Understanding the quark-gluon plasma via dimensional reduction

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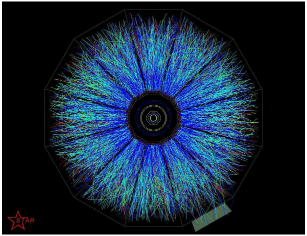
Why do we want to understand QGP??

RHIC/LHC: Deconfined QGP produced for the first time

- Ultimate goal: Quantitative understanding of the behavior of the collision from very early times to hadronization
- Success of hydrodynamic fits indicates early thermalization
- Hydrodynamic codes need equation of state, $p(T)$, as input
  - Problem in equilibrium thermodynamics



- Also: $p(T=0)$ at high T also relevant for precision cosmology

Preliminaries: Quantum fields in thermal equilibrium

All equilibrium thermodynamical information is encoded in the partition function.

$$Z = \mathrm{Tr}\exp\left(-\frac{\mathcal{H}}{T}\right) \equiv \sum_{\phi}\langle\phi|\exp\left(-\frac{\mathcal{H}}{T}\right)|\phi\rangle$$

For example:

$$\begin{aligned} p &= T\frac{\partial \log Z}{\partial V}, \qquad s = T\frac{\partial \log Z}{\partial T} \\ E &= pV + TS \end{aligned}$$

Preliminaries: Quantum fields in thermal equilibrium

All equilibrium thermodynamical information is encoded in the partition function.

- Can be written as a path integral

$$Z = \sum_{\phi} \langle \phi | \exp\left(-\frac{\mathcal{H}}{T}\right) | \phi \rangle = \int_{\phi(0,\mathbf{x})=\phi(1/T,\mathbf{x})} \mathcal{D}\phi \exp(-S^E)$$

with

$$S^E = \int_{-\infty}^{\infty} d^3x \int_0^{1/T} d\tau \mathcal{L}^E$$

- QFT in thermal bath = vacuum QFT in $\mathbb{R}^3 \times \mathrm{S}^1$
- Bosons periodic b.c., fermions anti-periodic b.c.

Various methods to study $Z = \int \mathcal{D}\phi \exp(-S)$

- Numerical lattice simulations
- Hadron resonance gas model
- Weak coupling expansion
- Dimensionally reduced effective field theories

- Numerical lattice simulations
  - The Euclidean action can be discretized on a $N^3 \times N_\tau$ lattice with lattice spacing a:
    - ★ Path integral → normal multi-dimensional integral.

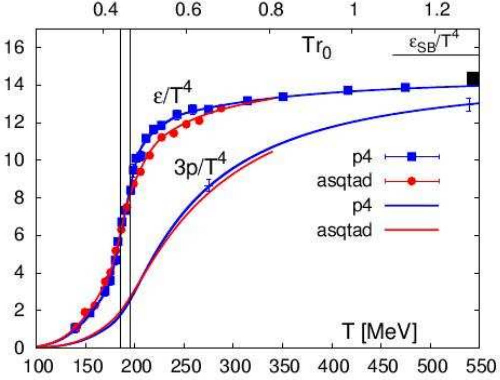
$$\int \mathcal{D}\phi \rightarrow \prod_{i=1}^{N^3 \times N_\tau} \int d\phi_i$$

  - Operator expectation values can be estimated using Monte Carlo simulations

$$\langle \mathcal{O} \rangle = \sum_{n=0}^{N_{MC}} \mathcal{O}(\phi_n) \overset{N_{MC} \rightarrow \infty}{\longrightarrow} \langle \mathcal{O} \rangle$$

    with $p(\phi) \propto \exp(-S(\phi))$
  - continuum limit $a \rightarrow 0$
  - Definitive non-perturbative method at $\mu = 0$.
  - Fails at finite μ due to the sign problem.
- Hadron resonance gas model
- Weak coupling expansion
- Dimensionally reduced effective field theories

- Numerical lattice simulations



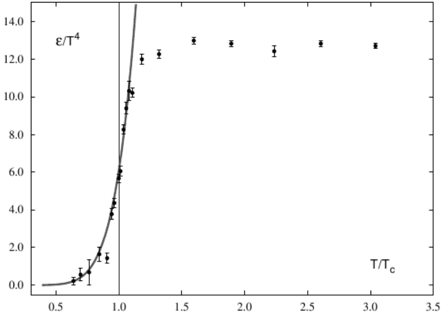
Karsch et al. 0903.4379

How should we understand this plot??

- Hadron resonance gas model
- Weak coupling expansion
- Dimensionally reduced effective field theories

- Numerical lattice simulations
- Hadron resonance gas model
    - At low T, the spectrum consists of almost free hadrons

$$\ln Z = \sum_i \ln Z_i^1, \quad \ln Z_i^1 = \eta V g_i \int_0^\infty \frac{d^3p}{(2\pi)^3} \ln(1+\eta e^{-\beta E_i})$$



Mean distance of particles

- $T = 140\text{MeV}$, $\Delta x = 2.2fm$.
- $T = 160\text{MeV}$, $\Delta x = 1.6fm$.
- $T = 200\text{MeV}$, $\Delta x = 0.9fm$.

$\rightarrow$ fails $\sim 200\text{MeV}$

Karsch et al. hep-ph/0303108

- Weak coupling expansion
- Dimensionally reduced effective field theories

- Numerical lattice simulations
- Hadron resonance gas model
- Weak coupling expansion
  - At high T asymptotic freedom makes the renormalized gauge coupling small $g(T) \sim 1/\ln T$

Finite temperature Feynman rules:

- Periodicity makes the p_0 discrete

$$\frac{1}{p^2} \longrightarrow \frac{1}{\mathbf{p}^2 + \omega_n^2},$$

  - Bosons: $\omega_n = (2n)\pi T$
  - Fermions: $\omega_n = (2n+1)\pi T$

Static mode: $n = 0$

- 4d integrals become 3+1d sum-integrals

$$\int \frac{d^4p}{(2\pi)^4} \to T \sum_n \int \frac{d^3p}{(2\pi)^3}$$

- Dimensionally reduced effective field theories

- Numerical lattice simulations
- Hadron resonance gas model
- Weak coupling expansion
  - At high T asymptotic freedom makes the renormalized gauge coupling small $g(T) \sim 1/\ln T$

Finite temperature Feynman rules:

- Leading order: Gas of non-interacting quarks and gluons

$$p_{\mathrm{SB}}/T^4 = \frac{\pi^2}{45}\left(N_c^2 - 1 + \frac{7N_cN_f}{4}\right)$$

- Recipe to compute higher order corrections: Vacuum diagrams

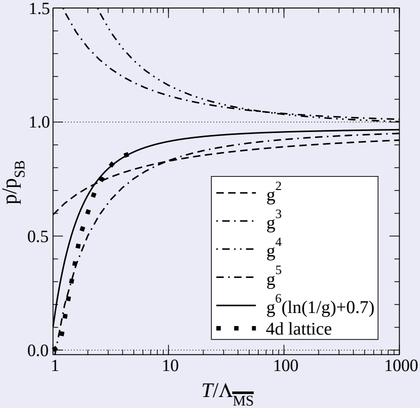
$$p/T^4 \sim 1 + g^2 + g^3 + g^4 \ln g + g^4 + g^5 + g^6 \ln g$$

- Non-analytic terms originate from IR divergences → resummations

- Dimensionally reduced effective field theories

- Numerical lattice simulations
- Hadron resonance gas model
- Weak coupling expansion

Finite temperature Feynman rules:



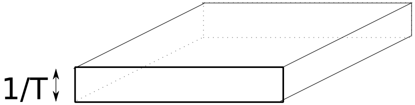
- Not straightforward to improve!! g^6-term non-perturbative.

- Dimensionally reduced effective field theories

Rest of this talk:

How to understand plasma at $T = T_c \ldots 10T_c$?

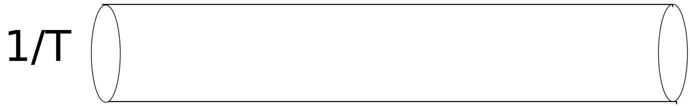
Can effective field theory and dimensional reduction provide answers?



$$S = \int_0^{1/T} \mathrm{d}\tau \int \mathrm{d}\mathbf{x} \mathcal{L}(\phi)$$

Dimensional reduction

- At high T: For long distance properties ($\Delta x \gg 1/T$), the system looks 3d.



- Degrees of freedom are static modes $\phi_0(\mathbf{x})$

$$\phi(\mathbf{x},\tau) = T\sum_{n=-\infty}^{\infty} \exp(i\omega_n\tau)\phi_n(\mathbf{x})$$

- Effective action: Integrate out non-static modes

$$Z = \int \mathcal{D}\phi_0 \mathcal{D}\phi_n \exp(-S_0(\phi_0) - S_n(\phi_0,\phi_n)) \quad \text{4d theory}$$

$$= \int \mathcal{D}\phi_0 \exp(-S_0(\phi_0) - S_{\text{eff}}(\phi_0)) \quad \text{3d theory}$$

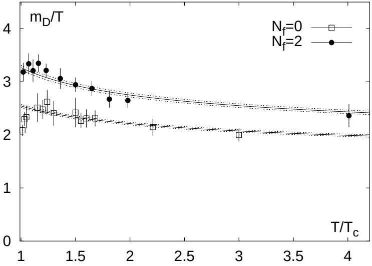
- In practice: Need scale separation between static and non-static modes to give a truncation to the effective action

Where does Dimensional Reduction work?

Scales in hot QCD:

- Perturbatively ($g \sim 1/\ln(T)$):
  - Hard scale: πT Typical thermal momentum, non-static modes
  - Soft scale: $m_D \sim gT$ Debye screening, A_0 static modes
  - Ultra soft scale: $m_M \sim g^2 T$ Color magnetic screening, A_i static modes

    $\Rightarrow$ Asymptotic dimensional reduction
- Non-perturbatively: $m(T_c) \sim 3T_c \overset{????}{\ll} \pi T_c$



Kaczmarek & Zantow hep-lat/0503017

- No scale separation below T_c, no DR

Electrostatic QCD Braaten & Nieto

Integrate out the hard scale to get EQCD
(=3d Yang-Mills + adjoint Higgs)

$$S_{\rm EQCD} = \underbrace{\frac{1}{g_3^2}}_{\sim g^2 T} \int \mathrm{d}^3x \Big[\underbrace{\frac{1}{2}\mathrm{Tr} F_{ij}}_{\text{spatial gluons}} + \underbrace{\mathrm{Tr}(D_i A_0)^2}_{\text{adjoint kinetic}} + \underbrace{\frac{1}{2}\overbrace{m_{\rm E}^2}^{g^2T^2}\,\mathrm{Tr} A_0^2 + \frac{1}{4}\lambda_E \mathrm{Tr} A_0^4}_{\text{interactions from integration out}} + \ldots$$

- Higher order terms suppressed by powers of the scale difference
- Eff. theory parameters $g_3(T), m_E(T), \lambda_E(T)$ via perturbative matching $\sim g^2(7T_c)/[4\pi]^2 \sim 0.05$.
- After integrating out the soft modes (A_0):

$$S_{\rm MQCD} = \frac{1}{g_3'^2} \int \mathrm{d}^3x \Big[\frac{1}{2}\mathrm{Tr} F_{ij} \Big]$$

- Philosophy: Integrate out heavy modes analytically, simulate low-energy theory numerically.

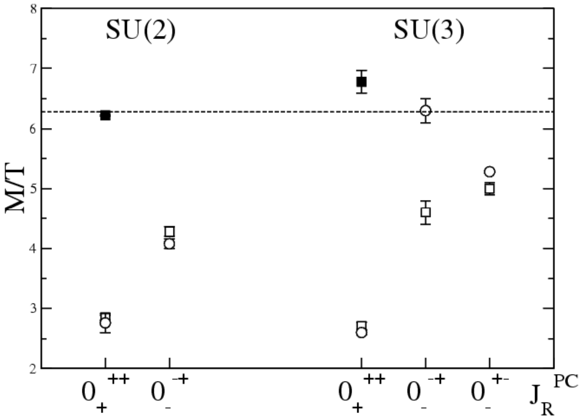
Example: g^6-coefficient from MQCD

The g^6-resummation can be done numerically in eff. theory framework:

- Match QCD $\rightarrow$ EQCD to sufficient depth in g
  - The coefficient of the unit operator not matched yet (4-loop computation).
- Match EQCD $\rightarrow$ MQCD to sufficient depth in g Kajantie et al. hep-ph/0211321
- Match the 3d continuum $\overline{\text{MS}}$ theory with lattice theory to order a^0
  - done using 4-loop Numerical Stochastic Perturbation Theory Di Renzo et al. 0808.0557
- Measure pressure of lattice theory numerically Hietanen et al. hep-lat/0509107, Hietanen & AK hep-lat/0609015

Some results from EQCD

Screening masses down to $\sim 2T_c$

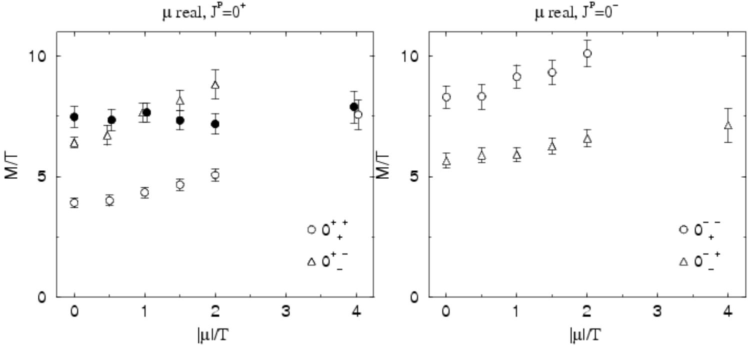


Circles 4d (no continuum or thermodynamic limit), squares dim.red.

Laermann, Philipsen, hep-ph/0303042

Some results from EQCD

Screening masses down to $\sim 2T_c$ at finite density

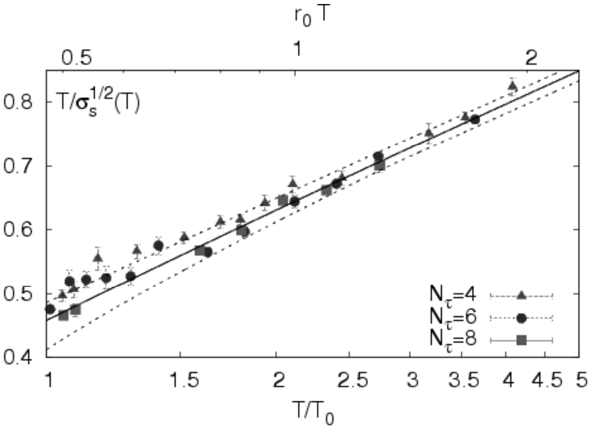


Hart et al., hep-ph/0004060

$$S_{\mathrm{EQCD}} = \int \mathrm{d}^3x \Big[\frac{1}{2}\mathrm{Tr}F_{ij} + \mathrm{Tr}(D_iA_0)^2 + \frac{1}{2}m_{\mathrm{E}}^2\mathrm{Tr}A_0^2 + i\gamma\mathrm{Tr}A_0^3 + \frac{1}{4}\lambda_E\mathrm{Tr}A_0^4\Big]$$

Some results from EQCD

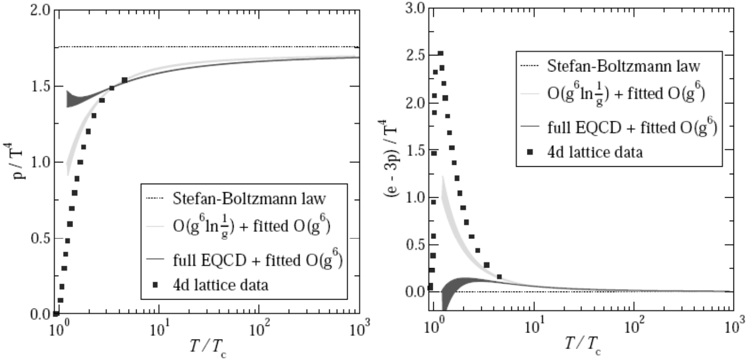
Spatial string tension to $\sim T_c$



Karsch et al., 0806.3264

Results from perturbative dimensional reduction

Pressure and trace anomaly fail near T_c



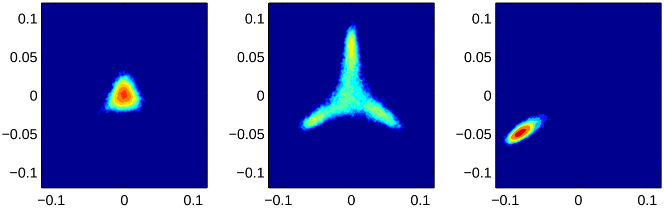
Hietanen et al., 0811.4664

Improving the low-T region Vuorinen & Yaffe hep-ph/0604100, AK 0704.1416 , de Forcrand & AK & Vuorinen 0801.1566

Reason for failure?

- Deconfinement transition breaks the (approximate) center symmetry.

Polyakov loop $\Omega = \mathrm{Tr}\left[P \exp\left(ig \int \mathrm{d}\tau A_0\right)\right]$, has N_c minima in the deconfined phase $\Rightarrow Z_N$ center symmetry.



$T < T_c$ $\quad$ $T \approx T_c$ $\quad$ $T > T_c$

- Center symmetry explicitely broken by EQCD

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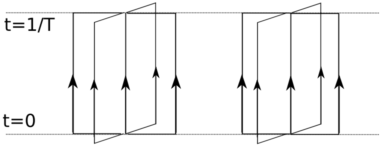
Vuorinen & Yaffe hep-ph/0604100, AK 0704.1416 , de Forcrand & AK & Vuorinen 0801.1566

$$\mathcal{Z}(\mathbf{x}) = \frac{T}{V_{\text{Block}}} \int_V \mathrm{d}^3 y U(\mathbf{x},\mathbf{y}) W(\mathbf{y}) U(\mathbf{y},\mathbf{x}), \notin SU(N_c)$$



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Example: SU(2) $\mathcal{Z} = \frac{1}{2}\Big\{ \underbrace{\Sigma \mathbb{1}}_{\text{Singlet}} + i \underbrace{\Pi_a \sigma_a}_{\text{Adjoint scalar}} \Big\}$

$$\mathcal{L}_{\mathrm{Z}(2)} = \underbrace{\frac{1}{2}\mathrm{Tr}\, F_{ij}^2}_{\text{spatial gluons}} + \underbrace{\mathrm{Tr}\left(D_i \mathcal{Z}^\dagger D_i \mathcal{Z}\right)}_{\text{Adjoint Kinetic}} + V(\mathcal{Z})$$

$$V(\mathcal{Z}) = \underbrace{b_1 \Sigma^2 + b_2 \Pi_a^2 + c_1 \Sigma^4 + c_2 (\Pi_a^2)^2 + c_3 \Sigma^2 \Pi_a^2}_{\text{interaction from integration out}}$$

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Example: SU(2) $\mathcal{Z} = \frac{1}{2}\Big\{ \underbrace{\Sigma \mathbb{1}}_{\text{Singlet}} + i \underbrace{\Pi_a \sigma_a}_{\text{Adjoint scalar}} \Big\}$

- Leads to the physical phase diagram
- Still relies on the scale differnece $m_D/(\pi T) \sim 1$.

Summary

- Dimensional reduction provides a bridge between the lattice computations and the perturbation theory.
- In DR setup, one can analytically deal with the heavy modes (non-static bosons and fermions) to get a theory which is more amenable for numerical simulations.
- The accuracy of description is limited by the truncation

however:

- DR is observed to give a good description to unexpectedly low temperatures $\sim T_c - 1.5T_c$.

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however:

- DR is observed to give a good description to unexpectedly low temperatures $\sim T_c - 1.5T_c$.
- Also: Work in progress to apply to 5d in order to get some non-perturbative info about KK theory.