Complete two-loop corrections to $H \to \gamma\gamma$

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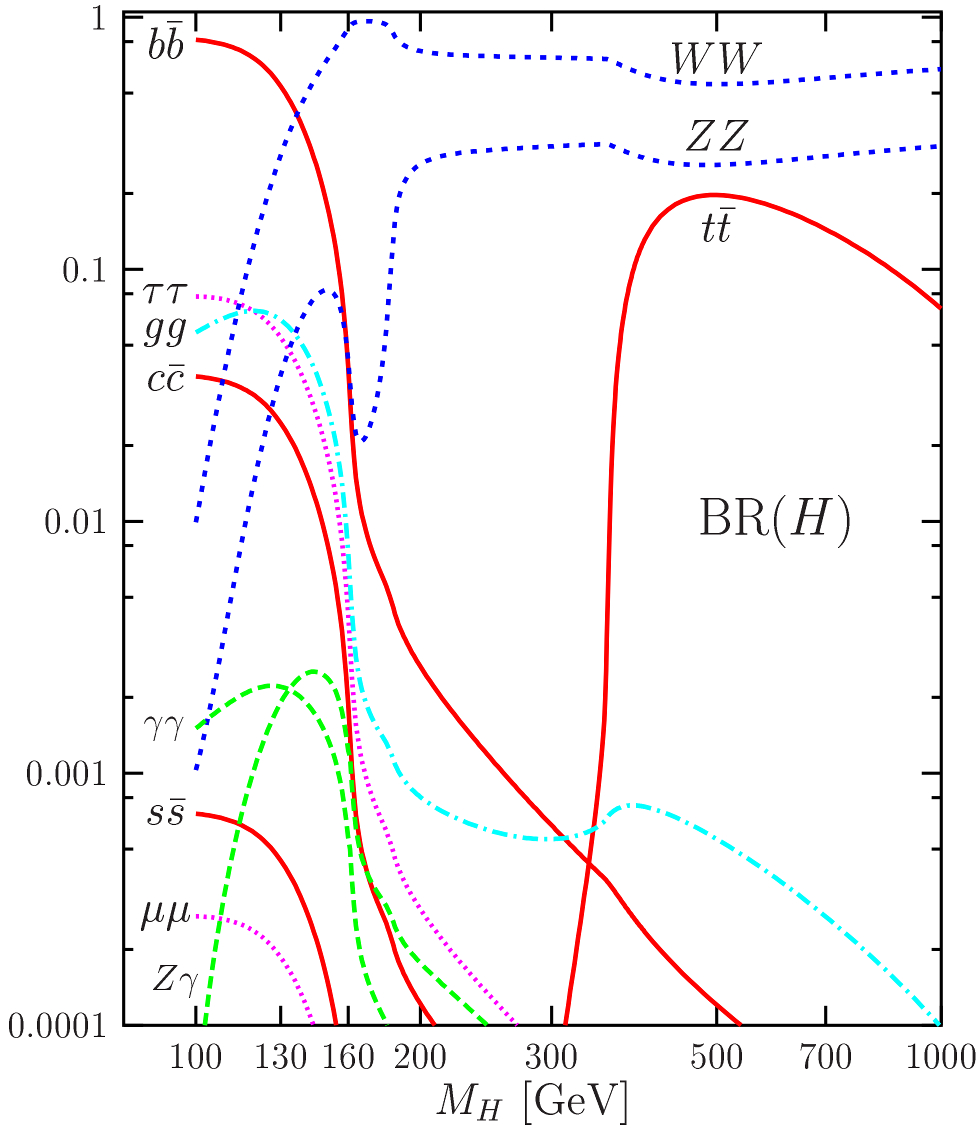
Higgs decays in the Standard Model



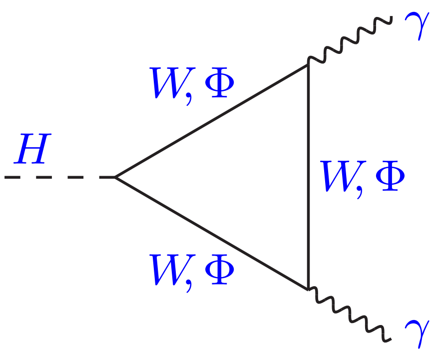
- $H \to b\bar{b}$:
  Dominant process for light Higgs, but huge QCD background.

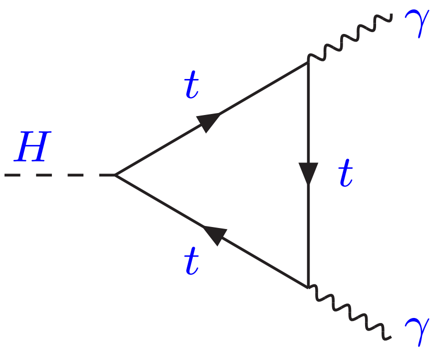
- $H \to \gamma\gamma$:
  Rare process, but experimentally clean.
  Discovery channel for light Higgs

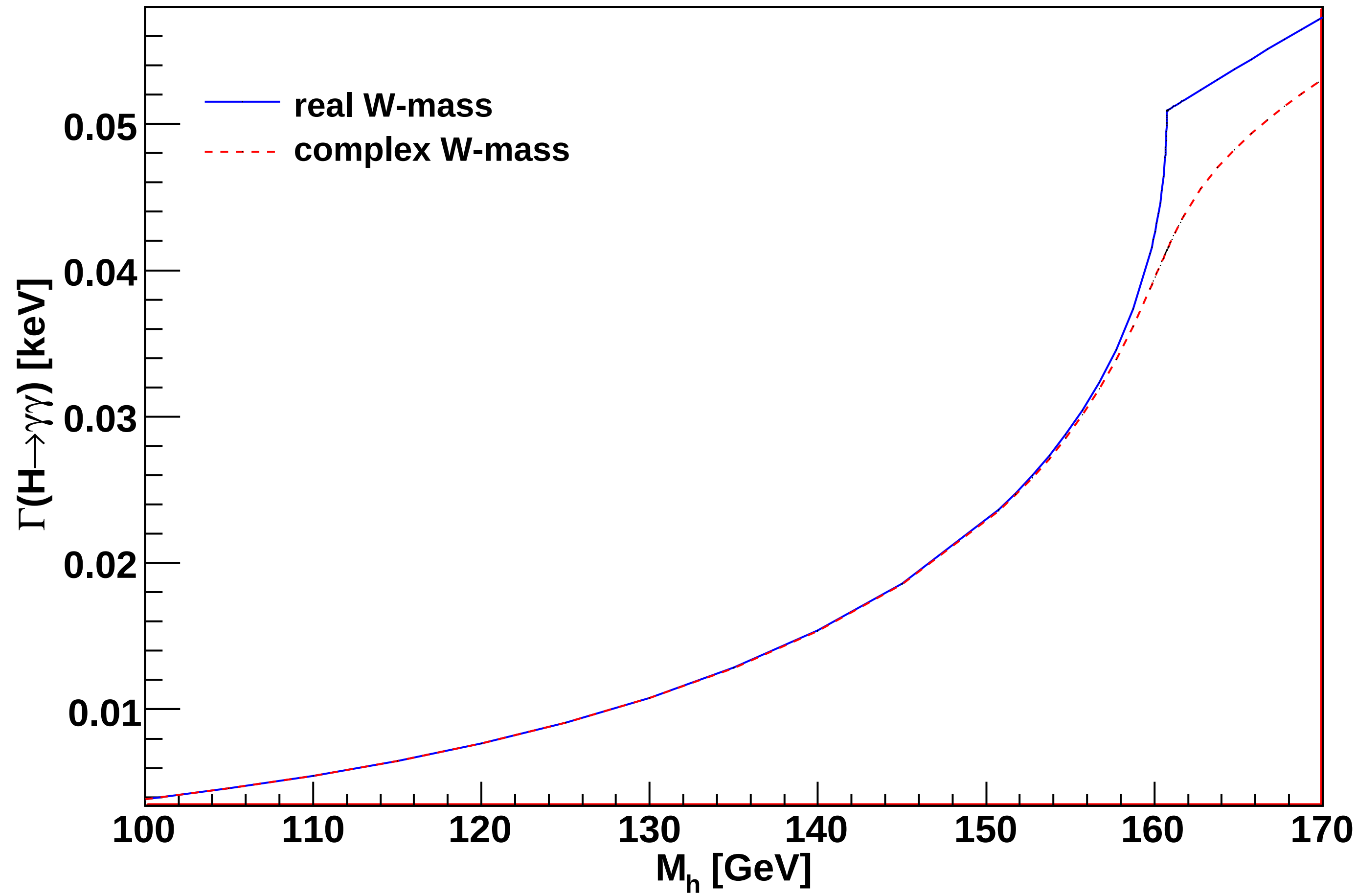
- $H \to WW, ZZ$:
  Discovery channels for heavy Higgs

Lowest order (one-loop) for $H \to \gamma\gamma$ (in SM)

- Well-known result
  - Ellis-Gaillard-Nanopoulos 1976, Shifman-Vainshtein-Voloshin-Zakharov 1979







Two-loop SM corrections to $H \to \gamma\gamma$

- QCD corrections
  - Zheng-Wu '90, Djouadi-Spira-van der Bij-Zerwas '91, Dawson-Kauffman '93, Melnikov-Yakovlev '93, Inoue-Najima-Oka-Saito '94, Steinhauser '96, Fleischer-Tarasov-Tarasov '04, Harlander-Kant '05, Aglietti-Bonciani-Degrassi-Vicini '06, Passarino-Sturm-U. '07
- EW corrections
  - corrections at $\mathcal{O}(G_\mu\, m_H^2)$ (Korner-Melnikov-Yakovlev '96)
  - corrections at $\mathcal{O}(G_\mu\, m_t^2)$ (Fugel-Kniehl-Steinhauser '04)
  - light-fermion contribution (Aglietti-Bonciani-Degrassi-Vicini '04)
  - top-quark and bosonic contributions for $m_H < 150$ GeV (Degrassi-Maltoni '05)
  - full EW contributions (Passarino-Sturm-U. '07)

The amplitude of $H(P) \to \gamma(p_1) + \gamma(p_2)$

$$\mathcal{A}^{\mu\nu}(H \to \gamma\gamma) = \frac{g^3\, s_\theta^2}{16\,\pi^2} \Big[F_{D}\, \delta^{\mu\nu} + \sum_{i,j=1}^{2} F_{P}^{(ij)}\, p_i^\mu\, p_j^\nu + F_\epsilon\, \epsilon(\mu, \nu, p_1, p_2) \Big].$$

$\Downarrow$ Interference with 1-loop
Bose symmetry
Ward identities

$$\mathcal{A}^{\mu\nu}(H \to \gamma\gamma) = \frac{g^3\, s_\theta^2}{16\,\pi^2} \left(F_{D}\, \delta^{\mu\nu} + F_{P}\, p_2^\mu\, p_1^\nu \right).$$

Ward identities: $F_{D} + p_1 \cdot p_2\, F_{P} = 0$ $\longrightarrow$ **Order by order in pertubation theory**

Introduce projectors:

$$F_{D} = P_{D}^{\mu\nu}\, \mathcal{A}^{\mu\nu}, \qquad P_{D}^{\mu\nu} = \frac{1}{n-2} \left(\delta^{\mu\nu} - \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} \right)$$

$$F_{P} \equiv F_{P}^{(21)} = P_{P}^{\mu\nu}\, \mathcal{A}^{\mu\nu}, \qquad P_{P}^{\mu\nu} = -\frac{1}{n-2}\, \frac{1}{p_1 \cdot p_2} \left(\delta^{\mu\nu} - \frac{(n-1)\, p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} \right)$$

The amplitude is computed with the

GraphShot package

- A FORM code to generate and manipulate the amplitudes in the SM
- A link to FORTRAN libraries for numerical computation
- Authors: G.Passarino, M.Passera, A.Ferroglia, S.Actis, C.Sturm, S.U.
- It is WORK IN PROGRESS (not yet available)

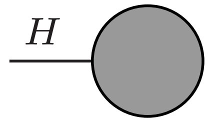
Let's discover the path to compute Feynman amplitudes ...

1. The Feynman rules

- The SM Lagrangian $\longrightarrow$ normal rules for propagators and vertices
- Special rules:
  - Higgs vacuum expectation value

normal : 

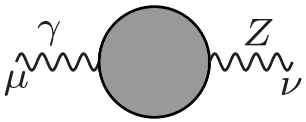
$= \; 0$

special : 

$= \; 0$

  - Z-Photon exchange ($g \to g\,(1+\Gamma)$):

normal : $\Gamma = 0$

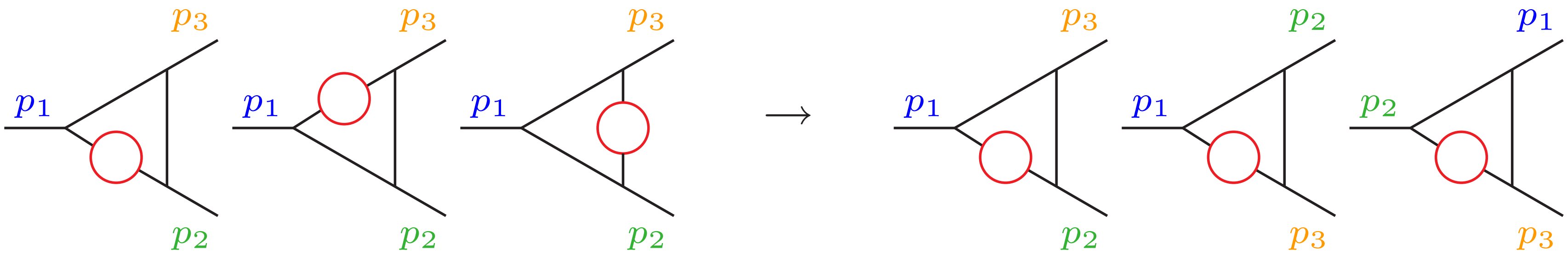
special : 

$= \; \mathcal{G}_d^{AZ}(p^2)\,\delta_{\mu\nu} + \mathcal{G}_{pp}^{AZ}(p^2)\,p_\mu p_\nu, \qquad \mathcal{G}_d^{AZ}(0) = 0$

- Renormalization $\longrightarrow$ $\overline{MS}$ scheme
  - Counterterms for couplings, masses, fields, ...
  - Finite Feynman amplitudes

2. Generate the amplitude

- Group the diagrams into families, paying attention to:
  - Permutation of external legs



  - Combinatorial factors (Goldberg strategy)
- Combine the topologies and the Feynman rules
- Introduce projectors
- Compute the trace of Dirac matrices

$$\Downarrow$$

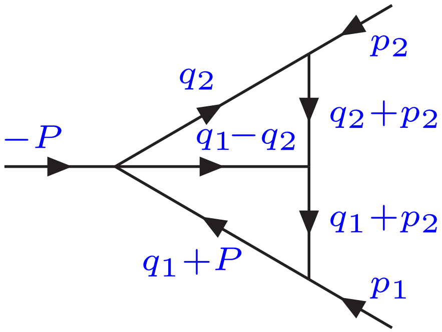
All loop momenta are contracted with other momenta

3. Reduction to Basic Integrals

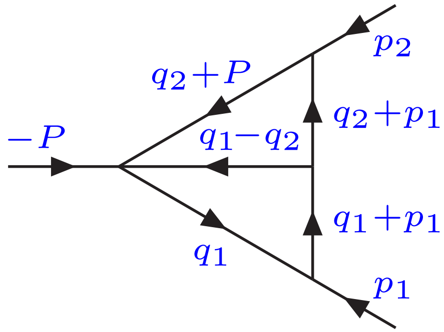
- Recursive application of:
  - Obvious reduction:

$$\frac{2\,q.p}{(q^2+m^2)\,[(q+p)^2+M^2]} = \frac{1}{q^2+m^2} - \frac{1}{(q+p)^2+M^2} - \frac{p^2-m^2+M^2}{(q^2+m^2)\,[(q+p)^2+M^2]}$$

  - Mapping on a fixed standard routing for loop momenta:

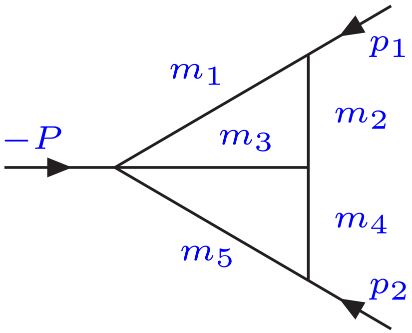


$\longrightarrow$

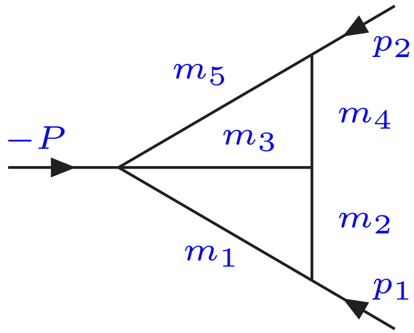


$$\left(\begin{array}{l} q_1 \rightarrow -q_1 - P \\ q_2 \rightarrow -q_2 - P \end{array} \right)$$

  - Symmetrization:



$\longrightarrow$

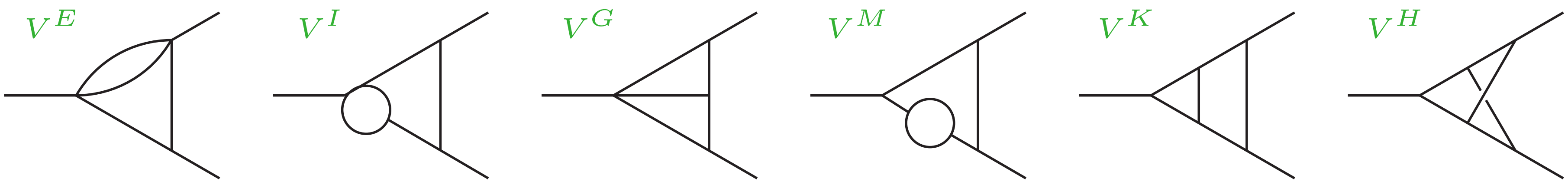


$$\left(\begin{array}{l} q_1 \rightarrow -q_2 - P \\ q_2 \rightarrow -q_1 - P \end{array} \right)$$

- We end with integrals up to rank 2:
  - 1-loop functions
  - 2-loop tadpoles (2 topologies)

  - 2-loop self-energies (4 topologies)

  - 2-loop vertices (6 topologies)

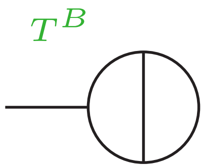


- Full scalarization of 2-loop self-energies
  - Reduction in sub-loops:

$$\int \frac{d^n q_1\, q_1^\mu}{(q_1^2 + m_1^2)[(q_1 - q_2)^2 + m_2^2]} = X\, q_2^\mu$$

A new propagator $\frac{1}{q_2^2}$ is introduced with spurious mass singularities.

  - New tadpoles with dots are generated
- Use integration by parts identities to reduce all tadpoles to:



- Full scalarization of 1-loop diagrams
- All 1-loop diagrams with dots are reduced wiht integration by parts identities

Feynman parametrization

Consider a general loop integral:

$$I_N^{\mu_1 \cdots \mu_R} = \int d^n q \, \frac{q^{\mu_1} \cdots q^{\mu_R}}{D_1 \, D_2 \cdots D_{N-1} \, D_N}, \qquad D_i = (q + k_i)^2 + m_i^2$$

Feynman parametrization

$$I_N^{\mu_1\cdots\mu_R} = \int d^n q \, \frac{q^{\mu_1}\cdots q^{\mu_R}}{\underbrace{D_1}_{(1-x_1)} \quad \underbrace{D_2}_{(x_1-x_2)} \quad \cdots \quad \underbrace{D_{N-1}}_{(x_{N-2}-x_{N-1})} \quad \underbrace{D_N}_{x_{N-1}}}, \qquad D_i = (q+k_i)^2 + m_i^2$$

The product of N propagators becomes one propagator to power N

$$I_N^{\mu_1\cdots\mu_R} = \Gamma(N) \int d^n q \, q^{\mu_1}\cdots q^{\mu_R} \int_0^1 dx_1 \int_0^{x_1} dx_2 \cdots \int_0^{x_{N-2}} dx_{N-1} \left[(q+K)^2 + M^2\right]^{-N}$$

$$\begin{aligned} K^\mu &= k_1^\mu (1-x_1) + k_2^\mu (x_1-x_2) + \ldots + k_{N-1}^\mu (x_{N-2}-x_{N-1}) + k_N^\mu \, x_{N-1} \\ M^2 &= (m_1^2+k_1^2)(1-x_1) + (m_2^2+k_2^2)(x_1-x_2) + \ldots \\ &\quad + (m_{N-1}^2+k_{N-1}^2)(x_{N-2}-x_{N-1}) + (m_N^2+k_N^2)\, x_{N-1} - K^2 \end{aligned}$$

Feynman parametrization

$$I_N^{\mu_1\cdots\mu_R} = \int d^n q\, \frac{q^{\mu_1}\cdots q^{\mu_R}}{D_1\, D_2 \cdots D_{N-1}\, D_N}, \qquad D_i = (q+k_i)^2 + m_i^2$$

$$= \Gamma(N) \int d^n q\; q^{\mu_1}\cdots q^{\mu_R} \int_0^1 dx_1 \int_0^{x_1} dx_2 \cdots \int_0^{x_{N-2}} dx_{N-1}\, \left[(q+K)^2 + M^2\right]^{-N}$$

$$K^\mu = k_1^\mu (1-x_1) + k_2^\mu (x_1 - x_2) + \ldots + k_{N-1}^\mu (x_{N-2} - x_{N-1}) + k_N^\mu\, x_{N-1}$$

$$M^2 = (m_1^2 + k_1^2)(1-x_1) + (m_2^2 + k_2^2)(x_1 - x_2) + \ldots$$
$$+ (m_{N-1}^2 + k_{N-1}^2)(x_{N-2} - x_{N-1}) + (m_N^2 + k_N^2)\, x_{N-1} - K^2$$

Integration in $d^n q$ is performed

$$I_N = i\,\pi^{\frac{n}{2}}\,\Gamma\left(N - \frac{n}{2}\right) \int_0^1 dx_1 \int_0^{x_1} dx_2 \cdots \int_0^{x_{N-2}} dx_{N-1}\, (M^2)^{\frac{n}{2}-N}$$

$$I_N^{\mu_1} = i\,\pi^{\frac{n}{2}}\,\Gamma\left(N - \frac{n}{2}\right) \int_0^1 dx_1 \int_0^{x_1} dx_2 \cdots \int_0^{x_{N-2}} dx_{N-1}\, (-K^{\mu_1})\,(M^2)^{\frac{n}{2}-N}$$

$$I_N^{\mu_1\mu_2} = i\,\pi^{\frac{n}{2}}\,\Gamma\left(N - \frac{n}{2}\right) \int_0^1 dx_1 \int_0^{x_1} dx_2 \cdots \int_0^{x_{N-2}} dx_{N-1} \left[K^{\mu_1} K^{\mu_2} + \frac{M^2\,\delta^{\mu_1\mu_2}}{2\,N - n - 2}\right] (M^2)^{\frac{n}{2}-N}$$

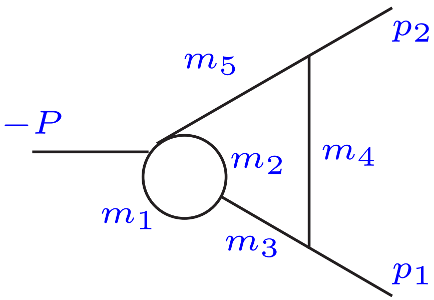
$$I_N^{\mu_1\mu_2\mu_3} = \ldots$$

4. Analytical cancellations of divergences

Extraction of the UV poles

- 1-loop diagrams $\longrightarrow$ trivial ($\Gamma(\epsilon/2)$)
- 2-loop diagrams:
  - Overall divergency $\longrightarrow$ trivial ($\Gamma(\epsilon)$)
  - Singularities coming from sub-loops $\longrightarrow$ hidden in the integrand

$V^I =$



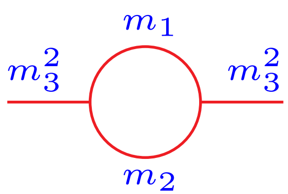
$$= \frac{1}{\pi^4}\int \frac{d^n q_1\, d^n q_2}{\underbrace{\underbrace{[1]\,[2]}_{x}\,[3]\,[4]\,[5]}_{y_1,y_2,y_3}},$$

$$[1] = q_1^2 + m_1^2$$
$$[2] = (q_1 - q_2)^2 + m_2^2$$
$$[3] = q_2^2 + m_3^2$$
$$[4] = (q_2 + p_1)^2 + m_4^2$$
$$[5] = (q_2 + P)^2 + m_5^2$$

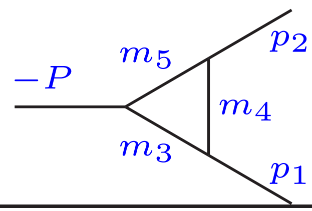
$$= \quad C_\epsilon \int_0^1 dx \int dS_3(y_1, y_2, y_3)\, [x\,(1-x)]^{-\epsilon/2}\, (1-y_1)^{\epsilon/2-1}\, V^{-1-\epsilon}$$

  - The single pole can always be expressed in terms of 1-loop functions.

$V^I =$



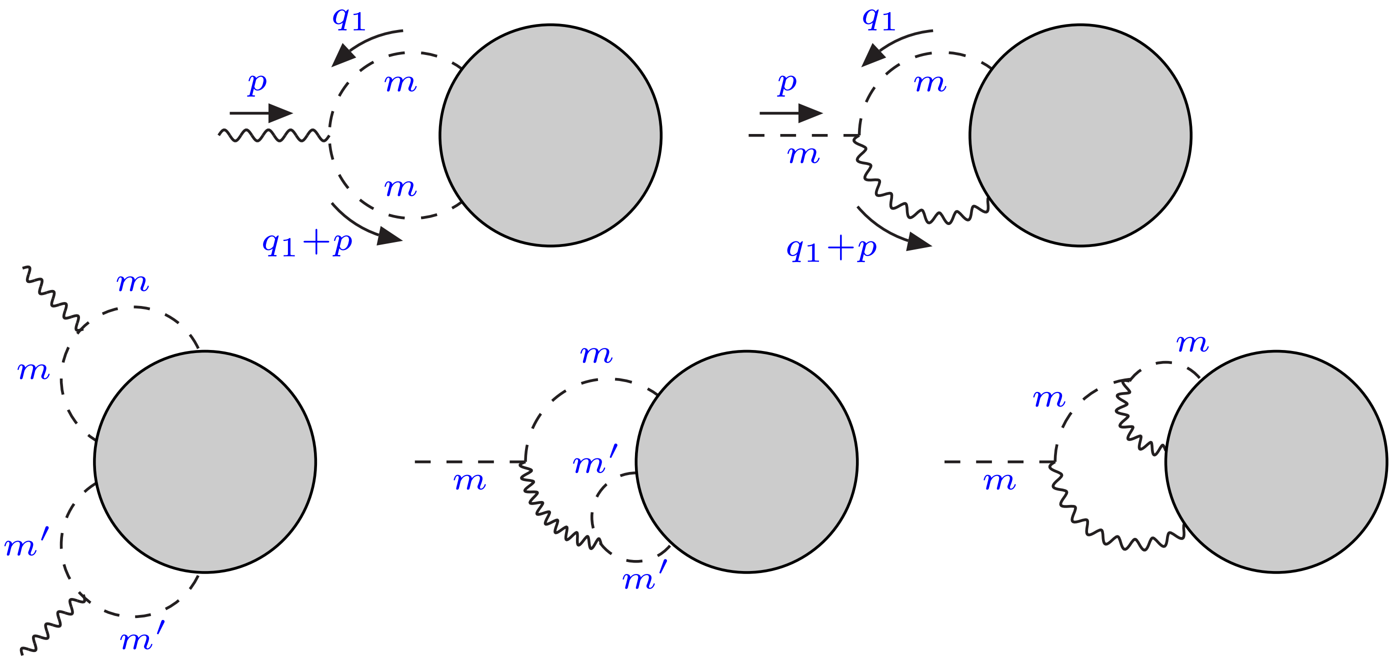
$\times$



$+$ finite part.

Collinear divergencies

They come from the coupling of light particles (m) with massless particles



$\Longrightarrow$ Single divergency

$\Longrightarrow$ Double divergency

- Single divergency: Subtraction method

$$J_1 = \frac{\mu^{4-n}}{i\,\pi^2} \int d^n q_1 \, \frac{1}{(q_1^2 + m^2)[(q_1+p)^2 + m^2][(q_1-q_2)^2 + M^2]}.$$

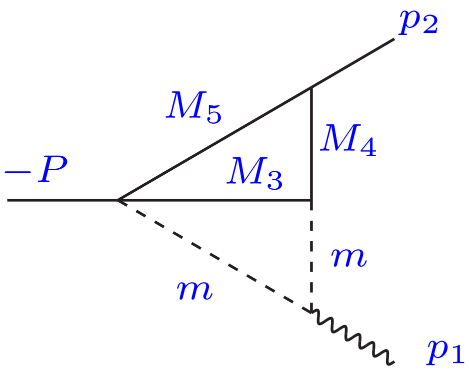
After parametrization

$$J_1 = \int_0^1 dz \int_0^z dy \, \frac{1}{V}, \qquad V = [\, A - y\,(q_2+p)^2\,]\, y + m^2\,(1-y), \qquad A = (q_2 + p\,z)^2 + M^2.$$

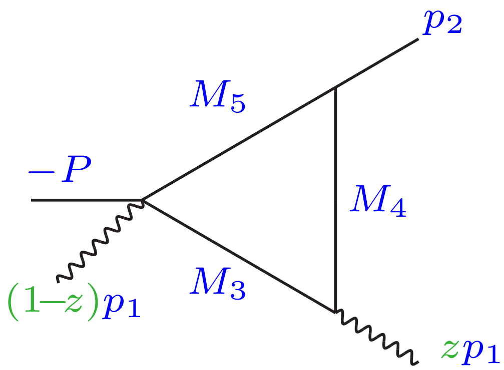
Add and subtract: $V_0^{-1} = (A\,y + m^2)^{-1}$

$$\begin{aligned} J_1 &= \int_0^1 dz \int_0^z dy \, \frac{1}{A\,y + m^2} + \int_0^1 dz \int_0^z dy \left(\frac{1}{V} - \frac{1}{V_0} \right) \\ &= -\ln\frac{m^2}{s} \int_0^1 dz \frac{1}{A} + \int_0^1 dz \frac{1}{A} \ln\frac{Az}{s} + \int_0^1 dz \int_0^z \frac{dy}{y} \left[\frac{1}{A - y\,(q_2+p)^2} - \frac{1}{A} \right] + \mathcal{O}(m^2). \end{aligned}$$

Example:



$$= \quad \ln\frac{m^2}{s} \int_0^1 dz$$



$+$ finite part

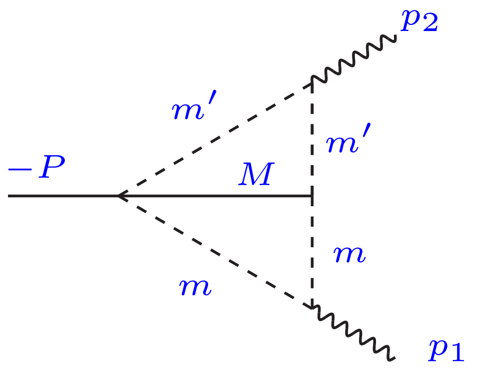
The coefficients of the log are 1-loop functions

- Double divergency: Double subtraction

$$\int_0^1 dxdy \frac{1}{xya(x,y)+\lambda b(x,y)} = \int_0^1 dxdy \Big\{ \frac{1}{xya(x,y)+\lambda b(x,y)}\Big|_{x,y}$$
$$+ \frac{1}{xya(x,0)+\lambda b(x,0)}\Big|_x + \frac{1}{xya(0,y)+\lambda b(0,y)}\Big|_y$$
$$+ \frac{1}{xya(0,0)+\lambda b(0,0)}\Big\}, \qquad \lambda \to 0$$

$$f(z)|_z = f(z) - f(z)|_{z^2=\lambda z=0}$$

- First term $\longrightarrow$ set $\lambda = 0$
- Second (third) term $\longrightarrow$ integrate in y (x) $\longrightarrow$ $\ln(\lambda)$
- Last term $\longrightarrow$ integrate in x and y $\longrightarrow$ $\ln^2(\lambda)$



$$= \ln\frac{m^2}{s}\ln\frac{m'^2}{s}\mathrm{Li}_2\left(\frac{s}{M^2}\right) + \left(\ln\frac{m^2}{s} + \ln\frac{m'^2}{s}\right)\Big[\mathrm{Li}_3\left(\frac{s}{M^2}\right)$$
$$+2\,S_{12}\left(\frac{s}{M^2}\right) - \ln\frac{M^2}{s}\mathrm{Li}_2\left(\frac{s}{M^2}\right)\Big] + \text{finite part}$$

5. Finite Renormalization (FR)

$$\mathcal{A}^{\mu\nu} = \mathcal{A}^{\mu\nu}_{(1)} \otimes (1 + \mathrm{FR}) + \mathcal{A}^{\mu\nu}_{(2)}$$

Take the 1-loop amplitude:

- Multiply by the wave-function factors $Z_H^{-1/2}\, Z_A^{-1}$
- Introduce the relations between renormalized and physical parameters:

$$m_B^2 = M_B^2 \left[1 + \frac{G_F M_W^2}{2\sqrt{2}\,\pi^2}\,\mathrm{Re}\,\Sigma^{(1)}_{BB}(M_B^2)\right], \quad B = W, H, \qquad m_t^2 = M_t^2 \left[1 + \frac{G_F M_W^2}{\sqrt{2}\,\pi^2}\,\mathrm{Re}\,\Sigma^{(1)}_{t}(M_t^2)\right]$$

$$g^2\, s_\theta^2\, Z_A^{-1} = 4\,\pi\,\alpha, \qquad g\, Z_H^{-1/2} = 2\,(\sqrt{2}\,G_F M_W^2)^{1/2}\left[1 - \frac{G_F M_W^2}{4\sqrt{2}\,\pi^2}\,\Pi_H(M_H^2)\right],$$

$$\Pi_H(s) = \frac{M_H^2}{s - M_H^2}\,\mathrm{Re}\left[\Sigma^{(1)}_{HH}(s) - \Sigma^{(1)}_{HH}(M_H^2)\right] - \mathrm{Re}\,\Sigma^{(1)}_{WW}(M_W^2) + \Sigma^{(1)}_{WW}(0) + \frac{7 - 4\,s_\theta^2}{2\,s_\theta^2}\,\ln c_\theta^2 + 6$$

6. Verify Ward identities

The simple-contracted Ward identity requires:

$$F_D + p_1 \cdot p_2 \, F_P = 0$$

After finite renormalization

$$F_D = F_D^{(1)} \otimes (1 + \mathrm{FR}) + F_D^{(2)} \qquad F_P = F_P^{(1)} \otimes (1 + \mathrm{FR}) + F_P^{(2)}$$

- one-loop level

$$F_D^{(1)} + p_1 \cdot p_2 \, F_P^{(1)} = 0$$

- two-loop level

$$F_D^{(2)} + p_1 \cdot p_2 \, F_P^{(2)} + (F_D^{(1)} + p_1 \cdot p_2 \, F_P^{(1)}) \otimes \mathrm{FR} \neq 0 \qquad \text{above WW-threshold}$$

$$\Downarrow$$

Remove the "Re" label in the FR of m_H^2

7. Numerical computation

Write the **finite part** in one of the following forms:

1) $\int dx \, \frac{Q(x)}{V(x)}$ $\qquad V(x)$ polinomial positive definite

2) $\frac{1}{B} \int dx \, Q(x) \, \ln^n V(x)$ $\qquad B$ constant $\neq 0$.

3) $\int dx \, \frac{Q(x)}{V(x)} \, f\left(\frac{V(x)}{P(x)}\right)$ $\qquad f(0) = 0,$ $\qquad f(x) = ln^n(1+x),\ Li_n(x),\ S_{n,p}(x)$

Typical integrand with k Feynman variables:

$$z_1^{n_1} \cdots z_k^{n_k} V^\mu(z_1, \ldots, z_k) \ln^m V(z_1, \ldots, z_k), \qquad \mu = -1, -2$$

- The integration domain is finite ($\subseteq [0,1]^k$)
- V is quadratic with respect to a subset of $\{z_1, \ldots, z_k\}$, in which ...
- ... each z_i^2 is proportional to one squared external momentum.

- The quadratic is not complete
  - $\mu = -1$ and $m = 0$ ($m > 0$ can be treated similarly)

$$\frac{1}{a\,x + b} = \partial_x \frac{1}{a}\;\ln\left(1 + \frac{a}{b}\,x\right)$$

  - $\mu = -2$ and $m = 0$ ($m > 0$ can be treated similarly)

$$\frac{1}{(a\,x\,y + b\,x + c\,y + d)^2} = -\partial_x\,\partial_y\,\frac{1}{a\,d - b\,c}\;\ln\left\{1 + \frac{(a\,d - b\,c)\,x}{b\,(axy + bx + cy + d)}\right\}$$

- The quadratic is complete

$$V(z) = z^t\,H\,z + 2\,K^t\,z + L = (z^t - Z^t)\,H\,(z - Z) + B = Q(z) + B$$

$$Z = -K^t H^{-1}, \quad B = L - K^t H^{-1} K, \qquad \mathcal{P}^t \partial_z\,Q(z) = -Q(z), \quad \mathcal{P} = -(z - Z)/2,$$

$$V^{\mu}(z) = \left(\beta - \mathcal{P}^t\,\partial_z\right)\int_0^1 dy\,y^{\beta-1}\left[Q(z)\,y + B\right]^{\mu}$$

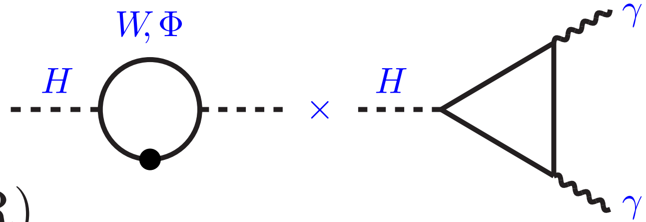
If $\mu = -1$ we choose: $\beta = 1$

$$V^{-1} = \left(1 - \mathcal{P}^t\,\partial_z\right)\frac{1}{Q}\;\ln\left(1 + \frac{Q}{B}\right)$$

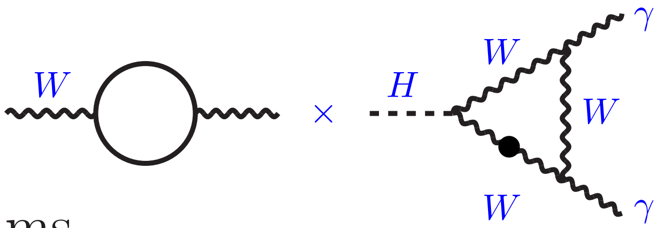
8. Behaviour at threshold

- Square root singularities $\rightarrow$ $1/\beta_W = 1/\sqrt{1 - 4\,M_W^2/s}$

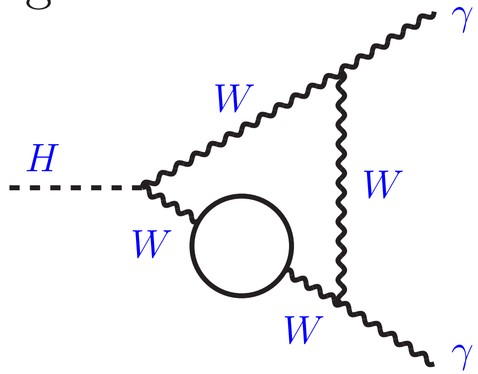
  I (1-loop diagrams) ⊗ (H wave-function FR)



  II (1-loop diagrams) ⊗ (W mass FR)



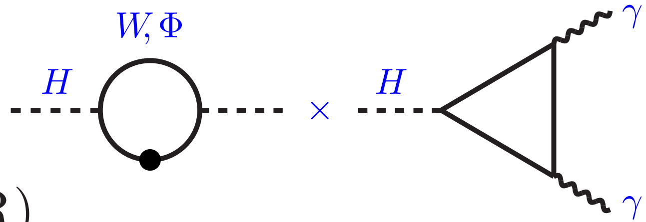
  III Pure 2-loop diagrams



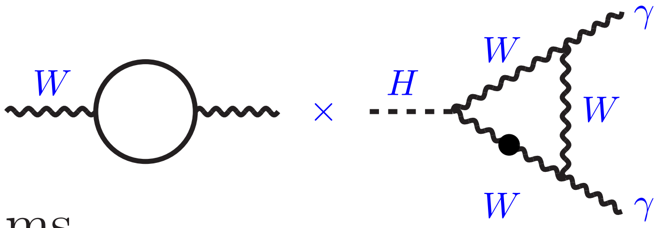
8. Behaviour at threshold

- Square root singularities $\longrightarrow$ $1/\beta_W = 1/\sqrt{1 - 4\,M_W^2/s}$

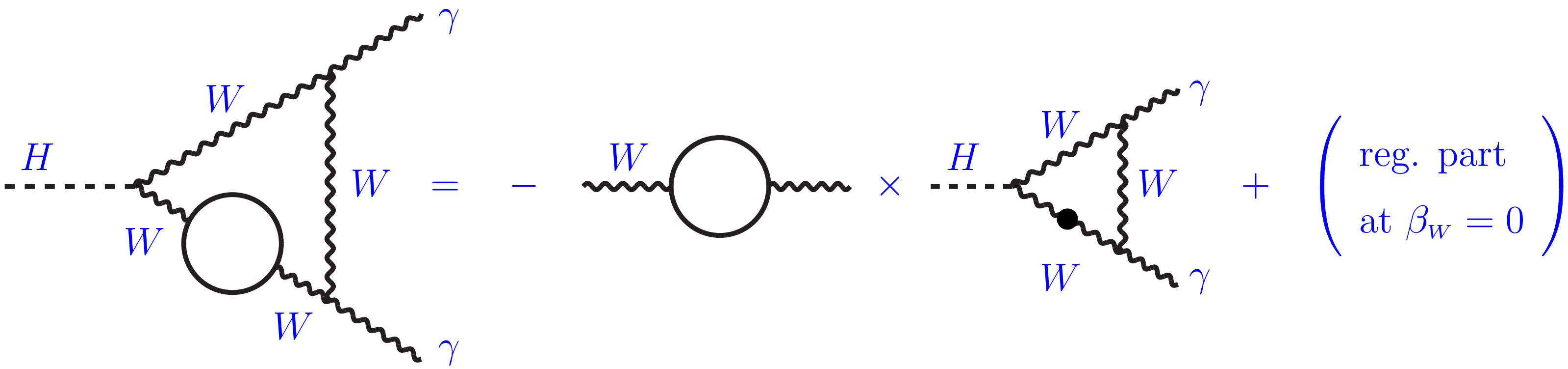
I (1-loop diagrams) $\otimes$ (H wave-function FR)



II (1-loop diagrams) $\otimes$ (W mass FR)



III Pure 2-loop diagrams



8. Behaviour at threshold

- Square root singularities $\rightarrow$ $1/\beta_W = 1/\sqrt{1-4\,M_W^2/s}$

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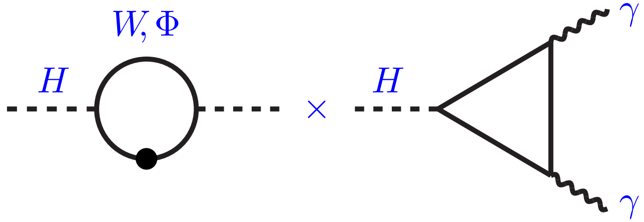
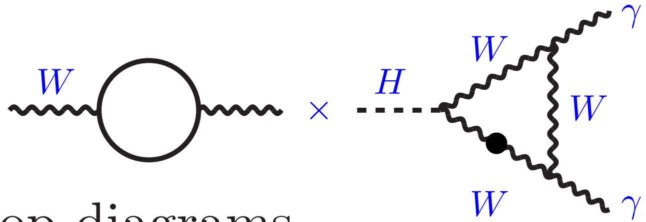


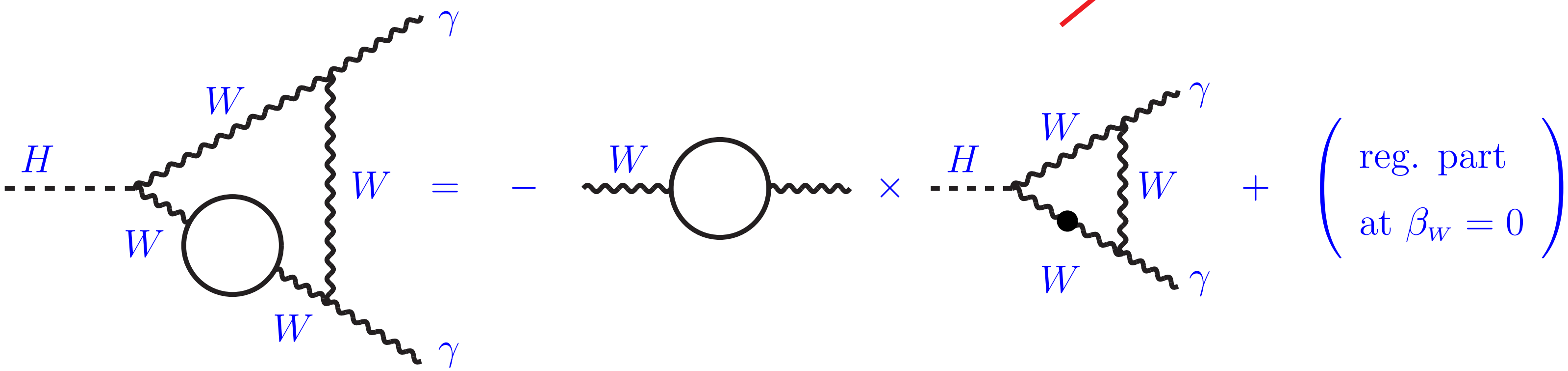
$\rightarrow$ **Complex W Mass**

II (1-loop diagrams) $\otimes$ (W mass FR)

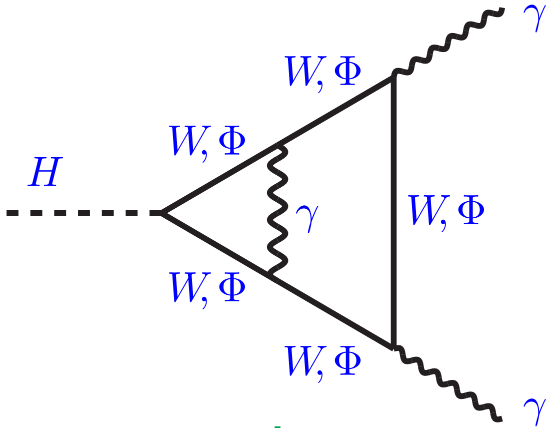


$\rightarrow$ **II + III reg. at $\beta_W = 0$**

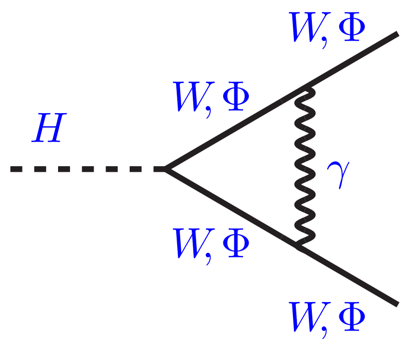
III Pure 2-loop diagrams



- Logarithmic singularities $\longrightarrow$ $\ln \beta_W = \ln(\sqrt{1 - 4\, M_W^2/s})$

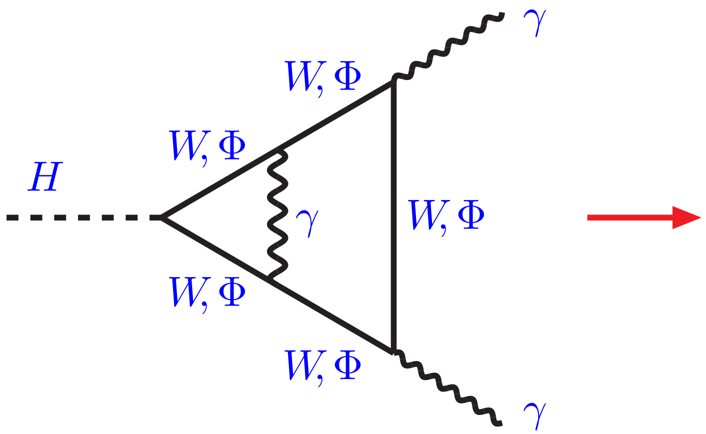


Remnant of Coulomb singularity

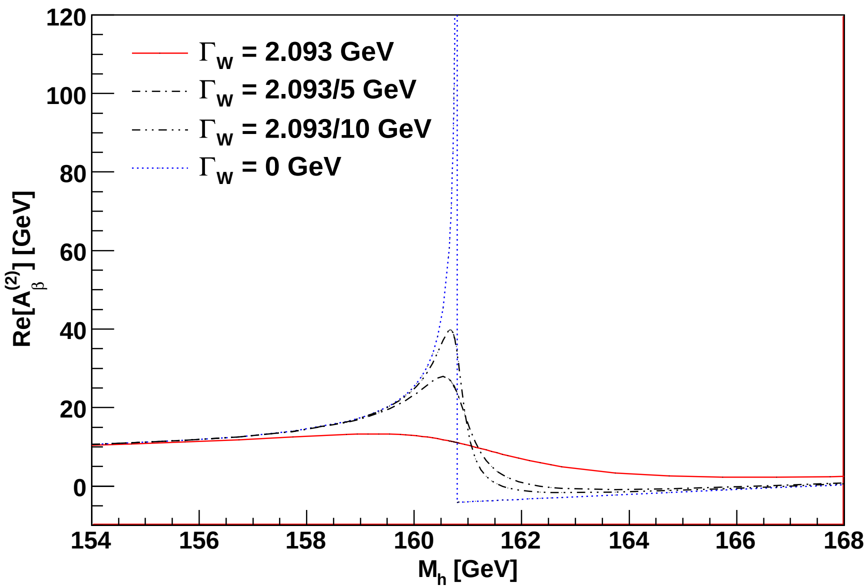


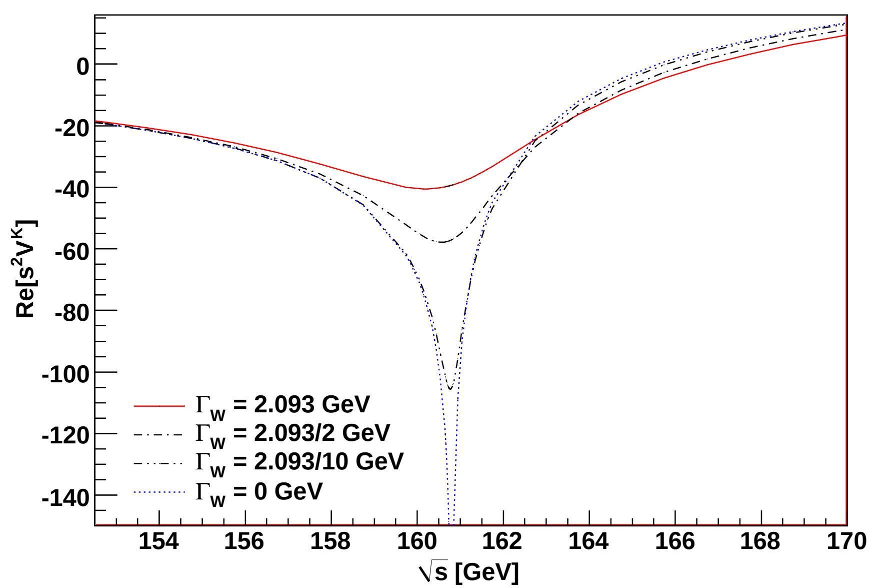
$\sim \ln \beta_W$ $\qquad$ $\sim 1/\beta_W$

- Logarithmic singularities $\rightarrow$ $\ln\beta_W = \ln(\sqrt{1 - 4\,M_W^2/s})$



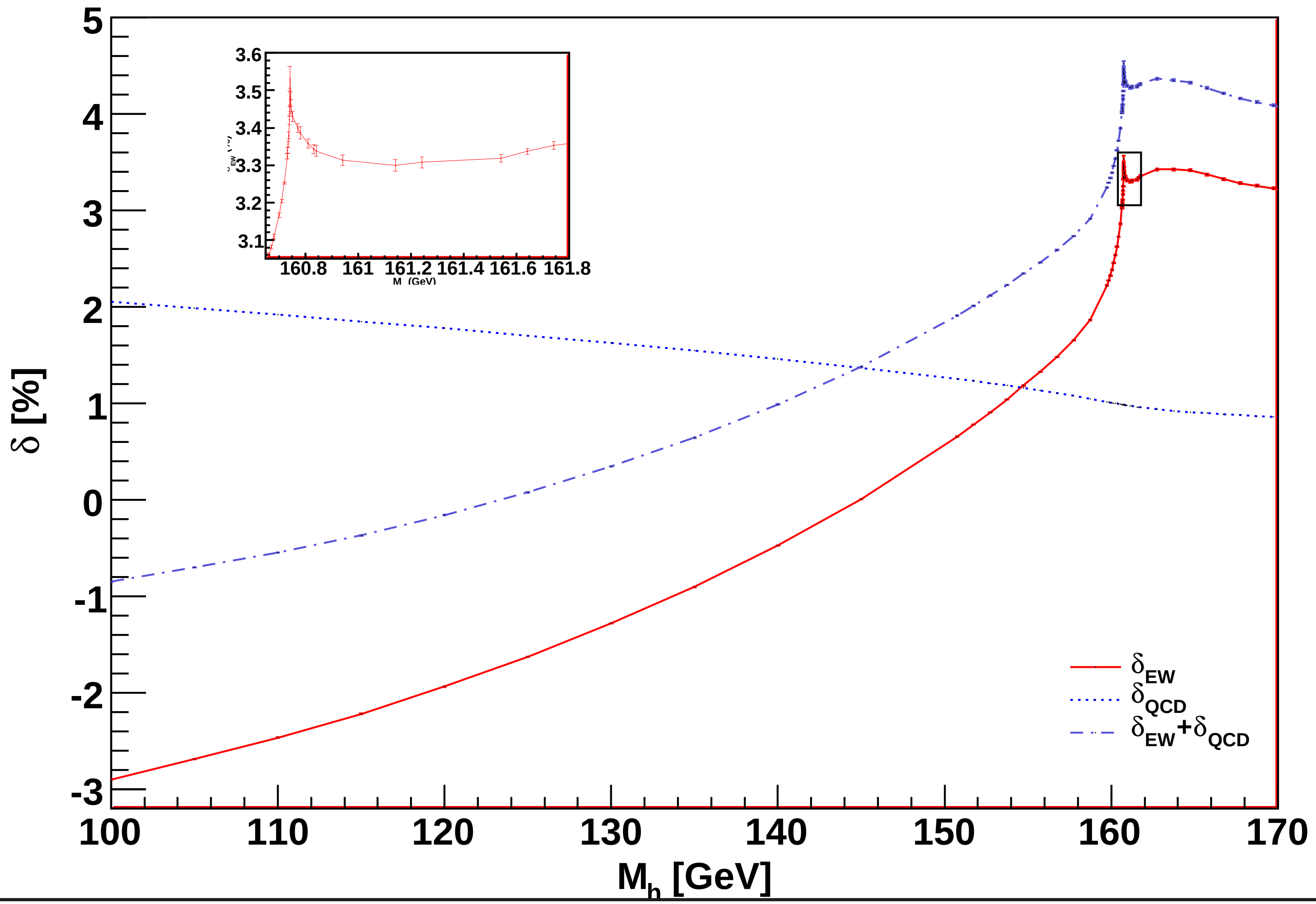
Complex
W Mass





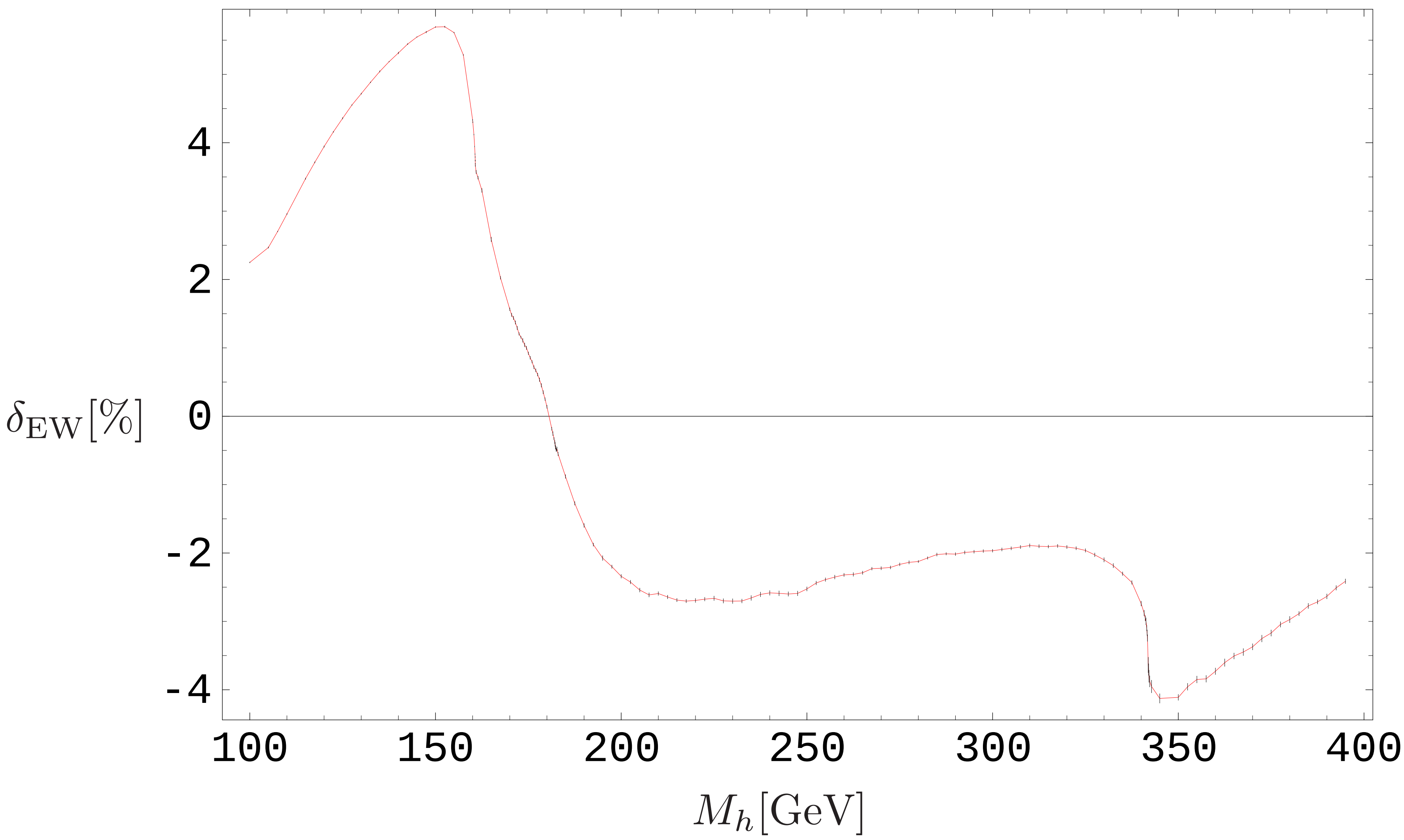
Results for the decay width of $H \to \gamma\gamma$

$$\Gamma(H \to \gamma\gamma) = \frac{|\mathcal{A}_{\rm phys}|^2}{16\,\pi\,M_H} = \Gamma_0\,(1+\delta)$$



Results for EW corrections to $H \to gg$

$$\Gamma(H \to gg) = \Gamma_0\,(1 + \delta_{\rm EW} + \delta_{\rm QCD})$$



Summary

- Completed the two-loop correction to $\Gamma(H \to \gamma\gamma)$:

$$\Gamma = \Gamma_0\,(1+\delta), \qquad -1\% \;<\; \delta \;<\; 4.5\%$$

- Analyzed the behaviour around the WW-threshold
- Studied the collinear singularities of two-loop diagrams
- Completed the two-loop EW correction to $\Gamma(H \to gg)$

$$\Gamma = \Gamma_0\,(1+\delta_{\mathrm{EW}}+\delta_{\mathrm{QCD}}), \qquad -4\% \;<\; \delta_{\mathrm{EW}} \;<\; 5.5\%$$

Next step

- Two-loop EW corrections to the production of an off-shell Higgs at LHC via

$$gg \to H$$