High energy limit of QCD: Heavy quark production and the effective action

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Outline

Perturbative QCD at high energies

- squared center-of-mass energy $s \gg$ all other occuring energy scales
- hard scale Q^2, $\alpha_s(Q^2) \ll 1$, $s \gg Q^2 \gg \Lambda_{\text{QCD}}$
- large logarithm compensate for smallness of the coupling $\alpha_s(Q^2) \ln s \sim 1$
- **L**eading **L**ogarithmic **A**pproximation (LLA): resummation of perturbative terms $(\alpha_s \ln s)^n$

→ **BFKL equation**

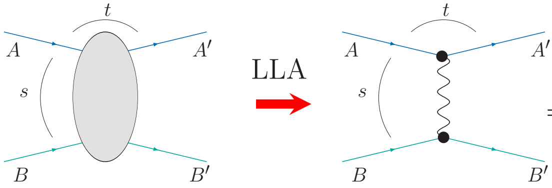
- LLA: [Balitsky, Fadin, Kuraev, Lipatov, 1975-1978]
- NLLA: [Fadin Lipatov, 1998; Ciafaloni, Camici 1998]

description strongly relies on the observation that the gluon reggeizes at high energies

High energy QCD amplitudes: gluon reggeization and BFKL

Reggeization of high energy QCD amplitudes

Scattering of colored objects, resum within LLA all order corrections [Fadin, Kuraev, Lipatov, 1975 - 1977]

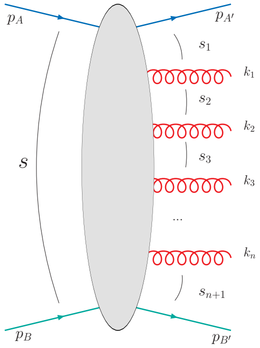


$$= 2gt^c_{AA'}\frac{s^{j(t)}}{t}gt^c_{BB'}$$

- all interaction gathered in collective excitation of gluon field
- Regge-pole with quantum number of the gluon, trajectory $j(t) = 1 + \beta(t)$, $j(t = 0) = 1$

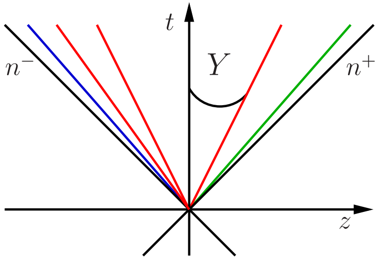
→ **the reggeized gluon**

n-gluon production amplitude within the LLA



MRK: particles strongly ordered in rapidity

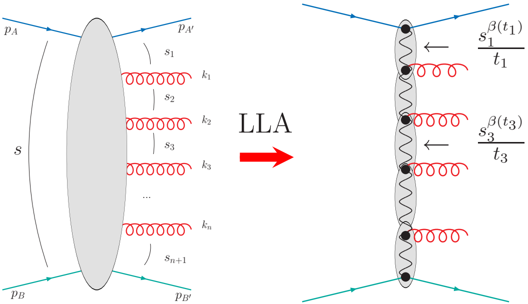
within LLA: production takes place in the Multi-Regge-Kinematics (MRK)
$s = (p_A + p_B)^2 \gg s_r = (k_r + k_{r-1})^2 \gg= -t_r$,
$t_r = -\boldsymbol{q}_r^2, k_r = q_{r+1} - q_r$



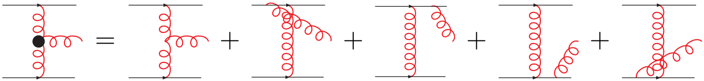
natural to define light-cone variables
$k^\pm = n^\pm \cdot k$
and rapidity $Y = \ln(k^+/k^-)/2$

n-gluon production amplitude within the LLA

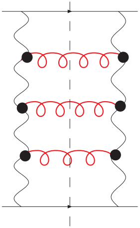
- Interaction mediated by the reggeized gluon ➡ effective degree of freedom in the t-channel of high energy QCD amplitudes



- interact with QCD particles by gauge invariant vertices contain 'non-local' contributions

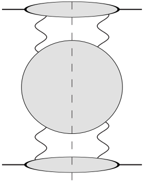


The BFKL-equation: state of two reggeized gluons



$$\sigma(s) = \int d^2\mathbf{k}_a \int d^2\mathbf{k}_b \phi_A(\mathbf{k}_a)\, f\left(\frac{s}{s_0}, \mathbf{k}_a, \mathbf{k}_b\right)\, \phi_B(\mathbf{k}_b)$$

- BFKL Green's function $f\left(\frac{s}{s_0}, \mathbf{k}_a, \mathbf{k}_b\right) = \int \frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega} f_\omega(\mathbf{k}_a, \mathbf{k}_b)$
- solution of BFKL-equation



$$\omega f_\omega(\mathbf{k}_a, \mathbf{k}_b) = \delta(\mathbf{k}_a - \mathbf{k}_b) + K_{\text{BFKL}}(\mathbf{k}_a, \mathbf{k}) \otimes f_\omega(\mathbf{k}, \mathbf{k}_b)$$

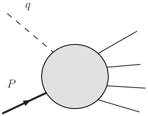
- BFKL-Kernel K_{BFKL} contains both real (production vertex) and virtual (gluon trajectory) contributions
- impact factors $\phi_A(\mathbf{k}_a)$, $\phi_B(\mathbf{k}_b)$: coupling of BFKL-Green's function to external particles

BFKL and HERA + LHC phenomenology

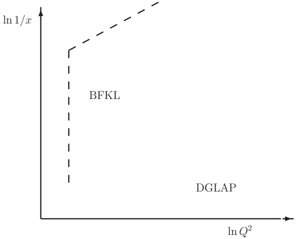
BFKL predicts powerlike rise of total cross-section

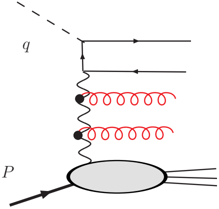
$\sigma_{\text{tot}} \sim s^{\lambda}$, $\lambda = \frac{\alpha_s N_c}{\pi} 4 \ln 2$, hard or LO BFKL Pomeron intercept

→connection to DIS and evolution equations



$F_2(x, Q^2)$, $x \sim Q^2/s$



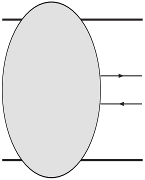


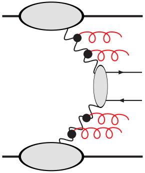
HERA: observe strong rise of F_2 for small x ($\equiv$ large s)

→believed to be driven by BFKL-evolution

$\leftrightarrow$all data well described by DGLAP-evolution

BFKL at hadron collider: heavy quark production





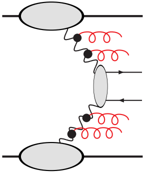
- heavy quark mass allows for perturbative treatment
- finite order calculations plagued by large logs which require resummation (threshold, small x)
- charm, bottom: proton probed at relatively small $x \sim M^2/s$, BFKL logarithms become important

previous studies:

- LO BFKL, inclusive study (total heavy flavor cross-section)
- good description of Tevatron data, large scale dependence (20 % - 50 %)

Exclusive heavy quark production at the LHC

goal: test BFKL predictions at the LHC; ideal testing ground as



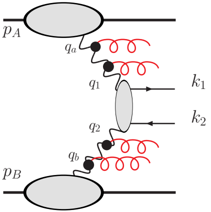
- high center of mass energy $\longrightarrow$ natural enviroment to apply BFKL
- high luminosity $\longrightarrow$ huge amount of heavy quarks produced

➡ allows for study of exclusive differential cross-sections

➡ stronger test than previous inclusive studies

- use NLO BFKL to increase of accuracy reduce large scale dependence
- give formulation that allows to keep additional gluon radiation exclusive

The differential cross-section in Regge factorization



$s_1 = (p_A + q_2)^2 = ys$, $s_2 = (p_B + q_1)^2 = xs$

$\alpha_i = \sqrt{\frac{M^2+\mathbf{k}_i^2}{s}} e^{\eta_i}$, $\beta_i = \sqrt{\frac{M^2+\mathbf{k}_i^2}{s}} e^{-\eta_i}$, i=1,2

η_i, $\mathbf{k}_i$, $i = 1, 2$: rapidity/transverse momentum of produced heavy quarks

M : heavy quark mass

$$\frac{d^6\sigma}{d\eta_1 d\eta_2 d\mathbf{k}_1 d\mathbf{k}_2} = \frac{1}{16\pi^2} \int dx \int dy \int \frac{d^2\mathbf{q}_1}{(2\pi)^3} \int \frac{d^2\mathbf{q}_2}{(2\pi)^3}$$
$$\left(\int d^2\mathbf{q}_a \frac{\Phi_A(\mathbf{q}_a)}{\mathbf{q}_a^2} f(\frac{s_1}{s_{0,2}}, \mathbf{q}_a, \mathbf{q}_1) \right) |\Gamma_{\mathrm{RR}\to\mathrm{Q}\bar{\mathrm{Q}}}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{k}_1, \mathbf{k}_2, z)|^2$$
$$\left(\int d^2\mathbf{q}_b \frac{\Phi_B(\mathbf{q}_b)}{\mathbf{q}_b^2} f(\frac{s_2}{s_{0,2}}, \mathbf{q}_2, \mathbf{q}_b) \right) (2\pi)^4 \delta^{(2)}(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{k}_1 - \mathbf{q}_2)$$
$$\delta(x - \alpha_1 - \alpha_2)\delta(y - \beta_1 - \beta_2)$$

The transverse energy scale $s_{0,1}$

BFKL Green's function: $f\left(\frac{s_1}{s_{0,1}}, \mathbf{k}_1, \mathbf{q}_1\right) = \int \frac{d\omega}{2\pi i} \left(\frac{s_1}{s_{0,1}}\right)^\omega f_\omega(\mathbf{k}_1, \mathbf{q}_1)$

f_ω as solution of BFKL equation

- choice of energy scale $s_{0,1}$ arbitrary for LO BFKL (source of large scale dependence), constrained at NLO
- theory: BFKL/Multi-Regge-Kineatics: all (transverse) scales are of the same order of magnitude

→ symmetric choice $s_{0,1} = \sqrt{\mathbf{k}_1^2 \Sigma}$, with $\mathbf{k}_1^2$, Σ : transverse scale of proton/heavy quark system, $\Sigma = (k_1 + k_2)^2 + (\mathbf{k}_1 + \mathbf{k}_2)^2 = xys$

for such a choice: $\left(\frac{s_1}{s_{0,1}}\right)^\omega = e^{\omega(y_A - y_{Q\bar{Q}})}$

$y_A, y_{Q\bar{Q}}$: rapidity of the proton/heavy quark system

Unintegrated gluon density at LO and NLO

hadronic collision: transverse scales asymmetric $\Sigma \gg \mathbf{q}_a^2$

here: natural choice: $s_{0,1} = \Sigma, \qquad \left(\frac{s_1}{s_{0,1}}\right)^{\omega} = x^{-\omega}$

→ leads to concept of unintegrated gluon density = probability to resolve an off-shell gluon carrying longitudinal momentum fraction x and transverse momentum k_T

LO: choice of $s_{0,1}$ not constrained

$$g^{\mathsf{LO}}(x, \mathbf{k}) = \int \frac{d^2\mathbf{q}}{2\pi} \frac{\Phi_P(\mathbf{q})}{\mathbf{q}^2} f(x, \mathbf{q}, \mathbf{k})$$

NLO: obtain modification of both proton impact factor and NLO Green's function

$$g^{\mathsf{NLO}}(x, \mathbf{k}) = \int \frac{d^2\mathbf{q}}{2\pi} \frac{\tilde{\Phi}_P(\mathbf{q})}{\mathbf{q}^2} \tilde{f}(x, \mathbf{q}, \mathbf{k})$$

NLO itself independent of choice, impact on higher, undetermined orders

Collinear correction terms due to change of energy scales

Modification of impact factors

$$\tilde{\Phi}_P^{\mathsf{NLO}}(\mathbf{q}) = \Phi_P^{\mathsf{NLO}}(\mathbf{q}) - \frac{1}{2}\mathbf{q}^2 \int d^2\mathbf{l} \frac{\Phi_P^{\mathsf{LO}}(\mathbf{l})}{\mathbf{l}^2} K_{\mathsf{BFKL}}^{\mathsf{LO}}(\mathbf{l}, \mathbf{q}) \ln \frac{\mathbf{l}^2}{\mathbf{q}^2},$$

BFKL-Kernel (which yields the modified Green's function $\tilde{f}(x_1, \mathbf{q}_a, \mathbf{q}_1)$)

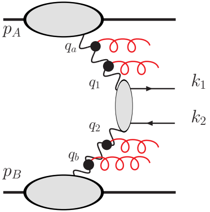
$$\tilde{K}_{\mathsf{BFKL}}^{\mathsf{NLO}}(\mathbf{l}_a, \mathbf{l}_b) = K_{\mathsf{BFKL}}^{\mathsf{NLO}}(\tilde{\mathbf{l}}_a, \mathbf{l}_b) - \frac{1}{2} \int d\mathbf{l}^2 K_{\mathsf{BFKL}}^{\mathsf{LO}}(\mathbf{l}_a, \mathbf{l}) K_{\mathsf{BFKL}}^{\mathsf{LO}}(\mathbf{l}, \mathbf{l}_b) \ln \frac{\mathbf{l}^2}{\mathbf{l}_b^2}.$$

and production vertex

$$\begin{aligned}
&|\tilde{\Gamma}_{\mathsf{RR}\to\mathsf{Q}\bar{\mathsf{Q}}}^{\mathsf{NLO}}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{k}_1, \mathbf{k}_2, z)|^2 = |\Gamma_{\mathsf{RR}\to\mathsf{Q}\bar{\mathsf{Q}}}^{\mathsf{NLO}}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{k}_1, \mathbf{k}_2, z)|^2 \\
&- \frac{\mathbf{q}_1^2}{2} \int d^2\mathbf{l} K_{\mathsf{BFKL}}^{\mathsf{LO}}(\mathbf{q}_1, \mathbf{l}) \frac{1}{\mathbf{l}^2} \ln \left(\frac{\mathbf{l}^2}{(\mathbf{q}_2 + \mathbf{l}^2)} \right) |\Gamma_{\mathsf{RR}\to\mathsf{Q}\bar{\mathsf{Q}}}^{\mathsf{LO}}(\mathbf{l}, \mathbf{q}_2; \mathbf{k}_1, \mathbf{k}_2, z)|^2 \\
&- \frac{\mathbf{q}_2^2}{2} \int d^2\mathbf{l} |\Gamma_{\mathsf{RR}\to\mathsf{Q}\bar{\mathsf{Q}}}^{\mathsf{LO}}(\mathbf{q}_1, \mathbf{l}; \mathbf{k}_1, \mathbf{k}_2, z)|^2 \frac{1}{\mathbf{l}^2} \ln \left(\frac{\mathbf{l}^2}{(\mathbf{q}_1 + \mathbf{l})^2} \right) K_{\mathsf{BFKL}}^{\mathsf{LO}}(\mathbf{l}, \mathbf{q}_2)
\end{aligned}$$

start of collinear/DGLAP evolution [Bartels, Sabio Vera, Schwennsen, 06]

The differential cross-section with unintgrated gluon densities



$\alpha_i = \sqrt{\frac{M^2+\mathbf{k}_i^2}{s}} e^{\eta_i}$, $\beta_i = \sqrt{\frac{M^2+\mathbf{k}_i^2}{s}} e^{-\eta_i}$, i=1,2

η_i, $\mathbf{k}_i$, $i = 1, 2$: rapidity/transverse momentum of produced heavy quarks

M : heavy quark mass

$$\frac{d^6\sigma}{d\eta_1 d\eta_2 d\mathbf{k}_1 d\mathbf{k}_2} = \frac{1}{4}\int \frac{d^2\mathbf{q}_1}{(2\pi)^2} g^{\mathrm{NLO}}(\alpha_1+\alpha_2, \mathbf{q}_1)$$
$$\times |\tilde{\Gamma}_{\mathrm{RR}\to\mathrm{Q\bar{Q}}}(\mathbf{q}_1, \mathbf{k}_1+\mathbf{k}_2-\mathbf{q}_1; \mathbf{k}_1, \mathbf{k}_2, z)|^2$$
$$\times g^{\mathrm{NLO}}(\beta_1+\beta_2, \mathbf{k}_1+\mathbf{k_2}-\mathbf{q}_1)$$

The NLO unintegrated gluon density

$$g^{\mathsf{NLO}}(x, \mathbf{k}) = \int \frac{d^2\mathbf{q}}{2\pi} \frac{\tilde{\Phi}_P(\mathbf{q})}{\mathbf{q}^2} \tilde{f}(x, \mathbf{q}, \mathbf{k})$$

- proton impact factors is non-perturbative; needs to be modeled and fitted to i.e. HERA DIS data
- LO Green's function: known analytically
- NLO Green's function: analytic form unknown, solve numerically using Monte-Carlo event generator

to this end: solve BFKL-equation iteratively:

BFKL kernel given as sum of virtual (trajectory) and real part:

$$\tilde{K}_{\mathsf{BFKL}}(\mathbf{q}_a, \mathbf{q}) = 2\omega(\mathbf{q}_a)\delta^{(2+2\epsilon)}(\mathbf{q}_a - \mathbf{q}) + K^{\mathsf{real}}(\mathbf{q}_a, \mathbf{q})$$

idea: treat real (gluon emission) and virtual (gluon trajectory, no gluon emission) parts on different grounds [Andersen, Sabio Vera, 03-04]

Iterative solution of the NLO BFKL-equation

i.e. rewrite BFKL-equation as

$$(\omega - 2\omega(\mathbf{q}_1))\,\tilde{f}_\omega(\mathbf{q}_a, \mathbf{q}_1) = \delta^{(2)}(\mathbf{q}_a - \mathbf{q}_1) + \int d^2\mathbf{q}\tilde{K}^{\text{real}}_{\text{BFKL}}(\mathbf{q}_a, \mathbf{q})\tilde{f}_\omega(\mathbf{q}, \mathbf{q}_1)$$

problem: real and virtual parts IR divergent, only sum is finite

→introduce phase space slicing parameter λ, to obtain finite real and virtual parts

$$K_\lambda(\mathbf{q}_a, \mathbf{q}) = \omega_\lambda(\mathbf{q})\delta^{(2)}(\mathbf{q}_a - \mathbf{q}) + K^{\text{real}}_\lambda(\mathbf{q}_a, \mathbf{q})$$

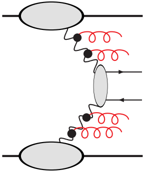
iterative solution of BFKL Green's function in (x, k_T) space given by

$$f(x, \mathbf{q}, \mathbf{k}) = x^{-\omega_\lambda(\mathbf{q})}\Bigg\{\delta^{(2)}(\mathbf{q} - \mathbf{k}) + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\int d^2\mathbf{l}_i\Bigg[K^{\text{real}}_\lambda(\mathbf{q}+\sum_j^{i-1}\mathbf{l}_j, \mathbf{q}+\sum_j^{i}\mathbf{l}_j)$$

$$\times\int_{x_{i-1}}^{1}\frac{dx_i}{x_i}x_i^{-\omega_\lambda(\mathbf{q}+\sum_{j=1}^{i}\mathbf{l}_j)+\omega_\lambda(\mathbf{q}+\sum_{j=1}^{i-1}+\mathbf{l}_j)}\Bigg]\delta^{(2)}(\mathbf{q}+\sum_{j=1}^{n}\mathbf{l}_j - \mathbf{k}),\Bigg\}$$

next task: numerical implementation + fit proton impact factors to HERA data

NLO matrix elements

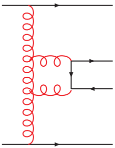


- NLO correction to heavy quark production vertex Γ^{NLO} currently unknown
- small x: main contribution from Green's function, restriction to LO production vertex good approximation
- prepare numerical study with Γ^{LO} alone

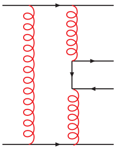
- Final goal: determine Γ^{NLO}
- problem: scattering of *off-shell reggeized gluons* ➡gauge invariance requires to include "non-local" diagrams

Sufficent: replace protons by quarks:

'local': low order reggeized gluon corresponds to single gluon



'non-local': reggeized gluon contains 2 gluon state

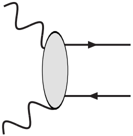


Can try to extract NLO matrix element from $2 \to 4$ process at 1-loop
➡systematic approach to calculate matrix elements: **effective action**

The gauge invariant effective action of high energy QCD

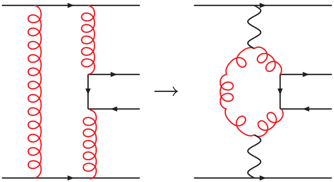
Systematic treatment: effective action

basic idea of th effective action: introduce new 'induced' vertices that allow to evaluate matrix elements *local in rapidity*



calculation of $g^*g^* \to Q\bar{Q}$ factorizes from scattering hadron, quark, gluon ...

achieve this by redistribution of diagrams in the $\mathbf{8_A}$-sector



→supplement QCD Lagrangian by induced term to achieve redistribution

The gauge invariant effective action

[Lipatov, 1995]

$$S_{\text{eff}} = \int d^4x \left[\mathcal{L}_{\text{QCD}}(v_\mu, \psi) + \mathcal{L}_{\text{ind}}(v_\mu, A_+, A_-)\right]$$

- describes interaction of reggeized gluons $(A^a_\pm)$ with quark- (ψ) and gluon-fields (v_μ)
- $A^a_\pm$ invariant under local gauge transformations, but transforms globally in the adjoint representation of $SU(N_c)$
- new induced part is given by

$$\mathcal{L}_{\text{ind}}(v_\pm, A_\pm) = \text{tr}\left[(A_-(v) - A_-)\,\partial^2_\sigma A_+\right] + \text{tr}\left[(A_+(v) - A_+)\,\partial^2_\sigma A_-\right]$$

where $A_\pm(v) = -\frac{1}{g}\partial_\pm U(v_\pm) = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + g^2 v_\pm \frac{1}{\partial_\pm} v_\pm \frac{1}{\partial_\pm} v_\pm - \ldots.$

- yields the induced vertices and reggeized gluon propagator

Gauge invariance and locality in rapidity

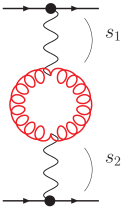
effective action factorizes amplitude into gauge invariant pieces, local in rapdidity

* interaction between particles and reggeized gluons restriced to small rapidity interval $\Delta Y < \eta$
* non-local interaction $\Delta Y > \eta$ mediated by gauge invariant reggeized gluon field

Effective action ready to calculate? NO!

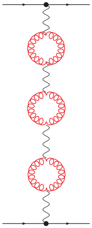
- need additional rules and a supplementary term

The trajectory of the reggeized gluon



- exchange of reggeized gluon only meaningful if s_1, s_2 are large
- simple cut-off regularization not sufficent ➡ improved regularization method:

$$\lim_{\nu\to 0}\int_{-i\infty}^{i\infty}\frac{d\omega}{4\pi i}\frac{1}{\omega+\nu}\left[\left(\frac{-s_1-i\epsilon}{\Lambda}\right)^{\omega}+\left(\frac{s_1-i\epsilon}{\Lambda}\right)^{\omega}\right]=\theta\left(\left|\frac{s_1}{\Lambda}\right|-1\right)$$



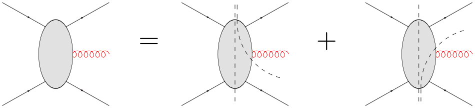
- allows to take into account imaginary part of the reggeized gluon
- Reobtain full reggeized gluon within the LLA with correct real and imaginary part

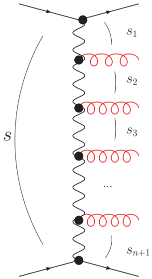
$$\mathcal{M}^{\text{tree}}(s,t)\int\frac{d\omega}{4\pi i}\frac{1}{\omega-\beta(t)}\left[\left(\frac{-s-i\epsilon}{s_R}\right)^{\omega}+\left(\frac{s-i\epsilon}{s_R}\right)^{\omega}\right]$$

Phase structure of the reggeized gluon

Production amplitudes: Constraint on discontinuities/imaginary parts due to the Steinmann relations: *no simultaneous discontinuities in overlapping channels*

Example:
$2 \to 3$ production amplitude:

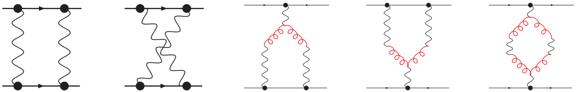




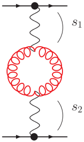
- LLA: both production vertex and reggeized gluon are real
- beyond: check by 1-loop correction to production vertex, that overall phase structure is correct - positive result

BFKL-equation: 2 reggeized gluons exchange

To reobtain the BFKL-equation from the effective action, consider t-channel state of 2 reggeized gluons:



- Diagrams contain contribution counted already in the trajectory



→remove overcounted terms by a supplementary term to the effective Lagrangian + yields convergence of integrals

$$\tilde{S}_{\text{eff}} = \int d^4x[\mathcal{L}_{\text{QCD}}(v_\mu, \psi) + \mathcal{L}_{\text{ind}}(v_\pm, A_\pm) - 2\mathcal{L}_{\text{ind}}(A_\pm, A_\pm)]$$

- not in conflict with the original derivation of the effective action
- allows to reobtain (LLA) BFKL-equation from effective action

Unitarization of the BFKL-Pomeron

Goals of effective action two-fold:

I Determination of $N^x LO$-corrections to the BFKL-equation and corresponding interactions with external particles

II systematic way to unitarize the BFKL-Pomeron ($=$ two reggeized gluon state in color singlet)

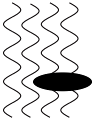
Why (II)? powerlike rise predicted by BFKL-Pomeron for total cross-section would finally violate unitarity, if infinitly continued
→ expect occurence of unitarity corrections for certain value of s

Effective action II: Transition vertices in QCD Reggeon-field-theory

Vertices in QCD Reggeon field theory

- most general approach: reformulate QCD as a $2+1$ dimensional Reggeon Field Theory (RFT) with both s-channel and t-channel unitarity build in
- know (from 'old' Regge theory), that a RFT of QCD exists [Gribov '68, . . .], however only some of its elements known so far

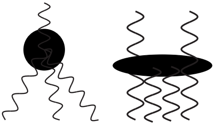
- BKP: states of n reggeized gluons [Bartels; Kwiecinski, Praszalowicz, '80]



  * ($\triangleq$ QM of n particles)
  * integrable in large N_c limit [Lipatov, '94; Faddeev, Korchemsky, '95]

- Number changing elements

  * ($\triangleq$ QFT)



  * Reggeon field theory

→ Use the effective action to derive these vertices

Transition vertices and signature conservation

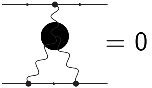
Also helpful for deeper understanding of the effective action

- proper treatment of higher order induced vertices
  $A(v_+) = v_+ - v_+ \frac{1}{\partial_+} v_+ + v_+ \frac{1}{\partial_+} v_+ \frac{1}{\partial_+} v_+$ - color and poles
- test the proposed supplementary term to the Lagrangian

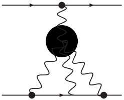
- additional conserved quantum number at high energies: signature
  $\tau = \pm 1$: $\mathcal{M}^{\tau=\pm}(-s,t) = \pm \mathcal{M}^{\tau=\pm}(s,t)$
- occurs naturally in the effective action

→transition from odd number state (negative signature) to even number state (positive signature) need to vanish inside elastic amplitude

- find: $U_{1\to 2}$ and $U_{2\to 3}$ vanish if inserted into elastic amplitude due to symmetry properties of loop integrals



- obtain non-zero transition $U_{1\to 3}$, $U_{2\to 4}$ inside the elastic amplitude



The $1-3$ transition in transverse momentum space

obtain as a result

$$U_{1\to 3}^{a;b_1b_2b_3}(\boldsymbol{q};\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \mathrm{tr}\left(T^aT^{b_1}T^{b_2}T^{b_3}\right)U(\boldsymbol{q};\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) + \mathrm{tr}\left(T^aT^{b_2}T^{b_1}T^{b_3}\right)U(\boldsymbol{q};\boldsymbol{k}_2,\boldsymbol{k}_1,\boldsymbol{k}_3) + \mathrm{tr}\left(T^aT^{b_1}T^{b_3}T^{b_2}\right)U(\boldsymbol{q};\boldsymbol{k}_1,\boldsymbol{k}_3,\boldsymbol{k}_2)$$

with

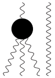
$$U(\boldsymbol{q};\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = g^4\frac{\boldsymbol{q}^2}{6}\int\frac{d^2\boldsymbol{l}}{(2\pi)^3}\left[\frac{\boldsymbol{q}^2}{\boldsymbol{l}^2(\boldsymbol{q}-\boldsymbol{l})^2} - \frac{(\boldsymbol{q}-\boldsymbol{k}_3)^2}{\boldsymbol{l}^2(\boldsymbol{q}-\boldsymbol{l}-\boldsymbol{k}_3)^2} + \frac{\boldsymbol{k}_1^2}{\boldsymbol{l}^2(\boldsymbol{l}-\boldsymbol{k}_1)^2}\right]$$

- symmetric under simultaneous exchange of transverse momenta and color
- $T^b_{cc'} = if^{cbc'}$ $SU(N_c)$ generator in the adjoint representation

The $2 \to 4$ transition vertex of the effective action

$$U_{2\to 4}^{a_1 a_2; b_1 b_2 b_3 b_4}(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = U_{\text{connect}}^{a_1 a_2; b_1 b_2 b_3 b_4}(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) + U_{\text{disconnect}}^{a_1 a_2; b_1 b_2 b_3 b_4}(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)$$

$$U_{\text{disconnect}}^{a_1 a_2; b_1 b_2 b_3 b_4}(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) =$$



$$+ \text{perm}$$

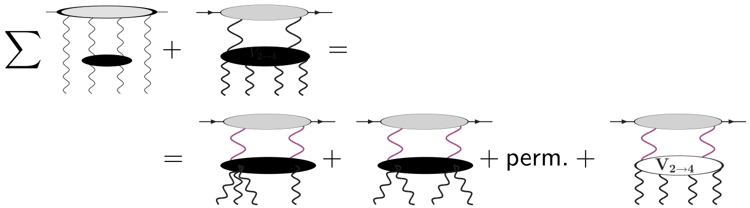
$$U_{\text{connect}}^{a_1 a_2; b_1 b_2 b_3 b_4}(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = \sum_{j_1, \ldots, j_4} \left(T^{b_{j_1}} T^{b_{j_2}} T^{b_{j_3}} T^{b_{j_4}}\right)_{a_1 a_2} U_C(\boldsymbol{l}_{i_1}, \boldsymbol{l}_{i_2}; \boldsymbol{k}_{j_1}, \boldsymbol{k}_{j_2}, \boldsymbol{k}_{j_3}, \boldsymbol{k}_{j_4})$$

$$U_C(\boldsymbol{l}_1, \boldsymbol{l}_2; \boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) = g^4 \boldsymbol{l}_1^2 \boldsymbol{l}_2^2 \left[\frac{1}{24} \frac{(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)^2}{\boldsymbol{l}_1^2 \boldsymbol{l}_2^2} - \frac{1}{12} \frac{(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)^2}{\boldsymbol{l}_1^2 (\boldsymbol{l}_2 - \boldsymbol{k}_4)^2} \right.$$
$$- \frac{1}{12} \frac{(\boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)^2}{\boldsymbol{l}_2^2 (\boldsymbol{l}_2 - \boldsymbol{k}_1)^2} + \frac{1}{8} \frac{(\boldsymbol{k}_2 + \boldsymbol{k}_3)^2}{(\boldsymbol{l}_1 - \boldsymbol{k}_1)^2 (\boldsymbol{l}_2 - \boldsymbol{k}_4)^2} + \frac{1}{24} \frac{(\boldsymbol{k}_1 + \boldsymbol{k}_2)^2}{\boldsymbol{l}_1^2 (\boldsymbol{l}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4)^2}$$
$$\left. - \frac{1}{24} \frac{\boldsymbol{k}_2^2}{(\boldsymbol{l}_2 - \boldsymbol{k}_3 - \boldsymbol{k}_4)^2 (\boldsymbol{l}_1 - \boldsymbol{k}_1)^2} + \frac{1}{24} \frac{(\boldsymbol{k}_3 + \boldsymbol{k}_4)^2}{\boldsymbol{l}_2^2 (\boldsymbol{l}_1 - \boldsymbol{k}_1 - \boldsymbol{k}_2)^2} - \frac{1}{24} \frac{\boldsymbol{k}_3^2}{(\boldsymbol{l}_1 - \boldsymbol{k}_1 - \boldsymbol{k}_2)^2 (\boldsymbol{l}_2 - \boldsymbol{k}_4)^2} \right]$$

The state of 4 reggeized gluons

- The transition vertex $U_{2\to4}$ has no good IR behavior by itself, not even in the overall color singlet
- Expect this only for the complete state of four reggeized gluons

for overall color singlet find following decomposition



- Additional reggeization in the symmetric sector
- $V_{2\to4}$ is the transition vertex of [Bartels, Wüsthoff, 1995]: infrared finite, 'triple Pomeron vertex'

Summary and Conclusions

- heavy flavor production :
  - set theoretical framework for heavy flavor production within NLO k_T factorization
  - increase accuracy and reduce scale dependence by NLO unintegrated gluon densities
  - iterative solution of BFKL-Green's function that allows for numerical solution
  - outlook: perform numerical implementation and obtain unintegrated gluon densities from fit to HERA data
- effective action:
  - requires supplementary rules + new term
  - allow rederivation of LLA reggeized gluon and BFKL-equation
  - derrived transition vertices of QCD Reggeon-field-theory and showed agreement with 'triple Pomeron vertex'
  - outlook: make action ready for NLO calculation