Testing the Standard Model with Kaon Decays

Joachim Brod

in collaboration with Martin Gorbahn

University of Karlsruhe

Theory Seminar PSI, Villigen, 23th July 2009

Phys.Rev.D78:034006,2008 [arXiv:0805.4119 [hep-ph]]

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Introduction

- The Standard Model is presumably only a low energy limit of a more complete theory
- The LHC might test extensions of the Standard Model directly
- Precision observables in Flavour Physics test high energy scales indirectly
- We will look at *CP* violation in the neutral Kaon system and rare Kaon decays

Contents

1. Quark Flavour Physics
2. CP Violation in the Neutral Kaon System
3. Rare Kaon Decays
4. Conclusion

Strangeness

In the 1940s, strangely long lived particles were discovered:

856 NATURE December 20, 1947 Vol. 160

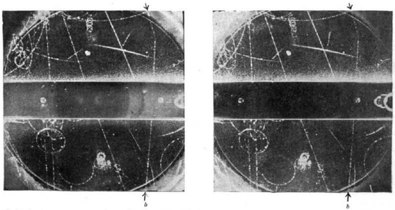


Fig. 2. Stereoscopic photographs showing an unusual fork (a b). The direction of the magnetic field is such that a positive particle coming downwards is deviated in a clockwise direction

[Rochester, Butler '47]

	$I = -\frac{1}{2}$	$I = \frac{1}{2}$
S $= 1$	$K^0 \sim \bar{s}d$	$K^+ \sim \bar{s}u$
S $= -1$	$K^- \sim s\bar{u}$	$\bar{K}^0 \sim s\bar{d}$

Strangeness [Gell-Mann '53]

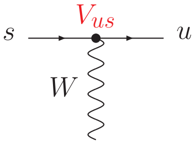
They are today known as K-Mesons.

- Production by strong interaction (e.g. $p + n \to p + \Lambda^0 + K^0$)
- Decay via weak interaction

Flavour Changing Transitions

- Mediated by the weak interaction via the exchange of charged W bosons
- Neutral gauge bosons (Z, photon, gluon) conserve flavour
- Couplings described by the unitary 3×3 Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$J_\mu^{CC} \propto (\bar{u}, \bar{c}, \bar{t})_L \gamma_\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L$$



The Physical Content of the CKM Matrix

After a phase transformation of the quarks field, three angles and one complex phase remain.

Nobel Prize for Physics 2008

Makoto Kobayashi

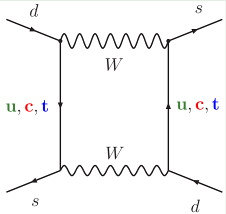
Toshihide Maskawa

$\mathcal{CP}$ violation is described by a $\mathcal{C}$omplex $\mathcal{P}$hase!

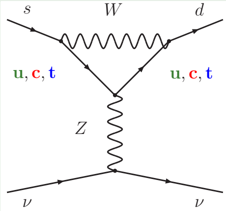
Flavour Changing Neutral Currents (FCNC)

- In the Standard Model, FCNC processes are forbidden at tree level and are thus induced by loop diagrams
- They are typically small
- They are sensitive to high energy scales

Neutral Kaon Mixing: $K^0 \leftrightarrow \bar{K}^0$



Rare Kaon decays: $K \longrightarrow \pi\nu\bar{\nu}$



Hierarchy of the CKM Matrix Elements

The CKM matrix is almost diagonal:

Wolfenstein Parameterisation ($\lambda \approx 0,22$ small)

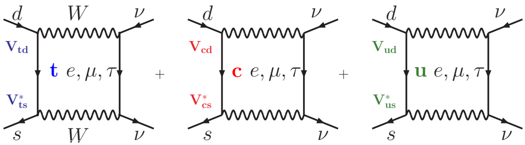
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$

In Kaon physics, top quark contribution is suppressed.

- charm quark is also important
- high sensitivity to high scales

GIM Mechanism

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \text{unitary} \Rightarrow V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$



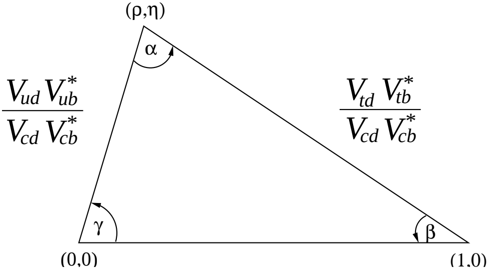
$= 0$

plus mass terms

- Historically: Prediction of the charm quark by Glashow, Iliopoulos, and Maiani in 1970
- FCNC processes suppressed also at loop level
- GIM relevant for hadronic uncertainties ($m_u \approx 0$)

Properties of the CKM Matrix: Unitarity Triangle

Multiplication of the first and last column of the CKM matrix leads to the Standard Unitarity Triangle



$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Summary

- In the Standard Model, flavour changing processes are mediated by the weak interaction via charged W bosons
- The CKM matrix contains a complex phase, which causes CP violation
- FCNC processes are highly supressed and sensitive to high energy scales ("New Physics")

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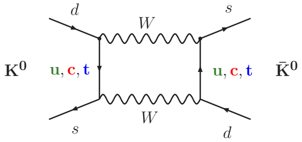
Introduction to the Neutral Kaon System (K^0–$\bar{K}^0$)

CP Transformation

$$CP|K^0\rangle = -|\bar{K}^0\rangle \quad CP|\bar{K}^0\rangle = -|K^0\rangle$$

The weak interaction causes transitions between K^0, $\bar{K}^0$ via



CP eigenstates: $|K_{1/2}\rangle := (|K^0\rangle \mp |\bar{K}^0\rangle)/\sqrt{2}.$

Decay Modes

$K_1 \to \pi\pi$ $(CP = 1)$, ... $\tau_1 \approx 0.89 \times 10^{-10}$s $|K_1\rangle \approx |K_{short}\rangle$

$K_2 \to \pi\pi\pi$ $(CP = \text{-}1)$, ... $\tau_2 \approx 5.17 \times 10^{-8}$s $|K_2\rangle \approx |K_{long}\rangle$

CP Violation in the Neutral Kaon System

Observation of $K_{\text{long}} \longrightarrow \pi\pi$
(Christenson, Cronin, Fitch, and Turley 1964)

$\Rightarrow \mathcal{CP}$ symmetry is violated!

The parameter ϵ_K

$$\epsilon_K := \frac{\langle(\pi\pi)_{I=0}|K_L\rangle}{\langle(\pi\pi)_{I=0}|K_S\rangle}$$

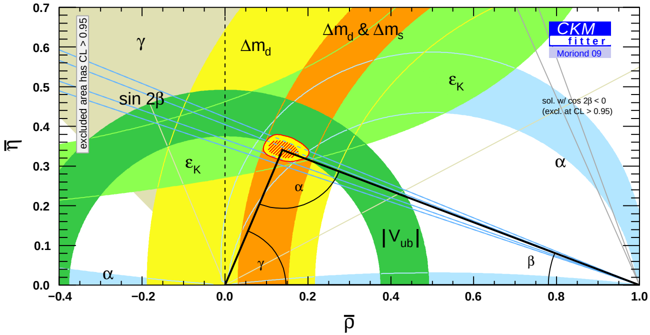
- ϵ_K measures indirect *CP* violation
- *CP* violation in decay is absent to very good approximation

ϵ_K in the Unitarity Triangle

Comparing the precisely measured value of

$$\epsilon_K = (2.229 \pm 0.012) \times 10^{-3} \times e^{i(43.5 \pm 0.7)^\circ}$$

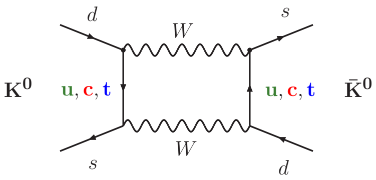
with the Standard Model prediction leads to a constraint on the CKM parameters: (Check of consistency!)



Effective Field Theory: Overview

Separation of scales

- M_W, m_t: $\mathcal{O}(100\text{GeV})$
- m_c: $\mathcal{O}(1.5\text{GeV})$
- Λ_{QCD}: $\mathcal{O}(200\text{MeV})$



$$\mathcal{M}_{12} = \mathcal{C} \langle K^0 | Q^{|\Delta S|=2} | \bar{K}^0 \rangle$$

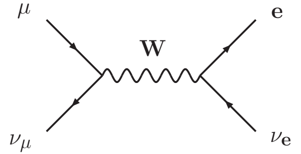
Wilson coefficients ("coupling constants"): Can be calculated precisely in perturbation theory

Effective operators ("interactions"): Induced by SM and beyond

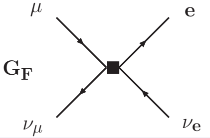
Matrix elements: Non-perturbative, hard to calculate...

Structure of the Effective Hamiltonian I

Analogy to Fermi theory



$\longrightarrow$

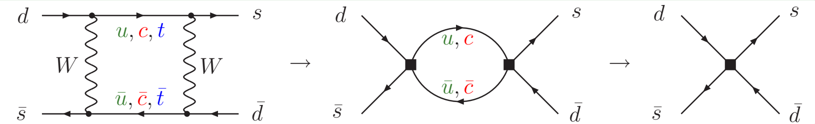


$$\frac{1}{p^2 - M_W^2}$$

$$\frac{-1}{M_W^2} + \mathcal{O}\left(\frac{p^2}{M_W^2}\right)$$

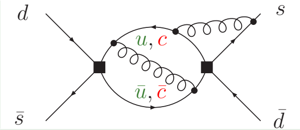
Structure of the Effective Hamiltonian II

Effective Theory for ϵ_K



- Resulting theory is non-renormalisable
- About 20 different (physical, non-physical) operators contribute in intermediate steps of the calculation
- Predictive power maintained by matching to Standard Model

Including QCD corrections



Effective Field Theory: Summation of Large Logarithms

Widely separate scales lead to large logarithms:

$$\log \frac{m_c^2}{M_W^2} \alpha_s(m_c) = \mathcal{O}(1)$$

Summation by the Renormalisation Group

$\sum_n \alpha_s^n \log^n$	leading-log (LL)
$\sum_n \alpha_s^{n+1} \log^n$	next-to-leading-log (NLL)
$\sum_n \alpha_s^{n+2} \log^n$	next-to-next-to-leading-log (NNLL)

- Running of Wilson coefficients: Anomalous Dimensions
- Initial conditions: Matching

ϵ_K: Different Contributions

$$\epsilon_K = \kappa_\epsilon \frac{\mathrm{Im}(\langle K^0 | \mathcal{H}^{|\Delta S|=2} | \bar{K}^0 \rangle)}{\Delta M_K}$$

[Nierste; Buras, Guadagnoli '08]

short distance:

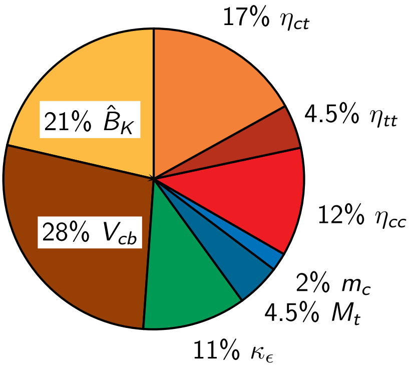
$$\mathcal{H}^{|\Delta S|=2} \propto \left[\eta_{cc} S_0(m_c^2) + \eta_{tt} S_0(m_t^2) + \eta_{ct} S_0(m_c^2, m_t^2)\right] Q^{|\Delta S|=2} ;$$

long distance:

$$\langle K^0 | Q^{|\Delta S|=2} | \bar{K}^0 \rangle \propto \hat{B}_K ,$$

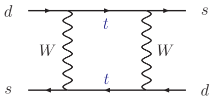
κ_ϵ ... remaining hadronic uncertainties .

ϵ_K: Error Budget



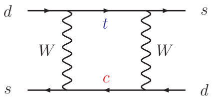
- Reduced error on $\hat{B}_K$ by lattice calculations [E.g. Aubin et al. '09]
- Better determination of V_{cb}
- 3-loop calculation reduces theoretical error of η_{cc}, η_{ct} [Brod, Gorbahn; Work in progress]

ϵ_K: Theoretical Status



- $S(\frac{m_t^2}{M_W^2})$
- NLL QCD

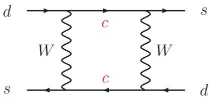
  [Buras, Jamin, Weisz '90]
- scale: 1.8 %
- ϵ_K: 75 %



- $S(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}) =$

  $\mathcal{O}(\frac{m_c^2}{M_W^2} \log \frac{m_c^2}{M_W^2})$
- NLL QCD

  [Herrlich, Nierste '96]
- scale: 7.5 %
- ϵ_K: 37 %



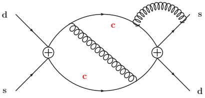
- $S(\frac{m_c^2}{M_W^2}) =$

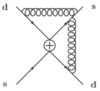
  $\mathcal{O}(\frac{m_c^2}{M_W^2})$
- NLL QCD

  [Herrlich, Nierste '93]
- scale: 17.7 %
- ϵ_K: -12 %

Calculation of η_{cc}, η_{ct} at NNLL

η_{cc}:

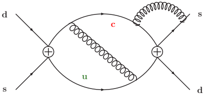


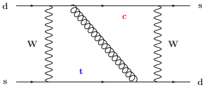


- Diagrams in effective theory finite by GIM
- Wilson Coefficients and ADM known to NNLO QCD at W scale [Chetyrkin et al. '98, Bobeth et al. '00; Gorbahn et al. '04]
- Three-loop Matching at the Charm scale

  [Brod, Gorbahn; Work in progress]

η_{ct}:





- Full Renormalisation Group analysis [Brod, Gorbahn '09]
- Calculate $\mathcal{O}(10\,000)$ Feynman diagrams $\to$ `qgraf` [Nogueira '93], `MATAD` [Steinhauser '01], `Mathematica`, `FORM` [Vermaseren '00]

Summary

- ϵ_K measures indirect CP violation in the neutral Kaon system
- Recent progress in lattice determination of B_K motivates calculation of NNLL QCD corrections
- Full analytic renormalisation group analysis of η_{ct} is complete
- η_{cc} and numerical analysis will follow soon ...

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3. Rare Kaon Decays
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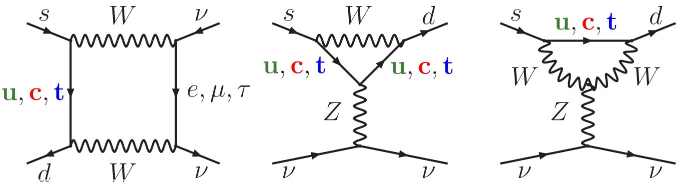
$K^+ \to \pi^+\nu\bar{\nu}$: Introduction



Dominated by $Q_\nu = (\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_L\gamma^\mu\nu_L)$

$$B\left(K^+ \to \pi^+\nu\bar{\nu}(\gamma)\right) \propto \kappa_+(1+\Delta_{\text{EM}})$$
$$\times \left|\lambda^5 X_t(m_t^2) + \lambda\left(P_c(m_c^2) + \delta P_{c,u}\right)\right|^2.$$

$K^+ \to \pi^+\nu\bar{\nu}$: Hadronic Matrix Elements

Isospin-Symmetry (Wigner-Eckart-Theorem)

$$\langle\pi^+|(\bar{s}d)_{V-A}|K^+\rangle = \sqrt{2}\langle\pi^0|(\bar{s}u)_{V-A}|K^+\rangle$$

Extract hadronic matrix elements from full set of well-measured $K_{\ell 3}$ ($K \to \pi\ell\nu_\ell$) decays, including isospin breaking effects.
The related theory error is now negligible.

$K^+ \to \pi^+\nu\bar{\nu}$: Theoretical Status

LD Contributions

- Uncertainty in matrix element reduced by a factor of 7
  [Mescia, Smith '07]
- QED radiative corrections included ($|\Delta_{\mathrm{EM}}| < 1\%$)
  [Mescia, Smith '07]
- $\delta P_{c,u}$ enhances branching ratio by 6%
  [Falk, Lewandowski, Petrov '01; Isidori, Mescia, Smith '05]

Charm Contribution P_c

Scale dependence reduced to $\pm 2.5\%$ (NNLL QCD)
[Buras, Gorbahn, Haisch, Nierste '06]

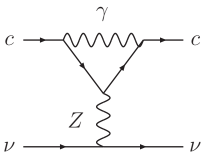
Top Contribution X_t

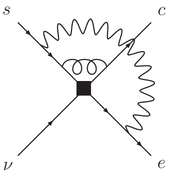
- Scale dependence reduced to $\pm 1\%$ (NLL QCD)
  [Misiak, Urban '99; Buchalla, Buras '99]
- Electroweak corrections to X_t in the large m_t limit $\approx 0.1\%$ ($\approx 2\%$ remaining) [Buchalla, Buras '98]

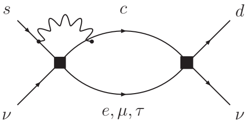
$K^+ \to \pi^+\nu\bar{\nu}$: Electroweak Corrections to P_c

Why Electroweak Corrections?

- Match precision achieved in hadronic matrix elements
- There is a large QED log
- Fix renormalisation scheme of electroweak input parameters!
- $\Rightarrow$ Normalise to G_μ







$K^+ \to \pi^+\nu\bar{\nu}$: Branching Ratio

Electroweak Corrections to P_c

P_c increases by up to 2%

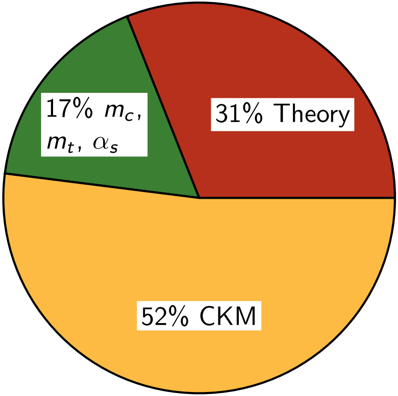
Our theoretical prediction [Brod, Gorbahn '08]

$$B^{\text{theo}}\left(K^+ \to \pi^+\nu\bar{\nu}(\gamma)\right) = (0.85 \pm 0.07) \times 10^{-10}.$$

Compare with current experimental value [E787, E949 '08]

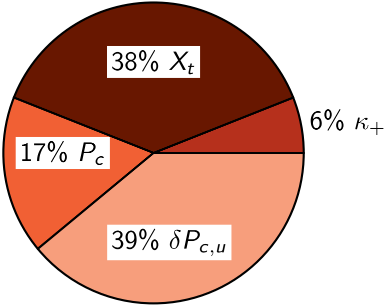
$$B^{\text{exp}}\left(K^+ \to \pi^+\nu\bar{\nu}(\gamma)\right) = \left(1.73\,^{+1.15}_{-1.05}\right) \times 10^{-10}.$$

$K^+ \to \pi^+\nu\bar{\nu}$: Error Budget



- Error dominated by CKM elements
- Theory error can still be reduced

$K^+ \to \pi^+\nu\bar{\nu}$: Theory Error Budget



- Calculate electroweak corrections to X_t [Brod, Gorbahn, Stamou; Work in progress]
- Improve on $\delta P_{c,u}$ by a lattice calculation [Isidori et al. '06]
- With better $K_{\ell 3}$ data improve on κ_+ [Mescia, Smith '07]

$K_L \to \pi^0\nu\bar{\nu}$: Overview

$K_L \to \pi^0\nu\bar{\nu}$ is now easy to discuss:

Contributions to the branching ratio

- (Almost) completely direct CP violating [Buchalla, Isidori '98]
- (Almost) completely short distance dominated
- $\to$ Only top quark contributes

$$B\left(K_L \to \pi^0\nu\bar{\nu}\right) \propto \kappa_L \left[\mathrm{Im}(V_{ts}^* V_{td})\, X_t(m_t^2)\right]^2$$

$K_L \to \pi^0\nu\bar{\nu}$: Theoretical Status

LD Contributions

- Uncertainty in matrix element reduced by a factor of 4 [Mescia, Smith '07]
- No further LD contributions

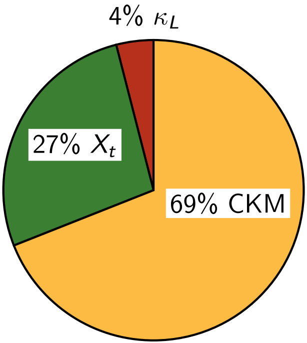
Charm Contribution

No charm contribution

Top Contribution X_t

- Scale dependence reduced to $\pm 1\%$ (NLL QCD) [Misiak, Urban '99; Buchalla, Buras '99]
- Electroweak corrections to X_t in the large m_t limit $\approx 0.1\%$ ($\approx 2\%$ remaining) [Buchalla, Buras '98]

$K_L \to \pi^0 \nu\bar{\nu}$: Error Budget



- Mainly parametric uncertainties
- Electroweak corrections to X_t

  [Brod, Estamou, Gorbahn; Work in progress]

$$B^{\text{theo}}\left(K_L \to \pi^0 \nu\bar{\nu}\right) = (2.76 \pm 0.40) \times 10^{-11}$$

$$B^{\text{exp}}\left(K_L \to \pi^0 \nu\bar{\nu}\right) < 6.7 \times 10^{-8}$$ [E391a '08]

$K \to \pi \nu \bar{\nu}$: Experimental Prospect

$K^+ \to \pi^+ \nu \bar{\nu}$

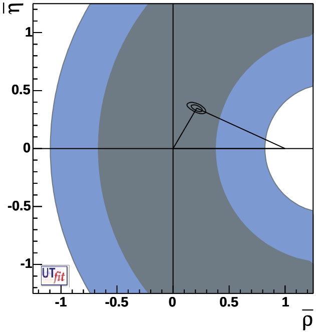
New CERN Experiment NA62 aiming at detecting 80 events, measuring $B(K^+ \to \pi^+ \nu \bar{\nu})$ with 10% error

$K_L \to \pi^0 \nu \bar{\nu}$

E14 experiment at J-PARC: Measuring $B(K_L \to \pi^0 \nu \bar{\nu})$ with uncertainty $< 10\%$

Experimental Situation: Today . . .

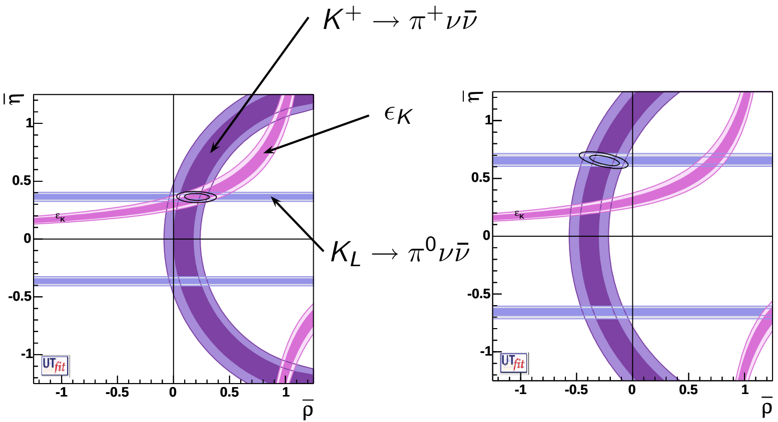
Seven events (BNL-787, BNL-949)



$B(K^+ \to \pi^+\bar{\nu}\nu) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$ Experiment

$B(K^+ \to \pi^+\bar{\nu}\nu) = (0.85 \pm 0.07) \times 10^{-10}$ Theory

. . . Future?



Standard Model

New physics?

Summary

- The two rare Kaon decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ are very sensitive to high energy scales
- Branching ratios are predicted with exceptional accuracy
- Error is mainly of parametrical origin

Conclusion

- *CP* violation in the neutral Kaon system and the rare Kaon decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ are FCNC processes
- These observables are very sensitive to high energy scales
- They can be predicted theoretically with high accuracy
- They constitute a very important consistency check of the Standard Model