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Field redefinitions and renormalization group equations in RχT

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Outline

- Formal $1/N_C$ expansion in RχT

- The example of the $\pi\pi$-VFF:

  -One-loop computation

  -EoM, Field redef. & redundant operators

  -RGE → Solutions → IR-fixed point

  -Perturbative regime →Perturbation theory in $1/N_C$

$1/N_C$ expansion

in $R\chi T$

- **RχT =** chiral invariant framework for the interaction of chiral NGB & Resonances

[Ecker et al.'89]

- The standard EFT expansion in powers of $(p^2)^m$, not valid in presence of heavy resonance loops

→ An alternative power counting is required

➔ RχT takes the formal $1/N_C$ expansion as a guiding principle: ['t Hooft'74, 75] [Witten'79]

- N_C scaling of the operators in $\mathcal{L}$ at LO in $1/N_C$:
  $\mathcal{L} = \ldots + \lambda\, \mathcal{O}$, with k meson fields → scales like $N_C^{1-k/2}$

- Subleading operators → subleading scaling in $1/N_C$ (with respect to the latter)

- For practical purposes, the mesonic lagrangian can be organised in the form

$$\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = \mathcal{L}^{\mathrm{GB}} + \mathcal{L}^{R_i} + \mathcal{L}^{R_i R_j} + \mathcal{L}^{R_i R_j R_k} + ..$$

- Usually, the operators are built with the lowest number of derivatives as

  - operators with more derivatives tend to violate the asymptotic high-energy behaviour prescribed by QCD for Green-fuctions and form-factors

[Ecker et al.'89]

[Rosell, Pich, SC'04]
[Xiao & SC'08]

  - in some cases, higher derivative operators can be removed by means of the EoM (field redefinitions)

At LO in $1/N_C$ (large N_C):

- The operators with only Goldstones (no Resonance) are those from χPT:

$$\mathcal{L}_{\rm LO}^{\rm GB} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle \, .$$

[Gasser & Leutwyler'84,85]

[Ecker et al.'89]

- In the case of the vectors (in the antisymmetric tensor form.),

$$\mathcal{L}_{\rm LO}^{V} = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{i\,G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

[Ecker et al.'89]

with the canonical free-field kinetic term given by

$$\mathcal{L}_{\rm Kin}^{V} = -\frac{1}{2}\langle V_{\lambda\nu}\nabla^\lambda\nabla_\rho V^{\rho\nu}\rangle + \frac{1}{4}M_V^2\langle V_{\mu\nu}V^{\mu\nu}\rangle$$

[Gasser & Leutwyler'84]

[Ecker et al.'89]

At NLO in $1/N_C$ (loops + NLO tree-level):

[Rosell, Pich, SC’04]

[Ruiz-Femenía, Portoles, Rosell’07]

[Kampf, Novotny, Trnka’09]

…

- The naive dimensional analysis tells us that

  - The LO amplitude scales like $\mathcal{M} \sim p^2$

  - The NLO one-loop amplitude → $\mathcal{M} \sim p^4 \ln(-p^2)$

➔UV-divergences $\mathcal{M} \sim \lambda_\infty\, p^4$ (NLO in $1/N_C$)

➔$O(p^4)$ subleading operators to renormalize

E.g. in the case of the ππ-VFF,

$$\mathcal{L}_{\rm NLO}^{\rm GB} = -i\,\widetilde{L}_9 \langle f_+^{\mu\nu} u_\mu u_\nu \rangle ,$$

$$\mathcal{L}_{\rm NLO}^{V} = X_Z \langle V_{\lambda\nu} \nabla^\lambda \nabla_\rho \nabla^2 V^{\rho\nu} \rangle + X_F \langle V_{\mu\nu} \nabla^2 f_+^{\mu\nu} \rangle + 2\,i\,X_G \langle V_{\mu\nu} \nabla^2 [u^\mu, u^\nu] \rangle .$$

[Rosell, Pich & SC’04]

- However, higher power corrections (like e.g. $\mathcal{M} \sim p^4$)

  may look potentially dangerous if $p^2 \sim M_R^2$

  ← as there no characteristic scale $\Lambda_{R\chi T}$

  that suppresses the NLO for $p \ll \Lambda_{R\chi T}$

- A solution to this issue will be achieved

  by means of meson field redefinitions

  for the case of the $\pi\pi$-VFF…

$\pi\pi-$Vector Form Factor

$\pi\pi$–Vector Form Factor:

Basic definitions

and one-loop computation

- The ππ-VFF is defined by the scalar quantitys $F(q^2)$,

$$\langle \pi^-(p_1)\pi^0(p_2)|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}\,(p_1^\mu - p_2^\mu)\,\mathcal{F}(q^2)\,,$$

- The VFF can be decomposed in 1PI contributions of the form

$$\mathcal{F}(q^2) = \mathcal{F}(q^2)_{1\mathrm{PI}} + \frac{\Phi(q^2)\Gamma(q^2)}{F^2}\frac{q^2}{M_V^2 - q^2 - \Sigma(q^2)}\,,$$

Example of the tree-level LO form factor $\mathcal{F}(q^2) = 1 + \frac{F_V G_V}{F^2}\frac{q^2}{M_V^2 - q^2}\,.$

Assumptions:

- Truncation of the infinite number of large-N_C hadronic states

- Only up to $O(p^2)$ NGB operators at LO

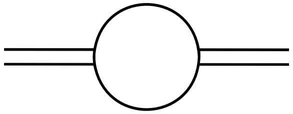
  \+ only up to $O(p^4)$ NGB operators at NLO

  (which can be justified under short-distance arguments)

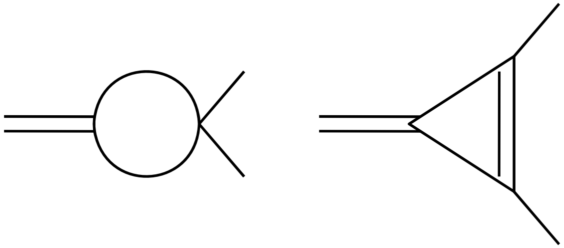
- Just the lowest thresholds ($\pi\pi$-cut) will be taken into account

(higher ones supposed to be renormalized in a decoupling scheme, with their contribution supressed below their production threshold)

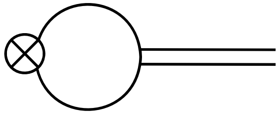
Explicit one-loop calculation → Loop contributions to the vertex functions,



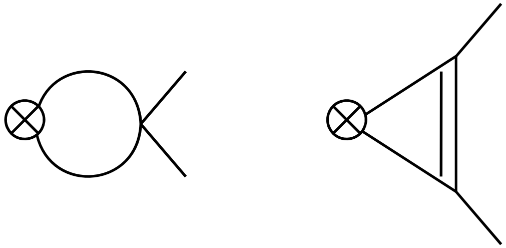
$\Sigma(q^2)$



$\Gamma(q^2)$



$\Phi(q^2)$



$\mathcal{F}(q^2)_{1\mathrm{PI}}$

•This requires the introduction of

$$X_Z\,, \qquad X_F\,, \qquad X_G\,, \qquad \widetilde{L}_9$$

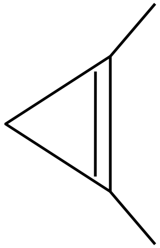
to make the amplitude finite

→Here ([Pich,Rosell, SC'04] [SC'09]) the MS(+constant) scheme was used

This leaves the renormalized vertex functions [Rosell, Pich & SC'04]

$$\begin{aligned}
\Sigma(q^2) &= -2q^4 X_Z - \frac{n_f}{2}\frac{2G_V^2}{F^2}\frac{q^4}{96\pi^2F^2}\ln\frac{-q^2}{\mu^2}\,, \\
\Gamma(q^2) &= -4\sqrt{2}X_G q^2 \\
&+G_V\left[1-\frac{n_f}{2}\left(1-\frac{G_V^2}{F^2}\right)\frac{q^2}{96\pi^2F^2}\ln\frac{-q^2}{\mu^2}+\Delta_t(q^2)\right], \\
\Phi(q^2) &= F_V - 2\sqrt{2}X_F q^2 - \frac{n_f}{2}\frac{2G_V}{F^2}\frac{q^2}{96\pi^2}\ln\frac{-q^2}{\mu^2}\,, \\
\mathcal{F}(q^2)_{\mathrm{1PI}} &= 1+\frac{2q^2\widetilde{L}_9}{F^2}+\Delta_t(q^2) \\
&\qquad -\frac{n_f}{2}\left(1-\frac{G_V^2}{F^2}\right)\frac{q^2}{96\pi^2F^2}\ln\frac{-q^2}{\mu^2}
\end{aligned}$$

with the finite triangle contribution,



$$\Delta_t(q^2) = \frac{n_f}{2}\frac{2G_V^2}{F^2}\frac{M_V^2}{16\pi^2F^2}\hat{\Delta}_t(q^2/M_V^2)\,,$$

being $\hat{\Delta}_t(x) = [\mathrm{Li}_2(1+x)-\mathrm{Li}_2(1)]\left(\frac{1}{x^2}+\frac{5}{2x}+1\right)+\ln(-x)\left(\frac{1}{x}+2\right)-\frac{1}{x}-\frac{9}{4}$

$$\left\{\begin{array}{l}
\hat{\Delta}_t \overset{x\to 0}{=} -\frac{1}{12}x\ln(-x)+\frac{35}{72}x+\mathcal{O}(x^2) \\[2ex]
\hat{\Delta}_t \overset{x\to\infty}{\sim} -\frac{1}{2}\ln^2 x.
\end{array}\right.$$

- The couplings in the renormalized amplitudes

  → Renormalized couplings

- The NLO running in $G_V(\mu)$

  → induces a residual NNLO μ-dependence

- This will allow us to use the RGE

  → to resum possible large radiative corrections

ππ–Vector Form Factor:

Field redefinitions

and redundant operators

- The $\mathcal{L}^{V}_{NLO}$ couplings are not physical by themselves

[Rosell, Pich & SC'04]

→Impossible to fix the $X_{Z,F,G}$ alone,

→just combinations of them and other couplings

- The reason is that they are proportional to the EoM,

$$(\nabla_\mu \nabla_\rho g_{\sigma\nu} - \nabla_\nu \nabla_\rho g_{\sigma\mu})\, R^{\rho\sigma} \;=\; -\, M_R^2\, R^{\mu\nu} \;+\; \underset{\substack{\text{at least}\\ \text{1 external}\\ \text{source}}}{\underset{\uparrow}{\mathcal{O}(J)}} + \underset{\substack{\text{at least}\\ \text{2 meson}\\ \text{fields}}}{\underset{\uparrow}{\mathcal{O}(\Phi^2)}}$$

and also obeying the KG equation,

$$\nabla^2 R^{\mu\nu} \;=\; -\, M_R^2\, R^{\mu\nu} \;+\; \mathcal{O}(J) + \mathcal{O}(\Phi^2)$$

- The $\left\{ \begin{array}{l} X_Z\langle V_{\lambda\nu}\nabla^\lambda\nabla_\rho\nabla^2 V^{\rho\nu}\rangle \\ + X_F\langle V_{\mu\nu}\nabla^2 f_+^{\mu\nu}\rangle \\ + 2\,i\,X_G\langle V_{\mu\nu}\nabla^2[u^\mu,u^\nu]\rangle \end{array}\right.$ can be transformed by a vector field redefinition $V\to V+\xi$

➔into other resonance operators with less derivatives: M_V, F_V, G_V

\+ the $-i\,\widetilde{L}_9\langle f_+^{\mu\nu}u_\mu u_\nu\rangle$ operator

\+ other operators that do not contribute to the ππ-VFF

- Similarly, higher derivative $\mathcal{L}_V$ operators for the VFF can be simplified

- Thus➔ the undesired O(p^4) resonance operators (and their O(p^4) contributions)

  can be removed from the theory

- We can remove the renormalized part of this operators through V➔ V + $\xi(X_Z^r, X_F^r, X_G^r)$

$$
\begin{aligned}
& X_{Z,F,G} \xrightarrow{\xi} 0\,, \\
& \widetilde{L}_9 \xrightarrow{\xi} \widetilde{L}_9 + \left(\sqrt{2} X_F G_V + 2\sqrt{2} F_V X_G - X_Z F_V G_V\right), \\
& F_V \xrightarrow{\xi} F_V + \left(2 X_Z F_V M_V^2 - 2\sqrt{2} X_F M_V^2\right), \\
& G_V \xrightarrow{\xi} G_V + \left(2 X_Z G_V M_V^2 - 4\sqrt{2} X_G M_V^2\right), \\
& M_V^2 \xrightarrow{\xi} M_V^2 + 2\, X_Z M_V^4\,.
\end{aligned}
$$

[Rosell, Pich & SC’04]

- This field transformation:

  →removes $X^r_{Z,F,G}$

  →encodes their running into $M_V^r, \quad G_V^r, \quad F_V^r, \quad \widetilde{L}_9^r$

- Although the field redefinition $\xi(X_Z^r, X_F^r, X_G^r)$

  depends on the particular μ under consideration (since $X^r_{Z,F,G}$ do),

  the resulting theory is still equivalent to the original one

- After removing $X^r_{Z,F,G}$,

  the remaining couplings are ruled by

$$\left\{ \begin{aligned} \frac{1}{M_V^2}\frac{\partial M_V^2}{\partial \ln \mu^2} &= \frac{n_f}{2}\frac{2G_V^2}{F^2}\frac{M_V^2}{96\pi^2 F^2}\,, \\ \frac{\partial G_V}{\partial \ln \mu^2} &= G_V\,\frac{n_f}{2}\frac{M_V^2}{96\pi^2 F^2}\left(\frac{3G_V^2}{F^2}-1\right)\,, \\ \frac{\partial F_V}{\partial \ln \mu^2} &= 2\,G_V\,\frac{n_f}{2}\frac{M_V^2}{96\pi^2 F^2}\left(\frac{F_V G_V}{F^2}-1\right)\,, \\ \frac{\partial \widetilde{L}_9}{\partial \ln \mu^2} &= \frac{n_f}{2}\frac{1}{192\pi^2}\left(\frac{F_V G_V}{F^2}-1\right)\left(1-\frac{3G_V^2}{F^2}\right) \end{aligned} \right.$$

- If now one sets the scale $\mu^2 = -q^2$ $(\equiv Q^2)$, the VFF takes the simple form

$$\mathcal{F}(q^2) = -\frac{2\,Q^2\widetilde{L}_9(Q^2)}{F^2} + [1+\Delta_t(q^2)]\left[1-\frac{F_V(Q^2)G_V(Q^2)}{F^2}\,\frac{Q^2}{M_V^2(Q^2)+Q^2}\right]$$

with the evolution of the amplitude with Q^2

→provided by the evolution of the couplings **(through the RGE)**

- For instace, the $M_V(\mu)$ would be related to the pole mass through

$$M^2_{V,pole} = M_V^2(\mu) + \frac{n_f}{2}\,\frac{2G_V^2}{F^2}\,\frac{M_V^4}{96\pi^2F^2}\ln\frac{M_V^2}{\mu^2} = M_V^2(M_V)$$

$\pi\pi$–Vector Form Factor:

RGE solutions

- The RGE for M_V and G_V provide a close system of equations,

  with solution of the form,

$$\left\{ \begin{array}{l} G_V^2 \;=\; \dfrac{F^2}{3}\left(1 + \kappa^3\, M_V^6\right) , \\ \dfrac{1}{M_V^2} + \kappa\, f(\kappa M_V^2) \;=\; -\dfrac{2}{3}\,\dfrac{n_f}{2}\,\dfrac{1}{96\pi^2 F^2}\,\ln\dfrac{\mu^2}{\Lambda^2} \end{array} \right.$$

with the integration variables κ and Λ

$$f(x) = \frac{1}{6}\ln\left(\frac{x^2+2x+1}{x^2-x+1}\right) - \frac{1}{\sqrt{3}}\arctan\left(\frac{2x-1}{\sqrt{3}}\right) - \frac{\pi}{6\sqrt{3}} = \mathcal{O}(x)$$

$$-\frac{2\pi}{3\sqrt{3}} \leq f(x) \leq 0,$$

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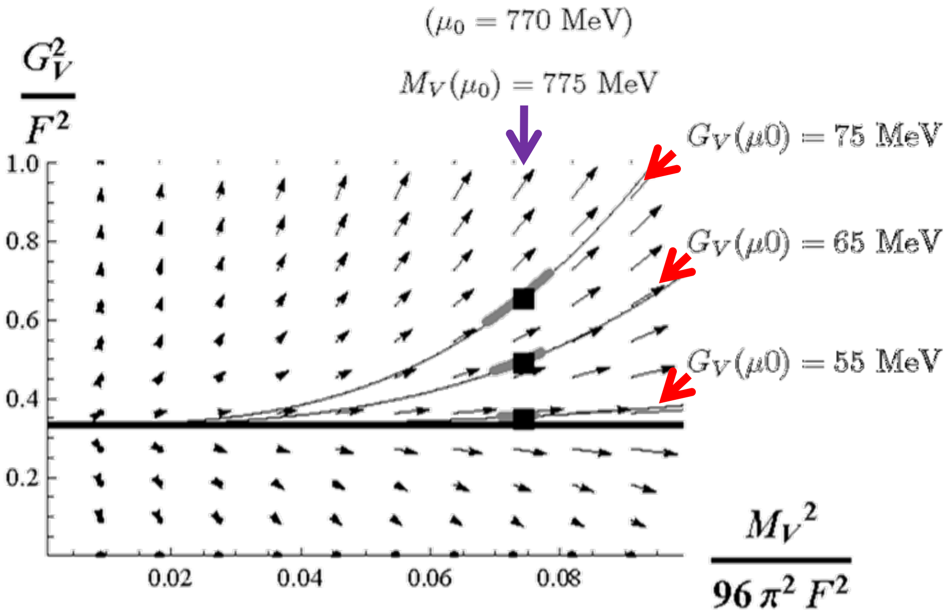
$$-\frac{2\pi}{3\sqrt{3}} \leq f(x) \leq 0,$$

- This produces the flux diagram:

$$\frac{1}{M_V^2}\frac{\partial M_V^2}{\partial \ln\mu^2} = \frac{n_f}{2}\frac{2G_V^2}{F^2}\frac{M_V^2}{96\pi^2F^2}\,,$$

$$\frac{\partial G_V}{\partial \ln\mu^2} = G_V\,\frac{n_f}{2}\frac{M_V^2}{96\pi^2F^2}\left(\frac{3G_V^2}{F^2} - 1\right)$$



- The RGE for M_V and G_V provide a close system of equations,

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$$
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\frac{1}{M_V^2}\,+\,\kappa\,f(\kappa M_V^2)\;&=\;-\,\frac{2}{3}\,\frac{n_f}{2}\,\frac{1}{96\pi^2F^2}\,\ln\frac{\mu^2}{\Lambda^2}
\end{aligned}
\right.
$$

with the integration variables κ and Λ

$$f(x)=\frac{1}{6}\ln\left(\frac{x^2+2x+1}{x^2-x+1}\right)-\frac{1}{\sqrt{3}}\arctan\left(\frac{2x-1}{\sqrt{3}}\right)-\frac{\pi}{6\sqrt{3}}=\mathcal{O}(x)$$

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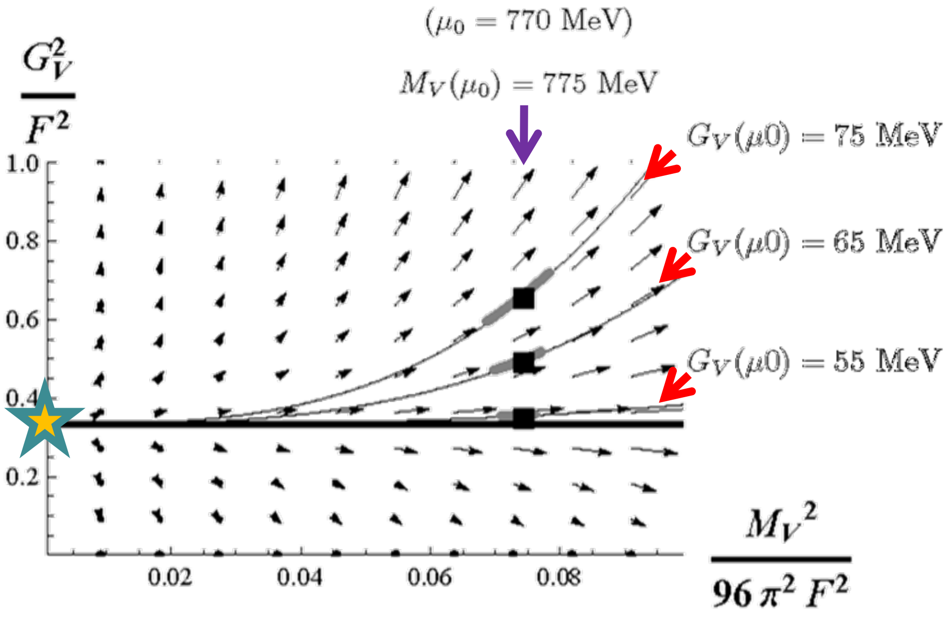

- The RGE for M_V and G_V provide a close system of equations,

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$$\left\{\begin{array}{l} G_V^2 \;=\; \frac{F^2}{3}\left(1+\kappa^3 M_V^6\right) , \\ \frac{1}{M_V^2}+\kappa f(\kappa M_V^2) \;=\; -\frac{2}{3}\frac{n_f}{2}\frac{1}{96\pi^2 F^2}\ln\frac{\mu^2}{\Lambda^2} \end{array}\right.$$

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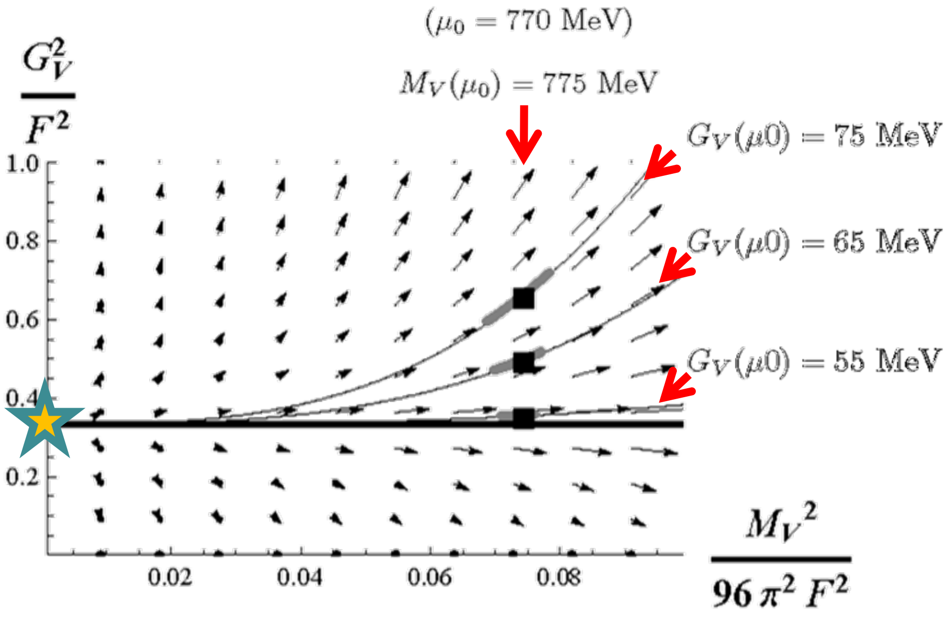


- There appears an IR fixed point

  at $M_V=0$, $3G_V^2=F^2$

where all the trajectories converge

- The same happens for the remaining couplings, which freeze out at μ→0

$$F_V(\mu) \xrightarrow{\mu\to 0} \sqrt{3}F\,, \qquad\qquad \widetilde{L}_9(\mu) \xrightarrow{\mu\to 0} \widetilde{L}_9(0)$$

→For any initial condition,

the vector parameters have always the same IR fixed point (μ→0),

$$M_V = 0\,, \qquad\qquad F_V G_V = F^2\,, \qquad\qquad 3G_V^2 = F^2$$

→ $\widetilde{L}_9$ is the only one that depends on the μ=0 condition $\widetilde{L}_9(0)$

- The theory evolves then
  - →from the IR fixed-point in the χPT domain,
  - →up to higher energies through the log running given by the RGE

Remarkable fact #1

•It is remarkable that

the values for the F_V and G_V IR-fixed points

agree with those obtained **at large-N_C**

after demanding the right behaviour at $Q^2 \rightarrow \infty$:

-For the ππ-VFF → $F_V G_V = F^2$ [Ecker et al.'89]

-For the ππ-partial wave scattering → $3 G_V^2 = F^2$ [Guo, Zheng & SC'07]

Remarkable fact #2

- Likewise, it is also interesting that the requirement $\mathcal{F}(Q^2) \xrightarrow{Q^2\to\infty} 0$

for the resummed VFF,

$$\mathcal{F}(q^2) = -\frac{2\,Q^2\widetilde{L}_9(Q^2)}{F^2} + [1+\Delta_t(q^2)]\left[1 - \frac{F_V(Q^2)G_V(Q^2)}{F^2}\frac{Q^2}{M_V^2(Q^2)+Q^2}\right]$$

leads to the same values as those from the IR-fixed point,

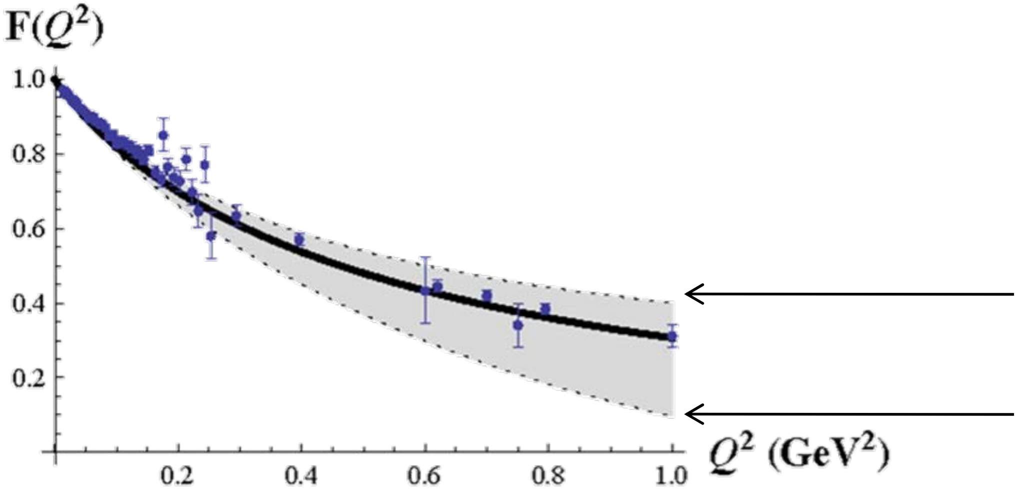
$F_V G_V = F^2\,,$ $3G_V^2 = F^2$ and, in addition, $\widetilde{L}_9 = 0$

for all μ (being μ–independent and with their running frozen out)

- The agreement of the resummed VFF and data is fairly good.

- Thus, for the inputs

$$M_V = 775 \text{ MeV}, \; F_V G_V = 3G_V^2 = F^2 \qquad \text{and } \bar{L}_9 = 0 \qquad \text{for } \mu_0 = 770 \text{ MeV}$$



$G_V(\mu_0)^2=0$ (same other inputs)

$G_V(\mu_0)^2=F^2$ (same other inputs)

- However,
  - →The non-zero π mass is responsible for a 20% decreasing of the ρ-width [Guo & SC'09]
  - →An accurate description of both space-like/time-like data needs the consideration of the pNGB masses

ππ–Vector Form Factor:

Perturbative regime

- Independently of possible high-energy matching,
  the RGE show the existence of a region in the space of parameters

  (close enough to the IR-fixed point at $\mu \to 0$)

  → where the loop corrections are small

  → and one has a slow logarithmic running

- This fills of physical meaning the formal $1/N_C$ expansion
  based on the formal N_C scaling of the lagrangian operators

  → The theory can be described perturbatively as far as one is close enough to the fixed point

- *ANALOGY:*

  -A fixed order QCD calculation is formally valid for any μ
  (and independent of it)

  -RGE → perturbation theory only valid for large μ

- In our case, the parameter that actually rules
  the strength of the Resonance-Goldstone interaction
  in the RGE is

$$\alpha_V = \frac{n_f}{2}\frac{2G_V^2}{F^2}\frac{M_V^2}{96\pi F^2}$$

- This parameter goes to 0 when $\mu \rightarrow 0$

  → Around the IR-fixed point ($\mu \rightarrow 0$) the couplings vary slowly

- Thus, → Although the formal expansion is $1/N_C$
  → α_V is the actual quantity suppressing subleading contributions

- Since at LO $\alpha_V \approx \Gamma_V/M_V \approx 0.2$ for $\mu \approx M_V$,
  the $1/N_C$ expansion meaningful in the resonance region
  as far as it is narrow enough (as it is the case)

- Nevertheless,

  - In case of broad states,

  - or more complicate processes,

→Less intuitive identification
of the parameter that characterizes
the strength of the interaction

- However, Perturbation Theory will make sense
  →as far as we are able to find a momentum region where
  - This parameter becomes small
  - One has a slow running of the couplings

Conclusions

•A priori, higher derivative operators needed for 1 loop renormalization

→However, there are **redundant operators**
(which can be removed through meson field redefinitions)

•**RGE→existence of an IR-fixed point**
+ slow log running in the region around $\mu\to 0$
where α_V is small enough

•This is where a **perturbative expansion in $1/N_C$** makes sense:

Amplitude given by the RGE **evolution of the couplings**

from the IR-fixed point at 0 up to Q^2

•These considerations are expected to be **relevant**

-for other QCD matrix elements

-In particular, for Scalars (where width and corrections are large)
e.g., the scalar form-factor