Supersymmetric Precision Calculations of Bottom Yukawa Couplings

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Outline

- Supersymmetry, MSSM and Higgs
- Bottom Yukawa Coupling
- Resummation
- Low Energy Theorem
- NLO Results
- Novel NNLO Corrections
- Results

SUSY transforms Fermions into Bosons and vice versa : $Q|F\rangle = |B\rangle$ and $Q|B\rangle = |F\rangle$

Duplication of Particle Spectrum : Every SM-Particle obtains a Superpartner

Superpartners :
- carry the same quantum numbers
- differ by spin 1/2

MSSM = Minimal Supersymmetric Extension of the Standard Model (SM)

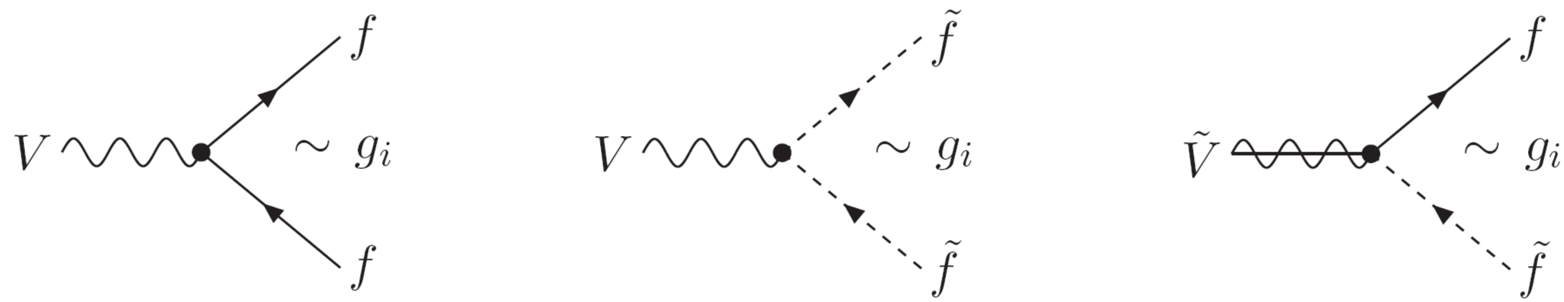
Chiral Multiplets

$J = \frac{1}{2}$	$J = 0$
Quarks q_L, q_R	Squarks $\tilde{q}_L, \tilde{q}_R$
Leptonen ℓ_L, ℓ_R	Sleptonen $\tilde{\ell}_L, \tilde{\ell}_R$
Higgsinos $\tilde{H}_1, \tilde{H}_2$	Higgs H_1, H_2

Vector Multiplets

$J = 1$	$J = \frac{1}{2}$
Gluon g	Gluino $\tilde{g}$
$W^{\pm}, W^3$	Wino $\tilde{W}^{\pm}, \tilde{W}^3$
B	Bino $\tilde{B}$

Coupling of Gauge Bosons and Gauginos to Fermions and Sfermions



$$g_1 = g' = e/\cos(\theta_W)$$
$$g_2 = g = e/\sin(\theta_W)$$
$$g_3 = g_s$$

In order for the Theory to be invariant under SUSY couplings to Particles and Sparticles must be identical.

Invariance under SUSY also requires masses of Particles and Sparticles to be degenerated.

Mass-degenerated particles not observed $\rightarrow$ SUSY must be broken

Supersymmetry Breaking through explicit soft-breaking terms in Lagrangian

These soft-breaking terms parametrise all possible soft SUSY-breaking mechanisms.

MSSM Higgs Sector

Electroweak Symmetry Breaking (EWSB) in MSSM requires 2 Higgs doublets

MSSM Higgs Potential :

$$V = \left(|\mu|^2 + m_1^2 \right) |H_1|^2 + \left(|\mu|^2 + m_2^2 \right) |H_2|^2 - \mu B \epsilon_{ij} \left(H_1^i H_2^j + \text{h.c.} \right)$$
$$+ \frac{g^2 + g'^2}{8} \left(|H_1|^2 - |H_2|^2 \right)^2 + \frac{g^2}{2} |H_1^\dagger H_2|^2$$

After EWSB 5 physical Higgs Bosons remain in the theory :

2 neutral scalar : h, H , 1 neutral pseudoscalar : A , 2 charged : $H^\pm$

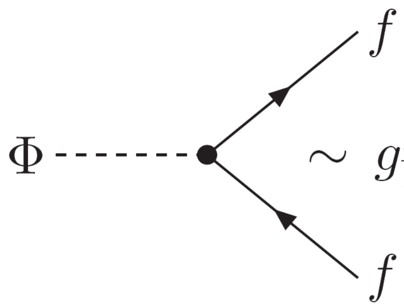
$$\begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} [v_1 + H \cos\alpha - h \sin\alpha + i\ A \sin\beta - i\ G^0 \cos\beta] \\ H^- \sin\beta - G^- \cos\beta \end{pmatrix}$$

$$\begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} H^+ \cos\beta + G^+ \sin\beta \\ \frac{1}{\sqrt{2}} [v_2 + H \sin\alpha + h \cos\alpha + i\ A \cos\beta + i\ G^0 \sin\beta] \end{pmatrix}$$

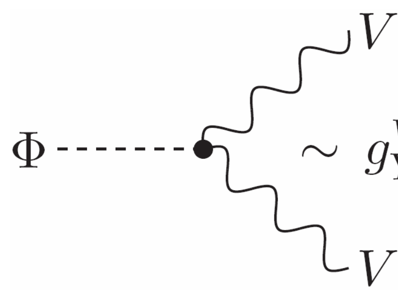
3 remaining Goldstone bosons $G^0, G^\pm$ can be removed by a gauge transformation

Higgs Potential depends on 2 parameters : $M_A\ ,\ \text{tg}\beta = \frac{v_2}{v_1}$

Higgs couples to Fermions and Gauge Bosons via modified Yukawa couplings



$\sim g_Y^f \cdot g_f^\Phi$



$\sim g_Y^V \cdot g_V^\Phi$

$$g_Y^f = m_f/v$$

$$g_Y^V = M_V^2/v$$

$$v = \sqrt{v_1^2 + v_2^2}$$

Φ		g_u^Φ	g_d^Φ	g_V^Φ
SM	H	1	1	1
MSSM	h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\sin(\beta-\alpha)$
	H	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos(\beta-\alpha)$
	A	$1/\tan\beta$	$\tan\beta$	0

Decoupling Limit : $\lim M_A \to \infty \Rightarrow \beta - \alpha = \pi/2$

$$g_{f,V}^h = 1\ ,\ g_u^H = -\cot\beta\ ,\ g_d^H = \tan\beta\ ,\ g_V^H = 0$$

In the decoupling limit the light MSSM Higgs boson *h* couples to fermions and gauge bosons like the SM Higgs boson.

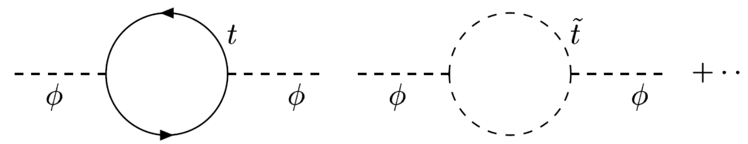
→ experimental distinction between SM and MSSM difficult

Radiative Corrections to Higgs Masses :

$$M^2_{H^\pm} = M^2_A + M^2_W \quad , \quad M^2_H = M^2_A + M^2_Z - M^2_h + \epsilon$$

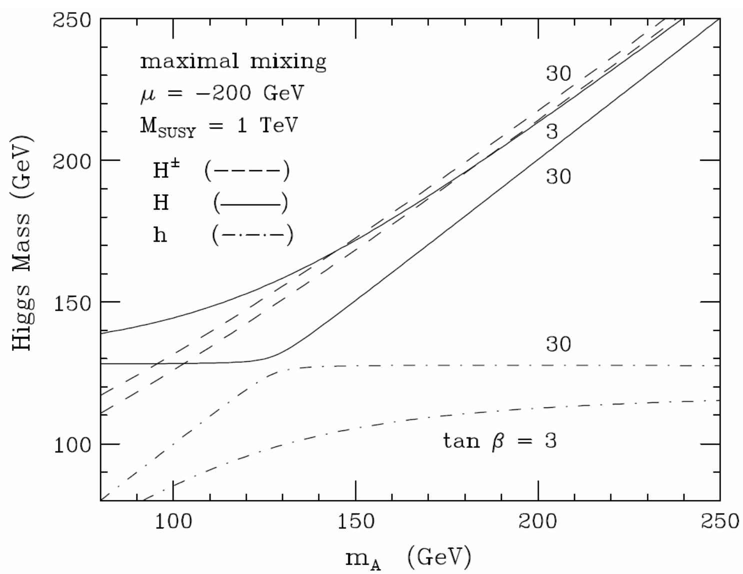
$$M^2_h = \frac{1}{2}\left[M^2_A + M^2_Z + \epsilon - \sqrt{[(M^2_A - M^2_Z)\cos 2\beta + \epsilon]^2 + (M^2_A + M^2_Z)^2 \sin^2 2\beta}\right]$$

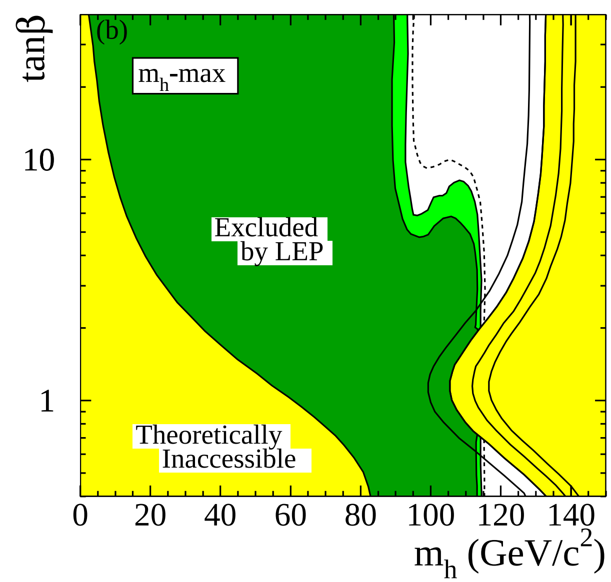
$$\epsilon = \frac{3 G_F m_t^4}{\sqrt{2}\pi^2 s_\beta^2} \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} + \text{Stop–Mixing } [\mu, A_t]$$



Haber, Carena, Heinemeyer, Zhang, Slavich, …

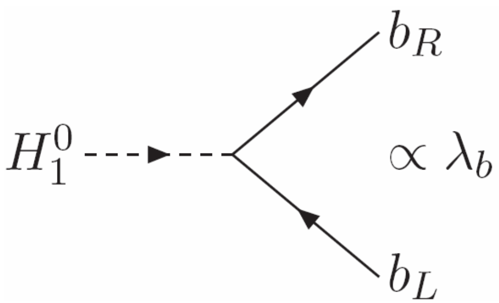
New bound : $M_h < M_Z \rightarrow \boxed{M_h \lesssim 140 \text{ GeV}}$





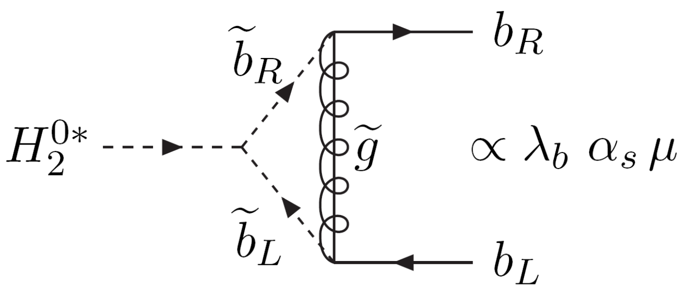
Maximal Mixing Scenario : most conservative bound on the light Higgs mass

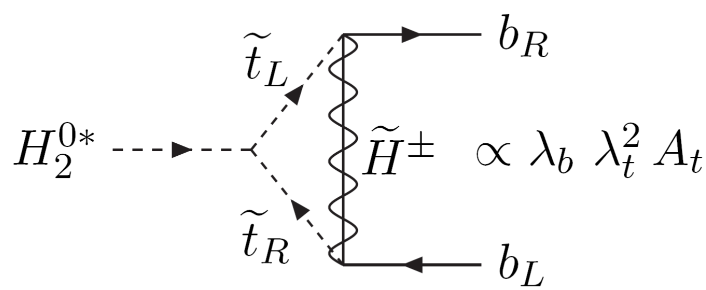
Bottom Yukawa Coupling at Leading Order (LO) :



$$\mathcal{L} = -\lambda_b \; \bar{b}_R \; H_1^0 \; b_L \; + \; \text{h.c.}$$

Next to Leading Order (NLO) :





Dominant Contributions : large $\text{tg}\beta$ $\rightarrow$ large corrections $\lesssim$ 100%

$$\mathcal{L}_{eff} = -\lambda_b \; \bar{b}_R \; [H_1^0 \; + \; \Delta_b \, H_2^{0*}] \; b_L \; + \; \text{h.c.} \; \rightarrow \; \boxed{m_b = \frac{\lambda_b}{\sqrt{2}} v_1 [1 + \underbrace{\Delta_b \, \text{tg}\beta}_{=\Delta m_b}]}$$

Higher Order corrections lead to a modification of the relation between bottom mass and bottom Yukawa coupling

Effective Lagrangian

We use $\boxed{m_b = \frac{\lambda_b}{\sqrt{2}} v_1 \, [1 + \Delta m_b]}$ to replace the bottom Yukawa coupling by the bottom mass.

$$\mathcal{L}_{eff} = -\lambda_b \, \bar{b}_R \, [H_1^0 \, + \, \frac{\Delta m_b}{\text{tg}\beta} \, H_2^{0*}] \, b_L \, + \, \text{h.c.}$$

$$\lambda_b \, \rightarrow \, \sqrt{2} \, \frac{m_b/v_1}{1 + \Delta m_b}$$

$$\mathcal{L}_{eff} = -\sqrt{2} \, \frac{m_b/v_1}{1 + \Delta m_b} \, \bar{b}_R \, [H_1^0 \, + \, \frac{\Delta m_b}{\text{tg}\beta} \, H_2^{0*}] \, b_L \, + \, \text{h.c.}$$

Rotation to physical Higgs fields

$$H_1^0 \rightarrow \frac{1}{\sqrt{2}}[v_1 + H \cos\alpha - h \sin\alpha + i \; A \sin\beta - i \; G^0 \cos\beta]$$

$$H_2^0 \rightarrow \frac{1}{\sqrt{2}}[v_2 + H \sin\alpha + h \cos\alpha + i \; A \cos\beta + i \; G^0 \sin\beta]$$

$$\mathcal{L}_{eff} = - \, m_b \, \bar{b} \left[1 + i\gamma_5 \frac{G^0}{v}\right] b - \frac{m_b/v}{1 + \Delta m_b} \, \bar{b} \Bigg[g_b^h \left(1 - \frac{\Delta m_b}{\text{tg}\alpha \; \text{tg}\beta}\right) h + g_b^H \left(1 + \Delta m_b \frac{\text{tg}\alpha}{\text{tg}\beta}\right) H - g_b^A \left(1 - \frac{\Delta m_b}{\text{tg}^2\beta}\right) i\gamma_5 A \Bigg] b$$

Φ	g_b^Φ
h	$-\sin\alpha/\cos\beta$
H	$\cos\alpha/\cos\beta$
A	$\tan\beta$

This defines effective couplings :

$$\mathcal{L}_{eff} = -\frac{m_b}{v}\,\bar{b}\,[\,\tilde{g}_b^h\;h + \tilde{g}_b^H\;H - \tilde{g}_b^A\;i\gamma_5\;A\,]\,b$$

with
$$\tilde{g}_b^h = \frac{g_b^h}{1+\Delta m_b}\left[1 - \Delta m_b \frac{1}{\mathrm{tg}\alpha\mathrm{tg}\beta}\right]$$

$$\tilde{g}_b^H = \frac{g_b^H}{1+\Delta m_b}\left[1 + \Delta m_b \frac{\mathrm{tg}\alpha}{\mathrm{tg}\beta}\right]$$

$$\tilde{g}_b^A = \frac{g_b^A}{1+\Delta m_b}\left[1 - \Delta m_b \frac{1}{\mathrm{tg}^2\beta}\right]$$

Without corrections Δm_b we would have $\tilde{g}_b^\Phi = g_b^\Phi$

Higher orders are automatically resummed :

$$\frac{1}{1+\Delta m_b} = 1 - \Delta m_b + \Delta m_b^2 + \cdots$$

Carena, Garcia, Nierste, Wagner
Guasch, Häfliger, Spira

Low Energy Theorem

Δm_b calculated in the limit of vanishing external momenta

This corresponds to an expansion in heavy loop masses

Low Energy Theorem : serve to calculate loop amplitudes with external Higgs bosons which are light compared to the loop particles

Ellis et al.
Kniehl, Spira
Kilian

Effective Lagrangian can be derived by means of the replacements

$$v_1 \to \sqrt{2} H_1^0 \quad , \quad v_2 \to \sqrt{2} H_2^{0*}$$

in the bottom mass operator

$$\bar{b}_R \, m_b \, b_L \; + \; \mathrm{h.c.} \qquad\qquad m_b = m_b^0 + \Sigma_b(m_b) = \frac{\lambda_b}{\sqrt{2}} \left(v_1 + \frac{\Delta m_b}{\mathrm{tg}\beta} v_2 \right)$$

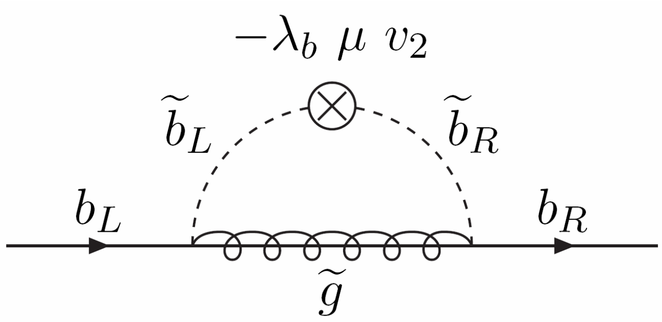
This yields the same effective Lagrangian as the full calculation with external Higgs

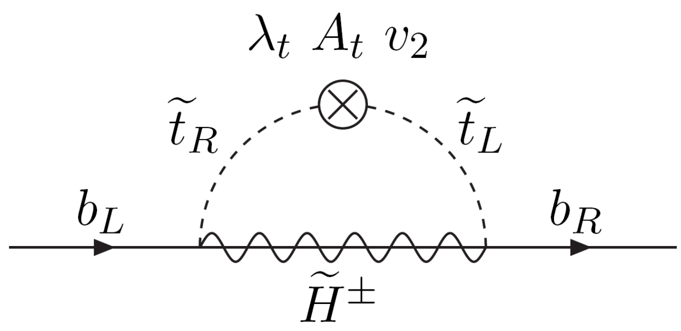
$$\mathcal{L}_{eff} = -\lambda_b \, \bar{b}_R \, [H_1^0 \; + \; \frac{\Delta m_b}{\mathrm{tg}\beta} \, H_2^{0*}] \, b_L \; + \; \mathrm{h.c.}$$

Guasch, Häfliger, Spira

→ For the derivation of the effective Lagrangian it suffices to calculate Self-Energies

Dominant NLO contributions to bottom quark Self-Energy stem from the diagrams :





$$\Sigma_b(m_b) = \frac{\lambda_b}{\sqrt{2}} v_1\ \Delta m_b$$

$$\Delta m_b = \Delta m_b^{QCD} + \Delta m_b^{EW}$$

Squark Mass Matrix :

$$\mathbf{M}_{\tilde{q}}^2 = \begin{pmatrix} M_{\tilde{q}_L}^2 + m_q^2 & m_q(A_q - \mu r_q) \\ m_q(A_q - \mu r_q) & M_{\tilde{q}_R}^2 + m_q^2 \end{pmatrix}$$

$$r_b = \mathrm{tg}\beta\ ,\ r_t = \cot\beta$$

NLO results :

$$\Delta m_b^{QCD\,(1)} = \frac{C_F}{2}\frac{\alpha_s}{\pi} M_{\tilde{g}}\,\mu\,\mathrm{tg}\beta\ I(M_{\tilde{b}_1}^2, M_{\tilde{b}_2}^2, M_{\tilde{g}}^2)$$

$$\Delta m_b^{EW\,(1)} = \frac{\lambda_t^2}{(4\pi)^2} A_t\,\mu\,\mathrm{tg}\beta\ I(M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2, \mu^2)$$

$$I(a,b,c) = -\frac{ab\log\frac{a}{b} + bc\log\frac{b}{c} + ca\log\frac{c}{a}}{(a-b)(b-c)(c-a)}$$

$$\mathrm{tg}\beta = \frac{v_2}{v_1}$$

Carena et al.
Hall et al.
Carena, Garcia, Nierste, Wagner
Guasch, Häfliger, Spira

Validity of Low Energy Approximation

The quantity Δm_b has been calculated

in the Low-Energy-Limit $M_\Phi^2, M_Z^2, m_b^2 \ll M_{SUSY}^2$

Question:
How reliable does this approximation work in phenomenological applications?

Compare full 1-loop result C_Φ with approximate result $C_\Phi^{LE} \sim \Delta m_b^{(1)}$

$$\Gamma(\Phi \to b\bar{b}) = \Gamma_{LO} \left(1 + (C_{QCD} + C_\Phi)\frac{\alpha_s}{\pi}\right)$$

$$\lim_{m_b, M_\Phi \to 0} C_\Phi = C_\Phi^{LE}$$

$$\tilde{g}_b^\Phi = g_b^\Phi \, [1 + \frac{1}{2} C_\Phi^{LE} + \ldots]$$

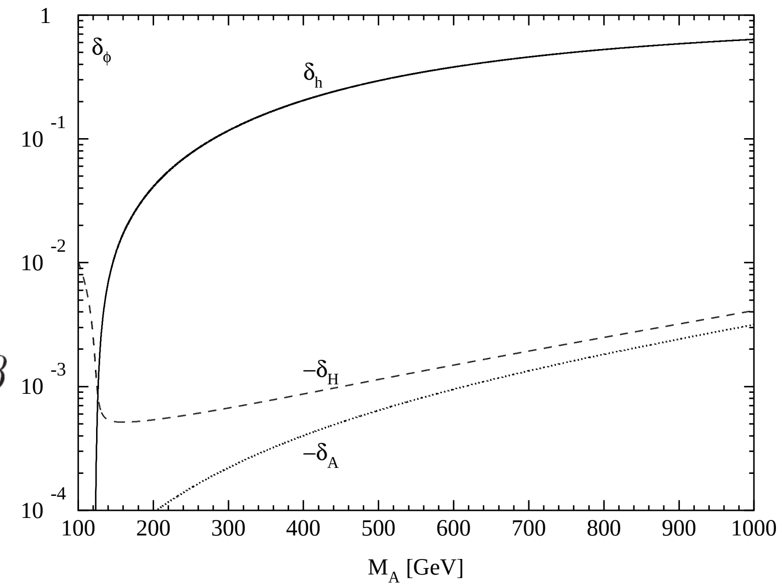
Measure for deviation from exact result :

$$\delta_\Phi = \frac{C_\Phi - C_\Phi^{LE}}{C_\Phi}$$

Approximation good for H and A but fails for h in the decoupling limit, but $\mathrm{tg}\alpha \to -1/\mathrm{tg}\beta$

$$\tilde{g}_b^h = \frac{g_b^h}{1 + \Delta m_b}\left(1 - \frac{\Delta m_b}{\mathrm{tg}\alpha \; \mathrm{tg}\beta}\right) \to g_b^h$$

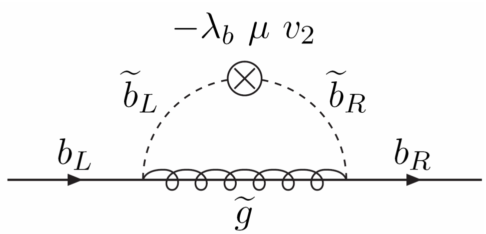
small α_{eff} scenario



Validity of Resummation

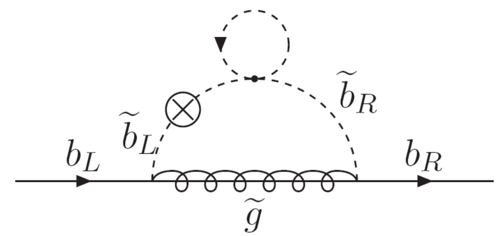
$\mathcal{L}_{eff}$ resums automatically all terms $\quad \frac{1}{1+\Delta m_b} = 1 - \Delta m_b + \Delta m_b^2 + \cdots$

This Resummation is valid, because it contains all leading terms.



$$\alpha_s\ \lambda_b\ \mu\ v_2\ M_{\tilde{g}}\ C_0(0,0;M_{\tilde{b}_1},M_{\tilde{b}_2},M_{\tilde{g}})\ \sim\ \alpha_s\ m_b\ M_{\tilde{g}}\ \frac{\mu\,\mathrm{tg}\beta}{M_{SUSY}^2}$$

$$\alpha_s^2\ \lambda_b\ \mu\ v_2\ M_{\tilde{g}}\ A_0(M_{\tilde{b}_i})\ D_0(0,0,0;M_{\tilde{b}_1},M_{\tilde{b}_2},M_{\tilde{b}_j},M_{\tilde{g}})\ \sim\ \alpha_s^2\ m_b\ M_{\tilde{g}}\ \frac{\mu\,\mathrm{tg}\beta}{M_{SUSY}^2}$$



→ No terms of order $\mathcal{O}\left(\mu^2\mathrm{tg}\beta^2\right)$ are produced.

Due to the Kinoshita-Lee-Nauenberg theorem, irreducible diagrams do not develop power-like divergences in the bottom mass for $m_b \longrightarrow 0$

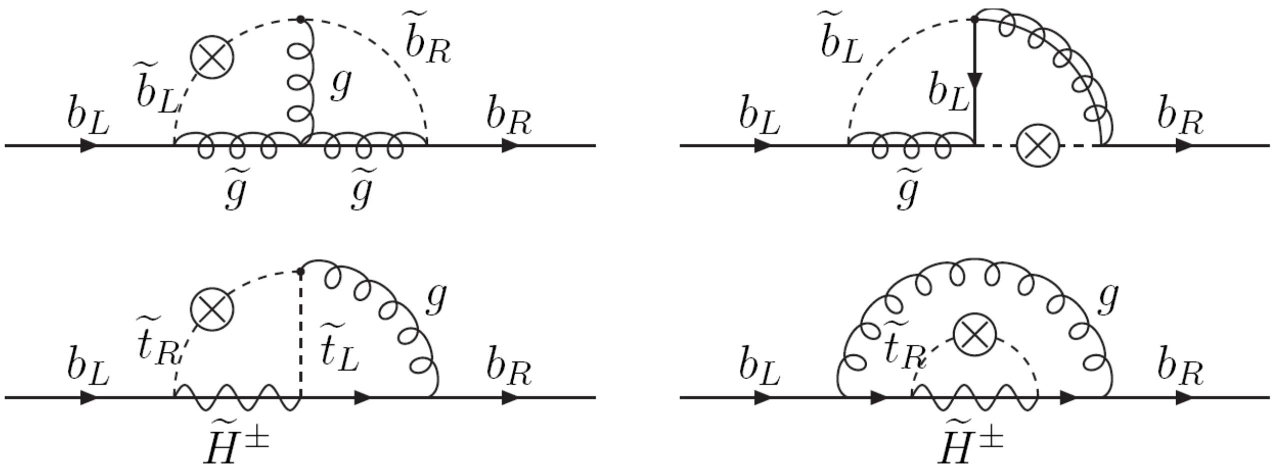
→ Any further mass-insertion $\quad \frac{1}{q^2-M_{\tilde{b}_i}^2} \to \frac{1}{q^2-M_{\tilde{b}_1}^2}\, m_b\,\mu\,\mathrm{tg}\beta\, \frac{1}{q^2-M_{\tilde{b}_2}^2} \sim -\frac{m_b\,\mu\,\mathrm{tg}\beta}{M_{SUSY}^2}\,\frac{1}{q^2-M_{\tilde{b}_i}^2}$

is suppressed by another power of m_b/M_{SUSY} and is therefore non-leading.

Hence, given a diagram with *n* loops, the leading contributions will be of order

$$\mathcal{O}\left(\frac{\alpha_s^n}{M_{SUSY}}\ \mu\,\mathrm{tg}\beta\right)\quad,\quad \mathcal{O}\left(\frac{\alpha_s^{n-1}}{M_{SUSY}}\ \lambda_t^2\ A_t\,\mathrm{tg}\beta\right)$$

Typical Diagrams at NNLO :



Self-Energy can be decomposed into a scalar, pseudoscalar, vectorial and axial vectorial part :

$$\Sigma(p) = m\,\Sigma_S(p) + m\gamma_5\,\Sigma_P(p) + \not{p}\,\Sigma_V(p) + \not{p}\gamma_5\,\Sigma_A(p)$$

The pseudoscalar vanishes and the vectorial and axial vectorial parts are non-leading

→ Only scalar part contributes : $\Sigma_S(m_b) \propto \Delta m_b = \Delta m_b^{(1)} + \Delta m_b^{(2)}$

$$\Delta m_b^{QCD\,(2)} \sim \mathcal{O}\left(\alpha_s^2 \mu \mathrm{tg}\beta / M_{SUSY}\right), \; \Delta m_b^{EW\,(2)} \sim \mathcal{O}\left(\alpha_s \lambda_t^2 A_t \mathrm{tg}\beta / M_{SUSY}\right)$$

Novel NNLO Corrections – Calculational Details

2-loop integrals in NNLO corrections can be reduced to products of 1-loop integrals and one 2-loop master integral :

$$T_{134}(m_1, m_3, m_4) = \int \frac{d^n k}{(2\pi)^n} \frac{d^n q}{(2\pi)^n} \frac{1}{(k^2 - m_1^2)((k-q)^2 - m_3^2)(q^2 - m_4^2)}$$

Berends, Davydychev, Tausk

NNLO corrections are divergent and need to be renormalised

$$\Delta m_b = \Delta m_b^{(1)}(\bar{p}^0) + \Delta m_b^{(2)}(\bar{p}^0) = \Delta m_b^{(1)}(\bar{p}) + \sum_p \frac{\partial \Delta m_b^{(1)}}{\partial p} \delta p + \Delta m_b^{(2)}(\bar{p}) + \mathcal{O}\left(\alpha_s^3, \alpha_s^2 \lambda_t\right)$$

$$= \Delta m_b(\bar{p}) + \mathcal{O}\left(\alpha_s^3, \alpha_s^2 \lambda_t\right)$$

$$p = \{\alpha_s, \lambda_t, A_t, M_{\tilde{b}_1}^2, M_{\tilde{b}_2}^2, M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2, M_{\tilde{g}}\}$$

All masses and the trilinear coupling are renormalised in the On-Shell scheme :

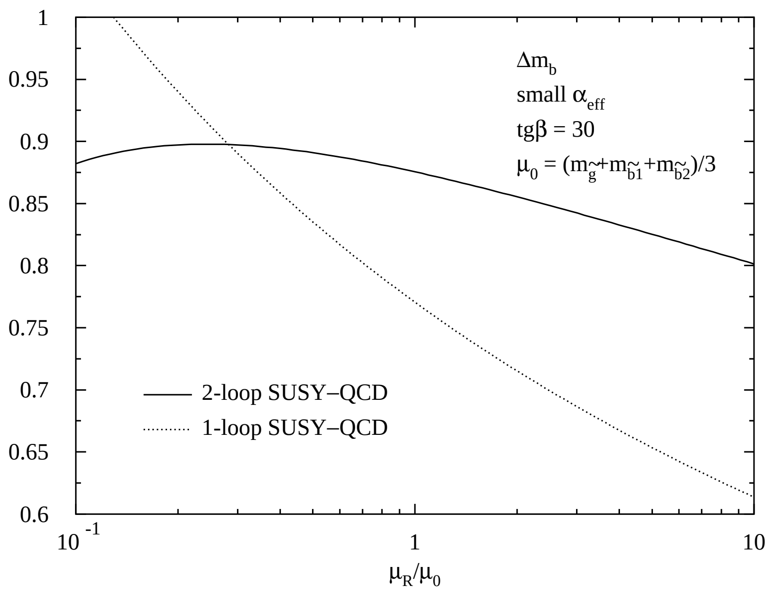
$$A_t, M_{\tilde{b}_1}^2, M_{\tilde{b}_2}^2, M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2, M_{\tilde{g}}$$

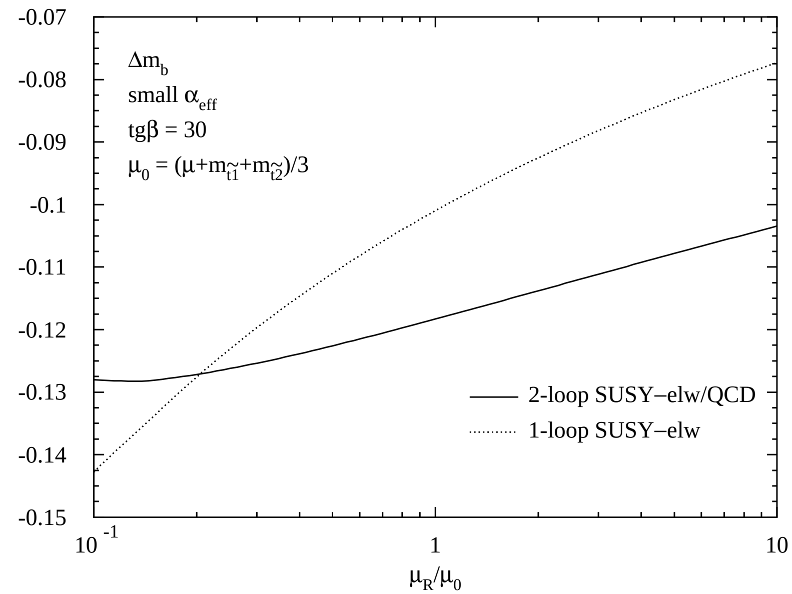
The strong coupling constant and the top Yukawa coupling are renormalised in the renormalisation scale dependent Collins-Wilczek-Zee scheme with 5 active flavours :

$$\alpha_s(\mu_R^2) \qquad \lambda_t(\mu_R^2)$$

Results – Renormalisation Scale Dependance

Due to the couplings $\alpha_s(\mu_R^2)$ and $\lambda_t(\mu_R^2)$, the corrections Δm_b depend on the renormalisation scale μ_R





At the 2-loop order (NNLO), the renormalisation scale dependence is reduced and a plateau appears at ~ 1/4 of the central scales

$$\mu_0^{QCD} = \frac{(M_{\tilde{b}_1} + M_{\tilde{b}_2} + M_{\tilde{g}})}{3} \quad , \quad \mu_0^{EW} = \frac{(M_{\tilde{t}_1} + M_{\tilde{t}_2} + \mu)}{3}$$

Results – Partial Decay Width $\Gamma(\Phi \to b\bar{b})\,,\ \Phi = h, H, A$

Partial Decay Width with QCD corrections :

$$\Gamma_{QCD}(\Phi \to b\bar{b}) = \frac{3 G_F M_\Phi}{4\sqrt{2}\pi}\, \overline{m}_b^2(M_\Phi)\ (g_b^\Phi)^2\ [1 + \Delta_{QCD}]$$

Braaten et al., Sakai, Inami et al., Gorishny et al., Drees et al., Kataev et al., Surguladze, Chetyrkin et al., Vermaseren et al.

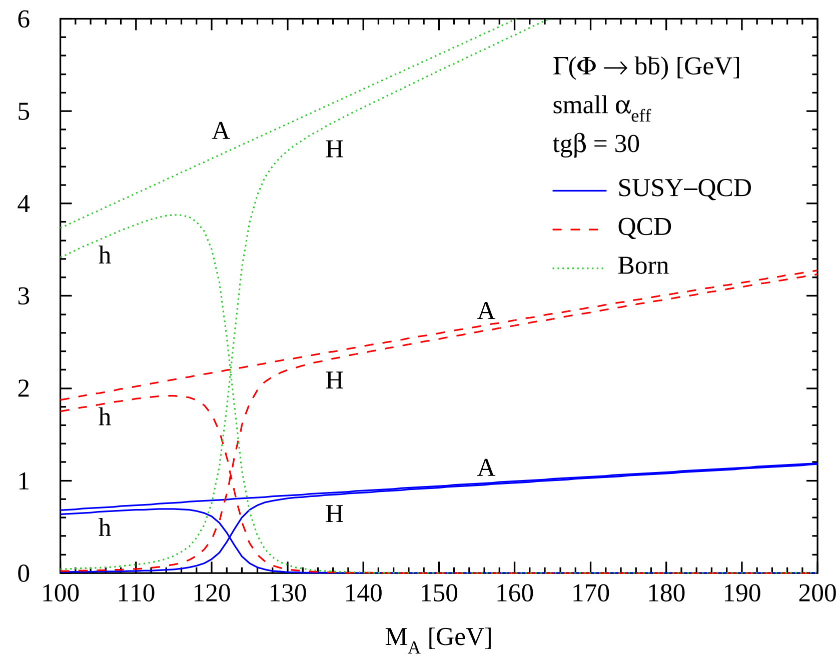
Partial Decay Width with SUSY-QCD corrections : $g_b^\Phi \to \tilde{g}_b^\Phi$

$$\tilde{\Gamma}(\Phi \to b\bar{b}) = \frac{3 G_F M_\Phi}{4\sqrt{2}\pi}\, \overline{m}_b^2(M_\Phi)\ (\tilde{g}_b^\Phi)^2\ [1 + \Delta_{QCD}]$$

NLO :
Carena et al., Hall et al.,
Carena, Garcia, Nierste, Wagner,
Guasch, Häfliger, Spira

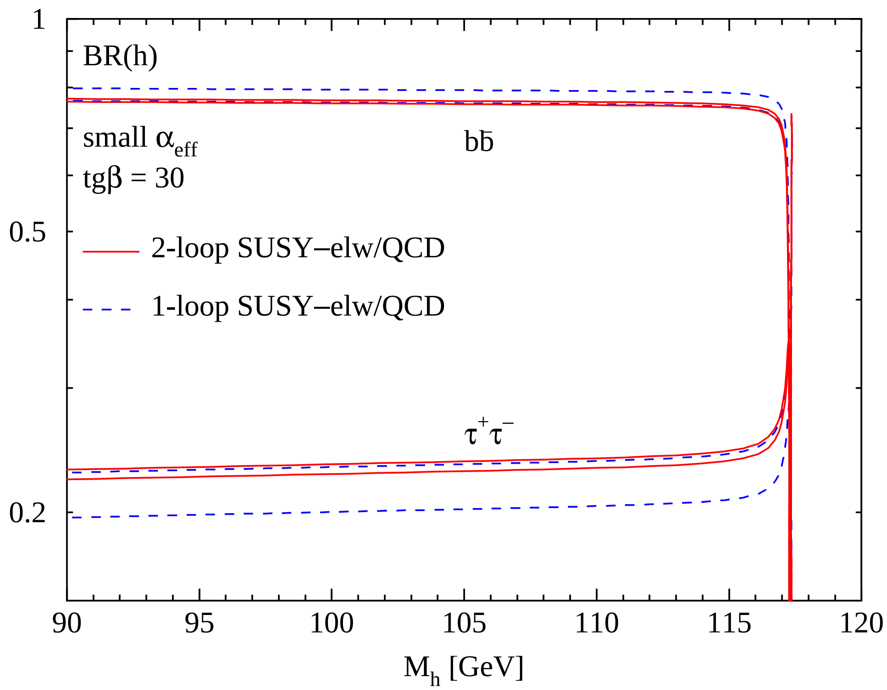
The partial decay width $\tilde{\Gamma}(\Phi \to b\bar{b})$ with the novel NNLO couplings $\tilde{g}_b^\Phi$ was included in the Computer Program HDECAY.

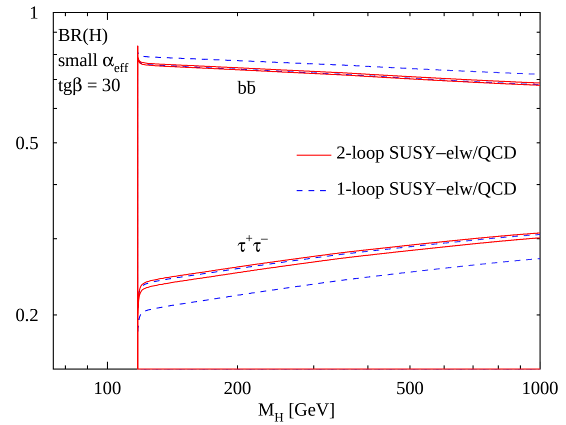
Djouadi, Kalinowski, Spira

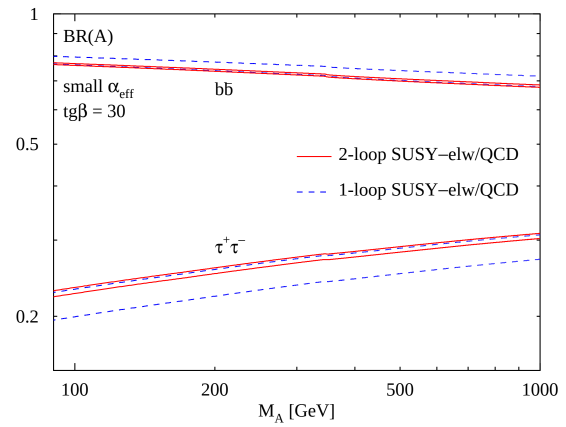


Results – Branching Ratios of the neutral Higgs bosons $\Phi = h, H, A$

Branching Ratio : $BR_j = \frac{\Gamma_j}{\sum_i \Gamma_i}$







Blue and red bands indicate the renormalisation scale dependence at 1-loop and 2-loop order when varying μ_R between 1/3 and 3 times the central scales

$$\mu_0^{QCD} = \frac{(M_{\tilde{b}_1} + M_{\tilde{b}_2} + M_{\tilde{g}})}{3} \quad , \quad \mu_0^{EW} = \frac{(M_{\tilde{t}_1} + M_{\tilde{t}_2} + \mu)}{3}$$

The renormalisation scale dependence and therefore the theoretical uncertainty is reduced from $\mathcal{O}$(10%) at NLO to $\mathcal{O}$(1%) at NNLO

We have calculated the NNLO corrections to the effective bottom Yukawa couplings in the MSSM.

- Significant reduction of the theoretical uncertainty from $\mathcal{O}$(10%) down to $\mathcal{O}$(1%)
- This reduces the theoretical uncertainty of all processes and parameters that involve the bottom Yukawa coupling, as e.g.

$b\bar{b}\Phi$	associated Higgs production
$gg \rightarrow \Phi$	gluon fusion with b-loops
$tbH^{\pm}$	associated charged Higgs production

- The obtained results can be implemented easily in the corresponding programs ($g_b^{\Phi} \rightarrow \tilde{g}_b^{\Phi}$)

Thank You!