Standard Model Yukawa corrections to $b\bar{b}H$ production at the LHC

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partly based on ref. arXiv:hep-ph/0711.2005; Phys. Rev. D in press

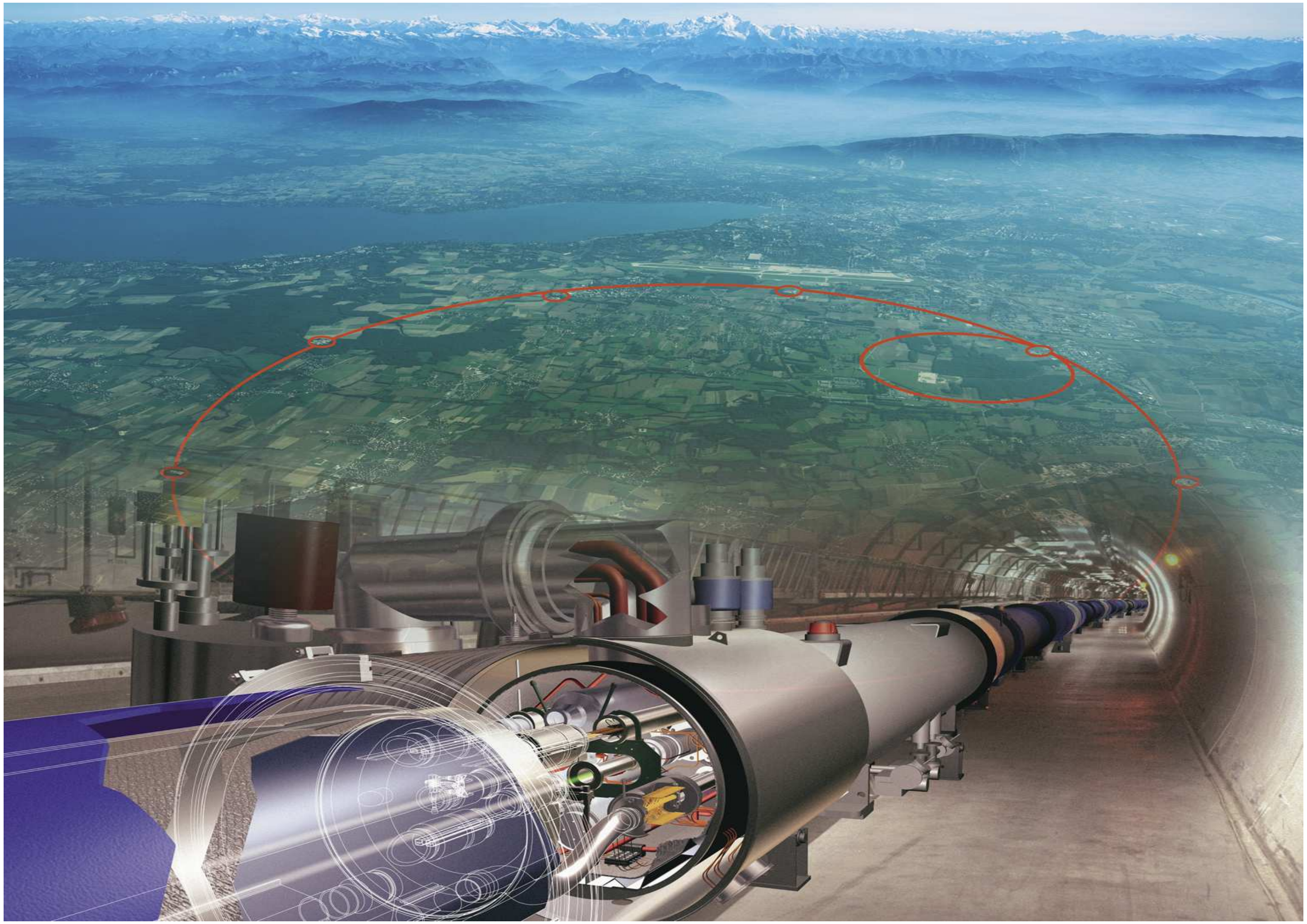
(work in collaboration with F. Boudjema)

Outline

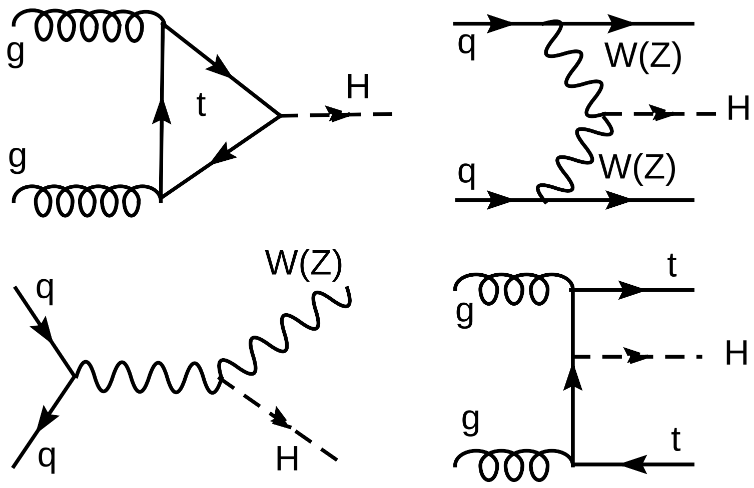
- Why $pp \to b\bar{b}H$?
- Tree level (LO).
- QCD correction at NLO.
- EW correction at NLO.
- EW correction when $\lambda_{bbH} = 0$.
- Landau singularities.
    - The problem
    - Conditions for Landau singularities
    - Nature of the singularities
    - How to solve the problem?
    - Examples
- Conclusions and outlooks.

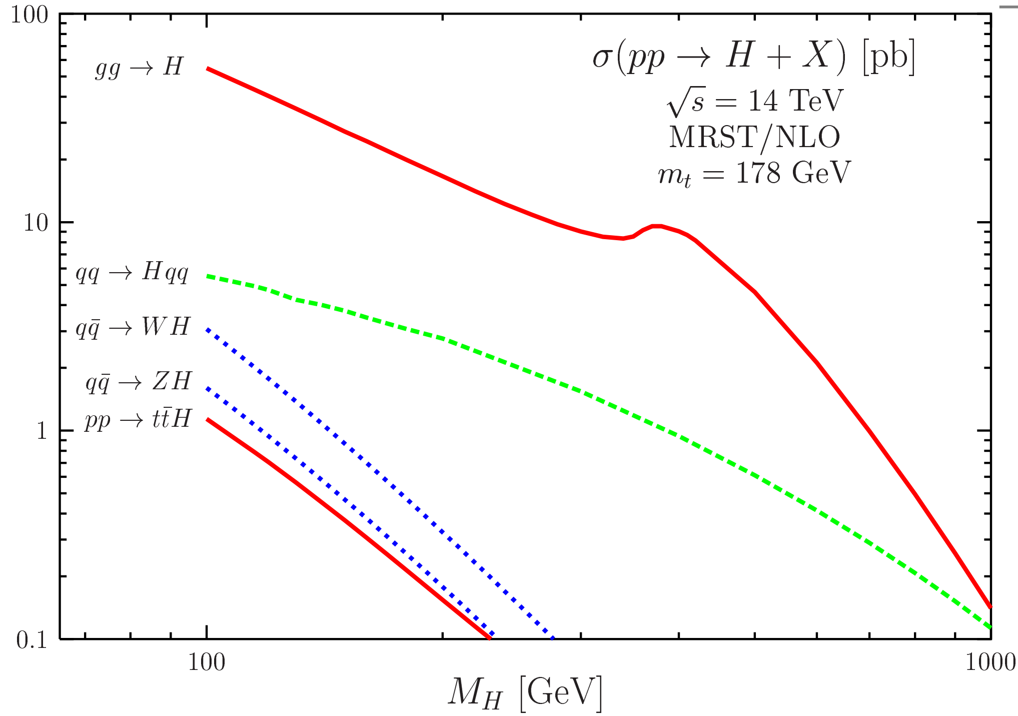
At last, the LHC will start in a few months

Primary goal: Discover the Higgs (+ surprises: SUSY?, Extra-dim, ??.??)



Higgs production at the LHC

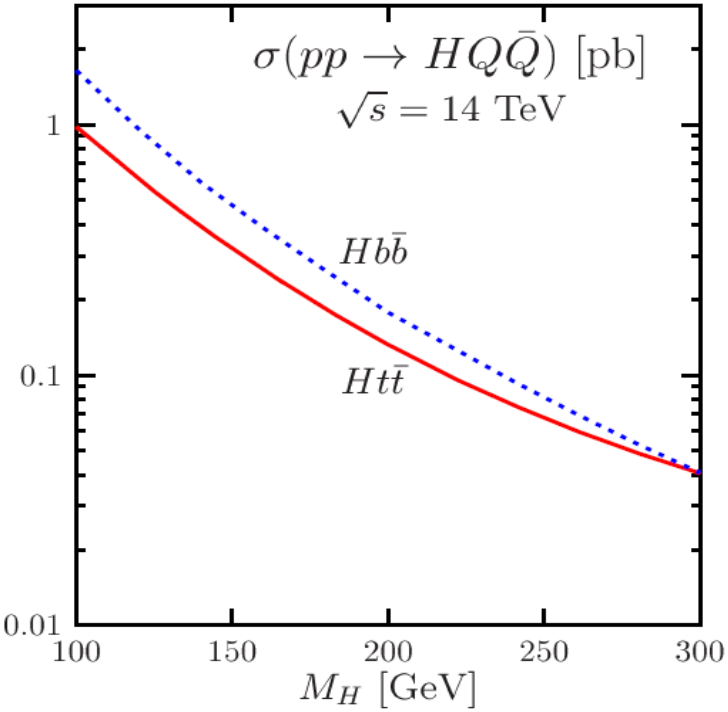


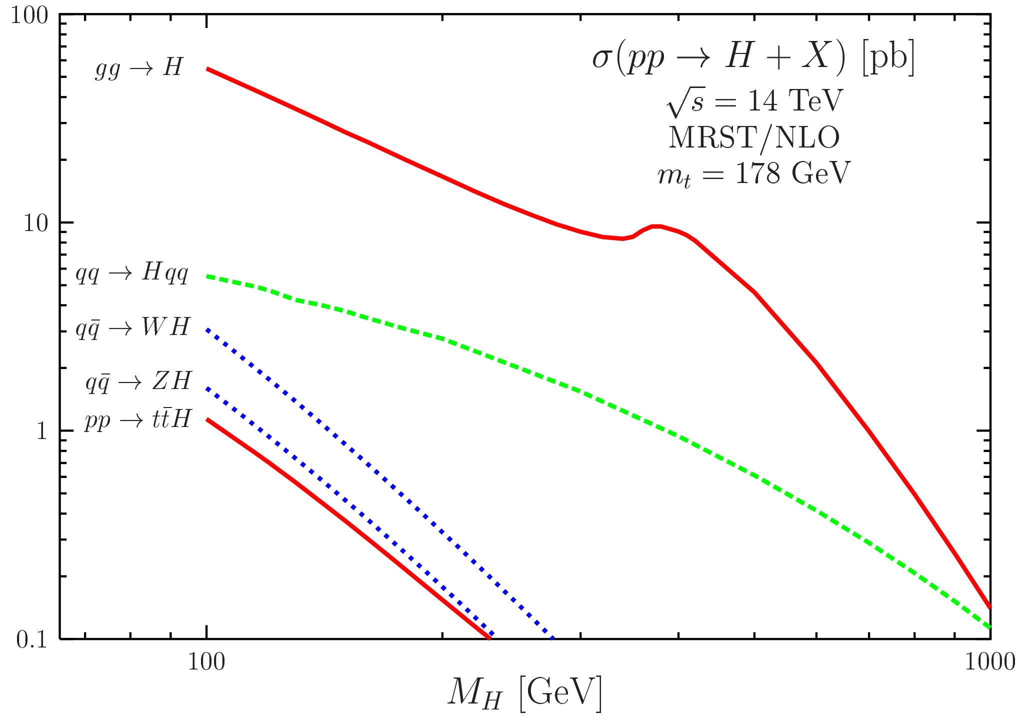


M. Spira, A. Djouadi

- The Higgs couples mainly to heavy particles, e.g. t, Z, W, b, τ ...
- Higgs production associated with heavy quarks can provide a direct measurement of the quark-Higgs Yukawa coupling.

Why $pp \to b\bar{b}H$? (I)

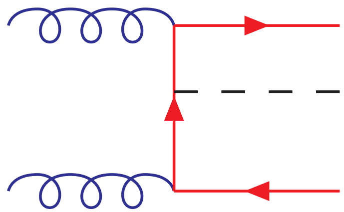




M. Spira, A. Djouadi

- At the LHC, $M_H < 300$GeV: $\sigma(pp \to b\bar{b}H) > \sigma(pp \to t\bar{t}H)$ because of large phase space and participation of small-x gluons.
- One-loop $2 \to 3$ process at the LHC: example of one-loop multileg processes incorporating a lot of techniques. Interplay between QCD and EW corrections.

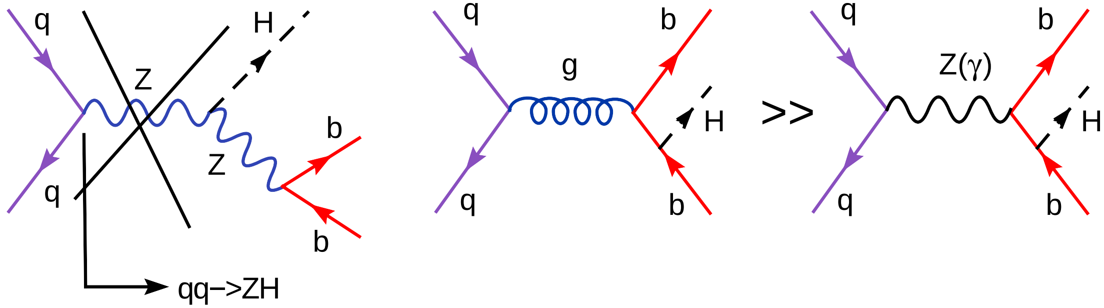
Why $pp \to b\bar{b}H$? (II)



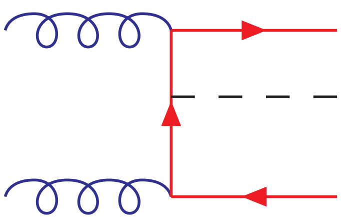
SM: $\lambda_{qqH} = -m_q/\upsilon$ with $\upsilon = 246$GeV.

- $\lambda_{bbH} =?$
- MSSM: if $\tan\beta \equiv \upsilon_1/\upsilon_2$ is large, the bottom-Higgs Yukawa coupling is enhanced, leading to large cross section.
- Tagging b-jets with high $\mathbf{p}_T$ to identify the process, QCD background is reduced.
- The final state observed in experiment depends on the value of the Higgs mass. If we want to look at photonic or leptonic production:
  - For $M_H < 140GeV$: $H \to \gamma\gamma$ ($BR \sim 10^{-3}$) $\Rightarrow pp \to 2b2\gamma$,
  - For $140GeV < M_H < 180$GeV: $H \to WW^* \to l\nu l\nu \Rightarrow pp \to 2b2l2\nu$,
  - For $M_H > 2M_Z$: $H \to ZZ \to 4l \Rightarrow pp \to 2b4l$.

$\sigma(q\bar{q})/\sigma(gg)$: neglecting $q\bar{q}$ contribution



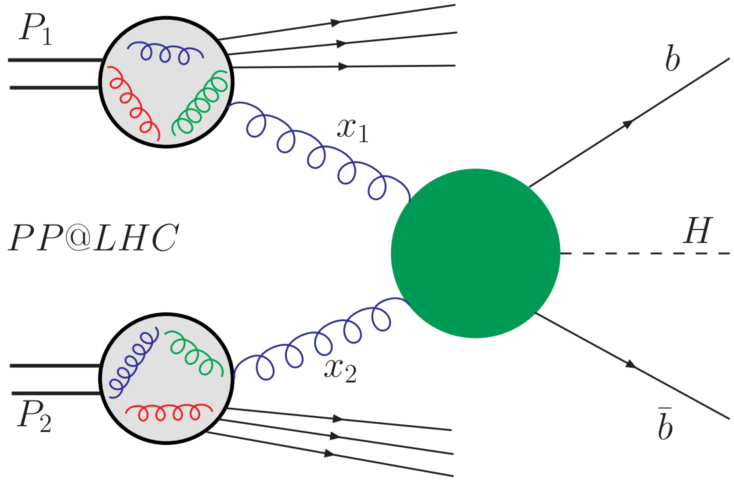
PDFs included; $\sqrt{s} = 14$TeV, $M_H = 120 GeV$; standard cut $=$ ($|\mathbf{p}_T^{b,\bar{b}}| > 20$GeV, $|\eta^{b,\bar{b}}| < 2.5$)



pp	$\sigma[fb]$
$u\bar{u}$	79.110×10^{-3}
$d\bar{d}$	56.716×10^{-3}
$s\bar{s}$	10.363×10^{-3}
gg	21.515
pp	21.6612

$\sigma(qq)/\sigma(gg) = 0.7\% \rightarrow qq$-contribution can be neglected.

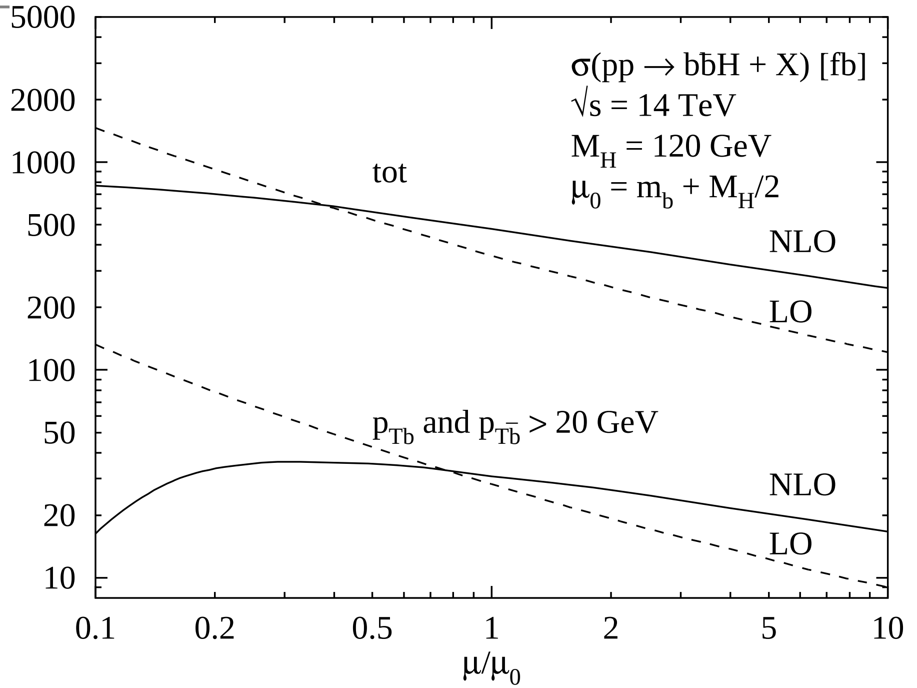
Total cross section

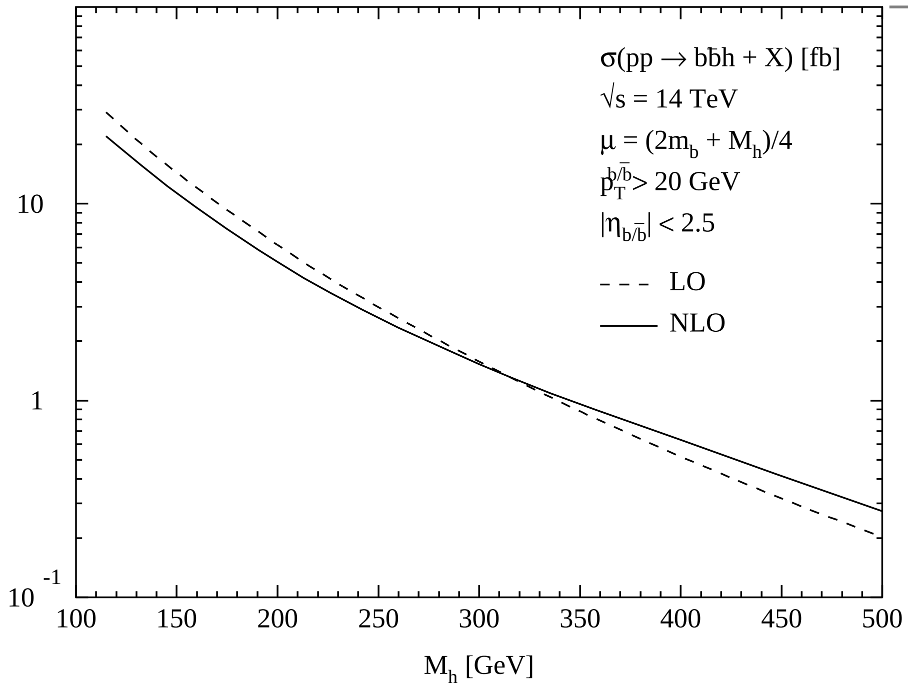


$$\sigma(pp \rightarrow b\bar{b}H) \approx \int_0^1 dx_1 g(x_1, Q) \int_0^1 dx_2 g(x_2, Q) \hat{\sigma}(g_1 g_2 \rightarrow b\bar{b}H)$$

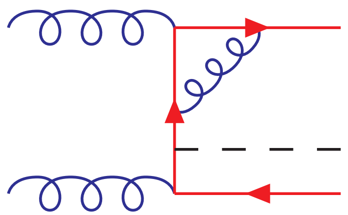
Q: arbitrary renormalisation/factorisation scale.

NLO QCD correction

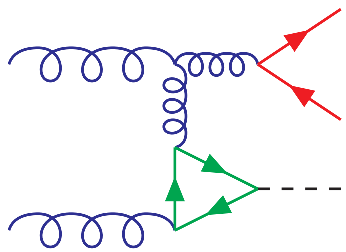




2 groups: *S. Dittmaier, M. Krämer, M. Spira, Phys. Rev. D70 (2004); S. Dawson* et al. *Phys. Rev. D69 (2004).*



$\sigma_{QCD} \sim \lambda_{bbH}^2$

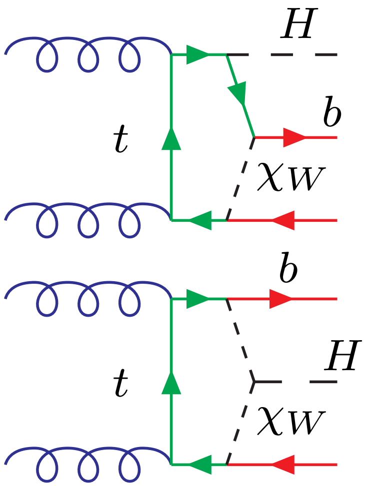


$\sigma_{top-loop} \sim \lambda_{ttH}\lambda_{bbH}$

- At NLO, the scale dependence is reduced by requiring $p_T^{b/\bar{b}} > 20$GeV.
- If $M_H = 120$GeV, $\mu = M_Z$: $\delta_{QCD}^{NLO} \approx -22\%$.
- If $\lambda_{bbH} = 0$: $\sigma_{QCD}^{NLO} = \sigma_0 = 0$.

Why EW correction?

* EW radiative correction: There are two dominant mechanisms to produce the Higgs via:



$\lambda_{ttH} \equiv -\frac{m_t}{\upsilon}$, $\lambda_t = -\sqrt{2}\lambda_{ttH} \approx g_s$

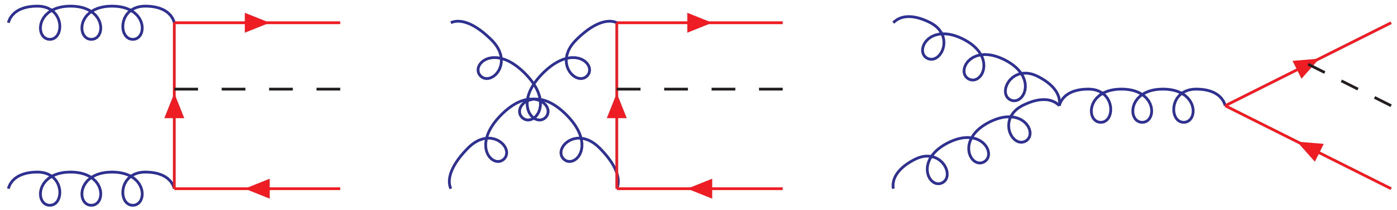
$\lambda_{\chi^+\chi^- H} \equiv \frac{M_H^2}{\upsilon}$, $\lambda_{tb\chi} = i\lambda_t(P_L - \frac{\lambda_b}{\lambda_t}P_R)$

* The questions:

- $\delta_{EW}^{NLO}/\delta_{QCD}^{NLO} = ?$
- If $\lambda_{bbH} = m_b = 0$ then $\delta_{EW} \neq 0$?

Helicity structures: Tree level

Process: $g(p_1,\lambda_1)+g(p_2,\lambda_2)\to b(p_3,\lambda_3)+\bar{b}(p_4,\lambda_4)+H(p_5)$.



- $m_b=0$ BUT $\lambda_{bbH}\neq 0$:
  - $\mathcal{A}_0(\hat{\lambda})=\bar{u}(\lambda_3)\Gamma^{\text{even}}_{\lambda_1,\lambda_2}v(\lambda_4)=\delta_{\lambda_3,-\lambda_4}\mathcal{A}_0^{\text{even}}$ (Chiral symmetry)
  - $\mathcal{A}_0(-\lambda_1,-\lambda_2;-\lambda_3,-\lambda_4)=-\mathcal{A}_0(\lambda_1,\lambda_2;\lambda_3,\lambda_4)^*$ (QCD Parity conservation)
  - $\Rightarrow\{\mathcal{A}_0(++-+),\mathcal{A}_0(+--+),\mathcal{A}_0(-+-+),\mathcal{A}_0(---+)\}$: even structure

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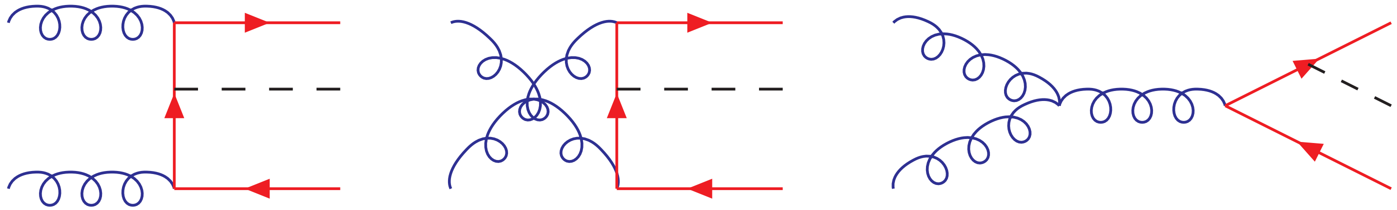

T U S

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- $m_b\neq 0$: mass insertion
  - $\mathcal{A}_0(\hat{\lambda})=\bar{u}(\lambda_3)\left(\Gamma^{\text{even}}_{\lambda_1,\lambda_2}+\Gamma^{\text{odd}}_{\lambda_1,\lambda_2}\right)v(\lambda_4)$
    $=\delta_{\lambda_3,-\lambda_4}\left(\mathcal{A}_0^{\text{even}}+m_b\tilde{\mathcal{A}}_0{}^{\text{odd}}\right)+\delta_{\lambda_3,\lambda_4}m_b\tilde{\mathcal{A}}_0{}^{\text{even}}$
  - $\mathcal{A}_0(-\lambda_1,-\lambda_2;-\lambda_3,-\lambda_4)=\lambda_3\lambda_4\mathcal{A}_0(\lambda_1,\lambda_2;\lambda_3,\lambda_4)^*$ (QCD Parity conservation)
  - $\Rightarrow \#4\ \mathcal{A}_0(\lambda_1,\lambda_2;\lambda,-\lambda)$ (even) AND $\#4\ \mathcal{A}_0(\lambda_1,\lambda_2;\lambda,\lambda)$ (odd)

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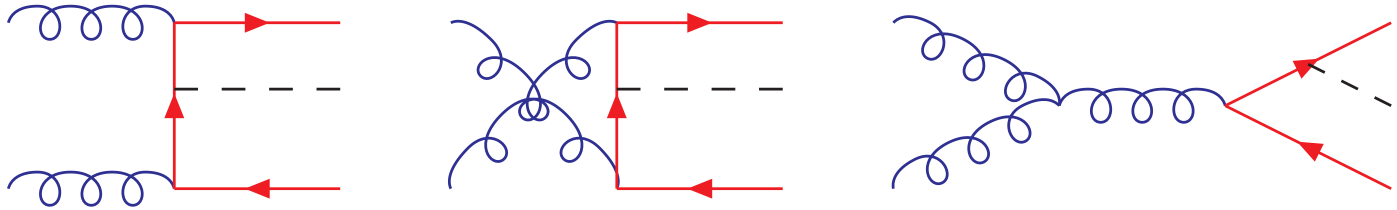

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    $= \delta_{\lambda_3,-\lambda_4}\left(\mathcal{A}_0^{\text{even}} + m_b\tilde{\mathcal{A}}_0{}^{\text{odd}}\right) + \delta_{\lambda_3,\lambda_4}m_b\tilde{\mathcal{A}}_0{}^{\text{even}}$
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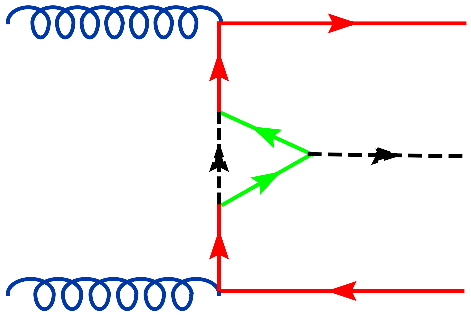
  - $\Rightarrow \#4\ \mathcal{A}_0(\lambda_1,\lambda_2;\lambda,-\lambda)$ (even) AND $\#4\ \mathcal{A}_0(\lambda_1,\lambda_2;\lambda,\lambda)$ (odd)
- $[\sigma(0)-\sigma(m_b)]/\sigma(m_b) = 3.7\%$ (1.1%) if $p_T^{b,\bar{b}} > 20$GeV (50GeV)

Helicity structures: One-loop



- $m_b = 0$ BUT $\lambda_{bbH} \neq 0$: m_t insertion

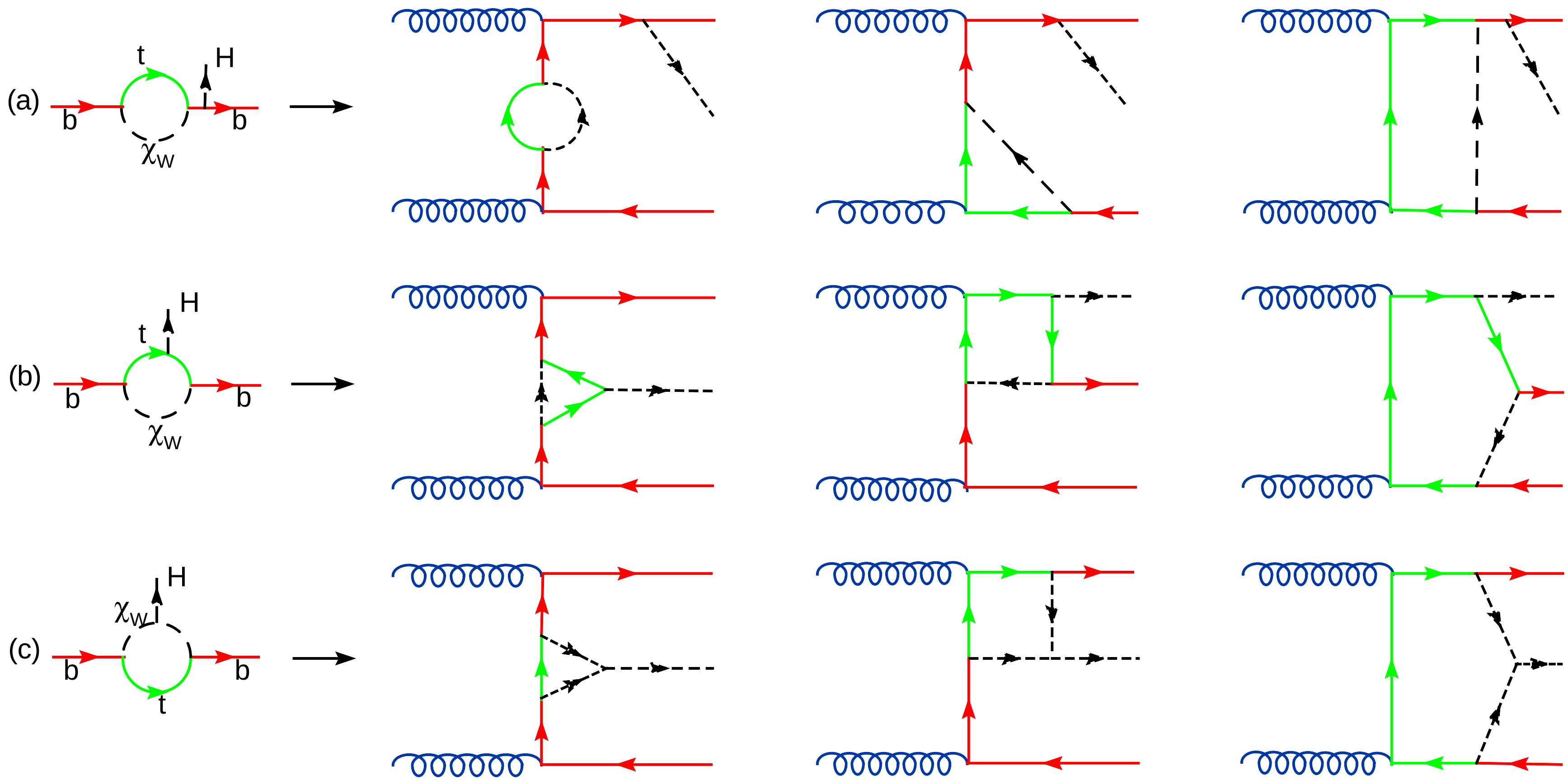
$$\mathcal{A}(\lambda_1, \lambda_2; \lambda_3, \lambda_4) = \bar{u}(\lambda_3) \left(\Gamma^{\text{even}}_{\lambda_1,\lambda_2} + \Gamma^{\text{odd}}_{\lambda_1,\lambda_2} \right) v(\lambda_4) = \delta_{\lambda_3,-\lambda_4} \mathcal{A}^{\text{even}} + \delta_{\lambda_3,\lambda_4} \mathcal{A}^{\text{odd}}$$

- $m_b \neq 0$: mass insertion

$$\mathcal{A}(\lambda_1, \lambda_2; \lambda_3, \lambda_4) = \delta_{\lambda_3,-\lambda_4} \left(\mathcal{A}^{\text{even}} + m_b \tilde{\mathcal{A}}^{\text{odd}} \right) + \delta_{\lambda_3,\lambda_4} \left(\mathcal{A}^{\text{odd}} + m_b \tilde{\mathcal{A}}^{\text{even}} \right)$$

one-loop correction $\rightarrow \mathcal{A}^{\text{odd}}$

One-loop EW correction: diagrams



- $\#$ diagrams: 115 (19 boxes, 8 pentagons)
- Each group is QCD gauge invariant
- $\lambda_{bbH} = 0 \rightarrow (a) = 0,\ (b, c) \neq 0$

λ_{bbH} expansion

- The total cross section as a function of λ_{bbH} can always be written in the form

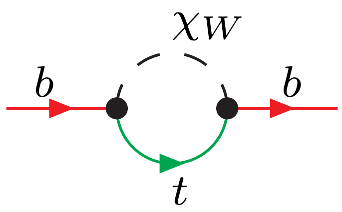
$$\begin{aligned}\sigma(\lambda_{bbH}) &= \sigma(\lambda_{bbH}=0) + \lambda_{bbH}^2\sigma'(\lambda_{bbH}=0) + \cdots \\ \lambda_{bbH}^2\sigma'(\lambda_{bbH}=0) &= \sigma_0[1+\delta_{EW}(m_t, M_H)], \\ \sigma(\lambda_{bbH}=0) &= \sigma_{EW}(\lambda_{bbH}=0).\end{aligned}$$

- Approximation: the leading EW contribution comes from the Feynman diagrams with the top quark and charged Goldstones ($W_L^{\pm}$) running in the loops (Yukawa correction).

	Γ^{even}	Γ^{odd}
tree-level	λ_{bbH}	0
(a)	$\lambda_t^2\lambda_{bbH}$	$\lambda_b\lambda_t\lambda_{bbH} \approx 0$
(b)	$\lambda_b\lambda_t\lambda_{ttH}$	$\lambda_t^2\lambda_{ttH}$, (P_R)
(c)	$\lambda_b\lambda_t\lambda_{\chi\chi H}$	$\lambda_t^2\lambda_{\chi\chi H}$, (P_R)

For $m_b = 0$: $\Gamma_{even} \to \lambda_{bbH}^2\sigma'(\lambda_{bbH}=0)$ and $\Gamma_{odd} \to \sigma(\lambda_{bbH}=0)$.

Renormalisation: On-shell

Bottom propagator: 

$\rightarrow$ $\delta m_b, \delta Z_{bL}, \delta Z_{bR} \propto \lambda_t^2$.

Vertices:

$$\delta^{\mu}_{bbg} = 2g_s\gamma^{\mu}(\delta Z_{b_L}^{1/2}P_L + \delta Z_{b_R}^{1/2}P_R)\,,$$

$$\delta_{bbH} = \lambda_{bbH}[\frac{\delta m_b}{m_b} + \delta Z_{b_L}^{1/2} + \delta Z_{b_R}^{1/2} + (\delta Z_H^{1/2} - \delta v)].$$

Remark: In the approximation we use, $(\delta Z_H^{1/2} - \delta v) = f(\lambda_{ttH}, \lambda_{\chi^+\chi^- H})$ is UV finite and can be seen as a universal correction to Higgs production processes.

One-loop calculation (I)

- Three groups of QCD gauge invariance ▶

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$$\mathcal{A}(\hat{\lambda})^{T,U,S} = CME(a,b) \times Cc \times FFE \times SME(\lambda_i),$$

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  - Cc, common coefficients (coupling constants, normalised factors, etc)
  - FFE, form factor element, $f(pi.pj, m_i^2)$ (loop integrals): time-consuming part
  - $SME(\hat{\lambda})$, standard matrix element, $f(\lambda_i, m_b, \gamma_5)$: $\#12$(tree) $\&$ $\#68$(1-loop) →

One-loop calculation (II)

$$
\begin{aligned}
SME_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4) &= [\bar{u}(\lambda_3,p_3)v(\lambda_4,p_4)] \times [\varepsilon_\mu(\lambda_1,p_1,p_2)p_4^\mu] \times [\varepsilon_\nu(\lambda_2,p_2,p_1)p_4^\nu], \\
&= BME_1(\lambda_3,\lambda_4) \times BME_2(\lambda_1) \times BME_3(\lambda_2),
\end{aligned}
$$

BME, basic matrix element, $\#31$(1-loop).

One-loop calculation (II)

$$SME_1(\lambda_1,\lambda_2,\lambda_3,\lambda_4) = [\bar{u}(\lambda_3,p_3)v(\lambda_4,p_4)] \times [\varepsilon_\mu(\lambda_1,p_1,p_2)p_4^\mu] \times [\varepsilon_\nu(\lambda_2,p_2,p_1)p_4^\nu],$$
$$= BME_1(\lambda_3,\lambda_4) \times BME_2(\lambda_1) \times BME_3(\lambda_2),$$

BME, basic matrix element, $\#31$(1-loop).

- Loop integrals: used library LoopTools (FF); $2-4$-point functions: Passarino and Veltman, 5-point function: Denner and Dittmaier.

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BME, basic matrix element, $\#31$(1-loop).

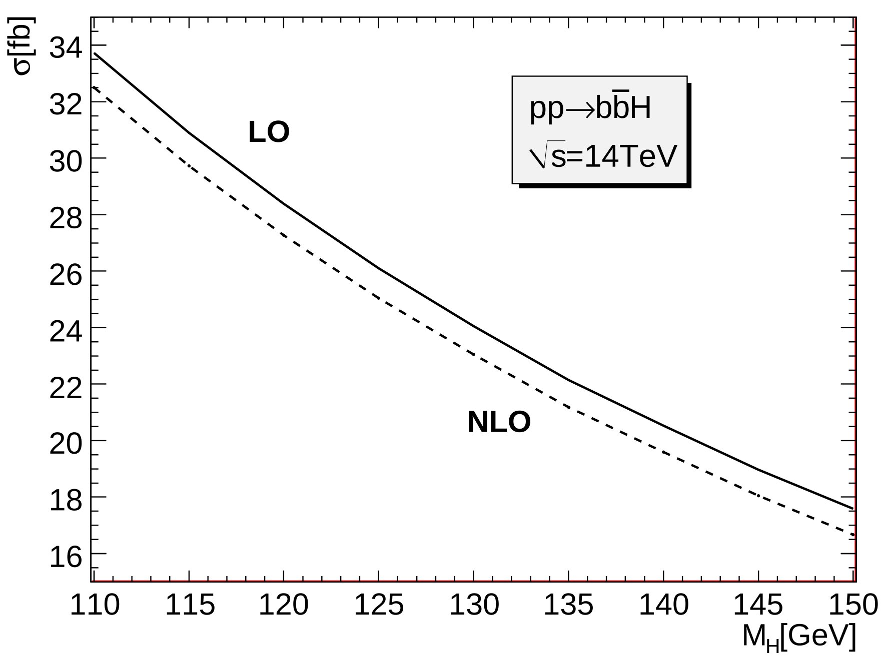
- Loop integrals: used library LoopTools (FF); $2-4$-point functions: Passarino and Veltman, 5-point function: Denner and Dittmaier.
- Phase space integration:
  - BASES (S. Kawabata, Monte Carlo with important sampling): no zero Gram determinant ($detG \equiv det(2p_i.p_j)$) detected, error $= 0.08\%$ for $N_{call} = 10^5$.
  - DADMUL (Genz and Malik, adaptive quadrature algorithm): detects zero Gram determinant associated with the 3pt and 4pt functions $\rightarrow$ imposing some tiny cuts, $\theta_{cut}^{b,\bar{b}} = |\sin\phi^{\bar{b}}|_{cut} = 10^{-6}$, $\rightarrow$ get stable results which agree with BASES.
  - 5pt functions: no problem (Caley determinant = Landau det., to appear later).

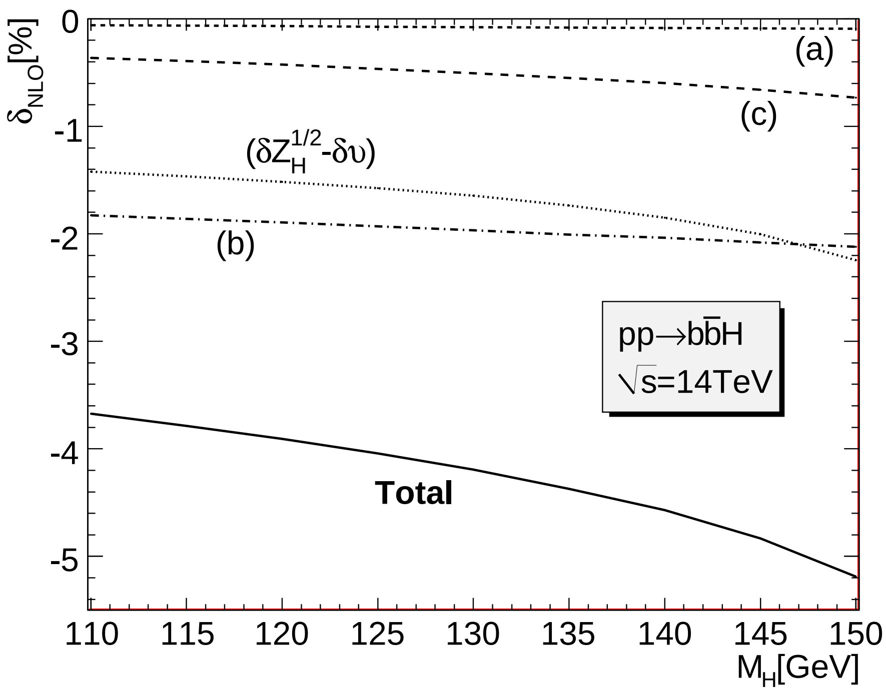
Checks

- Tree level:
  - QCD gauge invariant
  - checked against the results of CalcHEP
- One-loop
  - UV-finite
  - each helicity amplitude is QCD gauge invariant

$$\begin{aligned}
\epsilon_\mu(p_i,\lambda_i;q_i) &= \frac{\bar{u}(p_i,\lambda_i)\gamma_\mu u(q_i,\lambda_i)}{[4(p_i.q_i)]^{1/2}} \; (q_i^2=0), \\
\varepsilon^\mu(p,\lambda;q') &= e^{i\phi(q',q)}\varepsilon^\mu(p,\lambda;q) + \beta(q',q)p^\mu, \\
|\mathcal{A}(\lambda_1,\lambda_2;\lambda_3,\lambda_4;q_1,q_2)|^2 &= |\mathcal{A}(\lambda_1,\lambda_2;\lambda_3,\lambda_4;q_1',q_2')|^2.
\end{aligned}$$

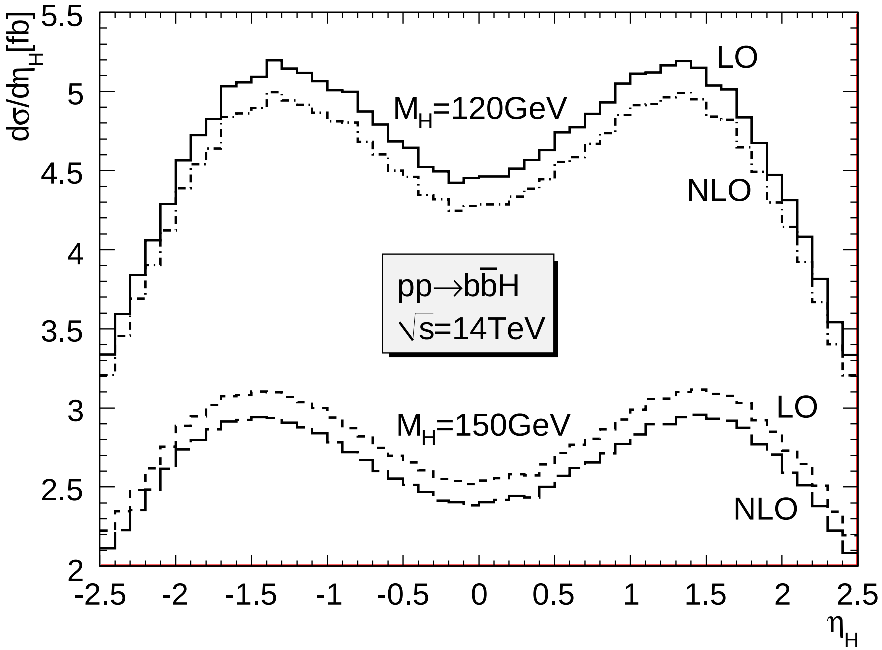
Cross section

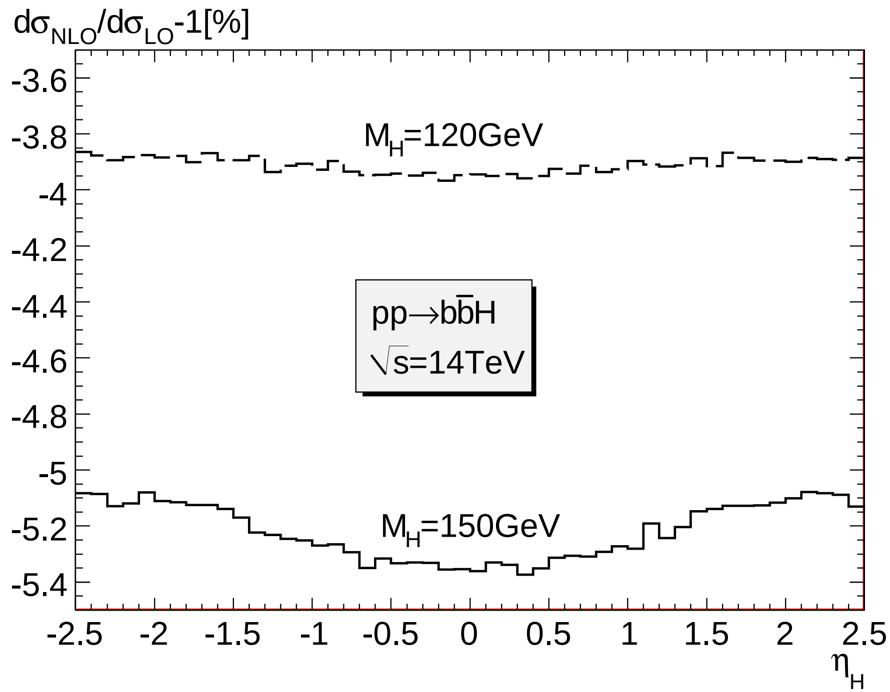




- Input parameters: $\sqrt{s} = 14$TeV, $m_b = 4.62$GeV, $m_t = 174$GeV, $Q = M_Z$.
- Cuts: $\mathbf{p}_T^{b,\bar{b}} > 20$GeV, $|\eta_{b,\bar{b}}| < 2.5$.
- $\delta_{EW}/\delta_{QCD} \approx 1/5$ ($M_H = 120$GeV).
- $\sigma_{LO,NLO}$ decrease as the Higgs mass increases. The contrary behaviour for δ_{EW}.

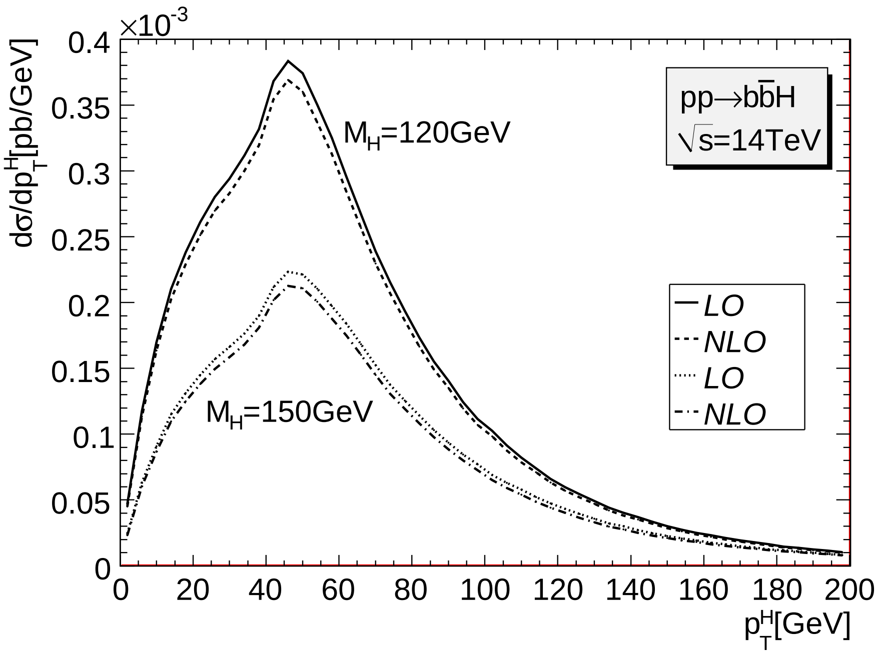
Higgs: pseudorapidity distributions

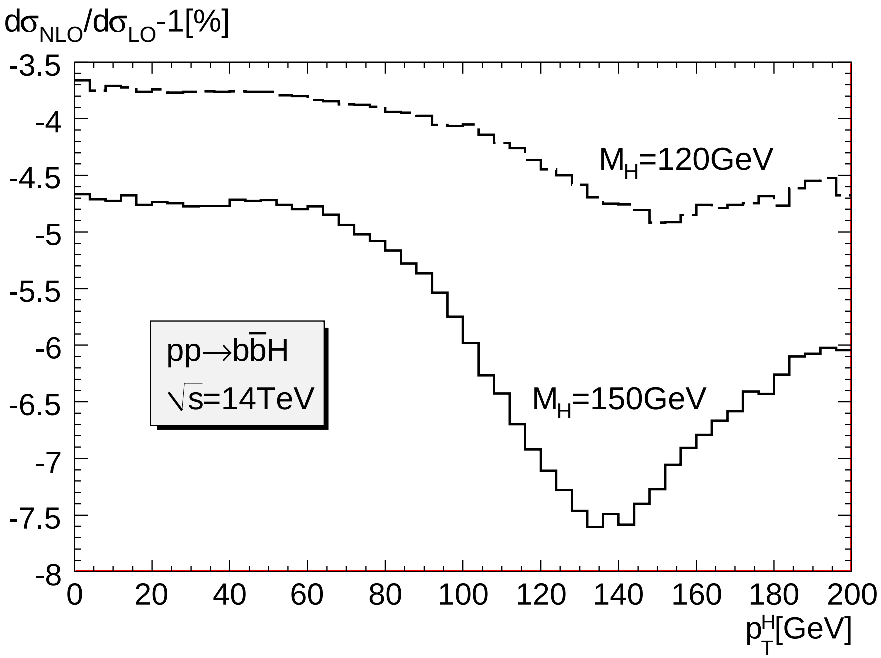




EW correction to the Higgs pseudorapidity distribution is also small.

Higgs: p_T-distributions





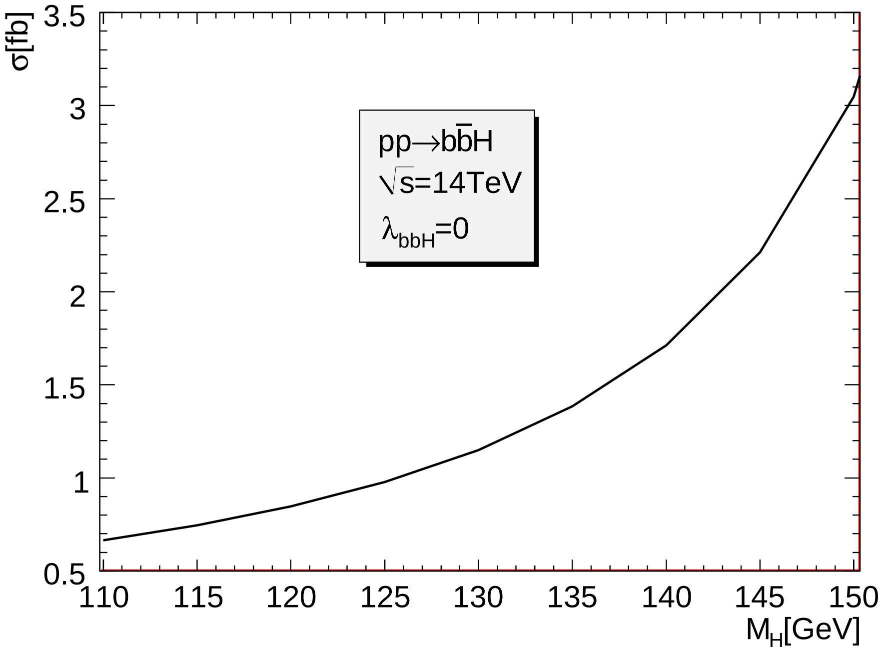
EW correction to the Higgs transverse momentum distribution is small (can be 8% but not in the interesting region).

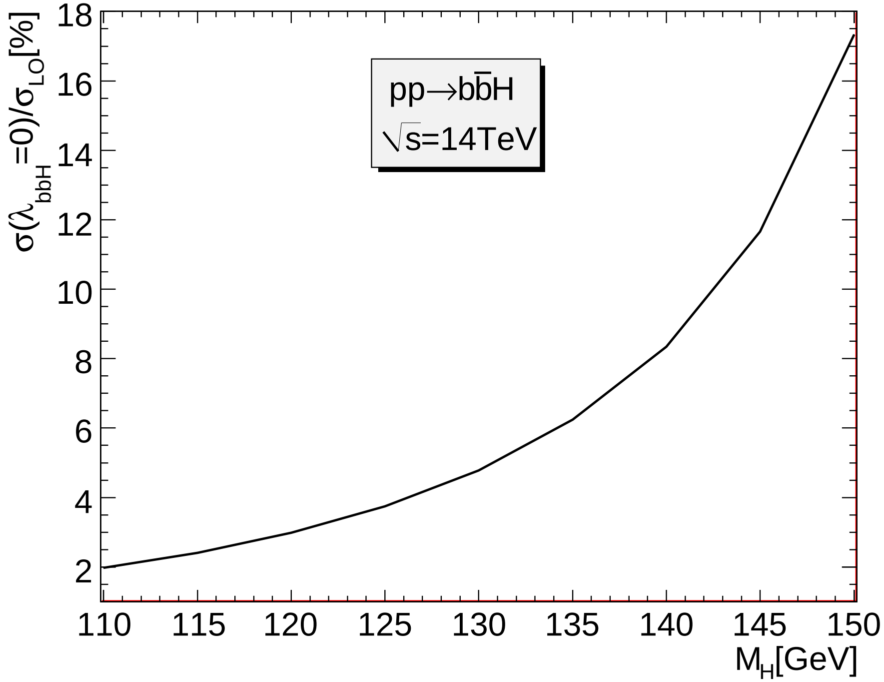
$\lambda_{bbH} = 0$

- LO: $\sigma_0 \propto |A_0|^2 \propto \lambda_{bbH}^2 = 0.$
- NLO: $\sigma_{NLO} \propto 2Re[A_0 A_1^*] \propto \lambda_{bbH} = 0.$
- NNLO: $\sigma_{NNLO} \propto |A_1|^2 + 2Re[A_0 A_2^*] \propto f(\lambda_{ttH}, \lambda_{\chi^+\chi^- H}).$

$$\boxed{\sigma(\lambda_{bbH} = 0) \propto |A_1|^2(M_H, m_t)}$$

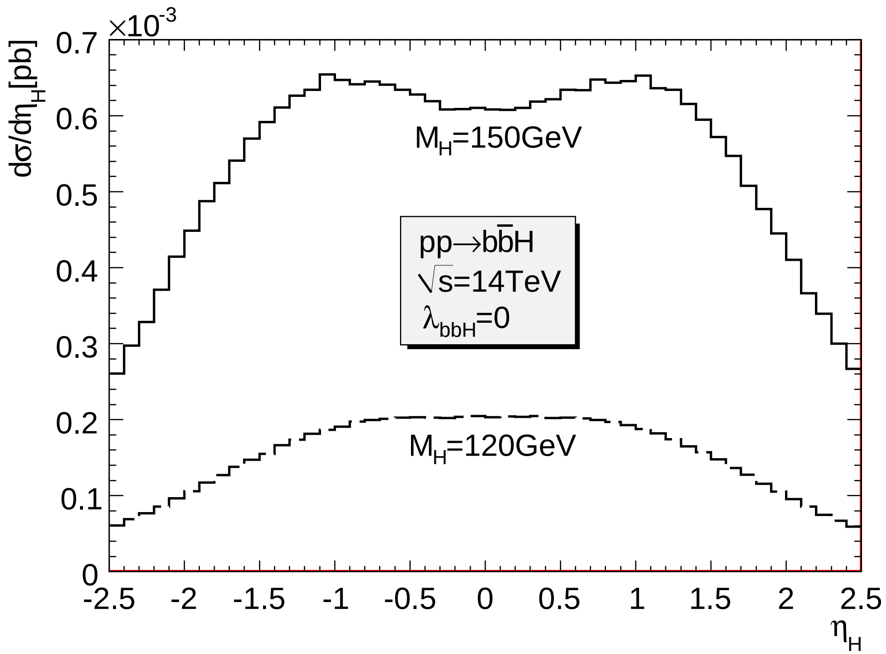
$\sigma_{EW}(\lambda_{bbH} = 0)$: $M_H < 2M_W$

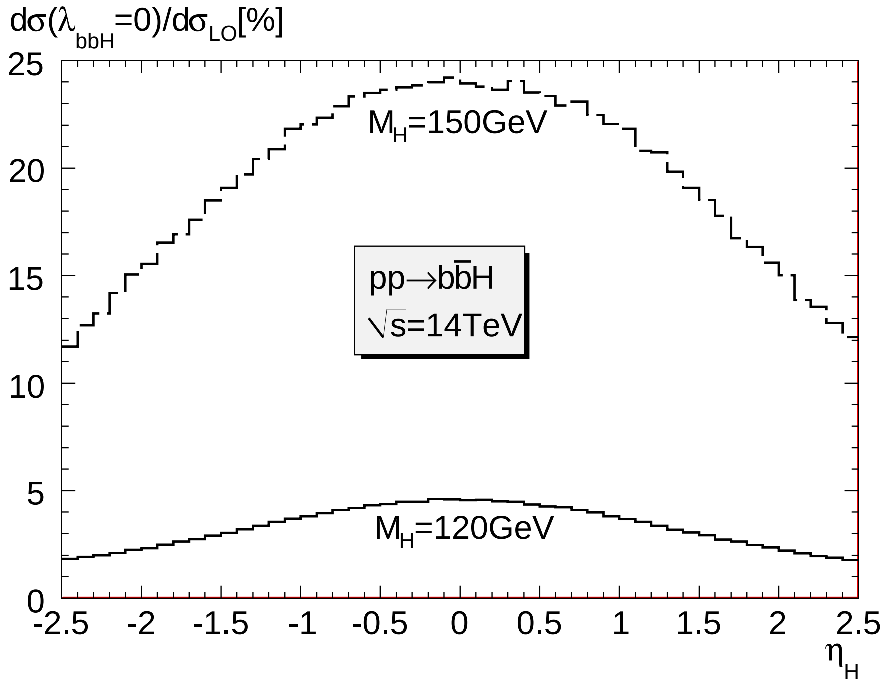




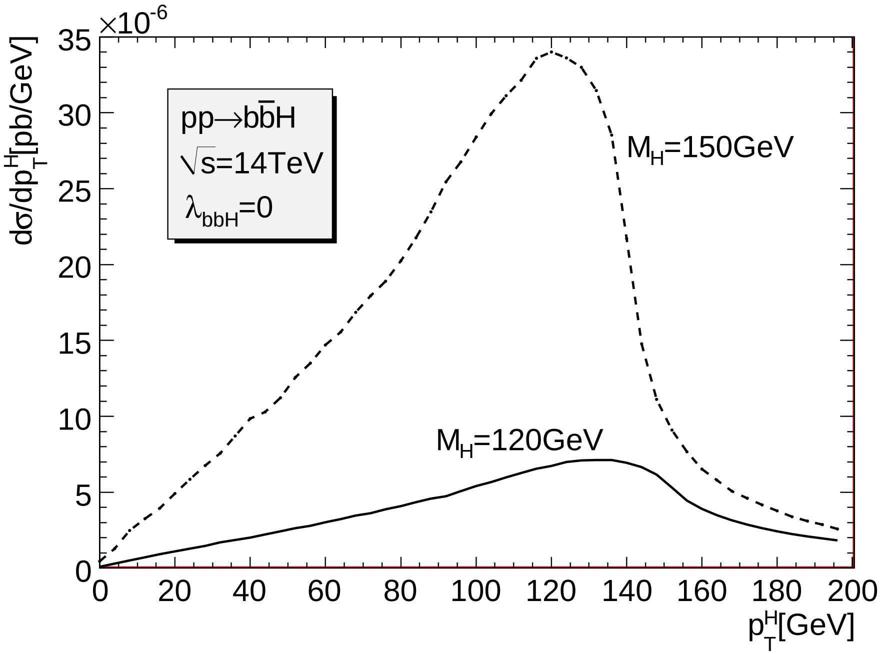
- it rapidly increases when $M_H \to 2M_W$.

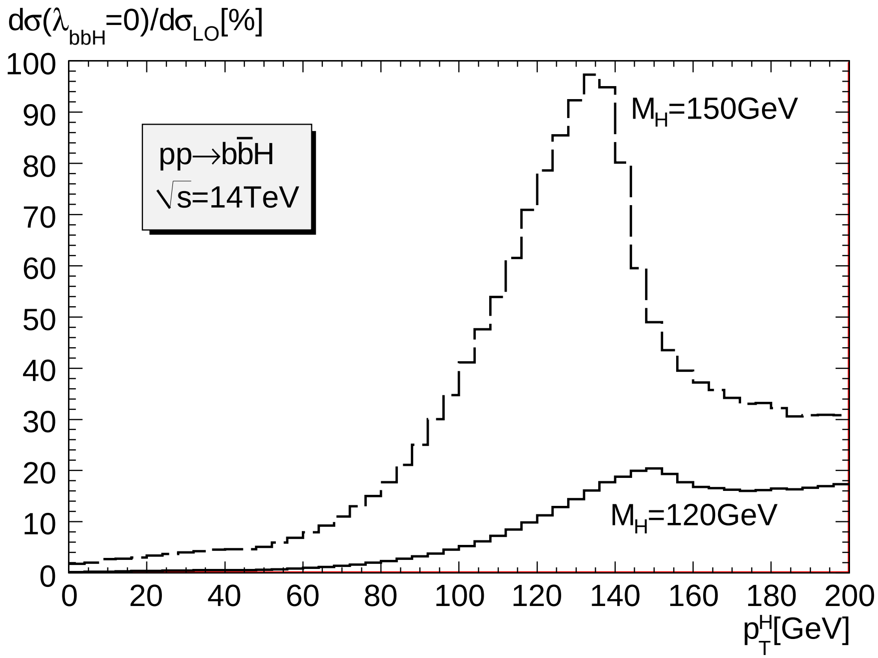
η_H-distributions($\lambda_{bbH} = 0$)@EW





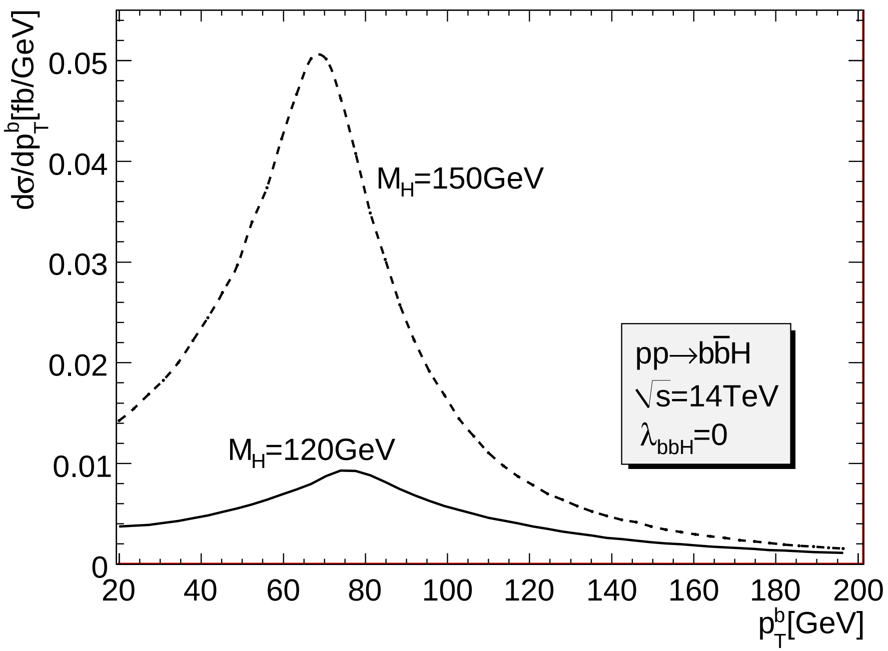
p_T^H-distributions($\lambda_{bbH} = 0$)@EW

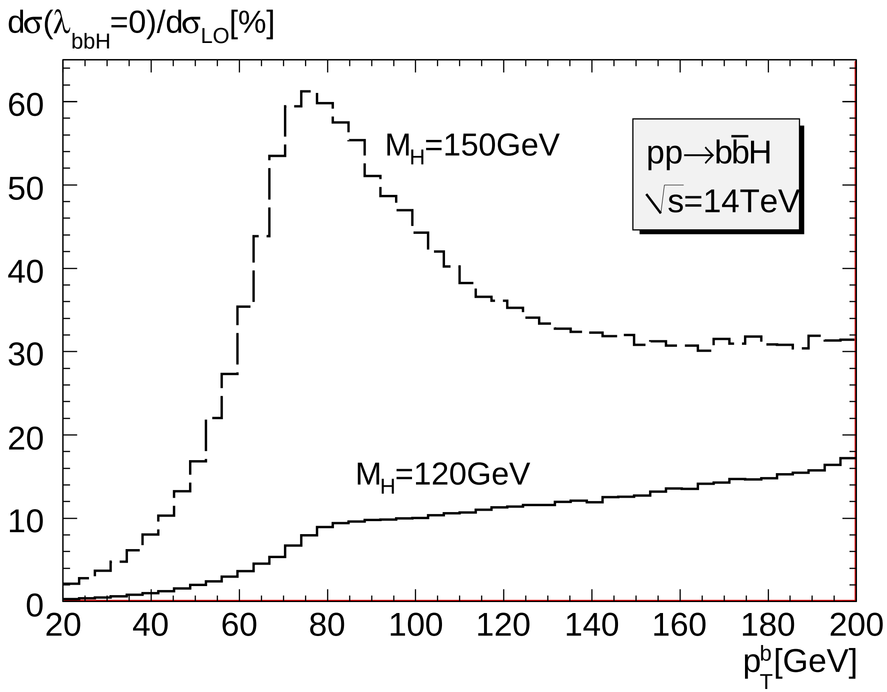




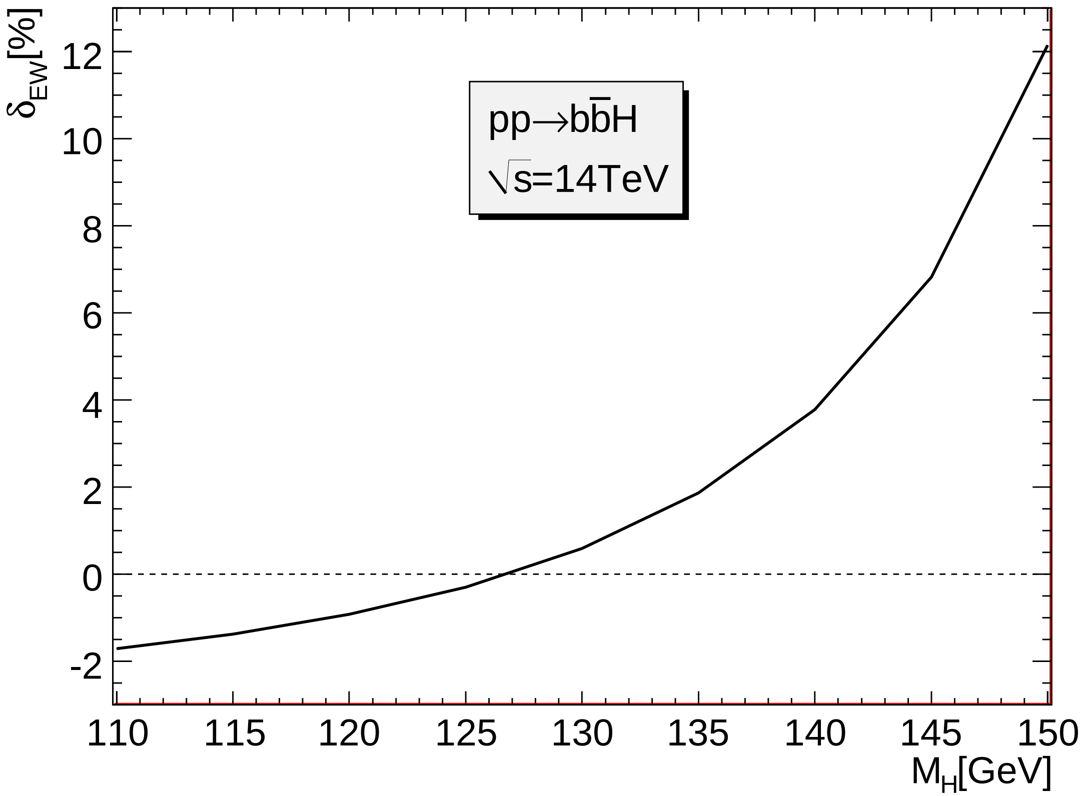
Distributions are very different from the tree level and NLO ones (helicity structures are completely different). ▶

p_T^b-distributions($\lambda_{bbH} = 0$)@EW



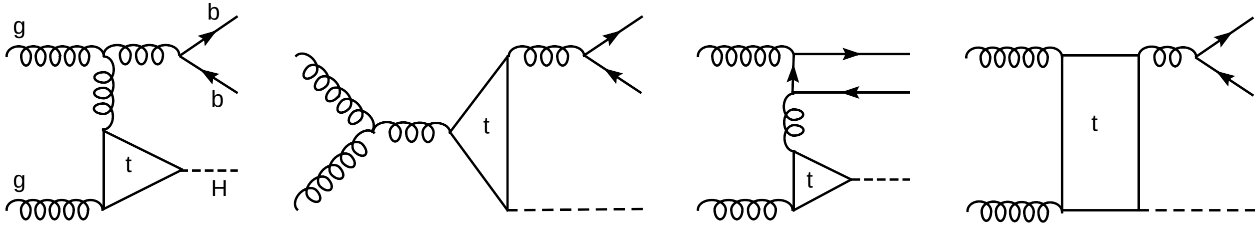


Final result @EW



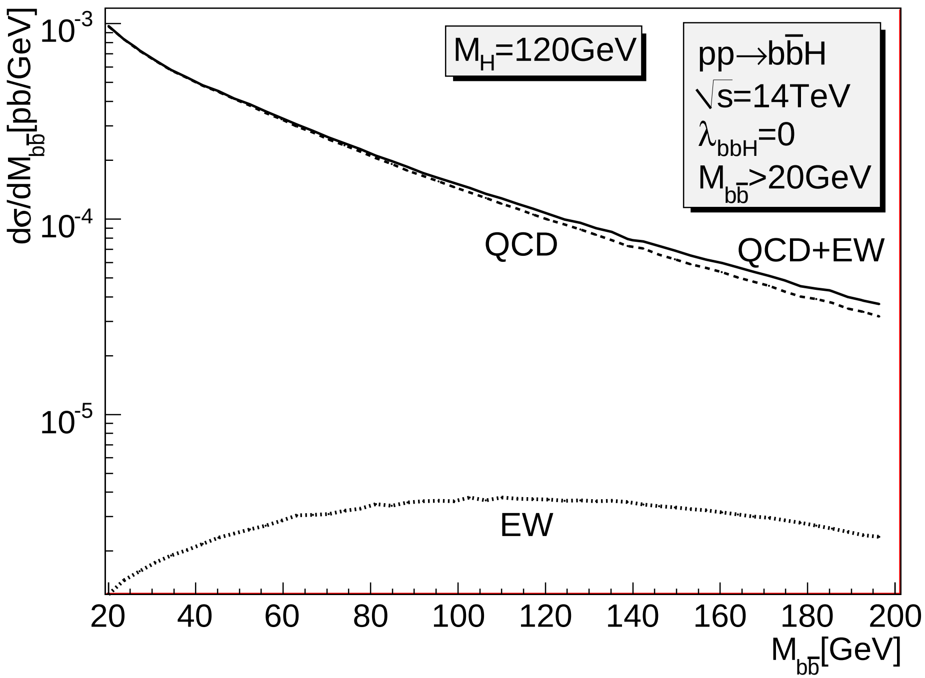
$$\delta_{EW} \quad \equiv \quad \delta_{NLO} + \frac{\sigma(\lambda_{bbH} = 0)}{\sigma_0}.$$

$\sigma_{top-loop}(\lambda_{bbH} = 0)$: part of inclusive H x-section

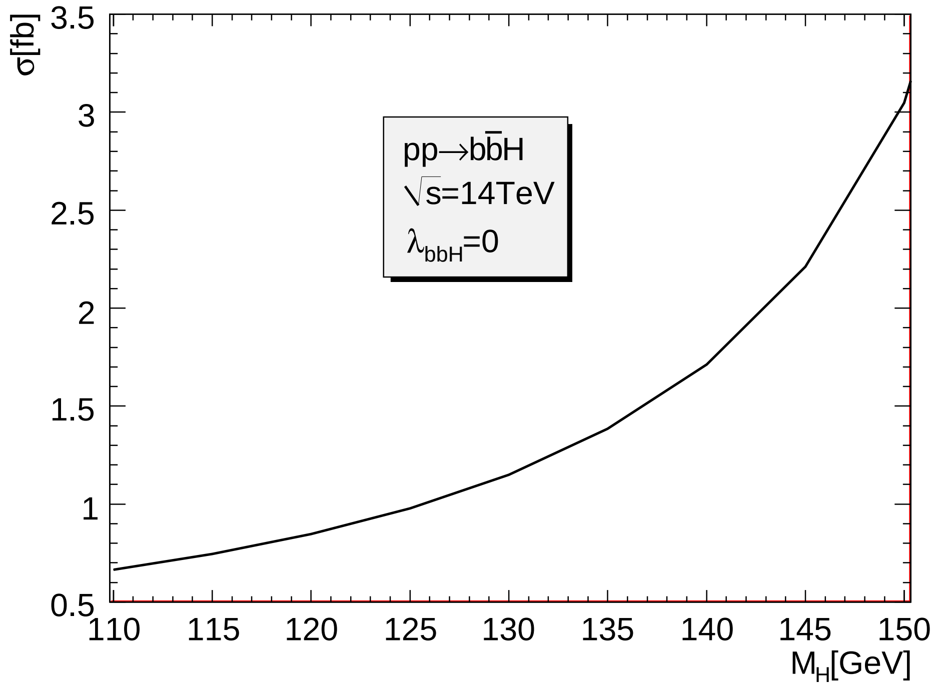


if $M_H = 120$GeV, standard cut & $M_{b\bar{b}} > 20$GeV:

$\sigma_0[fb]$	$\sigma_{EW}[fb]$	$\sigma_{top-loop}[fb]$	$\sigma_{(EW+top-loop)}[fb]$
28.095	0.8346	41.9864	43.773



$\sigma_{EW}(\lambda_{bbH} = 0) \propto |A_1|^2$: problem



Facts:

- $Re(A_1 A_0^*)$: Integration over the phase space gives stable results for $M_H \geq 2M_W$.
- $|A_1|^2$: Integration over the phase space becomes extremely unstable if $M_H \geq 2M_W$.

How to explain these observations?

$M_H \geq 2M_W$ AND $\sqrt{\hat{s}} \geq 2m_t$ (LHC)

look at the sub-process $gg \to b\bar{b}H$, more facts:

- if $M_H \geq 2M_W$ and $\sqrt{\hat{s}} < 2m_t \to$ no problem
- if $m_t = M_W \to$ no problem whatever the values of M_H and $\sqrt{\hat{s}}$ are

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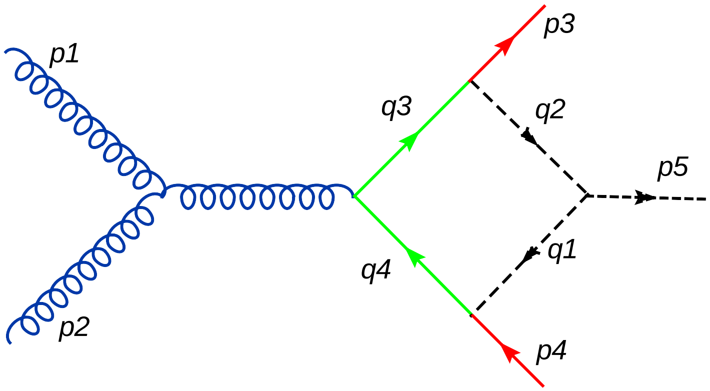
We found that this problem of numerical instabilities occurs in the 4pt and 5pt functions



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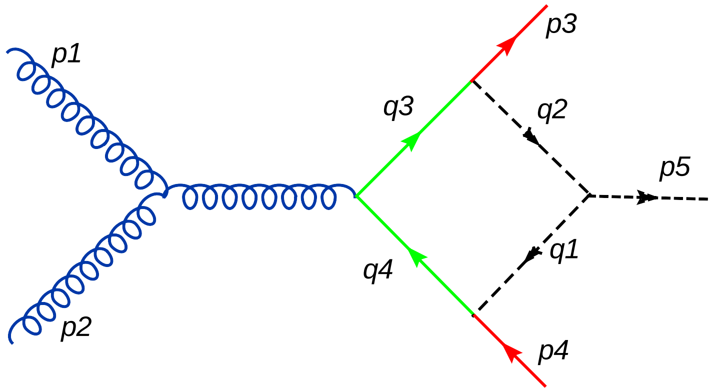
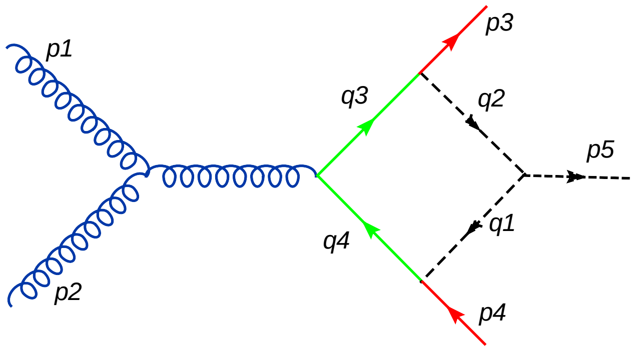


- Landau singularity occurs when $M_H \geq 2M_W$ and $\sqrt{\hat{s}} \geq 2m_t$, *i.e.* particles in the loop are simultaneously on-shell.
- H, W, t are unstable particles.

Landau equations

L.D. Landau, Nucl. Phys. 13 (1959) 181; Polkinghorne, Olive, Landshoff, Eden, The analytic S-Matrix (1966)



$$T_0^N \propto \int_0^\infty \prod_{i=1}^N dx_i \int \frac{d^D q}{(2\pi)^D} \frac{\delta(\sum_{i=1}^N x_i - 1)}{[\sum_{i=1}^N x_i(q_i^2 - m_i^2)]^N}$$

$$\begin{cases} \forall i \;\; x_i(q_i^2 - m_i^2) & = 0 \\ \sum_{i=1}^M x_i q_i & = 0 \end{cases}$$

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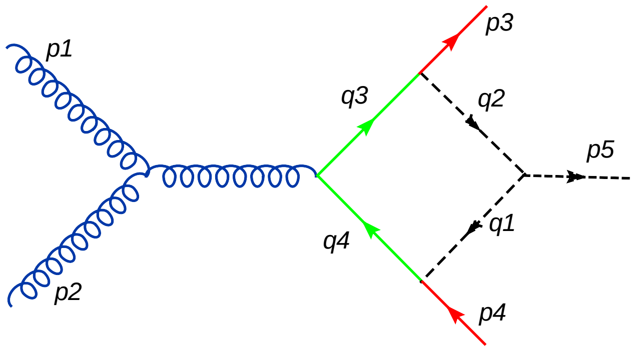

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- $g^* \to b\bar{b}H$:
  - $x_i > 0$, four-point function, the leading Landau singularity.
  - $x_i = 0$, three-point functions, anomalous thresholds (lower-order singularities).
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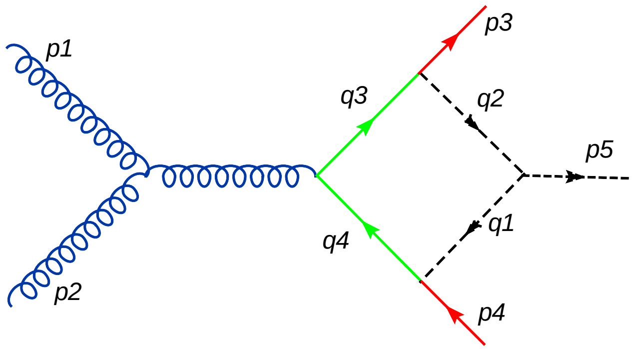

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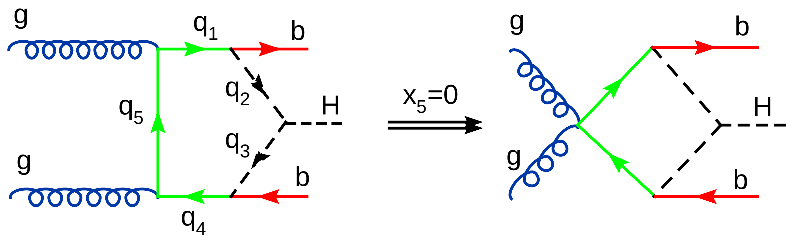
- It is interesting to know:
  - IR-divergence: $x_1 = \ldots x_{N-1} = 0$, $m_N = 0$
  - Collinear divergence: $x_1 = \ldots x_{N-2} = 0$, $m_{N-1} = m_N = p_N^2 = 0$
- To find singularities: look at the Landau equations.

5pt function



E_0 has no leading Landau singularity, but has several lower order Landau singularities. Another way to see is to look at the reduction formula (Denner and Dittmaier)

$$E_0 = -\sum_{i=1}^{5} \frac{det(Q_i)}{det(Q)} D_0(i),$$

$$Q_{ij} \equiv 2q_i.q_j = m_i^2 + m_j^2 - (q_i - q_j)^2; \;\; i,j \in \{1,\ldots,M\},$$

and Q_i is obtained by replacing all entries in the ith column with 1.

Landau matrix

2 conditions

Landau equations:

$$\begin{cases} Q_{11}x_1 + Q_{12}x_2 + \cdots Q_{1M}x_M & = 0, \\ Q_{21}x_1 + Q_{22}x_2 + \cdots Q_{2M}x_M & = 0, \\ \vdots & \\ Q_{M1}x_1 + Q_{M2}x_2 + \cdots Q_{MM}x_M & = 0. \end{cases}$$

2 conditions: implemented in a code to check for singularities in the scalar functions

- Landau determinant must vanish:

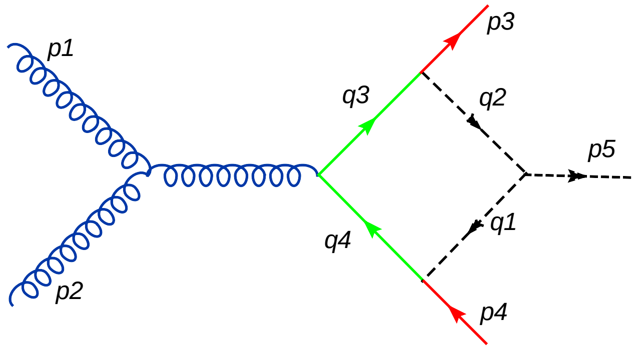
$$det(Q) = 0$$

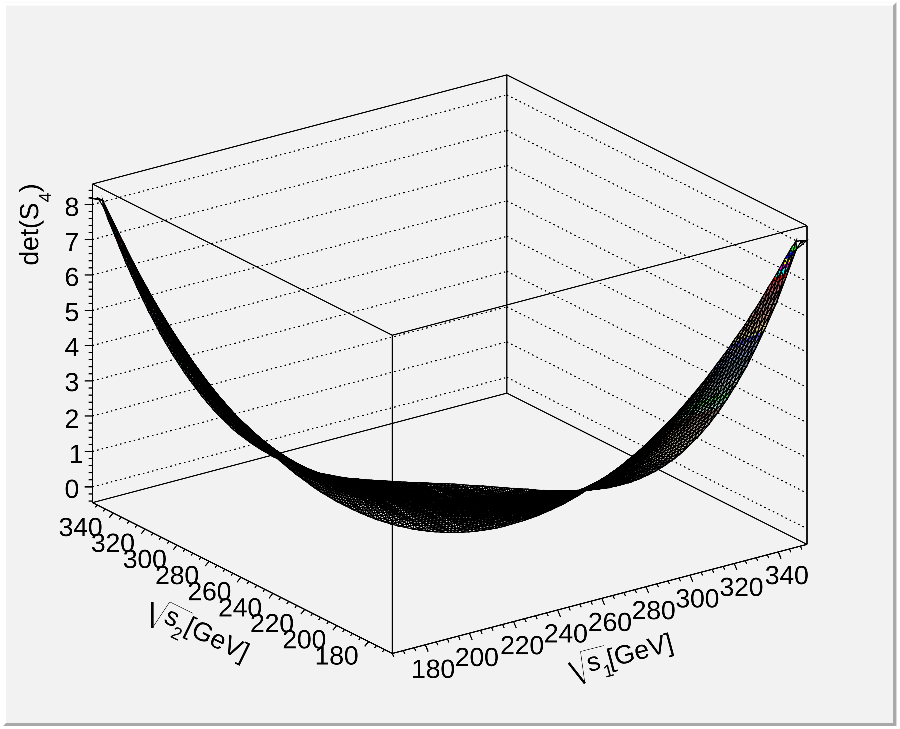
- Sign condition (occurring in the physical region):

$$x_i > 0,\ i = 1, \ldots, M \Longleftrightarrow x_j = det(\hat{Q}_{jM})/det(\hat{Q}_{MM}) > 0,\ j = 1, \ldots, M-1$$

$$det(\hat{Q}_{MM}) = d[det(Q)]/dQ_{MM},\ \ det(\hat{Q}_{1j}) = \frac{1}{2}d[det(Q)]/dQ_{1j}.$$

Landau determinant: $g^* \to b\bar{b}H$

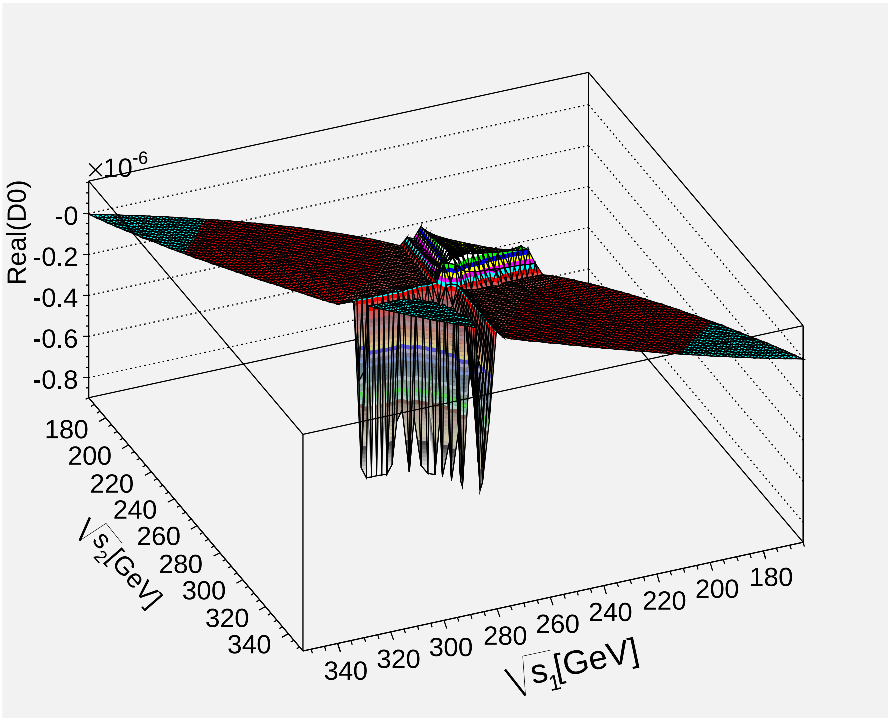


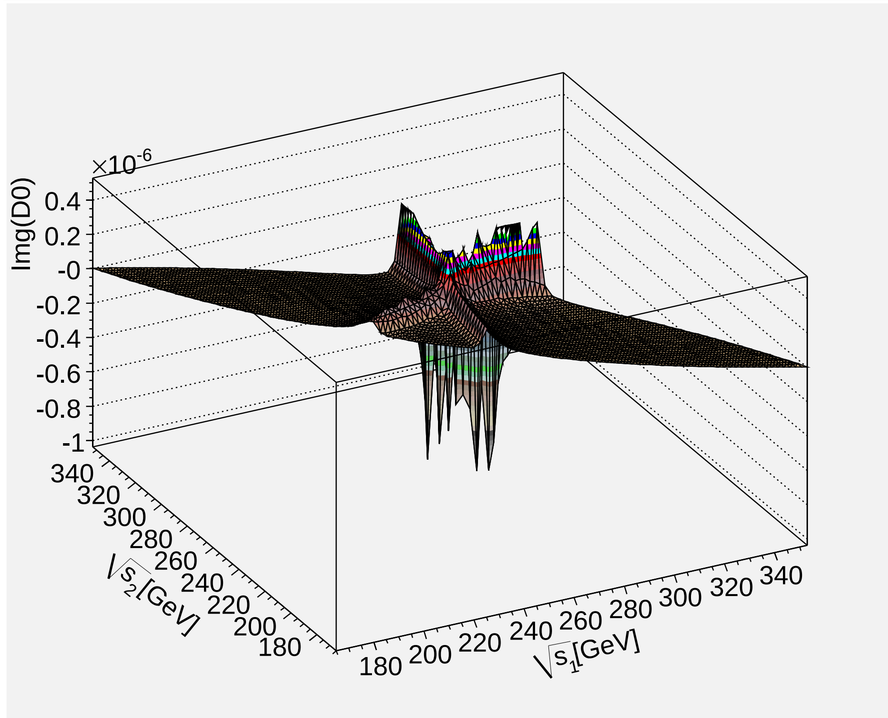


- $s_1 \equiv (p_3 + p_5)^2, \;\; s_2 \equiv (p_4 + p_5)^2$
- $det(S_4) \equiv \frac{det(Q_4)}{16 M_W^4 m_t^4} = a s_2^2 + 2 b s_2 + c$
  $a, b, c = f(m_t^2, M_W^2, s_1, s, M_H^2)$.
- Gram determinant: $det(G_3) = 2(s + M_H^2 - s_1 - s_2)(s_1 s_2 - s M_H^2)$
- The kinematically allowed region: $det(G_3) \geq 0$

$$M_H^2 \leq \quad s_1 \quad \leq s,$$
$$M_H^2 \frac{s}{s_1} \leq \quad s_2 \quad \leq M_H^2 + s - s_1.$$

Real & Imaginary parts: $g^* \to b\bar{b}H$



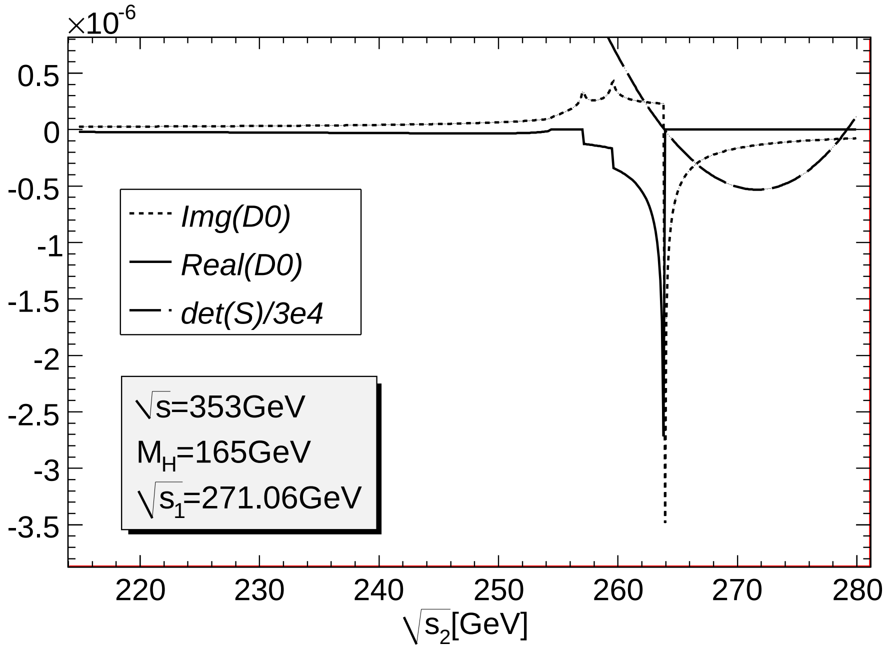


$$D_0 = D_0(M_H^2, 0, s, 0, s_1, s_2, M_W^2, M_W^2, m_t^2, m_t^2).$$

Input parameters: $\sqrt{s} = 353GeV > 2m_t$, $M_H = 165GeV > 2M_W$.

Take $\sqrt{s_1} = \sqrt{2(m_t^2 + M_W^2)} \approx 271.06GeV \to$

Nature of the singularity



$$det(S) = as_2^2 + bs_2 + c = 0$$

$$s_2 = \frac{1}{2a}(-b \mp \sqrt{b^2 - 4ac})$$

- $\sqrt{s_2} = 263.88$GeV:
  $\{x_1 \approx 0.53, x_2 \approx 0.75, x_3 \approx 0.78\}$
- $\sqrt{s_2} = 279.18$GeV:
  $\{x_1 \approx -0.74, x_2 \approx -0.75, x_3 \approx 1.07\}$

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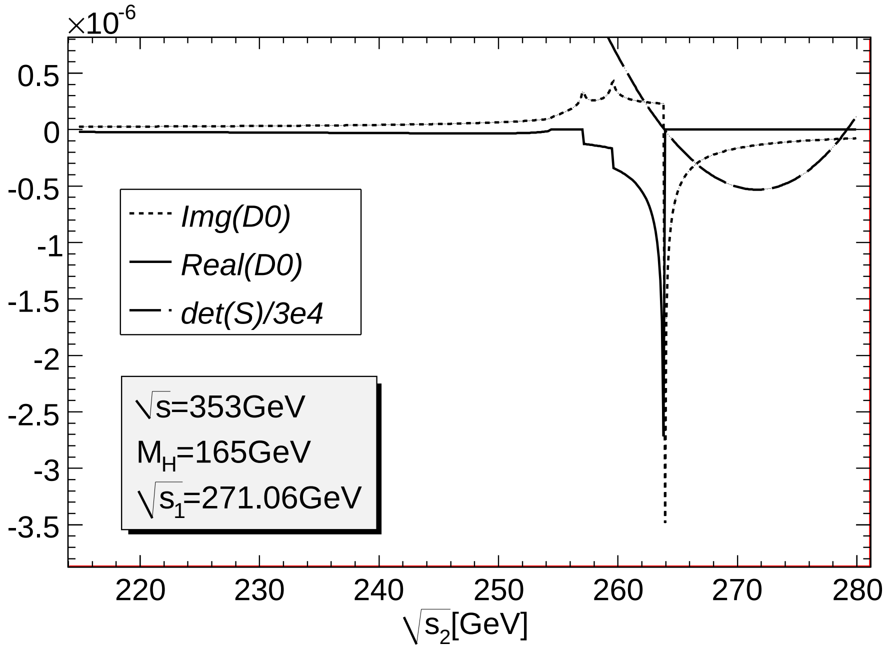

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$$D_0^{div} = \frac{e^{i\pi(3-K)/2}}{4\sqrt{(-1)^{3-K}det(Q) - i\varepsilon}} = -\frac{1}{4\sqrt{det(Q) - i\varepsilon}}, \quad (K=1)$$
$$C_0^{div} = \frac{-e^{i\pi(2-K)/2}\upsilon}{8\pi\sqrt{(-1)^{2-K}\lambda_1\lambda_2}} \ln(\lambda_3\upsilon^2 - i\varepsilon) \propto -i\ln(Q_3 - i\varepsilon), \quad (K=1)$$

λ_i: eigenvalues, K: # positive eigenvalues, $\upsilon = \sqrt{V_N.V_N}$: V_N eigenvector ($\lambda_N \propto det(Q)$).

- D_0^{div} is integrable but its square is not $\Rightarrow$ $\Gamma_{t,W}$ must be taken into account.
- C_0^{div} and its square are integrable. ▶

Complex masses: $g^* \to b\bar{b}H$

LoopTools(FF) with complex masses: up to 3pt functions.

$$D_0(\Gamma_t, \Gamma_W) \;=\; \frac{1}{\sqrt{det(Q)}} \sum_{i=1}^{2} \sum_{j=1}^{4} (-1)^{i+j} \int_0^1 dy \frac{1}{y - y_i} \ln(A_j y^2 \;+\; B_j y \;+\; C_j)$$

written in terms of 32 spence functions. Carefully checked:

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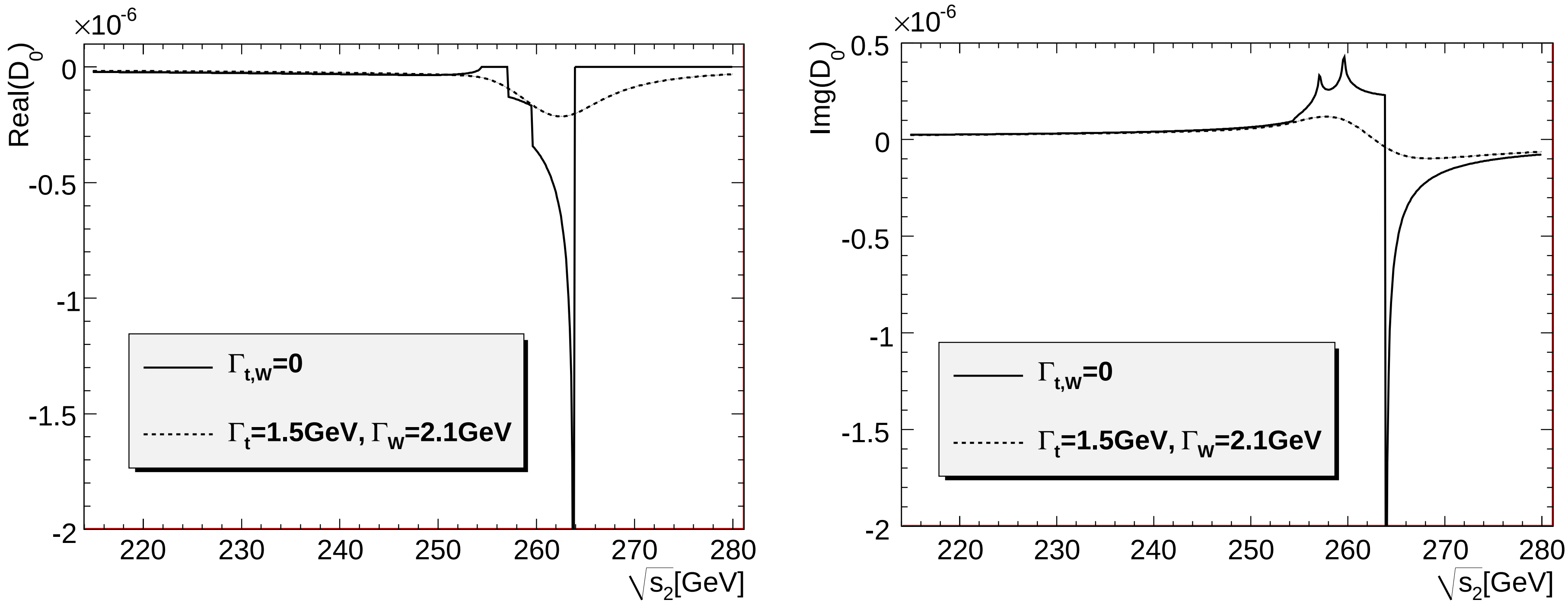
written in terms of 32 spence functions. Carefully checked:

- $\Gamma_{t,W} \approx 0$: compared to the real mass case
- $\Gamma_{t,W}$ very large: compared with numerical integration method
- Boundary of phase space (segmentation): $D_0(\Gamma_t, \Gamma_W) \propto \sum_{i=1}^{4} C_0^{(i)}(\Gamma_t, \Gamma_W)$ confirmed

Complex masses: $g^* \to b\bar{b}H$



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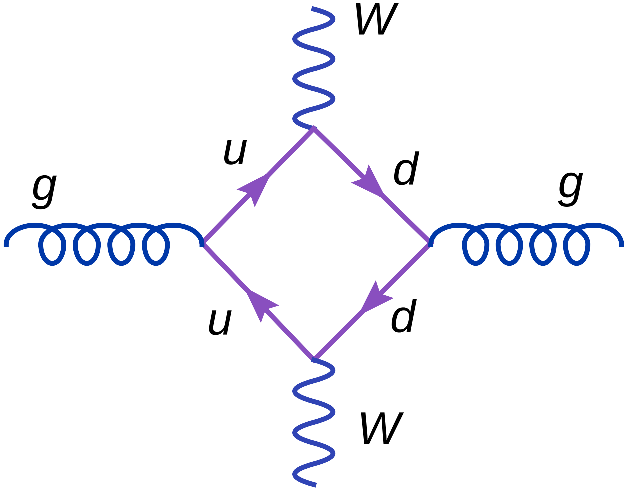
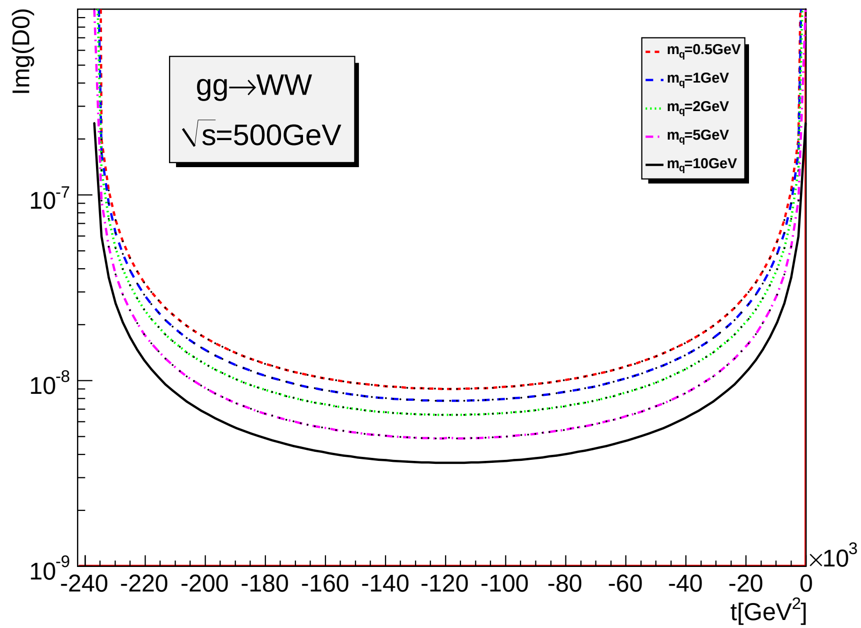
- Boundary of phase space (segmentation): $D_0(\Gamma_t, \Gamma_W) \propto \sum_{i=1}^{4} C_0^{(i)}(\Gamma_t, \Gamma_W)$ confirmed

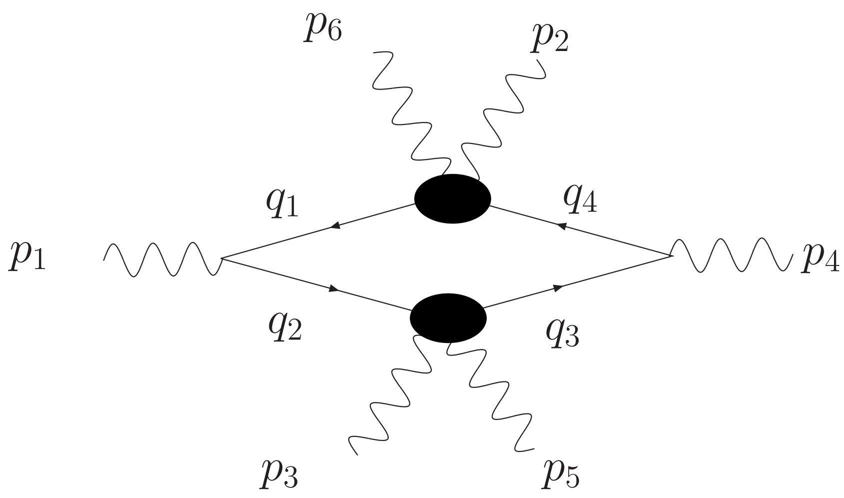
$gg \to WW$

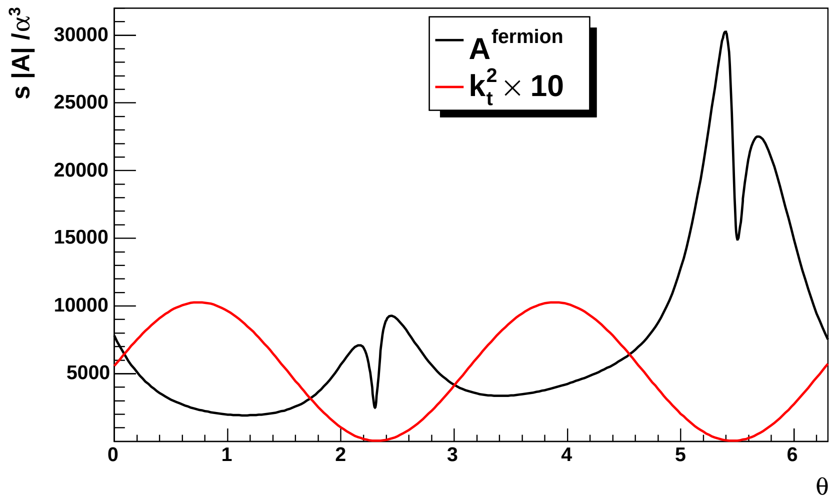
- $m_u = m_d = 0$ (*T. Binoth, M. Ciccolini, N. Kauer and M. Krämer; hep-ph/0611170; $gg \to W^*W^*$*).
  Leading Landau singularity occurs in the scalar 4pt function (see diagram) when
  $det(Q_4) = (tu - M_W^4)^2 = 0$ (and $t < 0, u < 0$)
  Gram determinant: $det(G_3) = 2s(tu - M_W^4) = -2s^2 k_t^2$.
  On the boundary: $det(Q_4) = det(G_3)^2/(4s^2) = 0 \to$ more complicated.
  Fact: double precision $\longrightarrow$ problem with numerical instabilities, quadruple precision $\longrightarrow$ no problem!
  $\Longrightarrow$ cancellation between the numerator and denominator.
- $m_u = m_d = m$. The leading Landau singularity is regularised.
  $det(Q_4) = (tu - M_W^4)(tu - M_W^4 + 4m^2 s) = 0$ but $x_1 + x_2 = \frac{(tu - M_W^4) + 2m^2 s}{2m^2(t - M_W^2)} < 0$





$2\gamma \rightarrow 4\gamma$





(*C. Bernicot and J.-Ph. Guillet arXiv: hep-ph/0711.4713; also Nagy and Soper*)

- $m_e = 0$, the same phenomenon as $gg \rightarrow WW$.
- The deep can be explained by looking at the 2 conditions for Landau singularity (it is not really singular as the Landau determinant is not exactly zero).
- If $\det(Q) = 0$, there is no problem because the numerator vanishes.
- Kinematical structure: W(unstable) = 2γ (stable).

Multi-leg ($N \geq 6$) $\rightarrow$ interesting structures will appear.

$V_1 V_1 \to V_2 V_2$

$$
\begin{aligned}
det(Q_4) &= [tu - (M_2^2 - M_1^2)^2][(t-4m^2)(u-4m^2) - (M_2^2 + M_1^2 - 4m^2)^2] \\
det(G_3) &= 2s[tu - (M_2^2 - M_1^2)^2] \geq 0
\end{aligned}
$$

- $M_1 = M_2 = M_Z$ (*Denner, Dittmaier and Hahn; hep-ph/9612390*):
  Leading Landau singularity happens: $(t-4m^2)(u-4m^2) = (2M_Z^2 - 4m^2)^2$ ($x_i > 0$)
  Solution: calculate the fully inclusive cross-section.
- $M_1^2 \leq 0$ and $M_2 = M_Z$ then $(t-4m^2)(u-4m^2) > (2M_Z^2 - 4m^2)^2$.
  No leading Landau singularity.

Conclusions and outlooks

- $pp \longrightarrow b\bar{b}H$ at LHC: NLO EW correction is small in the SM, typically -4% for $M_H = 120$GeV.
- $\sigma(\lambda_{bbH} = 0)$ can be large, about $0.17\sigma_{LO}$ for $M_H \approx 150$GeV.
- To deal with the leading Landau singularity and also $\delta Z_H^{1/2}$, $\Gamma_{t,W}$ must be taken into account:
  - $D_0(\Gamma \neq 0)$: analytical method ('t Hooft and Veltman?, xloops: parallel/orthogonal decomposition + expansion of hypergeometrical function, . . .) and numerical integration (contour deformation, ε extrapolation, . . .)
  - Renormalisation scheme with complex masses.
- Problem with Landau singularities: be careful!
  - 2 conditions for Landau singularities are re-formulated.
  - Nature of the singularities is explained, exact formulae are given.
  - Unstable internal particles: solved by introducing widths.
  - Massless configurations like $gg \rightarrow WW$ (or $2\gamma \rightarrow 4\gamma$): numerator vanishes at the singular point. OR keep the mass.