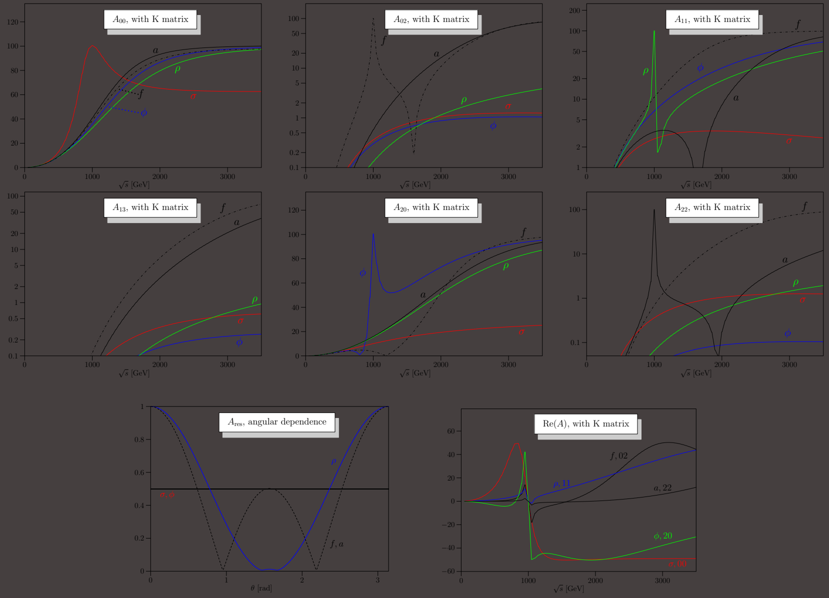




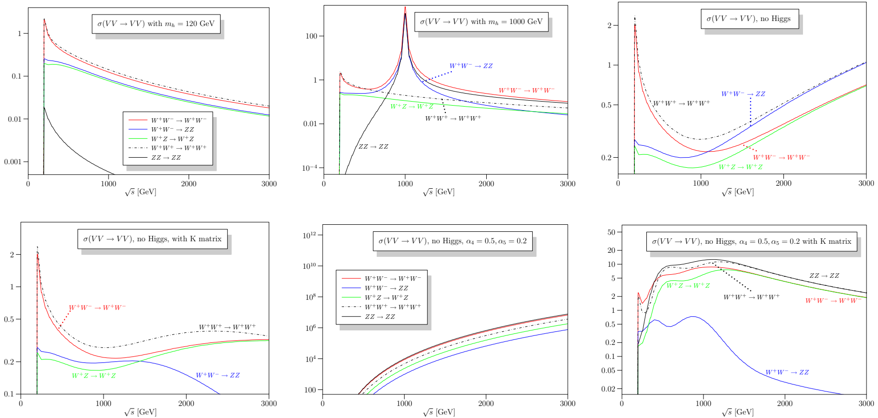




Eigenamplitudes

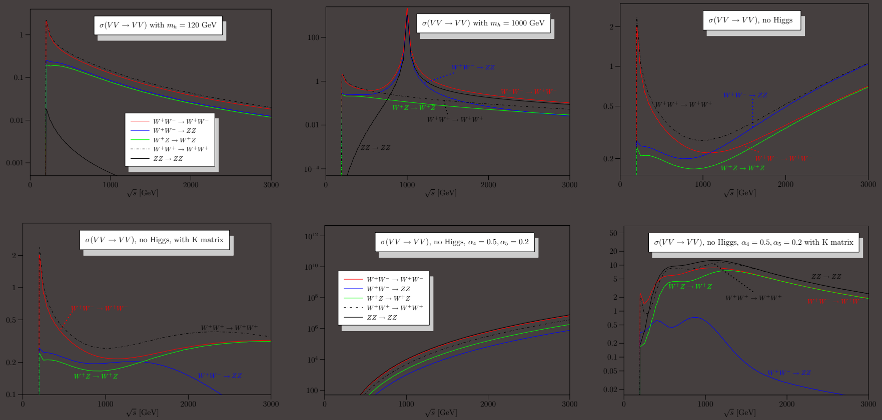


"Partonic" cross sections (I)



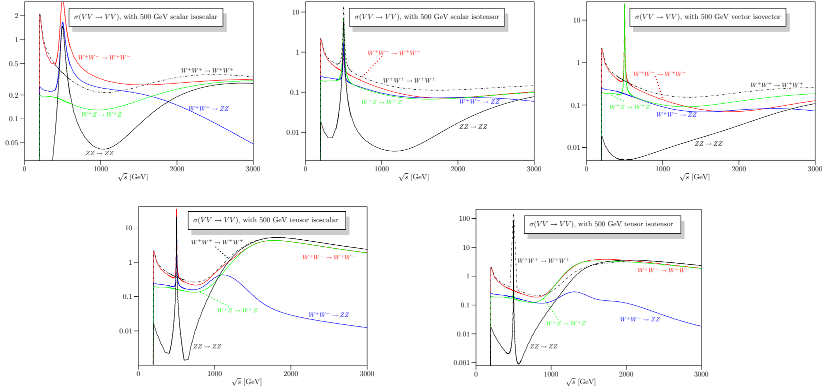
- Cross sections (in nb)

"Partonic" cross sections (I)



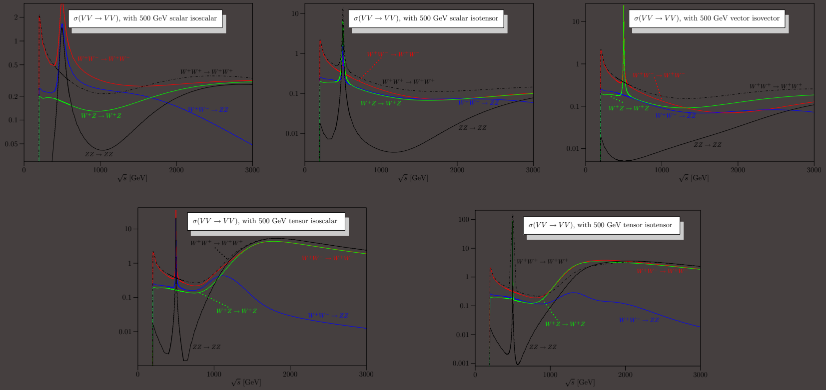
- Cross sections (in nb)

"Partonic" cross sections (II)



- $\sigma(\mathcal{V}\mathcal{V} \to \mathcal{V}\mathcal{V})$ in nb $\qquad M_R = 500$ GeV
- all amplitudes K-matrix unitarized
- Cut of $15°$ around the beam axis

"Partonic" cross sections (II)



- $\sigma(\mathcal{VV} \to \mathcal{VV})$ in nb $\qquad M_R = 500$ GeV
- all amplitudes K-matrix unitarized
- Cut of $15°$ around the beam axis

The Effective W approximation

- $M_{\mathcal{V}}$, $\hat{t}_i$ small corrections, $\mathcal{V}$ nearly onshell:

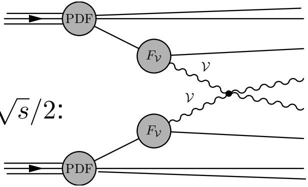
$$\sigma(q_1 q_2 \to q_1' q_2' \mathcal{V}_1' \mathcal{V}_2') \approx \sum_{\lambda_1,\lambda_2} \int dx_1\, dx_2\, F^{\lambda_1}_{q_1 \to q_1' \mathcal{V}_1}(x_1)\, F^{\lambda_2}_{q_2 \to q_2' \mathcal{V}_2}(x_2)\, \sigma^{\lambda_1 \lambda_2}_{\mathcal{V}_1 \mathcal{V}_2 \to \mathcal{V}_1' \mathcal{V}_2'}(x_1 x_2 s)$$

- In addition to Weizsäcker-Williams: longitudinal polarisation

$$F^{+}_{q \to q' \mathcal{V}}(x) = \frac{(V-A)^2 + (V+A)^2 (1-x)^2}{16\pi^2 \; x} \left[\ln \left(\frac{p^2_{\perp,\max} + (1-x) m^2_{\mathcal{V}}}{(1-x) m^2_{\mathcal{V}}} \right) - \frac{p^2_{\perp,\max}}{p^2_{\perp,\max} + (1-x) m^2_{\mathcal{V}}} \right]$$

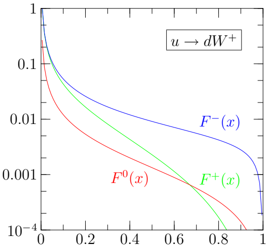
$$F^{-}_{q \to q' \mathcal{V}}(x) = \frac{(V+A)^2 + (V-A)^2 (1-x)^2}{16\pi^2 \; x} \left[\ln \left(\frac{p^2_{\perp,\max} + (1-x) m^2_{\mathcal{V}}}{(1-x) m^2_{\mathcal{V}}} \right) - \frac{p^2_{\perp,\max}}{p^2_{\perp,\max} + (1-x) m^2_{\mathcal{V}}} \right]$$

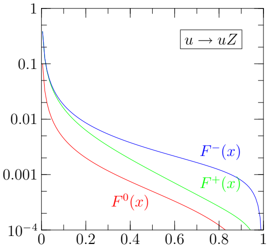
$$F^{0}_{q \to q' \mathcal{V}}(x) = \frac{V^2 + A^2}{8\pi^2} \, \frac{2(1-x)}{x} \, \frac{p^2_{\perp,\max}}{p^2_{\perp,\max} + (1-x) m^2_{\mathcal{V}}}$$



- Dominant contribution from small $\mathcal{V}$ virtualities
- Transverse momentum cutoff $p_{\perp,\max} \leq (1-x)\sqrt{s}/2$:
  - longitudinal pol.: finite for $p_{\perp,\max} \to \infty$
  - Transversal pol.: logarithmic singularity

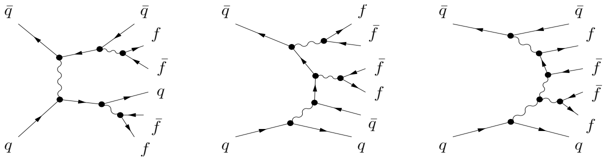
- EWA structure functions: W (left) and Z (right)





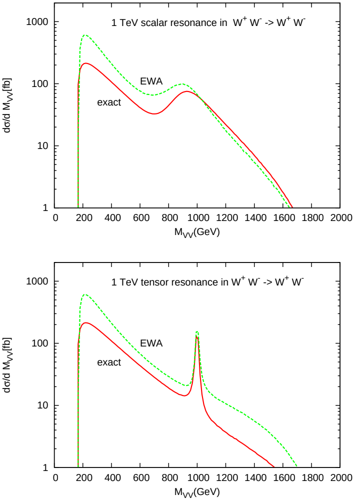
  - Emission from u, $\sqrt{s} = 2$ TeV
  - preferred at high energy: transversal emission

- Problem: Irreducible background to weak-boson scattering



  - Double ISR/FSR
  - t-channel like diagrams

- Coulomb-singularity (peak): cut on $p_{T,V} \gtrsim 30$ GeV



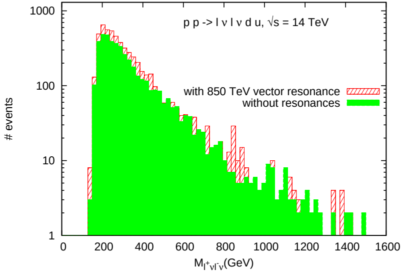
- Effective W approx. vs. WHIZARD full matrix elements
- Shapes/normalization of distributions heavily affected
- **EWA: Sideband subtraction completely screwed up!**

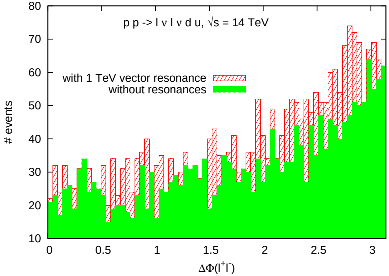
LHC Example: Vector Isovector

Alboteanu/Kilian/JR, 2008

- Example: 850 GeV vector resonance, coupling $g_\rho = 1$
- (Theory) Cuts:
  - $p_\perp(\ell\nu) > 30$ GeV
  - $|\delta R(\ell\nu)| < 1.5$
  - $\theta(u/d) > 0.5°$
- Integrated luminosity: 225 fb^{-1}
- Discriminator: angular correlations $\Delta\phi(\ell\ell)$
- Ongoing ATLAS study

  Kobel/JR/Schumacher
  - Cut analysis/NN
  - More kinematic observables
  - Geant4 FullSim (special points)
  - all resonances, parameter scans

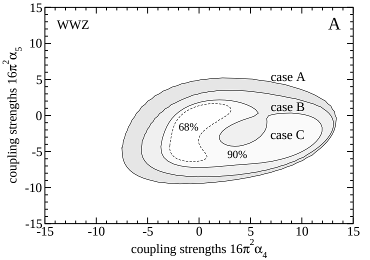




ILC Results: Triboson production

$e^+e^- \to WWZ/ZZZ$, dep. on $(\alpha_4+\alpha_6)$, $(\alpha_5+\alpha_7)$, $\alpha_4+\alpha_5+2(\alpha_6+\alpha_7+\alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR

1 TeV, $1\,\text{ab}^{-1}$, full 6-fermion final states, SIMDET fast simulation

Observables: M^2_{WW}, M^2_{WZ}, $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e^-_R, C) 80% e^-_R, 60% e^+_L

$16\pi^2\times$	WWZ no pol.	WWZ e^- pol.	WWZ both pol.	ZZZ no pol.	best
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	−4.40	−3.34	−1.71	−3.53	−1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	−7.10	−6.40	−2.19	−3.53	−1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \to 6$ jets

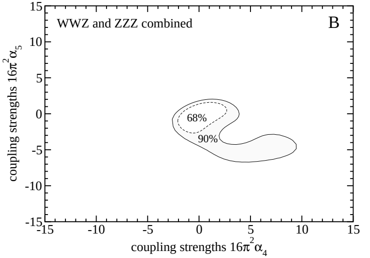
Veto against $E^2_{\text{mis}} + p^2_{\perp,\text{mis}}$

No angular correlations yet

ILC Results: Triboson production

$e^+e^- \to WWZ/ZZZ$, dep. on $(\alpha_4+\alpha_6)$, $(\alpha_5+\alpha_7)$, $\alpha_4+\alpha_5+2(\alpha_6+\alpha_7+\alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR

1 TeV, $1\,\text{ab}^{-1}$, full 6-fermion final states, SIMDET fast simulation

Observables: M^2_{WW}, M^2_{WZ}, $\sphericalangle(e^-,Z)$

A) unpol., B) 80% e^-_R, C) 80% e^-_R, 60% e^+_L

$16\pi^2\times$	WWZ no pol.	WWZ e^- pol.	WWZ both pol.	ZZZ no pol.	best
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \to 6$ jets

Veto against $E^2_{\text{mis}} + p^2_{\perp,\text{mis}}$

No angular correlations yet

Vector Boson Scattering

1 TeV, $1\ \mathrm{ab}^{-1}$, full $6f$ final states, 80 % e_R^-, 60 % e_L^+ polarization, binned likelihood

Contributing channels: $WW \to WW$, $WW \to ZZ$, $WZ \to WZ$, $ZZ \to ZZ$

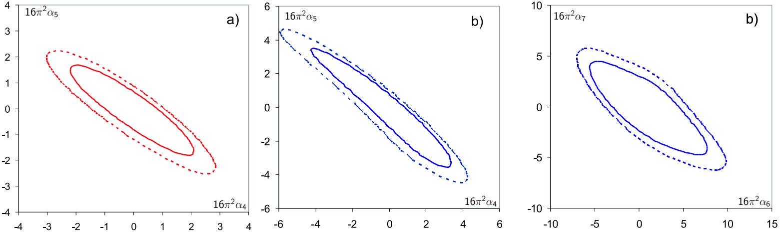
Process	Subprocess	σ [fb]
$e^+e^- \to \nu_e\bar\nu_e q\bar q q\bar q$	$WW \to WW$	23.19
$e^+e^- \to \nu_e\bar\nu_e q\bar q q\bar q$	$WW \to ZZ$	7.624
$e^+e^- \to \nu\bar\nu q\bar q q\bar q$	$V \to VVV$	9.344
$e^+e^- \to \nu e q\bar q q\bar q$	$WZ \to WZ$	132.3
$e^+e^- \to e^+e^- q\bar q q\bar q$	$ZZ \to ZZ$	2.09
$e^+e^- \to e^+e^- q\bar q q\bar q$	$ZZ \to W^+W^-$	414.
$e^+e^- \to b\bar b X$	$e^+e^- \to t\bar t$	331.768
$e^+e^- \to q\bar q q\bar q$	$e^+e^- \to W^+W^-$	3560.108
$e^+e^- \to q\bar q q\bar q$	$e^+e^- \to ZZ$	173.221
$e^+e^- \to e\nu q\bar q$	$e^+e^- \to e\nu W$	279.588
$e^+e^- \to e^+e^- q\bar q$	$e^+e^- \to e^+e^- Z$	134.935
$e^+e^- \to X$	$e^+e^- \to q\bar q$	1637.405

$SU(2)_c$ conserved case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-1.41	1.38
$16\pi^2\alpha_5$	-1.16	1.09

$SU(2)_c$ broken case, all channels

coupling	$\sigma-$	$\sigma+$
$16\pi^2\alpha_4$	-2.72	2.37
$16\pi^2\alpha_5$	-2.46	2.35
$16\pi^2\alpha_6$	-3.93	5.53
$16\pi^2\alpha_7$	-3.22	3.31
$16\pi^2\alpha_{10}$	-5.55	4.55

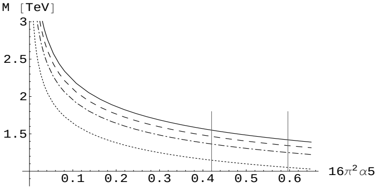


Interpretation as limits on resonances

Consider the width to mass ratio, $f_\sigma = \Gamma_\sigma / M_\sigma$

$SU(2)$ conserving scalar singlet

$$M_\sigma = v \left(\frac{4\pi f_\sigma}{3\alpha_5} \right)^{\frac{1}{4}}$$

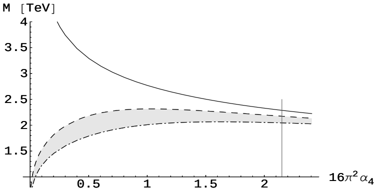


$f = 1.0$ (full), 0.8 (dash), 0.6 (dot-dash), 0.3 (dot)

$SU(2)$ broken vector triplet

needs input from TGC covariance matrix

$$M_{\rho^\pm} = v \left(\frac{12\pi\alpha_4 f_{\rho^\pm}}{\alpha_4^2 + 2(\alpha_2^\lambda)^2 + s_w^2(\alpha_4^\lambda)^2/(2c_w^2)} \right)^{\frac{1}{4}}$$



upper/lower limit from λ_Z, grey area: magnetic moments

Final result:

Spin	$I=0$	$I=1$	$I=2$
0	1.55	–	1.95
1	–	2.49	–
2	3.29	–	4.30

Spin	$I=0$	$I=1$	$I=2$
0	1.39	1.55	1.95
1	1.74	2.67	–
2	3.00	3.01	5.84

Summary/Conclusions

- New Physics generically encoded in EW Chiral Lagrangian
- Triple/Quartic gauge couplings measured either
  - via triple boson production
  - via vector boson scattering
- interpreted as resonances coupled to EW bosons
- “Correct” description for first resonance (also [very] broad)
- Beyond that: assure unitarity (K matrix)
- Sensitivity rises with number of intermediate states:
  - LHC sensitivity limited in pure EW sector: $0.6 - 2$ TeV
  - ILC : $1.5 - 6$ TeV
- Full analysis including all channels/backgrounds with WHIZARD
- Complete ATLAS study is under way

One Ring to Find them ... One Ring to Rule them Out

One Ring to Find them ... One Ring to Rule them Out