NLO QCD corrections to the production of a weak boson pair with a jet

Grégory Sanguinetti - LAPTH

`sangui@lapp.in2p3.fr`

in collaboration with J.-P. Guillet, T. Binoth, S. Karg, N. Kauer

PSI Theory Seminar - January 24, 2008

Content

- Motivation
- Automatic computation of one-loop amplitudes: the GOLEM project
- Example: $pp \to VV + jet$
- Outlook

Theory: Higher order corrections in QCD perturbative theory

Experiments: Fermilab (Chicago), LHC (Cern)

What is the LHC ?

- ...

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Theory: Higher order corrections in QCD perturbative theory

Experiments: Fermilab (Chicago), LHC (Cern)

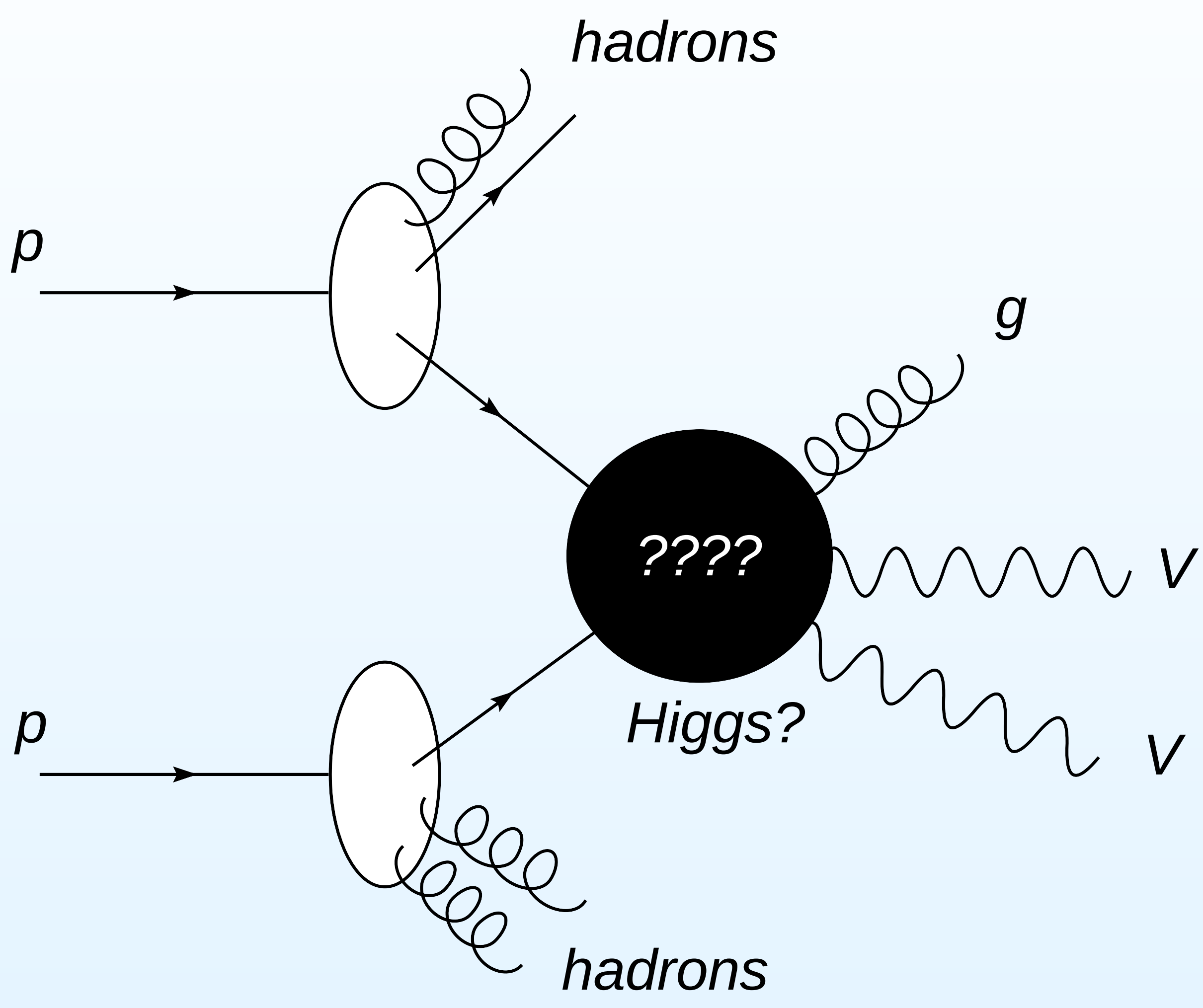
What is the LHC ?

- ...
- ... Lausanne Hockey Club ?
- ... Large Human Collider ??
- Large Hadron Collider !!!

About the LHC

- Proton-Proton collisions at $\sqrt{s} \simeq 14$ TeV
- 2 large beams of partons $\longrightarrow$ Beams of quarks, anti-quarks, gluons $\longrightarrow$ Hadrons
- QCD backgrounds have to be well known to discover new signals
- all the partonic processes evaluated at the tree level

About the LHC



Why do we need higher order corrections?

- LHC (pp) and Tevatron ($p\bar{p}$) need a precise phenomenological understanding of QCD signals and backgrounds
  $\longrightarrow$ Higgs boson, new physics searches (Supersymmetry, ...)

- Next-to-Leading Order (NLO) can be non negligible compared to the Leading Order (LO) predictions

- Standard techniques exist for partonic processes involving 4 particles (all NLO and some NNLO calculations already done)

So, what is the problem?

- For processes at NLO with more than 4 particles, an enormous growth of complexity (size, numerical instabilities)
  $\Rightarrow$ standard techniques are no longer applicable.
- But multi-particle backgrounds have to be known with high accuracy!
- Automated calculations for a numerically stable evaluation of multi-leg amplitudes are highly desirable...

Les Houches wishlist 2005

$$pp \to W^+W^- + jet \Rightarrow H + jet, t\bar{t}\,H, \text{new physics}$$

$$pp \to t\bar{t}\,b\bar{b} \Rightarrow t\bar{t}\,H$$

$$pp \to t\bar{t} + 2\,jets \Rightarrow t\bar{t}\,H$$

$$pp \to V\,V\,b\bar{b} \Rightarrow \text{vbf} \to H \to VV, t\bar{t}, \text{new physics}$$

$$pp \to V\,V + 2\,jets \Rightarrow \text{vbf} \to H \to VV$$

$$pp \to V + 3\,jets \Rightarrow t\bar{t}, \text{new physics}$$

$$pp \to V\,V\,V \Rightarrow \text{SUSY}$$

where $V \in \{Z, W^-, W^+, \gamma\}$

Motivations for $pp \to VV + jet$

- on the top of "Les Houches wishlist 2005" for important missing NLO predictions

  S. Dittmaier, S. Kallweit, P. Uwer arXiv [hep-ph]: 0710.1577v1

  J. Campbell, R.K. Ellis, G. Zanderighi arXiv [hep-ph]: 0710.1832v2

- important background for the production of $H + jet$, $t\bar{t}\,H$, and new physics
- useful for electro-weak gauge boson coupling analysis
- an important test before approaching more complicated many particle processes at NLO

For QCD & EW one-loop multi-leg processes ...

... Who you gonna call?

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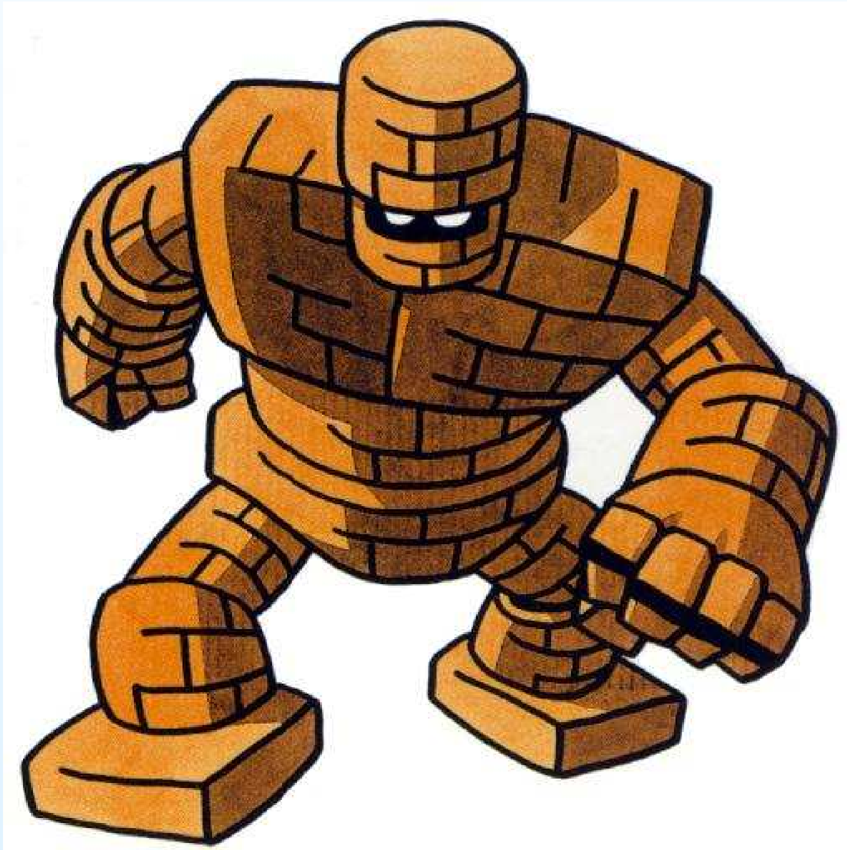


For QCD & EW one-loop multi-leg processes ...

... Who you gonna call?

G eneral

O ne

L oop

E valuator for

M atrix Elements

About GOLEM

- Contributors : T.Binoth, A.Guffanti, J.-P.Guillet, G.Heinrich, E.Pilon, C.Bernicot, T.Reiter, G.S.
- Golem paper: T.Binoth, J.-P.Guillet, G.Heinrich, E.Pilon, C.Schubert

  (JHEP 0510 (2005) 015 - arXiv: hep-ph/0504267)
- Aim : public Monte-Carlo codes for Standard Model predictions, available in particular for experimentalists
- Tools : computing, computer algebra system

  `FORM, Maple, Mathematica, Fortran 90`

Tests for GOLEM

$gg \to W^+W^-$	T.Binoth, M.Ciccolini, N.Kauer, M.Krämer JHEP 0612 (2006) 046
$gg \to \gamma\gamma g$	T.Binoth, J.-Ph.Guillet, F.Mahmoudi JHEP 0402 (2004) 057
$\gamma\gamma \to \gamma\gamma\gamma\gamma$	T.Binoth, T.Gehrmann, G.Heinrich, P.Mastrolia Phys. Lett. B 649 (2007) 422-426
$6\ quarks$	T.Binoth, T.Reiter, J.-Ph.Guillet in progress
$pp \to VV + jet$	T.Binoth, J.-Ph.Guillet, S.Karg, N.Kauer, G.S. in progress

Outline of $pp \longrightarrow VV + jet$

- final state: $g, q, \bar{q} \rightarrow$ jet (hadrons)
- $q\bar{q} \rightarrow VVg$, $\quad qg \rightarrow VV\bar{q}$, $\quad \bar{q}g \rightarrow VVq$
- $gg \rightarrow VVg$?

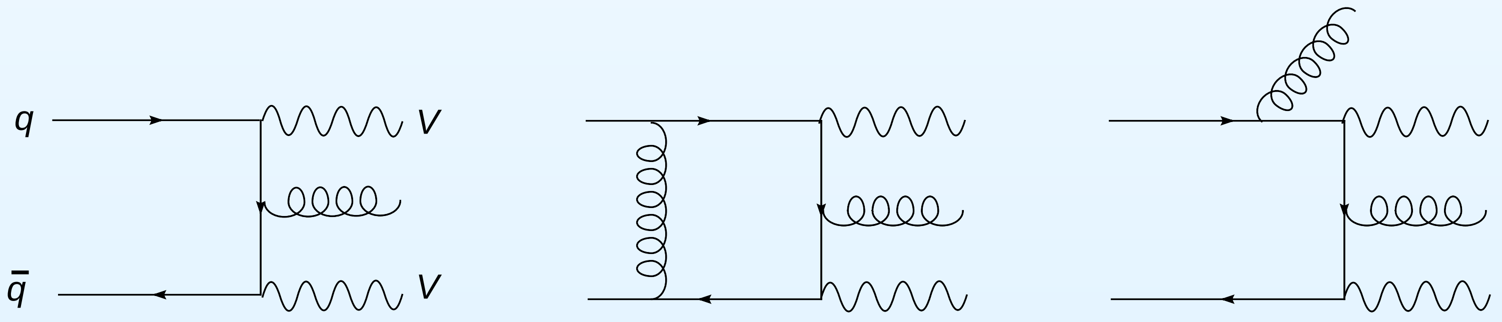
Outline of $pp \to VV + jet$

- final state: $g, q, \bar{q} \to$ jet (hadrons)
- $q\bar{q} \to VVg$, $qg \to VV\bar{q}$, $\bar{q}g \to VVq$ $\Rightarrow$ LO in α_s
- $gg \to VVg$? $\Rightarrow$ LO in ${\alpha_s}^3$ (no tree level)

Outline of $pp \to VV + jet$

- final state: $g, q, \bar{q} \to$ jet (hadrons)
- $q\bar{q} \to VVg$, $\quad qg \to VV\bar{q}$, $\quad \bar{q}g \to VVq$
- $gg \to VVg$?

- inclusive cross section: $pp \to VV + jet + X$
- 3 parts: tree level, virtual correction, real emission



Preliminaries

- $q(p_1,\lambda_1)+\bar{q}(p_2,\lambda_2)+V(p_3,\lambda_3)+\bar{V}(p_4,\lambda_4)+g(p_5,\lambda_5)\longrightarrow 0$
  Here $V\bar{V}\in\{ZZ,W^-W^+,\gamma\gamma\}$. For massless quarks
- $qg\longrightarrow VV\bar{q}$, $\bar{q}g\longrightarrow VVq \Leftarrow$ by momentum crossing
- Algebraic Feynman diagrams approach
  Semi-numerical method
- Helicity amplitudes formalism: $|\mathcal{M}^{\lambda_i}|^2\longrightarrow|\mathcal{M}|^2$
  $q\bar{q}\longrightarrow\lambda_1=\lambda_2=\pm$
  $V\longrightarrow\lambda_3,\lambda_4=\pm,0\Leftarrow M_V\neq 0$
  $g\longrightarrow\lambda_5=\pm$

  $\Rightarrow$ 36 helicities but 12 independent!
  ($\mathcal{B}$ose, $\mathcal{C}$harge, $\mathcal{P}$arity transformations)

Preliminaries

One preferably uses the spinor helicity formalism

But ${p_3}^2 = {p_4}^2 = {M_V}^2$

$\longrightarrow$ We introduce two auxiliary vectors k_3, k_4:

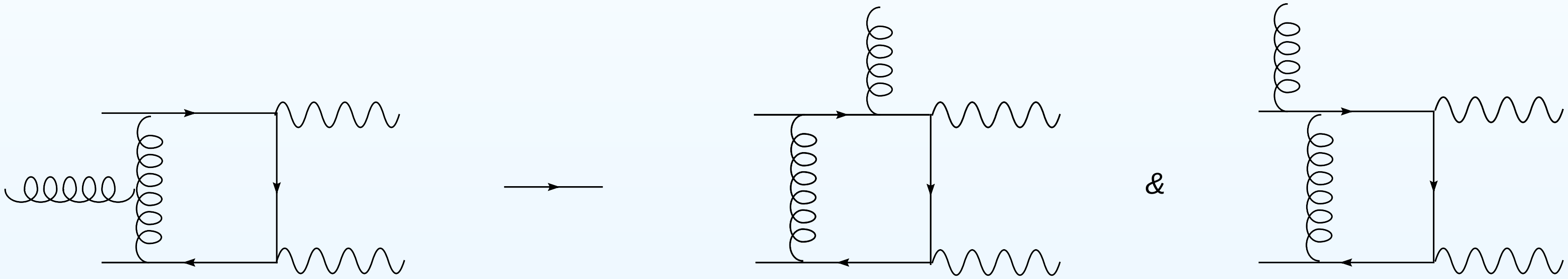
$p_3 + p_4 = k_3 + k_4, \quad {k_3}^2 = {k_4}^2 = 0$

$$\begin{aligned} k_3 &= \frac{1}{2\beta}\left[(1+\beta)\,p_3 - (1-\beta)\,p_4\right] \\ k_4 &= \frac{1}{2\beta}\left[(1+\beta)\,p_4 - (1-\beta)\,p_3\right] \end{aligned}$$

where $\beta = \sqrt{1 - \frac{4M_V^2}{s_{34}}}$

Preliminaries

- color factor decomposition $\rightarrow$ gauge independent set
  $\rightarrow$ large cancellation between Feynman diagrams

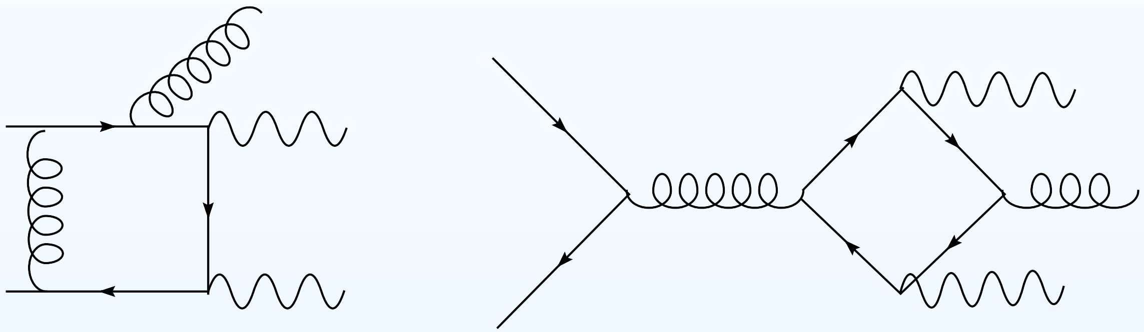


- Cross section: Virtual corrections & Real emission

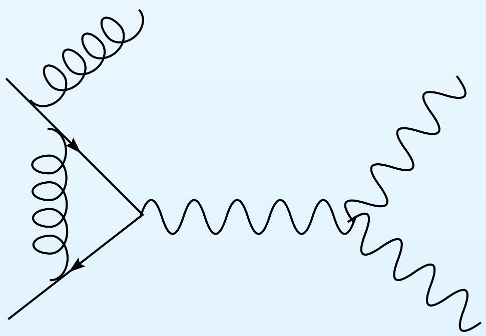
$$\sigma_{2\rightarrow n} = \sigma^{LO} + \sigma^{NLO}$$
$$= \sigma^{LO} + \int_{n+1,\text{1 or 2 jets}} d\sigma^{R} + \int_{n,\text{1 jet}} d\sigma^{V}$$

Differences between ZZ and W^+W^-

- ZZ case: 80 diagrams $\leftarrow$ Z bosons crossing



- W^+W^- case: 51 diagrams $\leftarrow$ additional diagrams with WWV vertex



Difficulties

- NLO calculation with 5 legs, 2 external masses
  10 pentagons, 22 boxes for $ZZ + jet$

  $\Rightarrow$ huge complexity / number of terms:
  `FORM` output around 100 Mbytes for a 5-points diagram!

- 36 helicities

- Gram determinants (cancellation?) $\longrightarrow$ numerical instabilities?

- Treatement of γ_5
  $\longrightarrow$ kept in 4 dim. with adequate (anti) commutation relations:

$$\{\hat{\gamma}^\mu, \gamma^5\} = 0, \quad [\tilde{\gamma}^\mu, \gamma^5] = 0$$

Divergences

- 2 types of divergences: UV & IR (soft and collinear singularities) divergences
- explicitely separated and extracted
- UV div. cancelled by QCD lagrangian renormalization
- IR div. cancelled by combining virtual and real amplitudes

How to check the code?

- UV & IR divergences cancellation
  (not difficult but not obvious)

- Gauge invariance for an external gauge boson:

$$
\begin{aligned}
\varepsilon_5^{\mu_5} &\rightarrow \varepsilon_5^{'\mu_5} = \varepsilon_5^{\mu_5} + \lambda\, p_5{}^{\mu_5} \\
\mathcal{M}(\varepsilon_5) &= \mathcal{M}'(\varepsilon_5')
\end{aligned}
$$

- Comparison of independent codes

Gram determinants

- spurious instability $\longrightarrow$ disappear for physical quantities
- Dilemma: slow numerical evaluation or reduction with instabilities?

$$I_5^{n,\mu_1,\mu_2,\mu_3,\mu_4}(a_1,a_2,a_3,a_4;S) = \int d\tilde{k}\, \frac{q_{a_1}^{\mu_1} \ldots q_{a_5}^{\mu_5}}{\prod\limits_{i=1,\ldots,5} (q_i^2 - m_i^2 + i\delta)}$$

$$\longrightarrow I_4^n(S), I_3^n(S), I_2^n(S)???$$

- Faith in "compact" analytical result...

- Origin: from the reduction

$$I_4^{n+2}(l;S) = \left(b_l\, I_4^{n+2}(S) + \frac{1}{2}\sum_{j\in S} S_{jl}^{-1}\, I_3^n(S\backslash\{j\})\right.$$
$$\left. - \frac{1}{2}\sum_{j\in S\backslash\{l\}} b_j\, S_{jl}^{-1}\, I_3^n(l, S\backslash\{j\})\right)$$

det G : polynomial in $s_{ij} = (p_i + p_j)^2$
$\rightarrow 0$ in the phase space?

- depend on a good choice of the function basis

- Origin: from the reduction

$$I_4^{n+2}(l;S) = -\frac{\det S}{\det G}\Big(b_l\, I_4^{n+2}(S) + \frac{1}{2}\sum_{j\in S} S^{-1}_{jl}\, I_3^n(S\backslash\{j\}) - \frac{1}{2}\sum_{j\in S\backslash\{l\}} b_j\, S^{-1}_{jl}\, I_3^n(l, S\backslash\{j\})\Big)$$

det G : polynomial in $s_{ij} = (p_i + p_j)^2$
$\rightarrow 0$ in the phase space?

- depend on a good choice of the function basis

Computation

- Sort amplitudes by helicity and colour properties
- Generate the Feynman diagrams analytical expressions (`FeynArts, QGRAPH`)
- Write a `FORM` program to develop the Feynman diagrams expressions
- Algebraic tensor reduction in `FORM`
  $I_5^{n,\mu_1,\mu_2,\mu_3,\mu_4} \longrightarrow I_5^{n,\mu_1}, I_4^{n,\mu\ldots}, I_3^{n,\mu\ldots}, I_2^{n,\mu..}$
- Sort into basis set of 2, 3, and 4 points scalar integrals using the `GOLEM` library

Computation

- using MAPLE
  $\rightarrow$ to reduce the size and cancel the inverse Gram determinants

  PROBLEM: expressions with $\neq$ denominators:

$$\frac{1}{(\det S)^n}, \quad \frac{1}{(\det G_{\{3,4\}})^m}, \quad \frac{1}{(\det S)^n (\det G)^m}$$

  in a common denominator, with a brutal factorization?
  13 Gb RAM memory not enough...

- using FORTRAN 90
  $\rightarrow$ numerical evaluation and phase space integration

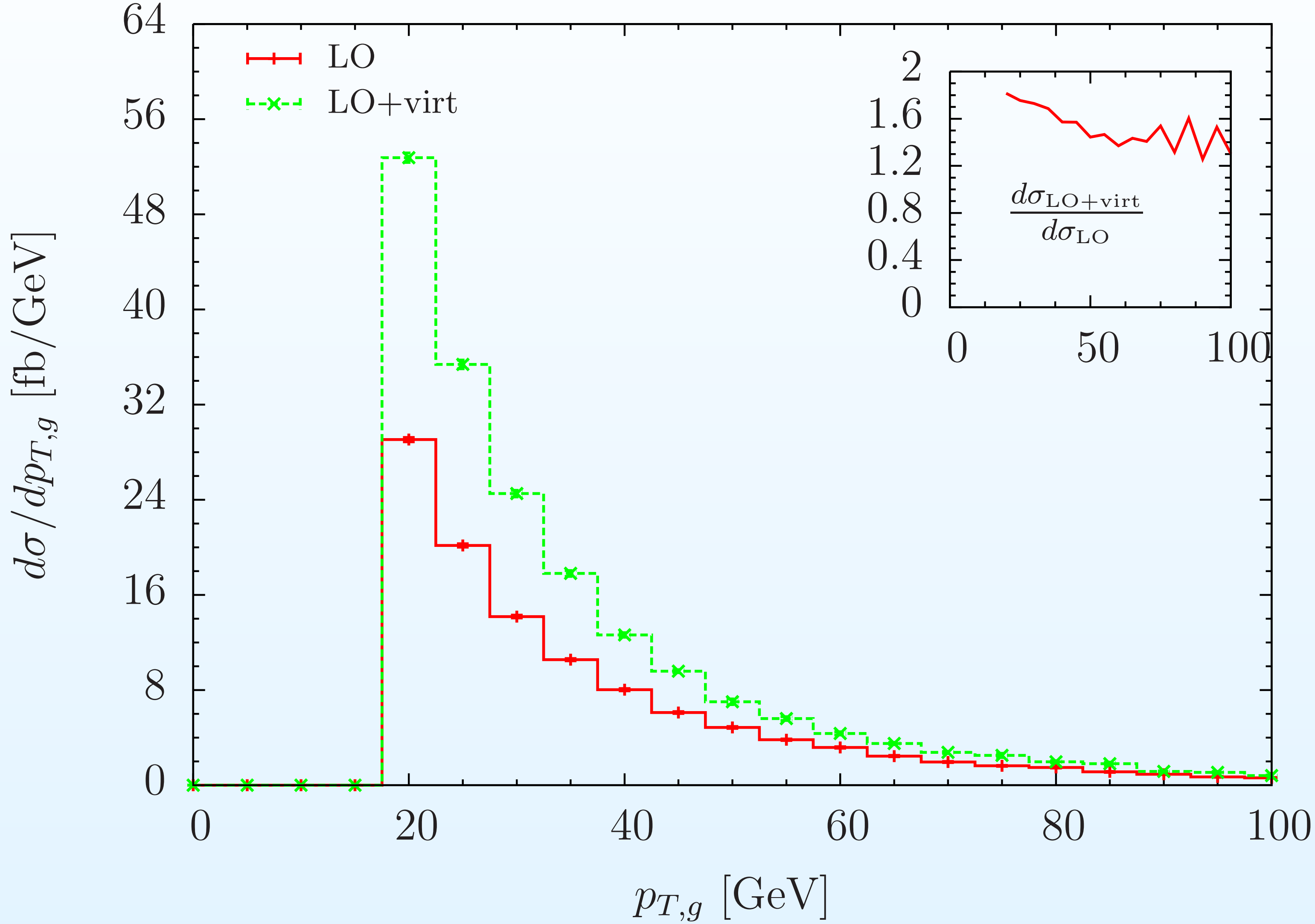
Numerical check

- $ZZ + jet$ and $W^+W^- + jet$ virtual corrections performed
- Comparison of 2 independent codes with S. Karg for $ZZ + jet$ and $W^+W^- + jet$
- Successful check of the virtual amplitudes

About the $ZZ + jet$ case:

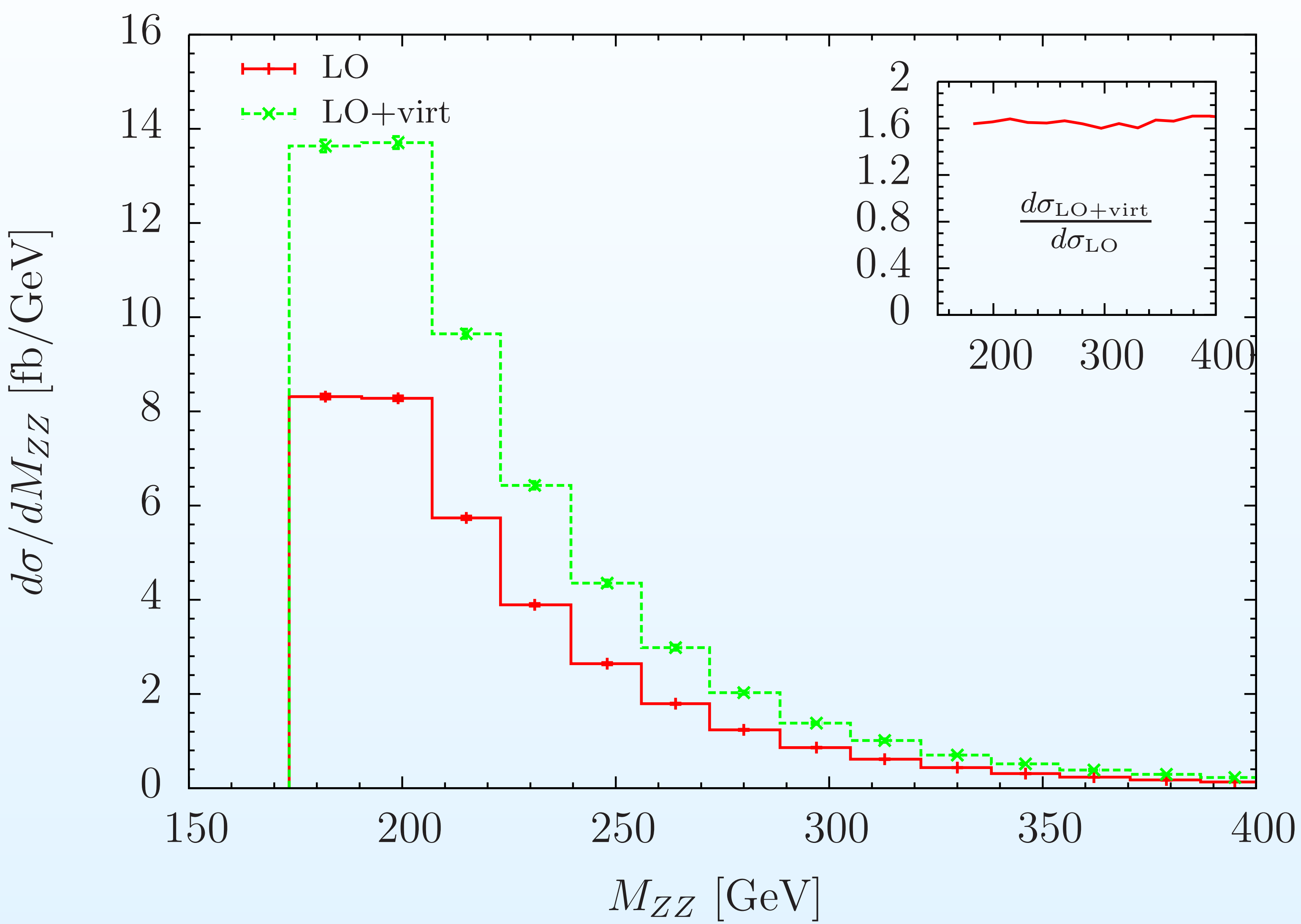
- Size files : $\simeq$ 100 Mbytes
- Runtime for 2000 phase space points : $\simeq$ 900 sec

Virtual cross section: $ZZ + jet$



from Stefan Karg's thesis

Virtual cross section: $ZZ + jet$



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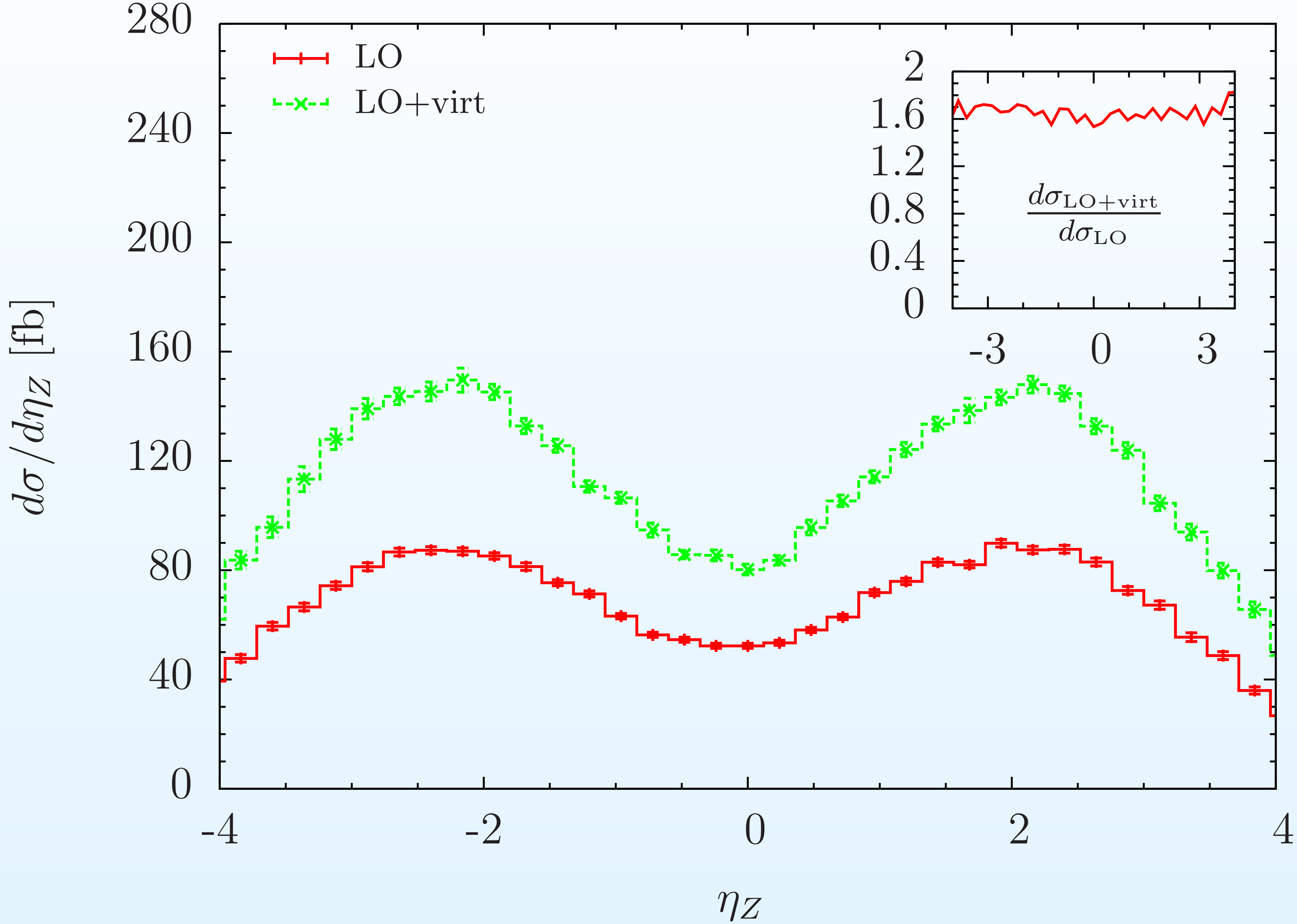
Virtual cross section: $ZZ + jet$



from Stefan Karg's thesis

Work in progress

- Comparison of numerical results for $W^+W^- + jet$
  with S. Dittmaier, S. Kallweit, P. Uwer
  and J. Campbell, R.K. Ellis, G. Zanderighi
  (Full agreement for LO and NLO virtual amplitude)
- improvement of the virtual amplitude (size)
- Calculation of the real emission with the Catani-Seymour dipole substraction

Thank You!