Conformal Symmetry and the Weak Scale

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[hep-th/0612165, arXiv:0710.2840, arXiv:0803.2814, arXiv:0809.1338[hep-th]]

NB: Conformal symmetry is an old subject!

[see e.g. H.Kastrup, arXiv:0808.2730 for an historical survey and references]

Mass Generation and Hierarchy

- **Fact: Standard Model (= SM) of elementary particle physics is conformally invariant at tree level except for explicit mass term $m^2\Phi^\dagger\Phi$ in potential $\rightarrow$ masses for vector bosons, quarks and leptons.**

- **Why $m^2 < 0$ rather than $m^2 > 0$?**

- **Quantum corrections $\delta m^2 \sim \Lambda^2 \Rightarrow$ why $m_H \ll M_{Pl}$? (with UV cutoff Λ = scale of 'new physics')**

  – ***stabilization/explanation of hierarchy?***

- **Most popular proposal: SM $\longrightarrow$ MSSM or NMSSM: use supersymmetry to control quantum corrections via cancellation of quadratic divergences $\Rightarrow$**

$$\delta m^2 \sim \Lambda_{SUSY}^2 \ln(\Lambda^2/\Lambda_{SUSY}^2)$$

Landau Poles

Large scalar self-coupling $\leftrightarrow$ Landau pole $(A > 0)$

$$\mu \frac{dy}{d\mu} = Ay^2 \quad \Longrightarrow \quad y(\mu) = \frac{y_0}{1 - Ay_0 \ln(\mu/\mu_0)}$$

Thus we are left with two possibilities:

- **Theory strongly coupled for $\ln(\mu/\mu_0) \sim (Ay_0)^{-1}$**
- **Or: theory *does not exist* (rigorously as a QFT)**

General features of RG evolution of couplings in SM:

- **Coupled RG equations (linking α_s to other couplings) also give rise to *infrared* (IR) Landau poles**
- **With SM-like bosonic and fermionic matter, UV and IR Landau poles are (generically) *unavoidable.***

The demise of relativistic quantum field theory
Or: Why we need quantum gravity!

- **Breakdown of *any* extension of the standard model (supersymmetric or not) that stays within the framework of relativistic quantum field theory is probably unavoidable [as it appears to be for $\lambda\phi_4^4$].**
- **Therefore the main challenge is to *delay* breakdown until M_{Pl} where a proper theory of quantum gravity is expected to replace quantum field theory.**
- **How the MSSM achieves this: scalar self-couplings tied to gauge coupling $\lambda \propto g^2$ by supersymmetry, and thus controlled by gauge coupling evolution.**
  $\Rightarrow m_H \leq \sqrt{2} m_Z$ in (non-exotic variants of) MSSM.

Conformal invariance and the Standard Model

Can classically unbroken conformal symmetry stabilize the weak scale w.r.t. the Planck scale? Claim: Yes, if

- **there are no intermediate mass scales between m_W and M_{Pl} ('grand desert scenario'); and**
- **the RG evolved couplings exhibit neither Landau poles nor instabilities (of the effective potential) over this whole range of energies.**

Thus: is it possible to explain all mass scales from a single scale v via the quantum mechanical breaking of conformal invariance (i.e. via conformal anomaly)

$\rightarrow$ Hierarchy 'natural' in the sense of 't Hooft?

[See also: W. Bardeen, FERMILAB-CONF-95-391-T, FERMILAB-CONF-95-377-T]

Evidence for large scales other than M_{Pl}?

- **(SUSY?) Grand Unification:** $m_X \geq \mathcal{O}(10^{15}\,\mathrm{GeV})$**?**
  - **But: proton refuses to decay (so far, at least!)**
  - **SUSY GUTs: unification of gauge couplings at** $\geq \mathcal{O}(10^{16}\,\mathrm{GeV})$
- **Light neutrinos** $(m_\nu \leq \mathcal{O}(1\,\mathrm{eV}))$ **and heavy neutrinos** $\rightarrow$ **most popular (and most plausible) explanation of observed mass patterns via seesaw mechanism:**

$$m_\nu^{(1)} \sim \frac{m_D^2}{M}\,,\; m_D = \mathcal{O}(m_W)\;\Rightarrow m_\nu^{(2)} \sim M \geq \mathcal{O}(10^{12}\,\mathrm{GeV})?$$

- **Resolution of strong CP problem** $\Rightarrow$ **need** ***axion*** $a(x)$**.** **Limits e.g. from axion cooling in stars** $\Rightarrow$

$$\mathcal{L} = \frac{1}{4f_a} a F^{\mu\nu}\tilde{F}_{\mu\nu} \qquad \textbf{with } f_a \geq \mathcal{O}(10^{10}\,\mathrm{GeV})$$

NB: axion is (still) an attractive CDM candidate.

Coleman-Weinberg Mechanism (1973)

- **Idea: spontaneous symmetry breaking by radiative corrections $\Longrightarrow$ can small mass scales be explained via *conformal anomaly* and *effective potential*?**

$$V(\varphi) = \frac{\lambda}{4}\varphi^4 \;\rightarrow\; V_{\text{eff}}(\varphi) = \frac{\lambda}{4}\varphi^4 + \frac{9\lambda^2\varphi^4}{64\pi^2}\left[\ln\left(\frac{\varphi^2}{\mu^2}\right) + C_0\right]$$

- **But: when can we trust one-loop approximation?**
  - **Radiative breaking spurious for pure φ^4 theory**
  - **Scalar electrodynamics: consistent for $\lambda \sim e^4$**

[See e.g.: Sher, Phys.Rep.179(1989)273; Ford,Jones,Stephenson,Einhorn, Nucl.Phys.B395(1993)17; Chishtie,Elias,Mann,McKeon,Steele, NPB743(2006)104]

- **And: can this be made to work for real world (=SM)?**
  - $m_H > 115\,\text{GeV}$ **and** $m_{top} = 174\,\text{GeV}$

Regularization and Renormalization

- **Conformal invariance *must be broken explicitly* for computation of quantum corrections via regulator mass scale with *any* regularization.**
- **Most convenient: dimensional regularization**

$$\int \frac{d^4k}{(2\pi)^4} \quad \rightarrow \quad v^{2\epsilon} \int \frac{d^{4-2\epsilon}k}{(2\pi)^{4-2\epsilon}}$$

- **Renormalize by requiring *exact* conformal invariance of the *local* part of the effective action $\Rightarrow$ preserve *anomalous Ward identity* $T^{\mu}{}_{\mu}(\phi) = \beta(\lambda) O_4(\phi)$**
- **(Renormalized) effective action to any order:**
  - **no mass terms ($\propto v^2$) in divergent or finite parts**
  - **conformal symmetry broken only by logarithmic terms containing $L \equiv \ln(\phi^2/v^2)$ (to *any* order!)**

RG improved effective potential

One (real) scalar field φ coupled to non-scalar fields

$$W_{\text{eff}} \equiv W_{\text{eff}}(\varphi, g, v) = \varphi^4 f(L, g) \quad \text{for} \quad L \equiv \ln(\varphi^2/v^2)$$

Improved effective potential must obey RG equation

$$\left[v\frac{\partial}{\partial v} + \sum_j \beta_j(g)\frac{\partial}{\partial g_j} + \gamma(g)\varphi\frac{\partial}{\partial \varphi} \right] W_{\text{eff}}(\varphi, g, v) = 0$$

Therefore [see also: Curtright,Ghandour, Ann.Phys.112(1978)237]

$$\left[-2\frac{\partial}{\partial L} + \sum_j \tilde{\beta}_j(g)\frac{\partial}{\partial g_j} + 4\tilde{\gamma}(g) \right] f(L, g) = 0$$

with $\tilde{\beta}(g) \equiv \beta(g)/(1-\gamma(g))$ **and** $\tilde{\gamma}(g) \equiv \gamma(g)/(1-\gamma(g)) \quad \Rightarrow$

Running couplings $\hat{g}_j(L)$ **from** $2(\mathrm{d}\hat{g}_j(L)/dL) = \tilde{\beta}_j(\hat{g})$**.**

- **General solution (with arbitrary function F)**

$$f(L,g) \equiv F(\hat{g}_1(L), \hat{g}_2(L), \dots) \exp\left[2\int_0^L \tilde{\gamma}(\hat{g}(t))\mathrm{d}t\right]$$

The choice $F(L,g) = \hat{g}_1(L)$ (g_1 = scalar self-coupling) yields correct $\hbar \to 0$ limit.

- **The textbook example: pure (massless) ϕ^4 theory**

$$W_{\text{eff}}(\varphi) = \frac{1}{4}\hat{\lambda}(L)\varphi^4 = \frac{\lambda}{4}\cdot\frac{\varphi^4}{1-(9\lambda/16\pi^2)L} = V_{\text{eff}}(\varphi) + \mathcal{O}(\lambda^3 L^2)$$

captures leading log contributions to all orders.

- **Explains spuriousness of symmetry breaking for V_{eff} via restoration of convexity by RG improvement $\Rightarrow W_{\text{eff}}(\varphi)$ has only trivial minimum at $\langle\varphi\rangle = 0$!**

An almost realistic example

QCD coupled to colorless real scalar field ϕ

$$\mathcal{L} = -\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu}F^{\mu\nu} + i\bar{q}\gamma^{\mu}D_{\mu}q + \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi + g_Y\phi\bar{q}q - \frac{g}{4}\phi^4$$

Cancellations in β-functions

$$2\frac{\mathrm{d}\hat{y}}{\mathrm{d}L} = a_1\hat{y}^2 + a_2\hat{x}\hat{y} - a_3\hat{x}^2 \quad , \quad 2\frac{\mathrm{d}\hat{x}}{\mathrm{d}L} = b_1\hat{x}^2 - b_2\hat{x}\hat{z} \quad , \quad 2\frac{\mathrm{d}\hat{z}}{\mathrm{d}L} = -2c\hat{z}^2$$

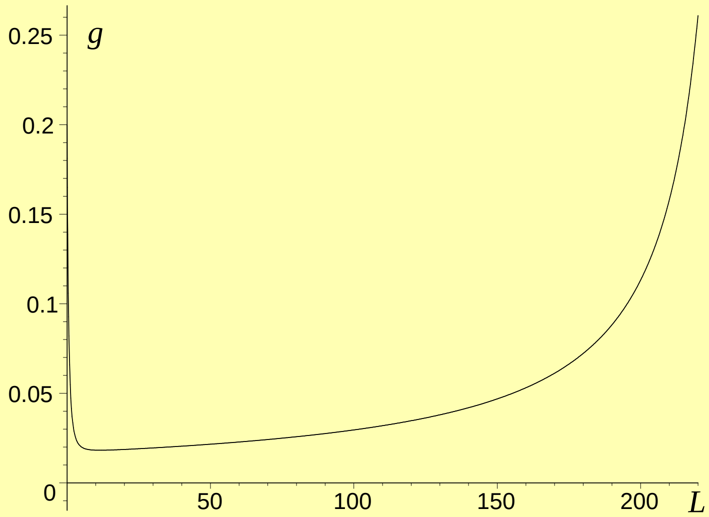
with

$$x \equiv \frac{g_Y^2}{4\pi^2} \quad , \quad y \equiv \frac{g}{4\pi^2} \quad , \quad z \equiv \frac{g_s^2}{4\pi^2} \equiv \frac{\alpha_s}{\pi}$$

Explicit closed form solutions of one-loop β-function equations available for general coefficients a_i, b_i, c

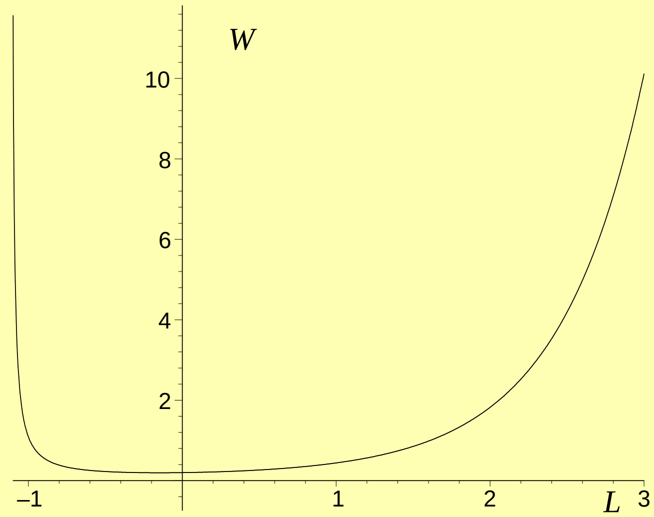
[Faivre,Branchina, PR D72 (2005) 065017; Chishtie et al., hep-ph/0701148; MN, arXiv:0809.1338]

Our general formula for W_{eff} allows more detailed study of range of validity of one-loop CW potential.



The scalar self-coupling $\hat{\lambda}(L)$

- $\hat{\lambda}(L)$ **remains small over large range of values for** L **in spite of large logarithms (for** $\hat{\lambda}(0)L$**)**
- **Landau pole at** $L > 200$ ***and*** **IR barrier** $\Lambda_{\rm IR} > 0$
- **Approximation can be trusted for** $\hat{\lambda}(L)$ **small**



The RG improved effective potential $W_{\text{eff}}(\varphi)$.

- **Convex function, unlike unimproved potential** V_{eff}**.**
- $\Lambda_{\text{IR}} > 0 \Rightarrow$ **enforces symmetry breaking** $\langle\varphi\rangle \neq 0$
- **Minimum safely within perturbative range**
- **Cancellations in** β**-functions are crucial**

A Minimalistic Proposal

- ***Minimal*** **extension of SM with classical conformal symmetry (i.e. no tree level mass terms) *and*:**
  - **right-chiral neutrinos**
  - **enlarged scalar sector: Φ and ϕ**
- **No large intermediate scales ('grand desert')**

  $\Rightarrow$ no grand unification, no low energy SUSY

  [also: M. Shaposhnikov, arXiv:0708.3550[hep-th]; R. Foot et al., arXiv:0709.2750[hep-ph]]
- **All mass scales from effective (CW) potential:**
  - **no new scales required to explain $m_\nu < 1$ eV if Yukawa couplings vary over $Y \sim \mathcal{O}(1) - \mathcal{O}(10^{-5})$**
  - **no new scales required to explain $f_a \geq \mathcal{O}(10^{12}\,\mathrm{GeV})$**

Minimally Extended Standard Model

- **Start from conformally invariant (and therefore renormalizable) Lagrangian $\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}'$ with:**

$$\begin{aligned}\mathcal{L}' \ := \ & \Big(\bar{L}^i \Phi Y_{ij}^E E^j + \bar{Q}^i \epsilon \Phi^* Y_{ij}^D D^j + \bar{Q}^i \epsilon \Phi^* Y_{ij}^U U^j + \\ & + \bar{L}^i \epsilon \Phi^* Y_{ij}^\nu \nu_R^j + \phi \nu_R^{iT} \mathcal{C} Y_{ij}^M \nu_R^j + \text{h.c.}\Big) - \\ & - \frac{\lambda_1}{4} (\Phi^\dagger \Phi)^2 - \frac{\lambda_2}{2} (\phi^\dagger \phi)(\Phi^\dagger \Phi) - \frac{\lambda_3}{4} (\phi^\dagger \phi)^2\end{aligned}$$

`[See also Shaposhnikov, Tkachev, PLB639(2006)104: the '`νMSM`']`

- **Besides usual $SU(2)$ doublet Φ: new scalar field $\phi(x)$**

$$\phi(x) = \varphi(x) \exp\left(\frac{ia(x)}{\sqrt{2}\mu}\right)$$

- **No mass terms, all coupling constants dimensionless**

- Y_{ij}^U, Y_{ij}^E, Y_{ij}^M **real and diagonal**
  Y_{ij}^D, Y_{ij}^ν **complex $\rightarrow$ parametrize family mixing (CKM)**

Effective potential at one loop

$$
\begin{aligned}
V_{\text{eff}}(H,\varphi) \;=\; & \frac{\lambda_1 H^4}{4} + \frac{\lambda_2 H^2\varphi^2}{2} + \frac{\lambda_3\varphi^4}{4} + \frac{9}{16\pi^2}\alpha_w^2 H^4 \ln\left[\frac{H^2}{v^2}\right] \\
& + \frac{3}{256\pi^2}(\lambda_1 H^2 + \lambda_2\varphi^2)^2 \ln\left[\frac{\lambda_1 H^2 + \lambda_2\varphi^2}{v^2}\right] \\
& + \frac{1}{256\pi^2}(\lambda_2 H^2 + \lambda_3\varphi^2)^2 \ln\left[\frac{\lambda_2 H^2 + \lambda_3\varphi^2}{v^2}\right] \\
& + \frac{1}{64\pi^2}F_+^2 \ln\left[\frac{F_+}{v^2}\right] + \frac{1}{64\pi^2}F_-^2 \ln\left[\frac{F_-}{v^2}\right] \\
& - \frac{6}{32\pi^2} g_t^4 H^4 \ln\left[\frac{H^2}{v^2}\right] - \frac{1}{32\pi^2} Y_M^4 \varphi^4 \ln\left[\frac{\varphi^2}{v^2}\right]
\end{aligned}
$$

with $H^2 \equiv \Phi^\dagger\Phi$ **and** $\varphi^2 \equiv \phi^\dagger\phi$

$$
F_\pm(H,\varphi) \equiv \frac{3\lambda_1+\lambda_2}{4}H^2 + \frac{3\lambda_3+\lambda_2}{4}\varphi^2 \pm \sqrt{\left[\frac{3\lambda_1-\lambda_2}{4}H^2 - \frac{3\lambda_3-\lambda_2}{4}\varphi^2\right]^2 + \lambda_2^2\varphi^2 H^2}
$$

Numerical analysis

Choice of parameters *strongly constrained* by experimental data and RGE analysis → 'trial and error' →

$$\lambda_1 = 3.77\,, \quad \lambda_2 = 3.72\,, \quad \lambda_3 = 3.73\,, \quad g_t = 1\,, \quad Y_M^2 = 0.4$$

Minimum lies at

$$\langle H \rangle = 2.74 \cdot 10^{-5}\, v\ , \quad \langle \varphi \rangle = 1.51 \cdot 10^{-4}\, v$$

Normalize this by setting $\langle H \rangle = 174$ GeV $\quad \Rightarrow$

$$H' = H\cos\beta + \varphi\sin\beta\ , \qquad \varphi' = -H\sin\beta + \varphi\cos\beta$$

$$m_{H'} = 207\ \text{GeV}, \qquad m_{\varphi'} = 477\ \text{GeV}; \qquad \sin\beta = 0.179$$

'Higgs mixing': only the components along H of the mass eigenstates couple to the usual SM particles.

Neutrino mass eigenvalues: with $|Y_\nu| < 10^{-5}$ we get

$$m_\nu^{(1)} = \frac{(Y_\nu \langle H \rangle)^2}{Y_M \langle \varphi \rangle} < 1\,\text{eV}\ \ , \quad m_\nu^{(2)} = Y_M \langle \varphi \rangle \sim 440\,\text{GeV}$$

Renormalization Group Equations

With

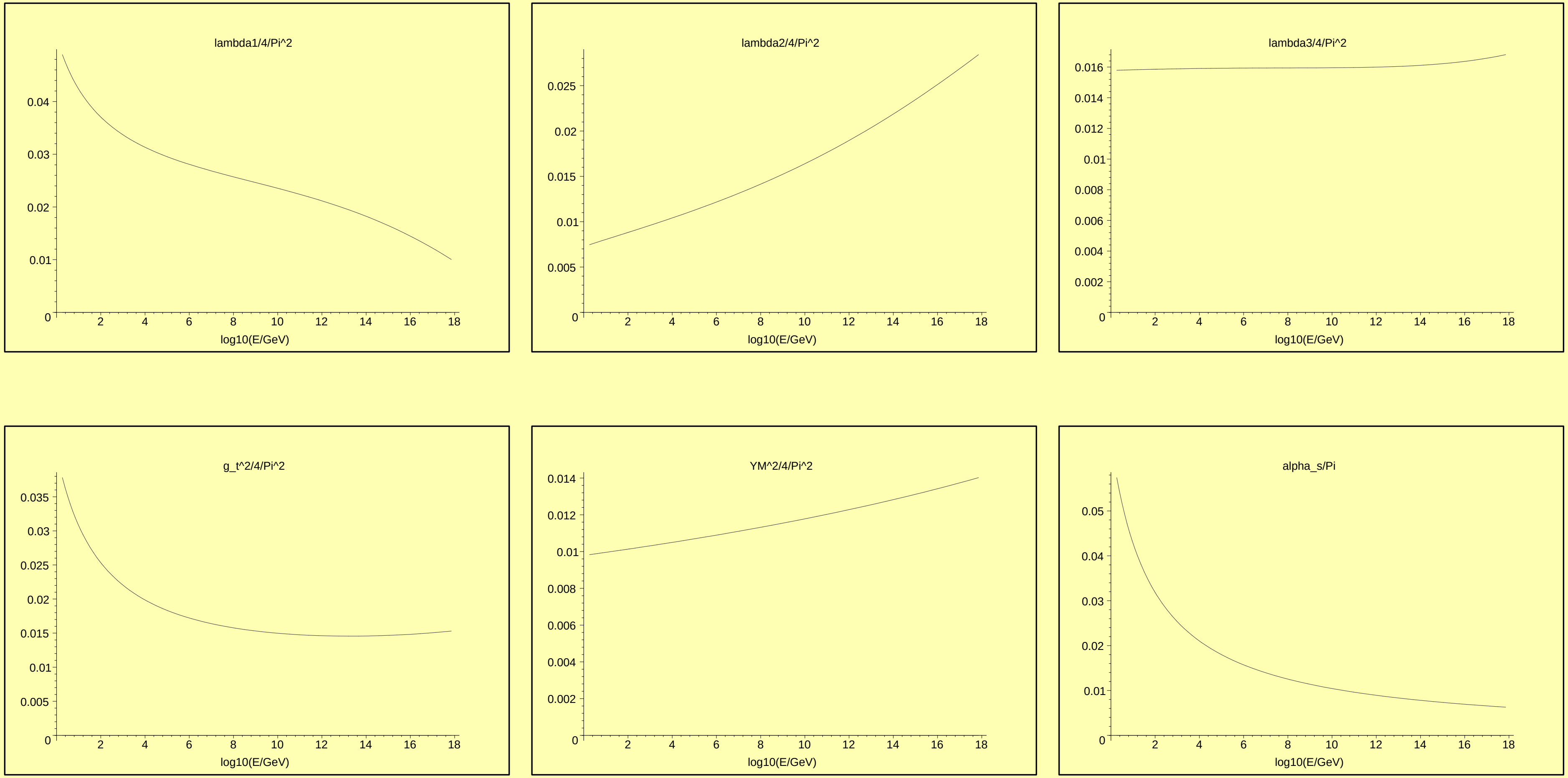
$$y_1 = \frac{\lambda_1^{\text{eff}}}{4\pi^2},\; y_2 = \frac{\lambda_2^{\text{eff}}}{4\pi^2},\; y_3 = \frac{\lambda_3^{\text{eff}}}{4\pi^2},\; x = \frac{g_t^2}{4\pi^2},\; u = \frac{Y_M^2}{4\pi^2},\; z_3 = \frac{\alpha_s}{\pi},\; z_2 = \frac{\alpha_w}{\pi}$$

we get

$$\begin{aligned}
\mu\frac{\mathrm{d}y_1}{\mathrm{d}\mu} &= \frac{3}{2}y_1^2 + \frac{1}{8}y_2^2 - 6x^2 + \frac{9}{8}z_2^2\;, \\
\mu\frac{\mathrm{d}y_2}{\mathrm{d}\mu} &= \frac{3}{8}y_2\left(2y_1 + y_3 + \frac{4}{3}y_2\right), \\
\mu\frac{\mathrm{d}y_3}{\mathrm{d}\mu} &= \frac{9}{8}y_3^2 + \frac{1}{2}y_2^2 - u^2, \qquad \mu\frac{\mathrm{d}u}{\mathrm{d}\mu} = \frac{3}{4}u^2 \\
\mu\frac{\mathrm{d}x}{\mathrm{d}\mu} &= \frac{9}{4}x^2 - 4xz_3, \qquad \mu\frac{\mathrm{d}z_3}{\mathrm{d}\mu} = -\frac{7}{2}z_3^2, \qquad \mu\frac{\mathrm{d}z_2}{\mathrm{d}\mu} = -\frac{19}{12}z_2^2
\end{aligned}$$

Start running at μ_0 for which $\lambda^{\text{eff}} \sim \lambda(\mu_0) \;\leftrightarrow\; \mu_0 \sim \langle H\rangle$.

RG Evolution of Coupling Constants

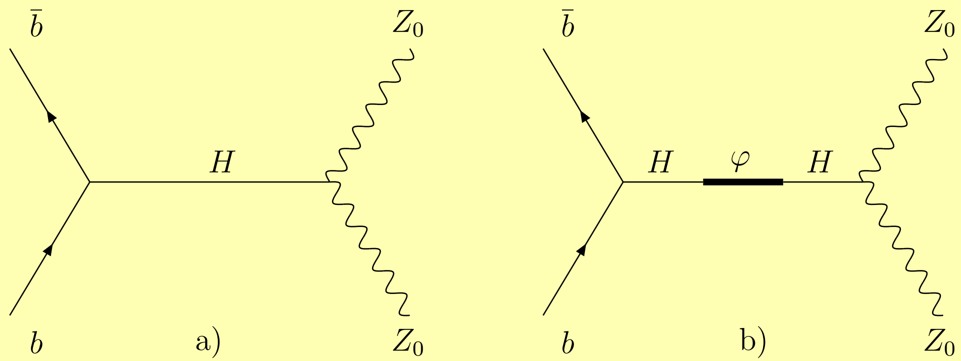


Discussion

- **All couplings stay bounded up to $\mathcal{O}(10^{20}\,\mathrm{GeV})$**
- **No instabilities up to $\mathcal{O}(10^{20}\,\mathrm{GeV})$**
- **Thus: model may remain viable up to Planck scale**
- **Caveats: Numerics? Higher order corrections? Expect better estimates from RG improvement $\rightarrow$ requires study of multi-field case**
- **Can we arrange $\Lambda_{\mathrm{IR}} \sim \mathcal{O}(1\,\mathrm{GeV})$ with $\langle H \rangle \sim \mathcal{O}(200\,\mathrm{GeV})$?**
- **Key question: do the observed values of couplings for (minimally extended) SM conspire to make this work? Could become an experimental question....**

An unmistakable signature?

New scalar φ = 'fat twin brother' of SM Higgs!



Resonant production for $m_{H'} > 2m_Z$ **and** $m_{\varphi'} > 2m_Z$

Identical branching ratios: 'shadow Higgs'

Decay widths $\Gamma_{H'} \propto \cos^2\beta$ **and** $\Gamma_{\varphi'} \propto \sin^2\beta \quad \Rightarrow$

for large $m_{\varphi'}$ **second resonance *narrow* if** β **small.**

Thus: 'twin peaks' at unusual mass values $> 200\,\mathrm{GeV}$**!**

Cf.: 'Veltman's window' for SM: $m_H \sim \mathcal{O}(190\mathrm{GeV})$ **; MSSM:** $m_H < \mathcal{O}(135\mathrm{GeV})$

Neutrinos and Axions

There are two global $U(1)$ symmetries, baryon number and (modified) lepton number symmetry with

$$\mathcal{J}_L^\mu = \sum_{i=1,2,3} \bar{e}^i \gamma^\mu e^i + \sum_{i=1,2,3} \bar{\nu}^i \gamma^\mu \nu^i - 2i\phi^\dagger \overleftrightarrow{\partial^\mu} \phi$$

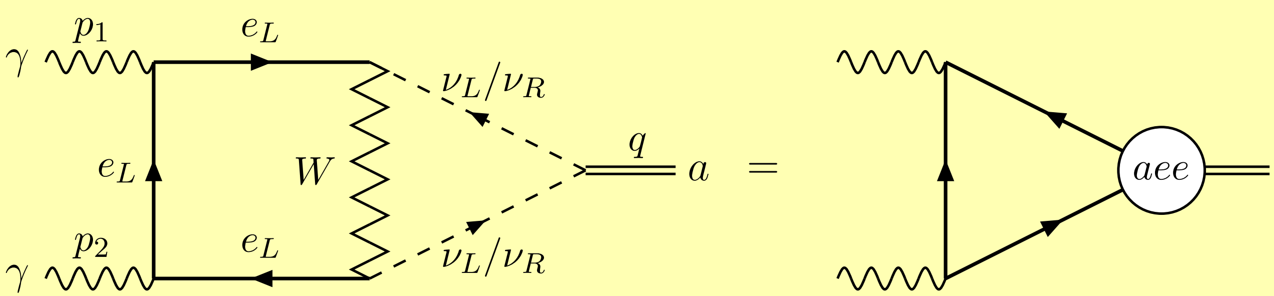
With $\mu = \langle \varphi \rangle \neq 0$ the 'leakage term' is $\propto \partial^\mu a + \ldots$ and $a(x)$ becomes a (pseudo-)Goldstone boson = 'Majoron'.

If we identify axion = Majoron we get (at two loops)

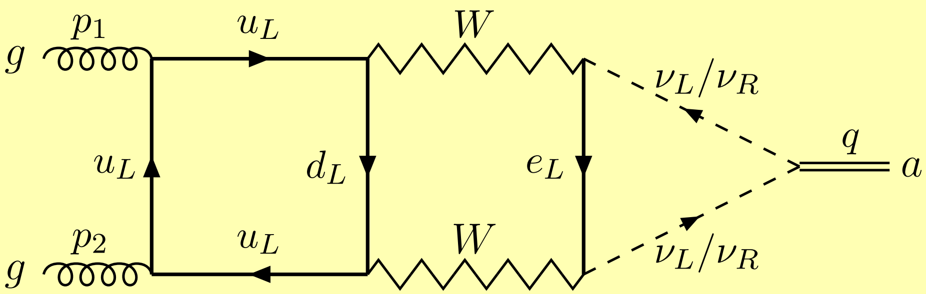
$$f_a = \frac{2\pi^2 m_W^2}{\alpha_w \alpha_{em} m_\nu Y_M} \sim \mathcal{O}(10^{15}\,\mathrm{GeV})$$

and analogous result for gluonic couplings of $a(x)$.

$\Rightarrow$ Smallness of axionic couplings to SM particles explained by neutrino mixing and smallness of m_ν.



Axion-photon-photon effective vertex



Axion-gluon-gluon effective vertex

Conformal invariance from gravity?

Here *not* from scale (Weyl) invariant gravity, but:

$N = 4$ supergravity $\qquad 1[2] \oplus 4[\frac{3}{2}] \oplus 6[1] \oplus 4[\frac{1}{2}] \oplus 2[0]$

coupled to n vector multiplets $\qquad n \times \{1[1] \oplus 4[\frac{1}{2}] \oplus 6[0]\}$

Gauged $N = 4$ SUGRA: [Bergshoeff,Koh,Sezgin; de Roo,Wagemans (1985)]

- **Scalars** $\phi(x) = \exp(L_I{}^A T^I{}_A) \in SO(6,n)/SO(6) \times SO(n)$
- **YM gauge group** $G_{\rm YM} \subset SO(6,n)$ **with dim** $G_{\rm YM} = n + 6$

[Example inspired by 'Groningen derivation' of conformal M2 brane ('Bagger-Lambert') theories from gauged $D = 3$ SUGRAs]

Although this theory is *not* conformally invariant, the conformally invariant $N = 4$ SUSY YM theory nevertheless emerges as a $\kappa \to 0$ limit, which 'flattens' spacetime (with $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$) and coset space

$$SO(6,n)/((SO(6) \times SO(n)) \;\longrightarrow\; \mathbb{R}^{6n} \ni \phi^{[ij]\,a}(x)$$

Exemplify this claim for scalar potential: with

$$C_{ai}{}^{j} = \kappa^2 f_{abc} \phi_{[ik]}{}^{b} \phi^{[jk]\,c} + \mathcal{O}(\kappa^3) \quad , \quad C_{ij} = \kappa^3 f_{abc} \phi_{[ik]}{}^{a} \phi^{b\,[kl]} \phi_{[lj]}{}^{c} + \mathcal{O}(\kappa^4)$$

potential of gauged theory is ($m, n = 1, \dots, 6$; $\kappa|z| < 1$)

$$V(\phi) = \frac{1}{\kappa^4} \frac{(1-\kappa z)(1-\kappa z^*)}{1-\kappa^2 z z^*} \left(C_{ai}{}^{j} C^{ai}{}_{j} - \frac{4}{9} C^{ij} C_{ij} \right) = \mathrm{Tr}\,[X_m, X_n]^2 + \mathcal{O}(\kappa)$$

Idem for all other terms in Lagrangian! Unfortunately

- $N = 4$ **SYM is quantum mechanically conformal theory** $\rightarrow$ **no conformal anomaly** $\rightarrow$ **no symmetry breaking!**
- **Thus need non-supersymmetric vacuum with** $\Lambda = 0$

Finiteness **of quantum (super)gravity** $\rightarrow$
Can gravity serve as a *universal regulator*?

Conformal anomaly as a finite 'remnant' of quantum gravity $\sim \kappa^s \int^{1/\kappa} (\dots) = \mathcal{O}(1)$**?**

Outlook

- **Scheme is consistent with all available data and seems *more economical* than MSSM-type models**
- **New features (for *classically conformal* theories):**
  - **restoration of convexity via RG improvement**
  - **an unsuspected link between weak scale and Λ_{QCD}**
  - **symmetry breaking becomes *mandatory* for $\Lambda_{\rm IR} > 0$**
- **Conformal invariance may still be the 'best' explanation why we live in $D = 4$ space-time dimensions.**
- **Main role of SUSY might be at M_{Pl} in rendering quantum gravity (perturbatively) consistent.**
- **Emergence of conformally invariant theory from gravity (which is *not* conformally invariant)?**
- **'Grand desert' may possibly provide us with an unobstructed view of Planck scale physics!**