Multi-leg one-loop amplitudes with D-dimensional unitarity cuts

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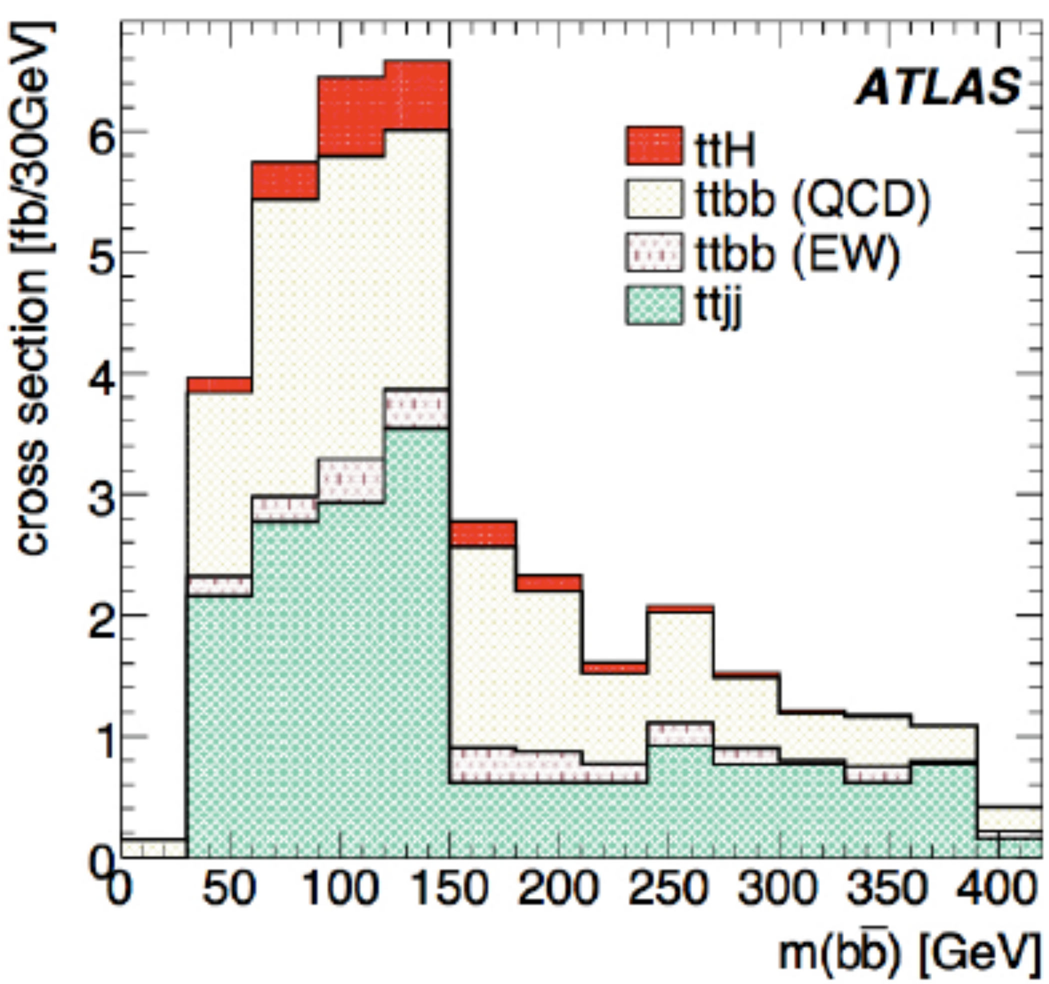
5 February 2009
PSI seminar

Motivation for NLO

- reduced theoretical uncertainty coming from higher order terms
- improved scale dependence
- contributions from new channels



- used to be important for low Higgs mass searches
- the ttbb LO uncertainty is huge
- a working NLO MC is necessary

experimentalist's ghost

An experimenter's wishlist

Run II Monte Carlo Workshop

Single Boson	Diboson	Triboson	Heavy Flavour
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}\leq 3j$	$W+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}\leq 3j$	$W+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$Z+b\bar{b}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{b}\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$	$ZZZ+\leq 3j$	$b\bar{b}+\leq 3j$
$\gamma+b\bar{b}\leq 3j$	$\gamma\gamma+b\bar{b}\leq 3j$		single top
$\gamma+c\bar{c}\leq 3j$	$\gamma\gamma+c\bar{c}\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}\leq 3j$		
	$WZ+c\bar{c}\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

The 'upgraded experimentalist's wishlist for LHC'

The NLO multileg working group summary report (Les Houches 2007) 0803.0494

Calculations remaining from Les Houches 2005	
4. $pp \to t\bar{t}\, b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to VV\, b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [10–12]
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures

The missing ingredient

- tree-level amplitude known
- real radiation treatment known
- phase-space generation in principle possible
- one-loop amplitudes the bottleneck

OPP reduction

- classical attack: reduce every one loop diagram to scalar integrals and evaluate those. Works with a limited number of external legs.
- OPP reduction: reduce the integrand of every one loop diagram.

Ossola, Papadopoulos, Pittau, hep-ph/0609007

OPP reduction

Reduced amplitude

$$A^{D_s=4}(p_1, p_2, \ldots, p_N) = \sum_{Q=\{i_1,i_2,i_3,i_4\}} d_Q \int [dl] \frac{1}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{T=\{i_1,i_2,i_3\}} c_T \int [dl] \frac{1}{d_{i_1} d_{i_2} d_{i_3}}$$
$$+ \sum_{B=\{i_1,i_2\}} b_B \int [dl] \frac{1}{d_{i_1} d_{i_2}} + \sum_{S=\{i_1\}} a_S \int [dl] \frac{1}{d_{i_1}}$$

Before the loop integration

$$A^{D_s=4}(p_1, p_2, \ldots, p_N) = \sum_{Q=i_1,i_2,i_3,i_4} \int [dl] \frac{\bar{d}_Q(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{T=i_1,i_2,i_3} \int [dl] \frac{\bar{c}_T(l)}{d_{i_1} d_{i_2} d_{i_3}}$$
$$+ \sum_{B=i_1,i_2} \int [dl] \frac{\bar{b}_B(l)}{d_{i_1} d_{i_2}} + \sum_{S=i_1} \int [dl] \frac{\bar{a}_S(l)}{d_{i_1}}$$

OPP reduction

$$\bar{d}(l) = d_0 + d_1(l \cdot n_1)$$

The functional form of coefficients known

$$\frac{N(l)}{d_1(l)\dots d_N(l)} = \sum_i \frac{\bar{d}_Q(l)}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}} + \sum_i \frac{\bar{c}_T(l)}{d_{i_1}d_{i_2}d_{i_3}} + \sum_i \frac{\bar{b}_B(l)}{d_{i_1}d_{i_2}} + \sum_i \frac{\bar{a}_S(l)}{d_{i_1}}$$

loop-momentum dependent

- Use numerical values for loop momentum to construct a system of equations for coefficients

OPP reduction

$$\frac{N(l)}{d_1(l)\ldots d_N(l)} d_i d_j d_k d_m = +\sum_i \frac{\bar{d}_Q(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} d_i d_j d_k d_m$$

$$+\sum_i \frac{\bar{c}_T(l)}{d_{i_1} d_{i_2} d_{i_3}} d_i d_j d_k d_m$$

$$+\sum_i \frac{\bar{b}_B(l)}{d_{i_1} d_{i_2}} d_i d_j d_k d_m$$

$$+\sum_i \frac{\bar{a}_S(l)}{d_{i_1}} d_i d_j d_k d_m$$

Set the loop momentum such that the four propagators vanish

OPP reduction

$$\frac{N(l)}{d_1(l)\dots d_N(l)} d_i d_j d_k d_m = +\sum_i \frac{\bar{d}_Q(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} d_i d_j d_k d_m$$

$$+\sum_i \frac{\bar{c}_T(l)}{d_{i_1} d_{i_2} d_{i_3}} d_i d_j d_k d_m$$

$$+\sum_i \frac{\bar{b}_B(l)}{d_{i_1} d_{i_2}} d_i d_j d_k d_m$$

$$+\sum_i \frac{\bar{a}_S(l)}{d_{i_1}} d_i d_j d_k d_m$$

Set the loop momentum such that the four propagators vanish

OPP reduction

$$\frac{N(\hat{l})}{d_1(\hat{l})\ldots d_N(\hat{l})} d_i(\hat{l}) d_j(\hat{l}) d_k(\hat{l}) d_m(\hat{l}) = \bar{d}_{i,j,k,m}(\hat{l})$$

$$\bar{d}(l) = d_0 + d_1(l \cdot n_1) \longrightarrow$$

We need two loop momenta that solve the constraints: a 2x2 system.

OPP reduction

$$\frac{N(l)}{d_1(l)\dots d_N(l)} = \sum_i \frac{\bar{d}_Q(l)}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}} + \sum_i \frac{\bar{c}_T(l)}{d_{i_1}d_{i_2}d_{i_3}} + \sum_i \frac{\bar{b}_B(l)}{d_{i_1}d_{i_2}} + \sum_i \frac{\bar{a}_S(l)}{d_{i_1}}$$

Proceed similarly to triangles, bubbles and tadpoles

OPP reduction

- relies on the knowledge of the functional form of the integrand coefficients.
- depends on the size of the numerator of the integrand.
- ε-dependent piece of numerator has to be explicitly put in (on a diagram by dagram base).

OPP reduction

- Implemented in CutTools Ossola, Papadopoulos, Pittau 0711.3596
- Tri-vector boson production without much sweat. Ossola, Papadopoulos, Pittau 0711.3596 (Melnikov, Petriello, AL : ZZZ ~25min/psp)
- Six-photons Binoth, Ossola, Papadopoulos, Pittau 0804.0350

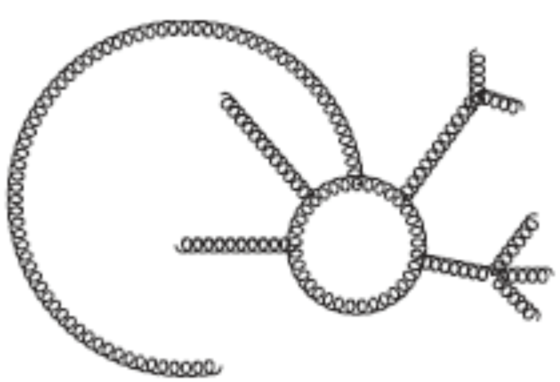
Unitarity cuts

Ellis, Giele, Kunszt 0708.2398

- Idea: the left hand side of OPP is actually a product of tree-level amplitudes.
- So one doesn't need Feynman diagrams at all.
- Instead a whole (gauge-invariant) class of color-ordered diagrams is computed.

Color decomposition

$$A_{N,full} = \sum_{c=1}^{[N/2]+1} \sum_{\sigma \in S_N/S_{N;c}} Gr_{N;c}(\sigma) A_{N;c}$$



$$Gr_{N;c}(\sigma) = Tr(t^{a_{\sigma(1)}} \ldots t^{a_{\sigma(c-1)}}) Tr(t^{a_{\sigma(c)}} \ldots t^{a_{\sigma(N)}})$$

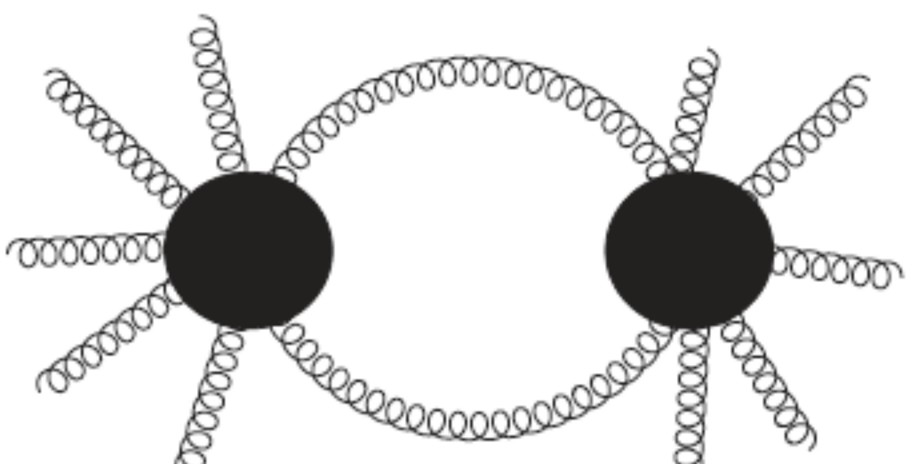
$$Gr_{N;1} = N_c Tr(t^{a_{\sigma(1)}} \ldots t^{a_{\sigma(N)}})$$

- Not knowing how to dress cuts with colors, let’s calculate color-ordered graphs
- Bonus from indistinguishable final state partons

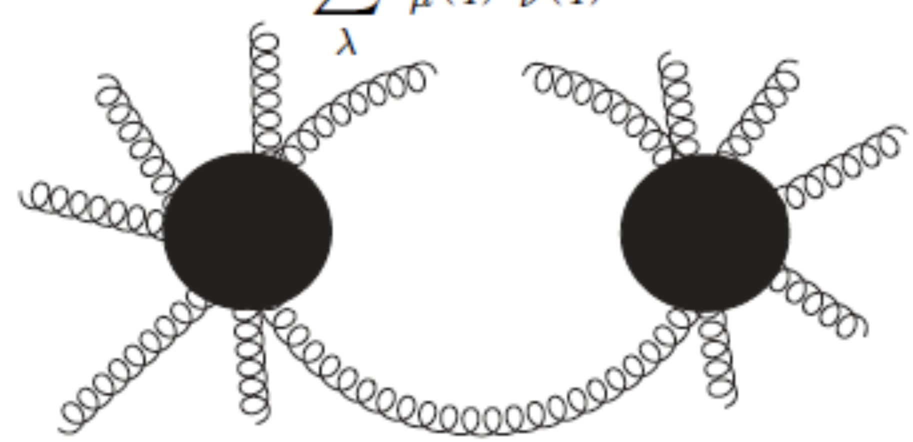
Unitarity cuts

$$\int [dl] A(p_1, p_2, \ldots, p_N; l) d_\alpha d_\beta d_\gamma d_\delta \; \delta^D(l - \hat{l}) = \sum_i \int [dl] \frac{\bar{d}_Q(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} d_\alpha d_\beta d_\gamma d_\delta \; \delta^D(l - \hat{l})$$
$$+ \sum_i \int [dl] \frac{\bar{c}_T(l)}{d_{i_1} d_{i_2} d_{i_3}} d_\alpha d_\beta d_\gamma d_\delta \; \delta^D(l - \hat{l})$$
$$+ \sum_i \int [dl] \frac{\bar{b}_B(l)}{d_{i_1} d_{i_2}} d_\alpha d_\beta d_\gamma d_\delta \; \delta^D(l - \hat{l})$$
$$+ \sum_i \int [dl] \frac{\bar{a}_S(l)}{d_{i_1}} d_\alpha d_\beta d_\gamma d_\delta \; \delta^D(l - \hat{l})$$

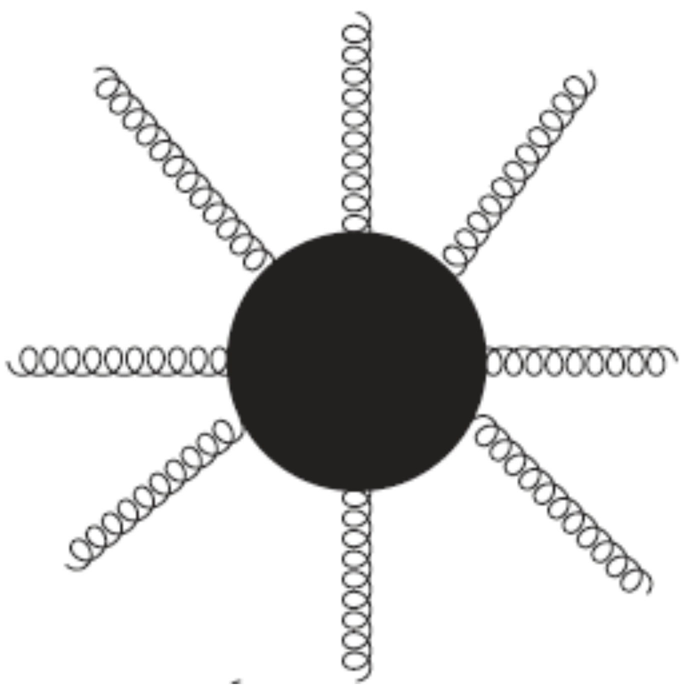
$$\frac{-g_{\mu\nu} + \frac{q_\mu n_\nu + q_\nu n_\mu}{q \cdot n}}{q^2} \cdot q^2 \delta(q^2)$$



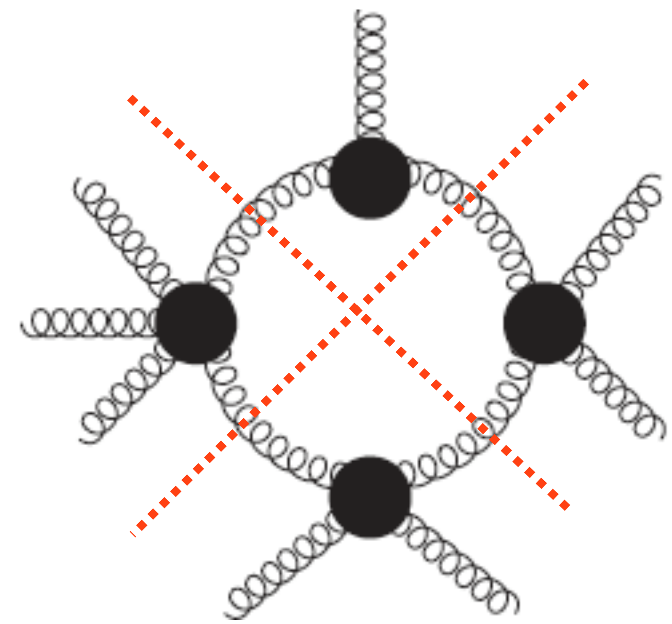
$$\sum_\lambda \epsilon^\lambda_\mu(q) \epsilon^\lambda_\nu(q)$$



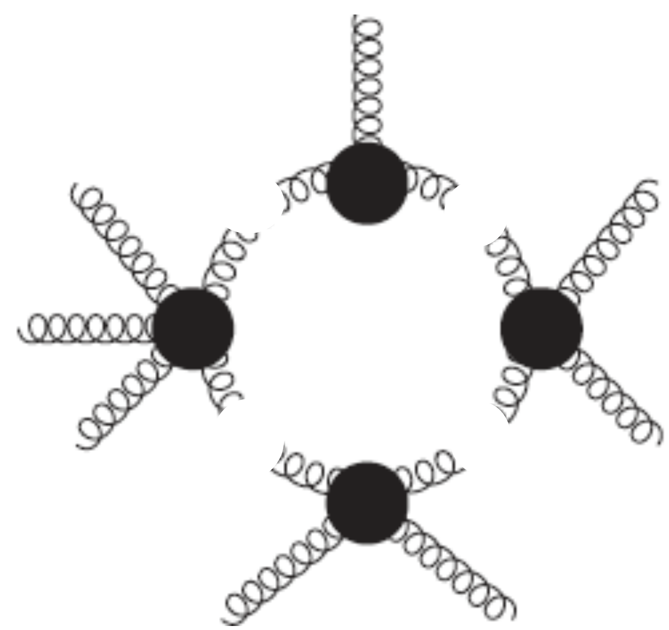
Unitarity cuts



Quadruple cut



$$\bar{d}_{\alpha\beta\gamma\delta}(\hat{l}) =$$



Unitarity cuts

$$A^{D_s}(p_1, p_2, \ldots, p_N) = \int [dl] \frac{N^{D_s}(p_1, p_2, \ldots, p_N; l)}{d_1 d_2 \ldots d_N}$$

$$d_1 = l^2 \quad d_{i>1} = (l + p_1 + p_2 + \ldots + p_{i-1})^2$$

- But unitarity cuts evaluate the branch-cut containing part of the amplitude.
- How to find the remaining, rational part without spoiling the simplicity of the algorithm ?

Ds-dim unitarity cuts

Ellis, Giele, Kunszt, Melnikov 0801.2237

$$A^{D_s}(p_1, p_2, \ldots, p_N) = \int [dl] \frac{N^{D_s}(p_1, p_2, \ldots, p_N; l)}{d_1 d_2 \ldots d_N}$$

$$d_1 = l^2 \quad d_{i>1} = (l + p_1 + p_2 + \ldots + p_{i-1})^2$$

$$A^{D_s} = A^0 + A^1 \cdot D_s$$

Perform the reduction twice, with integer Ds

Ds-dim unitarity cuts

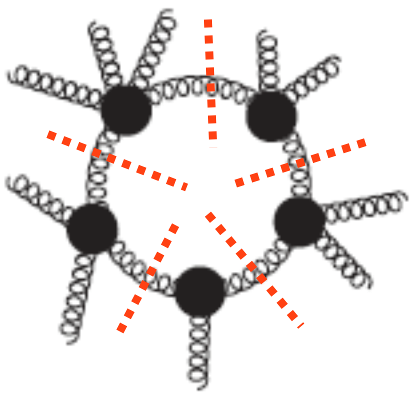
$$A^{D_s}(p_1, p_2, \ldots, p_N; l) = \sum_i \frac{\bar{e}_E(l)}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}d_{i_5}} + \sum_i \frac{\bar{d}_Q(l)}{d_{i_1}d_{i_2}d_{i_3}d_{i_4}}$$
$$+ \sum_i \frac{\bar{c}_T(l)}{d_{i_1}d_{i_2}d_{i_3}} + \sum_i \frac{\bar{b}_B(l)}{d_{i_1}d_{i_2}} + \sum_i \frac{\bar{a}_S(l)}{d_{i_1}}$$

Need pentagons (and pentuple cuts)

The algorithm for gluons

- color-ordered amplitudes
- helicities fixed
- external momenta fixed
- Ds=5,6

Pentuple cuts



$$\mathcal{X}^{D_s}_{\alpha\beta\gamma\delta\epsilon}(\hat{l}) = \bar{e}_{\alpha\beta\gamma\delta\epsilon}(\hat{l})$$

$$d_\alpha = l^2 \quad d_\beta = (l{+}Q_1)^2 \quad d_\gamma = (l{+}Q_1{+}Q_2)^2 \quad d_\delta = (l{+}Q_1{+}Q_2{+}Q_3)^2 \quad d_\epsilon = (l{+}Q_1{+}Q_2{+}Q_3{+}Q_4)^2$$

$$\hat{l}^\mu = V^\mu + \sqrt{-V^2} n_5^\mu \qquad V^\mu = x_i v_i^\mu \qquad v_i \cdot Q_j = \delta_{ij}$$

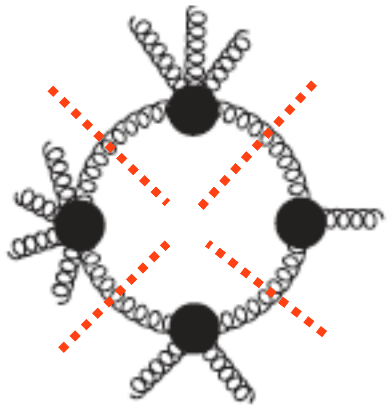
Vermaseren - Van Neerven

$$\bar{e}(l) = e_0$$

One $\hat{l}$ for pentuple cut

Scalar pentagon reduced to boxes

Quadruple cuts



$$\bar{d}_{\alpha\beta\gamma\delta}(\hat{l}) = \mathcal{X}^{D_s}_{\alpha\beta\gamma\delta}(\hat{l}) - \sum_{\epsilon\neq\alpha\beta\gamma\delta} \frac{e_{\alpha\beta\gamma\delta\epsilon}(\hat{l})}{d_\epsilon(\hat{l})}$$

$$d_\alpha = l^2 \quad d_\beta = (l+Q_1)^2 \quad d_\gamma = (l+Q_1+Q_2)^2 \quad d_\delta = (l+Q_1+Q_2+Q_3)^2$$

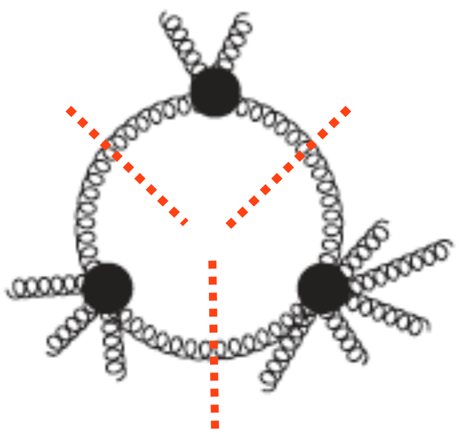
$$\hat{l}^\mu = V^\mu + a_1 n_1^\mu + a_5 n_5^\mu$$

$$a_1^2 + a_5^2 + V^2 = 0$$

$$\bar{d} = d_0 + d_1 s_1 + d_2 s_e^2 + d_3 s_1 s_e^2 + d_4 s_e^4$$

Five $\hat{l}$ for quadruple cut, of which three in D=5

Triple and Double cuts



$$\bar{c}_{\alpha\beta\gamma}(\hat{l}) = \mathcal{X}^{D_s}_{\alpha\beta\gamma}(\hat{l}) - \sum_{\delta,\epsilon\neq\alpha\beta\gamma} \frac{\bar{e}_{\alpha\beta\gamma\delta\epsilon}(\hat{l})}{d_\epsilon(\hat{l})d_\delta(\hat{l})} - \sum_{\delta\neq\alpha\beta\gamma} \frac{\bar{d}_{\alpha\beta\gamma\delta}(\hat{l})}{d_\delta(\hat{l})}$$

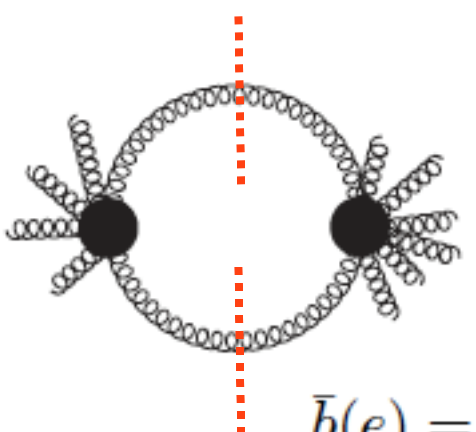
$$\hat{l}^\mu = V^\mu + a_1 n_1^\mu + a_2 n_2^\mu + a_5 n_5^\mu$$

$$s_i = l \cdot n_i$$

$$s_e^2 = \sum_{i=4..D} (l\cdot n_i)^2$$

$$\bar{c}(l) = c_0 + c_1 s_1 + c_2 s_2 + c_3(s_1^2 - s_2^2) + s_1 s_2(c_4 + c_5 s_1 + c_6 s_2) + c_7 s_1 s_e^2 + c_8 s_2 s_e^2 + c_9 s_e^4$$

10 $\hat{l}$ for triple cut of which three in D=5



$$\bar{b}_{\alpha\beta}(\hat{l}) = \mathcal{X}^{D_s}_{\alpha\beta}(\hat{l}) - \sum_{\gamma\delta,\epsilon\neq\alpha\beta} \frac{\bar{e}_{\alpha\beta\gamma\delta\epsilon}(\hat{l})}{d_\epsilon(\hat{l})d_\delta(\hat{l})d_\gamma(\hat{l})} - \sum_{\gamma\delta\neq\alpha\beta} \frac{\bar{d}_{\alpha\beta\gamma\delta}(\hat{l})}{d_\delta(\hat{l})d_\gamma(\hat{l})} - \sum_{\gamma\neq\alpha\beta} \frac{\bar{c}_{\alpha\beta\gamma}(\hat{l})}{d_\gamma(\hat{l})}$$

$$l^\mu = V^\mu + a_1 n_1^\mu + a_2 n_2^\mu + a_3 n_3^\mu + a_5 n_5^\mu$$

$$\bar{b}(e) = b_0 + b_1 s_1 + b_2 s_2 + b_3 s_3 + b_4(s_1^2 - s_3^2) + b_5(s_2^2 - s_3^2) + b_6 s_1 s_2 + b_7 s_1 s_3 + b_8 s_2 s_3 + b_9 s_e^2$$

10 $\hat{l}$ for double cut of which one in D=5

The result for fixed Ds

$$A_{cc} = \sum_Q \tilde{d}_{Q,0} I_Q + \sum_T c_{T,0} I_T + \sum_B b_{B,0} I_B$$

$$A_R = -\sum_Q \frac{d_{Q,4}}{6} - \sum_T \frac{c_{T,9}}{2} - \sum_B \frac{b_{B,9}}{6}$$

Terms proportional to $s_e^2 = \sum_{i=4..D_s} (l \cdot n_i)^2$ do not vanish upon integration.

They can be rewritten in terms of integrals in D+2,D+4. They contribute to the rational part.

Necessary ingredient: the one-loop scalar integrals

The current implementation

- C++ implementation of the EGKM algorithm
- from 4 to an arbitrary number of gluons
- only gluons internally (fermions will come soon)

FORTRAN implementation: Giele, Zanderighi 0805.2152

The current implementation

We need Ds=5,6 but we can keep D=5

$$A^6(p_1, \epsilon_1; \ldots; p_n, \epsilon_n) = \epsilon_1^\mu M^6_{\mu\nu} \epsilon_n^\nu$$

When loop momenta polarized along the 6th dimension

$$M^6_{\mu\nu} \to A^6_{sc} g_{\mu\nu}$$

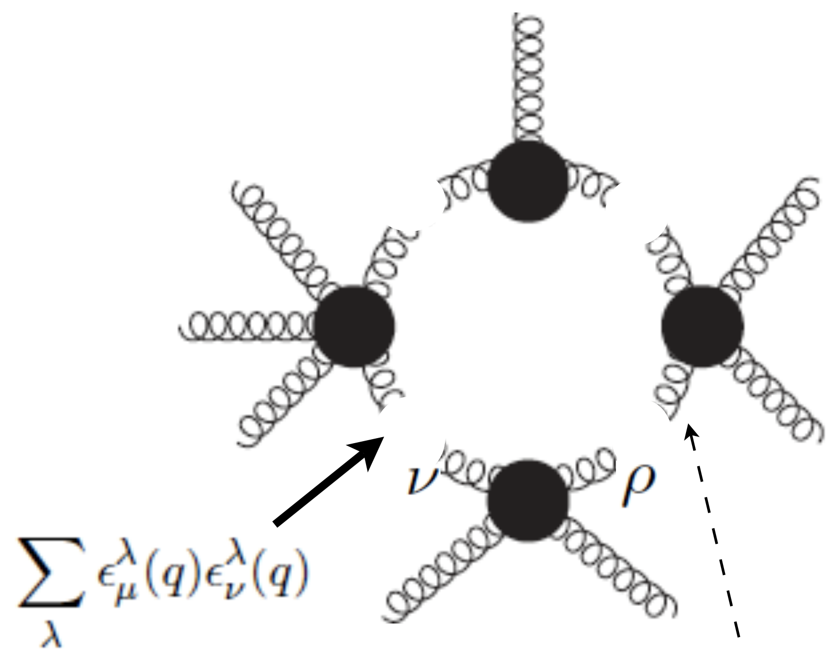
We only need five dimensional tree-level amplitudes with gluons or scalars!

Scalar Integrals: QCDLoop-1.4 Ellis, Zanderighi 0712.1851

The current implementation

Reduces the number of trees needed from X^4 to 4X !!

(X the # of polarization states)

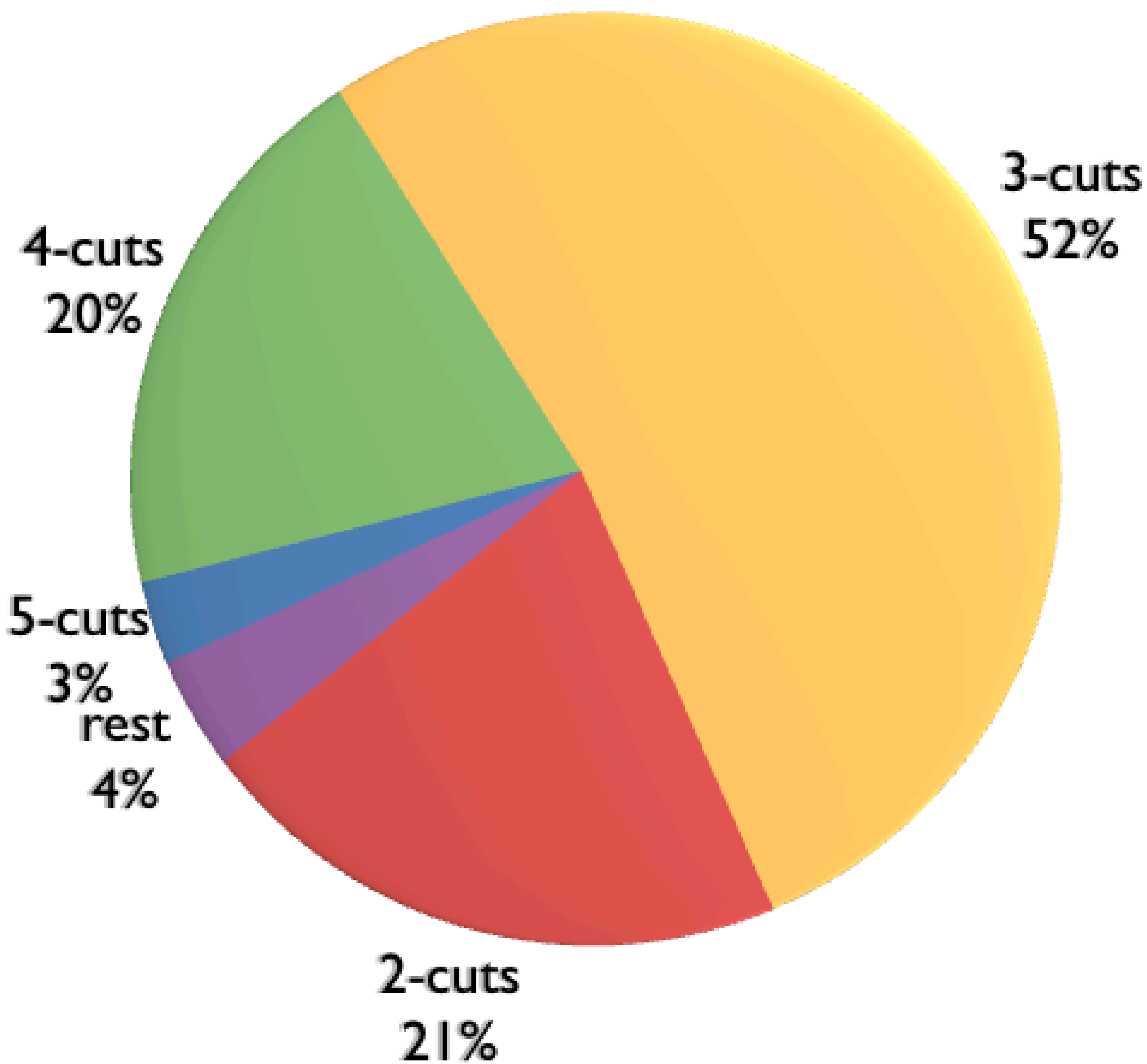


$$\epsilon_\nu^\lambda M^{\nu\rho} = V_1^\rho$$

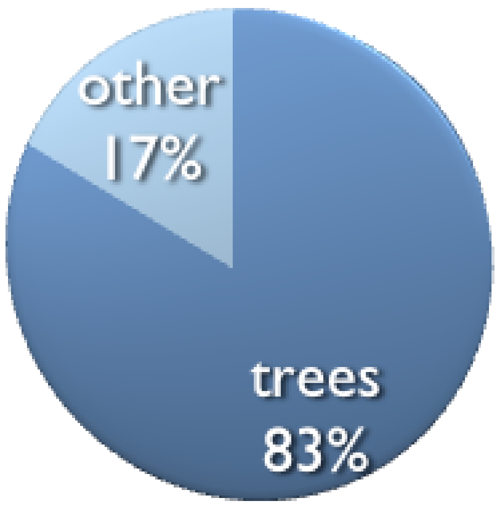
$$\sum \epsilon_\nu^\lambda(\hat{l})\epsilon_\mu^\lambda(\hat{l})M_1^{\mu\rho}D_{\rho\sigma}(\hat{l}+q_1)M_2^{\sigma\tau}D_{\tau\kappa}(\hat{l}+q_{12})M_3^{\kappa\xi}D_{\xi\phi}(\hat{l}+q_{123})M_4^{\phi\nu}$$

The 6 gluon case

CPU time share for 6 gluons



CPU time share of tree-level building blocks



trees	# needed
3	1150
4	850
5	520
6	280

The 6 gluon case

80% of the CPU time spent on tree color amplitudes

The current implementation

Most of the cpu time spent there

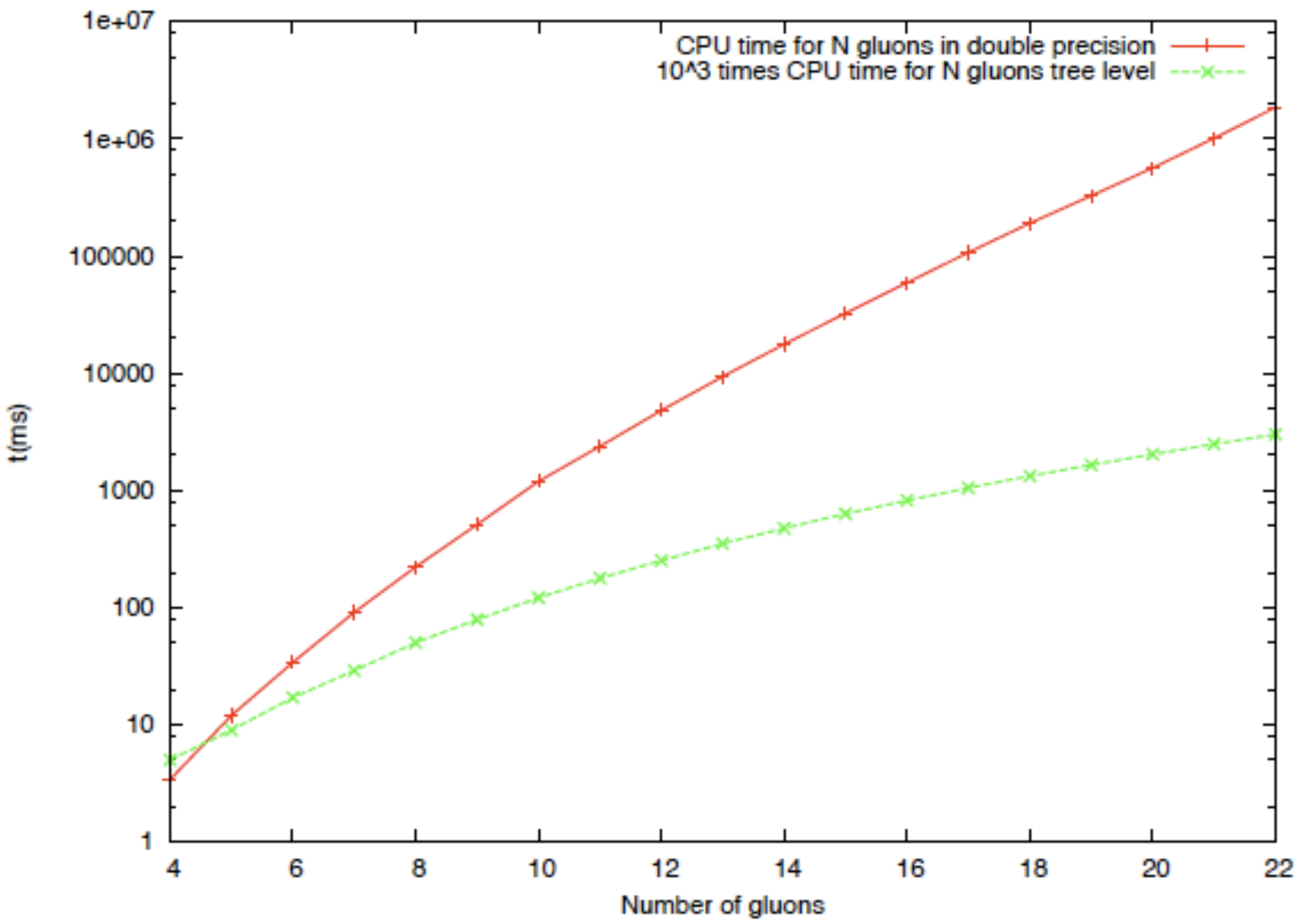
Time for a tree amplitude					
N	t(μs)	N	$t(\mu s)$	N	t(μs)
4	5	11	178	18	1326
5	9	12	252	19	1649
6	17	13	351	20	2032
7	29	14	475	21	2482
8	50	15	631	22	3004
9	79	16	818		
10	121	17	1048		

Fast recursive implementation

Tree-level amplitudes with fixed helicities and color-ordering

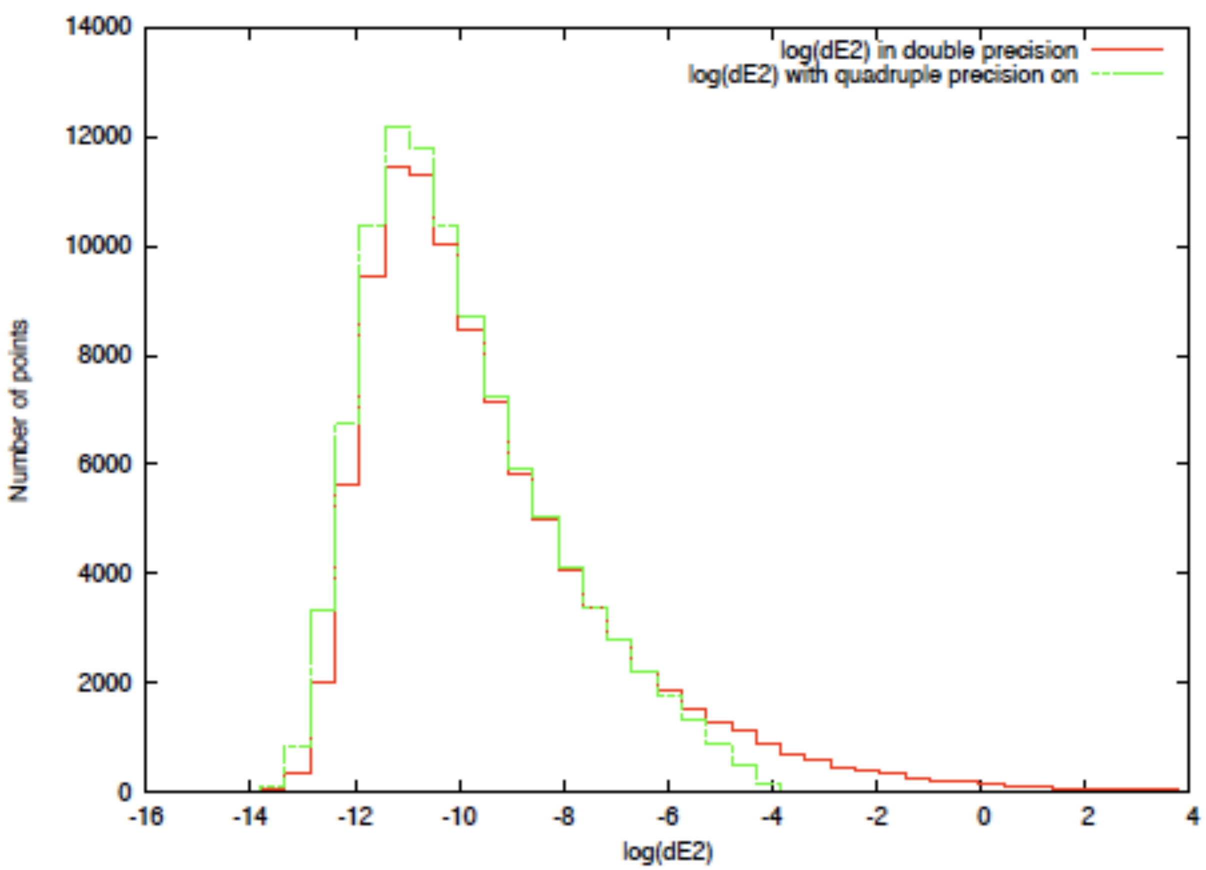
the current implementation

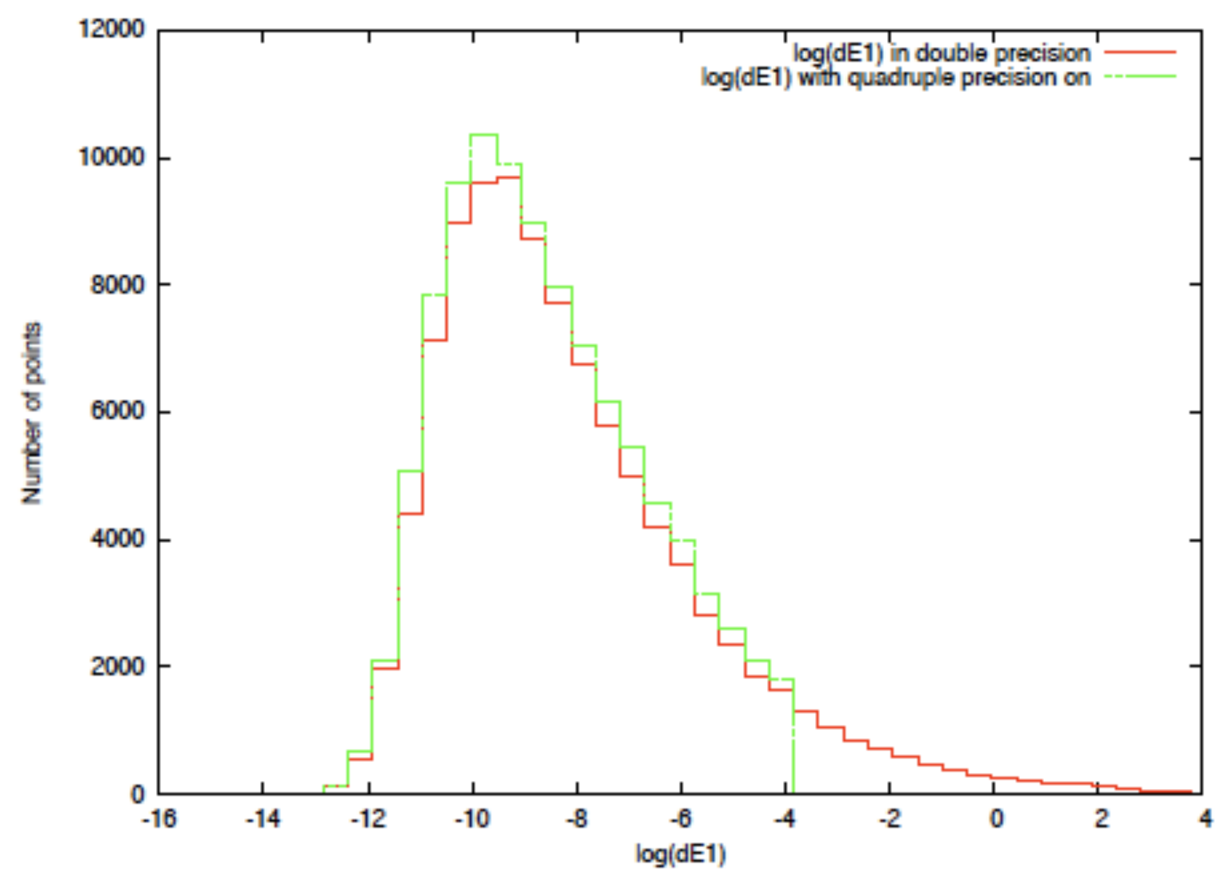
N	t(ms)
4	1.3
5	5
6	18
7	53
8	142
9	359
10	836



Intel Xeon X5450 @3.0GHz

the current implementation





When pole coefficients don't agree with analytic formula QP is switched on

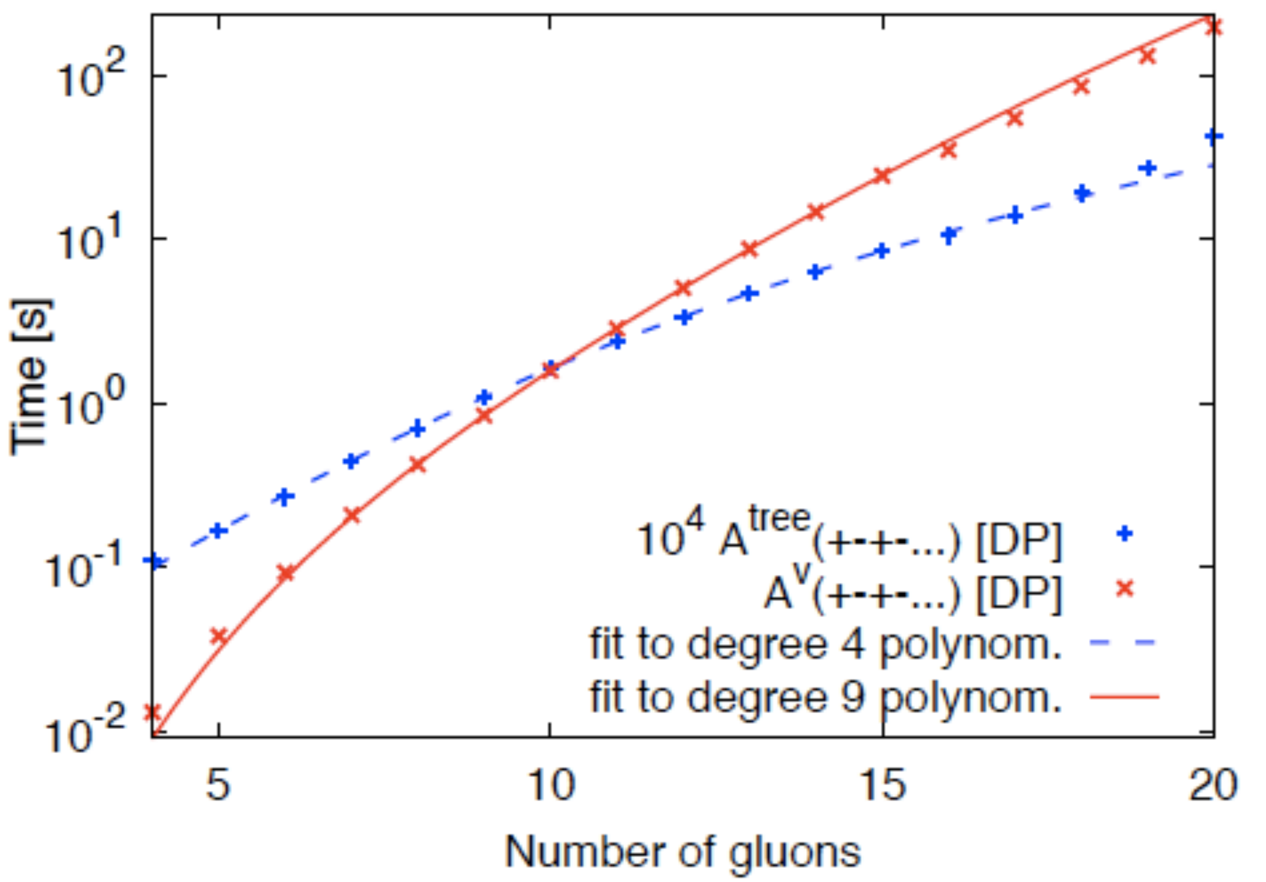
D. H. Bailey, Y. Hida, K. Jeyabalan, X. S. Li and B. Thompson, "ARPREC (C++/Fortran-90 arbitrary precision package)" (http://crd.lbl.gov/~dhbailey/mpdist/)

Quadruple precision

- ~20 times slower!
- not yet optimized (lots of margin for improvement)
- less than 5% of the points for 6 gluons (which would double evaluation time)

Comparisons

helicity	cut part	full amplitude double prec. only	full amplitude with multi-prec.
−−++++	2.4 ms	7 ms	11 ms
−−+++++	4.2 ms	11 ms	23 ms
−−++++++	6.1 ms	29 ms	43 ms
−+−+++	3.1 ms	18 ms	32 ms
−++−++	3.3 ms	61 ms	96 ms
−−−+++	4.4 ms	12 ms	22 ms
−−+−++	5.9 ms	47 ms	64 ms
−+−+−+	7.0 ms	72 ms	114 ms



Black Hat 2.3GHz

Berger, Bern, Dixon, Febres Codero, Forde, Ita, Kosower, Maitre 0803.4180

Rocket 2.8GHz

Giele, Zanderighi 0805.2152

23ms/primitive

90ms/primitive

Fermions

- Color-ordering becomes involved (see left-primitive amplitudes and leading color approximation)
- When e/w bosons are involved, parent diagrams result from primitive parent diagrams by inserting the e/w boson inall possible propagators.
- 6 and 8 dimensions (but 8 reduces trivially to a multiple of six - complicated book-keeping)

Fermion trees

preliminary times for ordered tree level graphs including fermions

	qq+(N-2)g	qq+QQ+(N-4)g	Ng
N	t(μs)	t(μs)	t(μs)
4	4	3.5	5
5	6.5	5.5	9
6	15	11.5	17
7	23	19	29
8	40	31	50
9	63	48	79
10	85	70	121

Fermions

- seems like parents with fermions evaluate as fast as parents with gluons.
- In the leading color approximation time with fermions is expected to be time with gluons multiplied by the number of different primitives per parent.

Time for a full channel

- for 6 gluons (without fermion loops):
  18ms x 64(helicities) x 5(ordering) = 5.7s/pt
  (in 100 machines : 16h for 1Mpts)
- for gg->ttgg there are 30 primitives:
  30 x 16h = 20days (assuming trees with massive fermions are not slower)
- Would it be better with more sophisticated color treatment ?

outlook

- light and heavy fermions soon implemented
- crash test on a physical process
- Together with a fully automatic treatment of real radiation: the way to go for upgrading to NLO existing matrix element generators.