MadGraph/MadEvent with spin-2 particles: Graviton plus monojet and dijet production at the LHC

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- MadGraph/MadEvent for Extra Dimension models

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Introduction

Extra Dimensions (ED)

- Search for extra dimensions has been one of the major objects at the LHC, since its physical effects can appear at the TeV energy scale.
- Two major classes of Extra Dimensions models:
  - **“Flat” (factorizable) ED**
    - Large ED (ADD) (Arkani-Hamed, Dvali & Dimopoulos)
    - TeV^{-1} ED (variation of ADD)
    - Universal ED(UED) (Appelquist, Cheng & Dobrescu)
    - ...
  - **“Warped” (non-factorizable) ED**
    - Randall-Sundrum(RS) model
    - ...

ADD Model (Arkani-Hamed, Dvali & Dimopoulos)

- Assuming the δ-extra dimensions are compacted into δ torus with the same radius r,the metrics in ADD Model are given by:

$$ds^2 = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu - r^2 d\Omega_\delta^2 + ...,$$

- From dimensional analysis, the scalar curvature $[R^{(4+\delta)}] = 2$, thus the D-dimensional(D=4+δ) Einstein-Hilbert (EH) action is

$$S_D = -\overline{M}_s^{\delta+2} \int d^{4+\delta}x \sqrt{|g^{(4+\delta)}|}[R^{(4+\delta)}],$$

compared with 4-dimensional EH action

$$S_4 = -\overline{M}_{Pl}^2 \int d^4x \sqrt{|g^{(4)}|}[R^{(4)}],$$

we have

$$\overline{M}_{Pl}^2 = \overline{M}_s^{\delta+2}(2\pi r)^\delta = r^\delta M_s^{\delta+2}$$

ADD Model: Large extra dimensions

- If r is quite small, at the order of Planck length, then $M_s \sim \overline{M}_{Pl}$. The effects of extra dimensions will be negligible. But...
- The Seattle Experiment probed directly gravity and tested Newton's law only at Submillimeter level (0.2mm). Thus the length of extra dimensions can be much larger than the Planck length.
- Possibility of TeV scale extra dimensions:
    - If $\delta = 1$ and $M_s \sim 1\text{TeV}$, $\rightarrow r \sim 10^{15}\text{cm}$, excluded,
    - If $\delta = 2$ and $r < 0.2\text{mm}$, $\rightarrow M_s > 1.5\text{TeV}$,
    - If $\delta > 2$ and $M_s \sim\text{TeV}$, $\rightarrow r < 10^{-6}\text{cm}$, thus it will be difficult to probe extra dimensions by direct gravity test: High Energy Colliders.

ADD Model: Kaluza-Klein (KK) tower

- In ADD model, there is an infinite tower of 4D KK modes.

$$\mathcal{L}_{int} = -\frac{1}{\overline{M}_{Pl}} \sum_{\vec{n}} (h^{(\vec{n})})^{\mu\nu} T_{\mu\nu},$$

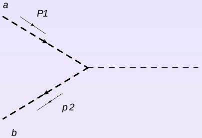
- The mass of the $\vec{n}$-th KK mode $h^{(\vec{n})}_{\mu\nu}$ is $|\vec{n}|/r$.

$$\Delta m \sim \frac{1}{r} = M_s \left(\frac{M_s}{\overline{M}_{Pl}}\right)^{2/\delta} \sim \left(\frac{M_s}{\mathrm{TeV}}\right)^{\frac{\delta+2}{2}} 10^{\frac{12\delta-31}{\delta}} \mathrm{eV}.$$

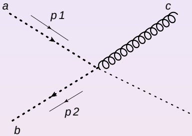
  - For $M_s = 1$TeV and $\delta =$4, 6 and 8, Δm=20KeV, 7MeV and 0.1GeV, respectively. Thus for $\delta \leq 6$, the KK tower can be looked as continuous.
  - Mass density function:

$$d\vec{n} = S_{\delta-1}|\vec{n}|^{\delta-1} d|\vec{n}| = S_{\delta-1}\frac{\overline{M}^2_{Pl}}{M_s^{2+\delta}} m^{\delta-1} dm, \mathrm{with}\ \mathrm{S}_{\delta-1} = \frac{2\pi^{\delta/2}}{\Gamma(\delta/2)}.$$

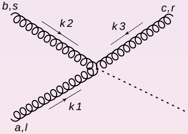
Examples on Feynman rules



$$-\frac{i}{\overline{M}_{Pl}}\delta^{ab}C_{\mu\nu,\rho\sigma}P_1^{\rho}P_2^{\sigma}$$



$$-\frac{g_s}{\overline{M}_{Pl}}f^{abc}C_{\mu\nu,\rho\sigma}P_2^{\sigma}$$



$$\frac{g_s}{\overline{M}_{Pl}}f^{abc}\Big[C_{\mu\nu,ls}(k_{1r}-k_{2r})+C_{\mu\nu,lr}(k_{3s}-k_{1s})$$
$$+C_{\mu\nu,sr}(k_{2l}-k_{3l})+F_{\mu\nu,lsr}(k_1,k_2,k_3)\Big]$$

$$C_{\mu\nu,\rho\sigma}=\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho}-\eta_{\mu\nu}\eta_{\rho\sigma},$$
$$F_{\mu\nu,\rho\sigma\lambda}(k_1,k_2,k_3)=\eta_{\mu\rho}\eta_{\sigma\lambda}(k_2-k_3)_\nu+\eta_{\mu\sigma}\eta_{\rho\lambda}(k_3-k_1)_\nu$$
$$+\eta_{\mu\lambda}\eta_{\rho\sigma}(k_1-k_2)_\nu+(\mu\leftrightarrow\nu).$$

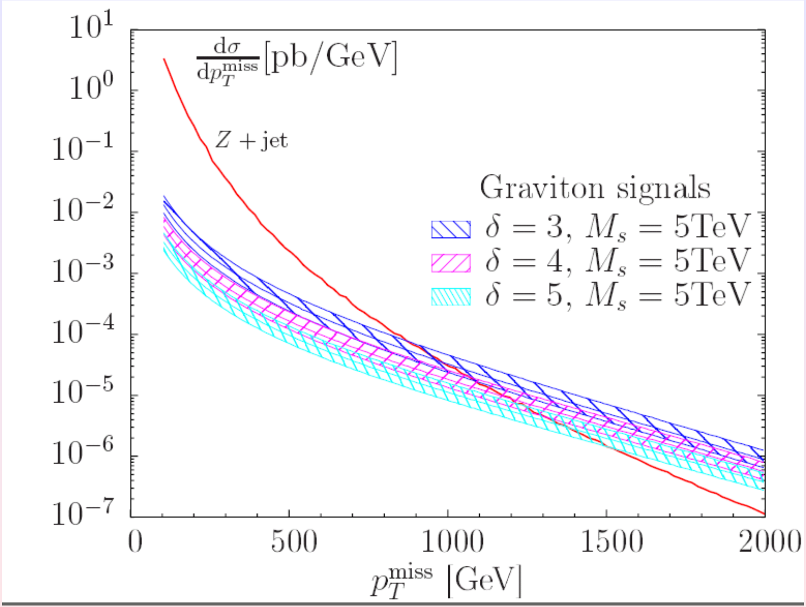
Constraints and Collider Phenomenology

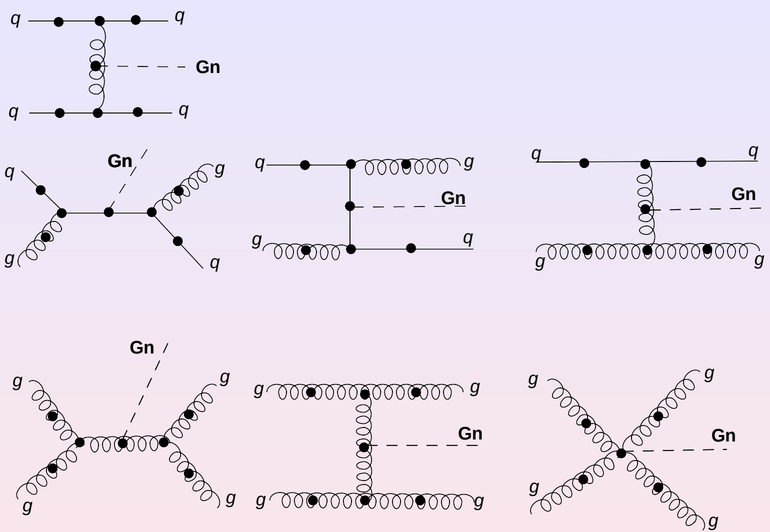
- Supernova (SN1987A) cooling constraints:
  $M_s > 30\text{TeV}$ (δ=2) and 4TeV (δ=3).
  However, the constraints can be relaxed easily without large change on collider phenomenology.

- Virtual KK-graviton exchange and direct KK-graviton production (Missing Energy):
  LEP and Tevatron datas lead to $M_s > 1.2\text{TeV}(\delta=3)$ and $0.83\text{TeV}(\delta=6)$.

Graviton production with jet(s)

- At the LHC, Graviton production with monojet has been studied and found to have strong ability to probe higher extra dimension scale: jet+missing Energy
- It is natural and important to study the Graviton dijets production: more soft jets, more information
- Moreover, to improve the theoretical accuracy, NLO QCD corrections needed.
- $\Delta R_{jj} > 0.7$, $|\eta_j| < 4.5$, $P_T^{\mathrm{miss}} > 500$ GeV, CTEQ6L1
  - $\mu_r = \mu_f = P_T^j/2$: 0.093pb for $\delta = 3$, 0.049pb for $\delta = 4$
  - $\mu_r = \mu_f = 2P_T^j$: 0.058pb for $\delta = 3$, 0.03pb for $\delta = 4$
- See also on next page the scale dependence of P_T^{miss} distributions for $\mu_r = \mu_f = 3\sqrt{\hat{s}}, \sqrt{\hat{s}}/3$


$\frac{d\sigma}{dp_T^{miss}}$[pb/GeV]
$Z+\text{jet}$
Graviton signals
$\delta=3,\ M_s=5\text{TeV}$
$\delta=4,\ M_s=5\text{TeV}$
$\delta=5,\ M_s=5\text{TeV}$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
500
1000
1500
2000
p_T^{miss} [GeV]



Large number of subprocesses and Feynman diagrams:
$\Longrightarrow$ Helicity amplitude method, MadGraph/MadEvent.

New HELAS Subroutines for spin-2 particles

New HELAS Subroutines

- HELAS (Helicity Amplitudes Subroutines) [MURAYAMA, WATANABE, HAGIWARA, 1992] is a set of Fortran77 subroutines which make it easy to compute the helicity amplitudes of an arbitrary tree-level Feynman diagram with a simple sequence of CALL SUBROUTINE statements.

- Calculating steps of Helicity amplitude:
  1. Getting the external particles' wave functions;
  2. Computing the off-shell lines;
  3. Calculating the helicity amplitude.

- Tensor's wavefunctions $\epsilon^{(*)}_{\mu\nu}(P, M)$:
  TXXXXX(P, M, λ(=0,±1, ±2), IF = ±, TWF)
  $\epsilon^{+}_{\mu}\epsilon^{+}_{\nu}$, $\frac{1}{\sqrt{2}}(\epsilon^{+}_{\mu}\epsilon^{0}_{\nu} + \epsilon^{0}_{\mu}\epsilon^{+}_{\nu})$,$\frac{1}{\sqrt{6}}(\epsilon^{+}_{\mu}\epsilon^{-}_{\nu} + \epsilon^{-}_{\mu}\epsilon^{+}_{\nu} + 2\epsilon^{0}_{\mu}\epsilon^{0}_{\nu})$, $\frac{1}{\sqrt{2}}(\epsilon^{-}_{\mu}\epsilon^{0}_{\nu} + \epsilon^{0}_{\mu}\epsilon^{-}_{\nu})$, $\epsilon^{-}_{\mu}\epsilon^{-}_{\nu}$.

- HELAS Subroutines for graviton's interaction:

  1. SST: SSTXXX, HSTXXX, USSXXX;
  2. FFT: IOTXXX, FTIKXX, FTOXXX, UIOXXX;
  3. FFVT: IOVTXX, FVTIXX, FVTOXX, JIOTXX, UIOVXX;
  4. VVT: VVTXXX, JVTXXX, UVVXXX;
  5. GGGT: GGGTXX, JGGTXX, UGGGXX;
  6. GGGGT: GGGGTX, JGGGTX, UGGGGX.

- Example: FVTIXX(fi, vc, tc, g, fmass, fwidth, fvti)

$$\frac{(\not{k})+m_f}{k^2-m_f^2+im_f\Gamma_f}[2\eta_{\mu\nu}\eta_{\rho\sigma}-\eta_{\mu\rho}\eta_{\nu\sigma}-\eta_{\mu\sigma}\eta_{\nu\rho}]\gamma^\sigma(g_1P_L+g_2P_R)\psi(k_1)$$
$$\times\epsilon^\rho(k_2)\epsilon^{\mu\nu}(k_3),$$

- Gauge invariance checking:
  $q\bar{q} \to ST$: SSTXXX, HSTXXX;
  $q\bar{q} \to gT$: IOTXXX, FTIXXX, FTOXXX, IOVTXX, VVTXXX;
  $gg \to gT$: GGGTXX, JVTXXX;
  $q\bar{q} \to ggT$: FVTIXX, FVTOXX, JIOTXX, UIOXXX, UVVXXX, UIOTXX;
  $gg \to ggT$: JGGGXX, UGGGXX, GGGGXX;

MadGraph/MadEvent for Extra Dimension models

MadGraph/MadEvent

- MadGraph (T. Stelzer and W. F. Long): Automatically generating the Feynman diagrams and a Fortran subroutine to calculate the squared amplitudes by calling Helas subroutine; Easy to implementing new models; Summing over protons, jets, leptons and others.
- MadEvent (F. Maltoni and T. Stelzer): Multi-purpose event generator; using the matrix elements and phase space mapping generated by MadGraph; Interface for further hadronization and detector simulation.

Modifying MG/ME for our purpose

- Use the User Model framework in MG, we make our new model directories for both the ADD and RS models, including the massive gravitons and their interactions with the SM particles.
- Insert all the new HELAS subroutines for spin-2 tensor bosons into the HELAS library in MG
- Modify the codes in MG to tell it how to generate the SST, FFT, VVT, VVVT and FFVT type of vertices and helicity amplitudes, and how to deal with the helicity of the spin-2 tensor bosons when they are external.
- Moreover, since MG can only generate Feynman diagrams with up to 4-point vertices, the amplitudes and their HELAS codes have been added by hand
- Modify the phase space generating codes in ME to add one more random number for graviton mass generating and implement the graviton mass integration.

Graviton plus two jets production

Background

We consider the most important ones

- *Zjj* production with subsequent decay $Z \to \nu\bar{\nu}$;
- *Wjj* with subsequent decay $W^{\pm} \to l^{\pm}\nu$ when the charged leptons $l = e, \mu, \tau$ are not identified.

We used the codes based on

V. D. Barger, T. Han, J. Ohnemus and D. Zeppenfeld, "Large p(t) Weak Boson Production at the Tevatron," Phys. Rev. Lett. **62**, 1971 (1989);

V. D. Barger, T. Han, J. Ohnemus and D. Zeppenfeld, Phys. Rev. D **40**, 2888 (1989) [Erratum-ibid. D **41**, 1715 (1990)],

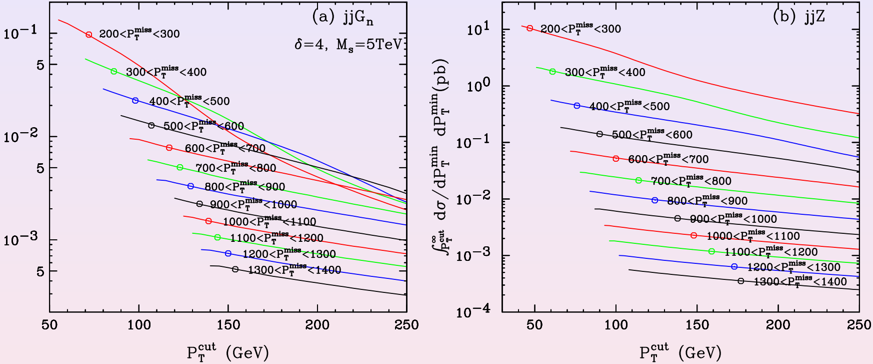
and checked by MadGraph/MadEvent.

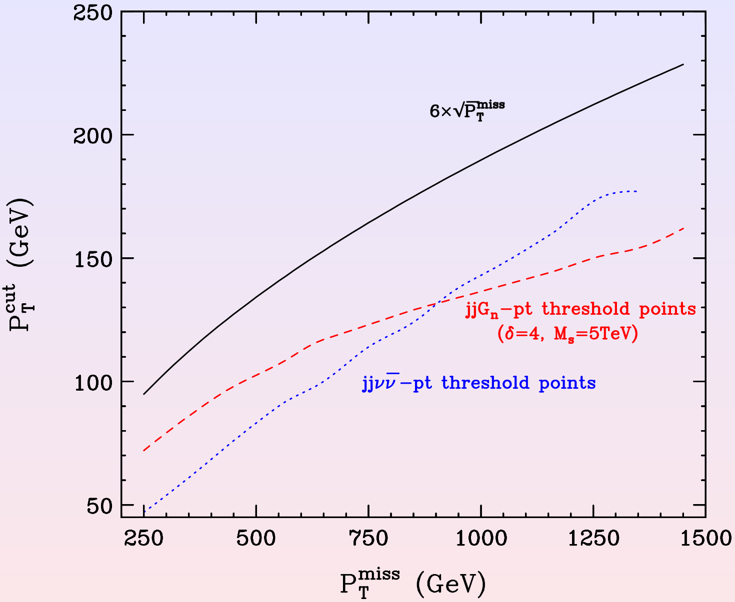
Two independent calculations for signal

- We performed two independent calculations, and found agreement between them. Both calculations used the helicity amplitude technique, and have been checked by making use of the invariance under the gauge and general coordinate transformations.
  - 1. Generating the MC codes for calculations of radiated gravitons ($e^+e^- \to l^+l^-G_n$) at linear colliders (hep-ph/0307117 and hep-ph/0509161). Since the calculation order can be chosen, so only several new Helas subroutines for Graviton added: *TXXXXX*, *VVTXXX*, *VVVTXX*, *VVVVTX*, *IOTXXX* and *IOVTXX*.
  - 2. Modified MadGraph/MadEvent.

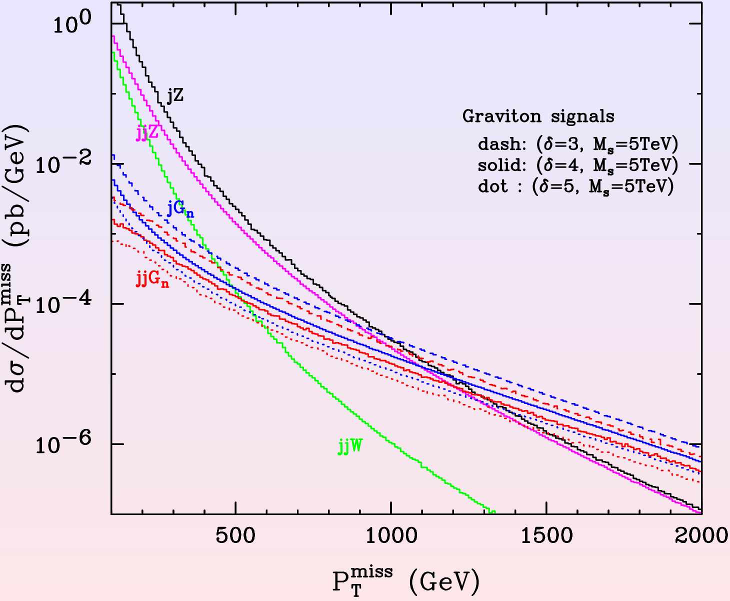
Inputs

- PDF: Cteq6L1, μ_f=min(P_T) of the jets,
- Using $\sqrt{\alpha_s(P_T^{j1})\alpha_s(P_T^{j2})}$, with $\alpha_s(m_Z) = 0.13$
- $\Delta R_{jj} > 0.7$, $|\eta_j| < 4.5$
- $P_T^{\mathrm{miss}} > 1$ TeV unless specified,
- P_T^j cut will be studied in detail to make results perturbatively reliable.
- Focus on $\delta = 4$ and $M_s = 5$ TeV first, then discuss the scale sensitivity (2 TeV$<$ M_s $<$10 TeV) and present the differential distributions for various δ (=3,4,5 and 6).


(a) jjG$_n$
δ=4, M$_s$=5TeV
200<P_T^{miss}<300
300<P_T^{miss}<400
400<P_T^{miss}<500
500<P_T^{miss}<600
600<P_T^{miss}<700
700<P_T^{miss}<800
800<P_T^{miss}<900
900<P_T^{miss}<1000
1000<P_T^{miss}<1100
1100<P_T^{miss}<1200
1200<P_T^{miss}<1300
1300<P_T^{miss}<1400
10^{-1}
5
10^{-2}
5
10^{-3}
5
50
100
150
200
250
P_T^{cut} (GeV)
(b) jjZ
200<P_T^{miss}<300
300<P_T^{miss}<400
400<P_T^{miss}<500
500<P_T^{miss}<600
600<P_T^{miss}<700
700<P_T^{miss}<800
800<P_T^{miss}<900
900<P_T^{miss}<1000
1000<P_T^{miss}<1100
1100<P_T^{miss}<1200
1200<P_T^{miss}<1300
1300<P_T^{miss}<1400
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$\int_{P_T^{cut}}^{\infty} d\sigma/dP_T^{min}\, dP_T^{min}$(pb)
50
100
150
200
250
P_T^{cut} (GeV)

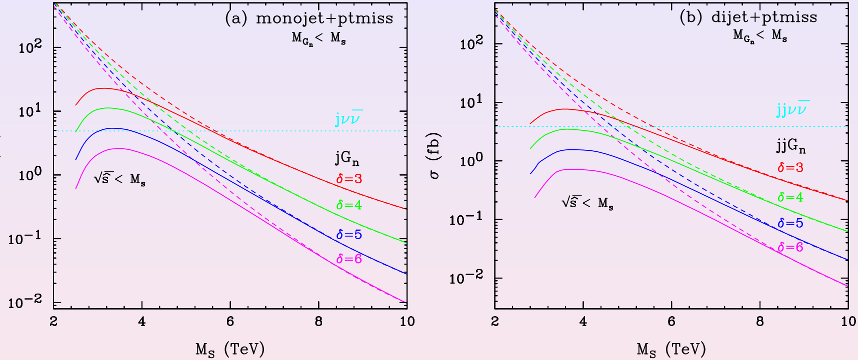


Below we will always use $P_T^j > 6 \times \sqrt{P_T^{\rm miss}/1{\rm GeV}}$ GeV.


Graviton signals
dash: (δ=3, M_s=5TeV)
solid: (δ=4, M_s=5TeV)
dot : (δ=5, M_s=5TeV)
jZ
jjZ
jG_n
jjG_n
jjW
10^0
10^{-2}
10^{-4}
10^{-6}
500
1000
1500
2000
$d\sigma/dP_T^{miss}$ (pb/GeV)
P_T^{miss} (GeV)

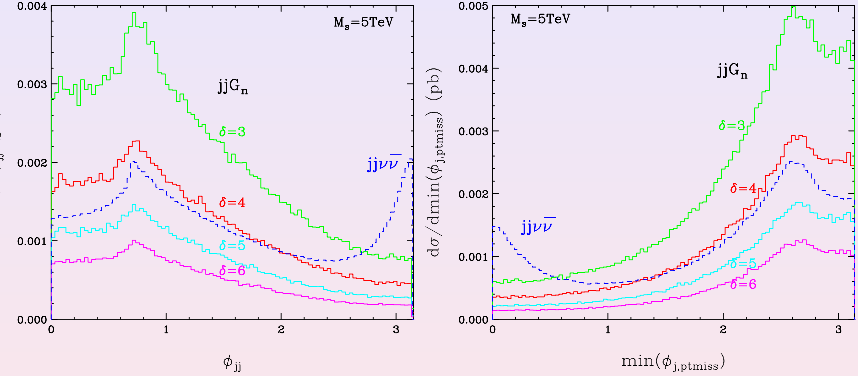
- Once dealing with this effective low-energy theory, one concern is its behavior above the ADD fundamental scale (M_s).
- We always use the unitarity criterion $M_{G_n} < M_s$, and furthur present the results within hard truncation scheme: $\sqrt{\hat{s}} < M_s$.
- We have also performed the simple sensitivity analysis as in the previous monojet study, considering the integrated luminosity $\mathcal{L} = 100\ \mathrm{fb}^{-1}$, where the systematic error in the background (assumed to be 10%) dominates over the statistical error. The sensitivity range is defined by

$$\sigma_{jjG_n}(\sigma_{jG_n}) > 5 \times 10\% \times \sigma_{\mathrm{background}} = 1.93\ (2.45)\,\mathrm{fb}.$$


(a) monojet+ptmiss
$M_{G_n} < M_s$
$j\nu\bar{\nu}$
jG_n
δ=3
δ=4
δ=5
δ=6
$\sqrt{\hat{s}} < M_s$
10^2
10^1
10^0
10^{-1}
10^{-2}
2
4
6
8
10
M_S (TeV)
(b) dijet+ptmiss
$M_{G_n} < M_s$
$jj\nu\bar{\nu}$
jjG_n
δ=3
δ=4
δ=5
δ=6
$\sqrt{\hat{s}} < M_s$
σ (fb)
10^2
10^1
10^0
10^{-1}
10^{-2}
2
4
6
8
10
M_S (TeV)

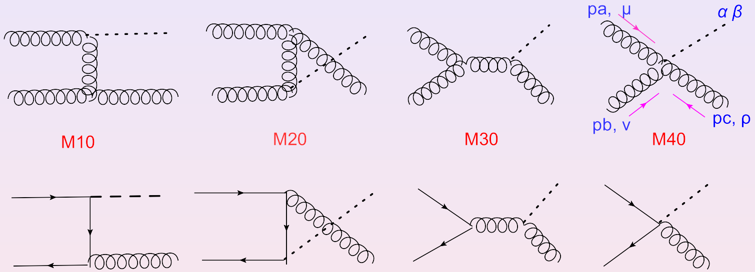
	Max M_s sensitivity	Max M_s sensitivity
	$\mathcal{L}$ =100 fb^{-1}	$\mathcal{L}$ =100 fb^{-1}
δ	No truncation	Hard truncation ($Q_{trunc} = \sqrt{\hat{s}} < M_s$)
3	6.4 (6.6) TeV	6.3 (6.5)TeV
4	5.6 (5.7)	5.1 (5.5)
5	5.2 (5.3)	- (4.8)
6	4.9 (5.0)	- (3.6)

TABLE I: Maximum ADD scale M_s sensitivity which can be reached by studying the 2 jets (1jet) and missing transverse momentum signal at the LHC, with integrated luminosity $\mathcal{L}$ =100 fb^{-1} or 10 fb^{-1}, assuming the systematic error to be 10%. The sensitivity range is defined by $\sigma_{jjG_n}(\sigma_{jG_n}) > 5(10\%)\sigma_{background} = 1.93(2.45)fb$.

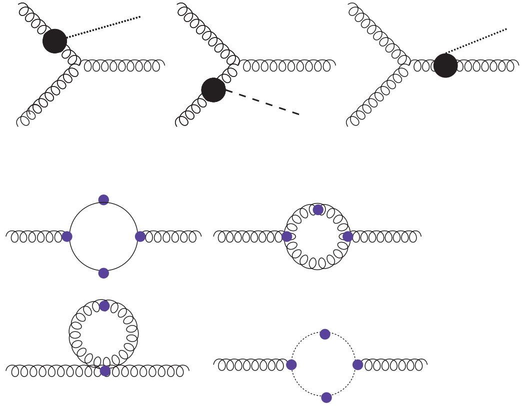

0.004
0.003
0.002
0.001
0.000
0
1
2
3
M_s=5TeV
jjG_n
δ=3
δ=4
δ=5
δ=6
$jj\nu\bar{\nu}$
ϕ_{jj}
0.005
0.004
0.003
0.002
0.001
0.000
$d\sigma/d\min(\phi_{j,ptmiss})$ (pb)
M_s=5TeV
jjG_n
δ=3
δ=4
δ=5
δ=6
$jj\nu\bar{\nu}$
$\min(\phi_{j,ptmiss})$

NLO QCD corrections to Graviton monojet production

LO Feynman Diagrams



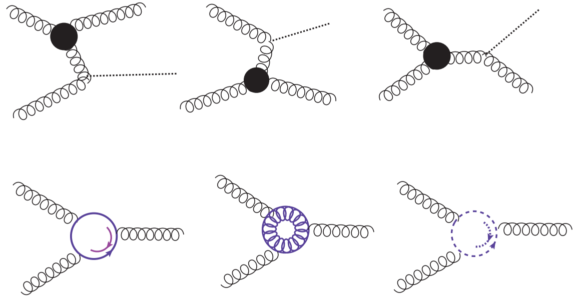
Set A: NLO QCD corrections to $gg\tilde{G}$ vertex.



$$M^A_{count} = \delta Z_g(M_{10} + M_{20} + M_{30}),$$

$$\delta Z_g = -\frac{\alpha_s}{2\pi}\Gamma(1+\epsilon)(4\pi)^\epsilon(\frac{n_f}{3} - \frac{5}{2})(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}).$$

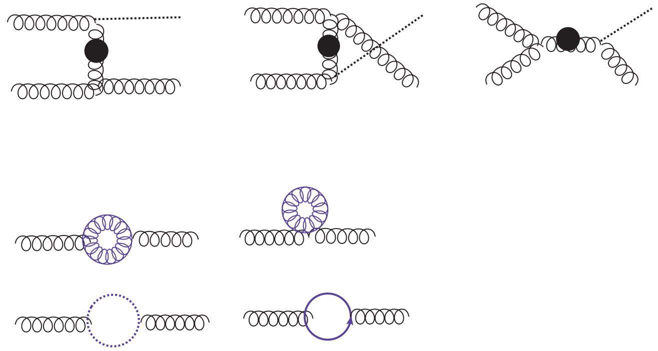
Set B: NLO QCD corrections to *ggg* vertex.



$$M^B_{count} = (\frac{3}{2}\delta Z_g + \frac{\delta g_s}{g_s})(M_{10} + M_{20} + M_{30}),$$

$$\frac{\delta g_s}{g_s} = -\frac{\alpha_s}{4\pi}\Gamma(1+\epsilon)(4\pi)^\epsilon \frac{\beta_0}{2\epsilon_{UV}}, \;\; \beta_0 = 11 - \frac{2n_f}{3}. \; (\overline{MS}\ scheme)$$

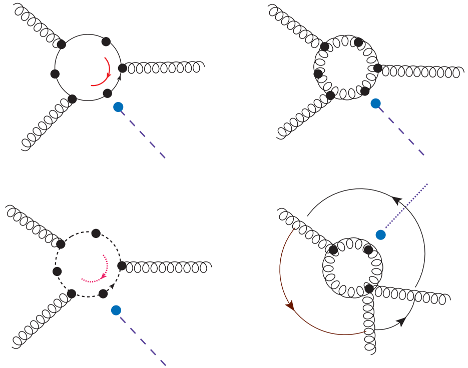
Set C: Self energy diagrams.



$$M^{C}_{count} = -\delta Z_g (M_{10} + M_{20} + M_{30}).$$

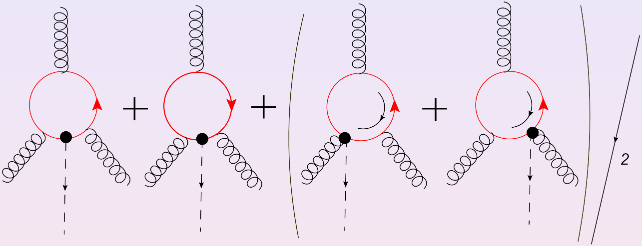
$$\Leftarrow M^{propagator}_{count} = -i\delta Z_g (g^{\mu\nu} p^2 - p^{\mu} p^{\nu})$$

Set D: "Box" diagrams.



$$M^D_{count} = (\frac{3}{2}\delta Z_g + \frac{\delta g_s}{g_s})M_{40}.$$

But of course, we do not need to calculate every diagrams. Take fermion loops as an example:



Others can be got by doing permutation:

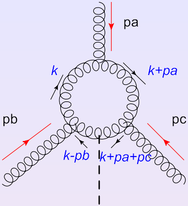
(1). $pa \leftrightarrow pb$, $pb \leftrightarrow pc$, $pc \leftrightarrow pa$,

(2). $pa \leftrightarrow pc$, $pb \leftrightarrow pa$, $pc \leftrightarrow pb$,

with color, Lorenz indices changing under the same way (Notice $f_{bca} = f_{cab} = f_{abc}$).

Calculating conventions

- Dimension regularization, with $D = 4 - 2\epsilon$,
  Set LimitTo4 = False, SetOptions[PaVeReduce, Dimension → D] and SetOptions[OneLoop, Dimension → D] in FeynCalc.
- Following the convention for the one-loop integrals of Fortschritte der Physik 1992, 41 by Denner, to be consistent with FeynCalc and FF packages.
- Using FeynCalc to preform the momentum integration, and reduction of tensor coefficient with rank up to 3.
- Higher rank 4-point tensor coefficients D_4 and D_5 are handled by hand writing codes.



$$(i\pi^2)^{-1}(2\pi\mu)^{2\epsilon}\int d^D k \frac{k^\alpha k^\beta k^\mu k^\rho k^\nu}{k^2(k-pb)^2(k+pa)^2(k+pa+pc)^2}$$
$$= D^{\alpha\beta\mu\rho\nu}(P_1=-pb, P_2=pa, P_3=pa+pc)$$

$$\begin{aligned} D^{\alpha\beta\mu\rho\nu} &= \sum_{j=1}^{3} g^{[[\alpha\beta}g^{\mu]\rho}p_j^{\nu]}D_{0000j} \\ &+ \sum_{j,k,l,=1}^{3}(g^{[\alpha\beta}p_j^{\mu}p_k^{\rho}p_l^{\nu]} + g^{[\alpha\mu}p_j^{\nu}p_k^{\beta}p_l^{\rho]})D_{00jkl} \\ &+ \sum_{j,k,l,m,n=1}^{3} p_j^{\alpha}p_k^{\beta}p_l^{\mu}p_m^{\rho}p_n^{\nu}D_{jklmn}. \end{aligned}$$

The UV and Quadratic divergences of D_4 and D_5 can be easily got by using the recursion formulae, or be found in hep-ph/0212259.

UV divergences (represented by $1/(D-4)$) of $D^{\alpha\beta\mu\rho\nu}$:

$$(D-4)D_{0000i} = \frac{1}{48}$$

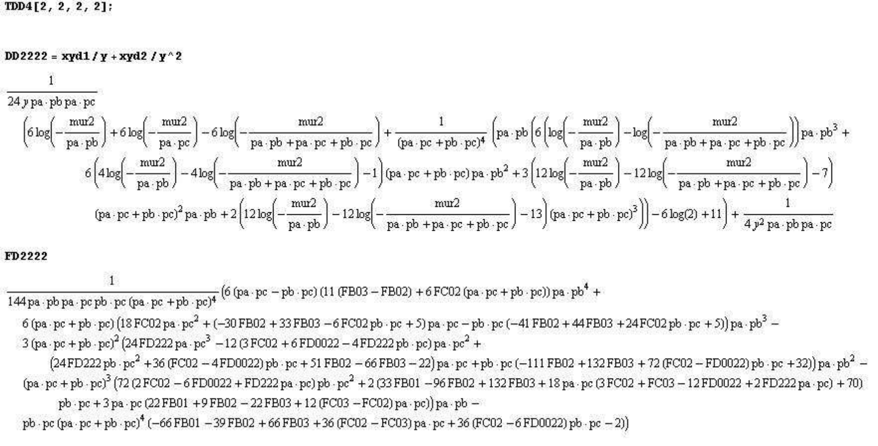
Quadratic divergences (represented by $1/(D-2)$, or A_0, if we add mass to internal particles) of $D^{\alpha\beta\mu\rho\nu}$:

$$D_{0000i},\ D_{00ijk}\ \mathrm{don't\ contain\ A_0}$$
$$D_{ijklm}\ \mathrm{contain\ (D-2)A_0}.$$

High rank tensor function reduction: D_4 and D_5

- Input all the B_0, C_0, D_0 needed (got from arXiv:0712.1851 by R.K.Ellis and G.Zanderighi, "Scalar one-loop integrals for QCD"), with divergent terms explicitly shown.
- Using FeynCalc to reduce C_i, C_{ij}, C_{ijk}, C_{ijkl}, D_i and D_{ij}, to get the divergent terms and finite terms (still quite short).
- Using recursion formulae to get the divergent and finite terms of D_{ijk}, D_{ijkl} and D_{ijklm} from the above things.
- Checking

Example 1: D2222



TDD4[2, 2, 2, 2];

DD2222 = xyd1 / y + xyd2 / y ^ 2

$$\frac{1}{24\, y\, pa\cdot pb\, pa\cdot pc}$$

$$\left(6\log\left(-\frac{mur2}{pa\cdot pb}\right)+6\log\left(-\frac{mur2}{pa\cdot pc}\right)-6\log\left(-\frac{mur2}{pa\cdot pb+pa\cdot pc+pb\cdot pc}\right)+\frac{1}{(pa\cdot pc+pb\cdot pc)^4}\left(pa\cdot pb\left(6\left(\log\left(-\frac{mur2}{pa\cdot pb}\right)-\log\left(-\frac{mur2}{pa\cdot pb+pa\cdot pc+pb\cdot pc}\right)\right)\right)pa\cdot pb^3+\right.\right.$$

$$6\left(4\log\left(-\frac{mur2}{pa\cdot pb}\right)-4\log\left(-\frac{mur2}{pa\cdot pb+pa\cdot pc+pb\cdot pc}\right)-1\right)(pa\cdot pc+pb\cdot pc)\,pa\cdot pb^2+3\left(12\log\left(-\frac{mur2}{pa\cdot pb}\right)-12\log\left(-\frac{mur2}{pa\cdot pb+pa\cdot pc+pb\cdot pc}\right)-7\right)$$

$$\left.\left.(pa\cdot pc+pb\cdot pc)^2\,pa\cdot pb+2\left(12\log\left(-\frac{mur2}{pa\cdot pb}\right)-12\log\left(-\frac{mur2}{pa\cdot pb+pa\cdot pc+pb\cdot pc}\right)-13\right)(pa\cdot pc+pb\cdot pc)^3\right)\right)-6\log(2)+11\Bigg)+\frac{1}{4\, y^2\, pa\cdot pb\, pa\cdot pc}$$

FD2222

$$\frac{1}{144\, pa\cdot pb\, pa\cdot pc\, pb\cdot pc\,(pa\cdot pc+pb\cdot pc)^4}(6\,(pa\cdot pc-pb\cdot pc)\,(11\,(FB03-FB02)+6\,FC02\,(pa\cdot pc+pb\cdot pc))\,pa\cdot pb^4+$$

$$6\,(pa\cdot pc+pb\cdot pc)\left(18\,FC02\,pa\cdot pc^2+(-30\,FB02+33\,FB03-6\,FC02\,pb\cdot pc+5)\,pa\cdot pc-pb\cdot pc\,(-41\,FB02+44\,FB03+24\,FC02\,pb\cdot pc+5)\right)pa\cdot pb^3-$$

$$3\,(pa\cdot pc+pb\cdot pc)^2\left(24\,FD222\,pa\cdot pc^3-12\,(3\,FC02+6\,FD0022-4\,FD222\,pb\cdot pc)\,pa\cdot pc^2+\right.$$

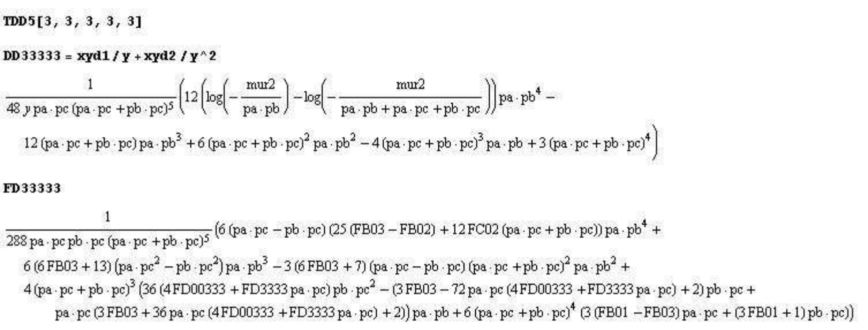
$$\left.\left(24\,FD222\,pb\cdot pc^2+36\,(FC02-4\,FD0022)\,pb\cdot pc+51\,FB02-66\,FB03-22\right)pa\cdot pc+pb\cdot pc\,(-111\,FB02+132\,FB03+72\,(FC02-FD0022)\,pb\cdot pc+32)\right)pa\cdot pb^2-$$

$$(pa\cdot pc+pb\cdot pc)^3\left(72\,(2\,FC02-6\,FD0022+FD222\,pa\cdot pc)\,pb\cdot pc^2+2\,(33\,FB01-96\,FB02+132\,FB03+18\,pa\cdot pc\,(3\,FC02+FC03-12\,FD0022+2\,FD222\,pa\cdot pc)+70)\right.$$

$$\left.pb\cdot pc+3\,pa\cdot pc\,(22\,FB01+9\,FB02-22\,FB03+12\,(FC03-FC02)\,pa\cdot pc)\right)pa\cdot pb-$$

$$pb\cdot pc\,(pa\cdot pc+pb\cdot pc)^4\,(-66\,FB01-39\,FB02+66\,FB03+36\,(FC02-FC03)\,pa\cdot pc+36\,(FC02-6\,FD0022)\,pb\cdot pc-2))$$

Example 2: D33333



TDD5[3, 3, 3, 3, 3]

DD33333 = xyd1 / y + xyd2 / y ^ 2

$$\frac{1}{48\,y\,\mathrm{pa}\cdot\mathrm{pc}\,(\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc})^5}\left(12\left(\log\left(-\frac{\mathrm{mur2}}{\mathrm{pa}\cdot\mathrm{pb}}\right)-\log\left(-\frac{\mathrm{mur2}}{\mathrm{pa}\cdot\mathrm{pb}+\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc}}\right)\right)\mathrm{pa}\cdot\mathrm{pb}^4 - 12\,(\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc})\,\mathrm{pa}\cdot\mathrm{pb}^3+6\,(\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc})^2\,\mathrm{pa}\cdot\mathrm{pb}^2-4\,(\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc})^3\,\mathrm{pa}\cdot\mathrm{pb}+3\,(\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc})^4\right)$$

FD33333

$$\frac{1}{288\,\mathrm{pa}\cdot\mathrm{pc}\,\mathrm{pb}\cdot\mathrm{pc}\,(\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc})^5}(6\,(\mathrm{pa}\cdot\mathrm{pc}-\mathrm{pb}\cdot\mathrm{pc})\,(25\,(\mathrm{FB03}-\mathrm{FB02})+12\,\mathrm{FC02}\,(\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc}))\,\mathrm{pa}\cdot\mathrm{pb}^4+ 6\,(6\,\mathrm{FB03}+13)\,(\mathrm{pa}\cdot\mathrm{pc}^2-\mathrm{pb}\cdot\mathrm{pc}^2)\,\mathrm{pa}\cdot\mathrm{pb}^3-3\,(6\,\mathrm{FB03}+7)\,(\mathrm{pa}\cdot\mathrm{pc}-\mathrm{pb}\cdot\mathrm{pc})\,(\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc})^2\,\mathrm{pa}\cdot\mathrm{pb}^2+ 4\,(\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc})^3\,(36\,(4\,\mathrm{FD00333}+\mathrm{FD3333}\,\mathrm{pa}\cdot\mathrm{pc})\,\mathrm{pb}\cdot\mathrm{pc}^2-(3\,\mathrm{FB03}-72\,\mathrm{pa}\cdot\mathrm{pc}\,(4\,\mathrm{FD00333}+\mathrm{FD3333}\,\mathrm{pa}\cdot\mathrm{pc})+2)\,\mathrm{pb}\cdot\mathrm{pc}+ \mathrm{pa}\cdot\mathrm{pc}\,(3\,\mathrm{FB03}+36\,\mathrm{pa}\cdot\mathrm{pc}\,(4\,\mathrm{FD00333}+\mathrm{FD3333}\,\mathrm{pa}\cdot\mathrm{pc})+2))\,\mathrm{pa}\cdot\mathrm{pb}+6\,(\mathrm{pa}\cdot\mathrm{pc}+\mathrm{pb}\cdot\mathrm{pc})^4\,(3\,(\mathrm{FB01}-\mathrm{FB03})\,\mathrm{pa}\cdot\mathrm{pc}+(3\,\mathrm{FB01}+1)\,\mathrm{pb}\cdot\mathrm{pc}))$$

Checking done

- Compared with FeynCalc PaVeReduce results
- Contracted $D^{\alpha\beta\mu\rho}$ and $D^{\alpha\beta\mu\rho\nu}$ with $g_{\alpha\beta}$, $P_{i\mu}$ and others, then compared, for example,

$$g_{\alpha\beta} D^{\alpha\beta\mu\rho\nu} = C^{\mu\rho\nu}(0)$$

with

$$g_{\alpha\beta} \Bigg\{ \sum_{j=1}^{3} g^{[[\alpha\beta} g^{\mu]\rho} p_j^{\nu]} D_{0000j}$$

$$+ \sum_{j,k,l,=1}^{3} (g^{[\alpha\beta} p_j^{\mu} p_k^{\rho} p_l^{\nu]} + g^{[\alpha\mu} p_j^{\nu} p_k^{\beta} p_l^{\rho]}) D_{00jkl}$$

$$+ \sum_{j,k,l,m,n=1}^{3} p_j^{\alpha} p_k^{\beta} p_l^{\mu} p_m^{\rho} p_n^{\nu} D_{jklmn} \Bigg\}.$$

Factorization into Born term

```
-------------nf --1/e--- ------------------

(AAf1 - (2 Pi ^2 * gs ^2 lo / 3)) // FCE // Collect[#, {Log[x_], MT[x_, y_]}, Simplify] &

0

------------gluon ghost --1/e--- --------------------

(temp + (11 Pi ^2 * gs ^2 lo)) // FCE // Collect[#, {Log[x_], MT[x_, y_]}, Simplify] &

0

------------gluon ghost Log term  1/e------------------

(temp2 + (3 Pi ^2 * gs ^2 lo)) // FCE // Collect[#, {Log[x_], MT[x_, y_]}, Simplify] &

0

(temp3 + (3 Pi ^2 * gs ^2 lo)) // FCE // Collect[#, {Log[x_], MT[x_, y_]}, Simplify] &

0

(temp4 + (3 Pi ^2 * gs ^2 llo)) // FCE // Collect[#, {Log[x_], MT[x_, y_]}, Simplify] &

0

--------------1/e^2---- -------------------------------

(AA2 + (9 Pi ^2 * gs ^2 llo)) // FCE // Collect[#, {Log[x_], MT[x_, y_]}, Simplify] &

0
```

- Collecting the above IR divergences, and including the counterterm contributions:

$$M^{virtual}|_{div} = \left(\frac{4\pi\mu_r^2}{2|pa\cdot pb|}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\frac{\alpha_s}{4\pi}M^{Born}\left[-\frac{3}{\epsilon^2}+\frac{1}{\epsilon}\left(\frac{n_f}{3}-\frac{11}{2}\right)\right]$$
$$+(pa\leftrightarrow pc)+(pb\leftrightarrow pc),$$

$$\Longrightarrow$$

$$2|M^{virtual}M^{Born*}|_{div}$$
$$= \left(\frac{4\pi\mu_r^2}{2|pa\cdot pb|}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\frac{\alpha_s}{2\pi}|M^{Born}|^2\left[-\frac{3}{\epsilon^2}+\frac{1}{\epsilon}\left(\frac{n_f}{3}-\frac{11}{2}\right)\right]$$
$$+(pa\leftrightarrow pc)+(pb\leftrightarrow pc).$$

The Substraction

- For hadronic state initial processes with $m+1$ partons in the final state, introduce dipole substraction term $d\sigma^A$ with the same singularity structure as $d\sigma^R$:

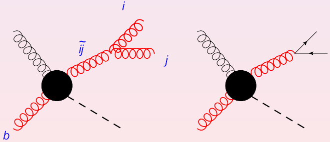
$$d\sigma^{NLO} = \int_{m+1}\left[d\sigma^R - d\sigma^A\right] + \int_{m+1} d\sigma^A + \int_m d\sigma^V + \int_m d\sigma^C,$$

$$\Longrightarrow$$

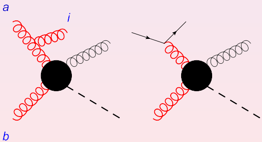
$$d\sigma^{NLO} = \int_{m+1}\left[d\sigma^R_{\epsilon=0} - d\sigma^A_{\epsilon=0}\right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A + d\sigma^C\right]_{\epsilon=0},$$

- $d\sigma^A$ is given by the sum of all possible dipole functions:

$$d\sigma^A = \Bigg[\sum_{k\neq i\neq j} \mathcal{D}_{ij,k} + \Bigg\{ \sum_{i\neq j} \mathcal{D}_{ij}^{a} + \sum_{k\neq i} \mathcal{D}_{k}^{ai} + + \sum_{i} \mathcal{D}^{ai,b} + (a \leftarrow b) \Bigg\} \Bigg] d\Phi_{m+1}$$



D^{b}_{ij}



$D^{ai,b}$

- After integrating the dipole term, we have

$$d\sigma^{\tilde{A}} + d\sigma^{C} = d\sigma^{B} \times \mathrm{I}(\epsilon) + \;......$$

and I is an universal operator (see Eq.(C.28) in hep-ph/9605323):

$$\mathrm{I}(\epsilon) = -\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\sum_{i}\frac{1}{T_i^2}\nu_i(\epsilon)\sum_{j\neq i}T_i\cdot T_j\left(\frac{4\pi\mu_r^2}{|2p_i\cdot p_j|}\right)^{\epsilon}$$

$$\nu_i(\epsilon) = T_i^2\left(\frac{1}{\epsilon^2}-\frac{\pi^2}{3}\right)+\gamma_i\frac{1}{\epsilon}+\;......,$$

In our case

$$T_g^2 = C_A,$$
$$2T_i\cdot T_j = T_k^2 - T_i^2 - T_j^2,\;(i\neq i\neq k)$$
$$\gamma_g = \frac{11}{6}C_A - \frac{2}{3}T_R n_f,$$

- So finally, we get the divergent parts of the substraction terms

$$d\sigma^{sub}|_{div} = \frac{\alpha_s}{4\pi}\frac{1}{\Gamma(1-\epsilon)}\sum_{i,j}\left(\frac{4\pi\mu_r^2}{|2p_i \cdot p_j|}\right)^{\epsilon}\left(\frac{3}{\epsilon^2}+\frac{11}{2\epsilon}-\frac{n_f}{3\epsilon}\right).$$

- Recall

$$\begin{aligned} &2|M^{virtual}M^{Born*}|_{div} \\ &= \left(\frac{4\pi\mu_r^2}{2|pa \cdot pb|}\right)^{\epsilon}\frac{1}{\Gamma(1-\epsilon)}\frac{\alpha_s}{2\pi}|M^{Born}|^2\left[-\frac{3}{\epsilon^2}+\frac{1}{\epsilon}\left(\frac{n_f}{3}-\frac{11}{2}\right)\right] \\ &+ (pa \leftrightarrow pc) + (pb \leftrightarrow pc). \end{aligned}$$

- Numerical works
    - Real corrections: on hand
    - Substraction terms: MadDipole
    - Loop corrections: on going

Summary

- We implemented Extra Dimension Models into MadGraph/MadEvent.
- We study graviton production in large extra dimension models via 2 jet plus missing transverse momentum signatures at the LHC.
- We present results for both the signal and the dominant Zjj and Wjj backgrounds, where we introduce missing P_T-dependent jet selection cuts that ensure the smallness of the 2-jet rate over the 1-jet rate to allow a perturbative fixed order analysis.
- Although the 2 jet results have slightly lower sensitivity to the scale of extra dimensions, the distributions of the two jets and their correlation with the missing transverse momentum provide additional evidence for the production of an invisible massive object.
- NLO QCD corrections to Graviton monojet production is on going, hopefully to be finished in next spring.