Electroweak corrections to three-jet production at e^+e^- colliders

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28.08.2008

Outline

- Einführung in die Teilchenphysik
- QCD at e^+e^- colliders
- Inventory of the calculation
- Results
- Summary and conclusions

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Grundlagen der (Teilchen)Physik

Fragestellung

- Woraus ist die Materie aufgebaut?
- Was sind die fundamentalen Bausteine?
- Welches sind die Kräfte, die alles zusammenhalten?
- Gibt es eine einheitliche Beschreibung des Ganzen?

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- Empirische Beobachtungen → Experiment
- Mathematische Beschreibung → Theorie

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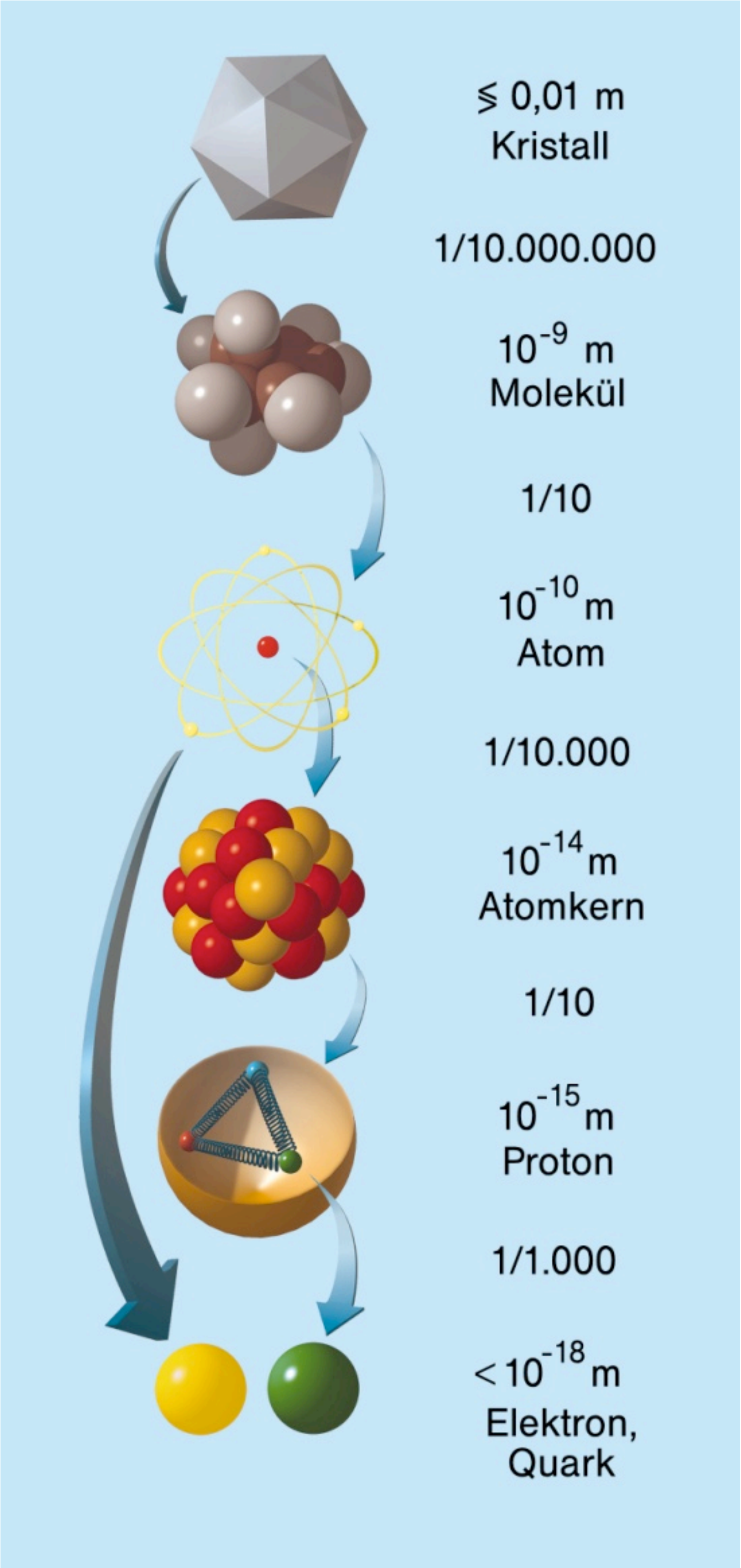
Aufbau der Materie



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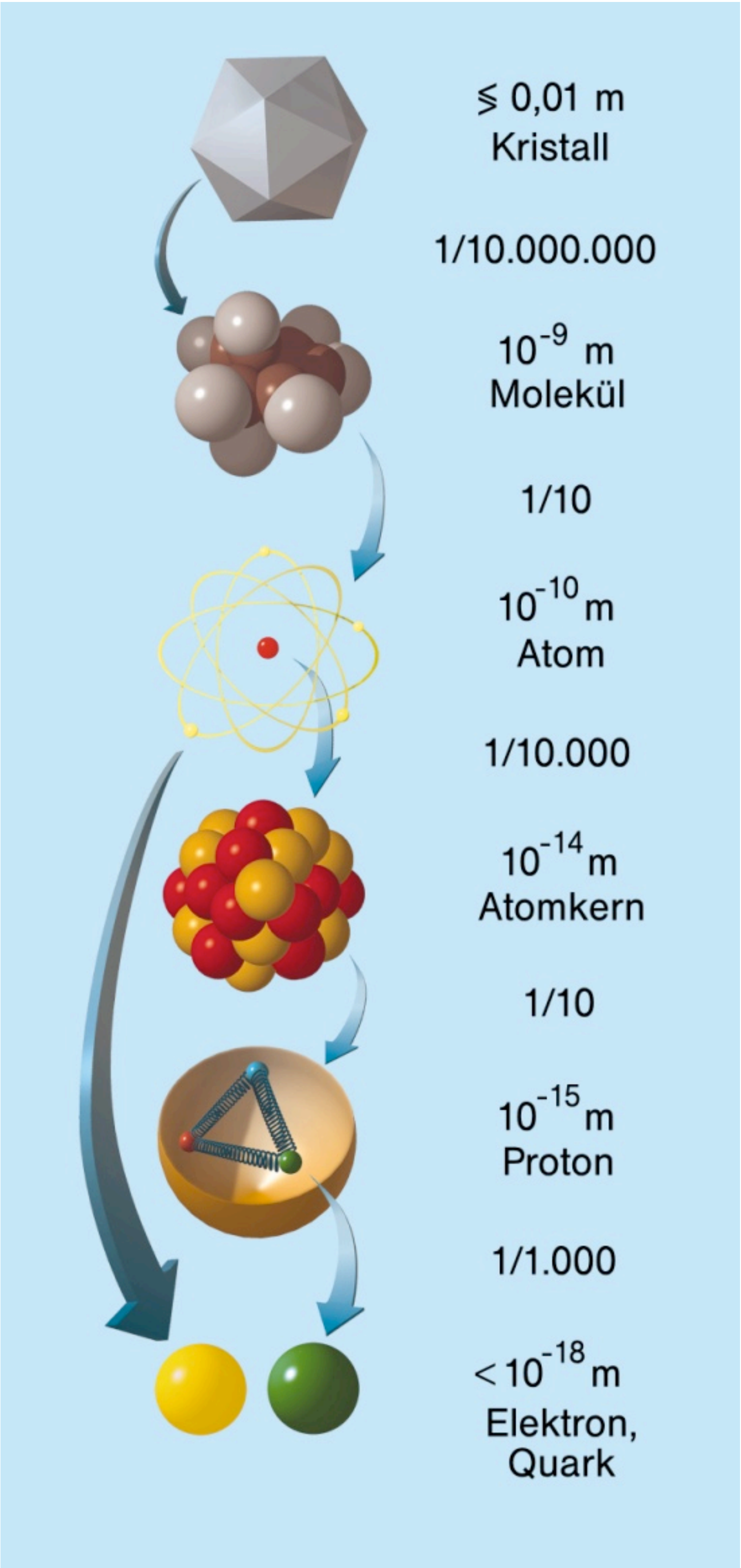

Elementarteilchen ...

				Ladung
Leptonen	ν_e	ν_μ	ν_τ	0
	e	μ	τ	-1
Quarks	u	c	t	$+2/3$
	d	s	b	$-1/3$
Familien	I	II	III	

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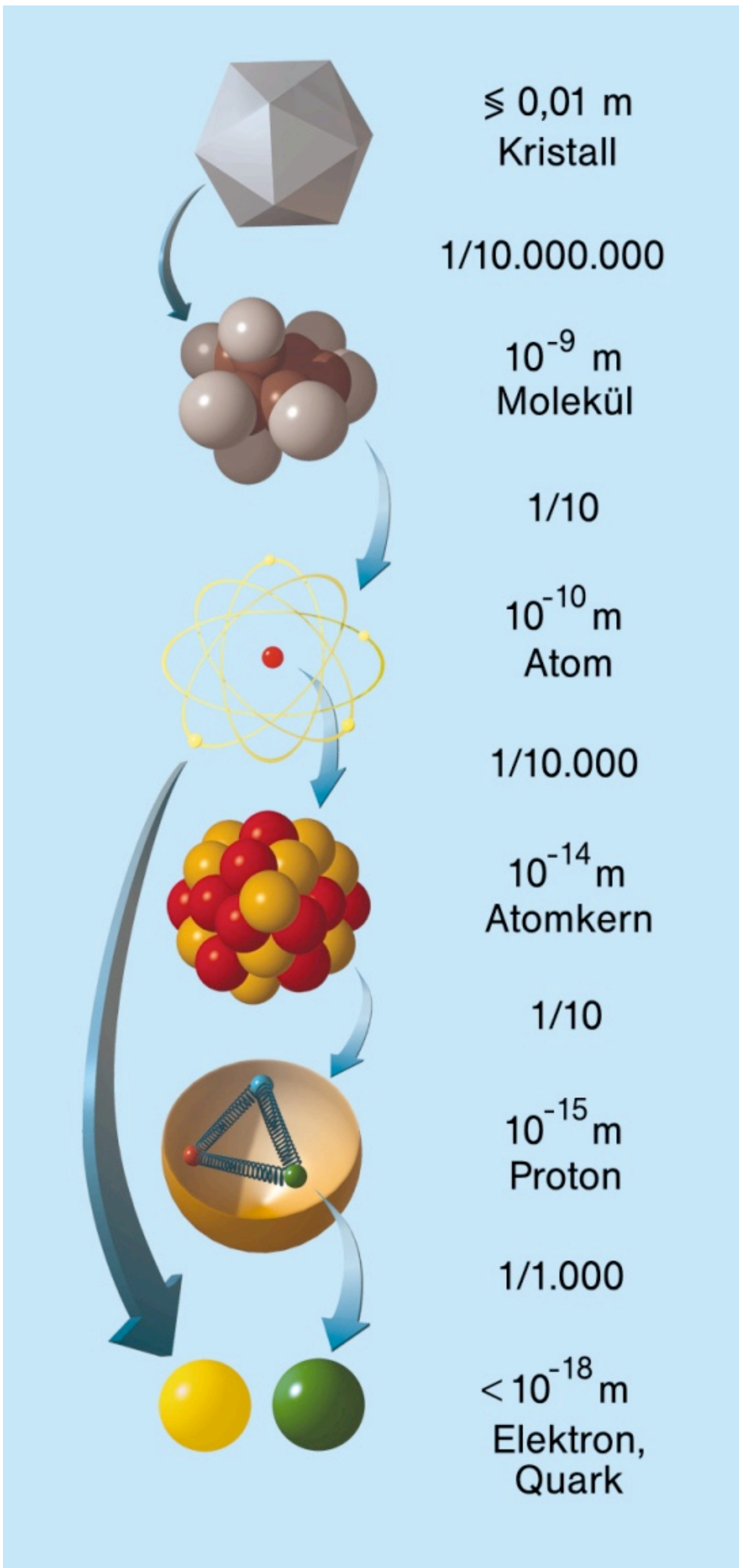

Elementarteilchen …

				Ladung
Leptonen	ν_e	ν_μ	ν_τ	0
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	d	s	b	-⅓
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… und ihre Antiteilchen

				Ladung
Anti-Leptonen	$\bar{\nu}_e$	$\bar{\nu}_\mu$	$\bar{\nu}_\tau$	0
	$\bar{e}$	$\bar{\mu}$	$\bar{\tau}$	+1
Anti-Quarks	$\bar{u}$	$\bar{c}$	$\bar{t}$	-⅔
	$\bar{d}$	$\bar{s}$	$\bar{b}$	+⅓
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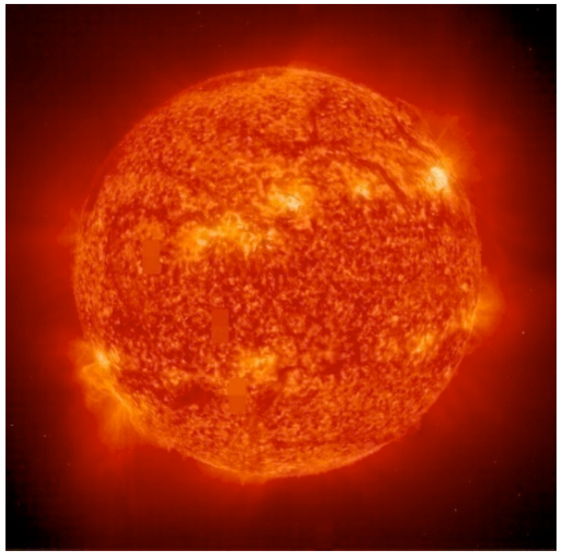
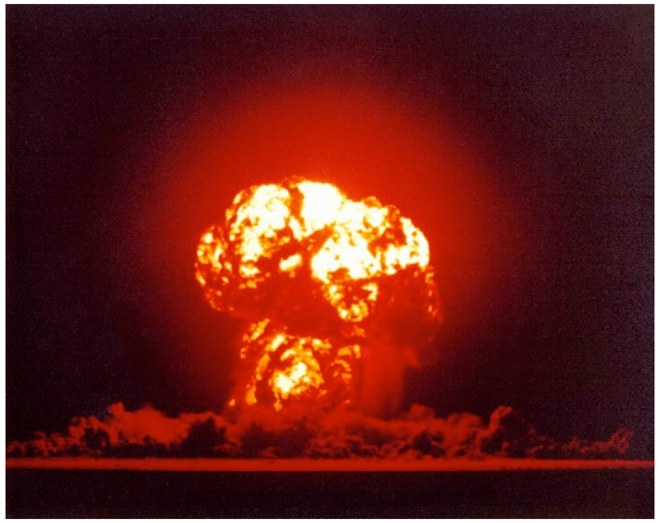
Kräfte/Wechselwirkungen

Elektromagnetische Wechselwirkung

schwache Wechselwirkung



starke Wechselwirkung



Gravitation

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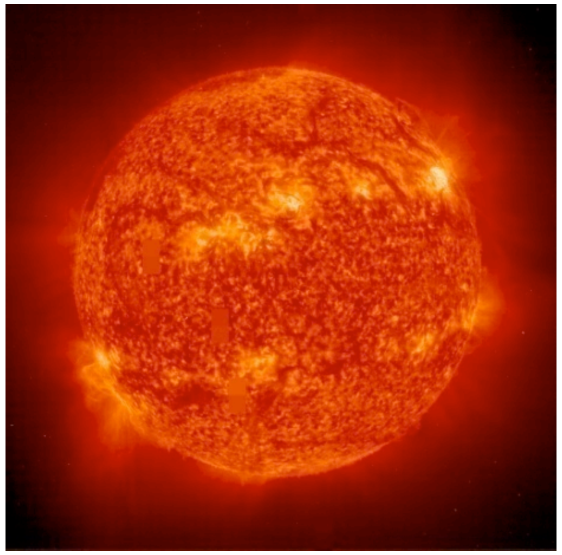

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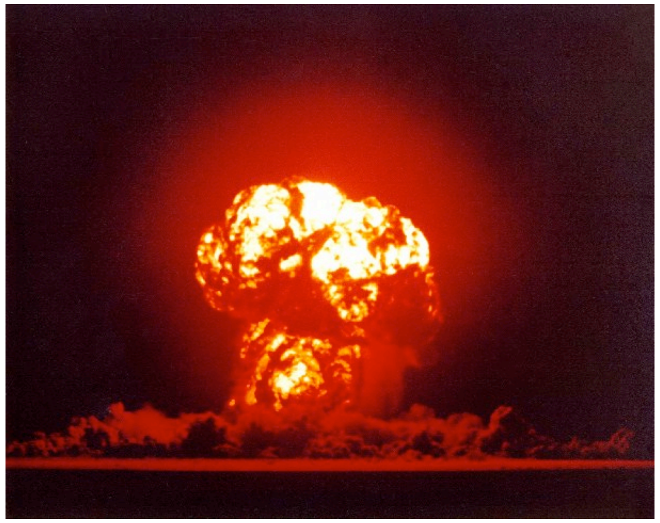

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- Zu jeder Wechselwirkung gehört eine Ladung
- Nur Teilchen, die entsprechende Ladung tragen, spüren Kraft

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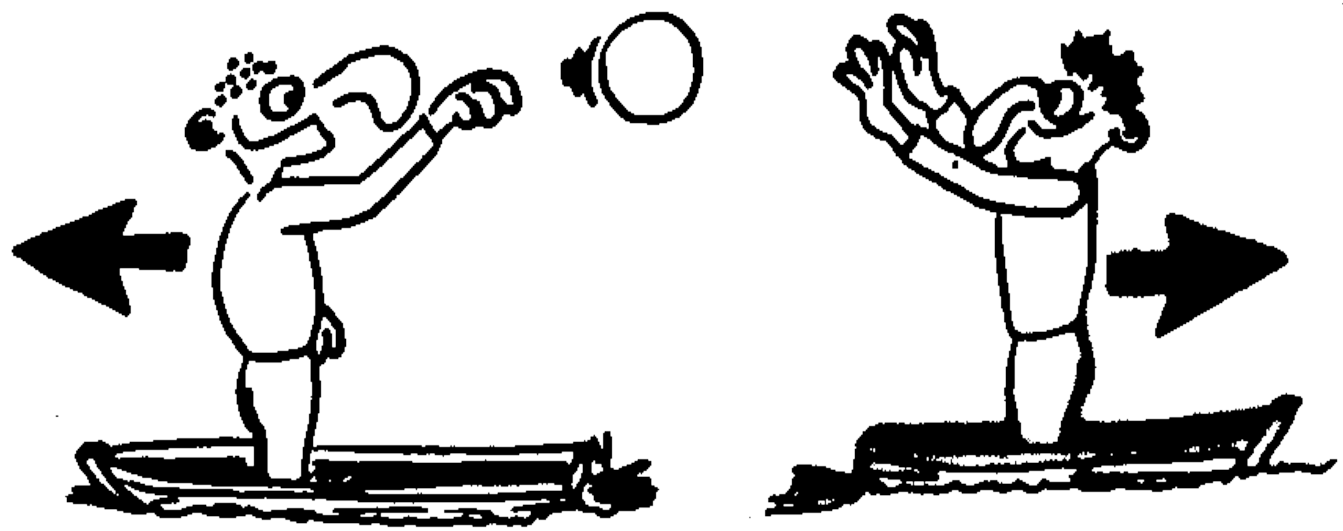
- Kraftübertragung erfolgt über Austausch von Botenteilchen



- aus Botenteilchen können paarweise Teilchen und Antiteilchen entstehen und umgekehrt

Teilchen und ihre Wechselwirkungen

	Austausch teilchen	Masse	Reichweite	rel. Stärke
EM	Photon γ	0	∞	$\sim 10^{-3}$
schwache Kraft	W^+, W^-, Z	~100 GeV	10^{-18}m	$\sim 10^{-5}$
starke Kraft	Gluon (8)	0	10^{-15}m	~1

				EM	schwach	stark
Quarks	up	charm	top	2/3	1/2	**3**
	down	strange	bottom	-1/3	-1/2	**3**
Leptons	e^-	μ	τ	-1	-1/2	1
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	υ_e	υ_μ	υ_τ	0	1/2	1

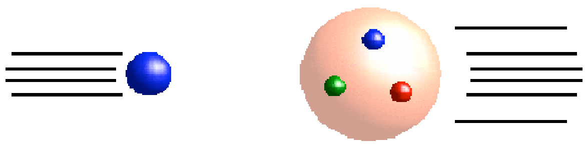
- Nach Einstein: $E = mc^2$

→ kollidiere hochenergetische Teilchen zur Entdeckung schwerer Teilchen

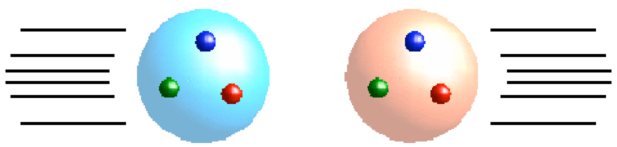
Teilchenbeschleuniger

- Elektron-Positron Collider
  SLAC Stanford 50 GeV Strahlenergie
  LEP Cern 100 GeV Strahlenergie

- Elektron-Proton Collider
  HERA Hamburg 27.5 GeV Elektronen, 920 GeV Protonen



- Hadron-Hadron Collider
  Tevatron Chicago 980 GeV Strahlenergie
  LHC Cern 7 TeV Strahlenergie



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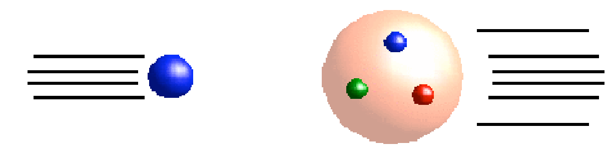

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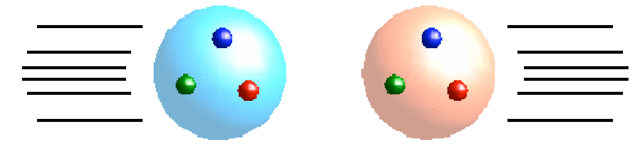
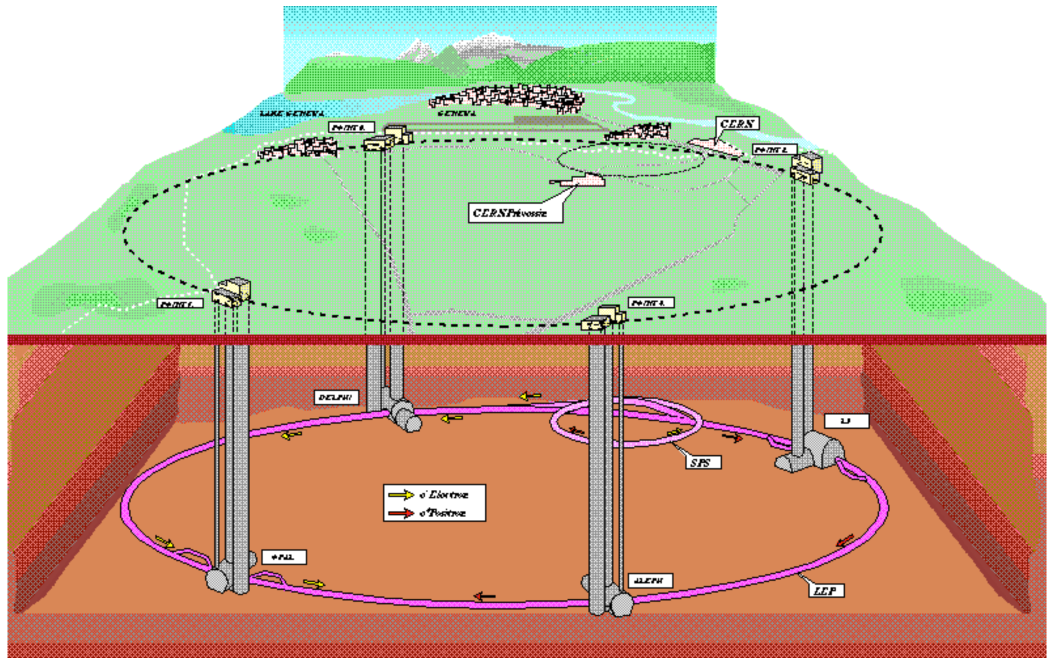


LEP Large Electron Positron Ring

1989 - 2000
bis zu 100 GeV pro Strahl
27km langer Tunnel
jetzt: LHC



Mathematische Beschreibung

- Mikroskopische Welt wird durch Quantenmechanik beschrieben
- Klassisch verboten → Quantenmechanisch nur "fast" verboten
- Heisenbergsche Unschärferelation $\Delta E \Delta t \gtrsim h$
- Entwicklung einer Quantenfeldtheorie aus Symmetrieprinzipien

Dirac Gleichung in QED $(\mathrm{i}\partial_\mu \gamma^\mu - e A_\mu \gamma^\mu - m_\mathrm{e}) \Psi_\mathrm{e} = 0$

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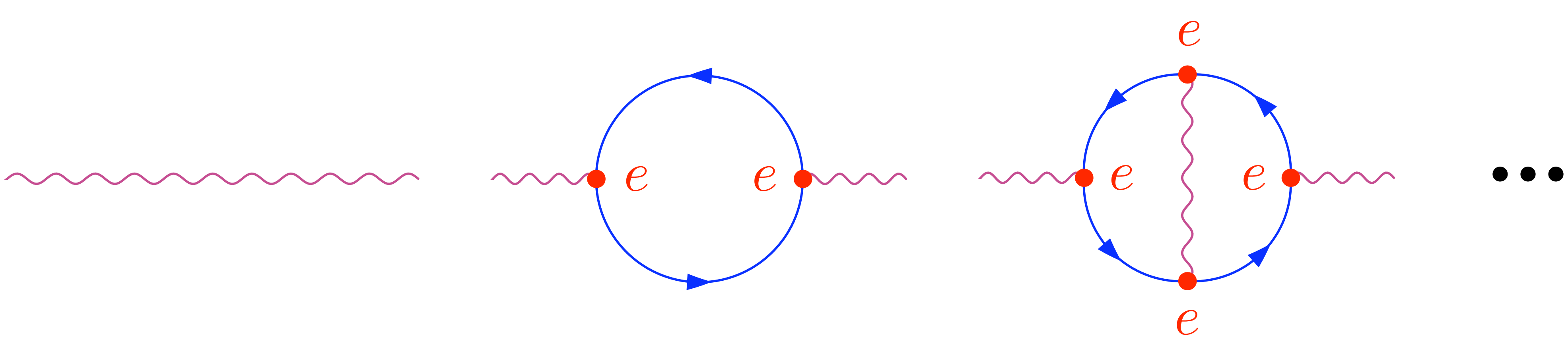

$\mathcal{O}(1)$	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha^2)$	Hilfsmittel:
LO	NLO	NNLO	Feynman Diagramme

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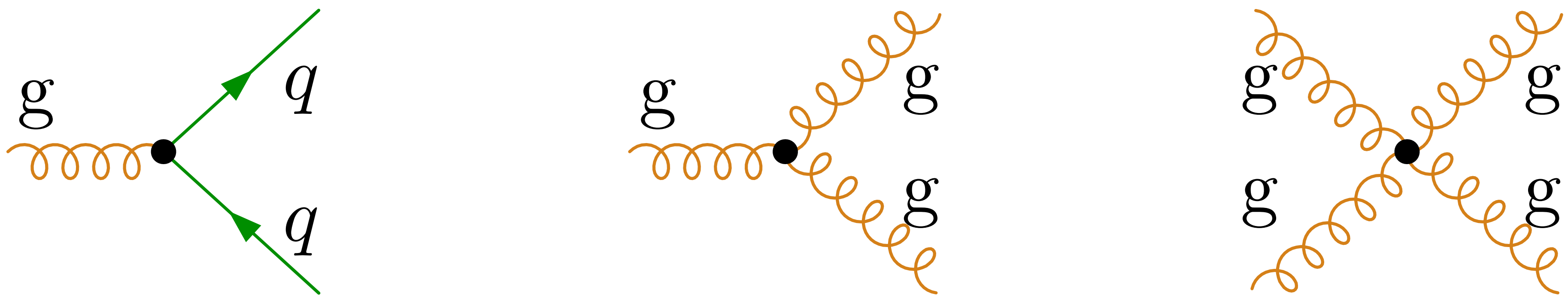
The QCD Lagrangian and its implications

- Gluon fields in adjoint, quark fields in fundamental rep. of SU(3)

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^A_{\mu\nu} G^{\mu\nu}_A + \sum_q \bar{q}_a \left(\mathrm{i}\gamma^\mu D_\mu - m_q\right)_{ab} q_b$$

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_{\mathrm{s}} f^{ABC} G^B_\mu G^C_\nu \qquad \left(D_\mu\right)_{ab} = \partial_\mu \delta_{ab} + \mathrm{i} g_{\mathrm{s}} (t^A G^A_\mu)_{ab}$$

- possible vertices



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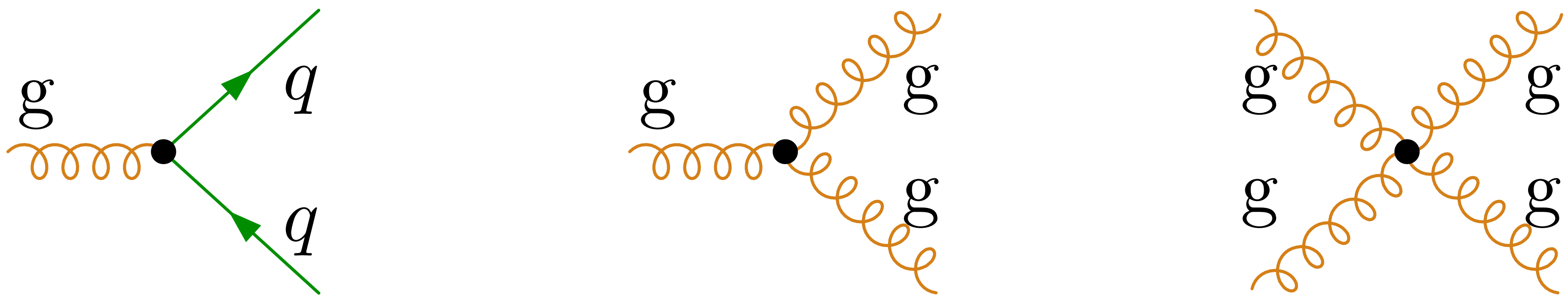

- strength of coupling depends on energy
  - large for small energies (infrared)
  - small for high energies (UV)

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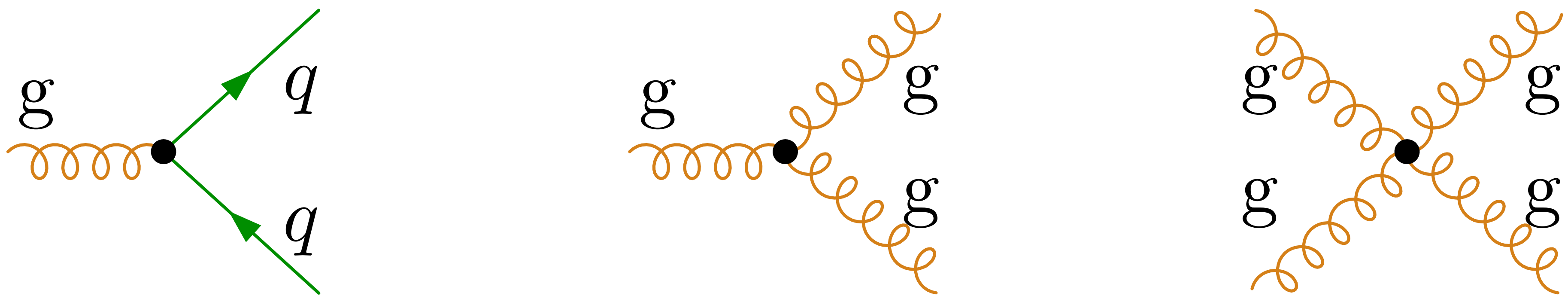
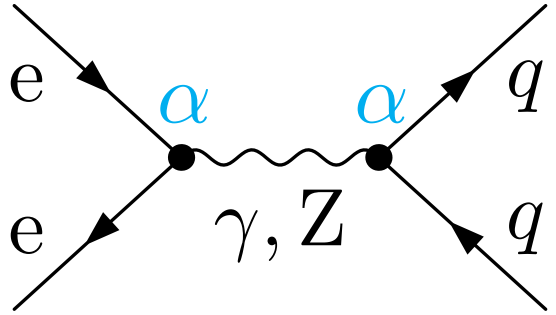


- strength of coupling depends on energy
  - large for small energies (infrared)
  - small for high energies (UV)

  $\Rightarrow$ perturbation theory only valid for high energies
- no free quarks and gluons (partons), only bound states (hadrons)

QCD at e^+e^- colliders

- annihilation of an electron and positron into a photon or a Z boson which decays into a quark-antiquark pair



$$\sim \alpha^2$$

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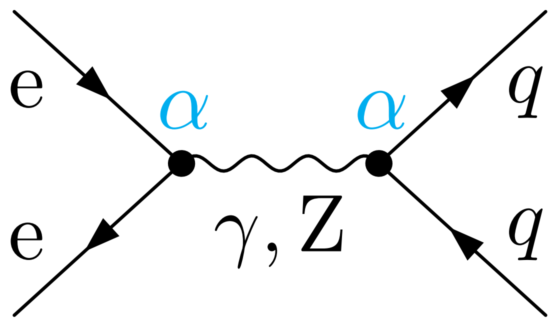
- first process where QCD plays a role is production of $q\bar{q}g$



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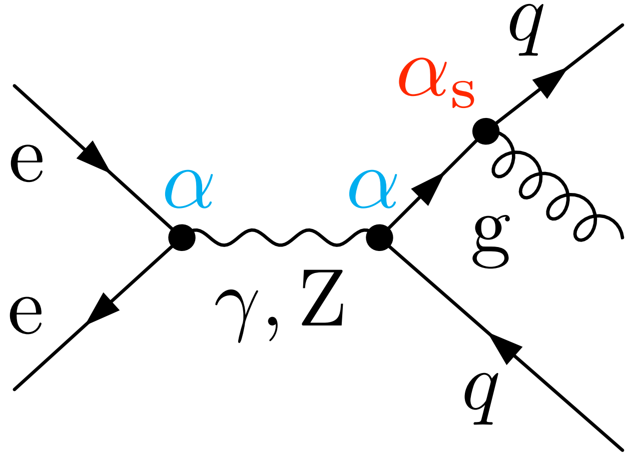

$\sim \alpha^2 \alpha_s$

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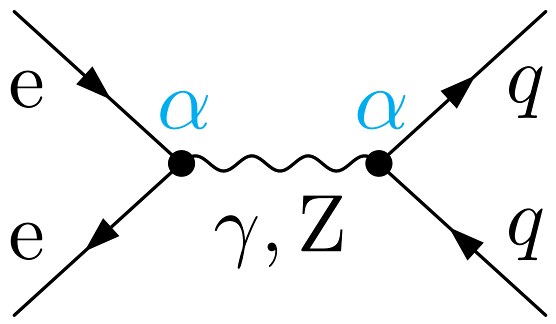

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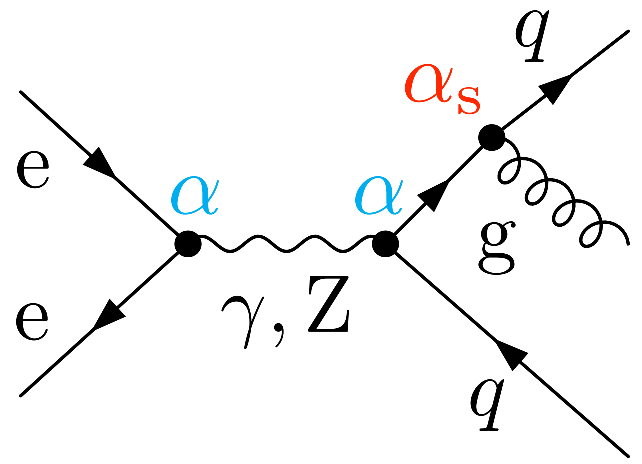
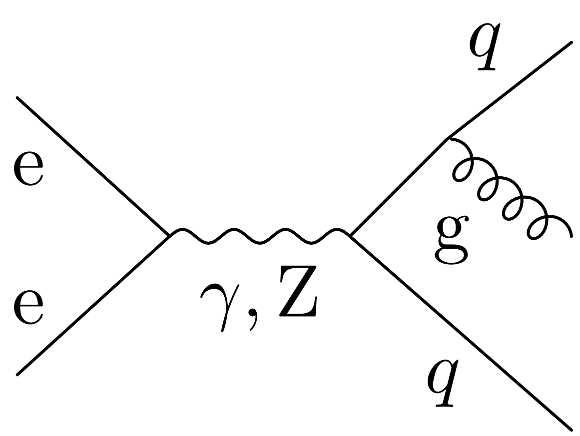


$\sim \alpha^2 \alpha_s$

- theory describes partons, experiments observe hadrons
- two possibilities
  - model parton → hadron transition
  - define appropriate final states → Jets
- Jets
  - experimentally hadrons with common momentum direction
  - theoretically partons with common momentum direction

Three-jet production: $e^+e^- \rightarrow 3$ jets

LO QCD $\mathrm{e}^+\mathrm{e}^- \rightarrow q\bar{q}\mathrm{g}$



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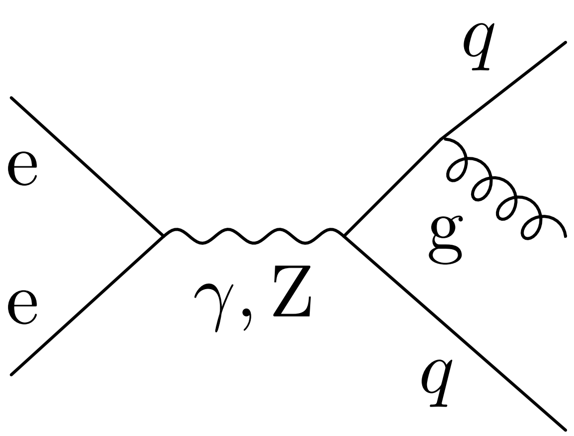

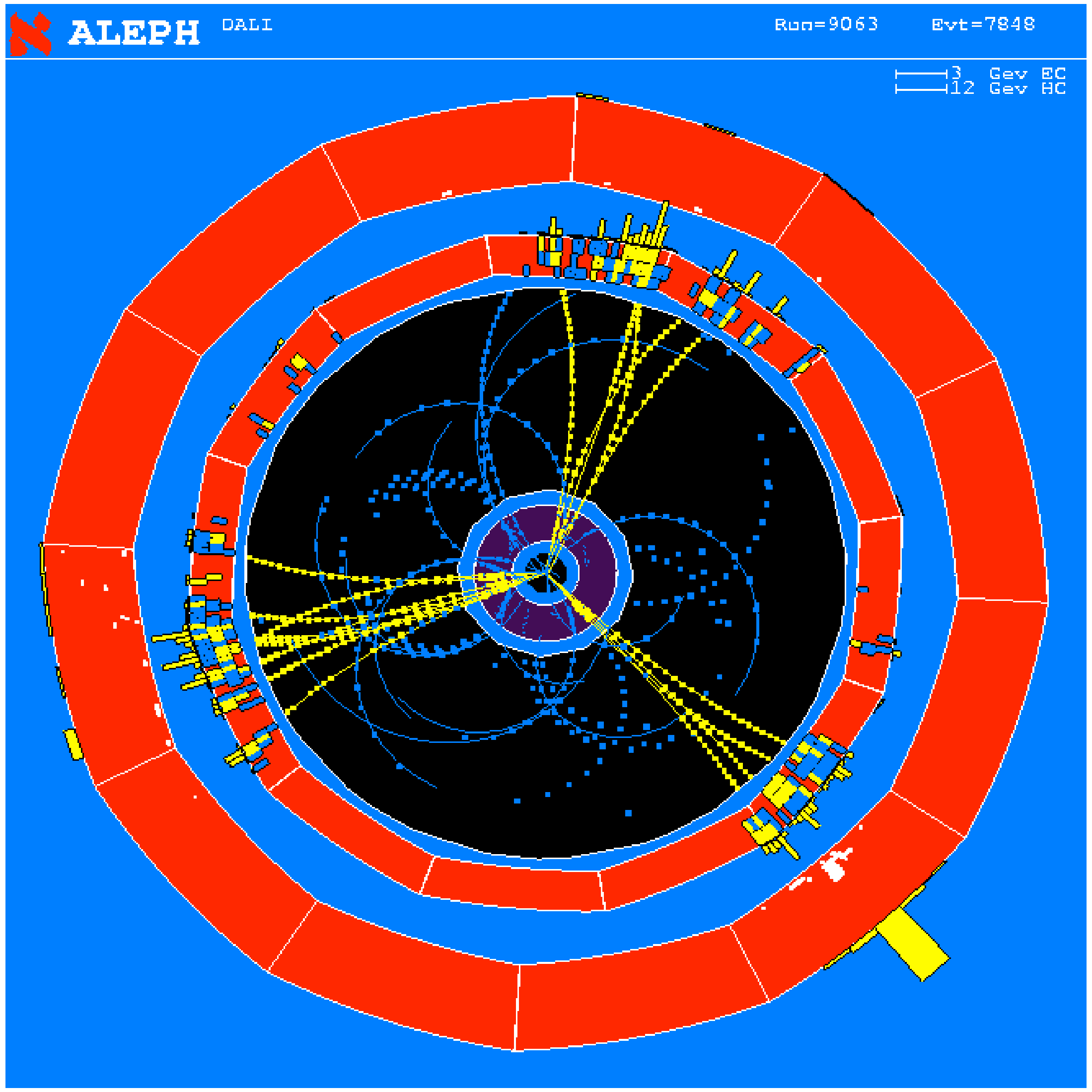

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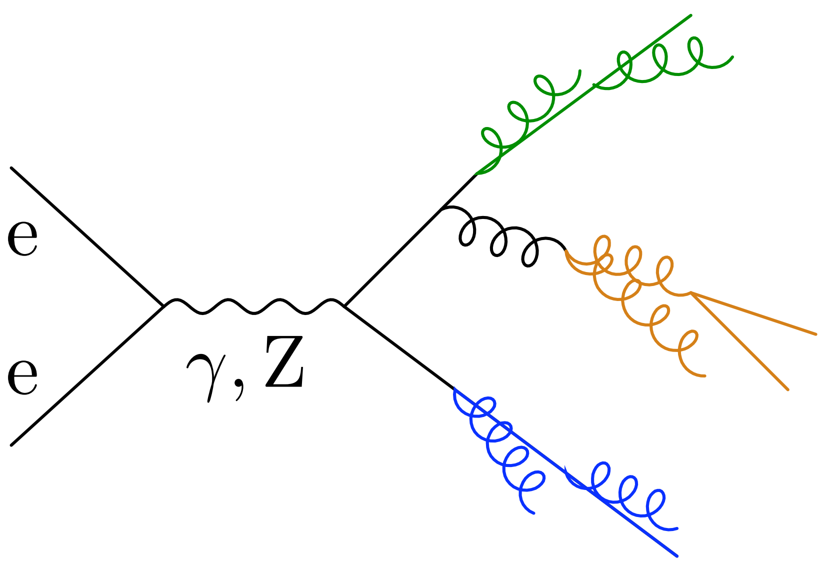

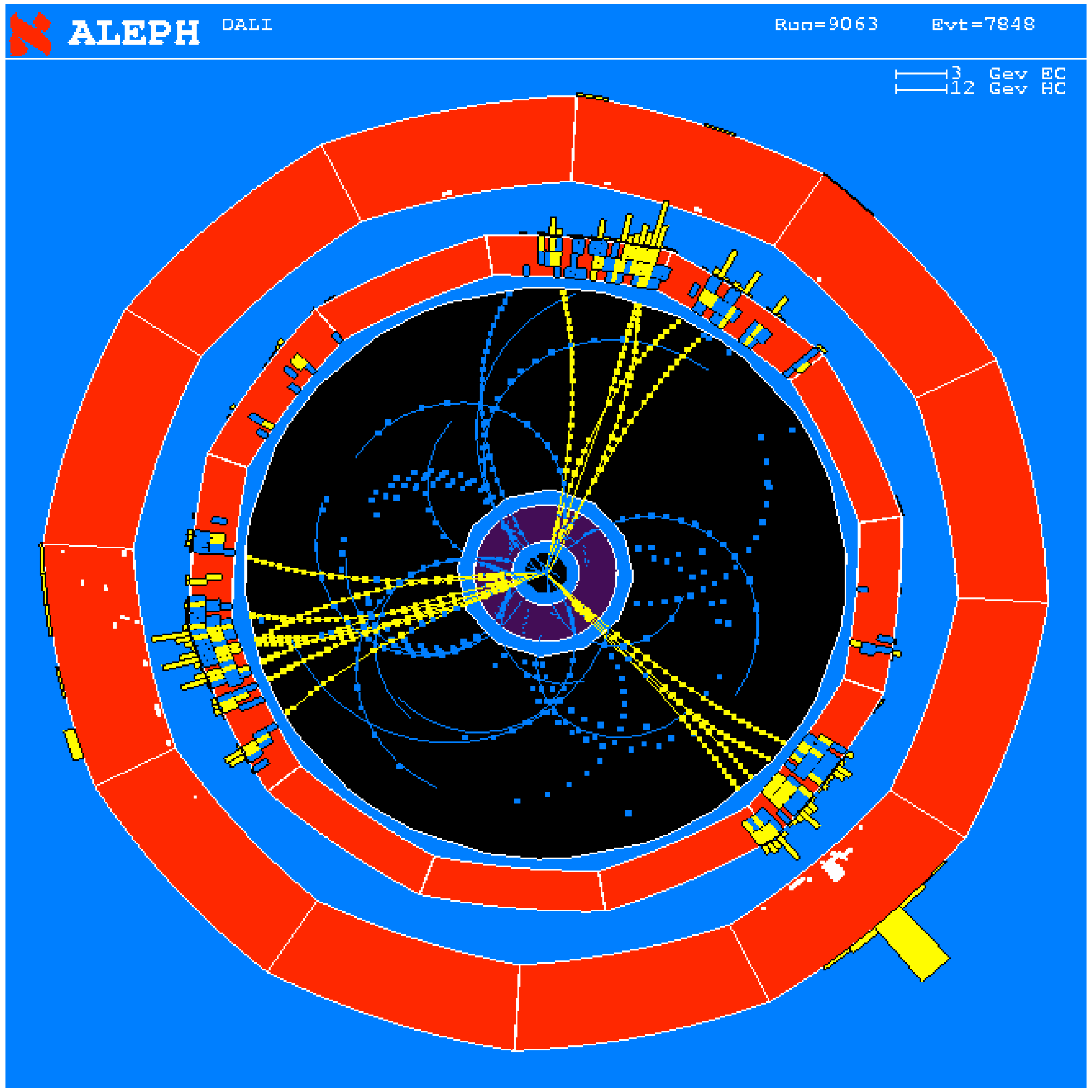

Jet observables

- Jet algorithm (Durham)

1. define minimum separation y_{cut}
2. calculate distance measure $y_{ij} = 2\min(E_i^2, E_j^2)\,(1 - \cos\Theta_{ij})$
3. particles are merged into clusters if $y_{ij} < y_{\mathrm{cut}}$
4. Go to 2 until no more pairs with $y_{ij} < y_{\mathrm{cut}}$ are left

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- Other algorithms differ in definition of y_{ij} (JADE, Geneva, ...)
- Measured observable: *n*-jet rate $R_n(y_{\mathrm{cut}}, \sqrt{s}) = \frac{\sigma_{n-\mathrm{jet}}}{\sigma_{\mathrm{had}}}$
- σ_{had}: totally inclusive cross section for $\mathrm{e^+e^-} \rightarrow \mathrm{hadrons}$

Event-shape observables

- Use information on geometry of final state
  → define mapping $\{p_i\} \to x$
- Example Thrust (used throughout this talk)

$$T = \max_{\vec{n}} \frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{\sum_i |\vec{p_i}|}$$

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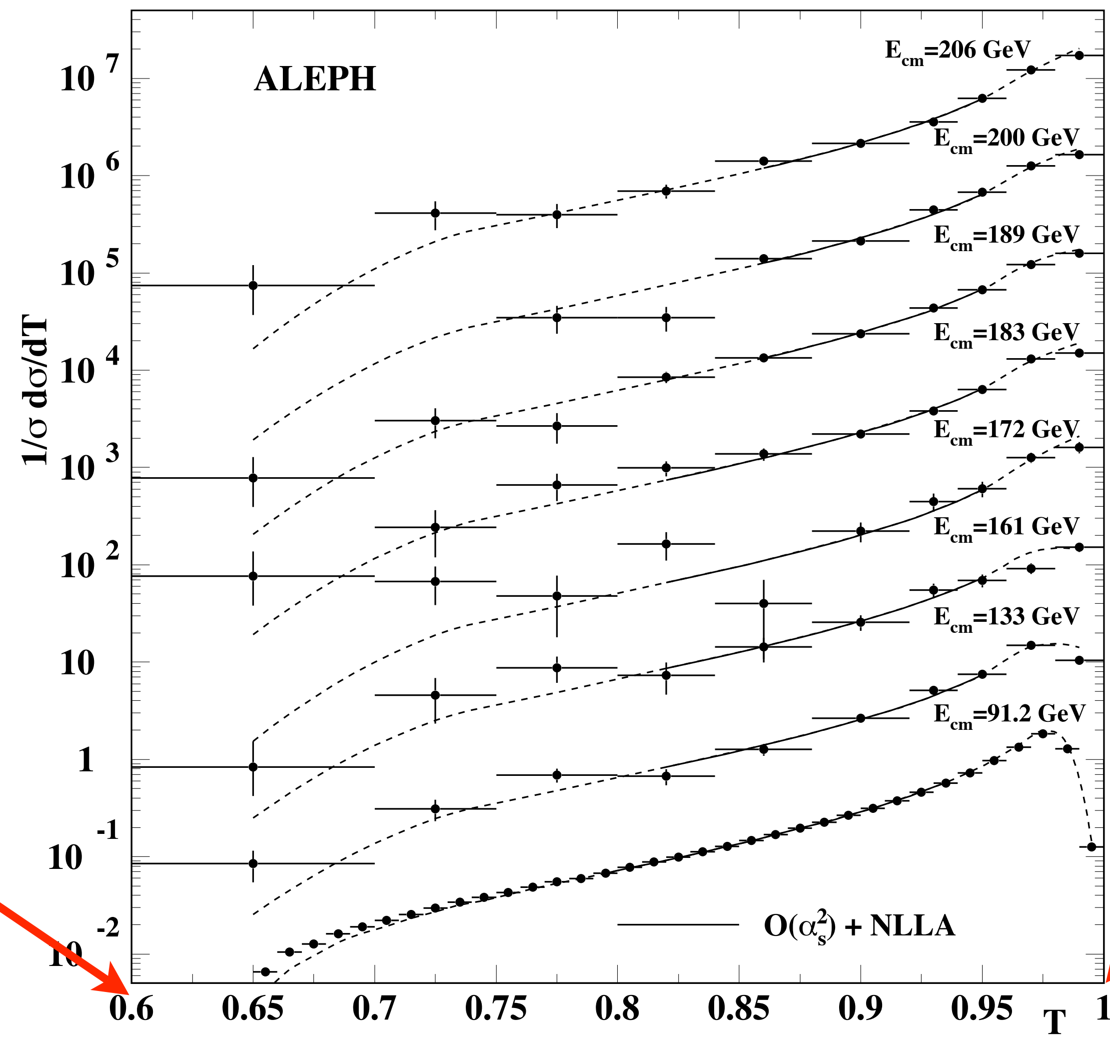
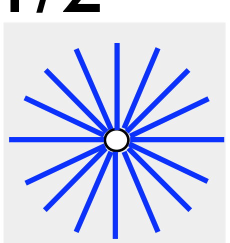
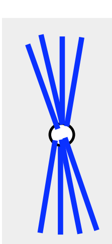


spherical event
T→1/2



pencil-like event
T→1



α_s from jet observables

- Jet rates and event-shape observables very well suited for determination of α_s from LEP data over large energy range
  - high statistics
  - clean environment for QCD calculations
- Enormous progress over the past 25 years, latest result up to NNLO+NNLL in QCD

$$\alpha_s\,(M_Z) = 0.1224 \pm 0.0009\,(\mathrm{stat}) \pm 0.0009\,(\mathrm{exp}) \pm 0.0012\,(\mathrm{had}) \pm 0.0035\,(\mathrm{theo})$$

Dissertori, Gehrmann, Luisoni, et.al. 06/2009
first NLO calculation Ellis, Ross, Terrano 1981

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- at this theo. precision also NLO electroweak effects become important: NNLO QCD $\alpha_s^2 \sim 0.01$, NLO EW $\alpha_{EW} \sim 0.008$
- more involved since photon and weak gauge bosons connect initial and final state → rest of the talk

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Ingredients of a NLO calculation

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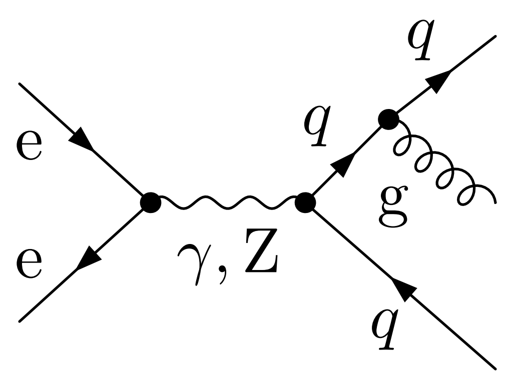
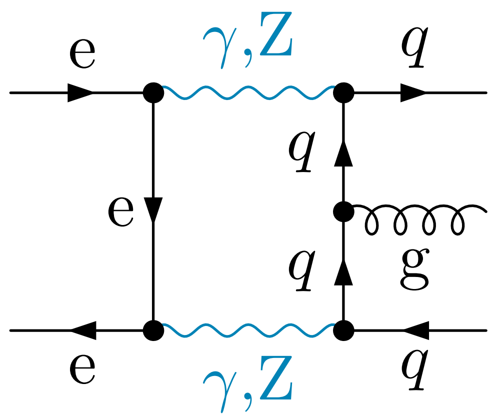


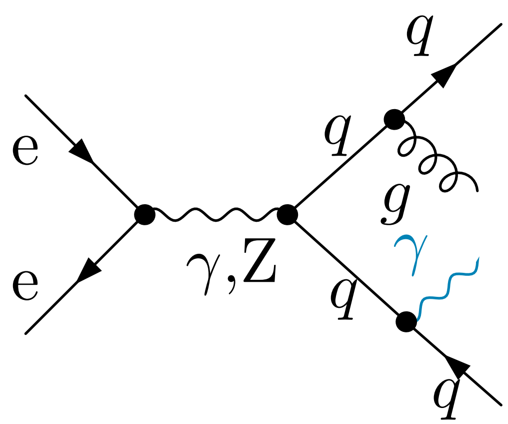
$\mathcal{O}(\alpha^2 \alpha_s)$

EW NLO virtual



$\mathcal{O}(\alpha^3 \alpha_s)$

EW NLO real



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Ingredients of a NLO calculation

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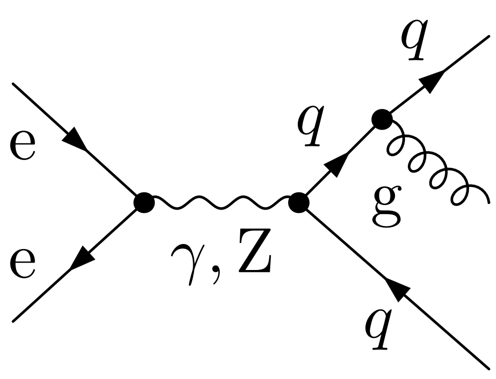
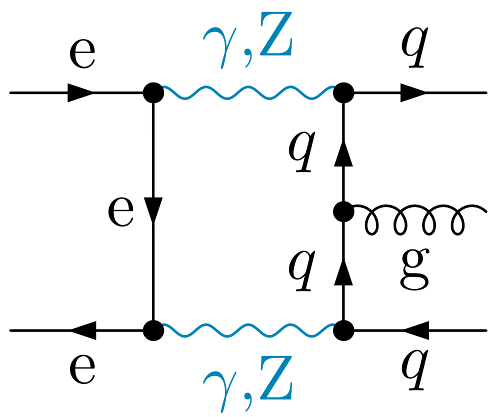


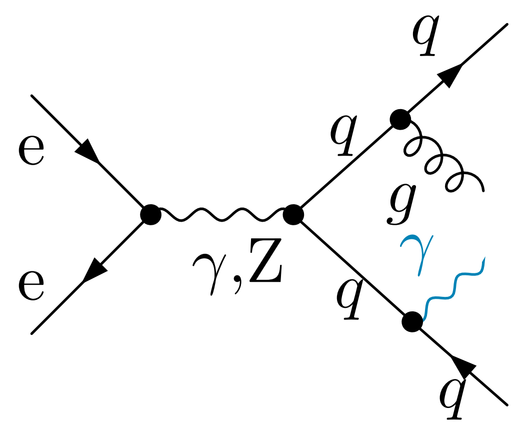
$\mathcal{O}(\alpha^2 \alpha_s)$

EW NLO virtual



$\mathcal{O}(\alpha^3 \alpha_s)$

EW NLO real



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- in jet observables no distinction between photon and gluon

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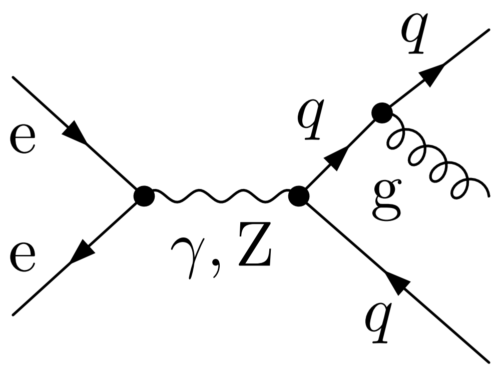	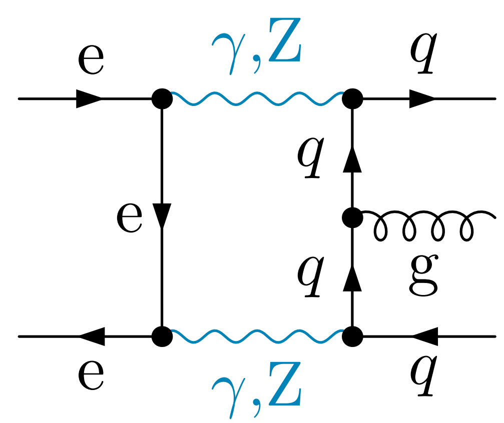	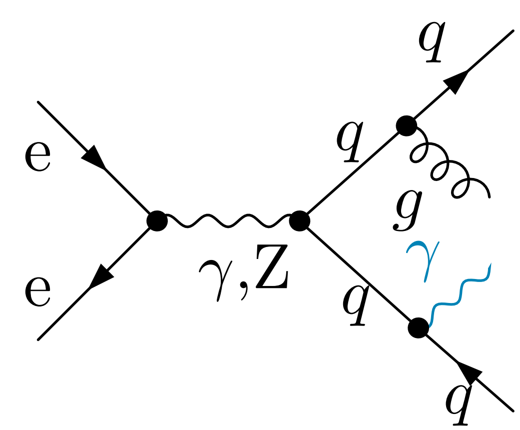
$\mathcal{O}(\alpha^2\alpha_s)$	$\mathcal{O}(\alpha^3\alpha_s)$	$\mathcal{O}(\alpha^3\alpha_s)$

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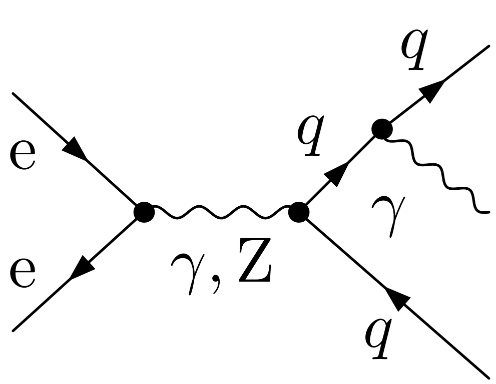	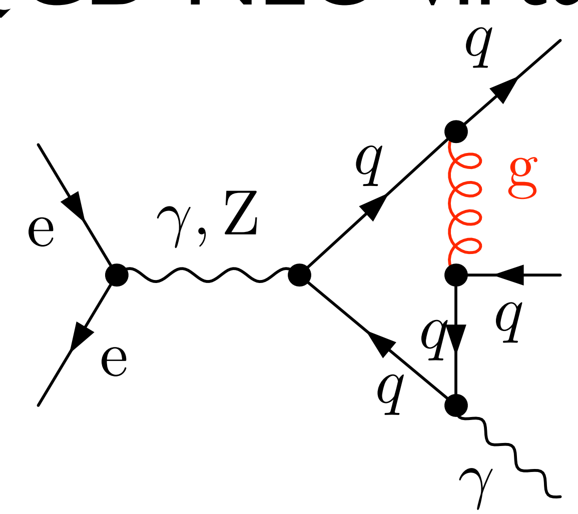	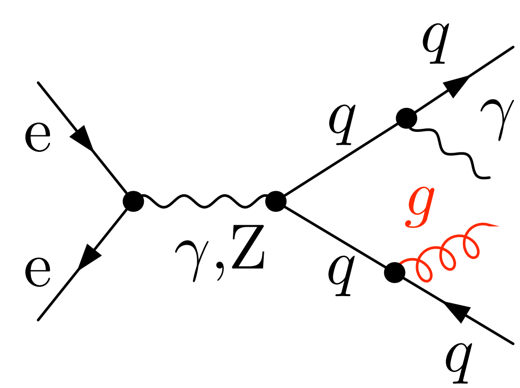
$\mathcal{O}(\alpha^3)$	$\mathcal{O}(\alpha^3\alpha_s)$	$\mathcal{O}(\alpha^3\alpha_s)$

Properties of the NLO corrections

- integrate over loop momentum l in virtual correction
  - UV divergencies for large l → renormalisation procedure, redef. of phys. parameters
  - IR divergencies for small l or special collinear configurations → regulate with small photon and fermion masses, leads to singular logarithms $\sim \ln(m_\gamma), \ln(m_f)$

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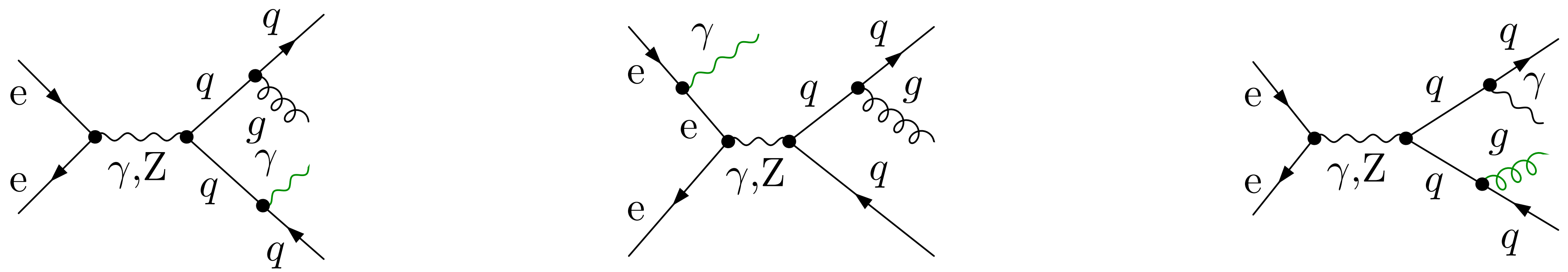
  - IR divergencies due to soft photon/gluon emission or collinear photon/gluon emission in the case of vanishing fermion masses



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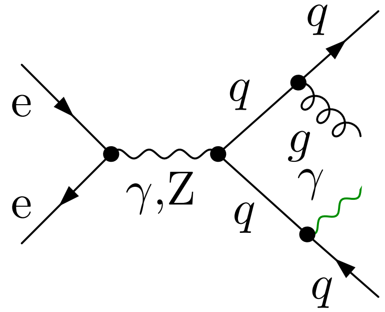

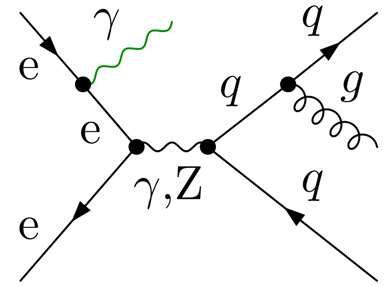

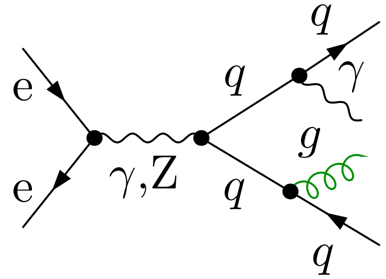

- for infrared-safe observables
  - BN theorem: soft divergencies cancel between virtual and real corrections
  - KLN theorem: collinear divergencies in final state cancel between virtual and real corrections
  - initial-state collinear radiation regularised by electron mass and suppressed by cut on production angle

Experimental event selection

- initial-state photons lead to difficulties in reconstruction of total energy of final state
- devise cuts to limit influence of initial-state photons (ALEPH)

1. accept only particles with production angle $\cos\theta_i < \cos\theta_{\rm cut}$
2. cluster particles according to Durham algorithm with $y_{\rm cut} = 0.002$
3. remove events where photonic energy in jet is > 90% ($z_{\rm cut}$)
4. calculate visible invariant mass s' of final state and accept event only if $s'/s > 0.81$ $(s_{\rm cut})$

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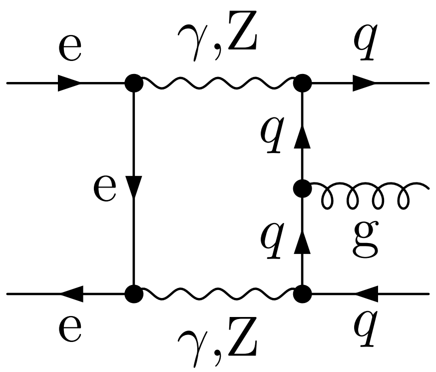
  4. calculate visible invariant mass s' of final state and accept event only if $s'/s > 0.81$ ($s_{\rm cut}$)
- step 3 leads to potential problems in perturbative calculations
  - remove events where photon and quark are collinear

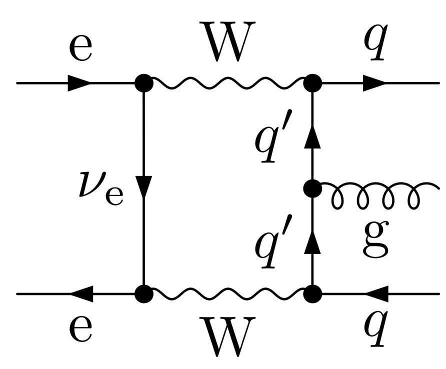
  → observable no longer infrared-safe

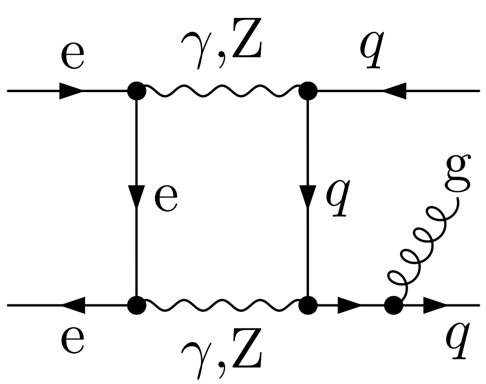
  → cancellation of collinear divergencies between virtual and real corrections no longer guaranteed
  - way out: use photon fragmentation function to restore infrared safety

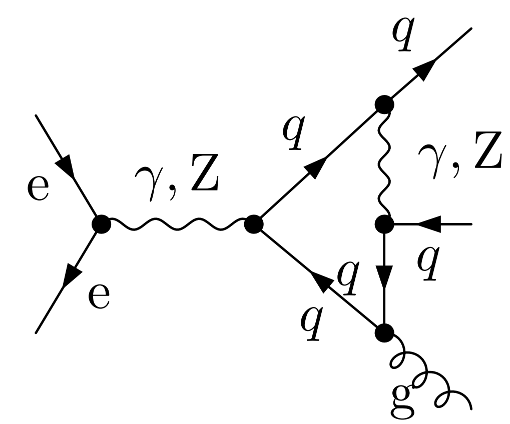
Virtual corrections - Survey of diagrams

- For EW corrections calculate ~ 200 different diagrams
  - 2 pentagons
  - 5 boxes
  - vertices, self-energies









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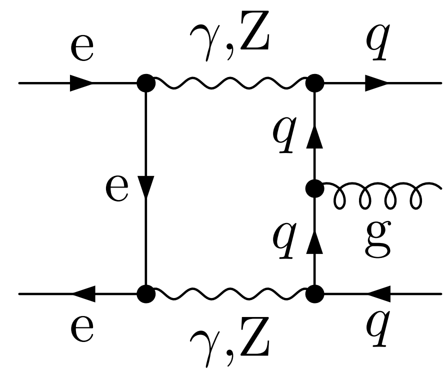
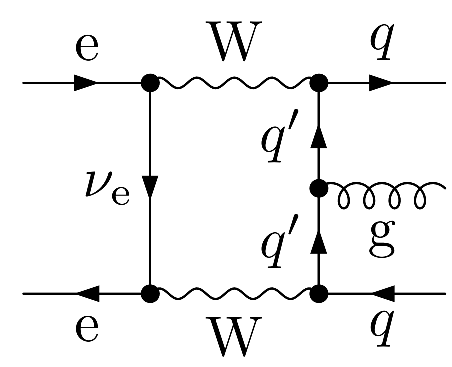
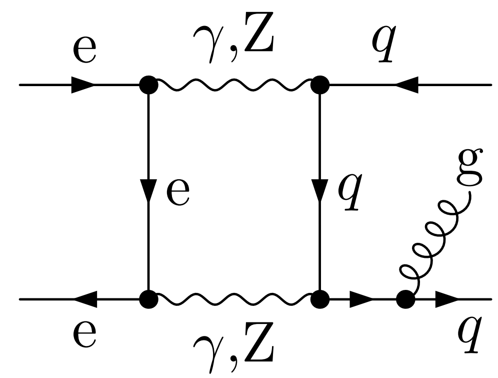
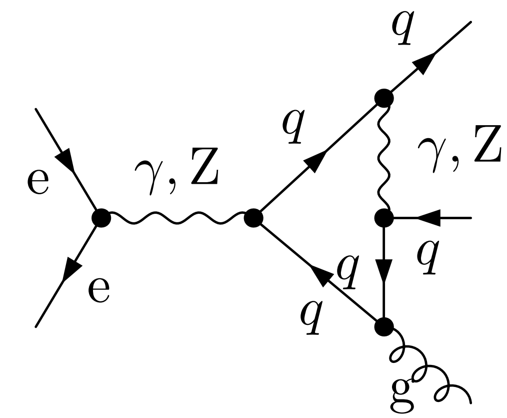
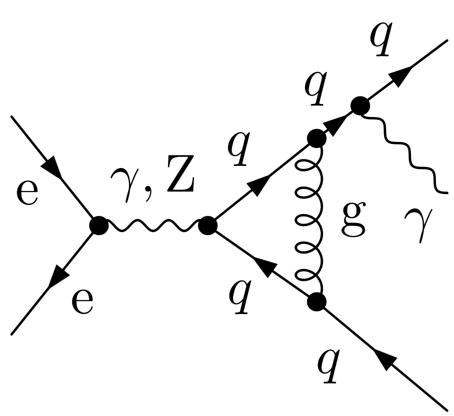


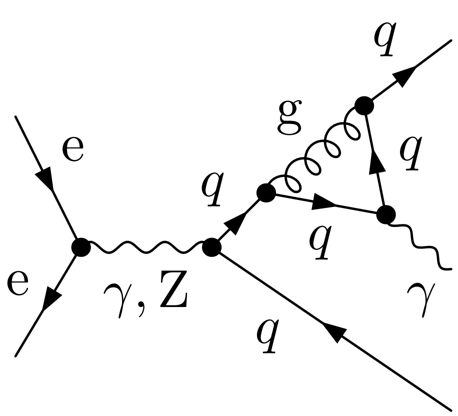


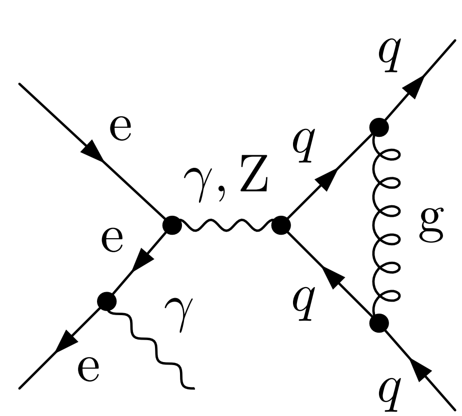


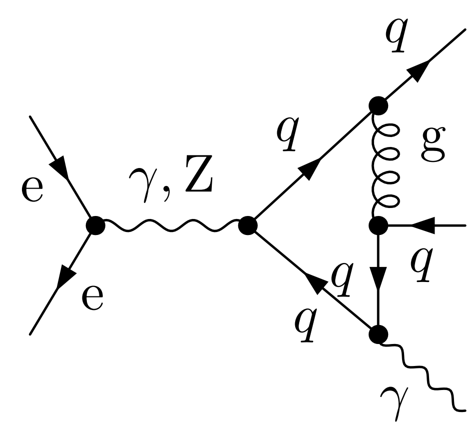


- For QCD corrections calculate ~ 20 different diagrams









Virtual corrections - Technical details

- generation of Feynman diagrams with FeynArts
- algebraic simplifications using FormCalc and Mathematica code
  - write 1-loop amplitude $\mathcal{M}_1^{\sigma\sigma'\lambda} = \sum_n F_n^{\sigma\sigma'\lambda}(\{s, s_{ij}, t_{li}\})\hat{\mathcal{M}}_n^{\sigma\sigma'\lambda}(k_1, k_2, k_3, k_4, k_5)$
  - Standard Matrix Elements $\hat{\mathcal{M}}_n^{\sigma\sigma'\lambda}$ contain all information on helicities
  - use 4D of space-time to write product of Dirac chains

$$\underbrace{\bar{v}_{k_2}\ \not\epsilon\gamma^\mu\gamma^\nu u_{k_1}}_{\text{DC 1}}\underbrace{\bar{u}_{k_3}\ \not k_2\gamma_\mu\gamma_\nu v_{k_4}}_{\text{DC 2}} = -\frac{64A_1A_2}{t_{13}^2 t_{14}t_{23}t_{24}}\epsilon\cdot k_2\underbrace{\bar{v}_{k_2}\ \not k_3 u_{k_1}}_{\text{DC 1}}\underbrace{\bar{u}_{k_3}\ \not k_1 v_{k_4}}_{\text{DC 2}}$$

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- reduce ~150 Dirac structures to ~20 SMEs → reduction of size of amplitude by factor 1/2
- efficient evaluation of SMEs using Weyl-van der Waerden formalism

Virtual corrections - Technical details

- for calculation of loop integrals use COLI library of A. Denner
  - tensor reduction according to Denner-Dittmaier algorithm
  - scalar integrals evaluated using standard techniques

  $\rightarrow$ numerically stable results also in exceptional phase-space points
  - use fermion masses only as regulators

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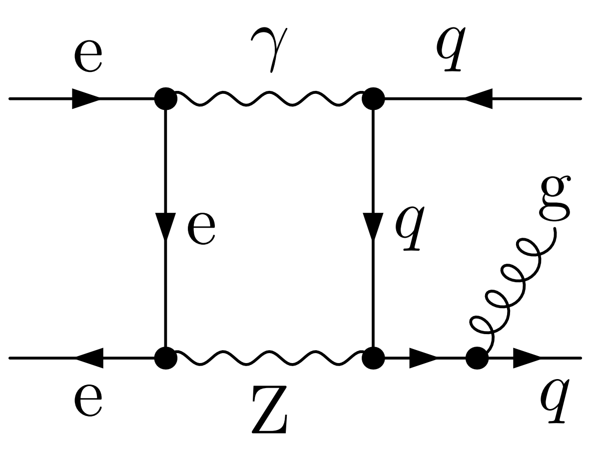
- gauge boson widths are treated in complex-mass scheme
  - replace $M_V^2 \to \mu_V^2 = M_V^2 - \mathrm{i} M_V \Gamma_V, \quad V = W, Z$
  - define complex weak mixing angle $\cos\theta_w = \frac{\mu_W^2}{\mu_Z^2}$

  ☑ gauge-invariant result

  ☑ valid everywhere in phase space



Real corrections - Overview

- calculate Feynman diagrams "by hand" using WvdW formalism
- perform phase-space integration numerically

→ cancellation of IR divergencies delicate

- divide corrections into finite and singular piece and treat singular piece analytically
  - exact cancellation of singularities between virtual and real corrections
  - possible to work in massless approximation in finite piece and use fermion masses only as regulators in singular piece

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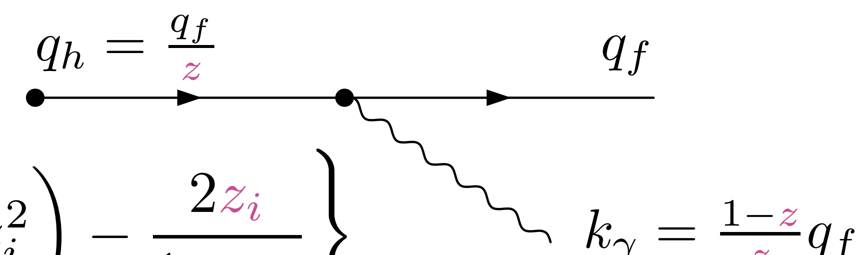
- divide corrections into finite and singular piece and treat singular piece analytically
  - exact cancellation of singularities between virtual and real corrections
  - possible to work in massless approximation in finite piece and use fermion masses only as regulators in singular piece
- two approaches: phase-space slicing and dipole subtraction
- Both algorithms rely on analytical integration over full photonic phase-space → need extension for event selection used in experiment

Real corrections - Technical details

- non-collinear-safe subtraction worked out by Dittmaier, Kasprzik
- for phase-space slicing consider collinear final-state radiation
  - without hard-photon cut



$$
\begin{aligned}
d\sigma^{\text{final}}_{\text{coll.}} &= \sum_{i=3}^{4} \frac{\alpha}{2\pi} Q_i^2 d\sigma_{\text{Born}}(q_i) \int_0^{1-\Delta E/E_i} dz_i \left\{ \frac{1+z_i^2}{1-z_i} \ln\left(\frac{4E_i^2 \delta_c}{2m_i^2} z_i^2 \right) - \frac{2z_i}{1-z_i} \right\} \\
&= \sum_{i=3}^{4} \frac{\alpha}{2\pi} Q_i^2 d\sigma_{\text{Born}}(q_i) \left(\left[\frac{3}{2} + 2\ln\left(\frac{\Delta E}{E_i}\right) \right] \left[1 - \ln\left(\frac{4E_i^2}{m_i^2} \frac{\delta_c}{2} \right) \right] + 3 - \frac{2\pi^2}{3} \right)
\end{aligned}
$$

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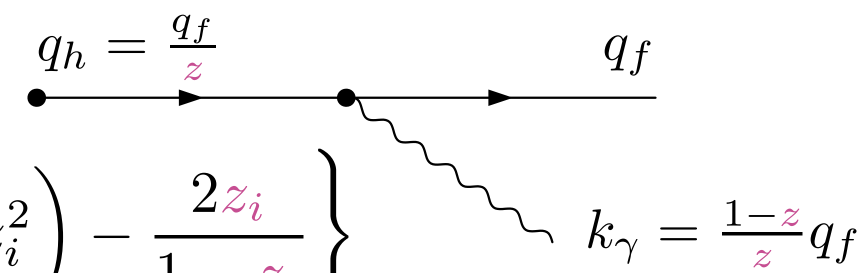

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$$= \sum_{i=3}^{4} \frac{\alpha}{2\pi} Q_i^2 \mathrm{d}\sigma_{\text{Born}}(q_i) \left(\left[\frac{3}{2} + 2\ln\left(\frac{\Delta E}{E_i}\right)\right] \left[1 - \ln\left(\frac{4E_i^2}{m_i^2}\frac{\delta_c}{2}\right)\right] + 3 - \frac{2\pi^2}{3} \right)$$

- with hard-photon cut (translates into soft-quark cut z'_{cut})

$$\mathrm{d}\sigma_{\text{coll.}}^{\text{final}}(z'_{\text{cut}}) = \sum_{i=3}^{4} \frac{\alpha}{2\pi} Q_i^2 \mathrm{d}\sigma_{\text{Born}}(q_i) \left\{ \int_{z'_{\text{cut}}}^{1-\Delta E/E_i} \mathrm{d}z_i \frac{1+z_i^2}{1-z_i} \ln\left(\frac{4E_i^2\delta_c}{2m_i^2} z_i^2\right) - \frac{2z_i}{1-z_i} \right\}$$

$$= \sum_{i=3}^{4} \frac{\alpha}{2\pi} Q_i^2 \mathrm{d}\sigma_{\text{Born}}(q_i) \left[\frac{9}{2} - 4z'_{\text{cut}} - \frac{{z'_{\text{cut}}}^2}{2} + \left(2z'_{\text{cut}} + {z'_{\text{cut}}}^2\right) \ln\left(z'_{\text{cut}}\right) \right.$$

$$+ \left(-\frac{3}{2} + z'_{\text{cut}} + \frac{1}{2}{z'_{\text{cut}}}^2 - 2\ln\left(\frac{\Delta E/E_i}{1-z'_{\text{cut}}}\right) \right) \ln\left(\frac{4E_i^2\delta_c}{2m_i^2}\right) - \frac{2\pi^2}{3}$$

$$\left. + 2\ln\left(\frac{\Delta E/E_i}{1-z'_{\text{cut}}}\right) + 4\ln\left(1-z'_{\text{cut}}\right)\ln\left(z'_{\text{cut}}\right) + 4\mathrm{Li}_2\left(z'_{\text{cut}}\right) \right]$$

Photon fragmentation function

- Idea: proceed as in parton distributions and factorise singular piece into experimentally determined photon fragmentation function

$$\Rightarrow \quad \int \mathrm{d}\sigma^{\mathrm{IR-safe}} = \int \mathrm{d}\sigma_{\mathrm{virt}} + \int \mathrm{d}\sigma_{\mathrm{real}}(z'_{\mathrm{cut}}) + \int \mathrm{d}\sigma_{\mathrm{frag}}(z'_{\mathrm{cut}})$$

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  - results in $d\sigma_{\mathrm{frag}}(z'_{\mathrm{cut}}) = \sum_{i=3}^{4}\frac{\alpha}{2\pi}Q_i^2 d\sigma_{\mathrm{Born}}(q_i)\int_0^{z'_{\mathrm{cut}}} dz_i D^{\mathrm{ALEPH,MR}}_{q\to\gamma}(z_i)$

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  - such that

$$\begin{aligned} d\sigma_{\mathrm{frag}}(z'_{\mathrm{cut}}) + d\sigma^{\mathrm{final}}_{\mathrm{coll.}}(z'_{\mathrm{cut}}) \quad = \quad & d\sigma^{\mathrm{final}}_{\mathrm{coll.}} - \sum_{i=3}^{4}\Bigg\{(4+C)z'_{\mathrm{cut}} \\ & + \left(z'_{\mathrm{cut}} + \frac{1}{2}{z'_{\mathrm{cut}}}^2\right)\ln\left(\frac{4E_i^2\delta_c}{2{\mu_0}^2}(1-z'_{\mathrm{cut}})^2\right) \\ & + \left[-\frac{3}{2} + \ln\left(\frac{4E_i^2\delta_c}{2{\mu_0}^2}(1-z'_{\mathrm{cut}})\right)\right]\ln\left((1-z'_{\mathrm{cut}})^2\right)\Bigg\} \end{aligned}$$

Event-shape observables in perturbation theory

- Experimentally measured quantity

$$\frac{1}{\sigma_{\mathrm{had}}}\frac{\mathrm{d}\sigma}{\mathrm{d}y}$$

- σ_{had} up to $\mathcal{O}(\alpha)$

$$\sigma_{\mathrm{had}} = \sigma_0 \left(1 + \left(\frac{\alpha}{2\pi}\right)\delta_{\sigma,1} + \mathcal{O}\left(\alpha^2\right)\right)$$

- leads to

$$\frac{1}{\sigma_{\mathrm{had}}}\frac{\mathrm{d}\sigma}{\mathrm{d}y} = \frac{\mathrm{d}A}{\mathrm{d}y} + \left(\frac{\alpha}{2\pi}\right)\left(\frac{\mathrm{d}\delta_\gamma}{\mathrm{d}y} + \frac{\mathrm{d}\delta_A}{\mathrm{d}y} - \frac{\mathrm{d}A}{\mathrm{d}y}\delta_{\sigma,1}\right) + \mathcal{O}\left(\alpha^2\right)$$

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$$\frac{1}{\sigma_{\mathrm{had}}}\frac{\mathrm{d}\sigma}{\mathrm{d}y} = \frac{\mathrm{d}A}{\mathrm{d}y} + \underbrace{\left(\frac{\alpha}{2\pi}\right)\left(\frac{\mathrm{d}\delta_\gamma}{\mathrm{d}y} + \frac{\mathrm{d}\delta_A}{\mathrm{d}y} - \frac{\mathrm{d}A}{\mathrm{d}y}\delta_{\sigma,1}\right)}_{\frac{\mathrm{d}\delta_{\mathrm{EW}}}{\mathrm{d}y}} + \frac{\mathrm{d}\delta_{\mathrm{EW,LL}}}{\mathrm{d}y} + \mathcal{O}\left(\alpha^2\right)$$

Event-shape observables in perturbation theory

- Experimentally measured quantity

$$\frac{1}{\sigma_{\text{had}}}\frac{\mathrm{d}\sigma}{\mathrm{d}y}$$

- σ_{had} up to $\mathcal{O}(\alpha)$

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$$\frac{\mathrm{d}\delta_{\text{EW,LL}}}{\mathrm{d}y} = \left(\frac{\mathrm{d}\delta_{A,\geq 2,\text{LL}}}{\mathrm{d}y} - \frac{\mathrm{d}A}{\mathrm{d}y}\delta_{\sigma,\geq 2,\text{LL}}\right) + \left(\frac{\mathrm{d}A}{\mathrm{d}y}\delta^2_{\sigma,1,\text{LL}} - \frac{\mathrm{d}\delta_{A,1,\text{LL}}}{\mathrm{d}y}\delta_{\sigma,1,\text{LL}}\right)$$

contains logarithms of the form $\alpha^n \ln^n\left(\frac{s}{m_{\text{e}}^2}\right)$

Implementation

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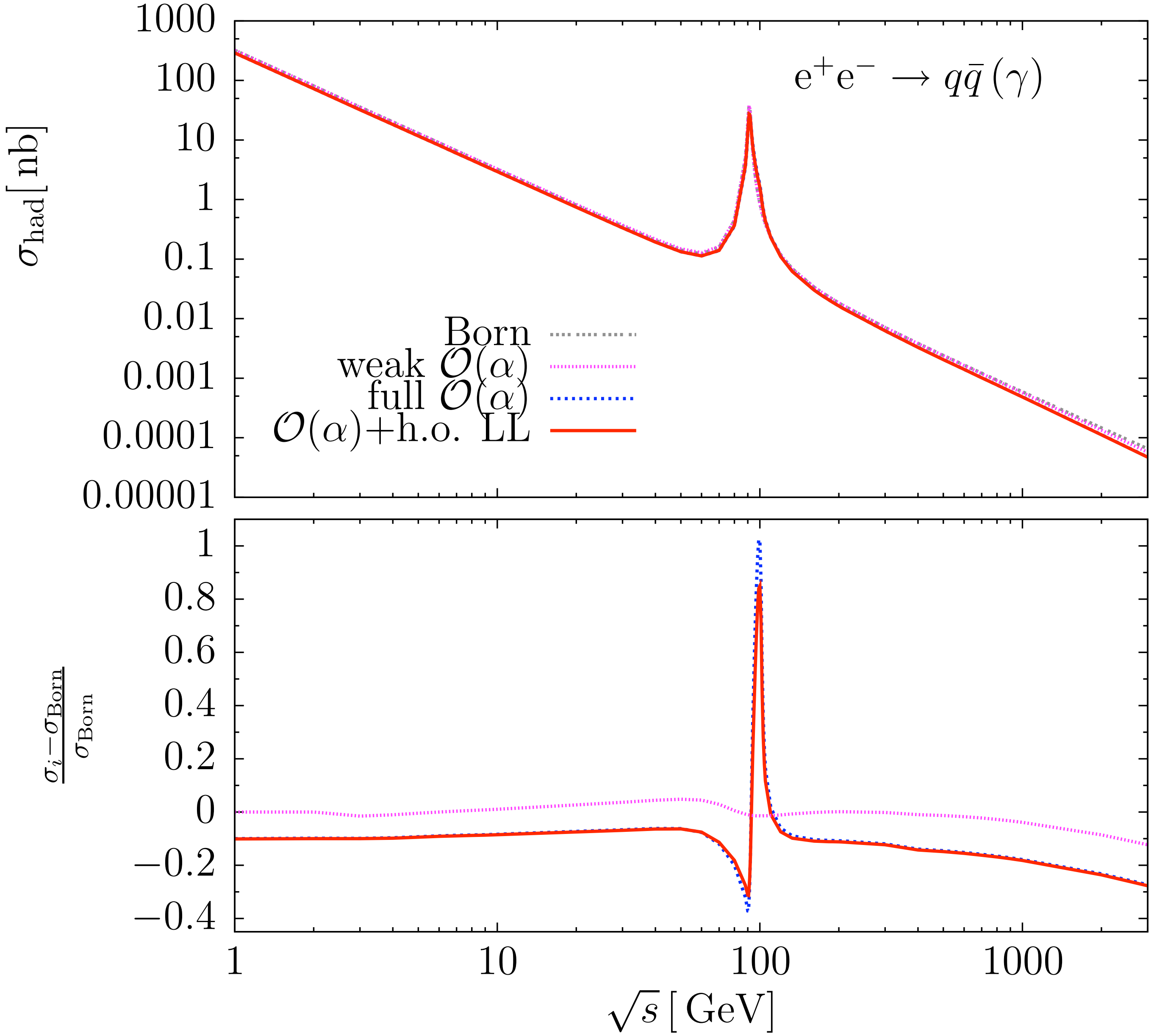
- Checks:
  - UV finiteness: vary scale μ of dim. reg. → result unchanged
  - IR finiteness: vary m_γ and small m_f → result unchanged
  - two completely independent calculations: one by S. Dittmaier and T. Gehrmann, the other by A. Denner and CK → full agreement

Outline

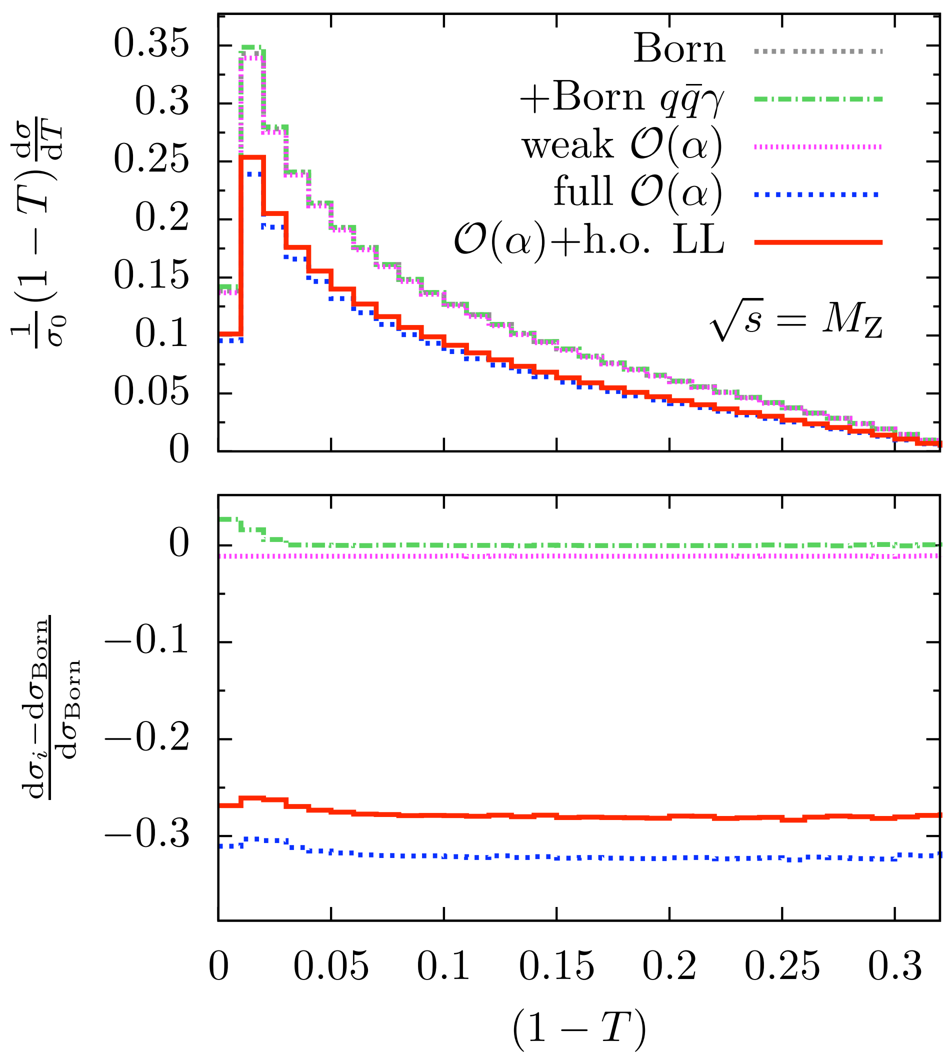
- Einführung in die Teilchenphysik
- QCD at e^+e^- colliders
- Inventory of the calculation
- Results
- Summary and conclusions

Results for σ_{had}



- weak $\mathcal{O}(\alpha)$ include weak and fermionic loops, contribute between -6% and +5%
- full $\mathcal{O}(\alpha)$ mostly between -10% and +30%, radiative return for $s > M_Z^2$
- h.o.LL increases corrections below 60 GeV and above 120 GeV, decrease between

Corrections for $s = M_Z^2$



- similar behavior as for σ_{had}
- onset of $q\bar{q}\gamma$ final states for $1 - T \lesssim 0.03$

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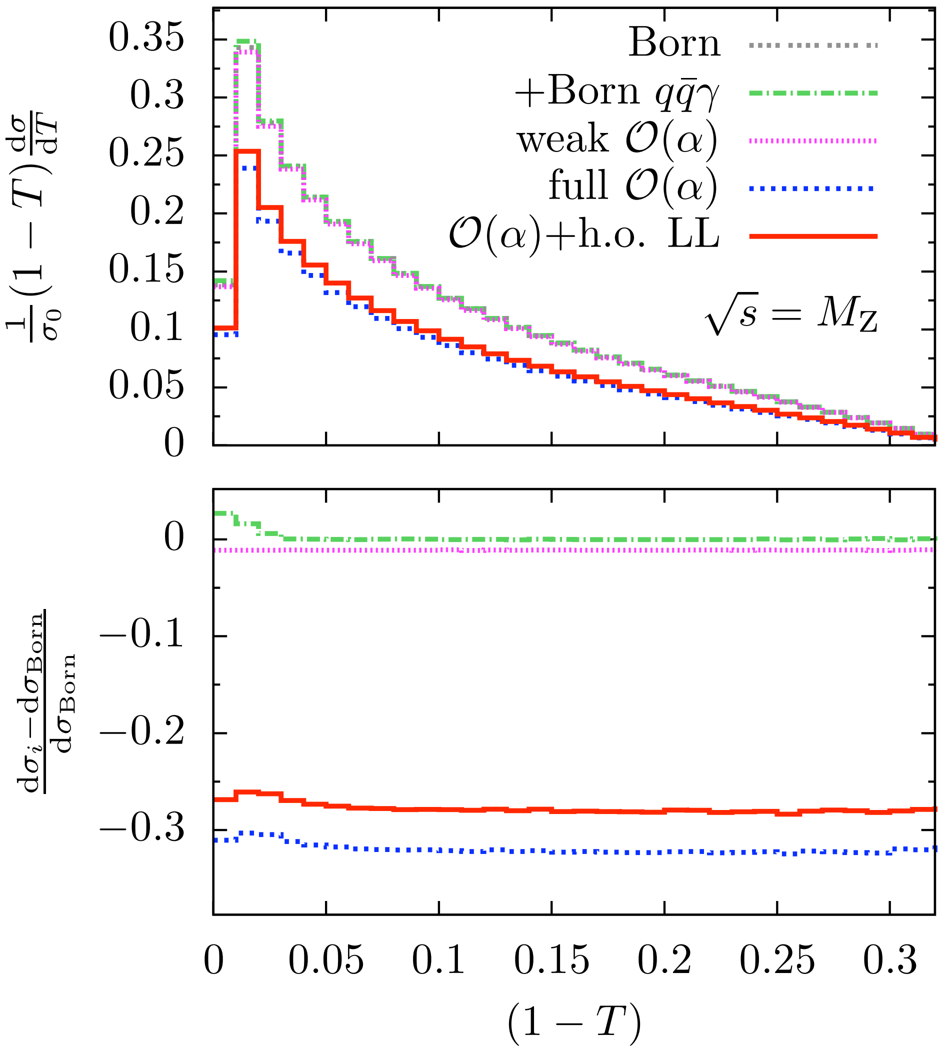

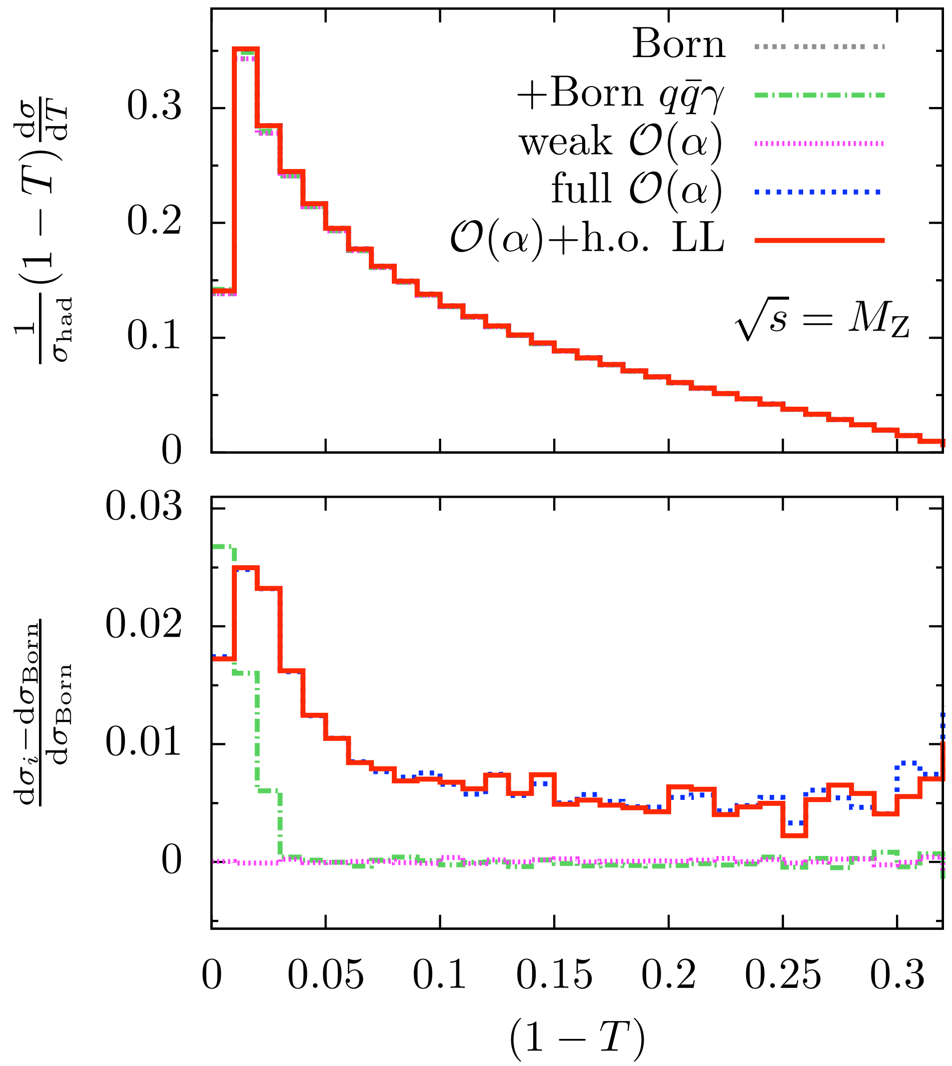

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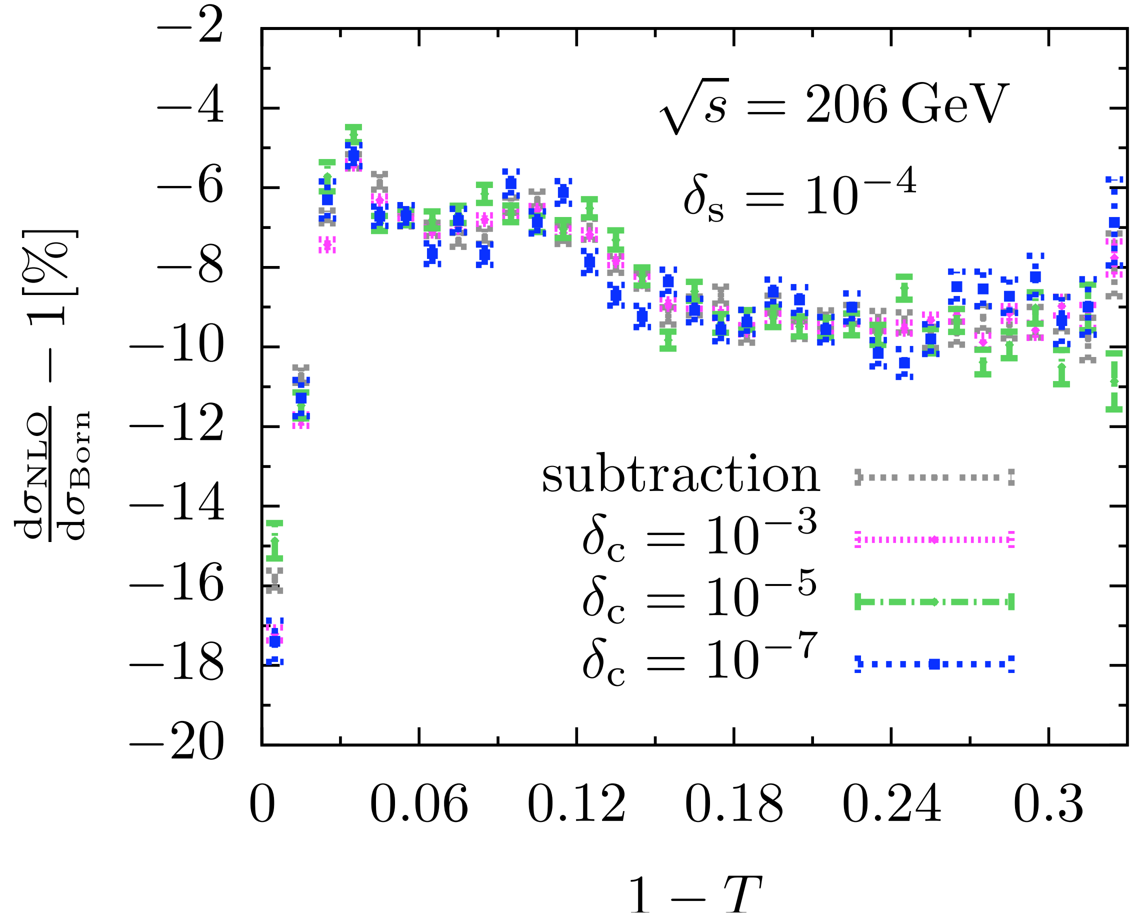
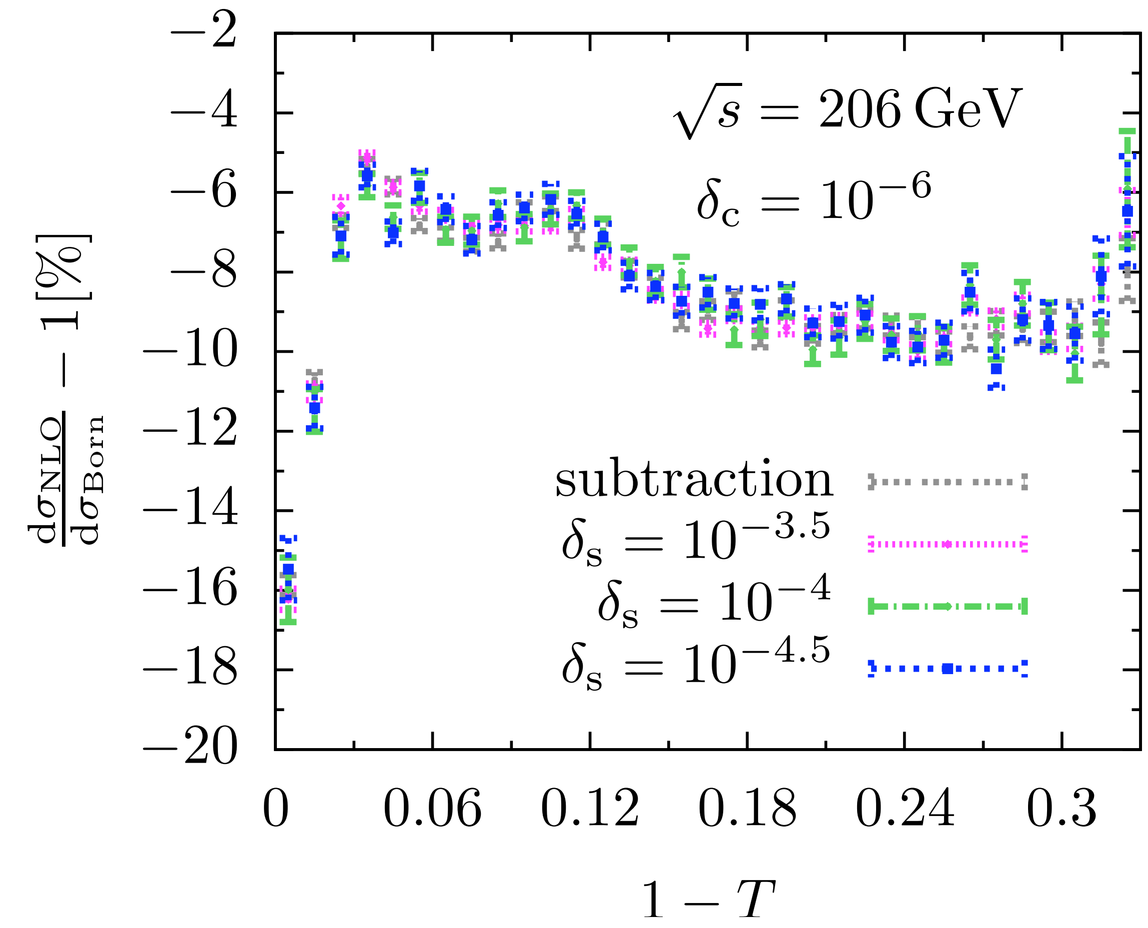
- cancellations of ISR contribution between distribution and σ_{had}
- weak corrections at per-mille level
- drop in first bin due to lower cut-off

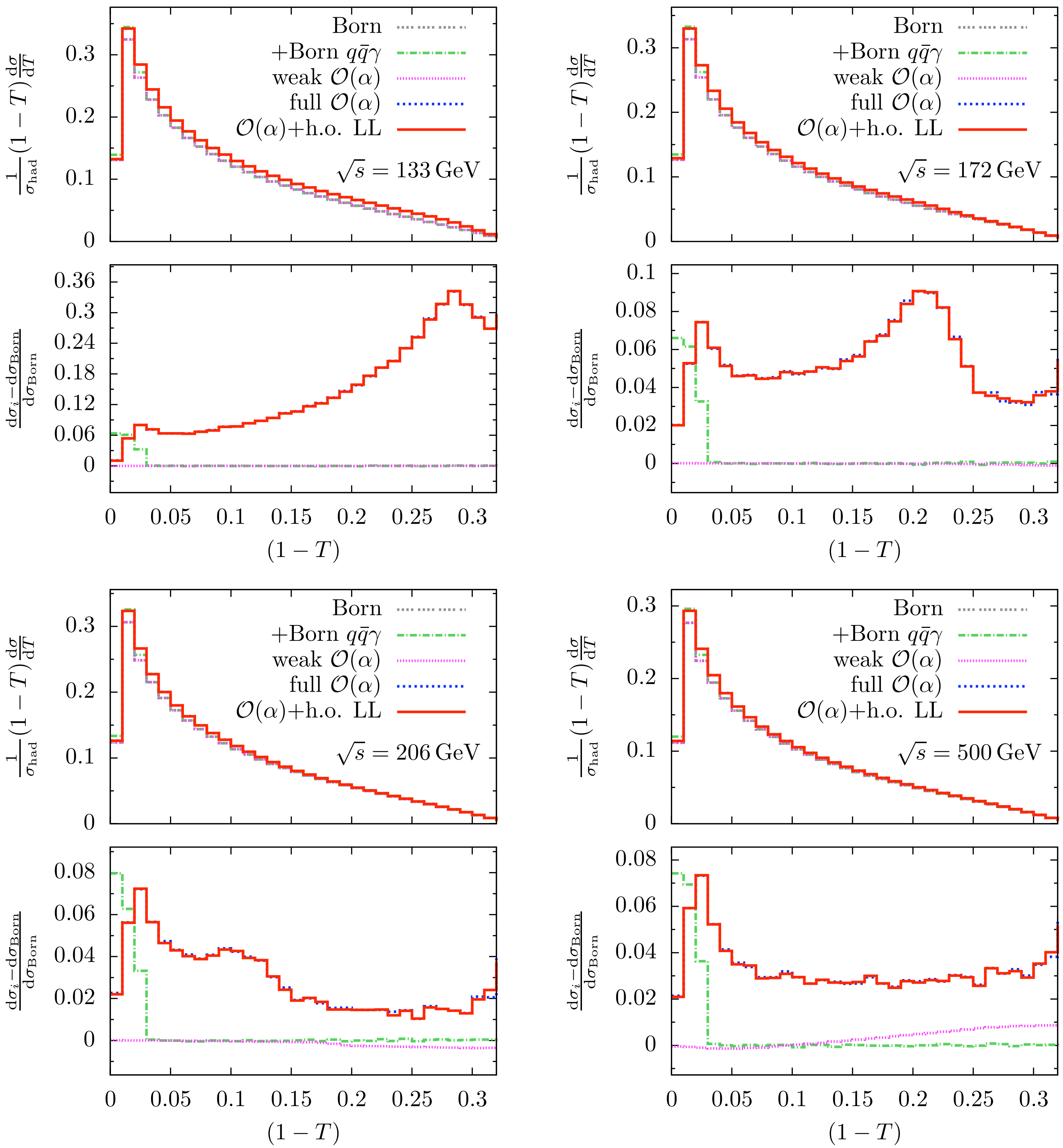
Phase-space slicing vs. subtraction





- vary slicing parameters → plateau for $\delta_s \lesssim 10^{-3}, \ \delta_c \lesssim 10^{-4}$
- subtraction and slicing agree perfectly
- subtraction more efficient:
  - 2×10^6 events for virtual, 5×10^9 events for real in slicing → 44h on single CPU
  - 2×10^6 events for virtual, 2×10^8 events for real in subtr. → 23h on single CPU

Corrections above Z peak



- emergence of peak for energies above M_Z
- peak moves to larger T for larger energies
- peak disappears for 500 GeV
- explained by radiative return
- crucial to implement exact experimental setup

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Summary and conclusions

- We have calculated the NLO EW corrections to event-shape observables and jet rates
- Results have been implemented into flexible Monte-Carlo parton-level event generator and are valid for arbitrary energies
- Experimental set up has been modeled as precisely as possible
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- We have calculated the NLO EW corrections to event-shape observables and jet rates
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- Include $q\bar{q}q\bar{q}$ contribution (QCD-EW interference)
- Use results for improved prediction of α_{s}

Backup slides

Input

- use standard set of input parameters
- use G_μ-scheme to derive electromagnetic coupling
- work in complex-mass scheme

$$
\begin{aligned}
G_\mu &= 1.16637 \times 10^{-5}\,\mathrm{GeV}^{-2}, & \alpha(0) &= 1/137.03599911, & \alpha_{G_\mu} &= 1/132.43421099 \\
\alpha_\mathrm{s}(M_\mathrm{Z}) &= 0.1176, \\
M_\mathrm{W}^\mathrm{LEP} &= 80.403\,\mathrm{GeV}, & \Gamma_\mathrm{W}^\mathrm{LEP} &= 2.141\,\mathrm{GeV}, \\
M_\mathrm{Z}^\mathrm{LEP} &= 91.1876\,\mathrm{GeV}, & \Gamma_\mathrm{Z}^\mathrm{LEP} &= 2.4952\,\mathrm{GeV}, \\
m_\mathrm{e} &= 0.51099892\,\mathrm{MeV}, & m_\mathrm{t} &= 171.0\,\mathrm{GeV}, & M_\mathrm{H} &= 120\,\mathrm{GeV}
\end{aligned}
$$

- conversion of on-shell LEP masses to pole masses

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M_V = M_V^\mathrm{LEP}/\sqrt{1+(\Gamma_V^\mathrm{LEP}/M_V^\mathrm{LEP})^2}, \qquad \Gamma_V = \Gamma_V^\mathrm{LEP}/\sqrt{1+(\Gamma_V^\mathrm{LEP}/M_V^\mathrm{LEP})^2}
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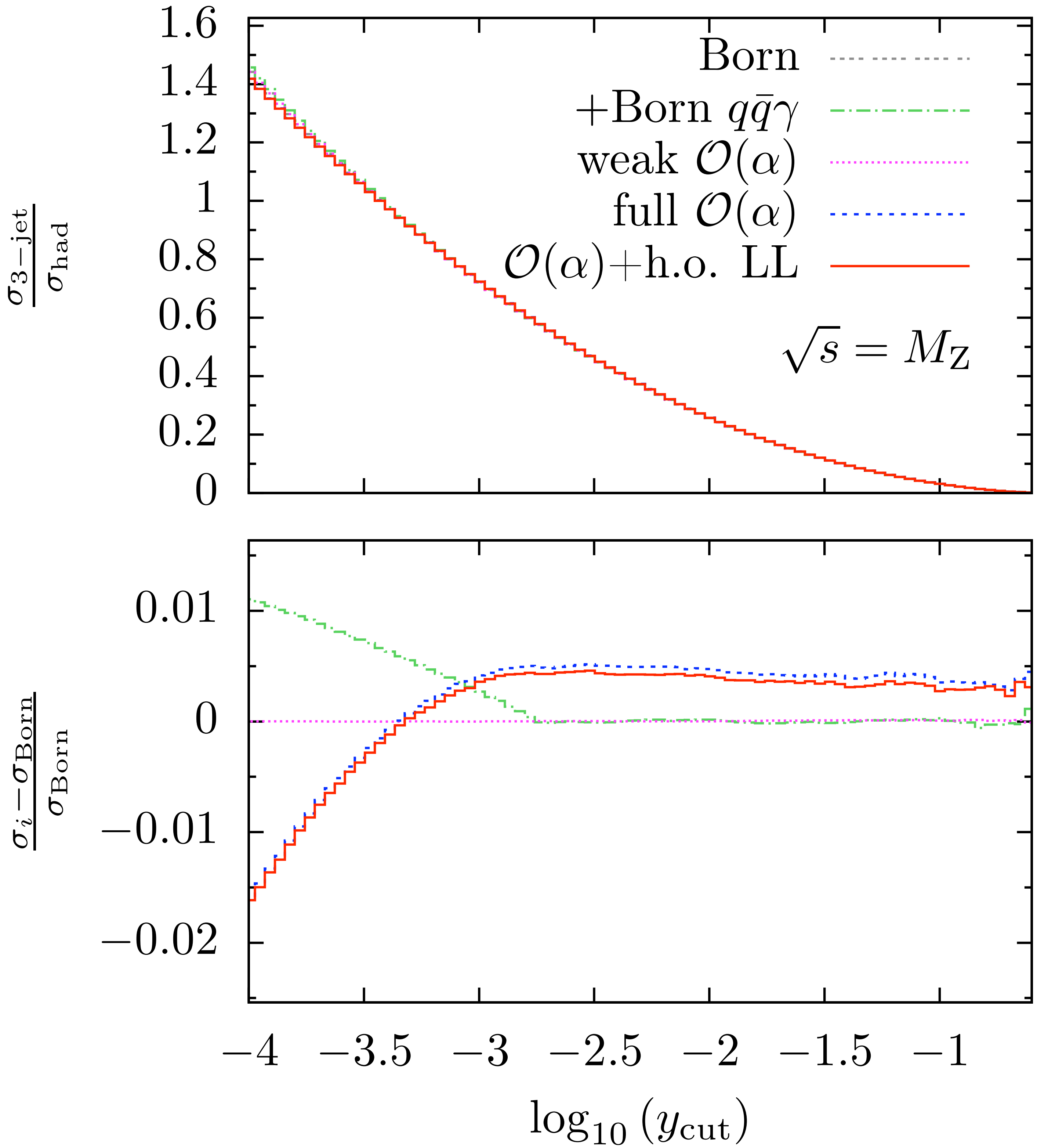
- event selection based on four parameters

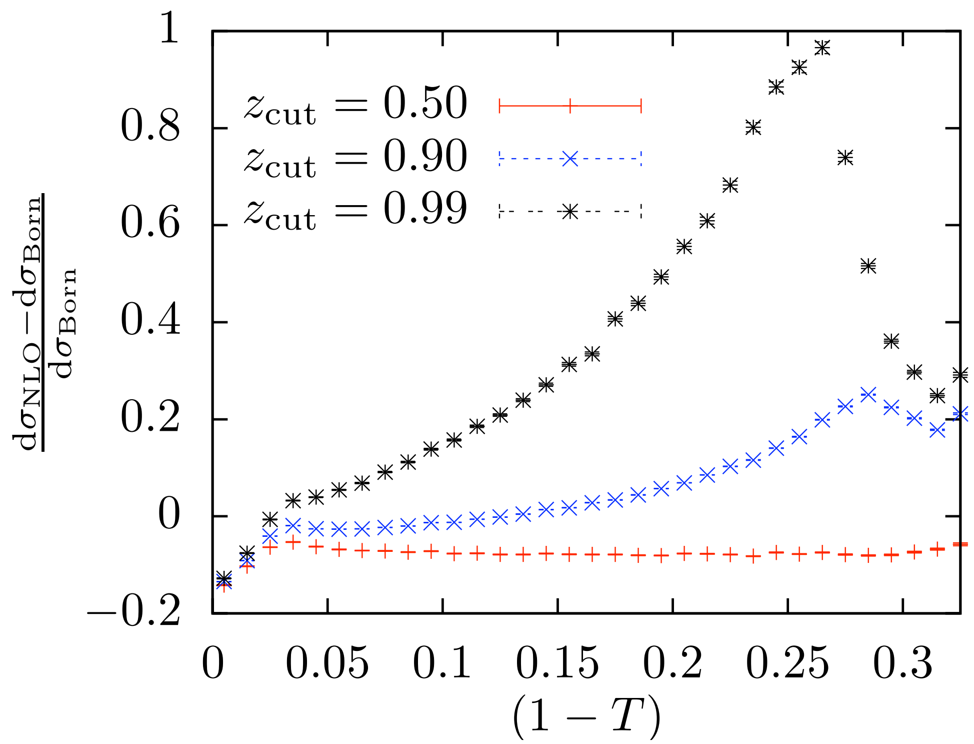
$$
\cos\theta_\mathrm{cut} = 0.965, \quad s_\mathrm{cut} = 0.81, \quad z_\mathrm{cut} = 0.9, \quad y_\mathrm{cut} = 0.002
$$

Three-jet rate

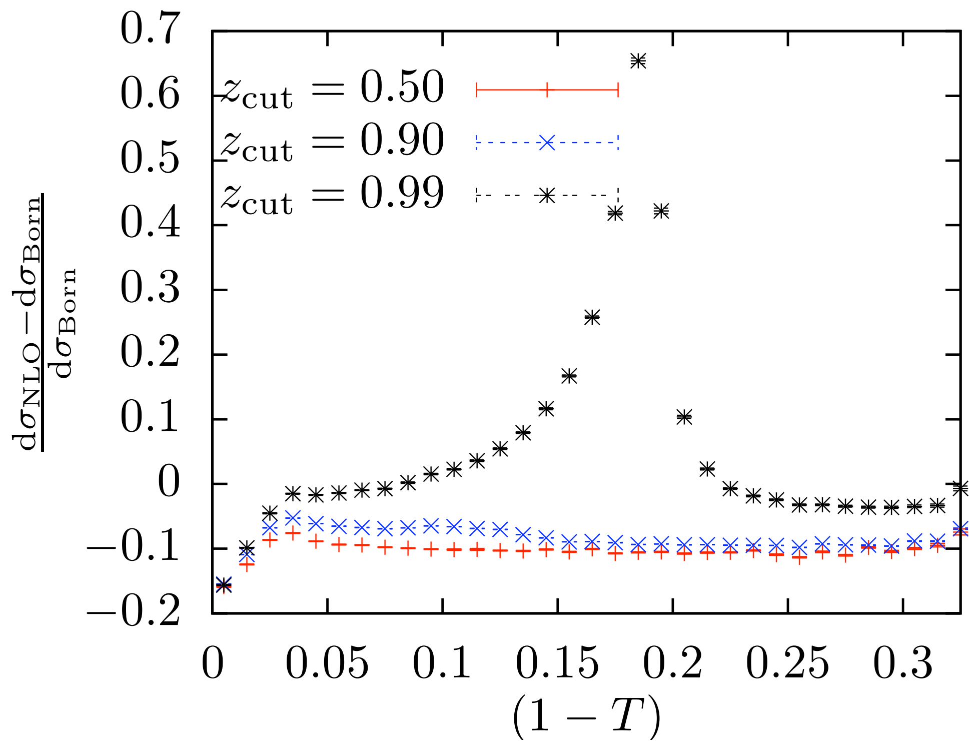


Dependence of results on es parameters

$\sqrt{s} = 133\,\mathrm{GeV}, s_{\mathrm{cut}} = 0.81, \cos\theta_{\mathrm{cut}} = 0.965, y_{\mathrm{cut}} = 0.002$



$\sqrt{s} = 206\,\mathrm{GeV}, s_{\mathrm{cut}} = 0.81, \cos\theta_{\mathrm{cut}} = 0.965, y_{\mathrm{cut}} = 0.002$



- peak can be explained by radiative-return phenomenon
- cluster energetic photon and soft gluon such that z_{cut} is not exceeded and $s_{q\bar{q}\gamma} = M_Z^2$
  - enhancement due to radiative return
  - logarithmic enhancement due to soft gluon
- analytic analysis shows perfect agreement with the observed behavior

Comparison to related work

- different group already has published results
  - do not consider $q\bar{q}\gamma$ final states
  - not normalised to σ_{had}
  - calculate event-shape observables from jet momenta and impose lower cut-off on jet energy

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  - do not consider $q\bar{q}\gamma$ final states
  - not normalised to σ_{had}
  - calculate event-shape observables from jet momenta and impose lower cut-off on jet energy

- not clear how event selection is realised in NLO calculation

  → difficult to perform tuned comparison

- agree on relative size of full $\mathcal{O}(\alpha)$ and h.o.LL improved results