Narrow-width approximation accuracy

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in collaboration with

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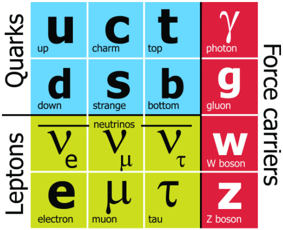
Outline

- Introduction
- Accuracy
- Limitations
- Improvements
- Conclusions

Search for the Higgs boson and BSM physics



- Unitarity-violating processes, e.g. $W_L^- W_L^+ \rightarrow W_L^- W_L^+$
- Consistent mass generation via SSB $\longrightarrow$ Higgs mechanism
- SUSY, composite Higgs, extra dimensions, ...

LHC at CERN: a powerful Terascale collider

- pp collisions at E_{CMS} = 14 TeV
- Luminosity $\mathcal{L}$ = 10–100 fb^{-1}/year

BSM physics via supersymmetry?

SUSY: invariance under boson $\leftrightarrow$ fermion transformation

Particle spectrum of the Minimal Supersymmetric Standard Model (MSSM):

gauge bosons $S = 1$ gluon, $W^\pm$, Z, γ	fermions $S = \frac{1}{2}$ $\binom{u_L}{d_L}, \binom{\nu_{eL}}{e_L}$ u_R, d_R, e_R	Higgs $S = 0$ $\binom{H_d^0}{H_d^-}, \binom{H_u^+}{H_u^0}$
gauginos $S = \frac{1}{2}$ gluino, $\widetilde{W}^\pm$, $\widetilde{Z}$, $\widetilde{\gamma}$	sfermions $S = 0$ $\binom{\widetilde{u}_L}{\widetilde{d}_L}, \binom{\widetilde{\nu}_{eL}}{\widetilde{e}_L}$ $\widetilde{u}_R, \widetilde{d}_R, \widetilde{e}_R$	Higgsinos $S = \frac{1}{2}$ $\binom{\widetilde{H}_d^0}{\widetilde{H}_d^-}, \binom{\widetilde{H}_u^+}{\widetilde{H}_u^0}$

$M_X \neq M_{\widetilde{X}} \Rightarrow$ SUSY broken

Unstable particles and QFT

"Ordinary" QFT: unstable particle fields $\longrightarrow$ asymptotic states ($t = \pm\infty$)

Eliminate these states $\longrightarrow$ Hilbert space of stable particle states

Theory still unitary, causal and renormalizable Veltman (1963)

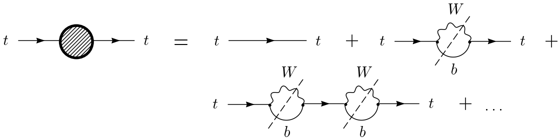
[No discussion of gauge invariance!]

Unstable particles and perturbation theory

LO propagators for massive gauge bosons and fermions:

$$\frac{-i}{p^2-M^2}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{M^2}\right)\,, \qquad \frac{i(\not{p}+M)}{p^2-M^2}$$

Fixed-order propagator: real pole at $p^2 = M^2$



$$\frac{i}{\not{p}-M_t}\sum_{n=0}^{\infty}\left((-i\Pi_t)\frac{i}{\not{p}-M_t}\right)^n = \frac{i}{\not{p}-M_t-\Pi_t}$$

[analytic continuation to resonant region]

$$\text{Im}\,\Pi_t = -\frac{1}{64\pi}\frac{g^2}{M_W^2}\left(1-\frac{M_W^2}{p^2}\right)^2(2M_W^2+p^2)\,\not{p}P_L$$

$p^2 \approx M_t^2$: $\text{Im}\,\Pi_t \approx -M_t\Gamma_t$

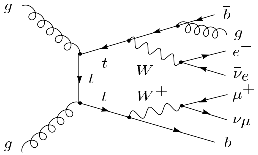
Breit-Wigner propagator:

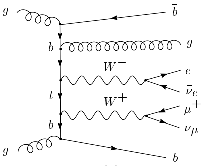
$$\frac{i(\not{p}+M_t)}{p^2-M_t^2+iM_t\Gamma_t}$$

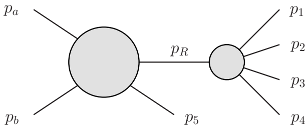
Dyson-resummed propagator: complex pole at $p^2 = M^2 - iM\Gamma$ [gauge invariance?]

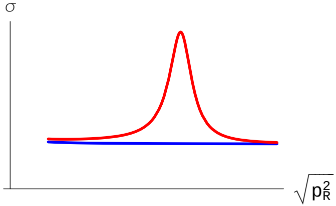
- Physical fixed-order amplitudes with unstable particles include contributions from all orders in perturbation theory

Generic resonant enhancement









Generic resonant enhancement: $$M^2 \int_{-\infty}^{\infty} \frac{dp^2}{(p^2 - M^2)^2 + (M\Gamma)^2} \sim \frac{\boldsymbol{M}}{\boldsymbol{\Gamma}}$$

$\Gamma/M \lesssim 1\% \longrightarrow$ nonresonant contribution typically negligible

Theoretically consistent method to extract dominant resonant contribution?

Resonant production and decay factorization

- **Off-shell** MC generators with **complete** $|\mathcal{M}|^2$ at tree level ✓
  use WHIZARD, MADEVENT, SHERPA, ... (but: often requires too much CPU time)
- Precise LHC/ILC predictions: need N^{n}LO calculations, $n \geq 1$
- (S)particle decay chains $\longrightarrow$ many-particle final states
- Already 4,5,6-leg one-/two-loop calculations tech. very demanding

Narrow-width approximation (NWA) to the rescue
Sub- and nonresonant/nonfactorizable contributions can be neglected in a theoretically consistent way (gauge inv., ...)

$$\Gamma \to 0: \quad D(p^2) \equiv \frac{1}{(p^2 - M^2)^2 + (M\Gamma)^2} \sim \frac{\pi}{M\Gamma} \cdot \delta(p^2 - M^2) \quad \boxed{\text{on-shell}}$$

Scales occurring in $D(p^2) \longrightarrow$ error estimate $\mathcal{O}(\Gamma/M)$

Branching ratio measurement implies NWA (and spin averaging):

$$\text{BR}_X \equiv \frac{\Gamma_X}{\Gamma} = \frac{\sigma_{\text{NWA}}}{\sigma_p} \approx \frac{\sigma_X}{\sum_X \sigma_X}$$

NWA accuracy: what to compare to?

Tree-level finite-width schemes [safe $\longrightarrow$ agreement up to $\mathcal{O}((\Gamma/M)^2)$]

Running-width scheme (Dyson-resummed propagator)

$$1/(p^2 - M^2) \longrightarrow 1/(p^2 - M^2 + ip^2\Gamma/M)$$

not gauge invariant, typically unsafe

Fixed-width scheme (FWS)

$$1/(p^2 - M^2) \longrightarrow 1/(p^2 - M^2 + iM\Gamma)$$

not gauge invariant, typically safe

Overall-factor scheme

$$\mathcal{M}_{\Gamma=0} \cdot (p^2 - M^2)/(p^2 - M^2 + iM\Gamma)$$

gauge invariant, unsafe for complex resonance structure

Baur, Vermaseren, Zeppenfeld (1992)

Complex-mass scheme

$M \longrightarrow \sqrt{M^2 - iM\Gamma} \;\Rightarrow$ complex $\cos\theta_W$, y_t

gauge invariant, no known practical problems

Lopez Castro, Lucio, Pestieau (1991); Denner, Dittmaier, Roth, Wackeroth (1999)

Complex-mass scheme at one-loop level

Denner, Dittmaier, Roth, Wieders (2005)

For each unstable particle:

$$\text{real bare mass} \;\longrightarrow\; \underbrace{\text{complex renormalized mass}}_{\longrightarrow\ \text{free propagator (resummed)}} \;+\; \underbrace{\text{complex counterterm}}_{\longrightarrow\ \text{CT vertex (not resummed)}}$$

- Gauge invariance relations fulfilled order by order
- No double counting
- 1-loop integrals with complex internal masses required
- Potential unitarity violations are of higher order

Applications: radiative corrections to

- $e^+e^-(\to W^+W^-) \to 4$ fermions Denner, Dittmaier, Roth, Wieders (2005)
- $H \to W^+W^-/ZZ \to 4$ fermions Bredenstein, Denner, Dittmaier, M. Weber (2006-7)
- $(q/\bar{q})^2 \to (q/\bar{q})^2 H$ Ciccolini, Denner, Dittmaier (2008)

Unstable-particle effective field theory

Chapovsky, Vy. Khoze, Signer, Stirling (2002)
Beneke, Chapovsky, Signer, Zanderighi (2004)

$\Gamma \ll M \rightarrow$ **hierarchy of scales** $\rightarrow$

Expand cross section in powers of α **and** $\delta \equiv (p^2 - M^2)/M^2 \sim \Gamma/M$

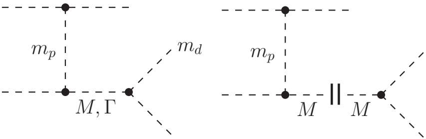
1. Integrate out **hard momenta** $k \sim M$ in underlying theory:
   underlying theory $\rightarrow$ effective theory with short-distance matching coefficients
2. Identify remaining dynamical modes (corresponding to momentum configurations near mass shell) $\rightarrow$ field operators $\rightarrow$ effective Lagrangian
3. Match coefficients (up to a certain order in α and δ) to underlying theory by calculating and comparing on-shell n-point functions

- Gauge invariance, resummation of self-energy insertions ✓
- Systematic extension to higher orders ✓, no double counting
- Well suited for inclusive calculations close to threshold
- Not well suited for fully differential calculations
- Underlying theory results with "$\Gamma \rightarrow 0$" required for matching

Corrections to $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$ at WW threshold Beneke, Falgari, Schwinn, Signer, Zanderighi (2008)

Limitations of the $\mathcal{O}(\Gamma/M)$ uncertainty estimate

Relative deviation $R \equiv \sigma_{\text{FWS}}/\sigma_{\text{NWA}} - 1 = R^{(1)} + \mathcal{O}(\Gamma^2)$ for scalar process:



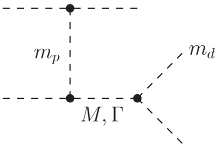
$$R^{(1)} = \left\{ \frac{M\left(s-m_d^2\right)}{\pi\left(m_d^2-M^2\right)\left(s-M^2\right)} + \left[\pi M\left(m_d^2-M^2\right)\left(s-M^2\right)\left(s+m_p^2\right)\left(s-M^2+m_p^2\right)\right]^{-1} \left[m_d^2\, s\left(s-M^2+m_p^2\right)^2 \ln\frac{s}{m_d^2} \right.\right.$$
$$\left.\left. + \left(s+m_p^2\right)\left(m_d^2\left(s\left(s+m_p^2\right)-2M^2 s+M^4\right)-M^4 m_p^2\right)\ln\frac{M^2-m_d^2}{s-M^2} + M^4 m_p^2\left(s-m_d^2+m_p^2\right)\ln\frac{s-m_d^2+m_p^2}{m_p^2} \right] \right\} \cdot \boldsymbol{\Gamma}$$

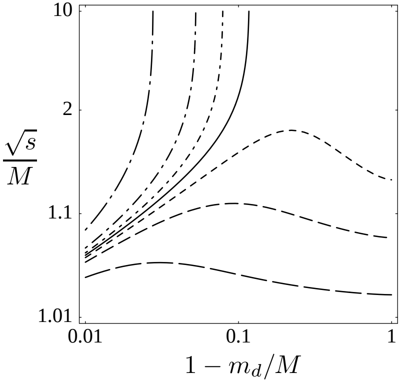
Note: additional scales ($\sqrt{s}$, particle masses in production or decay, ...)

Not $\{\ldots\} \approx 1/M$ when

$\sqrt{s} \to M$ (production threshold ✓) or $m_d \to M$

Scalar process





Contour lines (dashed, solid, dot-dashed) for $R/(\Gamma/M)$

$$\in \{-10, -3, -1, 0, 1, 3, 10\}$$

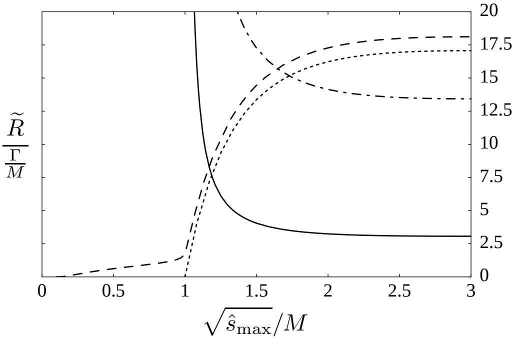
with $\Gamma/M = 1\%$ and $m_p = 0.02\,M$

Dash length increases with magnitude

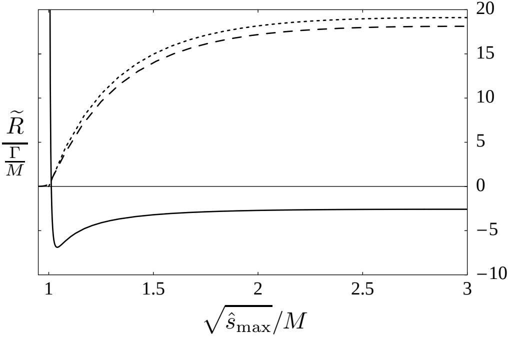
Scalar process with distributed $\sqrt{\hat{s}}$

Constant PDFs

$$\widetilde{\sigma}_{\text{[NWA]}}(\sqrt{\hat{s}_{\text{max}}}) \;\propto\; \frac{1}{s}\int_0^{\hat{s}_{\text{max}}} d\hat{s}\;\sigma_{\text{[NWA]}}(\hat{s})\cdot\ln\frac{s}{\hat{s}} \quad \text{and} \quad \widetilde{R} \equiv \frac{\widetilde{\sigma}}{\widetilde{\sigma}_{\text{NWA}}} - 1$$

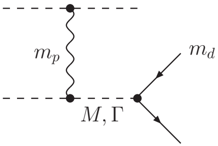


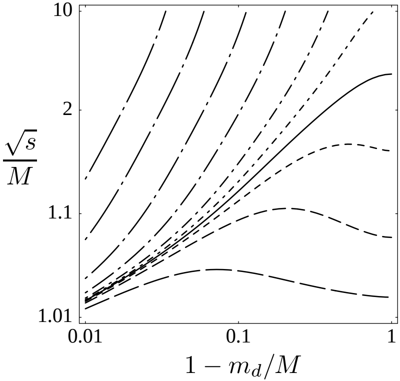
Cross sections $\widetilde{\sigma}_{\text{NWA}}$ (dotted) and $\widetilde{\sigma}$ (dashed) and relative deviation $\widetilde{R}/(\Gamma/M)$ for $m_d = 0.1\,M$ (solid) [and $m_d = 0.001\,M$ (dot-dashed)], $\sqrt{s} = 3\,M$, $\Gamma = 0.02\,M$, $m_p = 0.01\,M$



As before, but with $m_d = 0.9\,M$.

Nonscalar process





Contour lines (dashed, solid, dot-dashed) for $R/(\Gamma/M)$

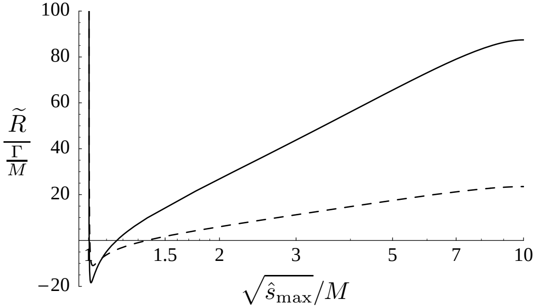
$$\in \{-10, -3, -1, 0, 1, 3, 10, 30, 100, 300\}$$

with $\Gamma/M = 1\%$ and $m_p = 0.02\,M$

Dash length increases with magnitude

Nonscalar process with distributed $\sqrt{\hat{s}}$

Constant PDFs



$\widetilde{R}/(\Gamma/M)$ for $m_d = 0.95\,M$ (solid) and $m_d = 0.9\,M$ (dashed),

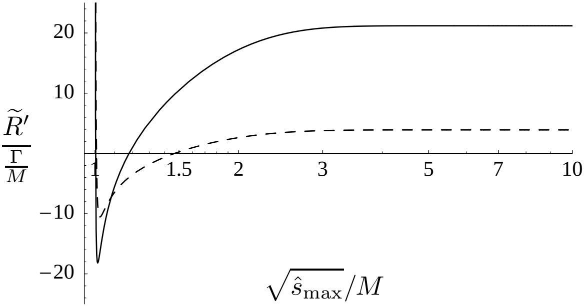
$\sqrt{s} = 10\,M$, $\Gamma = 0.02\,M$, $m_p = 0.01\,M$

Nonscalar process with distributed $\sqrt{\hat{s}}$

Proton-inspired PDFs

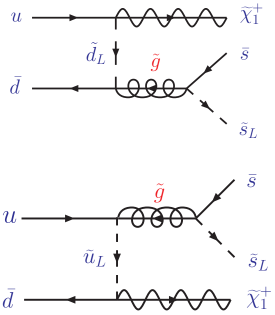
$$\widetilde{\sigma}'_{[\mathrm{NWA}]}(\sqrt{\hat{s}_{\mathrm{max}}}) \;\propto\; \frac{1}{s}\int_0^{\hat{s}\,\mathrm{max}} d\hat{s}\,\sigma_{[\mathrm{NWA}]}(\hat{s})\int_{\hat{s}/s}^{1}\frac{dx}{x}\,f(x)f\left(\frac{\hat{s}}{xs}\right),$$

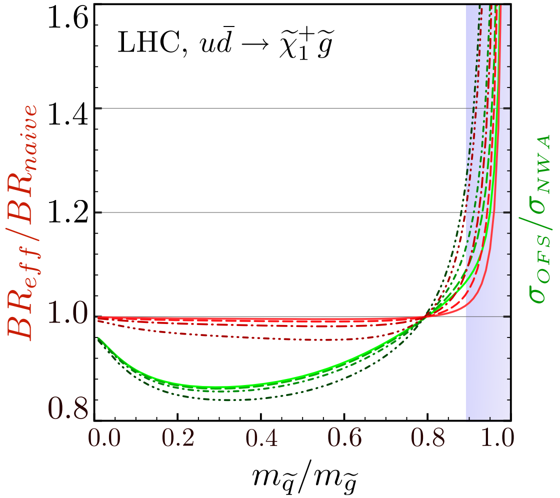
$f(x) \propto (1-x)^9/\sqrt{x}$ and $\widetilde{R}' \equiv \widetilde{\sigma}'/\widetilde{\sigma}'_{\mathrm{NWA}} - 1$



$\widetilde{R}'/(\Gamma/M)$ for $m_d = 0.95\,M$ (solid) and $m_d = 0.9\,M$ (dashed),
$\sqrt{s} = 10\,M$, $\Gamma = 0.02\,M$, $m_p = 0.01\,M$

MSSM example at the LHC





$\Gamma(\widetilde{g})/M(\widetilde{g}) \approx 6\,\text{GeV}/600\,\text{GeV} = 1\%$, SPS1a

Resonant 1 → 3 decays in the MSSM

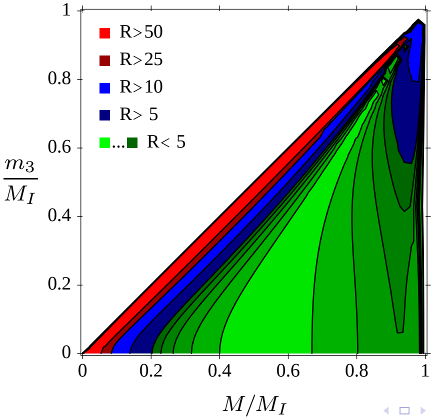
Systematic on-/off-shell deviation analysis for 48 generic processes

$$\boxed{T_I(P_I, M_I) \to T_1(p_1, m_1),\ T(q, M, \Gamma)} \text{ and}$$

$$\boxed{T(q, M, \Gamma) \to T_2(p_2, m_2),\ T_3(p_3, m_3)}$$

$T_I, T_1, T, T_2, T_3 \in$ {scalar (S), fermion (F), vector boson (V)}

Example: S → SS followed by S → SV



Large deviations at SPS points

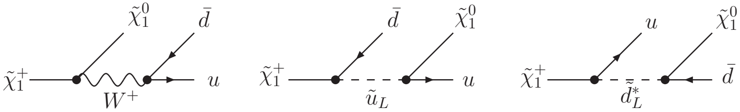
process	SPS	$R/(\Gamma/M)$	Γ/M [%]
$\tilde{g} \to d\tilde{d}_L^* \to d\bar{d}\tilde{\chi}_1^0$	$1a$	9.54	0.935
$\tilde{g} \to d\tilde{d}_L^* \to d\bar{d}\tilde{\chi}_1^0$	5	11.4	0.956
$\tilde{g} \to u\tilde{u}_L^* \to u\bar{u}\tilde{\chi}_1^0$	$1a$	5.98	0.976
$\tilde{g} \to u\tilde{u}_L^* \to u\bar{u}\tilde{\chi}_1^0$	5	9.46	0.975
$\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 W^+ \to \tilde{\chi}_1^0 u\bar{d}$	$1a$	5.21	2.49
$\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 W^+ \to \tilde{\chi}_1^0 e^+\nu_e$	$1a$	5.21	2.49
$\tilde{g} \to \bar{b}\tilde{b}_2 \to \bar{b}b\tilde{\chi}_1^0$	4	6.43	1.11
$\tilde{g} \to \bar{u}\tilde{u}_L \to \bar{u}d\tilde{\chi}_1^+$	9	114	1.19
$\tilde{g} \to d\tilde{d}_L^* \to d\bar{u}\tilde{\chi}_1^+$	9	209	1.19

and many 1 $\to$ 3 decay chain segments with $|R|/(\Gamma/M) > 5$

$\longrightarrow$ consider all amplitude contributions to $i \to f$

$\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 u \bar{d}$ at SPS 1a

BR $= 1.3\%$ (dominant decay modes: $\tilde{\chi}_1^+ \to \tilde{\tau}_1^+ \nu_\tau,\ \tilde{\nu}_{\tau 1}\tau^+,\ \tilde{\nu}_{\mu L}\mu^+,\ \tilde{\nu}_{eL} e^+$)



$\tilde{\chi}_1^+ \to \tilde{\chi}_1^0\ (W^+ \to u\bar{d})$: resonant intermediate state $\longrightarrow$ dominant contribution

1. decay stage: $\dfrac{m_1 + M}{M_I} = 0.975 \quad \to \quad R/(\Gamma/M) = 5.21$ with $\Gamma/M = 2.49\%$

No QCD corrections for 1. decay stage

Also small, nonresonant contributions from $\tilde{u}_L$ and $\tilde{d}_L$ decay channels

Total on-/off-shell decay rate deviation $R_{W+\tilde{u}_L+\tilde{d}_L} = 11\%$

Origin of amplified deviations

Off-shell vs. NWA cross section:

$$\sigma = \frac{1}{2s} \int \frac{dp^2}{2\pi} D(p^2) \int d\phi_p(p^2) \int d\phi_d(p^2) \, |\mathcal{M}_r(p^2)|^2$$

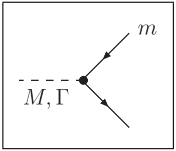
$$\sigma_{\text{NWA}} = \frac{1}{2s} K_{\text{NWA}} \int d\phi_p(M^2) \int d\phi_d(M^2) \, |\mathcal{M}_r(M^2)|^2$$

with $K_{\text{NWA}} = \frac{1}{2M\Gamma} = \int_{-\infty}^{\infty} \frac{dp^2}{2\pi} D(p^2)$

Deforming factors in phase space element and $|\mathcal{M}_d|^2$:

$$\frac{d\phi_d}{d\Omega^*} = \frac{1}{32\pi^2} \sqrt{\left[1 - \frac{(m_2+m_3)^2}{p^2}\right]\left[1 - \frac{(m_2-m_3)^2}{p^2}\right]}$$

$$= \frac{1}{32\pi^2} \frac{p^2 - m^2}{p^2} \quad \text{for } m \equiv m_2, m_3 = 0$$

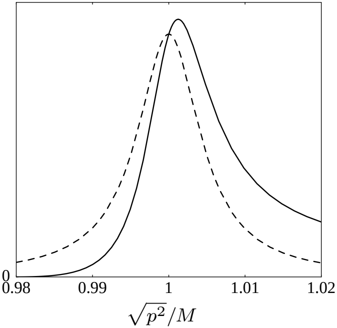


Strategy for improvements: absorb amplifying factors into K

Breit-Wigner shape deformation by threshold factors

$$\widetilde{R} \equiv K_{\text{INWA}}/K_{\text{NWA}} =$$

$$\left(\int_{m^2}^{p^2_{\text{max}}} \frac{dp^2}{2\pi} \frac{1}{(p^2 - M^2)^2 + (M\,\Gamma)^2} \frac{(p^2 - m^2)^2/p^2}{(M^2 - m^2)^2/M^2}\right) \Big/ \left(\int_{-\infty}^{\infty} \frac{dp^2}{2\pi} \frac{1}{(p^2 - M^2)^2 + (M\,\Gamma)^2}\right)$$



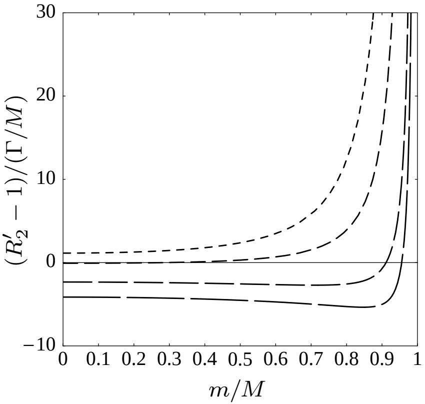
Integrand of numerator (solid) and denominator (dashed)

for $m = M - 2\Gamma$ and $\Gamma/M = 1\%$

Explicitly

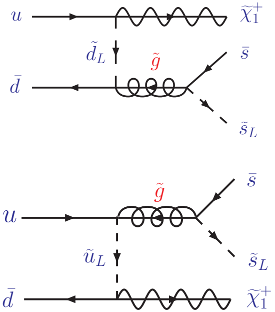
$$\widetilde{R}\,(\equiv R_2') \;=\; \frac{1}{\pi}\left[\tan^{-1}\frac{\beta^2}{\gamma}+\tan^{-1}\frac{\lambda}{\gamma}\right]+\frac{\gamma}{\pi}\left[\left(\frac{2}{\beta^2}-1\right)\ln\frac{\lambda}{\beta^2}+\left(\frac{1}{\beta^2}-1\right)^2\ln\frac{p^2_{\mathsf{max}}}{m^2}\right]$$

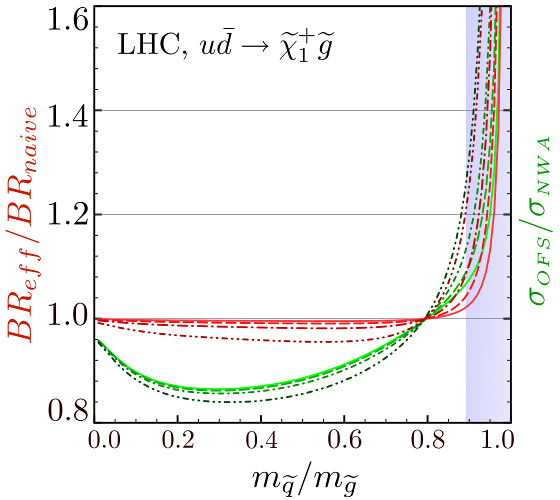
with $\gamma \equiv \Gamma/M$, $\beta = \sqrt{1-m^2/M^2}$ and $\lambda \equiv p^2_{\mathsf{max}}/M^2 - 1$



$\sqrt{p^2_{\mathsf{max}}}/M \in \{1.05, 1.1, 2, 10\}$, dash length decreases with increasing p^2_{max}, $\Gamma/M = 1\%$

MSSM example at the LHC

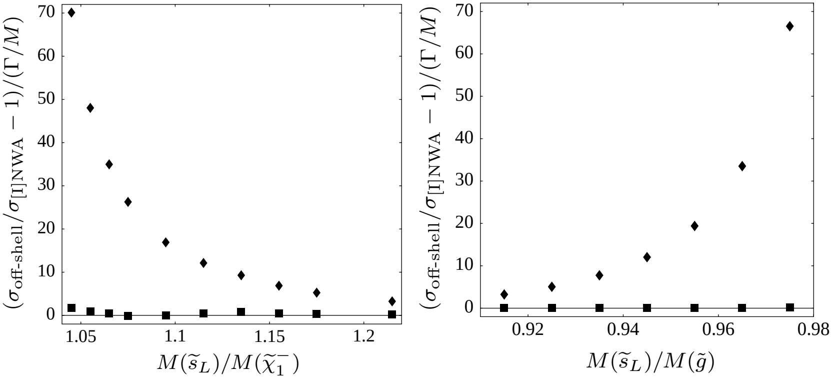




$\Gamma(\tilde{g})/M(\tilde{g}) \approx 6\,\text{GeV}/600\,\text{GeV} = 1\%$, SPS1a

Application to MSSM cascade decay at the LHC

$pp \to \tilde{g}\,\widetilde{u}_L$ followed by the cascade decay $\tilde{g} \to (\widetilde{s}_L \to \widetilde{\chi}_1^- c)\bar{s}$



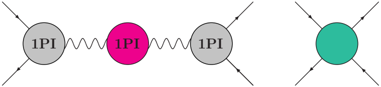
Variable strange squark mass approaching the chargino (left), gluino mass (right)

Deviations for the standard NWA (diamonds) and an improved NWA (boxes)

SPS1a

Pole Approximation

Stuart (1991); Aeppli, v. Oldenborgh, Wyler (1994)



Laurent series expansion of exact amplitude about exact complex pole M^2:

$$\mathcal{A}(s) = \frac{V_{ir}(s)V_{rf}(s)}{s - m^2 - \Pi_{rr}(s)} + \mathcal{A}'(s)\,,$$

$$M^2 - m^2 - \Pi_{rr}(M^2) = 0\,,$$

$$\mathcal{A}(s) = \frac{R(M^2)}{s - M^2} + N(s)$$

M^2, R and N are gauge invariant $\longrightarrow$ perturbative expansion

Off-shell phase space $\longrightarrow$ pole approximation better than NWA

But: amplifying factors in residual amplitude not mitigated $\longrightarrow$

Unexpectedly large deviations can also occur in pole approximation

Conclusions

- Unstable particles $\longrightarrow$ formal and practical issues in QFT
- Consistent perturbative methods for complete amplitudes exist, but such calculations are either very involved or not yet feasible
- For resonant cross sections the nonresonant contribution is suppressed
- Consistent methods to extract dominant resonant contribution exist

For configurations with kinematical bounds in an extended "threshold"-vicinity of resonances the conventional approximation uncertainty estimate $\mathcal{O}(\Gamma/M)$ is unreliable, because the p^2-dependence of the phase space elements and residual matrix elements causes a significant distortion of the Breit-Wigner peak and tail

Case study: MSSM benchmark scenarios feature decay chains with similar intermediate masses leading to approximation errors that are of the order of QCD corrections when on-shell intermediate states are used

A suggestive approach exists to improve the NWA in such cases