

Searching for new physics with a non decoupling effective theory

Theory group LTP, PSI
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Outline

1. The Low Energy Effective Theory beyond the Standard Model.
2. How to probe the electroweak couplings of quarks ?
3. $K_{\mu 3}^L$ decay : a stringent test of right-handed quark currents.
4. Tests of the electroweak couplings to Z boson.
5. Conclusion and outlook.

1. The Low Energy Effective Theory beyond the Standard Model

1.1 The Standard Model

- Theory of strong and electroweak interactions.

- The elementary particles:

- The quarks

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} b_L \\ t_L \end{pmatrix}$$

$$u_R, d_R, c_R, s_R, b_R, t_R$$

- The leptons

$$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \rightarrow \chi_L$$

$$e_R, \mu_R, \tau_R \rightarrow \chi_R$$

- The gauge bosons or Yang-Mills fields: W^\pm, Z^0, A

- Gauge group: $SU(2)_W \otimes U(1)_Y$

- Mass generation: Higgs Mechanism. A scalar particle remains in the spectrum H

- Theory renormalizable.

- A decade of experimental success:
 - Prediction of the quark top before its discovery.
 - CP violation measured in the B and K decays (CKM matrix).
- But hints of new physics (SM: theory valid up to a certain energy scale):
 - Not enough CP violation to explain the matter/antimatter asymmetry.
 - Neutrino oscillations: $m_\nu \neq 0$
 - m_ν very small compared to the other lepton masses, new scale ?
 - Mass origin ? (Standard Higgs or other ?)
 - Dark matter, dark energy.
- Extensions/Alternatives:
 - Supersymmetry, extra dimensions.
 - Higgsless theories (naturalness, non observed particle...).

1.2 Effective approach (framework) beyond the Standard Model

- The Standard Model, effective theory: Theory valid up to an energy scale Λ :
 - all observed particles
 - gauge group $SU(2)_W \otimes U(1)_Y$
 - renormalizability } **Standard Model**
- At higher energy ($\Lambda > \sim 1\text{TeV}$), : New symmetries and particles.
 - heavy states ($M > \Lambda$) beyond the Standard Model.
 - Larger symmetry group beyond the SM one : $S \supset SU(2)_W \otimes U(1)_Y$

Usual decoupling scenario

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_{D>4} \frac{1}{\Lambda^{D-4}} \mathcal{O}_D$$

with D: mass dimension.

SM degrees of freedom

New physics

- If Λ large enough, the new symmetries and particles are irrelevant at low energy $E \ll \Lambda$.
- At NLO, 80 independent invariant operators of D=6 dimension.
- Research of New Physics quasi impossible !

Construction of an effective theory a la ChPT.

- Build **all** the operators invariant under $SU(2)_W \otimes U(1)_Y$.
- Theory non renormalizable in the usual sense.
- Expansion in external momenta, coupling constants...

$$\mathcal{L}_{eff} = \sum_{d \geq 2} \mathcal{L}_d, \quad d = n_p + n_g + \frac{n_\chi}{2}$$

- Validity : $\mathbf{p} \ll \Lambda \sim 3 \text{ TeV}$.
- Renormalizable and unitarized order by order in the expansion.

Difficulties of an electroweak effective theory based on $SU(2)_W \otimes U(1)_Y$ symmetry

- At Leading Order operators $SU(2)_W \otimes U(1)_Y$ invariant .
 Unwanted operators.

- Example: right-handed currents.

→ Left-handed sector :

$$i \overline{\chi}_L^i \gamma^\mu (\Sigma D_\mu \Sigma^\dagger) \chi_L^j = \mathcal{O}(p^2)$$

→ Right-handed sector :

$$i \overline{\chi}_R^i \gamma^\mu (\Sigma^\dagger D_\mu \Sigma) \chi_R^j = \mathcal{O}(p^2)$$

$\Sigma \equiv e^{i\vec{\pi} \cdot \vec{\tau} / f_W}$: Goldstone bosons, χ_L, χ_R : fermions

- Suppressed in the SM case : operators of dimension D=6.

$$\frac{1}{\Lambda^2} \overline{\chi}_L^i \gamma^\mu \Phi (D_\mu \Phi^\dagger) \chi_L^j$$

$$\frac{1}{\Lambda^2} \overline{\chi}_R^i \gamma^\mu \Phi^\dagger (D_\mu \Phi) \chi_R^j$$

Couplings experimentally excluded at Leading Order !

- Difficulty: Leading Order ($\mathcal{O}(p^2)$) operators non present in the SM and experimentally excluded.
 - Solution: enlarge the symmetry $\mathcal{S}_{nat} \supset SU(2)_W \otimes U(1)_Y$ which becomes non-linearly realised:
 - not manifest in the low energy spectrum (the gauge fields associated to this symmetry do not all appear in the spectrum).
 - but constrains the effective interaction (cf. Custodial Symmetry).
- ➔ The minimal low-energy effective theory (LEET), a not quite decoupling theory:
- Heavy states present at $E > \Lambda$ still decouple at low energy.
 - BUT higher (local) symmetry $\mathcal{S}_{nat} \supset SU(2)_W \otimes U(1)_Y$ survives at low energy and becomes non linearly realised.

1.3 Minimal Higgs-less LEET

[J.Hirn & J.Stern, Phys.Rev.D73 '06]

- What is \mathcal{S}_{nat} ?


→ \mathcal{S}_{nat} can be inferred from \mathcal{L}_{SM} asking :

- At $\mathcal{O}(p^2)$, \mathcal{S}_{nat} selects all the (Higgs-less) vertices of the SM and nothing else.
- Minimality.

→
$$\mathcal{S}_{nat} = [SU(2) \otimes SU(2)]^2 \otimes U(1)_{B-L} \otimes Z_{2\nu_R}$$

- $Z_{2\nu_R} : \nu_R \rightarrow -\nu_R$: forbids Dirac neutrino masses and charged leptonic right-handed currents.

More concretely,

- At leading order, one unwanted operator: $\left[\text{Tr} \left(\tau_3 \Sigma^\dagger D_\mu \Sigma \right) \right]^2 = \mathcal{O} \left(p^2 \right)$
operator that can potentially induce a large modification of the SM gauge boson mixing: contribution to the « oblique parameter T ».
- Operator invariant under $SU(2)_W \otimes U(1)_Y$
 $\Sigma(\mathbf{x}) \rightarrow G_L(\mathbf{x}) \Sigma(\mathbf{x}) G_R^{-1}(\mathbf{x}), \quad G_L \in SU(2)_W, \quad G_R \in U(1)_Y.$
with $G_R(\mathbf{x}) \tau_3 G_R^{-1}(\mathbf{x}) = \tau_3$
- Enlarge the symmetry: $SU(2)_W \otimes U(1)_Y \rightarrow S_{elem} = SU(2)_{G_L} \otimes SU(2)_{G_R} \otimes U(1)_{G_B}^{B-L}$
 $\left[\text{Tr} \left(\tau_3 \Sigma^\dagger D_\mu \Sigma \right) \right]^2$ no more invariant under S_{elem} .
 $G_R(\mathbf{x}) \tau_3 G_R^{-1}(\mathbf{x}) \neq \tau_3$

Reduction of $S_{nat} \rightarrow SU(2)_W \otimes U(1)_Y$:
via the spurions.

$$\begin{array}{c}
 \underbrace{SU(2)_{G_L} \times SU(2)_{\Gamma_L}} \times \underbrace{SU(2)_{G_R} \times SU(2)_{\Gamma_R}} \times U(1)_{G_B}^{B-L} \\
 \begin{array}{ccc}
 X \rightarrow G_L X \Gamma_L^{-1} & X & Y_{u,d} \rightarrow G_R Y_{u,d} \Gamma_R^{-1} \\
 \downarrow & & \downarrow \\
 SU(2)_W & \times & \underbrace{SU(2)_{\Gamma_R} \times U(1)_{B-L}} \\
 \Gamma_{L\mu} = X g_L G_{L\mu} X^{-1} + i X \partial_\mu X^{-1} & & Z \rightarrow e^{i\alpha} \Gamma_R Z \Gamma_R^{-1} \\
 \downarrow & & \downarrow \\
 SU(2)_W & \times & U(1)_Y
 \end{array}
 \end{array}$$

- $S_{nat} / SU(2)_W \otimes U(1)_Y \equiv [SU(2)]^3$ populated by spurions
- X, Y, Z are called spurions ($D_\mu(\text{spurions}) = 0$).
- It exists a gauge where the spurions become constants \mathbf{K} : \Rightarrow 3 parameters which describe the hierarchical breaking of S_{nat} (\sim quark masses in ChPT).

Power counting scheme or how searching for physics beyond the SM

- Terms explicitly involving spurions \Rightarrow fermion masses and genuine effects beyond the SM.
- The operators are ordered in power of momenta and of explicit symmetry breaking parameters (spurions: \mathbf{K}) :

$$\mathcal{L}_{eff} = \underbrace{\mathcal{L}(p^2 \kappa^0) + \mathcal{L}(p^1 \kappa^2)}_{\text{LO : } \mathcal{O}(p^2)} + \underbrace{\mathcal{L}(p^2 \kappa^2)}_{\text{NLO : } \mathcal{O}(p^3)} + \underbrace{\mathcal{L}(p^4)}_{\text{NNLO : } \mathcal{O}(p^4)} + \dots$$

Loops, oblique corrections

- Counting rule: $\frac{m_t}{\Lambda} = \kappa^2 = \mathcal{O}(p)$

1.4 First effects beyond the SM

- At Leading Order (LO), $\mathcal{O}(p^2)$: we recover the Standard Model at tree level without a Higgs.
- At Next to Leading Order (NLO) $\mathcal{O}(p^2\kappa^2)$ new physics (contains spurions) arising before loops, oblique corrections..... only 2 operators :

→ Left-handed sector :

$$i\overline{\chi}_L^i \gamma^\mu (\Sigma D_\mu \Sigma^\dagger) \chi_L^j = \mathcal{O}(p^2)$$

→ Right-handed sector :

$$i\overline{\chi}_R^i \gamma^\mu (\Sigma^\dagger D_\mu \Sigma) \chi_R^j = \mathcal{O}(p^2)$$

$\Sigma \equiv e^{i\vec{\pi} \cdot \vec{\tau} / f_W}$: Goldstone bosons, χ_L, χ_R : fermions

Operators invariant under $SU(2)_W \otimes U(1)_Y$ but not invariant under S_{nat}

To make them invariant under S_{nat} , we insert spurions: 


spurions

$$i\overline{\chi}_L^i \gamma^\mu \mathcal{X}^\dagger (\Sigma D_\mu \Sigma^\dagger) \mathcal{X} \chi_L^j = \mathcal{O}(p^2\kappa^2)$$

spurions

$$i\overline{\chi}_R^i \gamma^\mu \mathcal{Y}_a^\dagger (\Sigma^\dagger D_\mu \Sigma) \mathcal{Y}_b \chi_R^j = \mathcal{O}(p^2\kappa^2)$$

1.4 First effects beyond the SM

- At leading order (LO) $\mathcal{O}(p^2)$, we recover the Standard Model at tree level without a Higgs.
- At Next to the Leading Order (NLO) $\mathcal{O}(p^2\kappa^2)$ before the loops only 2 operators instead of the 80 that appear in the decoupling scenario !
  new couplings of fermions to W and Z.

$$\mathcal{L}_{\mathcal{NLO}} \equiv \mathcal{L}_{cc} + \mathcal{L}_{NC}$$





W couplings

Z couplings

- Counting rule: NLO, potentially the most important effects of physics beyond the SM.

1.5 Conclusion

- New approach to go beyond the Standard Model : the minimal low-energy effective theory.
- Larger symmetry non linearly realised.
- Power counting scheme  classifies and hierarchizes the effects beyond the Standard Model.
- First predicted effects  modification and new electroweak couplings of fermions (quarks and leptons).

Couplings of the fermions to W : charged currents

$$\mathcal{L}_{cc} = \frac{1}{\sqrt{2}} g (1 - \xi^2 \rho_L) (J_\mu^{\bar{U}D} + J_\mu^{\bar{N}L}) W^\mu + h.c., \quad \mathbf{U} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \mathbf{D} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \mathbf{N} = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \mathbf{L} = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}$$



$$J_\mu^{\bar{U}D} = (1 + \delta) \overline{U}_L V_L \gamma_\mu D_L + \varepsilon \overline{U}_R V_R \gamma_\mu D_R$$

$$J_\mu^{\bar{N}L} = \overline{N}_L V_{MNS} \gamma_\mu L_L$$

- $\mathbf{V}_L, \mathbf{V}_R$: 2 unitary flavor mixing matrixes from the diagonalisation of the mass matrix of U and D quarks.
- 3 parameters which involve spurions arise :

$$(1 - \xi^2 \rho_L)$$

$$(1 + \delta)$$

$$\varepsilon$$

Charged Currents parameters at NLO

$(1 - \xi^2 \rho_L)$ Common factor absorbed in the G_F definition.

$(1 + \delta)$ Normalisation of the left-handed quarks with respect to the leptons.

ε Strength of right-handed quark currents.

Experimental probe

- Experimentally, we measure axial and vectorial currents.
- We write the charged current in term of axial and vectorial currents:

$$J_{\mu}^{\bar{U}D} = \frac{1}{2} \left[\bar{U} \mathcal{V}_{eff}^{\mu} D + \bar{U} \mathcal{A}_{eff}^{\mu} \gamma_5 D \right]$$

$$\mathcal{V}_{eff}^{ij} = (1 + \delta) V_L^{ij} + \varepsilon V_R^{ij} + \text{NNLO} \quad \text{and} \quad \mathcal{A}_{eff}^{ij} = - (1 + \delta) V_L^{ij} + \varepsilon V_R^{ij} + \text{NNLO}$$

- V_L and V_R unitary but not \mathcal{V}_{eff} and \mathcal{A}_{eff} !
- At LO (SM) : $\delta = \varepsilon = 0$, $\mathcal{V}_{eff} = -\mathcal{A}_{eff} = V_{CKM}$ (V-A interaction).
- At NLO: $\mathcal{V}_{eff} \neq -\mathcal{A}_{eff}$ charged right-handed currents appear.

Experimental probe

- Experimentally, we measure axial and vectorial currents.
- We write the charged current in term of axial and vectorial currents:

$$\mathbf{J}_\mu^{\bar{U}D} = \frac{1}{2} \left[\bar{U} \mathcal{V}_{eff} \gamma_\mu \mathbf{D} + \bar{U} \mathcal{A}_{eff} \gamma_\mu \gamma_5 \mathbf{D} \right]$$

$$\mathcal{V}_{eff}^{ij} = (1 + \delta) V_L^{ij} + \varepsilon V_R^{ij} + \text{NNLO} \quad \text{and} \quad \mathcal{A}_{eff}^{ij} = -(1 + \delta) V_L^{ij} + \varepsilon V_R^{ij} + \text{NNLO}$$

$$|\mathcal{V}_{eff}^{ij}|^2 = |V_L^{ij}|^2 \left\{ 1 + 2\delta + 2\varepsilon \operatorname{Re} \left(\frac{V_R^{ij}}{V_L^{ij}} \right) \right\} \quad |A_{eff}^{ij}|^2 = |V_L^{ij}|^2 \left\{ 1 + 2 \cdot \delta - 2\varepsilon \cdot \operatorname{Re} \left(\frac{V_R^{ij}}{V_L^{ij}} \right) \right\}$$

- In the light quark sector, 3 quantities to determine:

δ

$$\varepsilon_{NS} = \varepsilon \cdot \operatorname{Re} \left(\frac{V_R^{ud}}{V_L^{ud}} \right)$$

$$\varepsilon_S = \varepsilon \cdot \operatorname{Re} \left(\frac{V_R^{us}}{V_L^{us}} \right)$$

Order of magnitude of the parameters

- $\delta, \varepsilon \leq 1\%$ (quark top mass)

$$\varepsilon_{NS} = \varepsilon \operatorname{Re} \left(\frac{V_R^{ud}}{V_L^{ud}} \right)$$

$$\varepsilon_S = \varepsilon \operatorname{Re} \left(\frac{V_R^{us}}{V_L^{us}} \right)$$



- V_L close to V_{CKM} \Rightarrow experimental measurements $\left\{ \begin{array}{l} |V_L^{ud}| \sim 0.97 \\ |V_L^{us}| \sim 0.23 \end{array} \right.$
- V_R unitary $\Rightarrow \left\{ \begin{array}{l} |V_R^{ud}| \leq 1 \\ |V_R^{us}| \leq 1 \end{array} \right.$

$$\Rightarrow |\varepsilon_{NS}| \leq \varepsilon \sim 1\% \quad \text{and} \quad |\varepsilon_S| \leq 4.5 \varepsilon$$

- Possible enhancement of ε_S
- Expected effects of at most several percents.

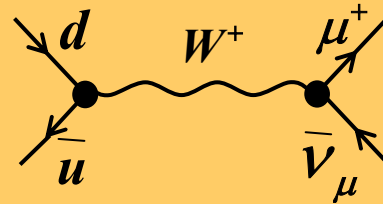
2. How to probe the electroweak couplings of quarks ?

2.1 Introduction

- Experiments of parity violation 50 years ago  adoption of the assumption that the weak interaction is V-A like.
- Have we probed that the weak interaction is V-A ?
- Several tests for the leptons (Muon, Tau polarisation measurements...)
- And for the quarks ?  Couplings difficult to test: we can not access directly to quarks because of the confinement !
- Experimentally, the measurements of the electroweak couplings $(\mathcal{V}_{eff}, \mathcal{A}_{eff})$ depend on the knowledge of the QCD observables and vice versa !

2.2 Dependence of the electroweak couplings and the low-energy QCD observables

- Example: Decay of the pion:



$$\Gamma \left[\pi^+ \rightarrow \mu^+ \nu (\gamma) \right] \rightarrow \mathcal{A}_{eff}^{ud} \langle 0 | \bar{u} \gamma_\mu \gamma_5 d | \pi \rangle \sim |F_\pi \mathcal{A}_{eff}^{ud}|$$

⇒ We do not measure F_π but a combination of F_π and \mathcal{A}_{eff}^{ud} .

- In the case of the electroweak couplings of the SM : $(\mathcal{V}_{eff}^{ud} = -\mathcal{A}_{eff}^{ud} = V_{CKM}^{ud})$

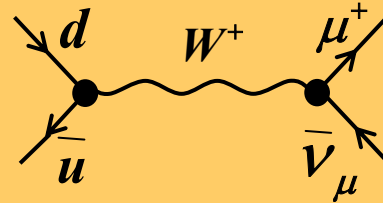
$\mathcal{A}_{eff}^{ud} \rightarrow V_{CKM}^{ud}$ very precisely measured from $(0^+ \rightarrow 0^+)$ superallowed β decays: $\mathcal{V}_{eff}^{ud} = 0.97418(26)$ [Towner and Hardy'07].

⇒ $\widehat{F}_\pi = (92.4 \pm 0.2) \text{ MeV}$

$$F_\pi = \widehat{F}_\pi (1 + 2\varepsilon_{NS})$$

2.2 Dependence of the electroweak couplings and the low-energy QCD observables

- Example: Decay of the pion:



$$\Gamma [\pi^+ \rightarrow \mu^+ \nu (\gamma)] \rightarrow \mathcal{A}_{eff}^{ud} \langle 0 | \bar{u} \gamma_\mu \gamma_5 d | \pi \rangle \sim |F_\pi \mathcal{A}_{eff}^{ud}|$$

➔ We do not measure F_π but a combination of F_π and \mathcal{A}_{eff}^{ud} .

- To be more precise

$$F_\pi = \underbrace{F_\pi |\mathcal{A}_{eff}^{ud}|}_{\text{Exp.}} \underbrace{\left| \frac{1}{\mathcal{V}_{eff}^{ud}} \right|}_{(0^+ \rightarrow 0^+)} \underbrace{\left| \frac{\mathcal{V}_{eff}^{ud}}{\mathcal{A}_{eff}^{ud}} \right|}_{1 + 2\varepsilon_{NS}}$$



$$F_\pi = \hat{F}_\pi (1 + 2\varepsilon_{NS})$$

unknown in the PDG

F_π extracted in the SM ($\mathcal{V}_{eff}^{ud} = -\mathcal{A}_{eff}^{ud} = V_{CKM}^{ud}$) ➔

$$\hat{F}_\pi = (92.4 \pm 0.2) \text{ MeV}$$

2.2 Dependence of the electroweak couplings and the low-energy QCD observables

- But also β -decay of the neutron :

$$\begin{aligned} \langle p | \mathcal{V}_{eff}^{ud} \bar{u} \gamma_\mu d + \mathcal{A}_{eff}^{ud} \bar{u} \gamma_\mu \gamma_5 d | n \rangle \sim & \left| \mathcal{V}_{eff}^{ud} \right| \left(g_V(q^2) \gamma_\mu - i \frac{g_M(q^2)}{m_n} \sigma_{\mu\nu} q^\nu \right) \\ & + \left| \mathcal{A}_{eff}^{ud} \right| \left(g_A(q^2) \gamma_\mu \gamma_5 + \frac{g_S(q^2)}{m_n} \gamma_5 q_\mu \right) \end{aligned}$$

- Measurement of the neutron life time and polarisation do not allow us any more to extract g_A !

The knowledge of the QCD parameters would allow to determine the electroweak couplings and vice versa !

2.3 How to probe these couplings ?

- Among all the experimental processes involving charged currents, we are looking for those whose QCD parameters (form factors, α_S , $m_q \dots$) are under control or enough small.
- We are looking for effects to first order in δ et ε .



Not a lot of processes...

2.4 Test of NLO effects

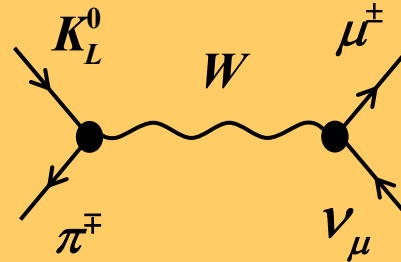
Experimental processes	Parameters extracted	Low Energy/QCD inputs
Nuclear β decays $0^+ \rightarrow 0^+$	$ V_{eff}^{ud} $	CVC + nuclear corrections [Towner & Hardy '07]
Hadronic τ decays $R_V, R_A, R_S,$ Moments ALEPH, OPAL	$\epsilon_{NS}, \delta + \epsilon_{NS}$	OPE [Braaten et al '92, Lediberder & Pich '92....] $\alpha_S(m_\tau), m_q,$ condensates
Γ_W LEP, TEVATRON	δ	Perturbative QCD $\alpha_S(m_W)$
DIS $\nu(\bar{\nu})$ on protons	δ	Normalized pdf
$K_{\mu 3}^L$ decay KTeV, NA48, KLOE	$\epsilon_S = \epsilon_{NS}$	$K\pi$ scattering phases [Buettiker, Descotes, Moussallam' 02] and $\Delta_{CT}: \chi PT$

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Experimental processes	Parameters extracted	Low Energy/QCD inputs
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Γ_W LEP, TEVATRON	δ	Perturbative QCD $\alpha_S(m_W)$
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$K_{\mu 3}^L$ decay KTeV, NA48, KLOE	$\epsilon_S = \epsilon_{NS}$	$K\pi$ scattering phases [Buettiker, Descotes, Moussallam' 02] and $\Delta_{CT}: \chi PT$

3. $K_{\mu 3}^L$ decay : a stringent test of right-handed quark currents.

3.1 Introduction



- $K_L^0 \rightarrow \pi^{\mp} \mu^{\pm} \nu_{\mu}$

Hadronic part :

$$\langle \pi^-(p_{\pi}) | \bar{s} \gamma_{\mu} u | K^0(p_K) \rangle = f_+(t) (p_K + p_{\pi})_{\mu} + f_-(t) (p_K - p_{\pi})_{\mu}$$

→ $f_+(t), f_-(t)$: facteurs de forme

→ $t = q^2 = (p_{\mu} + p_{\nu_{\mu}})^2 = (p_K - p_{\pi})^2$

- We consider the scalar form factor : $f_s(t) = f_+(t) + \frac{t}{m_K^2 - m_{\pi}^2} f_-(t)$

→ Normalisation : $\bar{f}_0(t) = \frac{f_s(t)}{f_+(0)}, f_0(0) = 1$

3.2 Theoretical knowledge: the Callan-Treiman relation

- Callan-Treiman Theorem: SU(2) x SU(2) theorem

$$C = \overline{f_0}(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+^{K^0}(0)} + \Delta_{CT}$$

$\Delta_{K\pi} = m_K^2 - m_\pi^2$

$\Delta_{CT}^{NLO} = -3.5 \cdot 10^{-3}$
[Gasser & Leutwyler]

- Corrections of order m_u, m_d
 - No chiral logarithms : $\Delta_{CT} \sim \mathcal{O}(m_{u,d} / 4\pi F_\pi)$
 - K^0 decay : no small denominators due to $\pi^0 - \eta$ mixing ($\mathcal{O}((m_d - m_u)/m_s)$).

$$C = \bar{f}_0(\Delta_{K\pi}) = \underbrace{\frac{F_K |A_{eff}^{us}|}{F_\pi |A_{eff}^{ud}|} \frac{1}{f_+(0) |V_{eff}^{us}|} |V_{eff}^{ud}|}_{B_{exp}} \frac{|A_{eff}^{ud}| |V_{eff}^{us}|}{|V_{eff}^{ud}| |A_{eff}^{us}|} + \Delta_{CT}$$

- B_{exp} is known from experiment using the measured Br: $Br(K_{12}/\pi_{12})$, $\Gamma(K_{e3})$ and $|V_{ud}|$.

- Standard Model case : $V_{eff} = -A_{eff} = V_{CKM} \Rightarrow \frac{|A_{eff}^{ud}| |V_{eff}^{us}|}{|V_{eff}^{ud}| |A_{eff}^{us}|} = 1$

$$\Rightarrow C_{SM} = 1.2439 \pm 0.0042 + \Delta_{CT} \quad \ln C_{SM} = 0.2183 \pm 0.0034 + \frac{\Delta_{CT}}{B_{exp}}$$

- Relation which tests the Standard Model very accurately.
If physics beyond the SM: $\sim 1\%$ difference between C and B_{exp} . Uncertainties from Δ_{CT} and B_{exp} on the permille level \Rightarrow opportunity to see a possible effect.

$$C = \bar{f}_0(\Delta_{K\pi}) = \underbrace{\frac{F_K |A_{eff}^{us}|}{F_\pi |A_{eff}^{ud}|} \frac{1}{f_+(0) |v_{eff}^{us}|} |v_{eff}^{ud}|}_{B_{exp}} \frac{|A_{eff}^{ud}| |v_{eff}^{us}|}{|v_{eff}^{ud}| |A_{eff}^{us}|} + \Delta_{CT}$$

- B_{exp} is known from experiment using the measured Br: $Br(K_{12}/\pi_{12})$, $\Gamma(K_{e3})$ and $|v_{ud}|$.

- Standard Model case : $v_{eff} = -A_{eff} = V_{CKM} \Rightarrow \frac{|A_{eff}^{ud}| |v_{eff}^{us}|}{|v_{eff}^{ud}| |A_{eff}^{us}|} = 1$

$$\Rightarrow C_{SM} = 1.2439 \pm 0.0042 + \Delta_{CT} \quad \ln C_{SM} = 0.2183 \pm 0.0034 + \frac{\Delta_{CT}}{B_{exp}}$$

- LEET case : Modification of the couplings $\Rightarrow \frac{|A_{eff}^{ud}| |v_{eff}^{us}|}{|v_{eff}^{ud}| |A_{eff}^{us}|} = 1 + 2(\epsilon_S - \epsilon_{NS})$

$$\Rightarrow \ln C = 0.2183 \pm 0.0034 + \Delta\epsilon \quad \text{with} \quad \Delta\epsilon = \tilde{\Delta}_{CT} + 2(\epsilon_S - \epsilon_{NS})$$

experimental uncertainties
 $\left[\tilde{\Delta}_{CT} = \frac{\Delta_{CT}}{B_{exp}} \right]$

$\Rightarrow \ln C = 0.2183 \pm 0.0034 + \Delta\varepsilon$ with $\Delta\varepsilon = \tilde{\Delta}_{CT} + 2(\varepsilon_S - \varepsilon_{NS})$

experimental uncertainties

$$\left[\tilde{\Delta}_{CT} = \frac{\Delta_{CT}}{B_{\text{exp}}} \right]$$

- $\Delta_{CT} \sim 10^{-3}$: to extract a quantitative information on right-handed currents (RHCs), $\ln C$ has to be measured with an accuracy of 10%.

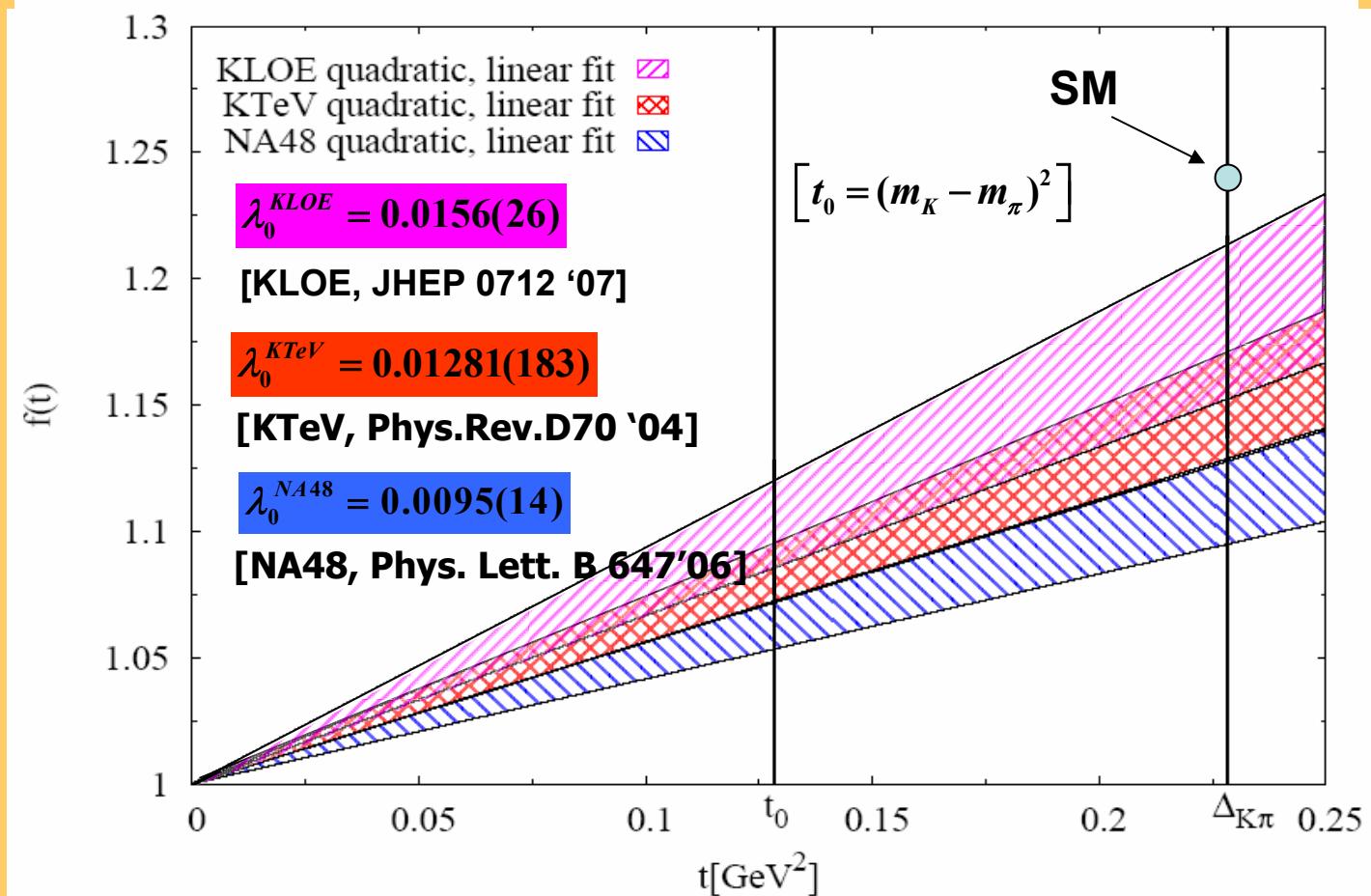
How to measure $\ln C$?

3.3 Experimental measurements

- Data available from KTeV, NA48 and KLOE.
- Necessity to parametrize the 2 form factors \overline{f}_+ and \overline{f}_0 to fit the measured distributions.
- Usually, use of linear parameterization or pole parameterization :

$$f_{lin}(t, \lambda) = 1 + \lambda \frac{t}{m_\pi^2}$$

$$f_{pol}(t, m_s) = \frac{m_s^2}{m_s^2 - t}$$



- Extrapolation with a linear parametrization: impossible to test the SM at the CT point with enough accuracy.
- A curvature exists and we can not neglect it !

How to improve the parametrization to measure $C=f(\Delta_{K\pi})$?

3.4 Dispersive parametrization for the scalar FF.

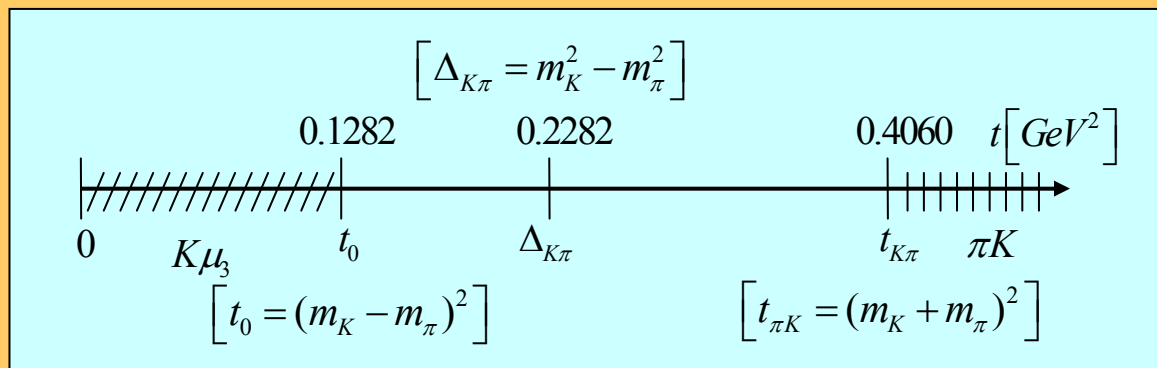
- Problem : How to construct a very precise representation of $\overline{f_0}(t)$ between 0 and $\Delta_{K\pi}$?
- Knowledge :
 - $\overline{f_0}(0) = 1$
 - $\overline{f_0}(\Delta_{K\pi}) = C$, Callan-Treiman point
 - $K\pi$ scattering phase
 - Asymptotic behaviour of the form factor : $\overline{f_0}(s) = \mathcal{O}(1/s)$
 $s \rightarrow -\infty$
- A dispersion relation with two subtractions at 0 and $\Delta_{K\pi}$ for $\ln(\overline{f_0}(t))$:

$$\overline{f_0}(t) = \exp \left[\frac{t}{\Delta_{K\pi}} (\ln C - G(t)) \right]$$

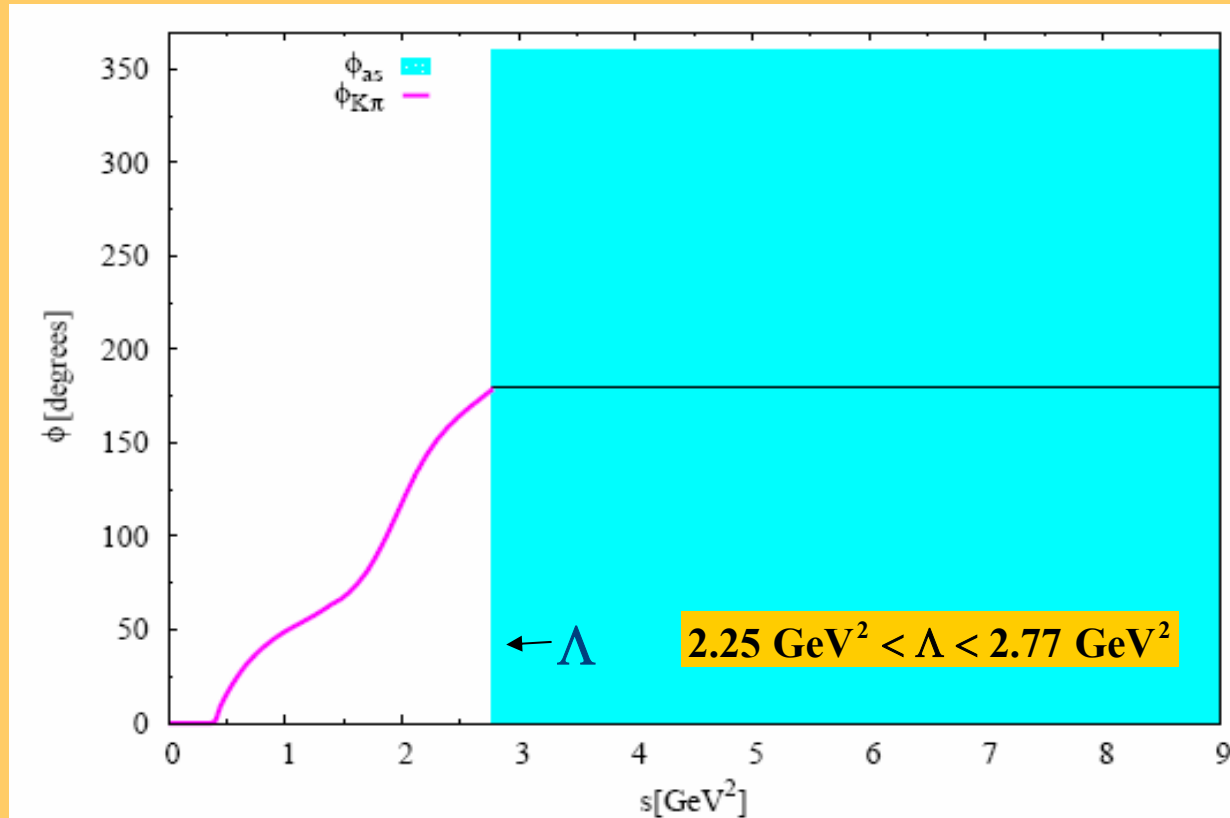
with

$$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi} \int_{t_{\pi K}}^{\infty} \frac{ds}{s} \frac{\phi(s)}{(s - \Delta_{K\pi})(s - t)}$$

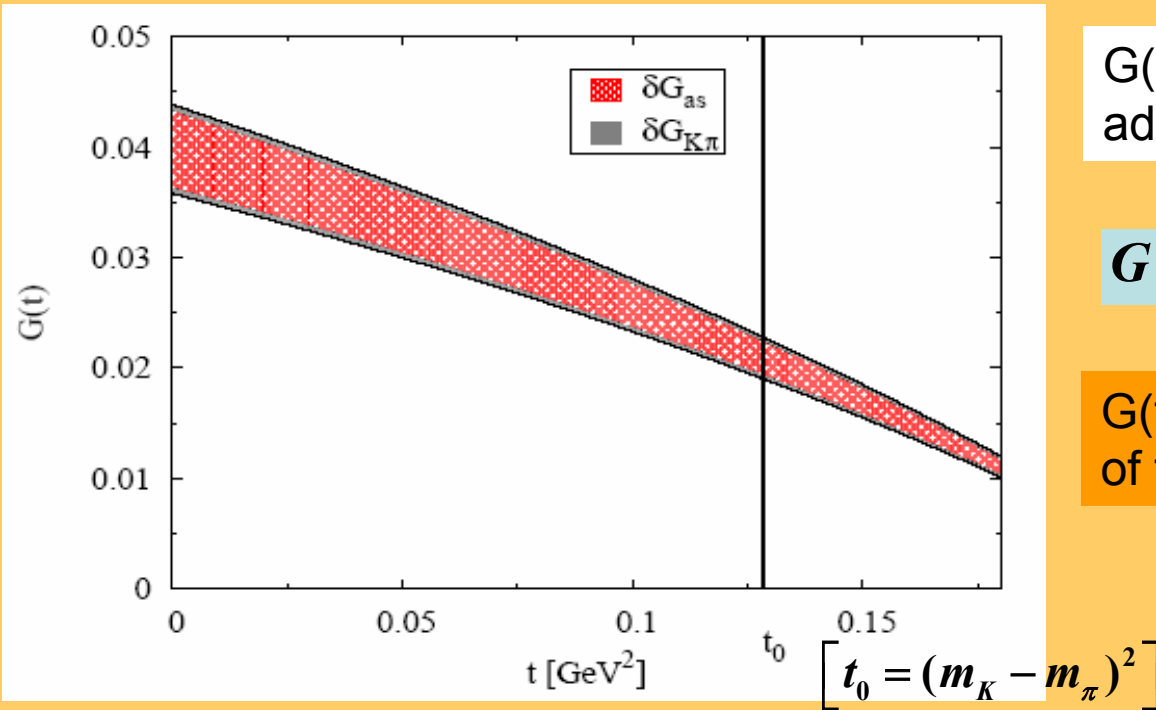
→ $\phi(t)$ phase of form factor : $\overline{f_0}(t) = |\overline{f_0}(t)| e^{i\phi(t)}$



- Phase used



- Elastic up to $\sim 1.5 \text{ GeV}$ \implies $t < \Lambda : \phi(t) = \phi_{K\pi}(t) = \delta_{\pi,K}^{s,\frac{1}{2}}(t) \pm \Delta \delta_{\pi,K}^{s,\frac{1}{2}}(t)$
 $t > \Lambda : \phi(t) = \phi_{as}(t) = \pi \pm \pi$



$G(t)$ with the uncertainties added in quadrature

$$G(0) = 0.0398 \pm 0.0040$$

$G(t)$ does not exceed 20% of the expected value of $\ln C$

$$\ln C \sim 0.20$$

- Apart from the parameter ($\ln C$) to be determined by a fit, very precise parametrization of the form factor in the physical region.

3.5 First measurement: a discrepancy with the SM ?

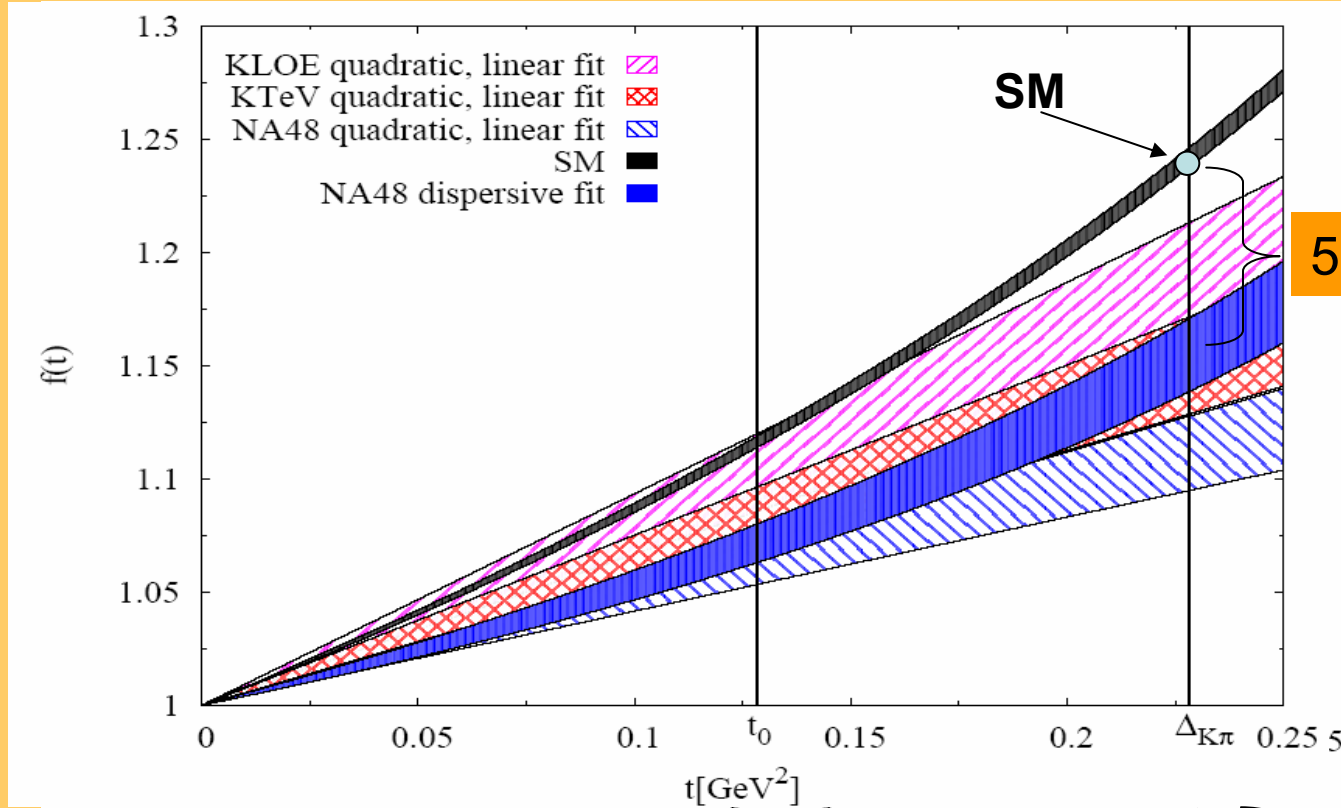
- Use of the dispersive parametrization to fit the measured distribution and to extract $\ln C$:

$$f(t) = \exp \left[\frac{t}{\Delta_{K\pi}} (\ln C - G(t)) \right]$$



$$\ln C_{NA48} = 0.1438 \pm 0.014$$

[NA48 Phys. Letter B 647 '06]



5 σ !

- With $\Delta_{CT}^{NLO} = -3.5 \cdot 10^{-3}$

$$\ln C_{SM} = 0.2183 \pm 0.0034 + \frac{\Delta_{CT}}{B_{exp}}$$

5 σ !

$$\ln C_{NA48} = 0.1438 \pm 0.014$$

Experimental result for $\Delta\varepsilon$

- $$\Delta\varepsilon = \frac{\Delta_{CT}}{B_{\text{exp}}} + 2(\varepsilon_S - \varepsilon_{NS}) \quad \Rightarrow \quad \Delta\varepsilon = -0.074 \pm 0.015$$

Possibility to disentangle Δ_{CT} from the RHCs by a matching with the ChPT two loop representation for the form factors [Bernard & E.P'07].

- $$\Delta\varepsilon = \frac{\Delta_{CT}}{B_{\text{exp}}} \text{ asks for } |\Delta_{CT}| \geq 20 \left| \Delta_{CT}^{NLO} \right| ! \text{ with } \Delta_{CT}^{NLO} \sim -3.5 \cdot 10^{-3}$$

- If it is not the case the measurement gives us an information on the right-handed mixing matrix : ε_S is enhanced and inverse hierarchy in the right-handed sector !

3.6 But other « contradictory » measurements

- Only combined Ke3 + K μ 3 result for KLOE (to reduce the uncertainties) :

$$\ln C_{KLOE} = 0.207 \pm 0.023$$

↖ large uncertainty


- Preliminary results for KTeV :

- K μ 3 alone : $\ln C_{KTeV} = 0.195 \pm 0.014$

- Combined Ke3 + K μ 3 : $\ln C_{KTeV} = 0.191 \pm 0.012$

- Consistent results at 1-1.5 σ from the SM, confirming the same direction but in disagreement with the NA48 result !
- K⁺ analysis in progress to solve this experimental puzzle.

3.7 Conclusion

- Stringent test of the V+A coupling in $K_{\mu 3}^L$ decays.
- Experimental situation not completely clear: from 1σ to 5σ away from the SM depending on the experiment.  Measurement of the direct coupling of right-handed quarks to W boson: $2(\varepsilon_S - \varepsilon_{NS})$
- Tendancy of an enhancement of ε_S , inverted hierarchy of V_R .
- No other experiments where ε_S is involved.

4. Tests of the electroweak couplings to the Z boson.

4.1 Neutral current interactions.

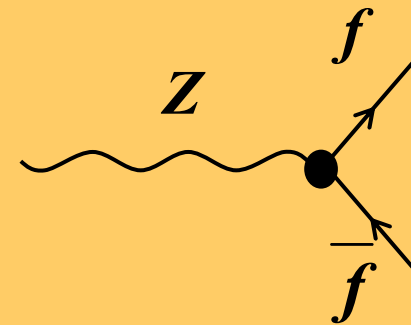
$$\mathcal{L}_Z = \frac{e}{2\cos\theta_w \sin\theta_w} (1 - \xi^2 \rho_L) \left[\bar{\mathbf{N}} \gamma_\mu (\mathbf{g}_V^N - \mathbf{g}_A^N \gamma_5) \mathbf{N} + \bar{\mathbf{L}} \gamma_\mu (\mathbf{g}_V^L - \mathbf{g}_A^L \gamma_5) \mathbf{L} \right. \\ \left. + \bar{\mathbf{U}} \gamma_\mu (\mathbf{g}_V^U - \mathbf{g}_A^U \gamma_5) \mathbf{U} + \bar{\mathbf{D}} \gamma_\mu (\mathbf{g}_V^D - \mathbf{g}_A^D \gamma_5) \mathbf{D} \right] \mathbf{Z}_\mu$$

Normalized factor
absorbed in G_F

$$\bullet \frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_w^2} (1 - \xi^2 \rho_L)^2 \quad \mathbf{N} = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \mathbf{L} = \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}, \mathbf{U} = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \mathbf{D} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\bullet g = \frac{e}{\sin\theta_w}$$

$$\bullet \sin^2\theta_w = 1 - \frac{m_w^2}{m_Z^2}$$



4.1 Neutral current interactions.

$$\mathcal{L}_Z = \frac{e}{2\cos\theta_w \sin\theta_w} (1 - \xi^2 \rho_L) \left[\bar{N} \gamma_\mu (\mathbf{g}_V^N - \mathbf{g}_A^N \gamma_5) N + \bar{L} \gamma_\mu (\mathbf{g}_V^L - \mathbf{g}_A^L \gamma_5) L \right. \\ \left. + \bar{U} \gamma_\mu (\mathbf{g}_V^U - \mathbf{g}_A^U \gamma_5) U + \bar{D} \gamma_\mu (\mathbf{g}_V^D - \mathbf{g}_A^D \gamma_5) D \right] \mathbf{Z}_\mu$$

Normalized factor
absorbed in G_F

- New couplings at NLO appearing in g_V^f and g_A^f :

$$\left\{ \begin{array}{l} \mathbf{g}_V^N = \frac{1}{2} + \frac{\varepsilon^v}{2} \\ \mathbf{g}_A^N = \frac{1}{2} - \frac{\varepsilon^v}{2} \end{array} \right. \quad \left\{ \begin{array}{l} \mathbf{g}_V^L = -\frac{1}{2} + 2\tilde{s}^2 - \frac{\varepsilon^e}{2} \\ \mathbf{g}_A^L = -\frac{1}{2} + \frac{\varepsilon^e}{2} \end{array} \right.$$

$$\left\{ \begin{array}{l} \mathbf{g}_V^U = \frac{1 + \delta}{2} - \frac{4}{3} \tilde{s}^2 + \frac{\varepsilon^u}{2} \\ \mathbf{g}_A^U = \frac{1 + \delta}{2} - \frac{\varepsilon^u}{2} \end{array} \right. \quad \left\{ \begin{array}{l} \mathbf{g}_V^D = -\frac{1 + \delta}{2} + \frac{2}{3} \tilde{s}^2 - \frac{\varepsilon^d}{2} \\ \mathbf{g}_A^D = -\frac{1 + \delta}{2} + \frac{\varepsilon^d}{2} \end{array} \right.$$

- At NLO 6 parameters:
 - modification of the left couplings: δ
 - modification of the right couplings :
 - For the neutrinos and electrons : $\varepsilon^{\nu}, \varepsilon^e$
 - For the quarks: $\varepsilon^u, \varepsilon^d$

- Normalisation factor: $\mathbf{1 - \xi^2 \rho_L} \Rightarrow \tilde{s}^2 = \frac{s^2}{1 - \xi^2 \rho_L}$

- Expectation: percent level.
- Universality of the couplings is assumed.

4.2 FIT to Z pole observables

- How to constrain these parameters ?

➔ Data from LEP and SLD.

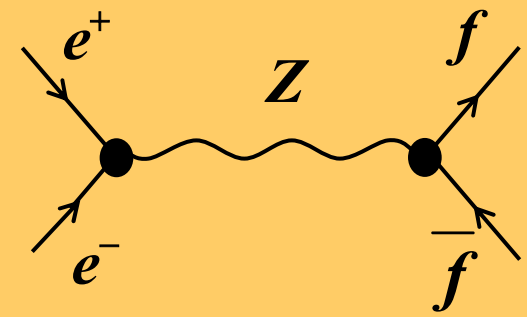
- Definition of « pseudo-observables » : $\Gamma_Z, \Gamma_{\text{had}}, \Gamma_{ff}, \sigma_f, A_{\text{FB}}, A_f \dots$

- Take the less correlated and the independent one for a FIT of the couplings: $\Gamma_Z, \sigma_{\text{had}}, R_e, R_b, A_{\text{FB}}^{e,b,c}$

$$\Gamma_f = N_c^f \frac{G_F}{6\sqrt{2}\pi} m_Z^3 \left[\left(g_V^f \right)^2 R_V^f + \left(g_A^f \right)^2 R_A^f \right]$$

Corrections QCD+QED

Observables of the FIT :



- Total decay width of Z : $\Gamma_Z = \sum_f \Gamma_f$

- Hadronic pole cross section : $\sigma_{had} = \frac{12\pi}{m_Z^2} \frac{\Gamma_e \Gamma_{had}}{\Gamma_Z^2}$

- Ratio R_b : $R_b = \frac{\Gamma_b}{\Gamma_{had}}$ ($3R_b + 2R_c = 1$)

- Ratio R_l : $R_l = \frac{\Gamma_{had}}{\Gamma_l}$

- The forward backward asymmetries: $A_{FB}^f = \frac{n_F(\theta_f < 90^\circ) - n_B(\theta_f > 90^\circ)}{n_F(\theta_f < 90^\circ) + n_B(\theta_f > 90^\circ)}$

$$A_{FB}^f = 3 \frac{g_V^e g_A^e}{[(g_V^e)^2 + (g_A^e)^2]} \frac{g_V^f g_A^f}{[(g_V^f)^2 + (g_A^f)^2]}$$

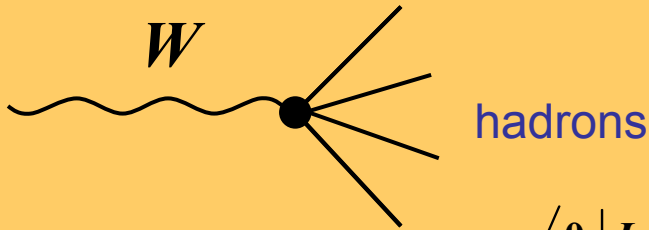
for f=e,b,c

↑
1/2 A_e

1/2 A_f
PSI, 6 March, 2008

W leptonic Branching Ratios

- Interface charged/neutral currents: $\Rightarrow \delta$



$$\Gamma(W^\pm \rightarrow h^\pm) \propto \langle \mathbf{0} | J^\mu | h \rangle \langle h | J^{\nu\dagger} | \mathbf{0} \rangle$$

$$\langle \mathbf{0} | J^\mu | h \rangle = \left\langle \mathbf{0} \left| \sum_{ij} (1 + \delta) V_L^{ij} \bar{u}_L^i \gamma_\mu d_L^j + \varepsilon V_R^{ij} \bar{u}_R^i \gamma_\mu d_R^j \right| h \right\rangle$$

$$\Gamma(W^\pm \rightarrow h^\pm) = (1 + 2\delta) \cdot \Gamma_{SM}(W^\pm \rightarrow h^\pm)$$

- Theoretical calculation: perturbative QCD

$$\Gamma_{W_{tot}} = \frac{G_F M_W^3}{6\sqrt{2}\pi} \left[3 + 6 \cdot (1 + 2\delta) \cdot \left[1 + \frac{\alpha_s(M_W)}{\pi} + 1.409 \left(\frac{\alpha_s(M_W)}{\pi} \right)^2 - 12.77 \left(\frac{\alpha_s(M_W)}{\pi} \right)^3 \right] \right]$$

- Take the leptonic branching ratio:

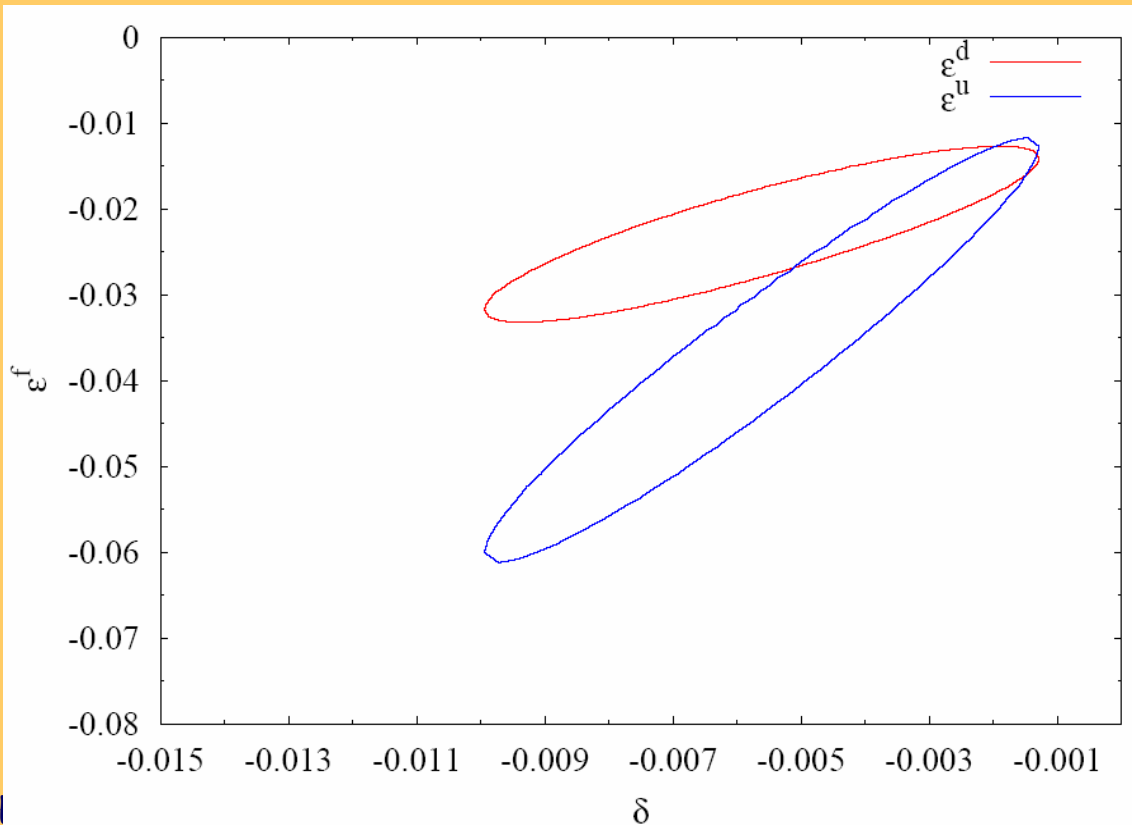
$$\text{Br}(W \rightarrow l\nu) = \frac{\Gamma_{W \rightarrow l\nu}}{\Gamma_{W_{tot}}}$$



very accurate measurement from
LEP : $\text{Br}(W \rightarrow l\nu) = 0.1084(9)$.
Very sensitive to δ .

Results

- Fit to first order in ε (NLO) \Rightarrow ε^v not present in the fit
- $\alpha_S(M_Z)=0.1190$ fixed, impossible to determine simultaneously with the EW parameters.
- $\delta=-0.0054(44)$, $\tilde{s} \approx 2 = 0.2308(4)$, $\varepsilon^e = -0.0024(5)$,
 $\varepsilon^u = -0.0223(104)$, $\varepsilon^d = -0.0355(257)$, $\chi^2/\text{dof}=3.09/3$.

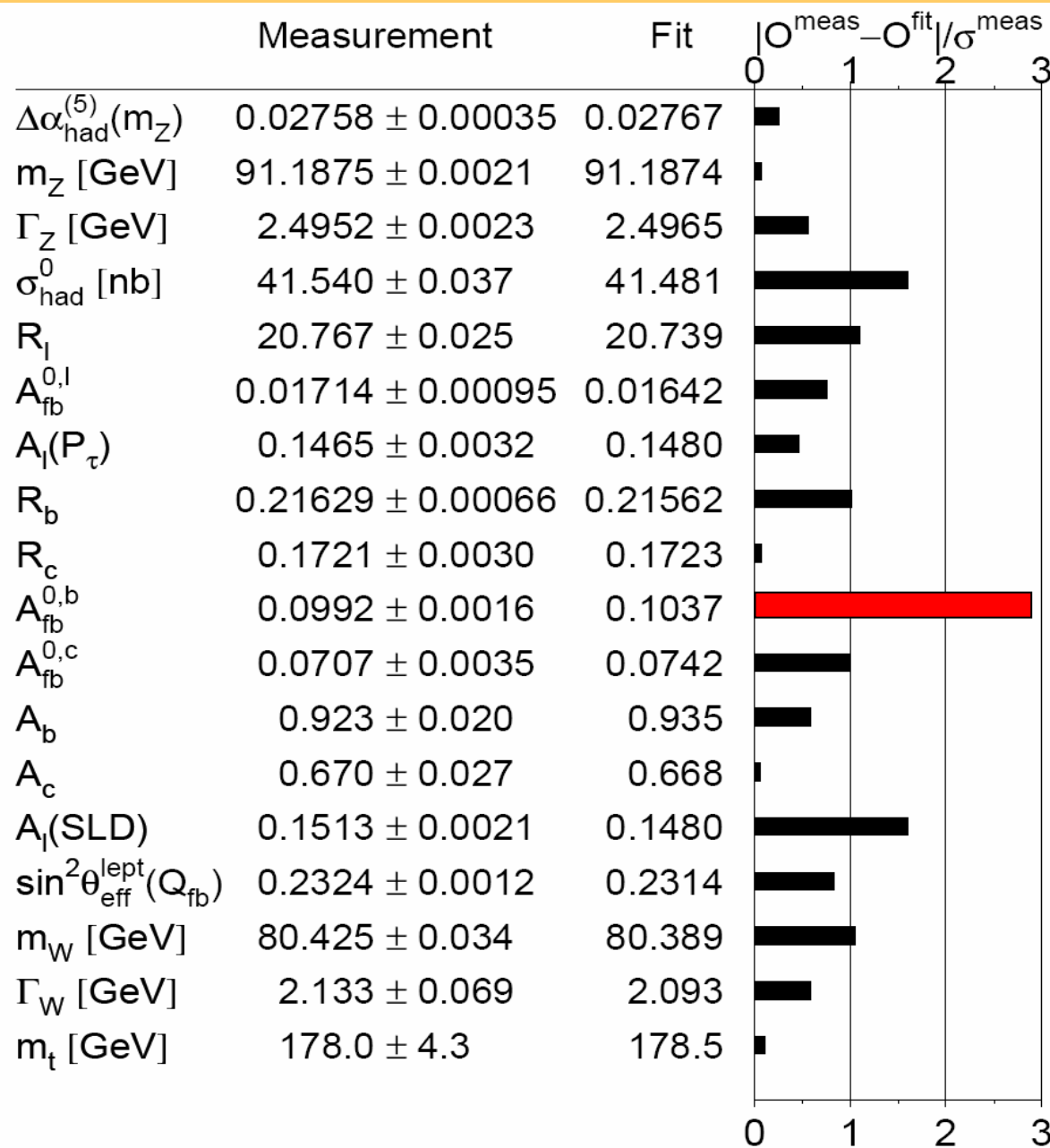


δ and ε^e very small
 \Rightarrow left-handed couplings
not very affected at NLO.

Results in agreement with
the order of magnitude

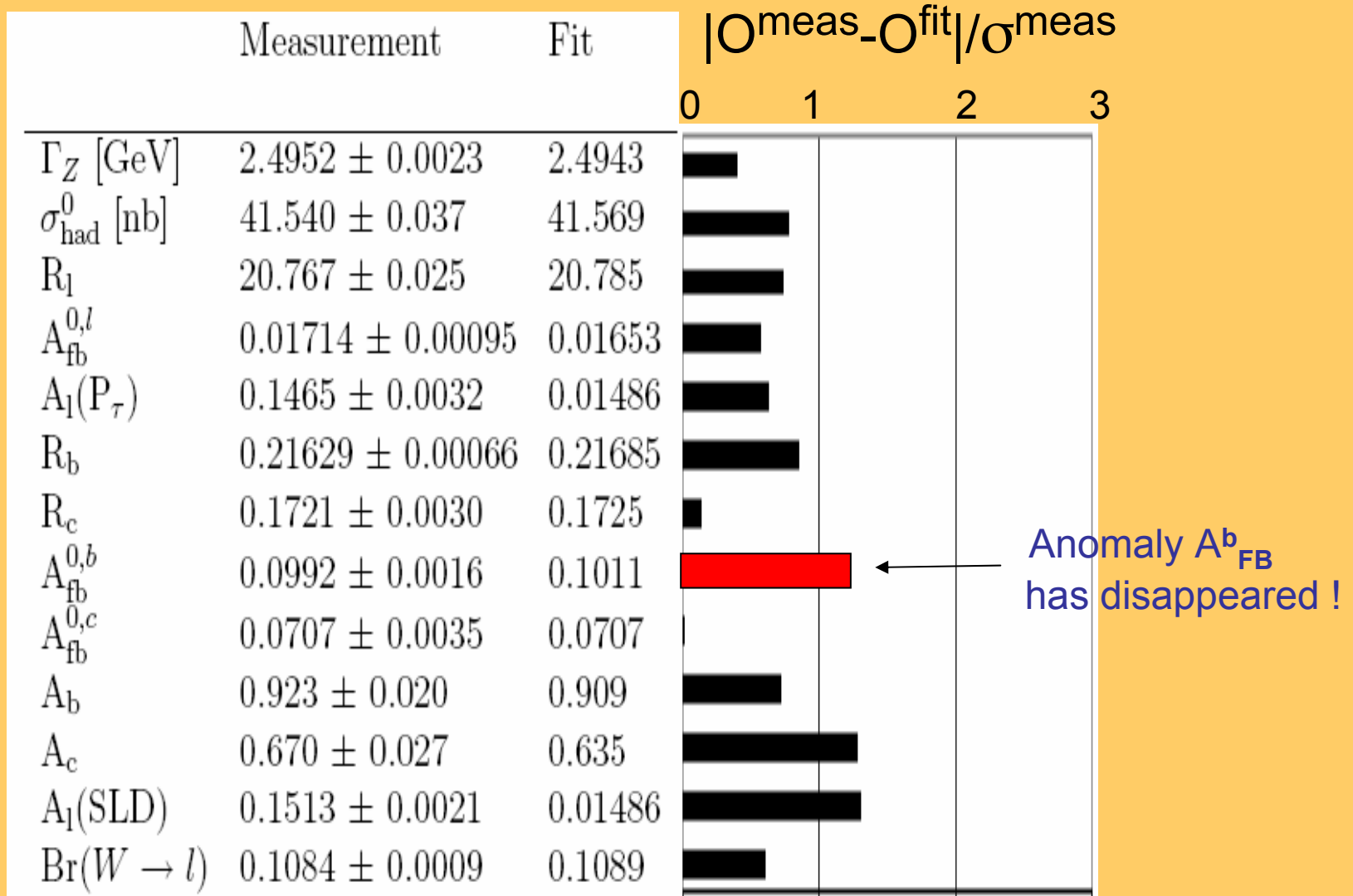
Uncertainties from
experiments only !

Results FIT SM (LEP'06)

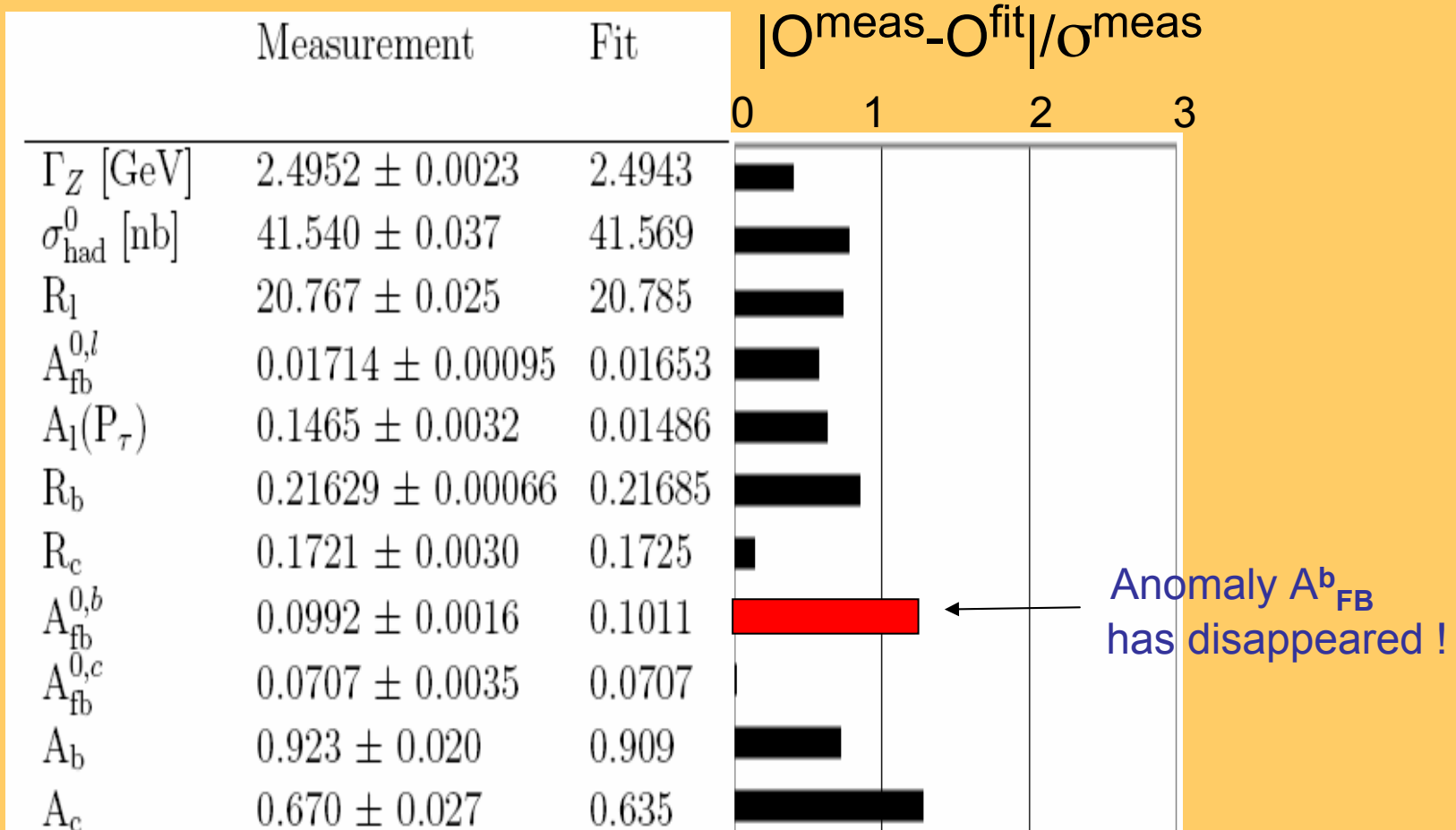


← Anomaly $A_{\text{FB}}^{0,b}$!

From the NLO fit

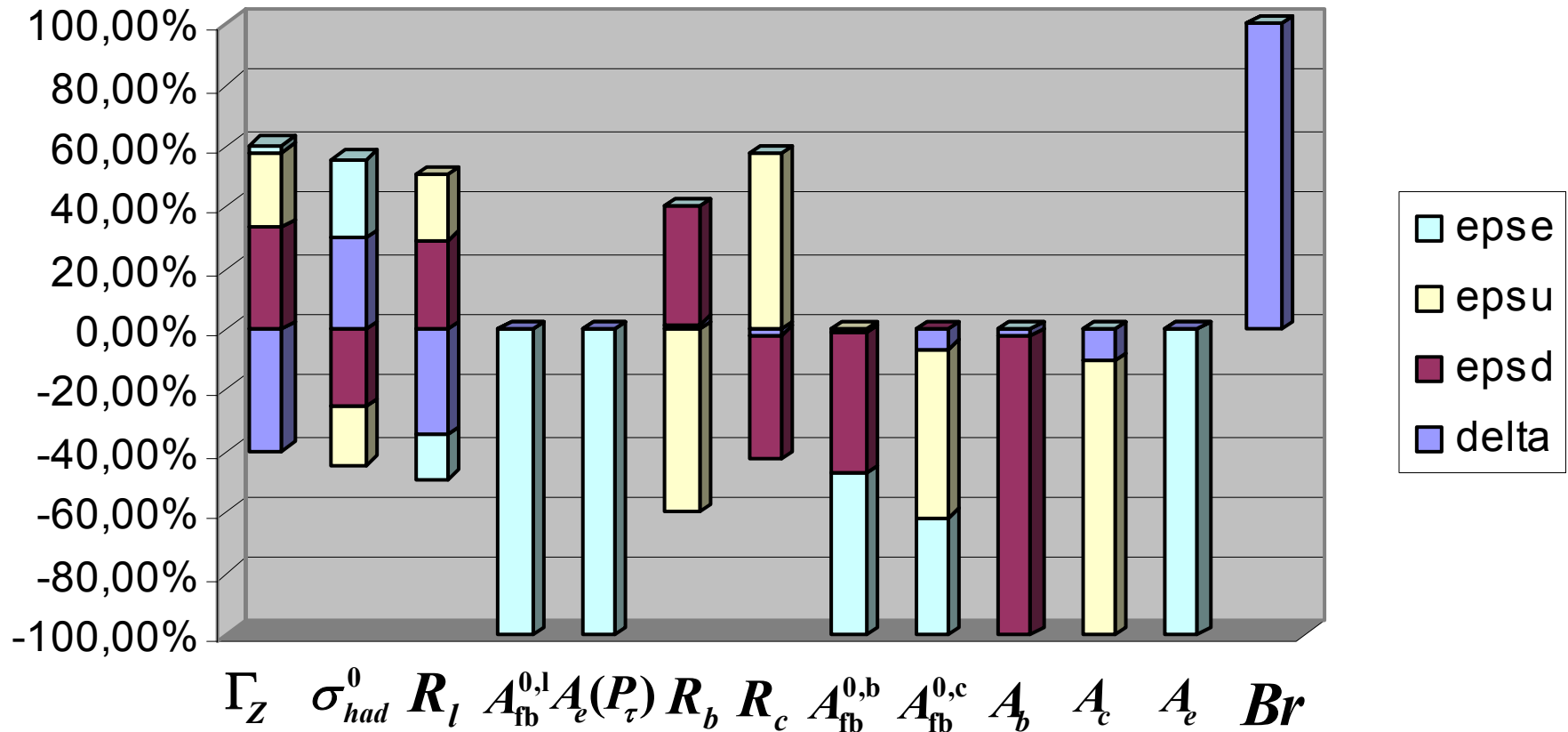


From the NLO fit



- Remarkable agreement at NLO (less than 1σ), the anomaly A_{FB}^b for b quarks disappears without breaking the universality.
- Parameters susceptible to be modified at NNLO (only loop corrections + counter terms meaningful within the LEET) but hardly imaginable that the nice agreement with data will be spoiled.

Contributions of the different parameters to observables



4.3 Low energy Experiments

- Using the values of the parameters determined in the FIT
⇒ predictions at low energy.
- Atomic parity violation: test the couplings of electrons to the quarks inside the nucleus via neutral current.
 - Violating part amplitude: 2 contributions ($A_e V_q$) and ($V_e A_q$).
 - Limitate the uncertainties, take vector couplings for quarks (CVC).

$$\mathcal{L}_{NC}^{lq} = \frac{G_F}{\sqrt{2}} 4 g_A^e \bar{e} \gamma_\mu \gamma^5 e \left(g_V^u \bar{u} \gamma^\mu u + g_V^d \bar{d} \gamma^\mu d \right)$$

$$\Rightarrow Q_W = 4 g_A^e \left[Z \left(2 g_V^u + g_V^d \right) + N \left(g_V^u + 2 g_V^d \right) \right]$$

- Parity violation in Polarized Moller Scattering
Measurement of the parity violating asymmetry (E-158)

$$A_{PV} = \frac{\sigma_R(e_R) - \sigma_L(e_L)}{\sigma_R(e_R) + \sigma_L(e_L)}$$

$$= -\mathcal{A}(Q^2, y) Q_W^e$$

Kinematic factor
($y = Q^2 / s$)

$$Q_W^e = 4g_A^e g_V^e$$

- Results :

Observable	Measurement	NLO prediction
$Q_W(^{133}\text{Cs})$	-72.62 ± 0.46	-70.73 ± 4.44
$Q_W(^{205}\text{Tl})$	-116.40 ± 3.64	-111.95 ± 7.47
Q_W^p	Qweak ?	0.060 ± 0.017
Q_W^e	0.041 ± 0.005	0.074 ± 0.02

Observable	Measurement	NLO prediction
$Q_W(^{133}\text{Cs})$	-72.62 ± 0.46	-70.73 ± 4.44
$Q_W(^{205}\text{Tl})$	-116.40 ± 3.64	-111.95 ± 7.47
Q_W^p	Qweak ?	0.060 ± 0.017
Q_W^e	0.041 ± 0.005	0.074 ± 0.02

5 σ !

- Good agreements except for weak charge of electron



$$Q_W^e = 1 - 4\tilde{s}^2(1 - \varepsilon^e)$$

Accidental cancelation at NLO !

$$\left(4\tilde{s}^2(1 - \varepsilon^e) \sim 1\right)$$

- We have to go to NNLO !

4.4 Conclusion

- Experimental tests to the Z pole successful !
- B quark asymmetry A_{FB} anomaly solved.
- Agreement with the low-energy experiments.

5. Conclusions and prospects

5.1 Conclusions

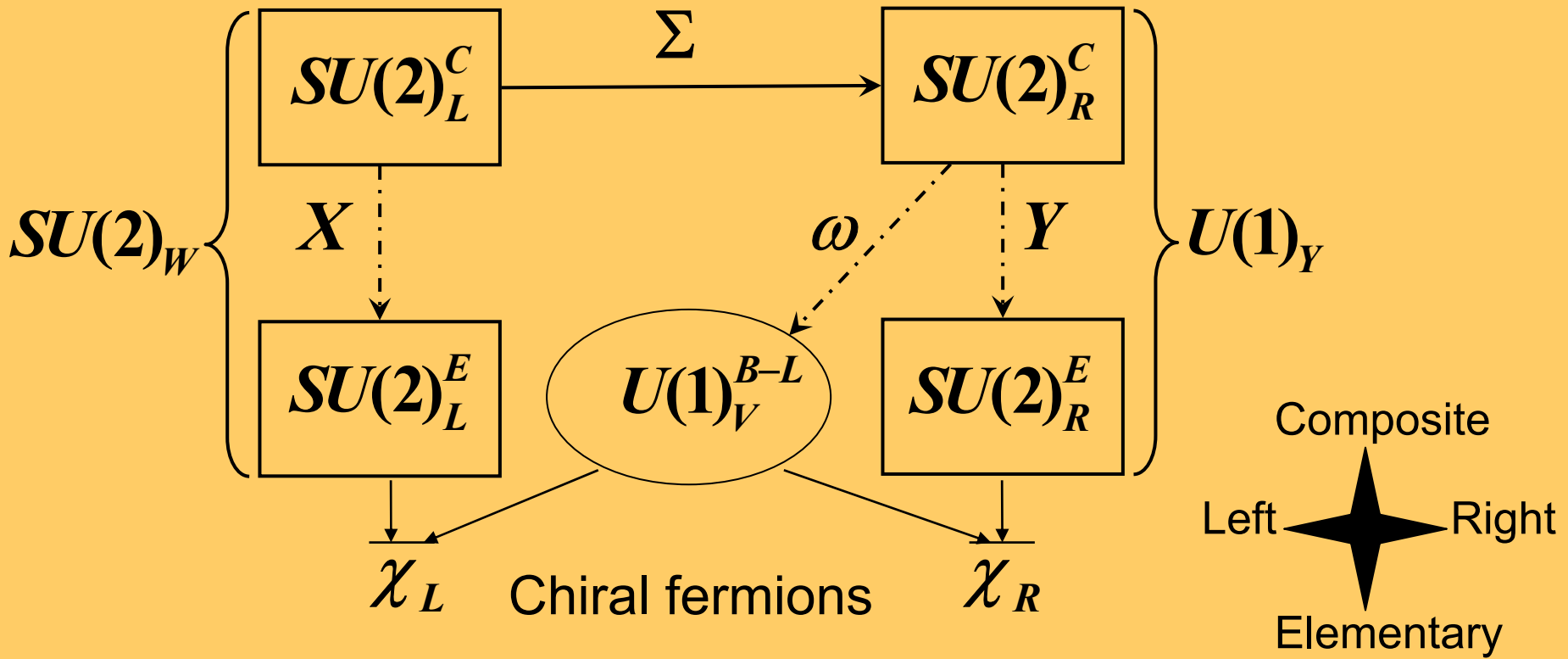
- Minimal low-energy effective theory \Rightarrow tool to look for physics beyond the Standard Model.
- Prediction of NLO non standard couplings of fermions to W and Z bosons.
- The most dramatic effect: right-handed quark coupling to W \Rightarrow ε .
- Quasi non-existent tests of the V-A couplings of quarks because of the confinement \Rightarrow use of precise hadronic physics, ChPT, short distances.
- $K_{\mu 3}^L$ decay : ideal decay \Rightarrow experimental puzzle: the 5 discrepancy found by NA48 not confirmed. Measurement of RHCs
- Precision tests to the Z pole successful !

5.2 Prospects

- NLO : heavy quark sector.
- NNLO, loop effects : Flavour Changing Neutral Currents, CP violation, Dipolaire Electric Moments

➔ V_R matrix completely unknown to constrain !

Additional slides



- GB Σ : link $L \longleftrightarrow R$
- Spurions X, Y, ω : link $C \longleftrightarrow E$
- Covariant constraints reducing the symmetry and the physical dof to

$$SU(2) \otimes U(1)_Y \Leftrightarrow \boxed{D_\mu X = D_\mu Y = D_\mu \omega = 0}$$

- $X \sim \xi$ $Y \sim \eta$ $\omega \sim \zeta$ small expansion parameters :

$$p.s. \quad \xi, \eta = \frac{m_{top}}{\Lambda_W} = O(p), \quad \zeta \ll \xi, \eta \dots LNV$$

3.2 Theoretical knowledge: the Callan-Treiman relation

- Callan-Treiman Theorem: SU(2) x SU(2) theorem

$$C = \overline{f_0}(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+^{K^0}(0)} + \Delta_{CT}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

- Corrections of order m_u, m_d

→ No chiral logarithms : $\Delta_{CT} \sim \mathcal{O}(m_{u,d} / 4\pi F_\pi)$

→ Isospin limit $m_d = m_u$: $\Delta_{CT}^{NLO} \sim -3.5 \cdot 10^{-3}$ [Gasser & Leutwyler]

→ K^0 decay : no small denominators due to $\pi^0 - \eta$ mixing ($\mathcal{O}((m_d - m_u) / m_s)$).

→ K^+ decay case : enhancement by $\pi^0 - \eta$ mixing in the final state

⇒ $\Delta_{CT}^{K^+} \sim \text{few } 10^{-2}$ (K^0 ideal decay)

- Estimations of the higher order terms: corrections in $\mathcal{O}(m_{u,d})$ and $\mathcal{O}(m_s)$

⇒ $\Delta_{CT} = (-3.5 \pm 8) \cdot 10^{-3}$

3.3 Experimental measurements

- Data available from KTeV, NA48 and KLOE.
- **In fix target experiments** (KTeV, NA48), it is impossible to measure directly the t -distribution (Initial energy of K_L unknown) : for each event, 2 possible values of t .
 - ➔ Trick: ➔ For NA48, choice of the most probable t value and bias accounted in the detector acceptance.
 - ➔ For KTeV: use of tranverse t .
- For KLOE: K_L produced at rest, the t -distribution is known but difficulty to separate the muons from the pions at low energy: use the neutrino distribution.
- Necessity to parametrize the 2 form factors \overline{f}_+ and \overline{f}_0 to fit the measured distributions.
- Usually, use of linear parameterization or pole parameterization :

$$f_{lin}(t, \lambda) = 1 + \lambda \frac{t}{m_\pi^2}$$

$$f_{pol}(t, m_s) = \frac{m_s^2}{m_s^2 - t}$$