

# Weak effects in heavy quark and di-jet production

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# Outline

- Motivation & Leading order processes
- Next-to-leading order corrections  $\mathcal{O}(\alpha_s^2 \alpha)$
- Results & Discussion
- Conclusion & Outlook

# **Motivation & Leading order processes**

# Discoveries and Precision at Colliders

- Recent Colliders (examples)
  - **Discovery**:  $W$  and  $Z$  boson at the **SPS**
  - **Precision**:  $Z$ -resonance at the **LEP** → three generations
  - **Discovery**: Top-quark at the **Tevatron**

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  - **Discovery**: Top-quark at the **Tevatron**

Theory  $\rightarrow$  Standard Model

# Discoveries and Precision at Colliders

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  - **Discovery**: Top-quark at the **Tevatron**

## Theory → Standard Model

- (Near) future Colliders (examples)
  - **LHC**: **Discovery** & **Precision** machine  
→ direct and indirect measurements
  - **ILC**: **High Precision** machine

# Why weak effects in hadronic collisions ?

## ● Hadron Colliders

- Provide high energy events
- Many observables will be measured at 10-20% accuracy

## Theory: NLO corrections

## ● QCD-corrections are dominant

## ● Weak corrections

- Smaller coupling:  $\alpha < \alpha_s$
- Large logarithms: Sudakov Logarithms

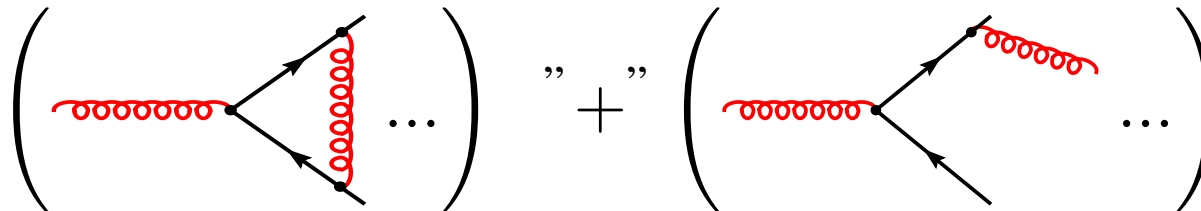
$$\ln^2 \left( \frac{E_{cm}}{M_w} \right), \ln \left( \frac{E_{cm}}{M_w} \right)$$

(Sudakov 1954)  
(Kühn, Penin, Smirnov 1999)  
(Ciafaloni, Comelli 1999)  
(Denner, Pozzorini 2001)

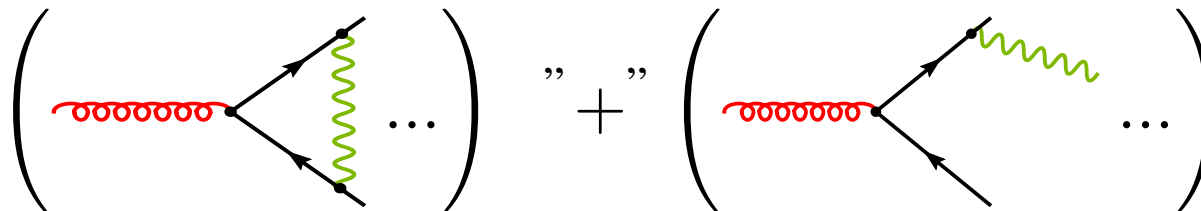
# Origin of the Sudakov Logarithms

- Analogy to theories with massless gauge bosons (QCD, QED)
  - Soft and collinear logarithms in virtual corrections
  - Cancellation in the sum of virtual and real contributions

(Bloch, Nordsieck 1937)  
(Kinoshita, Lee, Nauenberg 1964)



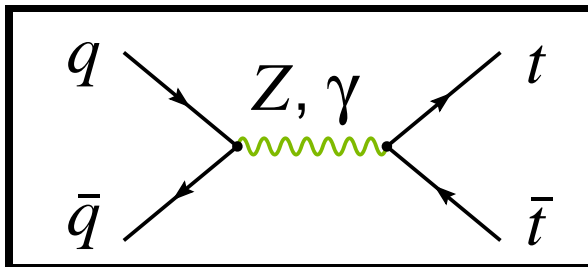
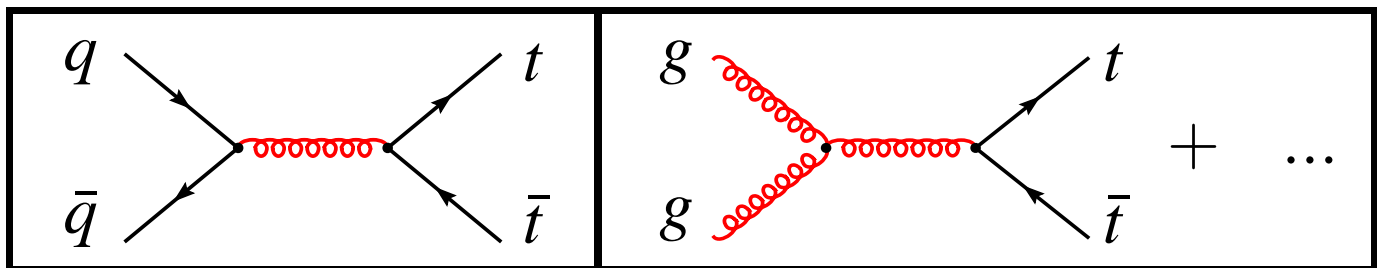
- Weak theory: massive bosons ( $Z, W$ )
  - Virtual and real corrections can be separated





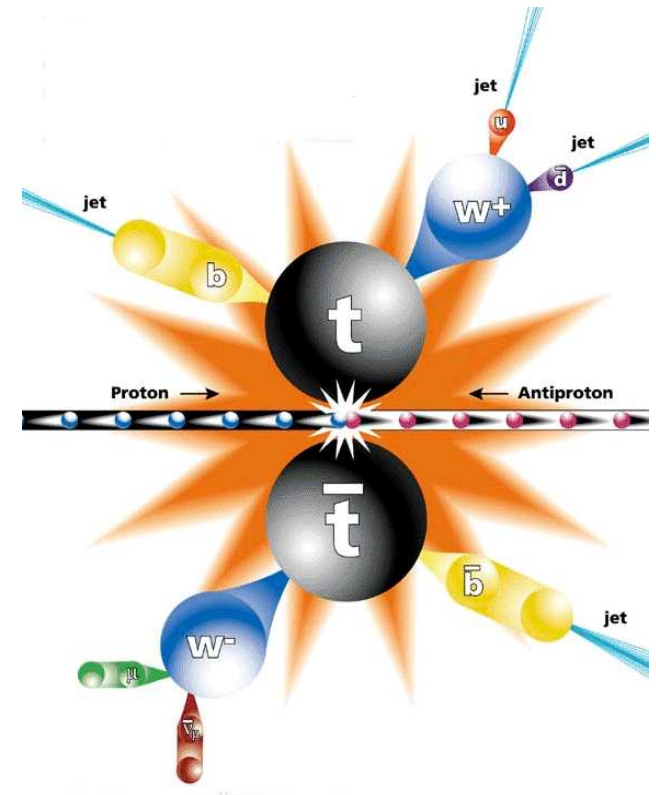
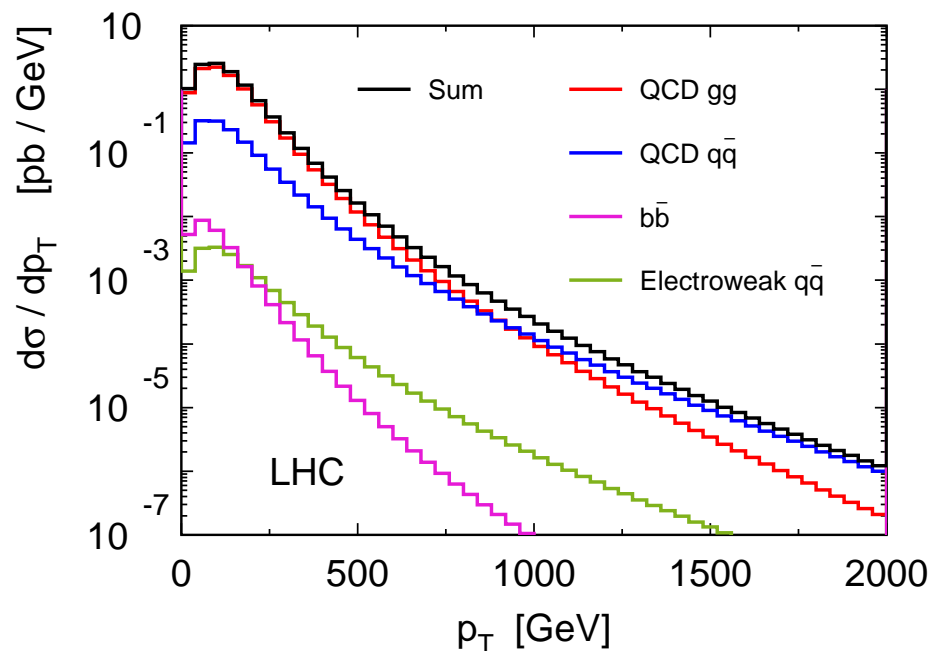
# Top-quark pair production

- Top-quark ( $m_{\text{top}} \simeq 172.7 \text{ GeV}$ )
  - Completes the fermion sector of the SM
  - Still not very well measured
  - Probes physics at high mass scale
  - Plays central role in many SM extensions
- $t\bar{t}$ -production at Hadron Colliders



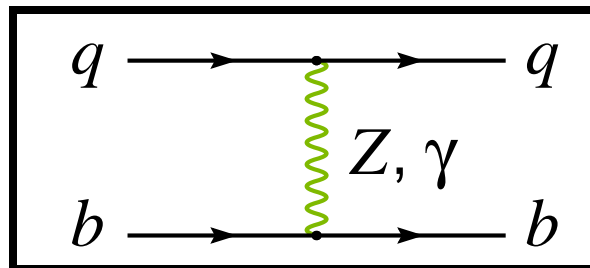
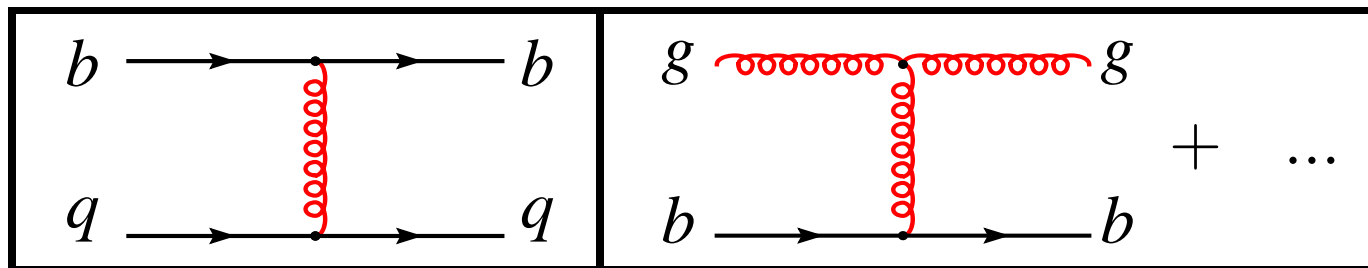
# Top-quark pair production

- Experimentally (e.g.  $p + \bar{p} \rightarrow t\bar{t} \rightarrow b\bar{b} + l\bar{\nu}_l + jj$ )
  - Decay channels: di-lepton, **semi-leptonic**, full hadronic
  - Tevatron:  $\frac{\Delta m_t}{m_t} \sim 1\%$
- Leading order  $p_T$  distribution



# Bottom jet production

- Bottom-quark ( $m_b = 0$ )
  - Events with well separated partons ( $p_T > 50$  GeV)
  - Background process ( $t\bar{t}$ , SUSY)
  - Testing the SM at high  $p_T$
- $b$ -jet production at Hadron Colliders



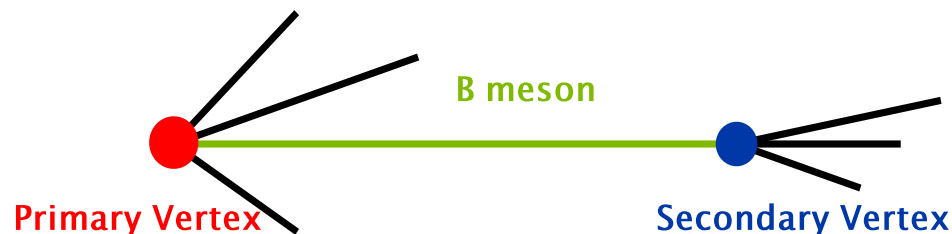
# Bottom jet production

- QCD, Mixed and Electroweak contributions

initial state	single $b$ -tag	
quark-induced	$qb \rightarrow qb, q\bar{b} \rightarrow q\bar{b}, \bar{q}b \rightarrow \bar{q}b, \bar{q}\bar{b} \rightarrow \bar{q}\bar{b}$	$q\bar{q} \rightarrow b\bar{b}$
gluon-induced	$gb \rightarrow gb, g\bar{b} \rightarrow g\bar{b}$	$gg \rightarrow b\bar{b}$
pure bottom-induced	$b\bar{b} \rightarrow b\bar{b}, bb \rightarrow bb, \bar{b}\bar{b} \rightarrow \bar{b}\bar{b}$	
	double $b$ -tag	

- Experimentally

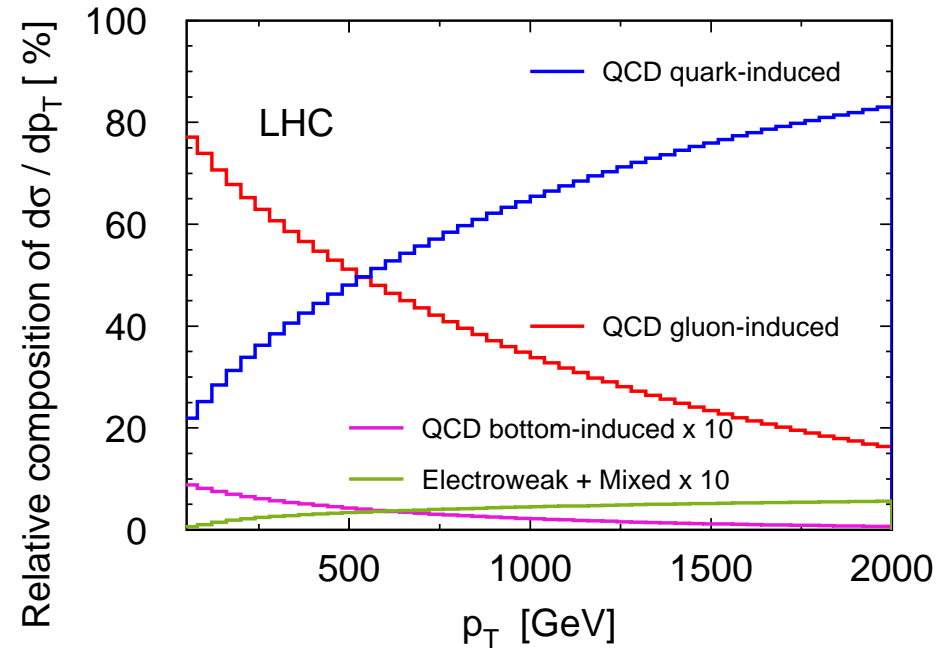
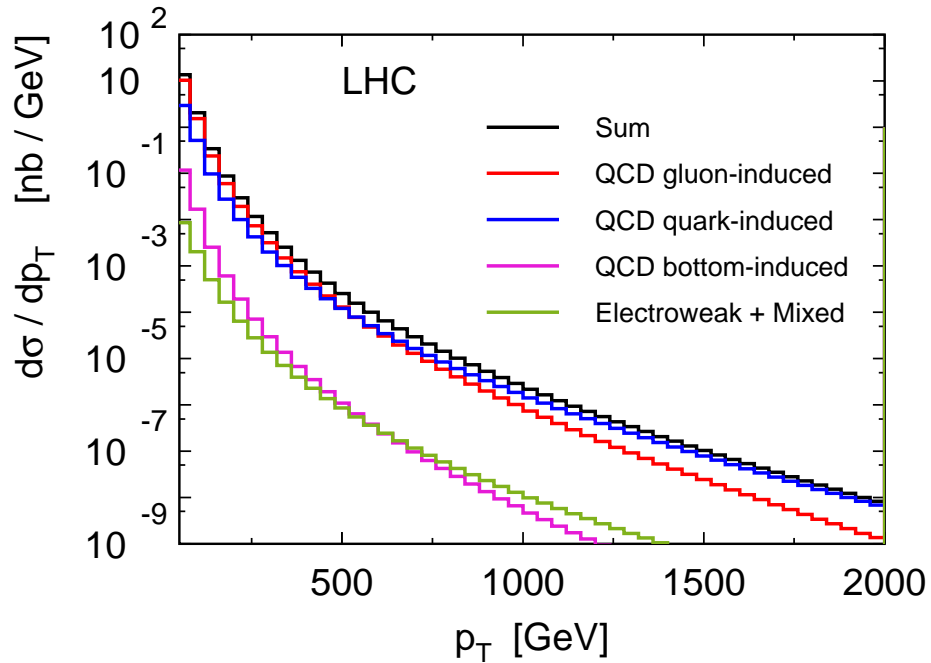
- Lifetime of  $B$  mesons  $\propto 1.5 \times 10^{-12} s$
- Decay length allows  $b$ -jet identification



# Bottom jet production

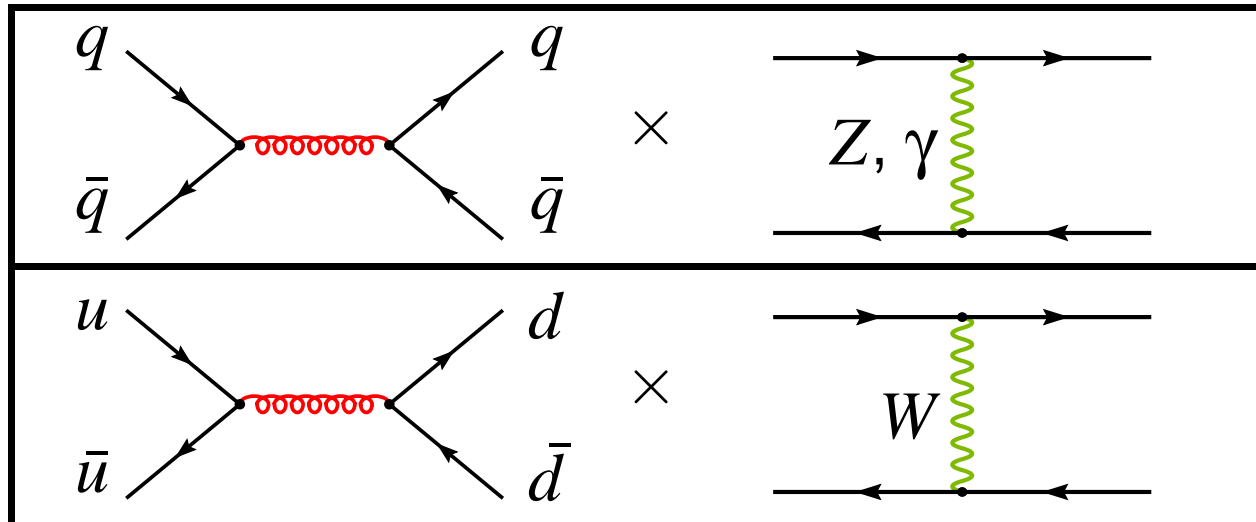
●  $p_T$ -distribution at leading order

● Single  $b$ -tag



# Di-jet production

- Gluon- & light quark-jets ( $m_u = m_d = m_s = m_c = 0$ )
  - Well separated jets  $p_T > 50$  GeV
  - Indirect new physics search, e.g.  $Z'$
  - Study of jet production rates in the LHC start-up phase
- Di-jet production at Hadron Colliders
  - Processes of  $O(\alpha_s^2)$  and  $O(\alpha_s\alpha)$  !



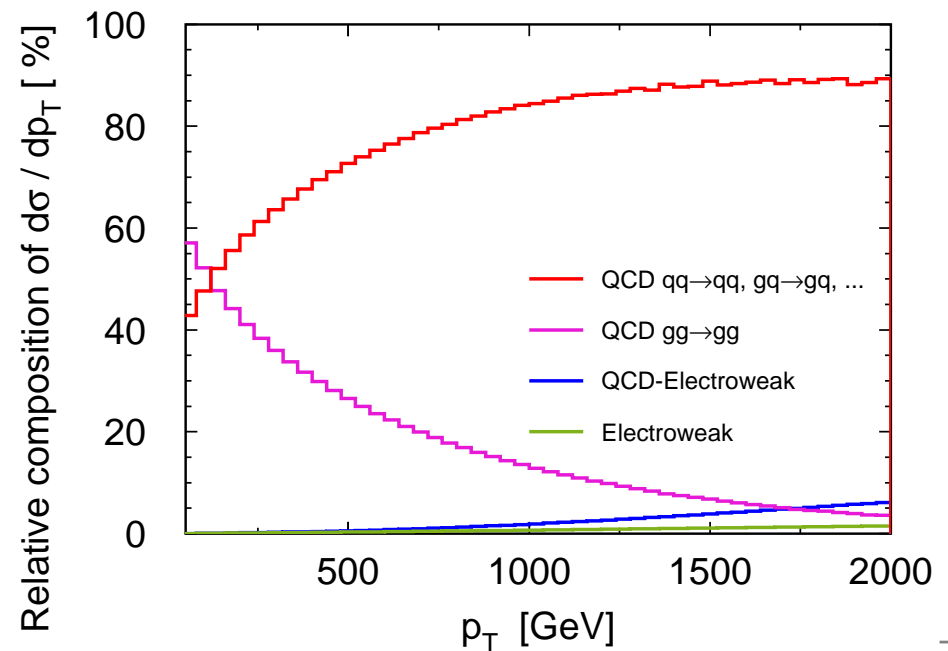
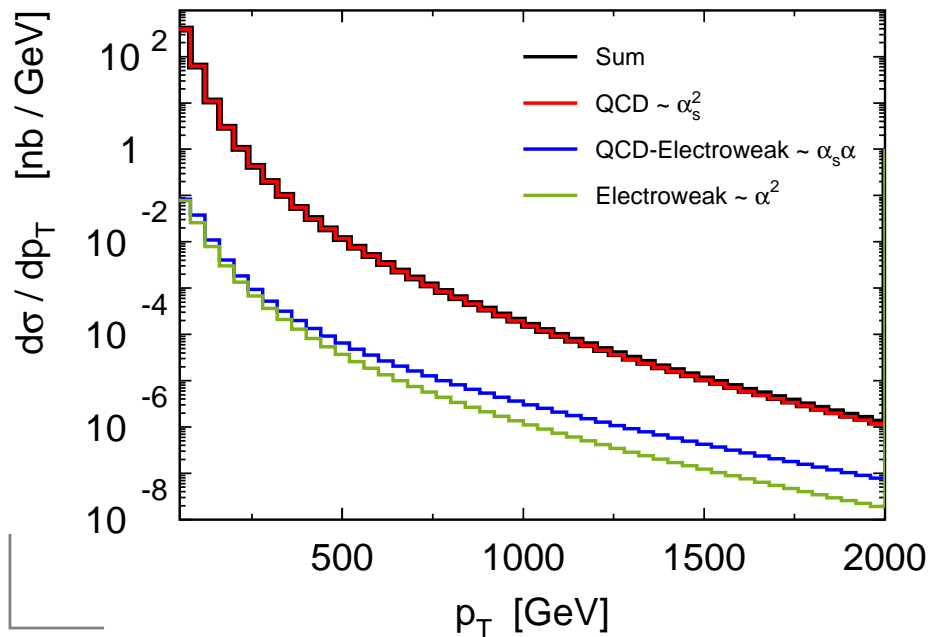
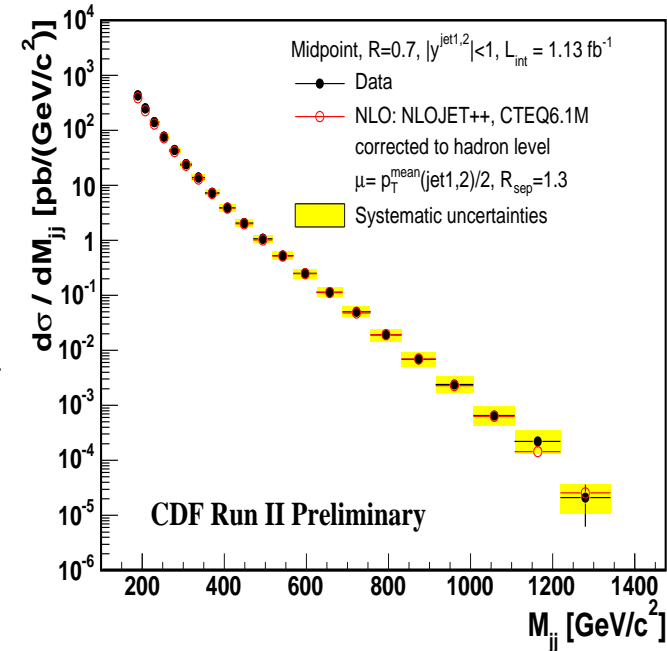
# Di-jet production

● Experimentally

● Tevatron: Di-jet-Masses up to 1 TeV

● LHC: Di-jet-Masses up to several TeV

●  $p_T$  distribution at leading order

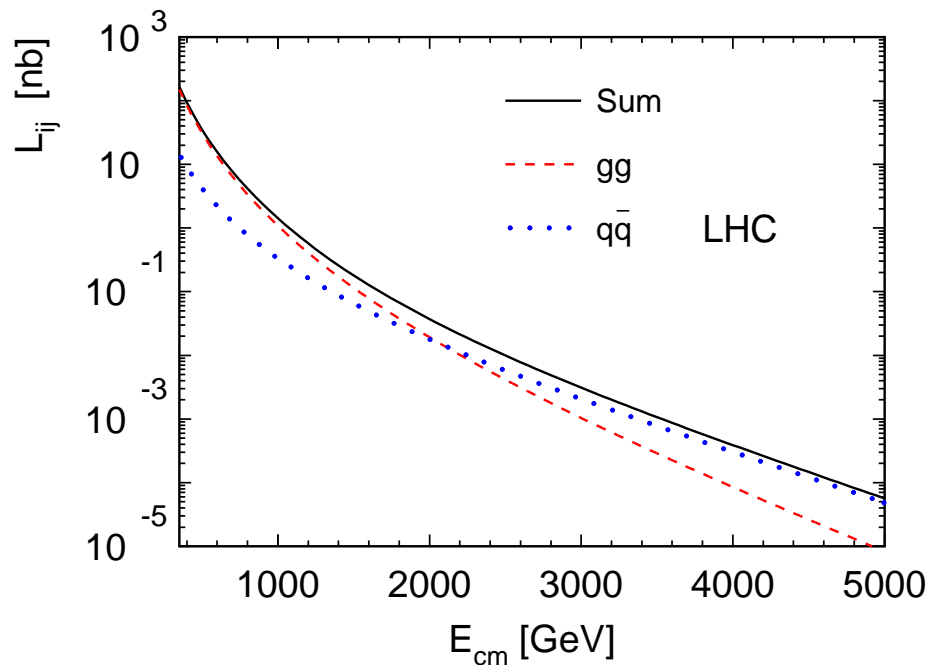


# Luminosity at the LHC

● Consider top-quark pair production at LO

● Define luminosity function:

$$L_{ij}(\tau, \mu_F) = \frac{1}{S} \int_{\tau}^1 \frac{1}{x_1} f_{i,p}(x_1, \mu_F) f_{j,p}\left(\frac{\tau}{x_1}, \mu_F\right) \quad \text{with } \tau = \frac{E_{cm}^2}{S}$$



Quark density dominates  
at high energies!

● Similar for  $b$ -jet and di-jet production !



**Next-to-leading order  
corrections  $O(\alpha_s^2\alpha)$**

# Status of NLO calculations

## ● Top-quark pair production

- QCD corrections  $O(\alpha_s^3)$

(Dawson, Nason, Ellis 1988)

(Beenakker, Kuijf, Neerven, Smith 1989)

(Bernreuther, Brandenburg, Si, Uwer 2004)

- (Electro-)weak corrections  $O(\alpha_s^2\alpha)$

(Beenakker, Denner, Hollik, Mertig, Sack, Wackerroth 1994)

(Bernreuther, Fückler, Si 2005, 2006)

(Kühn, A.S., Uwer 2005, 2006)

(Hollik, Kollar 2007)

## ● $b$ -jet production

- QCD corrections  $O(\alpha_s^3)$

(Dawson, Ellis, Nason 1988)

(Beenakker, Kuijf, Neerven, Smith 1989)

(Frixione, Mangano 1997)

- $b\bar{b}$  production  $O(\alpha_s^2\alpha)$

(Moretti et al 2003)

## ● Di-jet production

- QCD corrections  $O(\alpha_s^3)$

(Ellis, Sexton 1985)

(Aversa et al 1988, 1991)

- Weak corrections  $O(\alpha_s^2\alpha)$

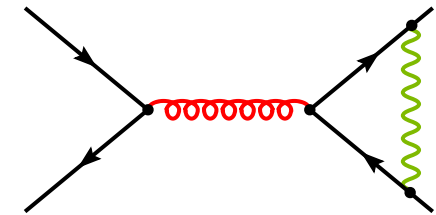
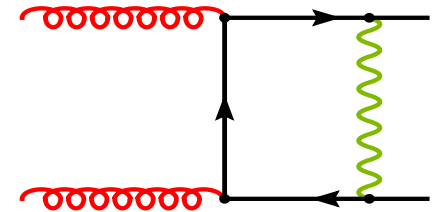
(Moretti et al 2006)

# General Remarks

- Corrections of  $O(\alpha_s^2\alpha)$  calculated in Feynman-'t Hooft Gauge
- Consider **only weak corrections** → **neglecting photonic contributions**
  - Gauge invariant subset
  - Photonic contributions involve no Sudakov Logarithms
- $t\bar{t}$  and  $b$ -jet production
  - $O(\alpha)$  corrections to LO  $\alpha_s^2$  processes:  $Z, W, \phi, (\chi, H)$
- Di-jet production
  - " $O(\alpha)$  corrections to LO  $\alpha_s^2$  processes:  $Z, W$  "
  - " $O(\alpha_s)$  corrections to LO  $\alpha_s\alpha$  processes"

# Methods: Overview

- Passarino-Veltman reduction
- Scalar Integrals
- Renormalisation
- Real corrections



$$\left( \text{Diagram 1} + \dots \right) \times \left( \text{Diagram 2} \right)^*$$

The first diagram in the parentheses is a loop diagram with two red wavy lines and one green wavy line. The second diagram is a tree-level diagram with a red wavy line and a green wavy line.

$$\left( \text{Diagram 3} + \dots \right) \times \left( \text{Diagram 4} + \dots \right)^*$$

The first diagram in the parentheses is a tree-level diagram with a green wavy line and a red wavy line. The second diagram is a tree-level diagram with two red wavy lines.

# Methods: Renormalisation

- Renormalisation: Counterterm formalism

(Denner 1991)

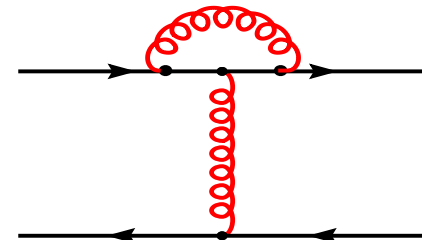
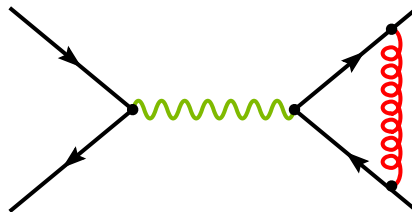
$$\mathcal{L}(\Psi_0, A_0, m_0, g_0) \rightarrow \mathcal{L}(\Psi_R, A_R, m_R, g_R) + \mathcal{L}_{ct}(\Psi_R, A_R, m_R, g_R)$$

- $t\bar{t}$  and  $b$ -jet production:  $O(\alpha)$  corrections

- On-shell scheme
- No coupling renormalisation is needed
- Only wave function and mass renormalisation

- Di-jet production

- On-shell scheme (like  $t\bar{t}$  and  $b$ -jet)
- $\overline{\text{MS}}$  scheme



# Methods: Real corrections

- IR singularities

- Phase-space-slicing

(Giele, Glover 1992)

(Giele, Glover, Kosower 1993)

- (Dipole-) subtraction method

(Catani, Seymour 1996)

(Catani, Dittmaier, Seymour, Trocsányi 2002)

- Virtual corrections considered here

- Only four quark processes are IR divergent

- IR divergent real corrections to four quark processes

- Only  $q(\bar{q}) \rightarrow q(\bar{q}) + g$  splitting must be considered


- $t\bar{t}$  and  $b$ -jet production


- Partonic results are independent from  $\mu_F$

# Methods: (Dipole-) Subtraction

- Subtraction method in  $e^+e^-$  collisions ( $e^+e^- \rightarrow m$ )

$$\sigma^{\text{NLO}} = \int_m d\sigma^V + \int_{m+1} d\sigma^R = \int_m \left[ d\sigma^V + \int_1 d\sigma^A \right] + \int_{m+1} \left[ d\sigma^R - d\sigma^A \right]$$

Integrated dipoles 

Dipoles 

# Methods: (Dipole-) Subtraction

- Subtraction method in  $e^+e^-$  collisions ( $e^+e^- \rightarrow m$ )

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Integrated dipoles →

Dipoles



- Dipole method in hadron collisions ( $h_1h_2 \rightarrow m$ )

$$\begin{aligned} \sigma^{\text{NLO}} &= \int_{m+1} d\sigma^R + \int_m d\sigma^V + \int_m d\sigma^C \quad \text{Has same singularity structure as } d\sigma^R \\ &= \int_{m+1} \left[ \left( d\sigma^R \right)_{\epsilon=0} - \left( \sum_{\text{dipoles}} d\sigma^{\text{LO}} \otimes dV_{\text{dipole}} \right)_{\epsilon=0} \right] \\ &\quad + \int_m \left[ d\sigma^V + d\sigma^{\text{LO}} \otimes \mathbf{I} \right]_{\epsilon=0} + \int dx \int_m d\sigma^{\text{LO}}(x) \otimes \left( \mathbf{K}(\mathbf{x}) + \mathbf{P}(\mathbf{x}) \right) \end{aligned}$$

Contains all  $\epsilon$  poles →

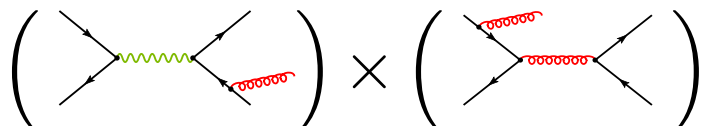
Finite remainder →



# Methods: Technical Details

- Goal: Compact analytic results for further studies
- Amplitudes, Passarino-Veltman reduction & Renormalisation
  - FORM, MATHEMATICA, MAPLE
- Scalar Integrals
  - FF-LIBRARY, literature, analytical computation
- Numerics & Dipoles
  - FORTRAN 77
  - C/C++

# Example

Real corrections for  $q\bar{q} \rightarrow b\bar{b}g$  

$$\begin{aligned}
 \frac{1}{4} \frac{1}{N^2} \sum_{\text{Spin}} \sum_{\text{Colour}} \left| M^{q\bar{q} \rightarrow b\bar{b}g} \right|^2 &= \alpha_s^2 \alpha (4\pi)^3 \frac{N^2 - 1}{N^2} \\
 &\times \left( g_v^q g_v^b (t_1^2 + t_2^2 + u_1^2 + u_2^2) - g_a^q g_a^b (t_1^2 + t_2^2 - u_1^2 - u_2^2) \right) \\
 &\times \frac{1}{s} \frac{1}{s - m_Z^2} \frac{1}{s + t_1 + t_2 + u_1 + u_2} \frac{1}{s + t_1 + t_2 + u_1 + u_2 + m_Z^2} \\
 &\times \frac{1}{s + t_1 + u_1} \frac{1}{s + t_2 + u_1} \frac{1}{s + t_1 + u_2} \frac{1}{s + t_2 + u_2} \\
 &\times \left( 2s^2 + (t_1 + t_2 + u_1 + u_2)(2s - m_Z^2) \right) \\
 &\times \left( (t_1 + t_2 - u_1 - u_2)s^2 + ((t_1 + t_2)^2 - (u_1 + u_2)^2)s \right. \\
 &\left. + (t_1 + t_2 + u_1 + u_2)(t_1 t_2 - u_1 u_2) \right)
 \end{aligned}$$

# Consistency

## ● $t\bar{t}$ production

- Cancellation of the IR and UV poles, ... ✓
- Independent calculation by Peter Uwer ✓
- Parallel work ✓
- Literature ✓

(W. Bernreuther, M. Fückler, Z.G. Si)

(Beenakker, Denner, Hollik, Mertig, Sack, Wackerath 1994)

(Grzadkowski, Kühn 1987)

(Kniehl, Kühn 1989)

(Jezábek, Kühn 1993)

→ No agreement with Moretti et al 2006

## ● $b$ -jet production

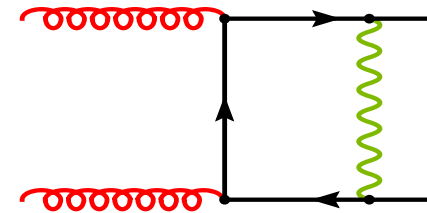
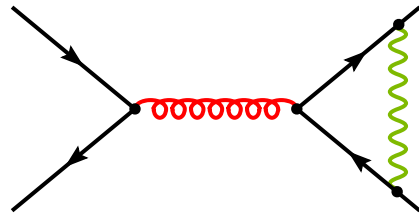
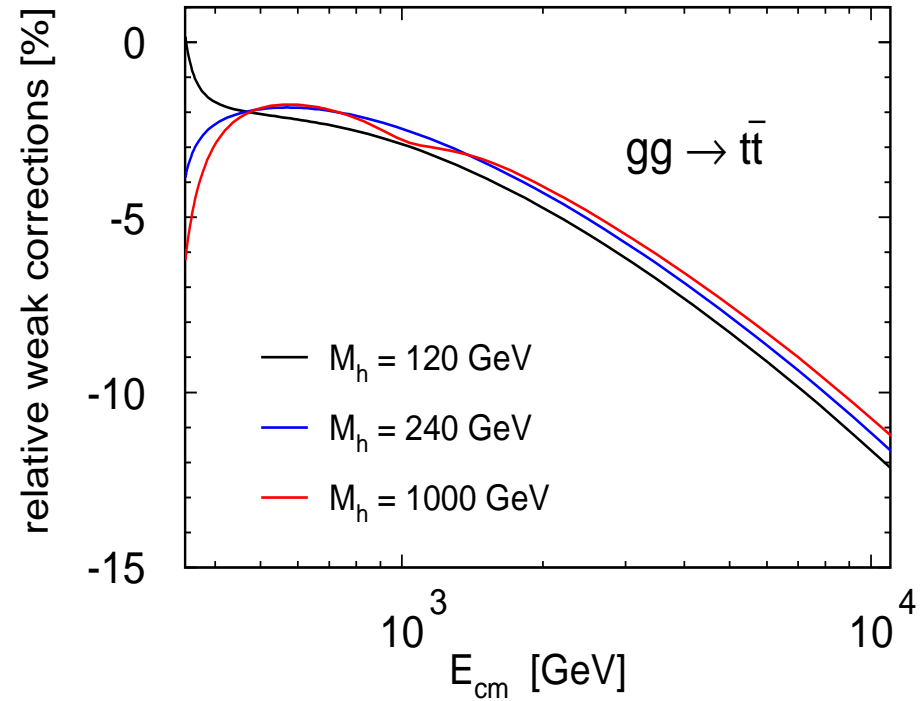
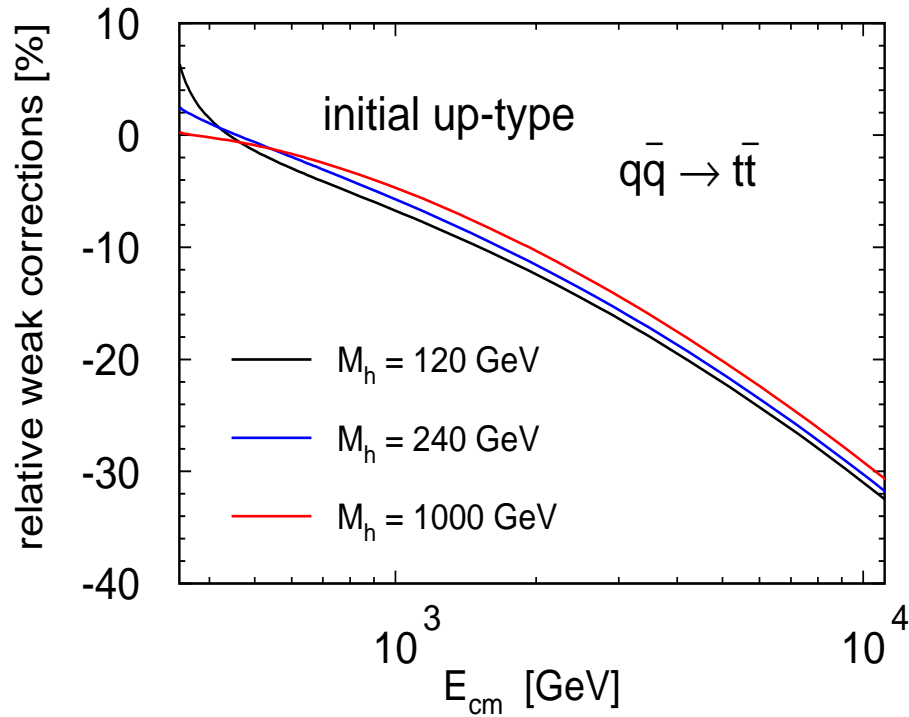
- Cancellation of the IR and UV poles, ... ✓
- Crossing symmetries ✓
- Independent calculation by Peter Uwer (✓)
- Literature

# Status of di-jet production

- Virtual Corrections ✓
- Real Corrections ✓
- Renormalisation ✓
- Subtraction method **in progress**

# Results & Discussion

# Partonic results for $t\bar{t}$ production

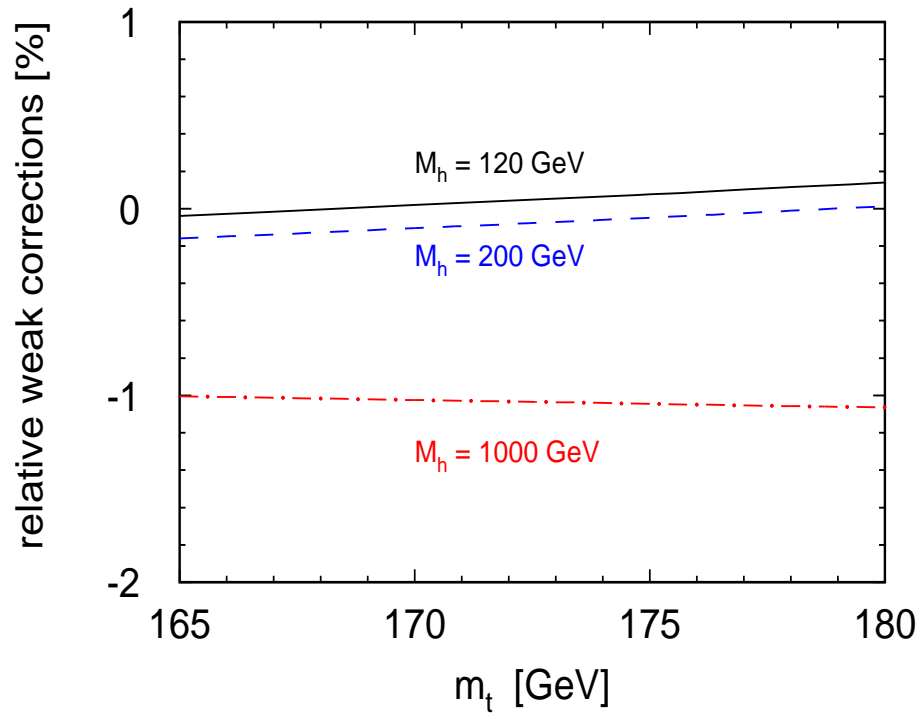


● Quark-induced processes receive larger corrections

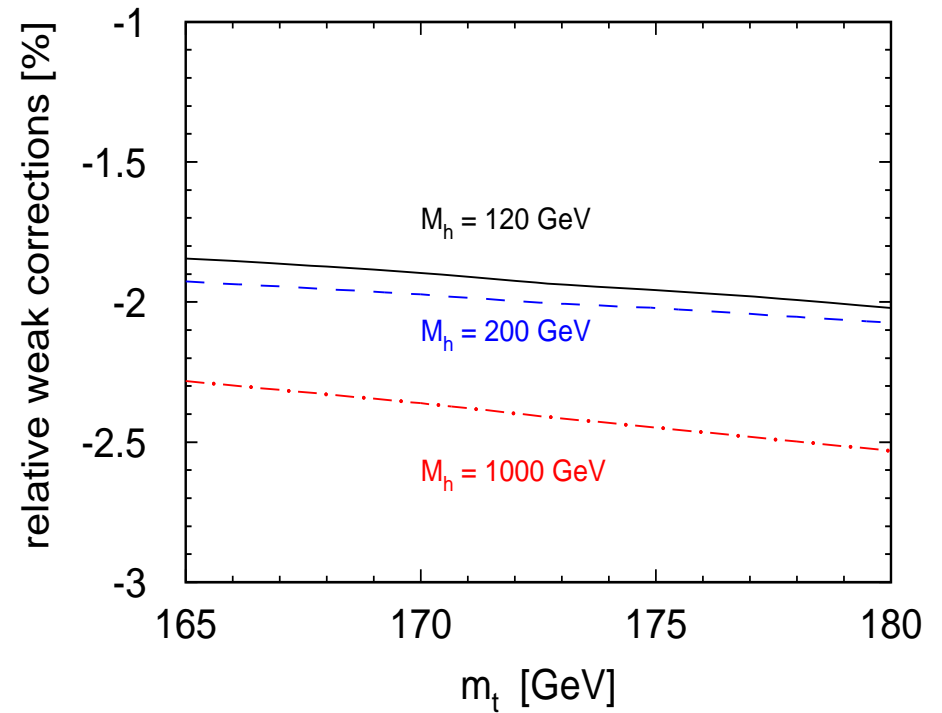
# Total hadronic cross section for $t\bar{t}$

● Small corrections

TEVATRON

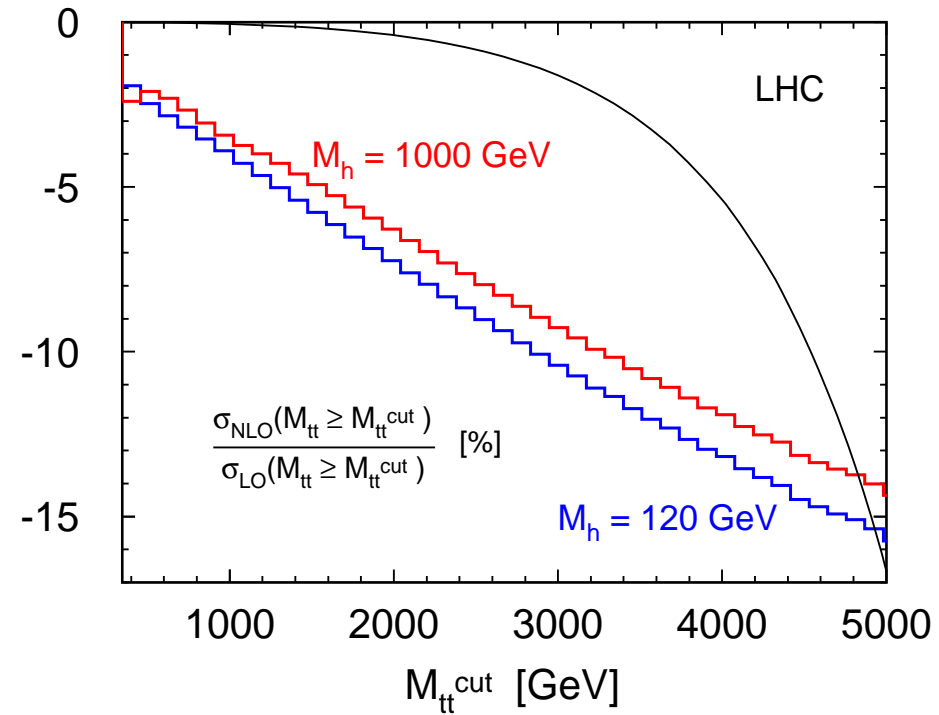
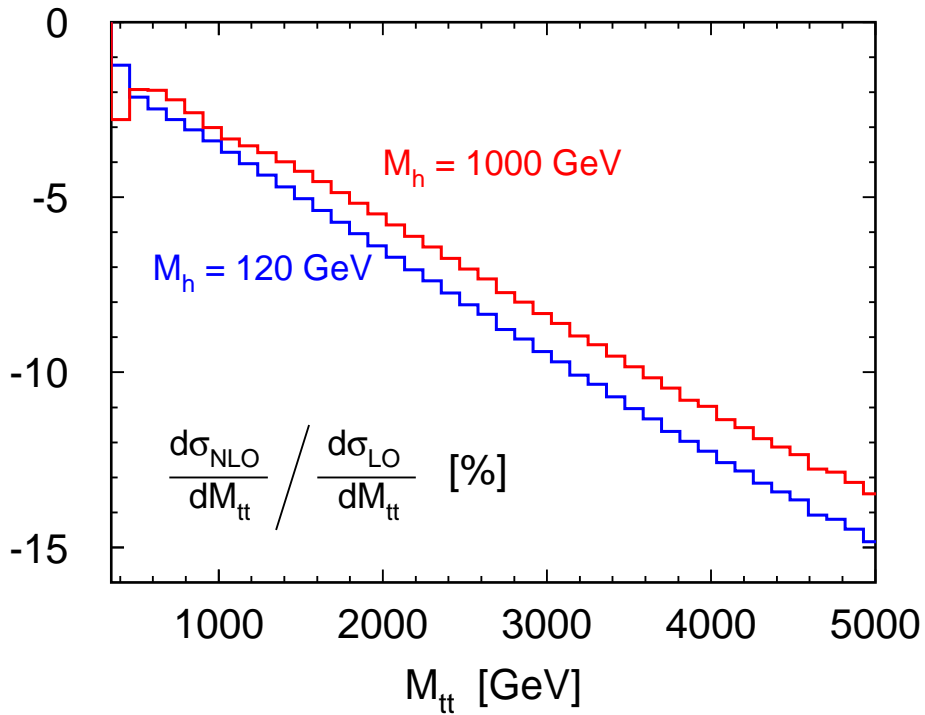


LHC



# $t\bar{t}$ production at the LHC

- Relative corrections to  $M_{t\bar{t}} = \sqrt{(k_t + k_{\bar{t}})^2}$

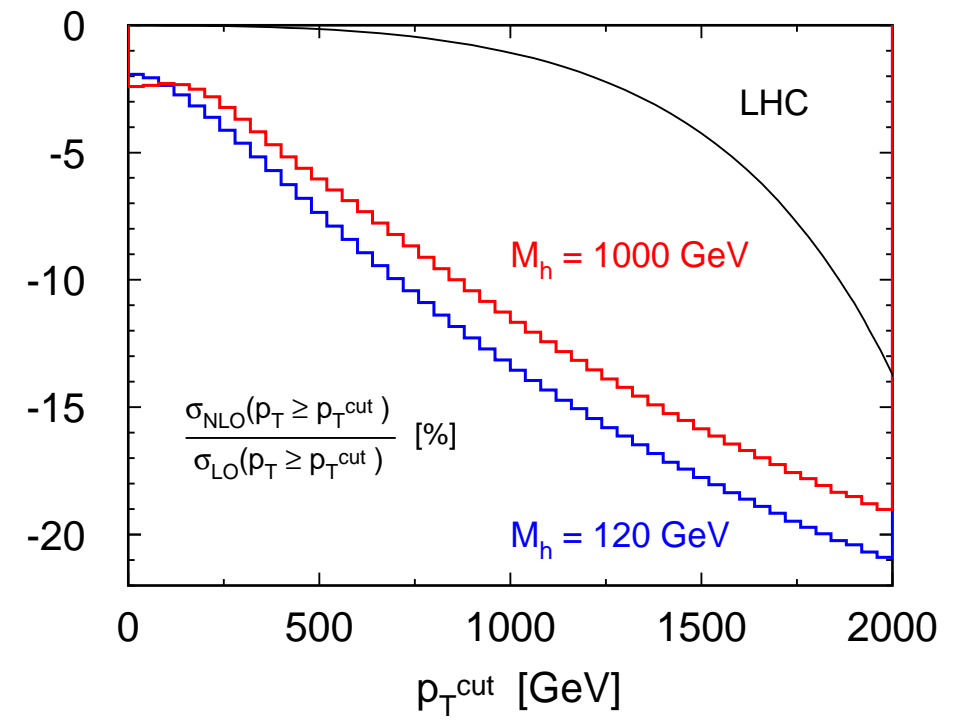
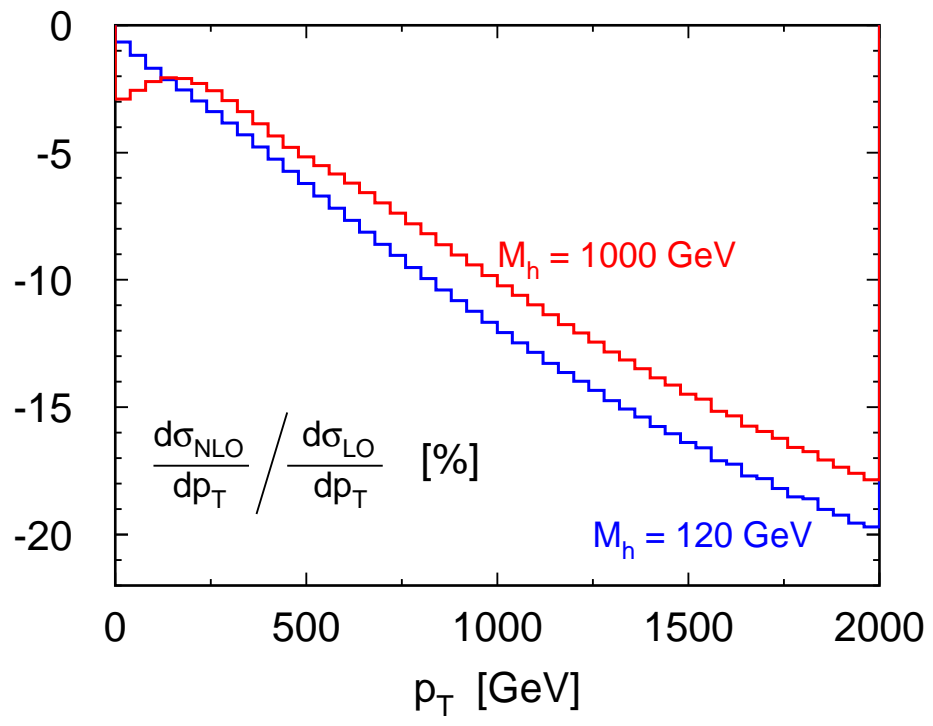


$$\int \mathcal{L} dt = 200\text{fb}^{-1}$$



# t $\bar{t}$ production at the LHC

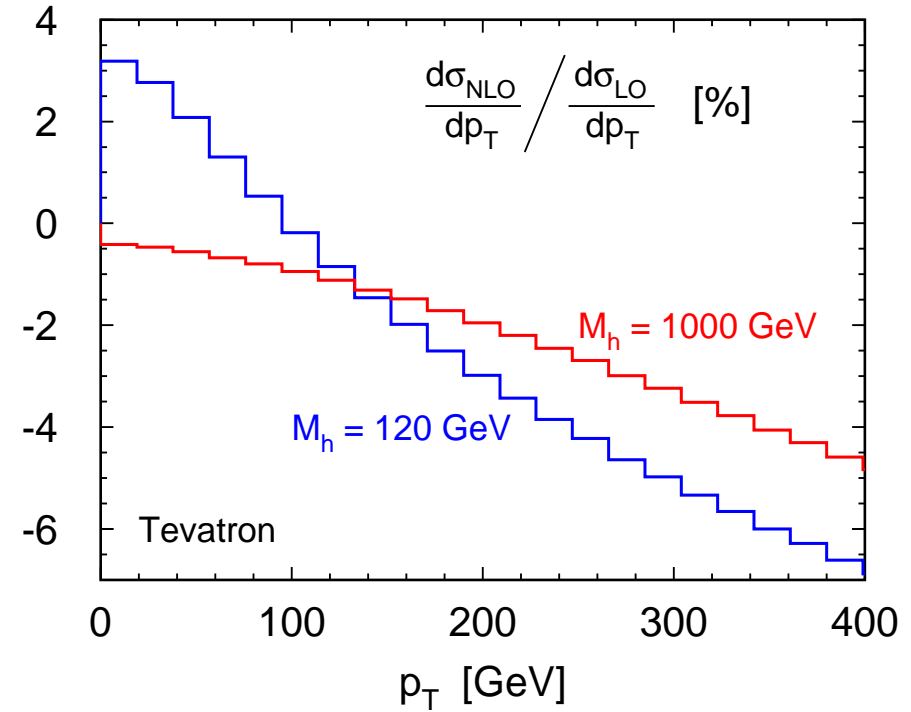
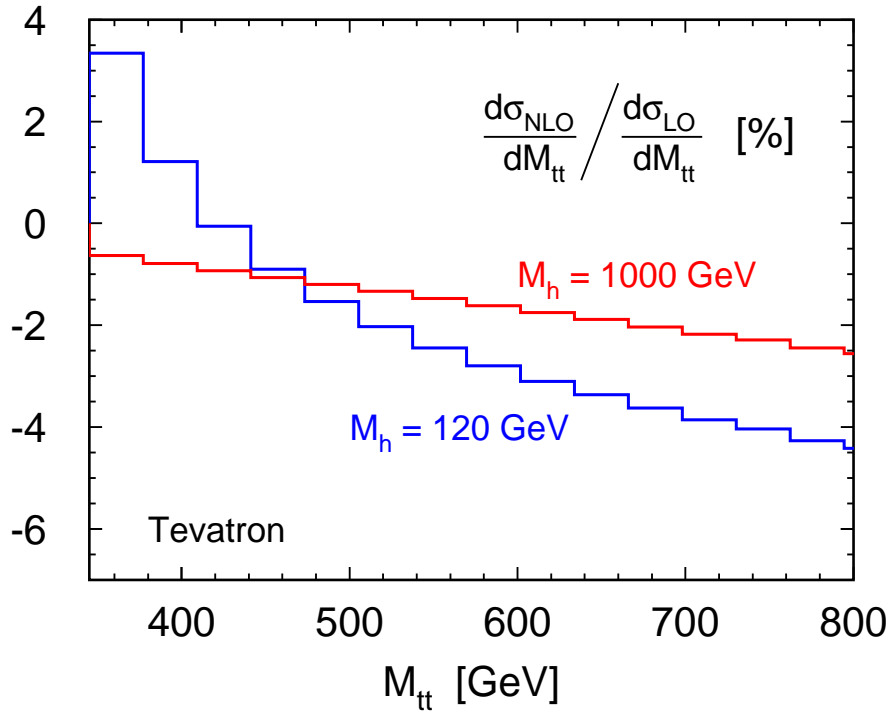
## ● Relative corrections to p<sub>T</sub>



$$\int \mathcal{L} dt = 200 \text{ fb}^{-1}$$

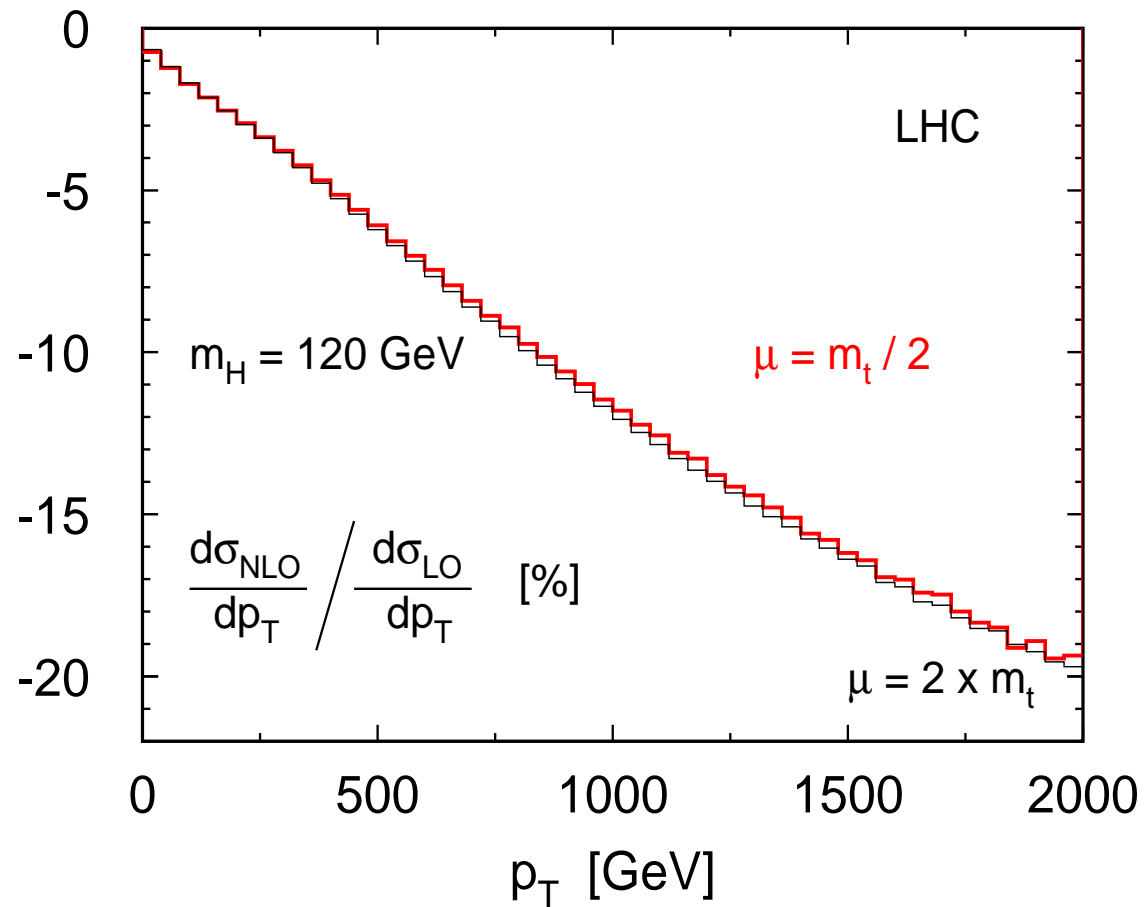
# $t\bar{t}$ production at the Tevatron

## ● Relative corrections



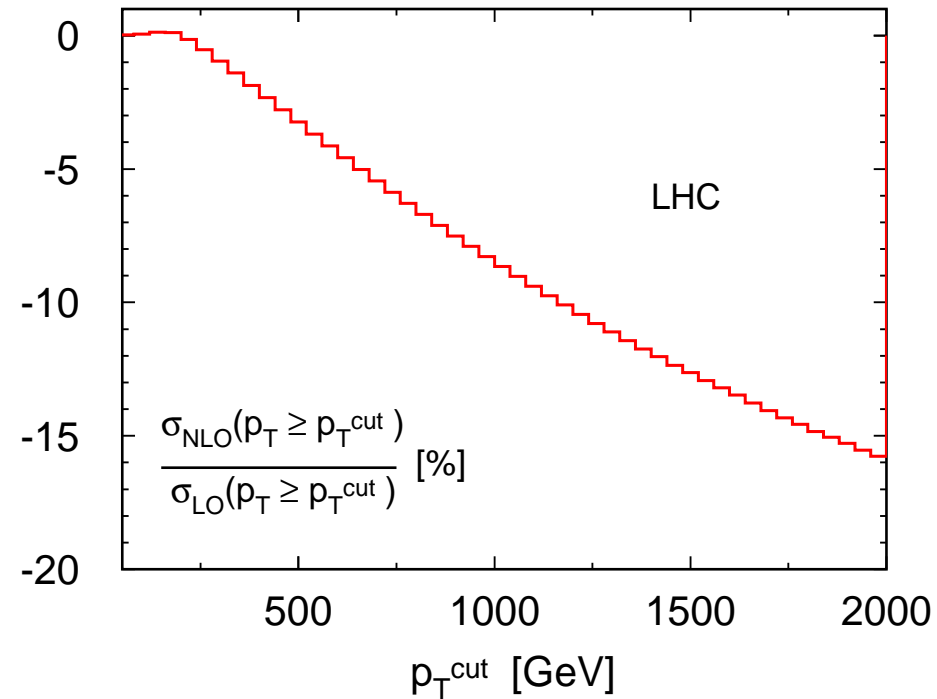
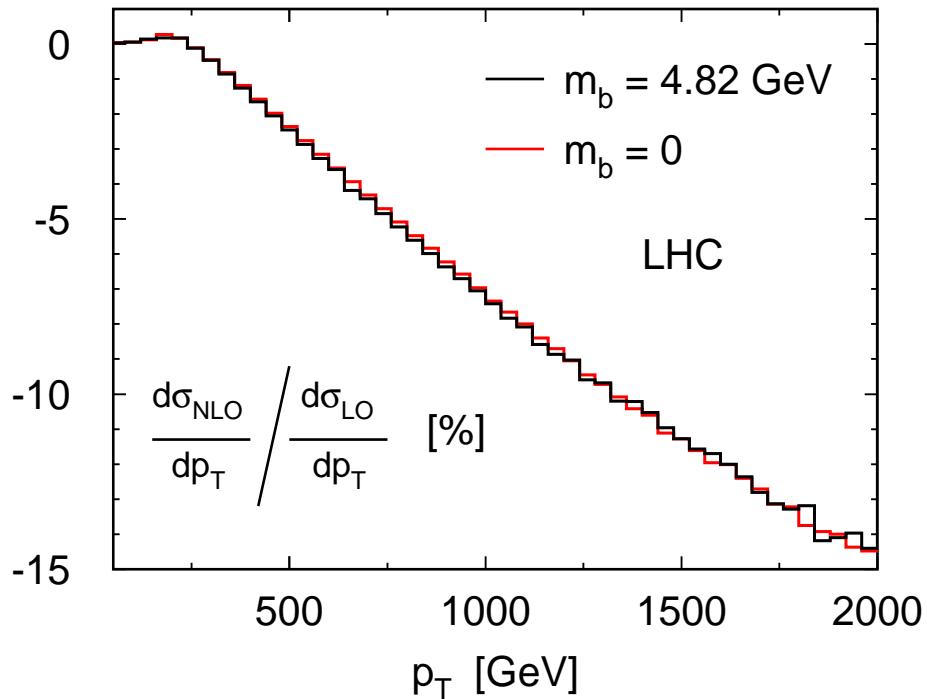
# Scale Dependence for $t\bar{t}$ production

●  $\mu = \mu_R = \mu_F$



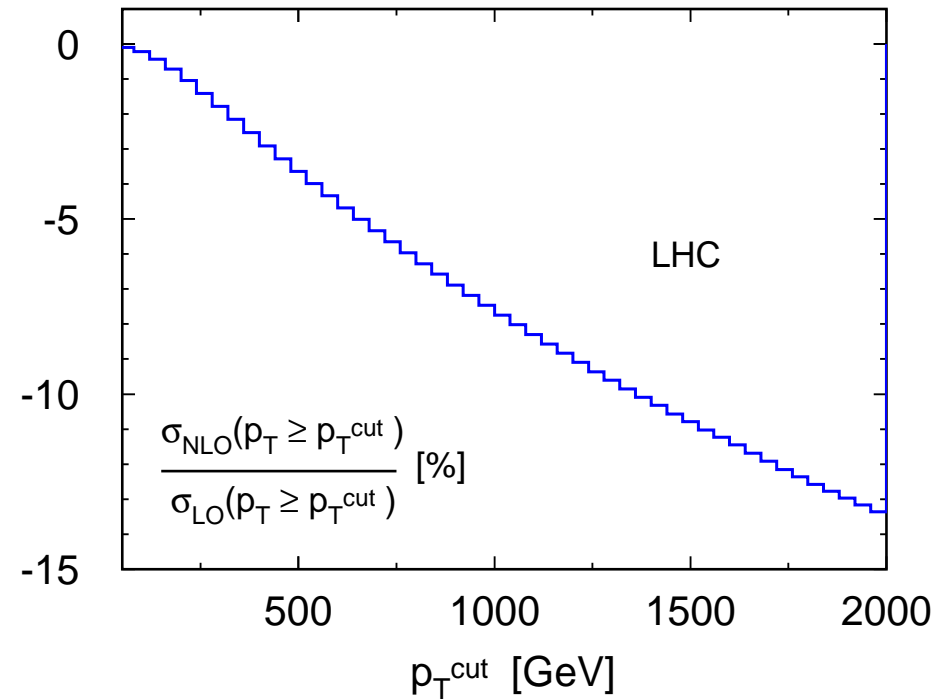
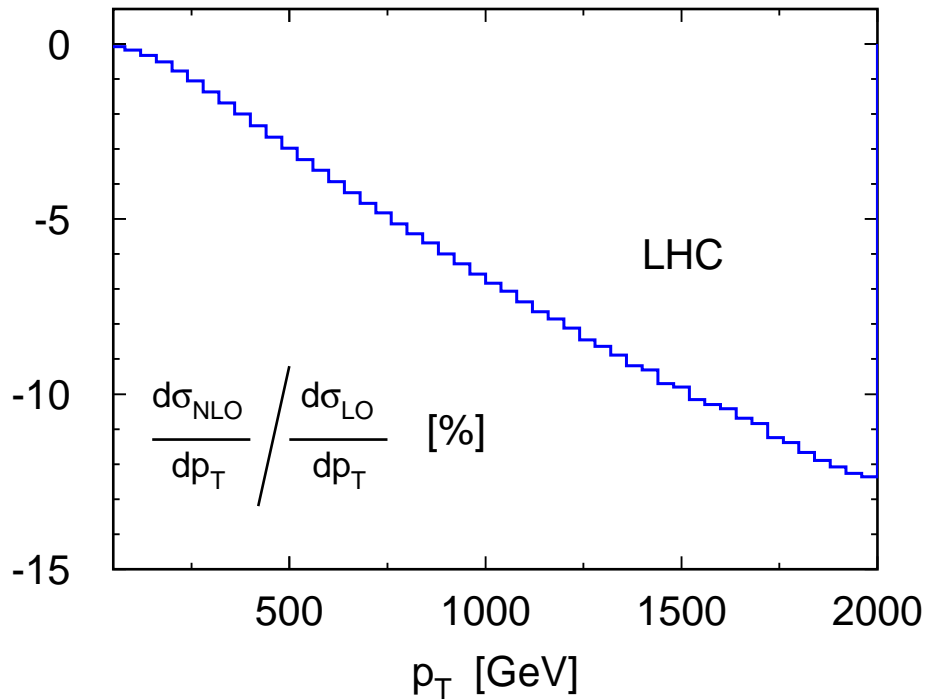
# *b*-jet production at the LHC

- Preliminary results
- Relative corrections to  $p_T$ : **double *b* tag**



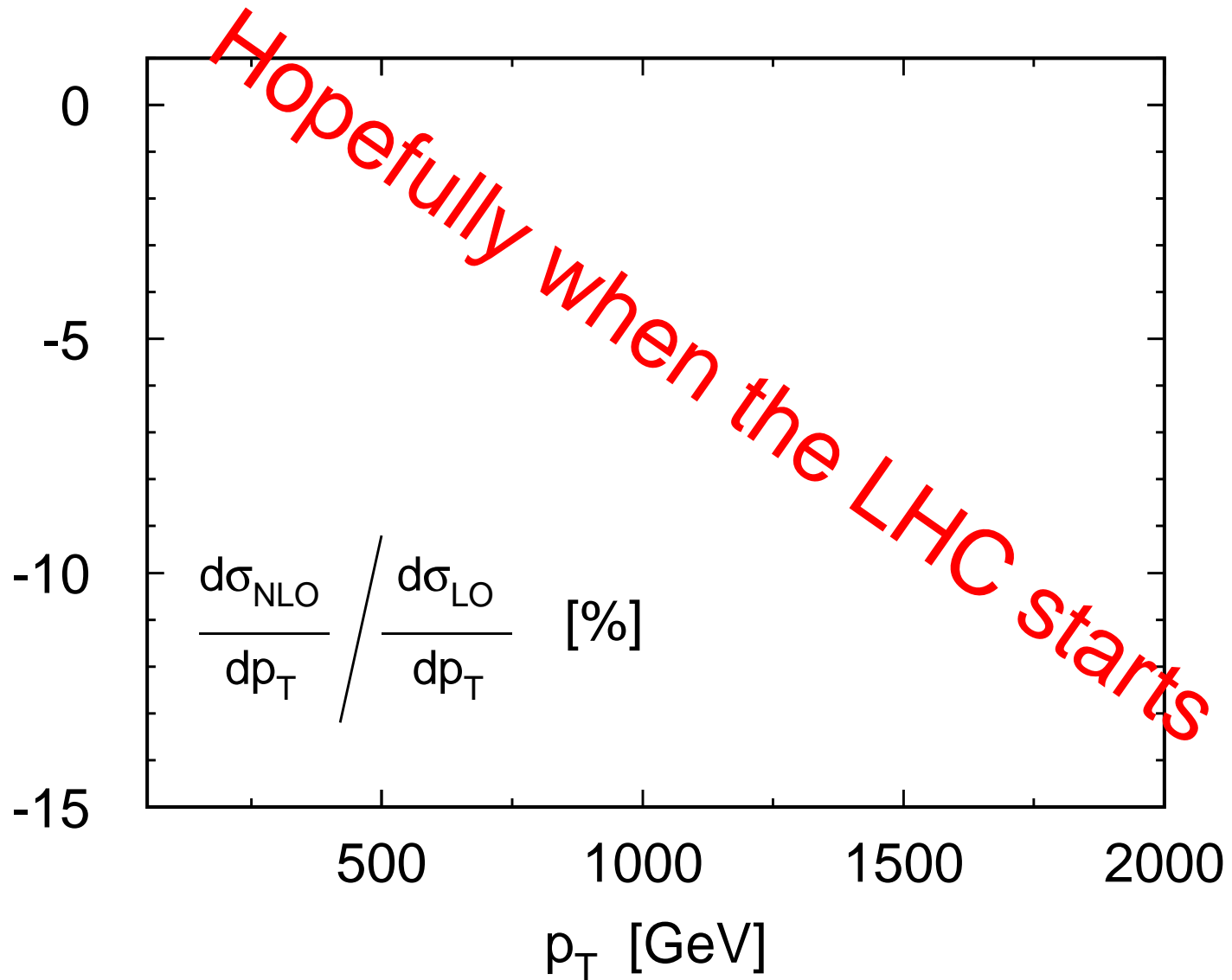
# *b*-jet production at the LHC

- Preliminary results
- Relative corrections to  $p_T$ : **single *b* tag**



# Di-jet production

- Preliminary prediction



## Discussion: General remarks

- Small corrections to total cross sections
- Important corrections for differential distributions at high energies
  - Statistics ?
  - Precision ?
- Weak boson emission ?
- Parton distribution functions
  - Error for light **quark-PDF's**  $\sim 10\%$  (2 TeV)
  - Error for **gluon-PDF's**  $\sim 50\%$  (2 TeV)

(Baur 2006)

# Discussion

- $t\bar{t}$  production

- NLO corrections to the "real" process  
( $p + p \rightarrow t\bar{t} \rightarrow b\bar{b} + l\bar{\nu}_l + jj$ ) ?
- NNLO corrections ?

- $b$ -jet production

- Bottom-quark PDF's are **not** measured up to now
- Uncertainty on the jet energy-scale

- Di-jet production

- Uncertainty on the jet energy-scale

- ... .



# Conclusion & Outlook

# Conclusion

- Heavy quark and di-jet production
  - Standard Model ?
  - New physics ?
- Weak corrections are important for differential distributions
  - Top-quark pair production: 15 – 20%
  - $b$ -jet production: 10 – 15%
  - Analytic results for further studies

# Outlook

- $b$ -jet production
  - Study further observables
- Di-jet production
  - Data taking starts already in the LHC warm up phase
  - Weak corrections should be known