NLO QCD corrections to WW+jet production at hadron colliders

Stefan Kallweit[1]

in collaboration with S. Dittmaier[1] and P. Uwer[2]

based on Phys. Rev. Lett. **100**, 062003 (2008)

[1]Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), München

[2]Institut für Theoretische Teilchenphysik, Universität Karlsruhe

May 8, 2008, PSI particle theory seminar

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Outline

1. Introduction
2. Calculation of NLO QCD corrections
   - Virtual corrections
   - Real corrections
   - Collinear subtraction counterterm
3. Numerical results & comparison with other groups
4. Implementation of W decays
5. Preliminary results
6. Conclusions & Outlook

Motivation

Why is $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{WW} + \mathrm{jet} + \mathrm{X}$ interesting ?

- Important background process at the LHC (and for Tevatron Higgs searches)
  "Les Houches experimenter's wishlist '05" for important missing NLO predictions:

process	**background for**
$\mathrm{pp} \to \mathrm{WW} + \mathrm{jet}$	H+jet, $\mathrm{t\bar{t}H}$, new physics (leptons + $\not{E}_\mathrm{T}$)
Dittmaier/S.K./Uwer '07, Campbell/Ellis/Zanderighi '07	
Binoth/Guillet/Karg/Kauer/Sanguinetti (in progress)	
$\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$	$\mathrm{t\bar{t}H}$
$\mathrm{pp} \to \mathrm{t\bar{t}} + 2\mathrm{jets}$	$\mathrm{t\bar{t}H}$
$\mathrm{pp} \to \mathrm{VVb\bar{b}}$	VBF $\to \mathrm{H} \to \mathrm{VV}$, $\mathrm{t\bar{t}H}$, new physics
$\mathrm{pp} \to \mathrm{VV} + 2\mathrm{jets}$	VBF $\to \mathrm{H} \to \mathrm{VV}$
VBF: Jäger et al. '06, Bozzi et al. '07	
$\mathrm{pp} \to \mathrm{V} + 3\mathrm{jets}$	$\mathrm{t\bar{t}}$, new physics
$\mathrm{pp} \to \mathrm{VVV}$	SUSY tri-lepton
ZZZ: Lazopoulos et al. '07, WWZ: Hankele et al. '08, VVV: Binoth et al. '08	

Motivation

Why is $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{WW} + \mathrm{jet} + \mathrm{X}$ interesting ?

- Important background process at the LHC (and for Tevatron Higgs searches)
  "Les Houches experimenter's wishlist '05" for important missing NLO predictions:

process	**background for**
$\mathrm{pp} \to \mathrm{WW} + \mathrm{jet}$	H+jet, $\mathrm{t\bar{t}H}$, new physics (leptons + $\not{E}_\mathrm{T}$)
Dittmaier/S.K./Uwer '07, Campbell/Ellis/Zanderighi '07	
Binoth/Guillet/Karg/Kauer/Sanguinetti (in progress)	
$\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$	$\mathrm{t\bar{t}H}$
$\mathrm{pp} \to \mathrm{t\bar{t}} + 2\mathrm{jets}$	$\mathrm{t\bar{t}H}$
$\mathrm{pp} \to \mathrm{VVb\bar{b}}$	VBF $\to \mathrm{H} \to \mathrm{VV}$, $\mathrm{t\bar{t}H}$, new physics
$\mathrm{pp} \to \mathrm{VV} + 2\mathrm{jets}$	VBF $\to \mathrm{H} \to \mathrm{VV}$
VBF: Jäger et al. '06, Bozzi et al. '07	
$\mathrm{pp} \to \mathrm{V} + 3\mathrm{jets}$	$\mathrm{t\bar{t}}$, new physics
$\mathrm{pp} \to \mathrm{VVV}$	SUSY tri-lepton
ZZZ: Lazopoulos et al. '07, WWZ: Hankele et al. '08, VVV: Binoth et al. '08	

- A large fraction of W-pair events at the LHC show additional jet activity.
  ↪ EW gauge-boson coupling analysis

Motivation

Why is $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{WW} + \mathrm{jet} + \mathrm{X}$ interesting ?

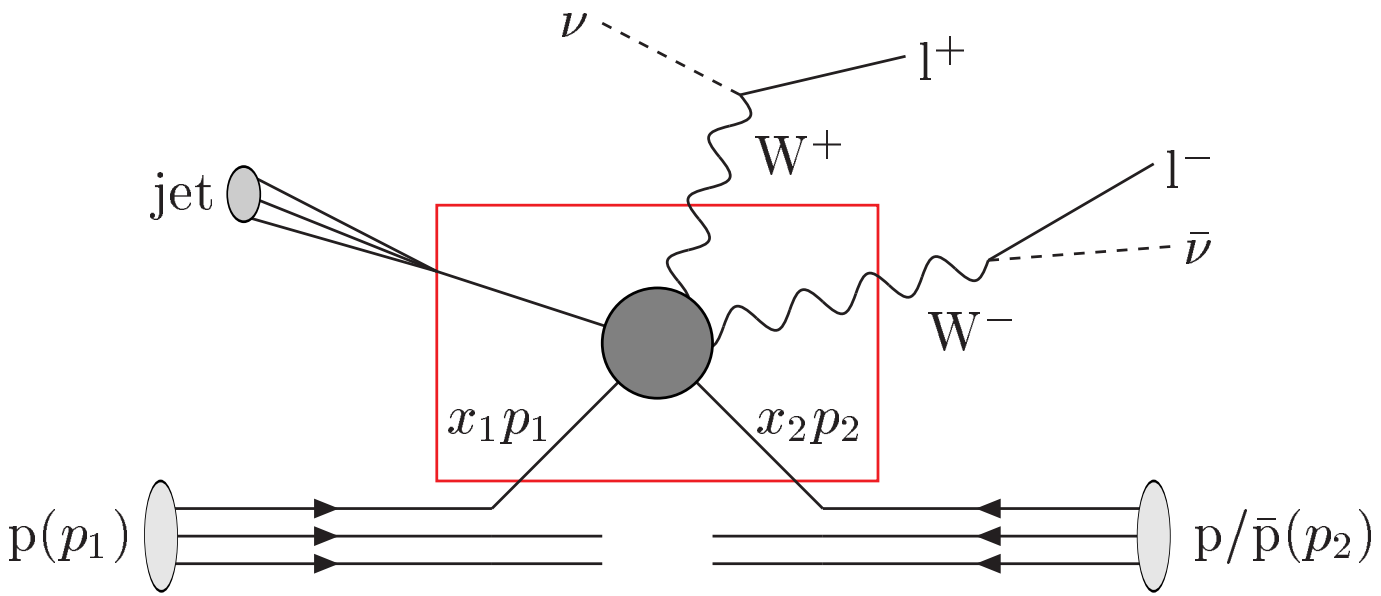
- Important background process at the LHC (and for Tevatron Higgs searches)
  "Les Houches experimenter's wishlist '05" for important missing NLO predictions:

process	**background for**
$\mathrm{pp} \to \mathrm{WW} + \mathrm{jet}$	H+jet, $\mathrm{t\bar{t}H}$, new physics (leptons + $\not{E}_\mathrm{T}$)
Dittmaier/S.K./Uwer '07, Campbell/Ellis/Zanderighi '07	
Binoth/Guillet/Karg/Kauer/Sanguinetti (in progress)	
$\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$	$\mathrm{t\bar{t}H}$
$\mathrm{pp} \to \mathrm{t\bar{t}} + 2\mathrm{jets}$	$\mathrm{t\bar{t}H}$
$\mathrm{pp} \to \mathrm{VVb\bar{b}}$	VBF $\to \mathrm{H} \to \mathrm{VV}$, $\mathrm{t\bar{t}H}$, new physics
$\mathrm{pp} \to \mathrm{VV} + 2\mathrm{jets}$	VBF $\to \mathrm{H} \to \mathrm{VV}$
VBF: Jäger et al. '06, Bozzi et al. '07	
$\mathrm{pp} \to \mathrm{V} + 3\mathrm{jets}$	$\mathrm{t\bar{t}}$, new physics
$\mathrm{pp} \to \mathrm{VVV}$	SUSY tri-lepton
ZZZ: Lazopoulos et al. '07, WWZ: Hankele et al. '08, VVV: Binoth et al. '08	

- A large fraction of W-pair events at the LHC show additional jet activity.
  $\hookrightarrow$ EW gauge-boson coupling analysis
- Process is an important test ground before approaching more complicated many-particle processes at NLO.

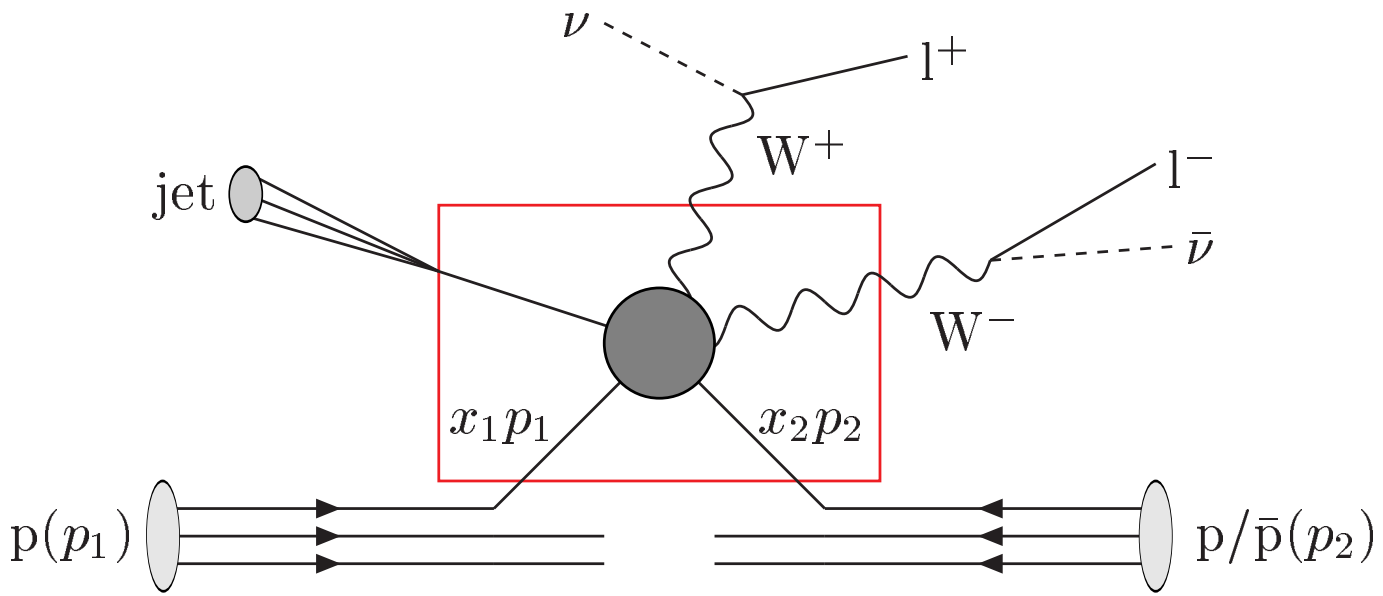
Hadronic cross section

Schematic illustration of the hadronic process $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{W^+W^-} + \mathrm{jet} + \mathrm{X}$:



Hadronic cross section

Schematic illustration of the hadronic process $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{W^+W^-} + \mathrm{jet} + \mathrm{X}$:



Hadronic cross section:

$$\sigma^{\mathrm{pp}/\mathrm{p\bar{p}}}(p_1, p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 \underbrace{f_{a(\mathrm{p})}(x_1, \mu_\mathrm{F})\, f_{b(\mathrm{p}/\bar{\mathrm{p}})}(x_2, \mu_\mathrm{F})}_{\text{PDF's of parton } a/b \text{ in } \mathrm{p}/\bar{\mathrm{p}}} \underbrace{\hat{\sigma}^{ab}(x_1 p_1, x_2 p_2)}_{\text{partonic cross section}}$$

Treatment of quarks

All light quarks $(\mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c}, \mathrm{b})$ are treated as massless particles.
$\hookrightarrow$ IR singularities are treated in dimensional regularization.

Treatment of quarks

All light quarks $(\mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c}, \mathrm{b})$ are treated as massless particles.
$\hookrightarrow$ IR singularities are treated in dimensional regularization.

For external particles only the two light generations $(\mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c})$ are taken into account.

- initial state: negligible contributions due to small b-quark pdf's
- final state: selection by anti-b-tagging

Treatment of quarks

All light quarks $(\mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c}, \mathrm{b})$ are treated as massless particles.
$\hookrightarrow$ IR singularities are treated in dimensional regularization.

For external particles only the two light generations $(\mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c})$ are taken into account.

- initial state: negligible contributions due to small b-quark pdf's
- final state: selection by anti-b-tagging

The CKM matrix reduces to Cabibbo matrix (mixing only between the two light generations). Many subprocesses are not influenced by CKM matrix (unitarity).

Treatment of quarks

All light quarks $(\mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c}, \mathrm{b})$ are treated as massless particles.
$\hookrightarrow$ IR singularities are treated in dimensional regularization.

For external particles only the two light generations $(\mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c})$ are taken into account.

- initial state: negligible contributions due to small b-quark pdf's
- final state: selection by anti-b-tagging

The CKM matrix reduces to Cabibbo matrix (mixing only between the two light generations). Many subprocesses are not influenced by CKM matrix (unitarity).

The running of α_S is solely generated by light quark and gluon loops.
(The top-quark loop is subtracted at zero momentum.)
$\hookrightarrow$ 5-flavour-running of α_S

Subprocesses contributing at leading order

6 partonic channels in LO (12 flavour channels for 2 generations, b quarks negligible):

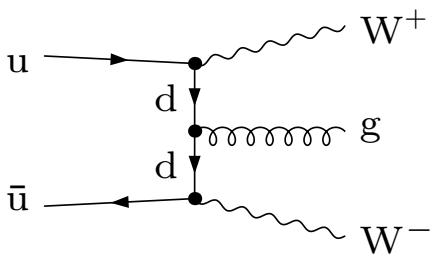
$$\mathrm{u\bar{u}} \to \mathrm{W^+W^-g}, \quad \mathrm{ug} \to \mathrm{W^+W^-u}, \quad \mathrm{g\bar{u}} \to \mathrm{W^+W^-\bar{u}},$$
$$\mathrm{d\bar{d}} \to \mathrm{W^+W^-g}, \quad \mathrm{dg} \to \mathrm{W^+W^-d}, \quad \mathrm{g\bar{d}} \to \mathrm{W^+W^-\bar{d}}$$

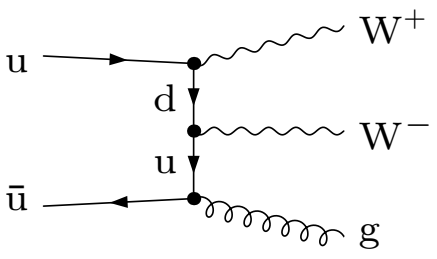
Subprocesses contributing at leading order

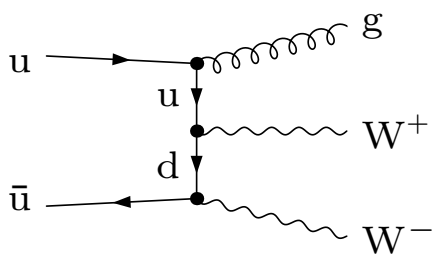
6 partonic channels in LO (12 flavour channels for 2 generations, b quarks negligible):

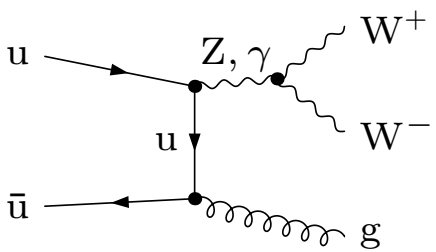
$$\mathrm{u\bar{u}} \to \mathrm{W^+W^-g}, \quad \mathrm{ug} \to \mathrm{W^+W^-u}, \quad \mathrm{g\bar{u}} \to \mathrm{W^+W^-\bar{u}},$$
$$\mathrm{d\bar{d}} \to \mathrm{W^+W^-g}, \quad \mathrm{dg} \to \mathrm{W^+W^-d}, \quad \mathrm{g\bar{d}} \to \mathrm{W^+W^-\bar{d}}$$

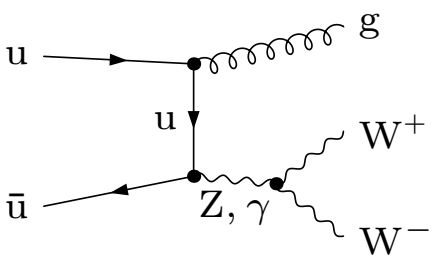
Diagrams for $\mathrm{u\bar{u}}$ initial state:











- all light quarks massless
- CKM matrix $\to$ Cabibbo matrix

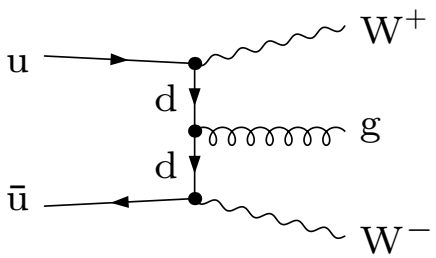
$\hookrightarrow$ no CKM dependence in LO (unitarity)

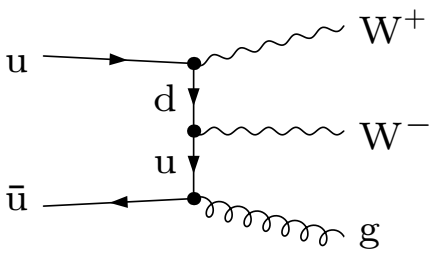
Subprocesses contributing at leading order

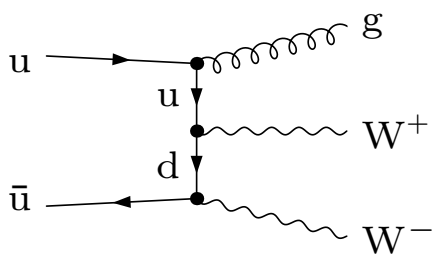
6 partonic channels in LO (12 flavour channels for 2 generations, b quarks negligible):

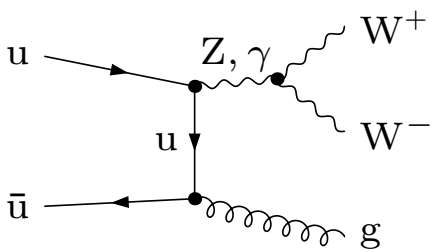
$$\mathrm{u\bar{u}} \to \mathrm{W^+W^-g}, \quad \mathrm{ug} \to \mathrm{W^+W^-u}, \quad \mathrm{g\bar{u}} \to \mathrm{W^+W^-\bar{u}},$$
$$\mathrm{d\bar{d}} \to \mathrm{W^+W^-g}, \quad \mathrm{dg} \to \mathrm{W^+W^-d}, \quad \mathrm{g\bar{d}} \to \mathrm{W^+W^-\bar{d}}$$

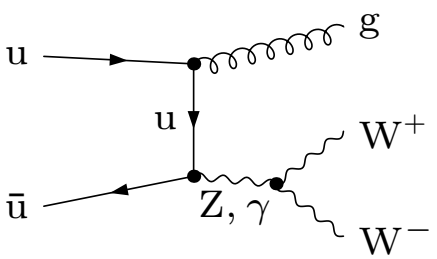
Diagrams for $\mathrm{u\bar{u}}$ initial state:











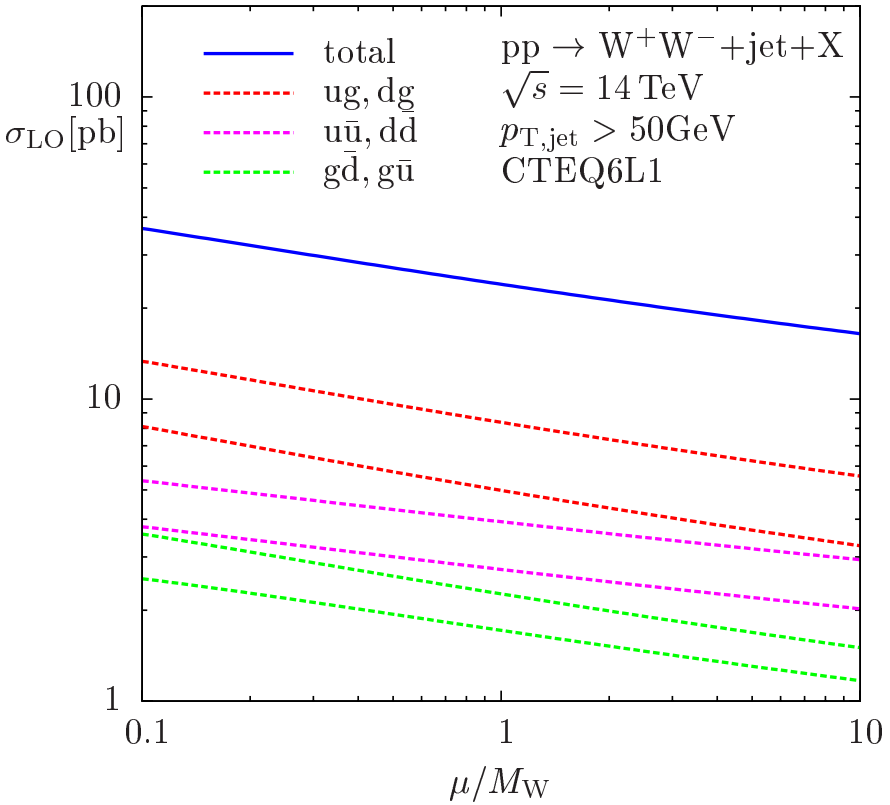
- all light quarks massless
- CKM matrix $\to$ Cabibbo matrix

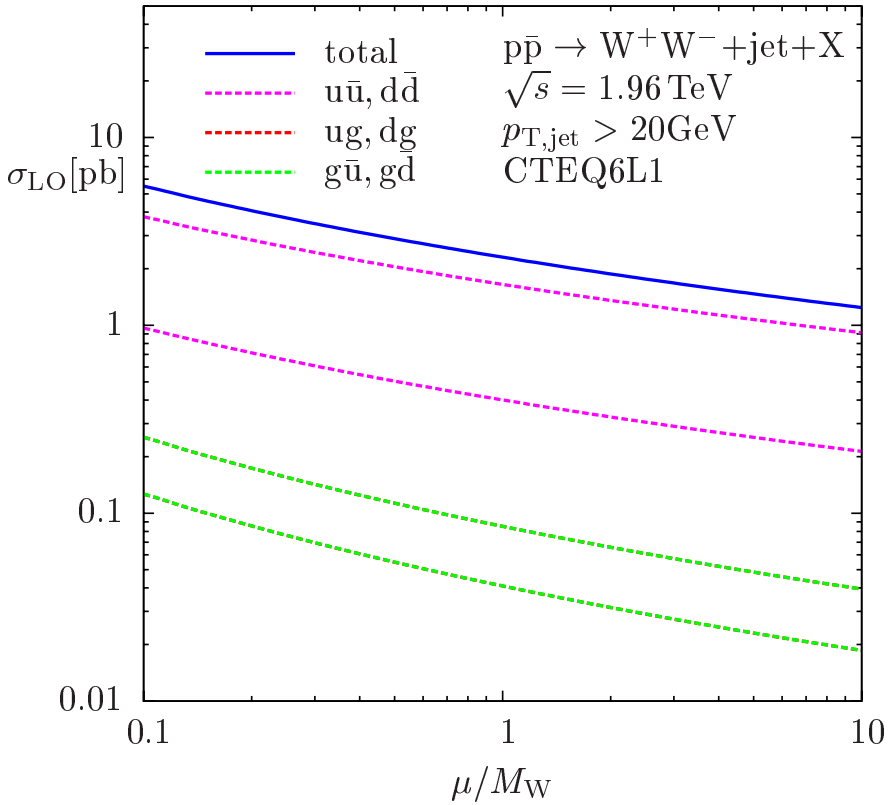
$\hookrightarrow$ no CKM dependence in LO (unitarity)

The amplitudes for all other channels are generated by crossing the gluon into the initial state and by SU(2) symmetry ($\mathrm{u} \leftrightarrow \mathrm{d}, \mathrm{W^+} \leftrightarrow \mathrm{W^-}$).

Leading-order prediction

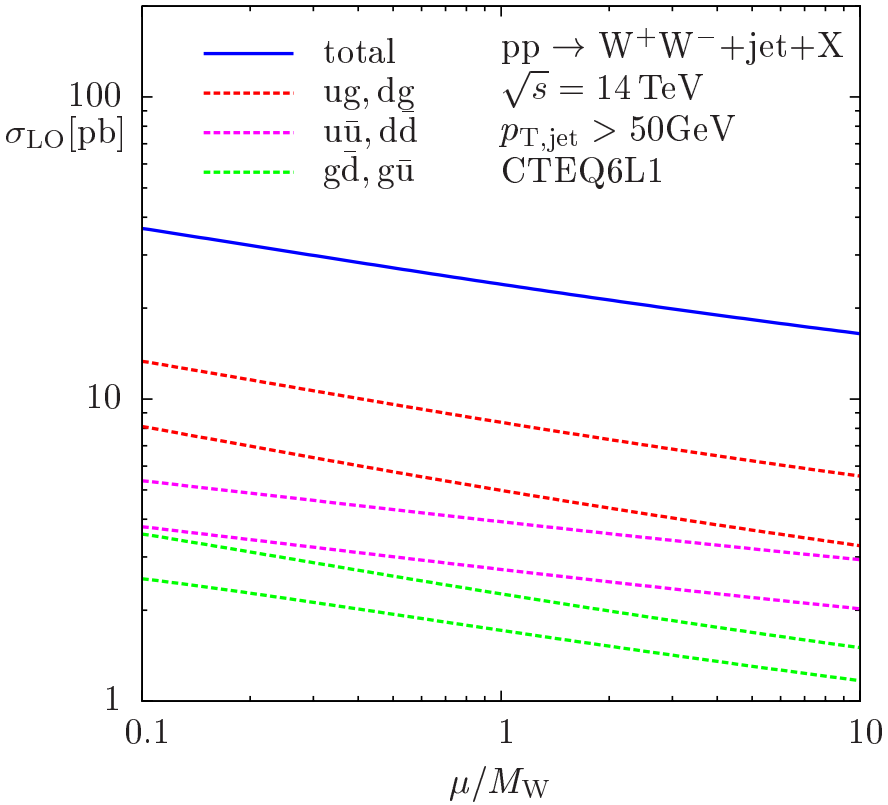
Scale dependence of LO cross section (with $\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$):

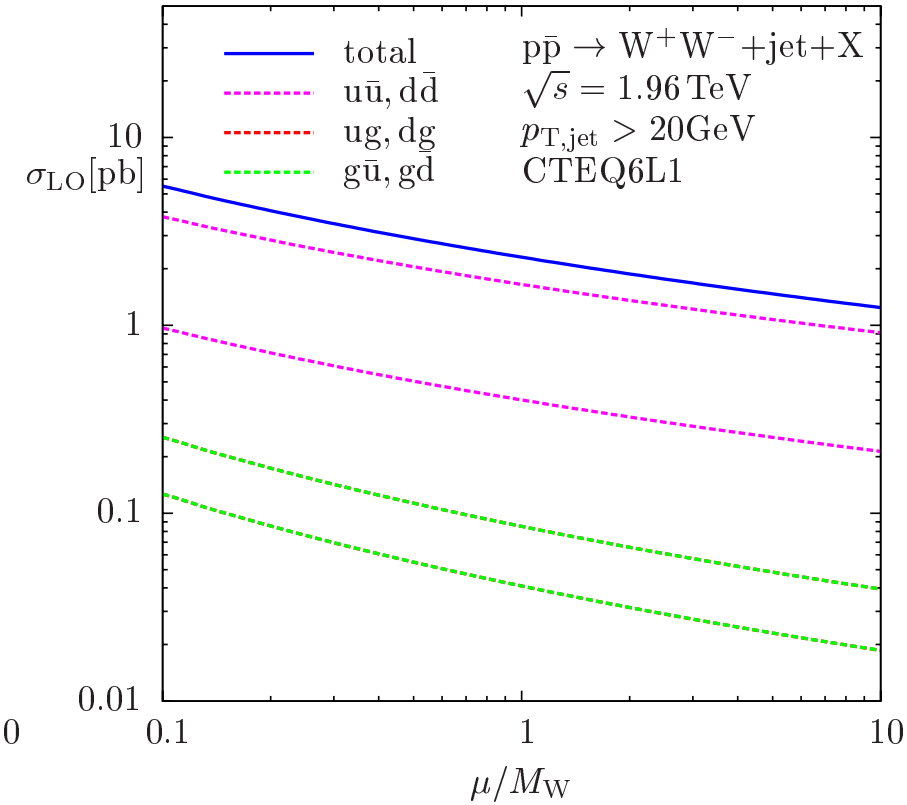




Leading-order prediction

Scale dependence of LO cross section (with $\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$):





LHC: Cross section changes by 12% (30%) when scaling μ by a factor of 2 (5).
Tevatron: Cross section changes by 25% (75%) when scaling μ by a factor of 2 (5).

$\hookrightarrow$ For precise predictions the calculation of NLO QCD corrections is required.

General form of NLO QCD corrections

The NLO cross section gets contributions which are separately IR divergent:

- Real corrections (emission of another QCD parton)
- Virtual corrections (exchange of virtual particles)

For jet observables (sufficiently inclusive via jet algorithm) factorization theorems state:

Sum of divergences is universal and connected to initial state. $\longrightarrow$ PDF redefinition

- Collinear-subtraction counterterm (factorization scale dependence)

$\hookrightarrow$ Sum over contributions is finite.

General form of NLO QCD corrections

The NLO cross section gets contributions which are separately IR divergent:

- Real corrections (emission of another QCD parton)
- Virtual corrections (exchange of virtual particles)

For jet observables (sufficiently inclusive via jet algorithm) factorization theorems state:

Sum of divergences is universal and connected to initial state. $\longrightarrow$ PDF redefinition

- Collinear-subtraction counterterm (factorization scale dependence)

$\hookrightarrow$ Sum over contributions is finite.
But: Numerical cancellations between different phase spaces required.

Technical solution: Dipole subtraction formalism [Catani, Seymour '96]:
Add and subtract an approximation of the cross section $d\sigma^A$:

- $d\sigma^A$ exactly matches the singular behaviour of $d\sigma^R$ in $d = 4 - 2\varepsilon$ dimensions.
- $d\sigma^A$ is integrable analytically over the single-parton subspaces in $d = 4 - 2\varepsilon$ dimensions leading to soft and collinear divergences (like in $d\sigma^V$ and $d\sigma^C$).

NLO cross section with the dipole subtraction formalism

Schematic formula for the NLO cross section in the situation of two initial-state hadrons (LHC and Tevatron):

$$\sigma^{\mathrm{NLO}} = \underbrace{\int_{m+1} d\sigma^R}_{\text{real corrections}} + \underbrace{\int_m d\sigma^V}_{\text{virtual corrections}} + \underbrace{\int_0^1 dx \int_m d\sigma^C}_{\text{collinear-subtraction counterterm}}$$

NLO cross section with the dipole subtraction formalism

Schematic formula for the NLO cross section in the situation of two initial-state hadrons (LHC and Tevatron):

$$\sigma^{\mathrm{NLO}} = \underbrace{\int_{m+1} d\sigma^R}_{\text{real corrections}} + \underbrace{\int_m d\sigma^V}_{\text{virtual corrections}} + \underbrace{\int_0^1 dx \int_m d\sigma^C}_{\text{collinear-subtraction counterterm}} - \int_{m+1} d\sigma^A + \int_{m+1} d\sigma^A,$$

$$d\sigma^A = \sum_{\text{dipoles}} d\sigma^B \otimes dV_{\text{dipole}}$$

NLO cross section with the dipole subtraction formalism

Schematic formula for the NLO cross section in the situation of two initial-state hadrons (LHC and Tevatron):

$$\sigma^{\mathrm{NLO}} = \underbrace{\int_{m+1} d\sigma^R}_{\text{real corrections}} + \underbrace{\int_m d\sigma^V}_{\text{virtual corrections}} + \underbrace{\int_0^1 dx \int_m d\sigma^C}_{\text{collinear-subtraction counterterm}} - \int_{m+1} d\sigma^A + \int_{m+1} d\sigma^A,$$

$$d\sigma^A = \sum_{\text{dipoles}} d\sigma^B \otimes dV_{\text{dipole}}$$

$$= \int_{m+1} \left[d\sigma^R - d\sigma^A \right]_{\epsilon=0} \qquad \Rightarrow \mathrm{R} - \mathrm{A}$$

$$+ \int_m \left[d\sigma^V + \sum_{\text{dipoles}} d\sigma^B \otimes V_{\text{dipole}}(1) \right]_{\epsilon=0} \qquad \Rightarrow \mathrm{V} + \mathrm{A}$$

$$+ \int_0^1 dx \int_m \left[d\sigma^C + \sum_{\text{dipoles}} \int_1 d\sigma^B(x) \otimes \left[dV_{\text{dipole}}(x) \right]_+ \right]_{\epsilon=0} \qquad \Rightarrow \mathrm{C} + \mathrm{A}$$

$$dV_{\text{dipole}}(x) = \left[dV_{\text{dipole}}(x) \right]_+ + dV_{\text{dipole}}(1)\delta(1-x)$$

Virtual corrections (V+A term)

For each channel $\mathcal{O}(100)$ 1-loop diagrams contribute, which can be classified as

- "bosonic" corrections (exchange of an additional gluon)
- "fermionic" corrections (closed quark loops)

Renormalization leads to counterterm diagrams contributing at the 1-loop level.

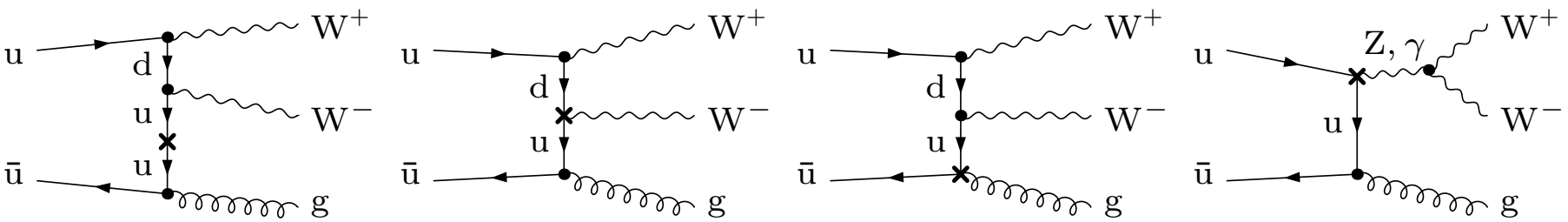
Virtual corrections (V+A term)

For each channel $\mathcal{O}(100)$ 1-loop diagrams contribute, which can be classified as

- "bosonic" corrections (exchange of an additional gluon)
- "fermionic" corrections (closed quark loops)

Renormalization leads to counterterm diagrams contributing at the 1-loop level.

Representative sample of counterterm diagrams:



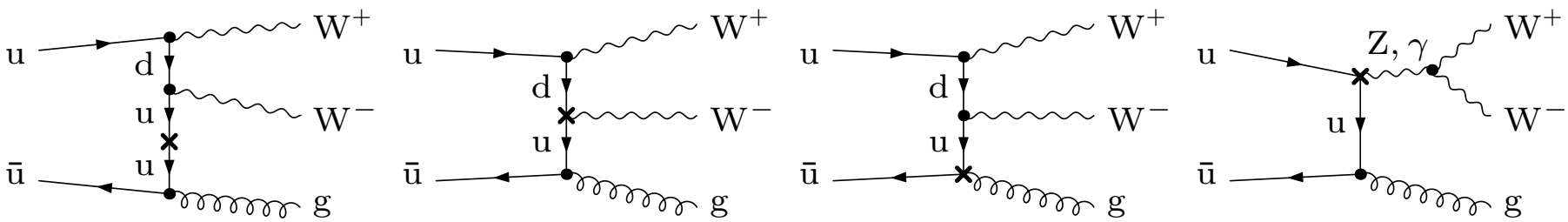
Virtual corrections (V+A term)

For each channel $\mathcal{O}(100)$ 1-loop diagrams contribute, which can be classified as

- "bosonic" corrections (exchange of an additional gluon)
- "fermionic" corrections (closed quark loops)

Renormalization leads to counterterm diagrams contributing at the 1-loop level.

Representative sample of counterterm diagrams:



Sum over all counterterm diagrams is proportional to tree-level amplitude:

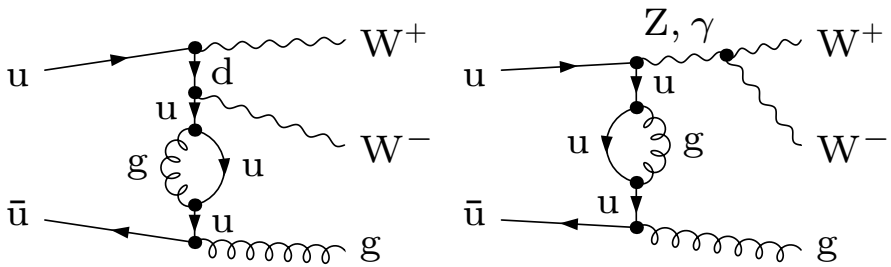
$$\mathcal{M}_{\text{born}} \times \frac{\alpha_s}{4\pi}\left[\underbrace{-\frac{13}{3}\Delta_{\text{UV}}(\mu)-\frac{7}{6}\Delta_{\text{IR}}(\mu)+\frac{N_f}{3}\Delta_{\text{IR}}(\mu)}_{\text{cancel against loops or real corrections}}+\underbrace{\left(\frac{11}{2}-\frac{N_f}{3}\right)}_{\propto\beta(\alpha_s)}\log\frac{\mu^2}{M_W^2}\right],$$

$$\Delta_{\text{IR}}(\mu)=\frac{\Gamma(1+\varepsilon_{\text{IR}})}{\varepsilon_{\text{IR}}}\left(\frac{4\pi\mu^2}{M_W^2}\right)^{\varepsilon_{\text{IR}}},\quad \Delta_{\text{UV}}(\mu)=\frac{\Gamma(1+\varepsilon_{\text{UV}})}{\varepsilon_{\text{UV}}}\left(\frac{4\pi\mu^2}{M_W^2}\right)^{\varepsilon_{\text{UV}}}$$

Virtual corrections (V+A term)

Bosonic corrections can be further subdivided to

- self-energy contributions (2-point functions)

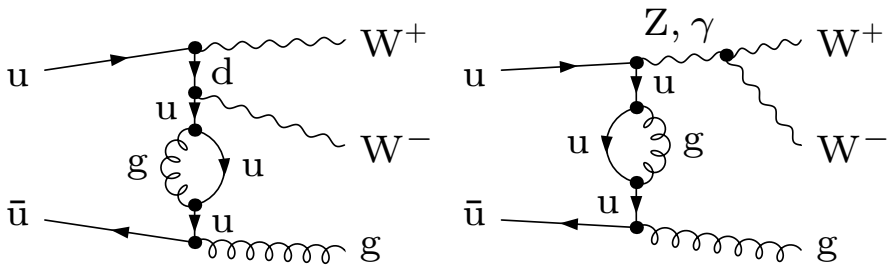


$\mathcal{M}_{\text{self-energy}} \overset{\text{UV}}{\sim} \int d^4q \frac{q^\mu}{(q^2)^2} \sim \int_0^\infty d|q| \rightarrow$ UV-divergent, but IR-finite

Virtual corrections (V+A term)

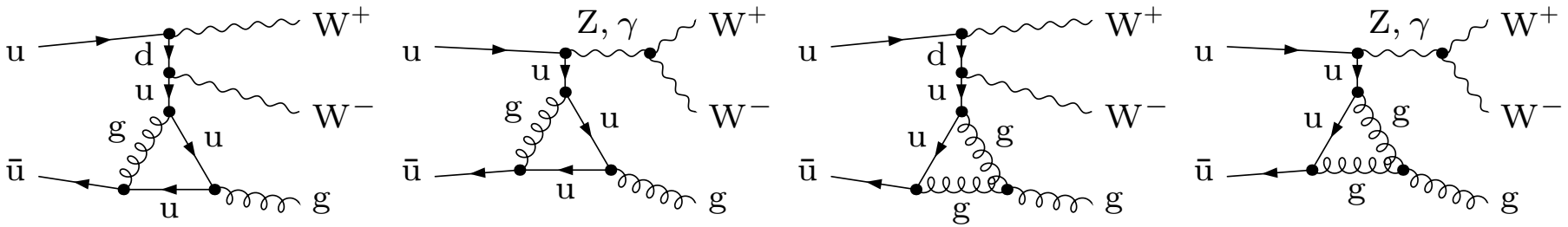
Bosonic corrections can be further subdivided to

- self-energy contributions (2-point functions)



$\mathcal{M}_{\text{self-energy}} \overset{\text{UV}}{\sim} \int d^4q \frac{q^\mu}{(q^2)^2} \sim \int_0^\infty d|q| \rightarrow$ UV-divergent, but IR-finite

- vertex contributions (3-point functions)

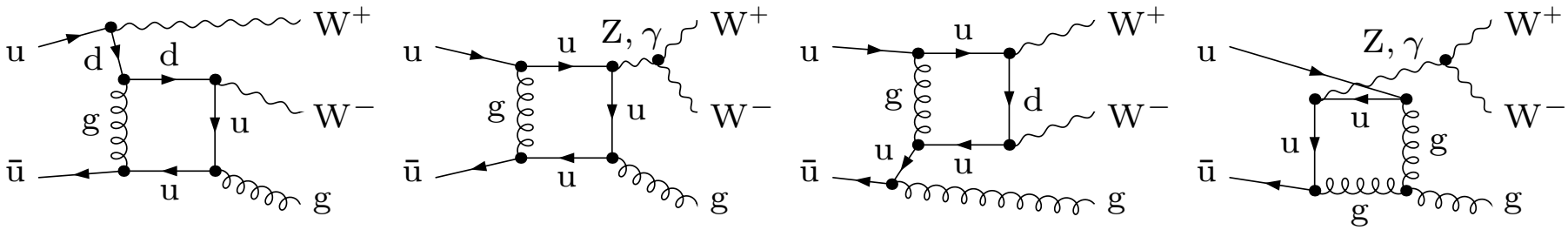


$\mathcal{M}_{\text{vertex}} \overset{\text{UV}}{\sim} \int d^4q \frac{q^\mu q^\nu}{(q^2)^3} \sim \int_0^\infty d|q|\,|q|^{-1} \rightarrow$ UV-divergent, also IR-divergent

Virtual corrections (V+A term)

Further bosonic corrections:

- box contributions (4-point functions)

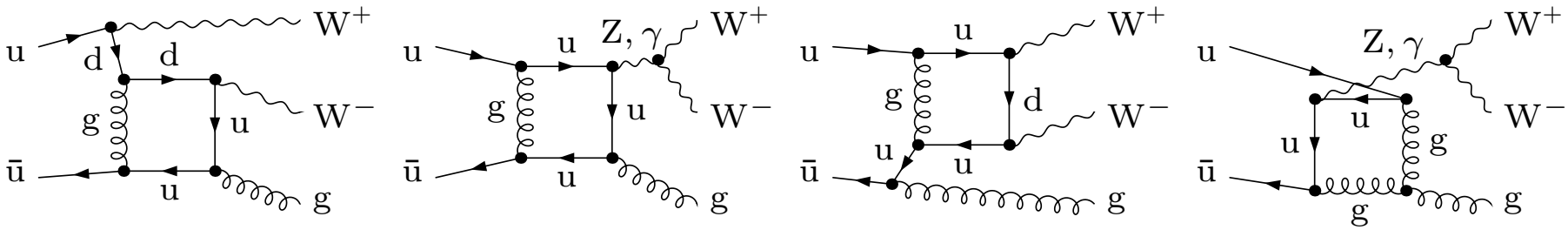


$\mathcal{M}_{\text{box}} \overset{\text{UV}}{\sim} \int d^4q \frac{q^\mu q^\nu q^\rho}{(q^2)^4} \sim \int_0^\infty d|q|\,|q|^{-2} \rightarrow$ UV-finite, but IR-divergent

Virtual corrections (V+A term)

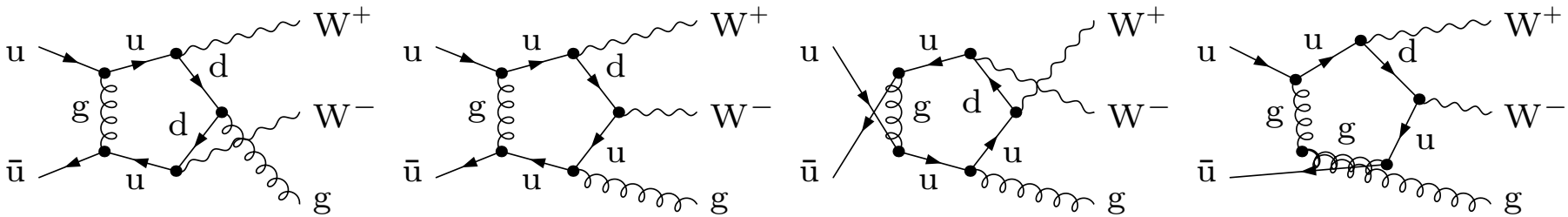
Further bosonic corrections:

- box contributions (4-point functions)



$$\mathcal{M}_{\text{box}} \overset{\text{UV}}{\sim} \int d^4q \frac{q^\mu q^\nu q^\rho}{(q^2)^4} \sim \int_0^\infty d|q|\,|q|^{-2} \rightarrow \text{UV-finite, but IR-divergent}$$

- pentagon contributions (5-point functions)



$$\mathcal{M}_{\text{pentagon}} \overset{\text{UV}}{\sim} \int d^4q \frac{q^\mu q^\nu q^\rho q^\sigma}{(q^2)^5} \sim \int_0^\infty d|q|\,|q|^{-3} \rightarrow \text{UV-finite, but IR-divergent}$$

Virtual corrections (V+A term)

Strategy for extracting or translating IR (soft / collinear) singularities

Idea: integrals $I^{(\varepsilon)}$ in $d = 4-2\varepsilon$ dim. $\leftrightarrow$ 4-dim. integrals $I^{(\lambda)}$ with mass regulator λ

Procedure: Consider finite and regularization-scheme-independent difference:

$$\left[I^{(\varepsilon)} - I^{(\varepsilon)}_{\text{sing}}\right]\Big|_{\varepsilon\to 0} = \left[I^{(\lambda)} - I^{(\lambda)}_{\text{sing}}\right]\Big|_{\lambda\to 0}$$

$$\Rightarrow\ I^{(\varepsilon)} = I^{(\varepsilon)}_{\text{sing}} + \left[I^{(\lambda)} - I^{(\lambda)}_{\text{sing}}\right]\Big|_{\lambda\to 0} + \mathcal{O}(\epsilon)$$

Virtual corrections (V+A term)

Strategy for extracting or translating IR (soft / collinear) singularities

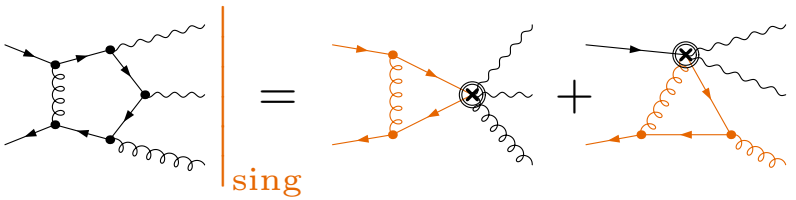
Idea: integrals $I^{(\varepsilon)}$ in $d = 4-2\varepsilon$ dim. $\leftrightarrow$ 4-dim. integrals $I^{(\lambda)}$ with mass regulator λ

Procedure: Consider finite and regularization-scheme-independent difference:

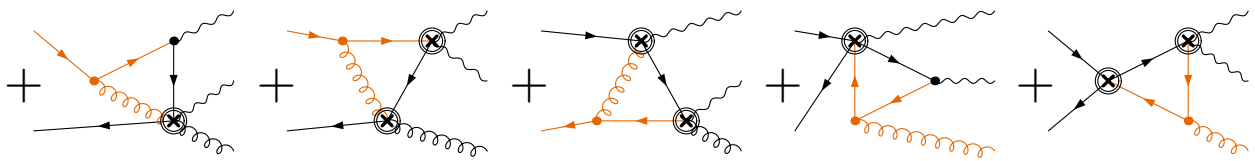
$$\left[I^{(\varepsilon)} - I^{(\varepsilon)}_{\text{sing}}\right]\Big|_{\varepsilon\to 0} = \left[I^{(\lambda)} - I^{(\lambda)}_{\text{sing}}\right]\Big|_{\lambda\to 0}$$

$$\Rightarrow\ I^{(\varepsilon)} = I^{(\varepsilon)}_{\text{sing}} + \left[I^{(\lambda)} - I^{(\lambda)}_{\text{sing}}\right]\Big|_{\lambda\to 0} + \mathcal{O}(\epsilon)$$

Note: Mass-singular part can be universally constructed from 3-point integrals.
$\hookrightarrow$ general result known explicitly [Dittmaier '03] [Beenakker et al. '02]



$\left.\begin{matrix}\\\\\end{matrix}\right\}$ $\frac{1}{\varepsilon^2}, \frac{1}{\varepsilon}$, $\ln^2\lambda, \ln\lambda$

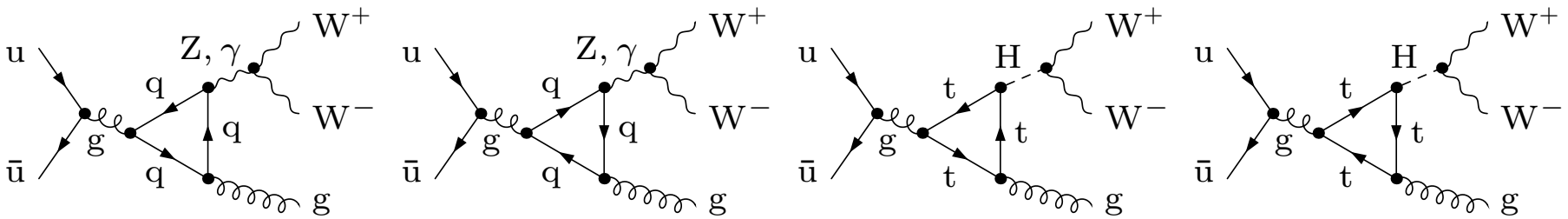


$\left.\begin{matrix}\\\\\end{matrix}\right\}$ $\frac{1}{\varepsilon}$, $\ln\lambda$

Virtual corrections (V+A term)

Fermionic corrections can be subdivided to

- vertex contributions (3-point functions)

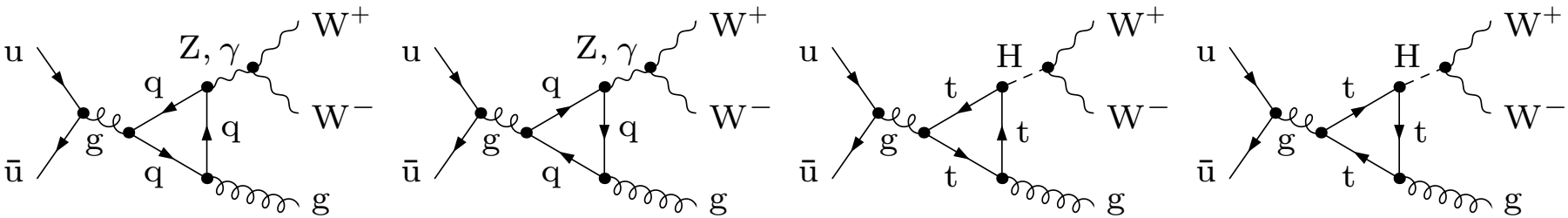


→ UV-finite because no $\mathrm{Zgg}, \gamma\mathrm{gg}, \mathrm{Hgg}$ vertex exists in LO, also IR-finite

Virtual corrections (V+A term)

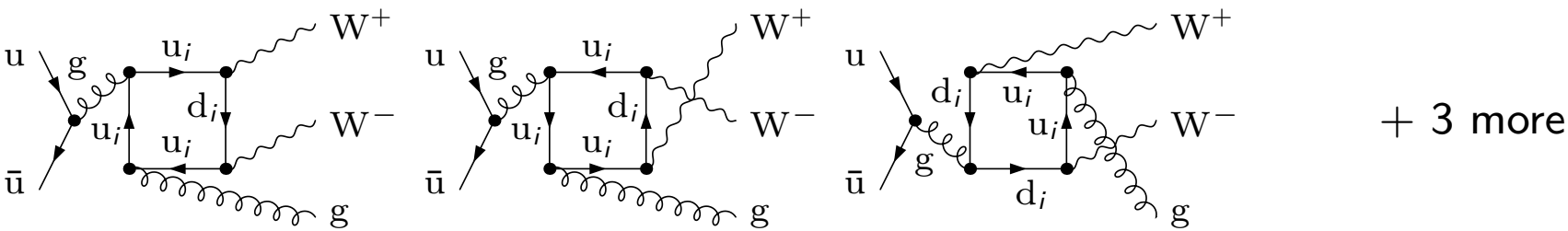
Fermionic corrections can be subdivided to

- vertex contributions (3-point functions)



→ UV-finite because no $Zgg, \gamma gg, Hgg$ vertex exists in LO, also IR-finite

- box contributions (4-point functions)



+ 3 more

→ UV-finite (no $ggWW$ vertex in LO), also IR-finite

Virtual corrections (V+A term)

Two different strategies for evaluation of loop amplitudes
(realized in two independent calculations!)

- Analogous to NLO calculation for
  $pp \to t\bar{t}H$ [Beenakker et al. '02] and $pp \to t\bar{t} + jet$ [Dittmaier, Uwer, Weinzierl '07]
  - diagrams generated with FeynArts 1.0 [Küblbeck, Böhm, Denner '90] and reduced with in-house Mathematica routines → Fortran
  - analytical extraction of IR singularities [Beenakker et al. '02, Dittmaier '03]
  - reduction of 5-point to 4-point integrals according to [Denner, Dittmaier '02]

Virtual corrections (V+A term)

Two different strategies for evaluation of loop amplitudes
(realized in two independent calculations!)

- Analogous to NLO calculation for
  $pp \to t\bar{t}H$ [Beenakker et al. '02] and $pp \to t\bar{t} + jet$ [Dittmaier, Uwer, Weinzierl '07]
  - diagrams generated with FeynArts 1.0 [Küblbeck, Böhm, Denner '90] and reduced with in-house Mathematica routines → Fortran
  - analytical extraction of IR singularities [Beenakker et al. '02, Dittmaier '03]
  - reduction of 5-point to 4-point integrals according to [Denner, Dittmaier '02]

- Alternative calculation with available tools
  - diagrams generated with FeynArts 3.2 [Hahn '00]
  - algebraic reduction / numerical evaluation with FormCalc 5.2/LoopTools: [Hahn, Perez-Victoria '98]
    - reduction of 5-point integrals à la [Denner, Dittmaier '02]
    - regular scalar integrals with FF [v.Oldenborgh '91]
  - dimensionally regularized singular integrals implemented into LoopTools:
    - box integrals checked against result of [Bern, Dixon, Kosower '93]

Real corrections (R−A term)

Contributing processes are generated by two types of generic amplitudes,

- $0 \to \mathrm{W}^+\mathrm{W}^-\mathrm{q}\bar{\mathrm{q}}\mathrm{g}\mathrm{g}$,
- $0 \to \mathrm{W}^+\mathrm{W}^-\mathrm{q}\bar{\mathrm{q}}\mathrm{q}'\bar{\mathrm{q}}'$,

and crossing any two partons into the initial state.
$\hookrightarrow$ Large number of contributions (136 flavour channels for 2 generations)!

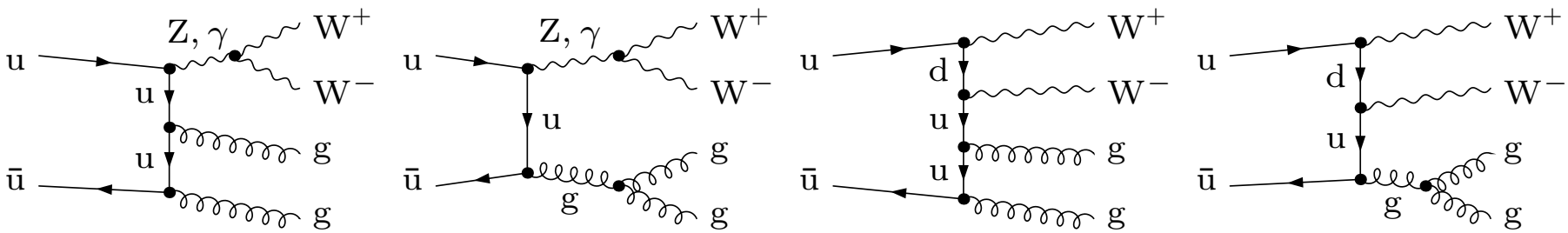
Real corrections (R−A term)

Contributing processes are generated by two types of generic amplitudes,

- $0 \to \mathrm{W}^+\mathrm{W}^-\mathrm{q}\bar{\mathrm{q}}\mathrm{g}\mathrm{g}$,
- $0 \to \mathrm{W}^+\mathrm{W}^-\mathrm{q}\bar{\mathrm{q}}\mathrm{q}'\bar{\mathrm{q}}'$,

and crossing any two partons into the initial state.
$\hookrightarrow$ Large number of contributions (136 flavour channels for 2 generations)!

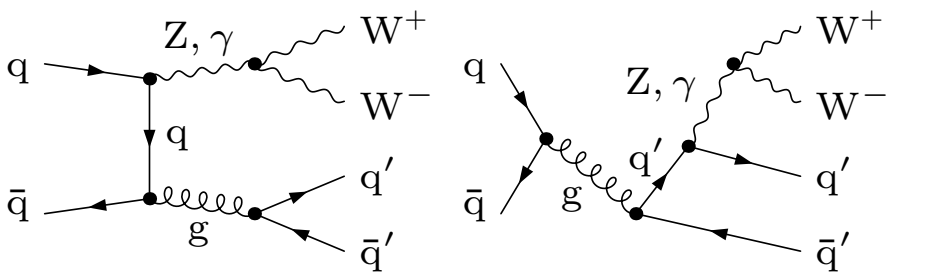
Representative diagrams of the subprocess $\mathrm{u}\bar{\mathrm{u}} \to \mathrm{W}^+\mathrm{W}^-\mathrm{g}\mathrm{g}$ (31 diagrams in total):



- Amplitudes for $\mathrm{u}\mathrm{g} \to \mathrm{W}^+\mathrm{W}^-\mathrm{u}\mathrm{g}$, $\mathrm{g}\bar{\mathrm{u}} \to \mathrm{W}^+\mathrm{W}^-\mathrm{g}\bar{\mathrm{u}}$ and $\mathrm{g}\mathrm{g} \to \mathrm{W}^+\mathrm{W}^-\mathrm{u}\bar{\mathrm{u}}$ are achieved from this amplitude by applying crossing symmetry.
- Amplitudes with external d-type quarks are generated via SU(2) symmetry ($\mathrm{u} \leftrightarrow \mathrm{d}, \mathrm{W}^+ \leftrightarrow \mathrm{W}^-$).

Real corrections (R−A term)

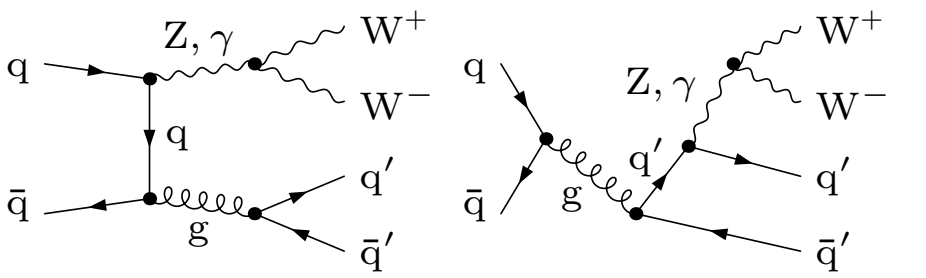
Sample of diagrams of the subprocesses $q\bar{q} \to W^+W^-q'\bar{q}'$ (up to 28 diagrams):



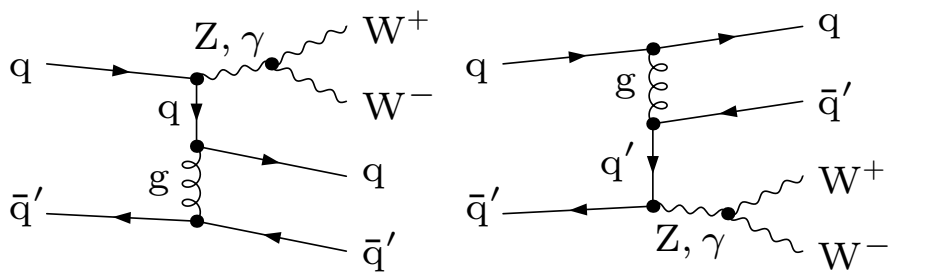
$u\bar{u} \to W^+W^-c\bar{c}$, $u\bar{u} \to W^+W^-d\bar{d}$, ...

Real corrections (R−A term)

Sample of diagrams of the subprocesses $q\bar{q} \to W^+W^-q'\bar{q}'$ (up to 28 diagrams):



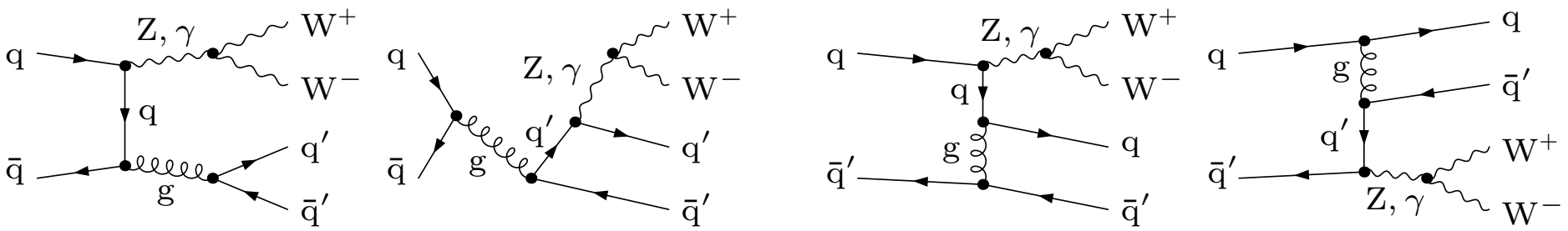
$u\bar{u} \to W^+W^-c\bar{c}$, $u\bar{u} \to W^+W^-d\bar{d}$, ...



$u\bar{c} \to W^+W^-u\bar{c}$, $u\bar{d} \to W^+W^-u\bar{d}$, ...

Real corrections (R−A term)

Sample of diagrams of the subprocesses $q\bar{q} \to W^+W^-q'\bar{q}'$ (up to 28 diagrams):



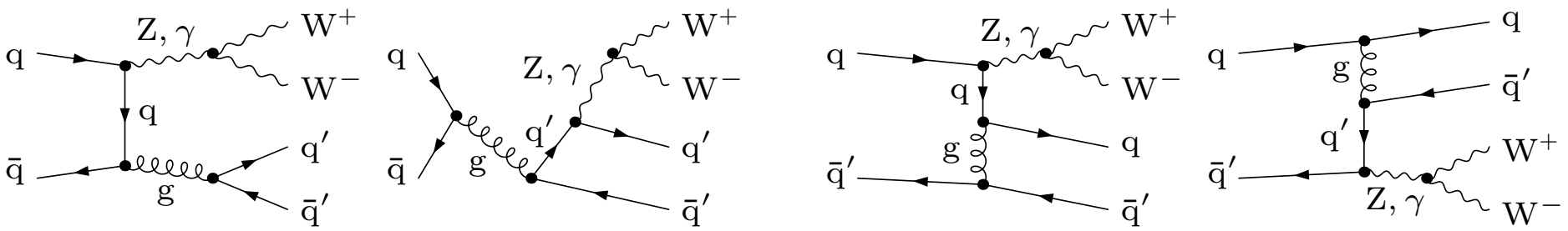
$u\bar{u} \to W^+W^-c\bar{c}$, $u\bar{u} \to W^+W^-d\bar{d}$, ...

$u\bar{c} \to W^+W^-u\bar{c}$, $u\bar{d} \to W^+W^-u\bar{d}$, ...

$u\bar{u} \to W^+W^-u\bar{u}$, $d\bar{d} \to W^+W^-d\bar{d}$, ...

Real corrections (R−A term)

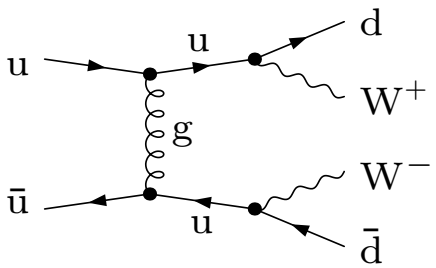
Sample of diagrams of the subprocesses $q\bar{q} \to W^+W^-q'\bar{q}'$ (up to 28 diagrams):



$u\bar{u} \to W^+W^-c\bar{c}$, $u\bar{u} \to W^+W^-d\bar{d}$, ... $u\bar{c} \to W^+W^-u\bar{c}$, $u\bar{d} \to W^+W^-u\bar{d}$, ...

$u\bar{u} \to W^+W^-u\bar{u}$, $d\bar{d} \to W^+W^-d\bar{d}$, ...

Further diagrams contribute in case of both external u-type and d-type quarks:



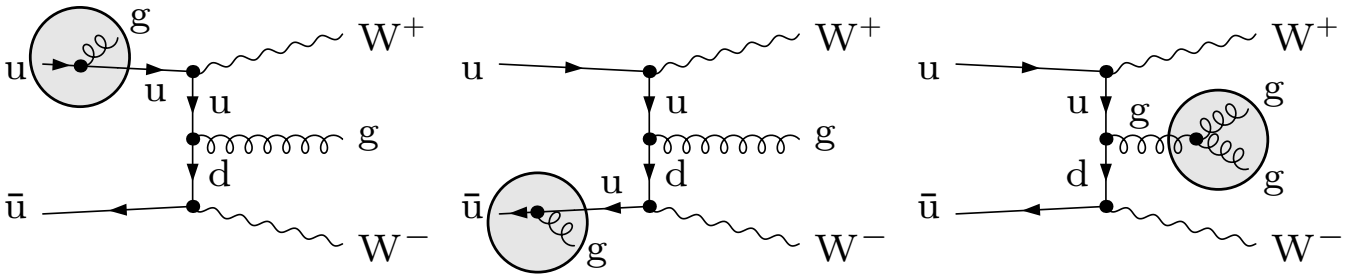
W bosons couple to different fermion lines.

↪ Subprocesses with up to 4 different quark flavours give non-vanishing contributions (e. g. $u\bar{c} \to W^+W^-d\bar{s}$).

Crossing symmetry leads to amplitudes for the subprocess classes $qq \to W^+W^-qq$ and $\bar{q}\bar{q} \to W^+W^-\bar{q}\bar{q}$.

Real corrections (R−A term)

For $u\bar{u} \to W^+W^-gg$ IR singularities are cured by the following subtraction terms:

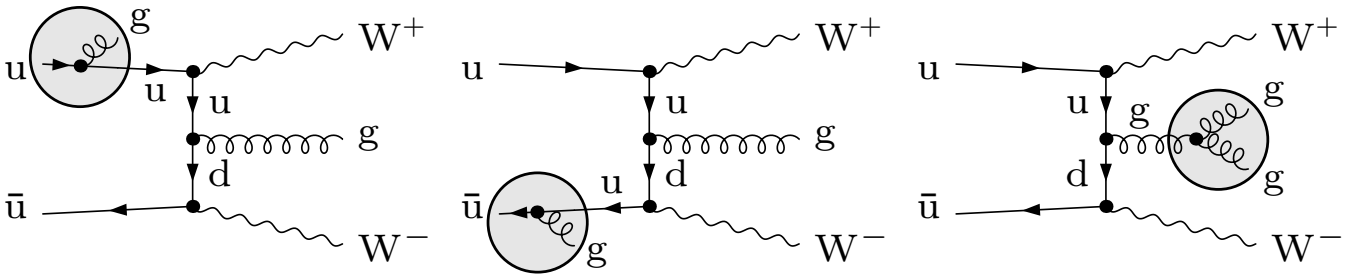


- process-independent part → dipole terms → IR (soft and collinear) singularities
- process-dependent part → on-shell amplitudes of LO subprocesses

Note: Spin and colour correlations have to be taken into account.

Real corrections (R−A term)

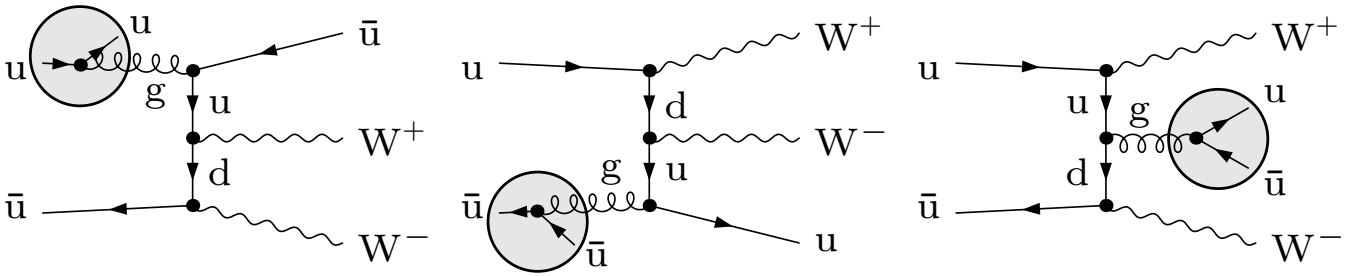
For $u\bar{u} \to W^+W^-gg$ IR singularities are cured by the following subtraction terms:



- process-independent part → dipole terms → IR (soft and collinear) singularities
- process-dependent part → on-shell amplitudes of LO subprocesses

Note: Spin and colour correlations have to be taken into account.

E. g. for $u\bar{u} \to W^+W^-u\bar{u}$ subtraction terms contain different $2 \to 3$-subprocesses:



Real corrections (R−A term)

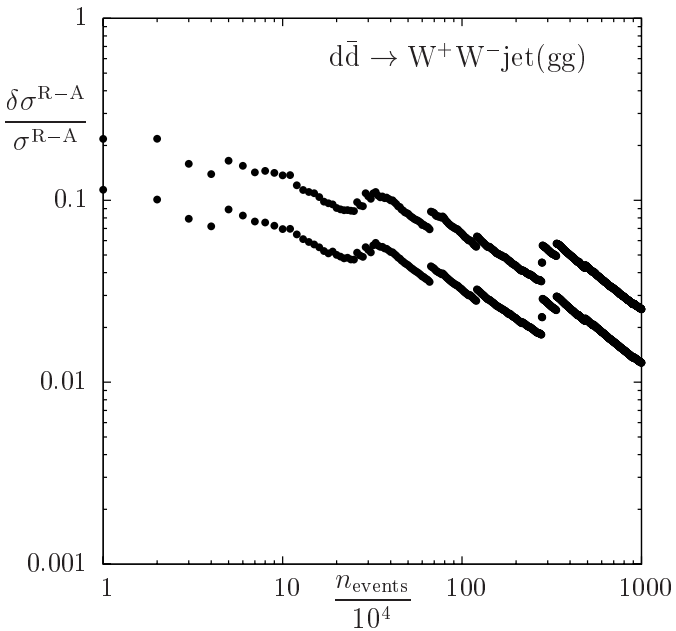
Subtraction-term phase spaces have the following features:

- Momentum conservation is maintained.
- A one-to-one-correspondence between subtraction-term phase space and real-correction phase space exists (degenerate in the relevant limits).

Numerical problem: R − A

event passes cuts

event is cut away (or vice versa)



Real corrections (R−A term)

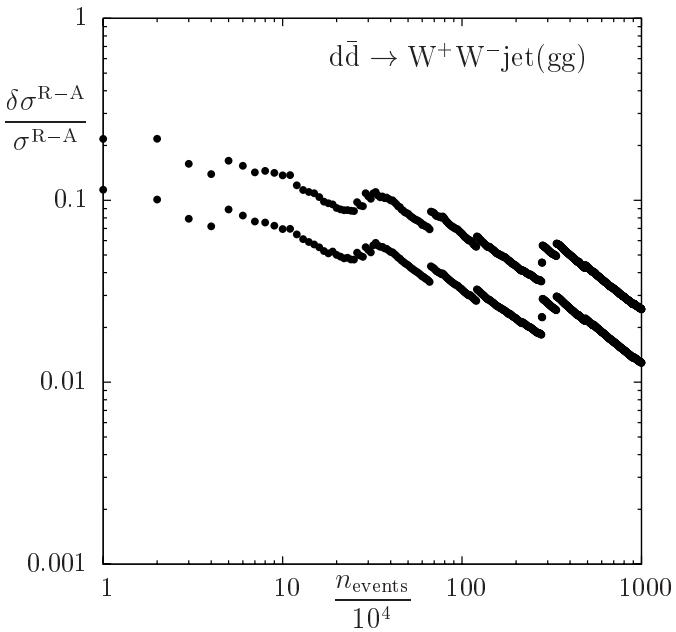
Subtraction-term phase spaces have the following features:

- Momentum conservation is maintained.
- A one-to-one-correspondence between subtraction-term phase space and real-correction phase space exists (degenerate in the relevant limits).

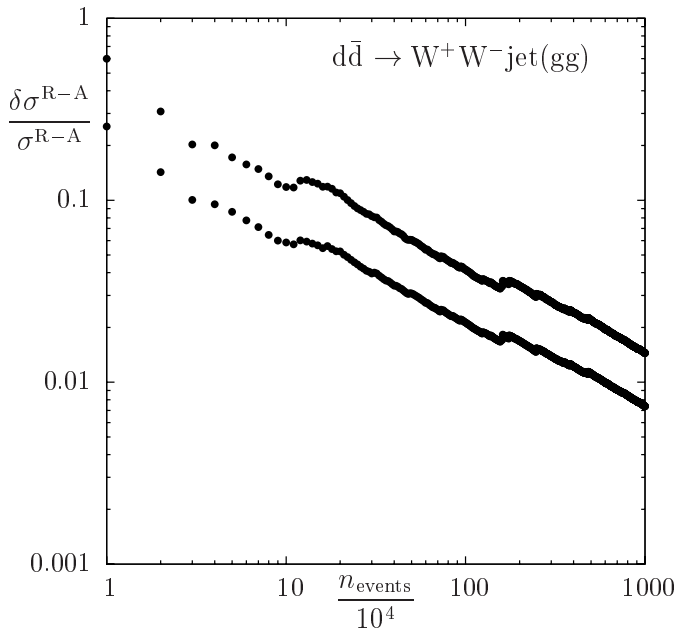
Numerical problem: R − A

event passes cuts

event is cut away (or vice versa)



Improved numerical integration by adding extra channels populating the critical region:



Real corrections (R−A term)

Two independent evaluations of helicity amplitudes:

- application of (4-dimensional) WvdW spinor techniques
- alternative evaluation based on MADGRAPH [Stelzer, Long '94]

Real corrections (R−A term)

Two independent evaluations of helicity amplitudes:

- application of (4-dimensional) WvdW spinor techniques
- alternative evaluation based on MADGRAPH [Stelzer, Long '94]

Two independent versions of Monte Carlo integrators:

- one entirely based on multi-channel MC technique [Berends, Pittau, Kleis '94] [Kleis, Pittau '94]
  - non-singular parts checked against SHERPA 1.0.8 [Gleisberg et al. '03] and WHIZARD 1.50 [Kilian '01] (details on comparison → [diploma thesis of S.K. '06])

process	σ[pb]	σ_{SHERPA}[pb]	$\Delta\sigma$/stat. error
$p\bar{p} \to WW + 1\text{jet}$	2.10456(94)	2.10562(78)	−0.87
$p\bar{p} \to WW + 2\text{jets}$	0.42431(22)	0.42437(13)	−0.23
$pp \to WW + 1\text{jet}$	46.453(16)	46.4399(94)	+0.70
$pp \to WW + 2\text{jets}$	31.555(17)	31.5747(63)	−1.08

  - extra channels included for subtraction terms
- second version based on simple mapping (phase space by sequential splitting)

Collinear subtraction counterterm (C+A term)

Incoming hadrons require collinear subtraction counterterms:

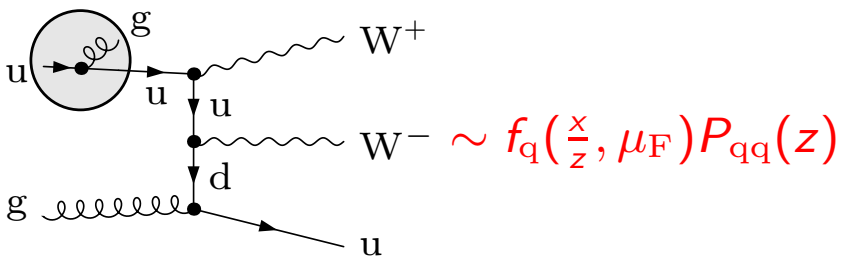
- divergences resulting from collinear emission off these identified partons
  $\hookrightarrow$ no cancellation against virtual corrections
- absorption of divergences in redefined parton distribution functions (PDF's)

Collinear subtraction counterterm (C+A term)

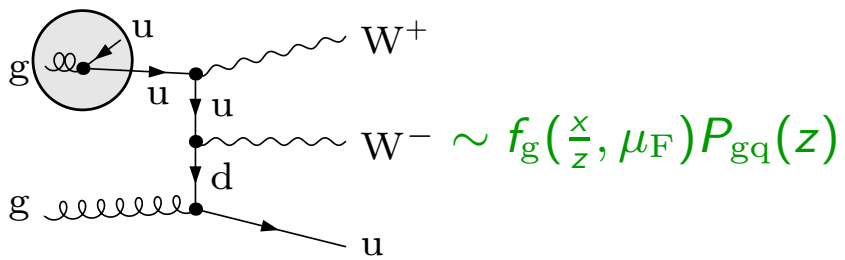
Incoming hadrons require collinear subtraction counterterms:

- divergences resulting from collinear emission off these identified partons
  $\hookrightarrow$ no cancellation against virtual corrections
- absorption of divergences in redefined parton distribution functions (PDF's)

Sample of diagrams showing collinear subtraction counterterms:



$\sim f_{\mathrm{q}}(\frac{x}{z},\mu_{\mathrm{F}})P_{\mathrm{qq}}(z)$



$\sim f_{\mathrm{g}}(\frac{x}{z},\mu_{\mathrm{F}})P_{\mathrm{gq}}(z)$

Corresponding redefinition of PDF's: $P_{ij}(z)$ = Altarelli–Parisi splitting functions

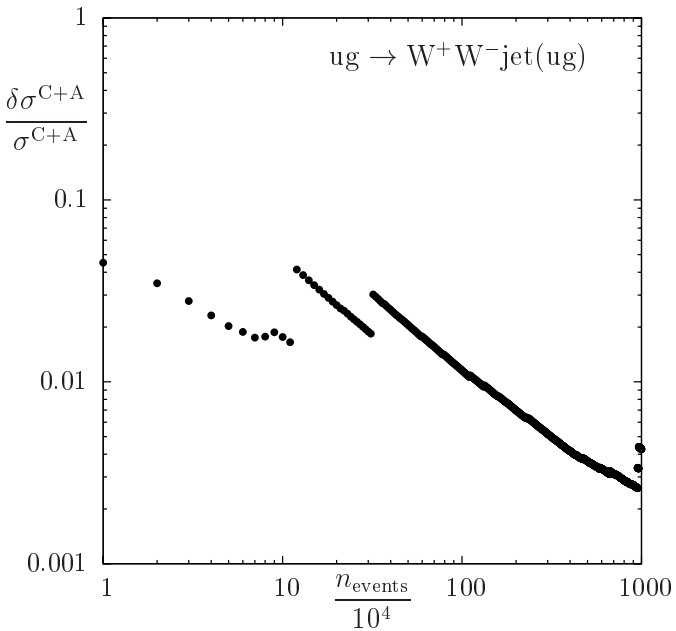
$$
\begin{aligned}
f_{\mathrm{q}}(x,\mu_{\mathrm{F}}) \quad \to \quad & f_{\mathrm{q}}(x,\mu_{\mathrm{F}}) \\
& + \frac{\alpha_{\mathrm{s}}}{2\pi}\int_x^1 \frac{dz}{z} f_{\mathrm{q}}\left(\frac{x}{z},\mu_{\mathrm{F}}\right)\left(\frac{\Gamma(1+\varepsilon)}{\varepsilon}(4\pi)^{\varepsilon}+\ln\frac{\mu^2}{\mu_{\mathrm{F}}^2}\right) C_{\mathrm{F}}\left[P_{\mathrm{qq}}(z)\right]_+ \\
& + \frac{\alpha_{\mathrm{s}}}{2\pi}\int_x^1 \frac{dz}{z} f_{\mathrm{g}}\left(\frac{x}{z},\mu_{\mathrm{F}}\right)\left(\frac{\Gamma(1+\varepsilon)}{\varepsilon}(4\pi)^{\varepsilon}+\ln\frac{\mu^2}{\mu_{\mathrm{F}}^2}\right) T_{\mathrm{R}} P_{\mathrm{gq}}(z) \\
f_{\mathrm{g}}(x,\mu_{\mathrm{F}}) \quad \to \quad & \cdots \text{ analogously}
\end{aligned}
$$

Collinear subtraction counterterm (C+A term)

Meaning of z: Momentum fraction of the radiating parton after splitting

$$\int_0^1 dz\, \hat{\sigma}(z\hat{s})\,[\mathcal{V}(z)]_+ = \int_0^1 dz \Big[\hat{\sigma}(z\hat{s}) - \hat{\sigma}(\hat{s})\Big]\mathcal{V}(z)$$

- Cancellations between different phase-space points (degenerate only for $z \longrightarrow 1$)!
- Spoiled cancellations if only one point passes phase-space cuts!

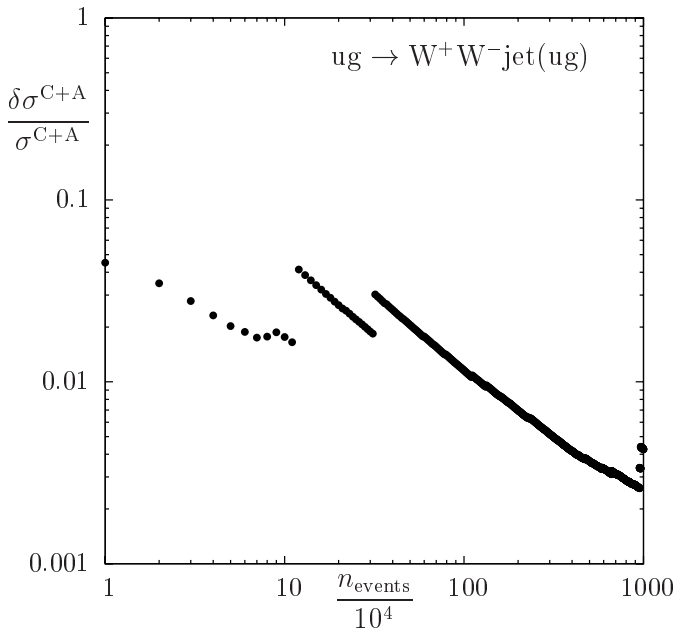


Collinear subtraction counterterm (C+A term)

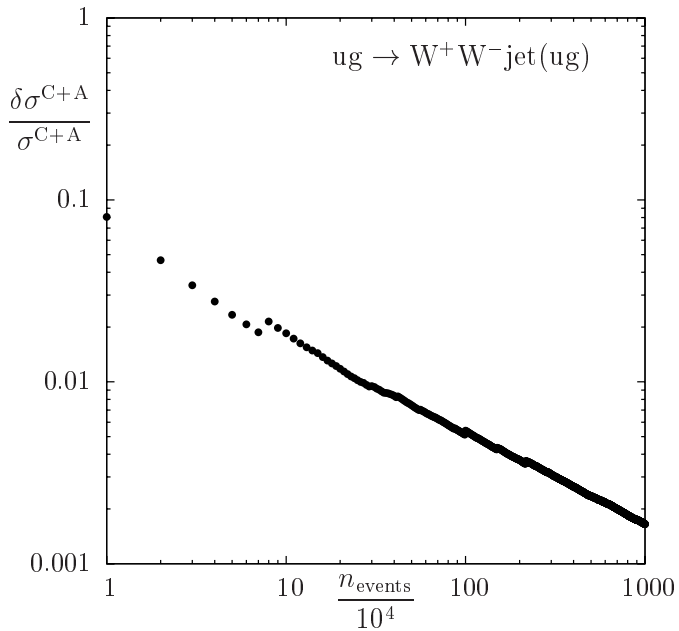
Meaning of z: Momentum fraction of the radiating parton after splitting

$$\int_0^1 dz\, \hat{\sigma}(z\hat{s})\,[\mathcal{V}(z)]_+ = \int_0^1 dz \Big[\hat{\sigma}(z\hat{s}) - \hat{\sigma}(\hat{s})\Big]\mathcal{V}(z)$$

- Cancellations between different phase-space points (degenerate only for $z \longrightarrow 1$)!
- Spoiled cancellations if only one point passes phase-space cuts!



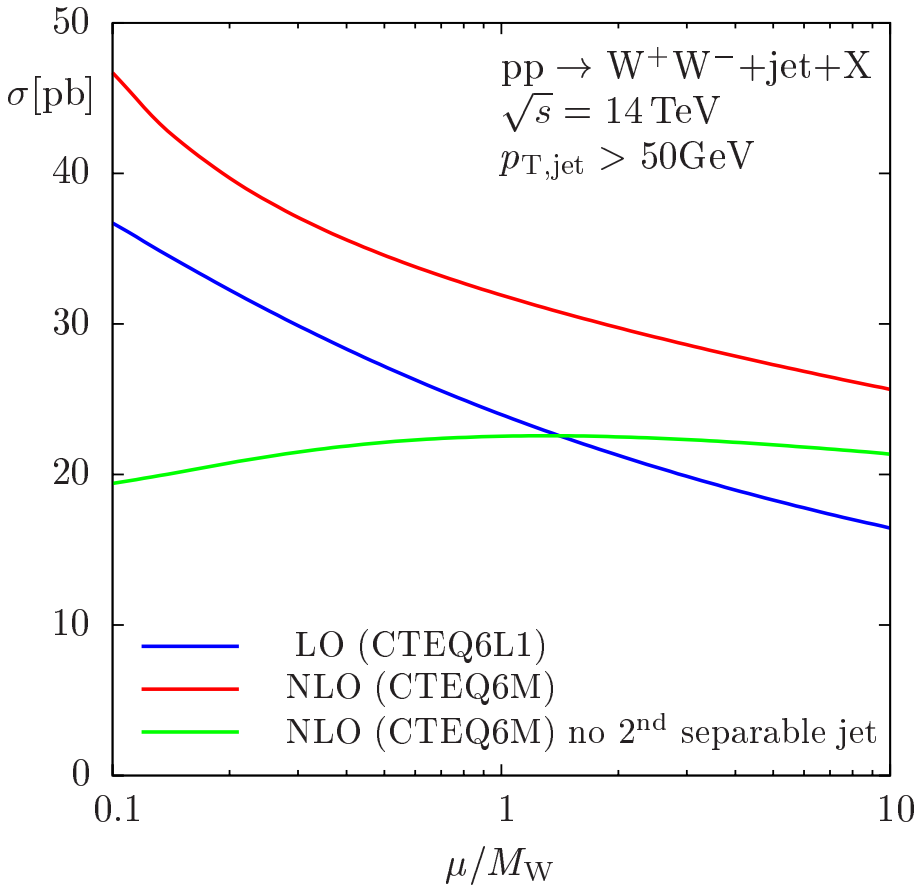
Improved numerical integration by adding extra channels populating the critical region:

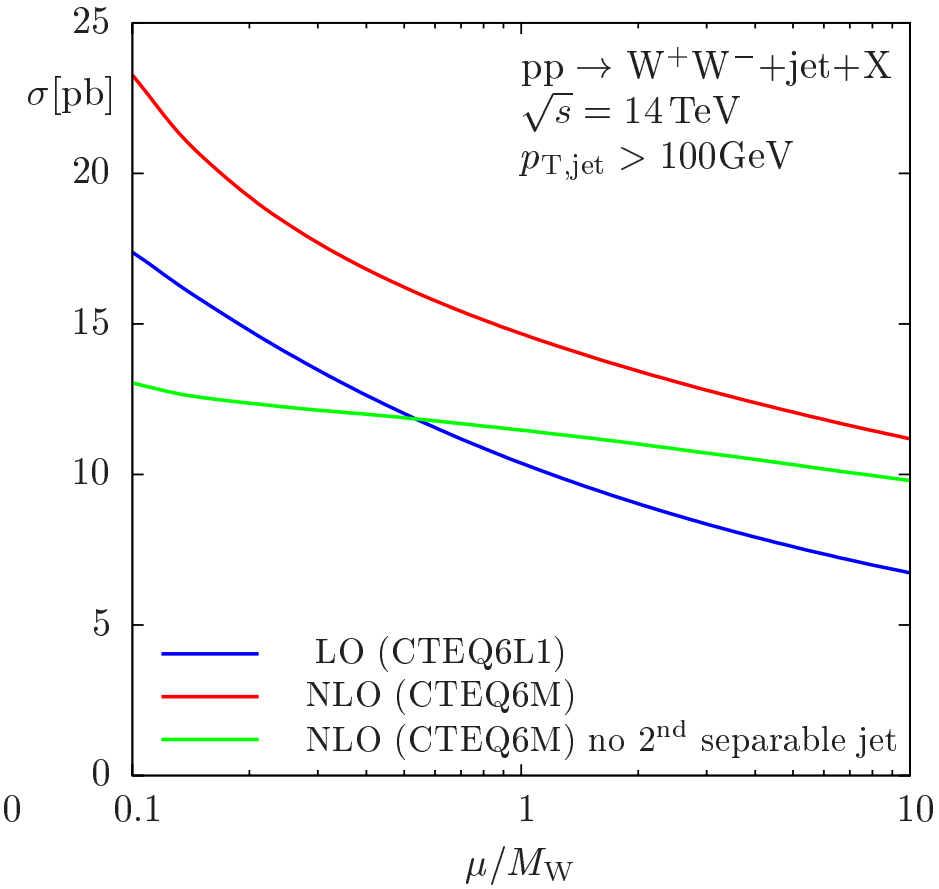


Numerical results

LO versus NLO cross section at the LHC ($\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$):

[Dittmaier, S.K., Uwer '07]





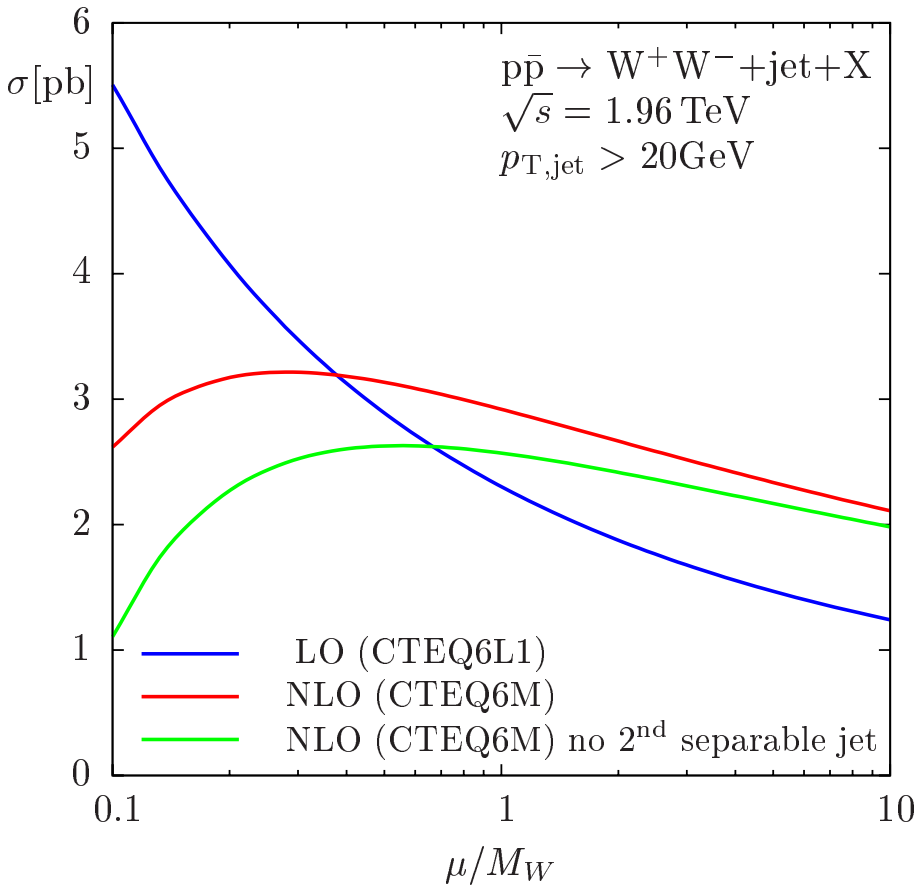
Jet definition: successive combination algorithm (with $R = 1$) [S.D.Ellis, Soper '93]

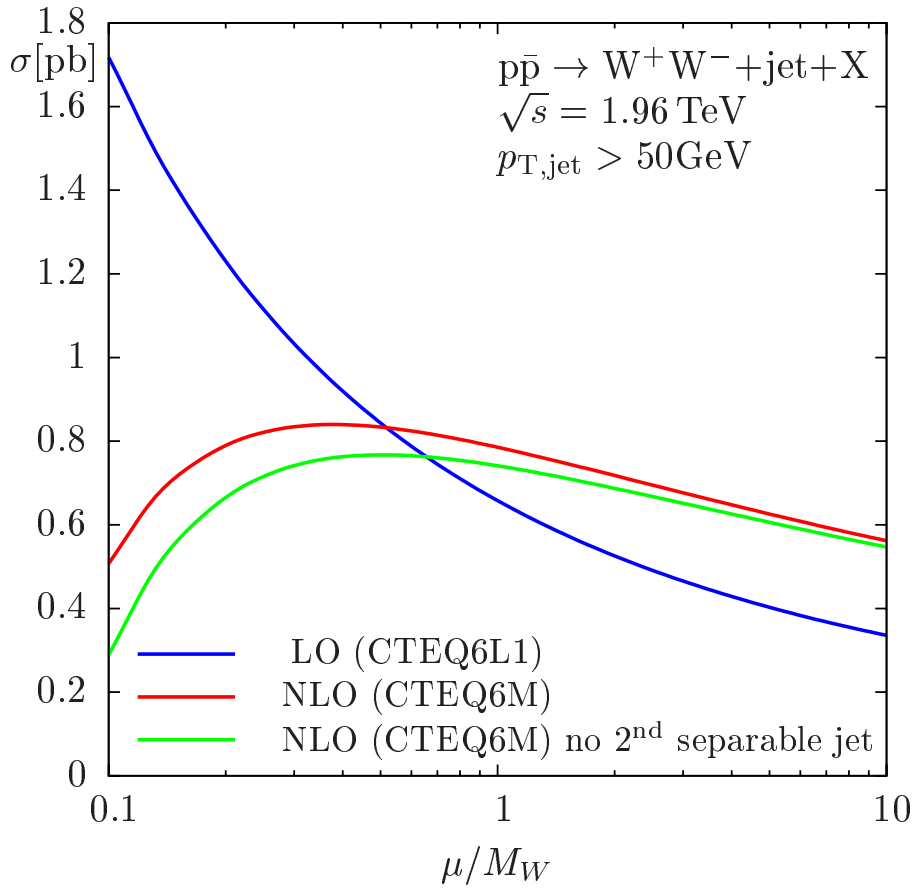
- Scale dependence stabilizes at NLO for genuine WW+jet production.
- But: Significant scale dependence is introduced by WW+2jets events.

Numerical results

LO versus NLO cross section at the Tevatron ($\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$):

[Dittmaier, S.K., Uwer '07]



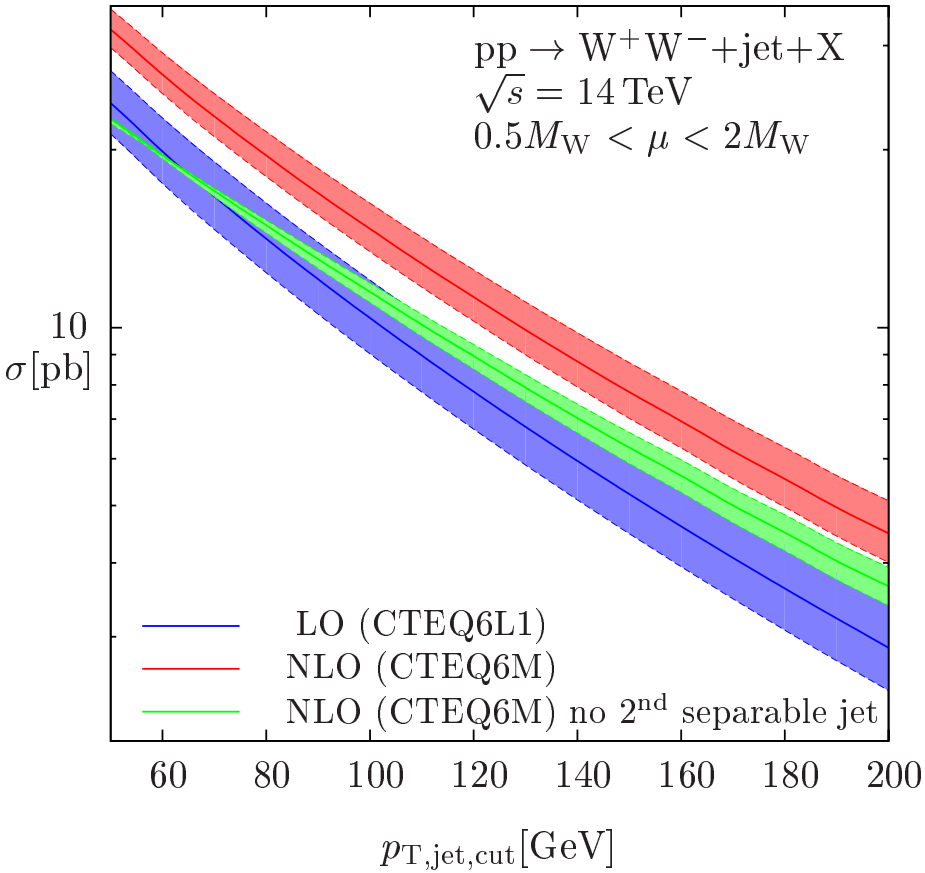


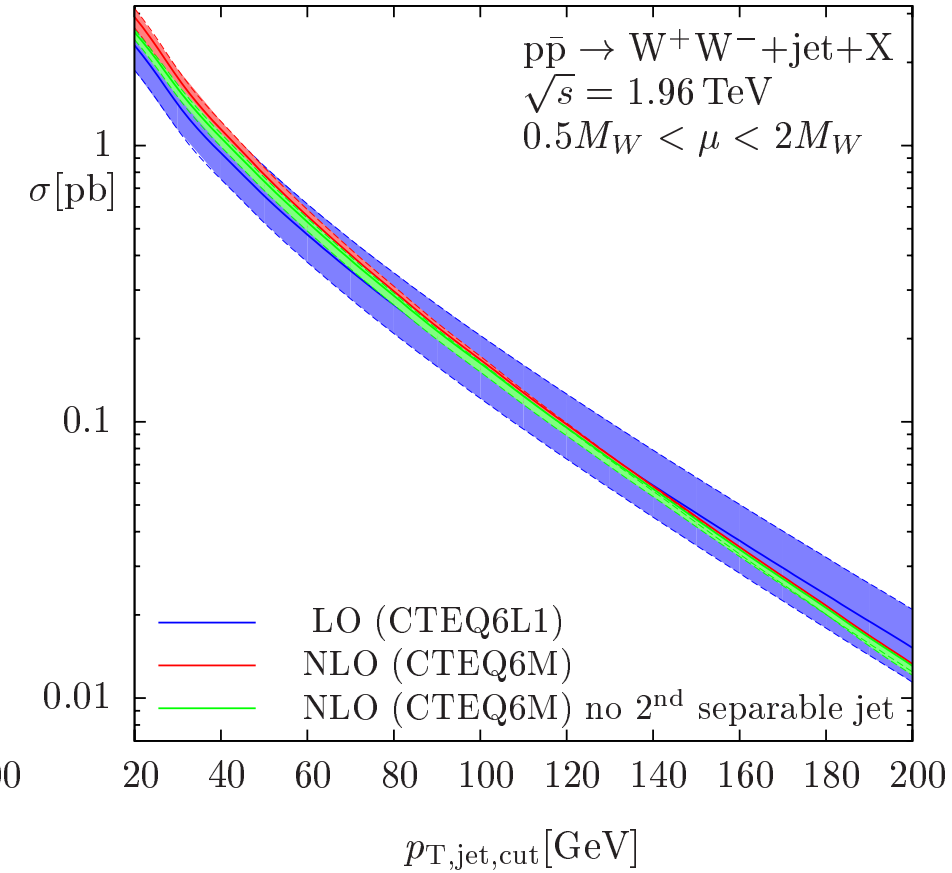
↪ Scale dependence stabilizes at NLO.
(only small influence from WW+2jets production)

Numerical results

LO versus NLO cross section for $p_{\mathrm{T,cut,jet}}$-variation at LHC and Tevatron:
($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$)

[Dittmaier, S.K., Uwer '07]





↪ Significant stabilisation of scale dependence is achieved at NLO
(at LHC especially for genuine WW+jet production)

Tuned comparison

Status of tuned comparison between results of [Dittmaier, S.K., Uwer '07], [Campbell, Ellis, Zanderighi '07] and [Binoth, Guillet, Karg, Kauer, Sanguinetti (in progress)]:

↪ [NLM Les Houches report '08]

- Integrated LO results checked:

$pp \to W^+W^-+jet+X$	σ_{LO}[fb]
DKU	10371.7(12)
CEZ	10372.26(97)
BGKKS	10371.7(11)

- Results for virtual corrections checked at one phase-space point:

$u\bar{u} \to W^+W^-g \qquad |\mathcal{M}_{LO}|^2/e^4g_s^2 = 0.9963809154477200 \cdot 10^{-3}$

$$2\mathrm{Re}\{\mathcal{M}_V^* \cdot \mathcal{M}_{LO}\} = e^4 g_s^2 \Gamma(1+\varepsilon)\left(\frac{4\pi\mu^2}{M_W^2}\right)^{\varepsilon}\left(\frac{1}{\varepsilon^2}c_{-2} + \frac{1}{\varepsilon}c_{-1} + c_0\right)$$

All bosonic contributions:

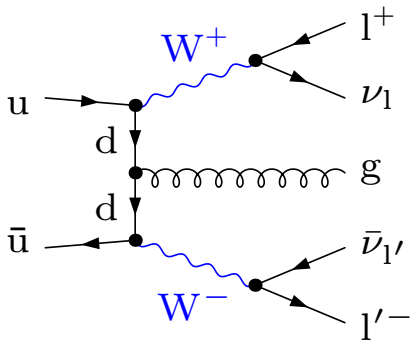
$u\bar{u} \to W^+W^-g$	c_{-2}[GeV^{-2}]	c_{-1}^{bos}[GeV^{-2}]	c_0^{bos}[GeV^{-2}]
DKU	$-1.080699305508758 \cdot 10^{-4}$	$7.842861905263072 \cdot 10^{-4}$	$-3.382910915425372 \cdot 10^{-3}$
CEZ	$-1.080699305505865 \cdot 10^{-4}$	$7.842861905276719 \cdot 10^{-4}$	$-3.382910915464027 \cdot 10^{-3}$
BGKKS	$-1.080699305508814 \cdot 10^{-4}$	$7.842861905263293 \cdot 10^{-4}$	$-3.382910915616242 \cdot 10^{-3}$

Fermionic contributions for 2 light generations in the loop:

$u\bar{u} \to W^+W^-g$	$c_{-1}^{ferm1+2}$[GeV^{-2}]	$c_0^{ferm1+2}$[GeV^{-2}]
DKU	$2.542821895320379 \cdot 10^{-5}$	$4.372323372044527 \cdot 10^{-7}$
CEZ	$2.542821895311753 \cdot 10^{-5}$	$4.372790378087550 \cdot 10^{-7}$
BGKKS	$2.542821895314862 \cdot 10^{-5}$	$4.372324288356448 \cdot 10^{-7}$

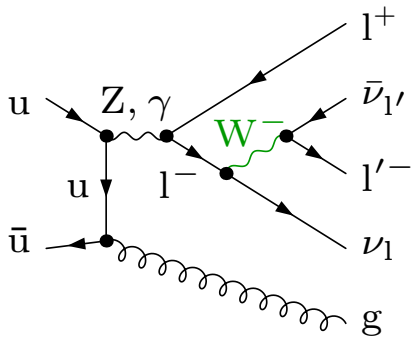
W decays via narrow-width approximation

Double-resonant diagrams:



$k_W^2 \longrightarrow M_W^2$

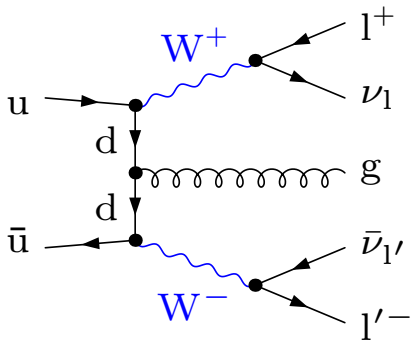
Single-resonant diagrams:



neglected!

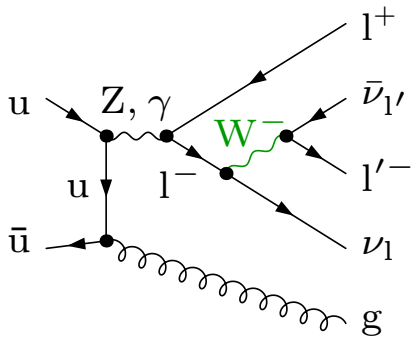
W decays via narrow-width approximation

Double-resonant diagrams:



$k_{\mathrm{W}}^2 \longrightarrow M_{\mathrm{W}}^2$

Single-resonant diagrams:



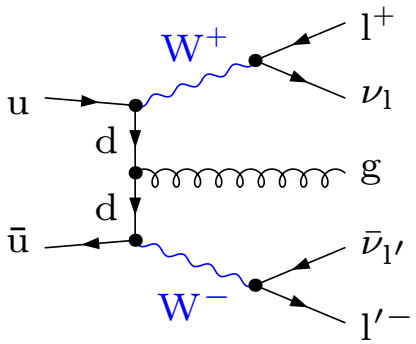
neglected!

- on-shell approximation for resonant matrix elements:

$$\mathcal{M}_{\mathrm{resonant}} \sim \sum_{\lambda_{\mathrm{W}}} \frac{\mathcal{M}^{\mathrm{W}}_{\mathrm{prod}}(k_{\mathrm{W}}^2 \longrightarrow M_{\mathrm{W}}^2, \lambda_{\mathrm{W}}) \times \mathcal{M}^{\mathrm{W}}_{\mathrm{decay}}(k_{\mathrm{W}}^2 \longrightarrow M_{\mathrm{W}}^2, \lambda_{\mathrm{W}})}{k_{\mathrm{W}}^2 - M_{\mathrm{W}}^2 + \mathrm{i}\Gamma_{\mathrm{W}} M_{\mathrm{W}}}$$

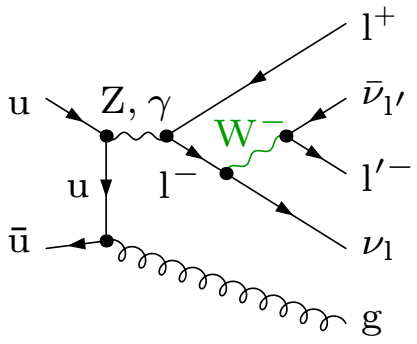
W decays via narrow-width approximation

Double-resonant diagrams:



$k_{\mathrm{W}}^2 \longrightarrow M_{\mathrm{W}}^2$

Single-resonant diagrams:



neglected!

- on-shell approximation for resonant matrix elements:

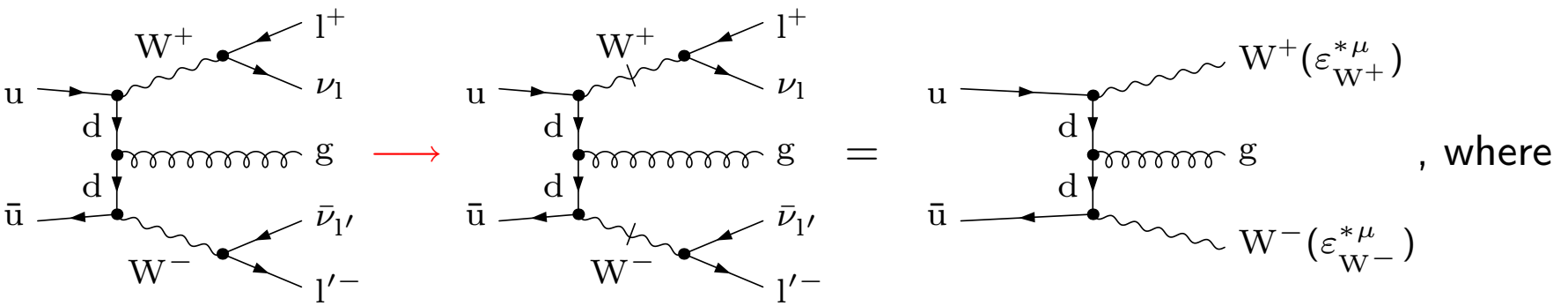
$$\mathcal{M}_{\mathrm{resonant}} \sim \sum_{\lambda_{\mathrm{W}}} \frac{\mathcal{M}^{\mathrm{W}}_{\mathrm{prod}}(k_{\mathrm{W}}^2 \longrightarrow M_{\mathrm{W}}^2, \lambda_{\mathrm{W}}) \times \mathcal{M}^{\mathrm{W}}_{\mathrm{decay}}(k_{\mathrm{W}}^2 \longrightarrow M_{\mathrm{W}}^2, \lambda_{\mathrm{W}})}{k_{\mathrm{W}}^2 - M_{\mathrm{W}}^2 + \mathrm{i}\Gamma_{\mathrm{W}} M_{\mathrm{W}}}$$

- replacement of Breit–Wigner-propagator by a δ-function approximation:

$$\int_{\mathrm{min}}^{\mathrm{max}} dk_{\mathrm{W}}^2 \int d\Phi^{\mathrm{W}}_{\mathrm{prod}}(k_{\mathrm{W}}^2) \frac{1}{(k_{\mathrm{W}}^2 - M_{\mathrm{W}}^2)^2 + \Gamma_{\mathrm{W}}^2 M_{\mathrm{W}}^2} \int d\Phi^{\mathrm{W}}_{\mathrm{decay}}(k_{\mathrm{W}}^2)$$

$$\xrightarrow[\Gamma_{\mathrm{W}} \to 0]{} \int_{-\infty}^{\infty} dk_{\mathrm{W}}^2 \int d\Phi^{\mathrm{W}}_{\mathrm{prod}}(M_{\mathrm{W}}^2) \frac{\pi}{\Gamma_{\mathrm{W}} M_{\mathrm{W}}} \delta(k_{\mathrm{W}}^2 - M_{\mathrm{W}}^2) \int d\Phi^{\mathrm{W}}_{\mathrm{decay}}(M_{\mathrm{W}}^2)$$

W decays via narrow-width approximation

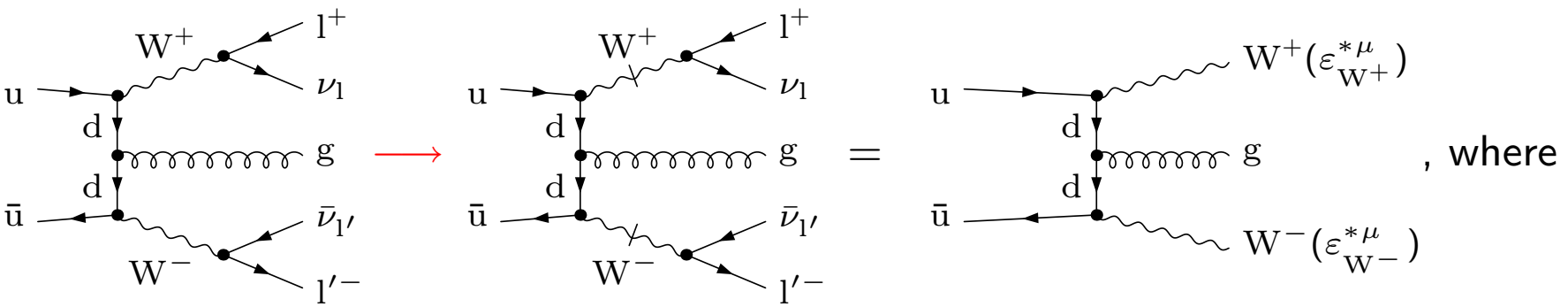


$$\varepsilon^{*\mu}_{\mathrm{W}^+} \longrightarrow \frac{1}{k^2_{\mathrm{W}^+} - M^2_{\mathrm{W}} + \mathrm{i}\Gamma_{\mathrm{W}} M_{\mathrm{W}}} \times eC^-_{\mathrm{W}}\bar{u}(k_{\nu_l})\gamma^\mu\omega_- v(k_{l^+}), \quad k_{\mathrm{W}^+} = k_{\nu_l} + k_{l^+}$$

$$\varepsilon^{*\mu}_{\mathrm{W}^-} \longrightarrow \underbrace{\frac{1}{k^2_{\mathrm{W}^-} - M^2_{\mathrm{W}} + \mathrm{i}\Gamma_{\mathrm{W}} M_{\mathrm{W}}}}_{\longrightarrow\text{phase-space integration}} \times eC^-_{\mathrm{W}}\bar{u}(k_{l'^-})\gamma^\mu\omega_- v(k_{\bar{\nu}_{l'}}), \quad k_{\mathrm{W}^-} = k_{l'^-} + k_{\bar{\nu}_{l'}}$$

Spin correlations respected!

W decays via narrow-width approximation



, where

$$\varepsilon^{*\mu}_{\mathrm{W}^+} \longrightarrow \frac{1}{k^2_{\mathrm{W}^+} - M^2_{\mathrm{W}} + \mathrm{i}\Gamma_{\mathrm{W}} M_{\mathrm{W}}} \times eC^-_{\mathrm{W}}\bar{u}(k_{\nu_l})\gamma^\mu\omega_- v(k_{l^+}), \quad k_{\mathrm{W}^+} = k_{\nu_l} + k_{l^+}$$

$$\varepsilon^{*\mu}_{\mathrm{W}^-} \longrightarrow \underbrace{\frac{1}{k^2_{\mathrm{W}^-} - M^2_{\mathrm{W}} + \mathrm{i}\Gamma_{\mathrm{W}} M_{\mathrm{W}}}}_{\longrightarrow\text{phase-space integration}} \times eC^-_{\mathrm{W}}\bar{u}(k_{l'^-})\gamma^\mu\omega_- v(k_{\bar{\nu}_{l'}}), \quad k_{\mathrm{W}^-} = k_{l'^-} + k_{\bar{\nu}_{l'}}$$

Spin correlations respected!

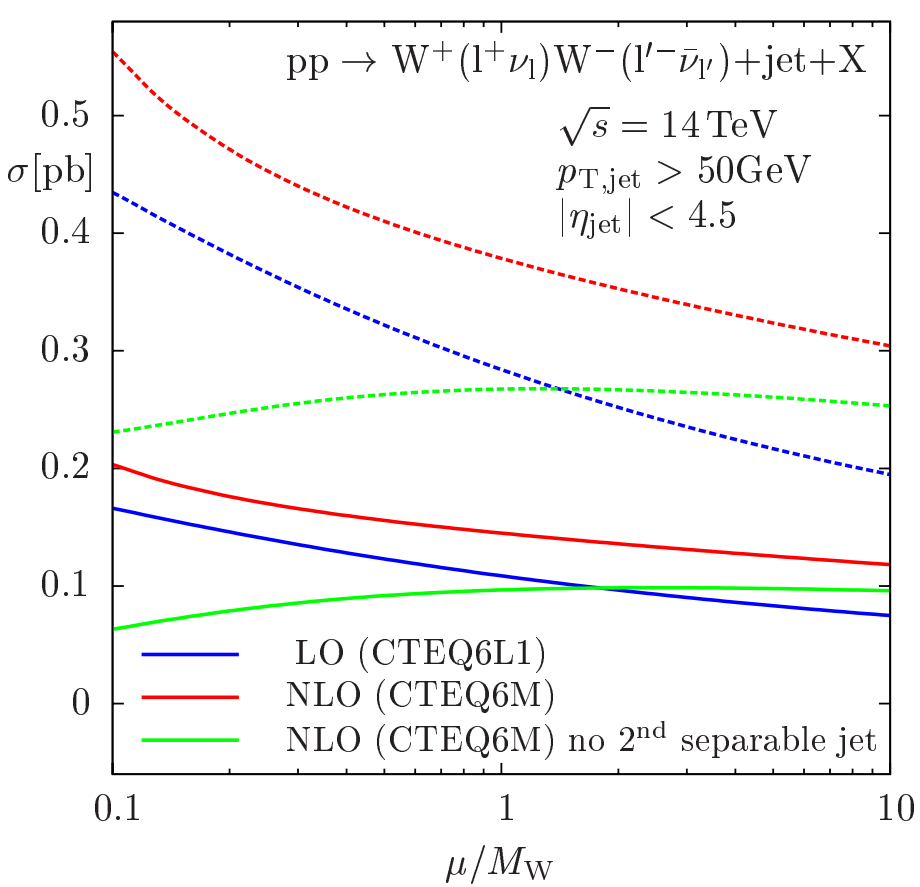
Phase-space factorization results from the approximation:

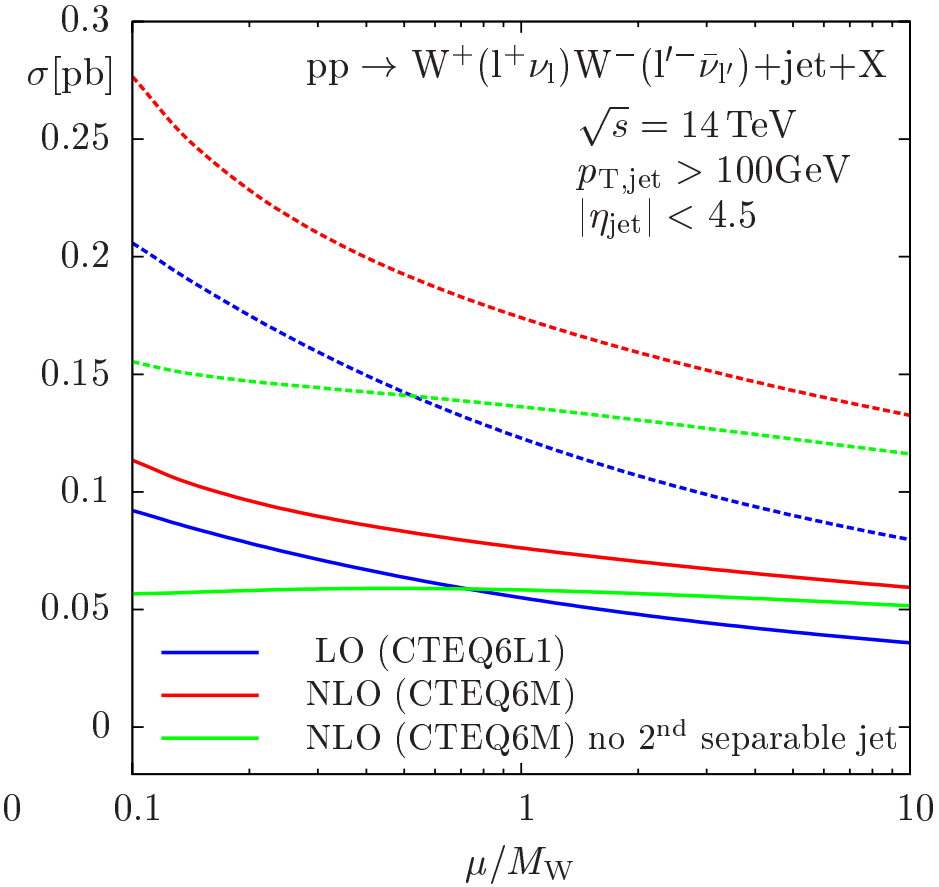
$$\int dk^2_{\mathrm{W}^+} \int dk^2_{\mathrm{W}^-} \frac{\int d\Phi^{\mathrm{WW}}_{\mathrm{prod}}(k^2_{\mathrm{W}^+}, k^2_{\mathrm{W}^-}, 0) \int d\Phi^{\mathrm{W}^+}_{\mathrm{decay}}(k^2_{\mathrm{W}^+}) \int d\Phi^{\mathrm{W}^-}_{\mathrm{decay}}(k^2_{\mathrm{W}^-})}{((k^2_{\mathrm{W}^+} - M^2_{\mathrm{W}})^2 + \Gamma^2_{\mathrm{W}} M^2_{\mathrm{W}})((k^2_{\mathrm{W}^-} - M^2_{\mathrm{W}})^2 + \Gamma^2_{\mathrm{W}} M^2_{\mathrm{W}})}$$

$$\longrightarrow \left(\frac{\pi}{\Gamma_{\mathrm{W}} M_{\mathrm{W}}}\right)^2 \int d\Phi^{\mathrm{WW}}_{\mathrm{prod}}(M^2_{\mathrm{W}}, M^2_{\mathrm{W}}, 0) \int d\Phi^{\mathrm{W}^+}_{\mathrm{decay}}(M^2_{\mathrm{W}}) \int d\Phi^{\mathrm{W}^-}_{\mathrm{decay}}(M^2_{\mathrm{W}})$$

Preliminary numerical results

LO versus NLO cross section at the LHC ($\mu = \mu_{\text{fact}} = \mu_{\text{ren}}$): PRELIMINARY!

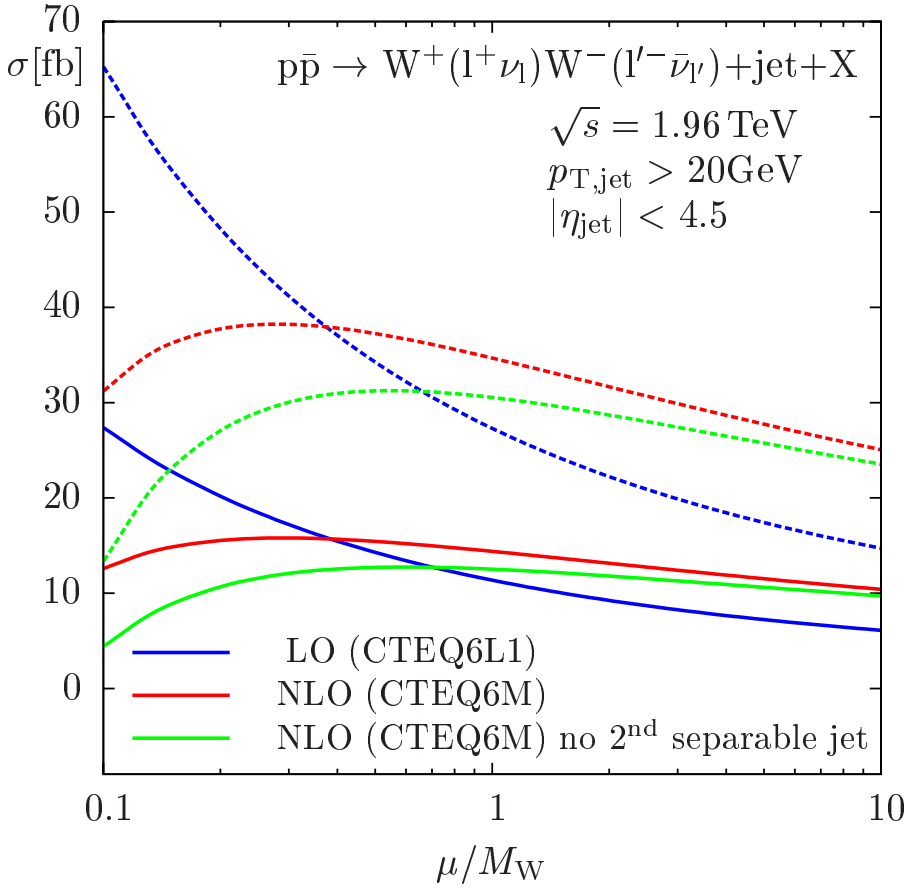


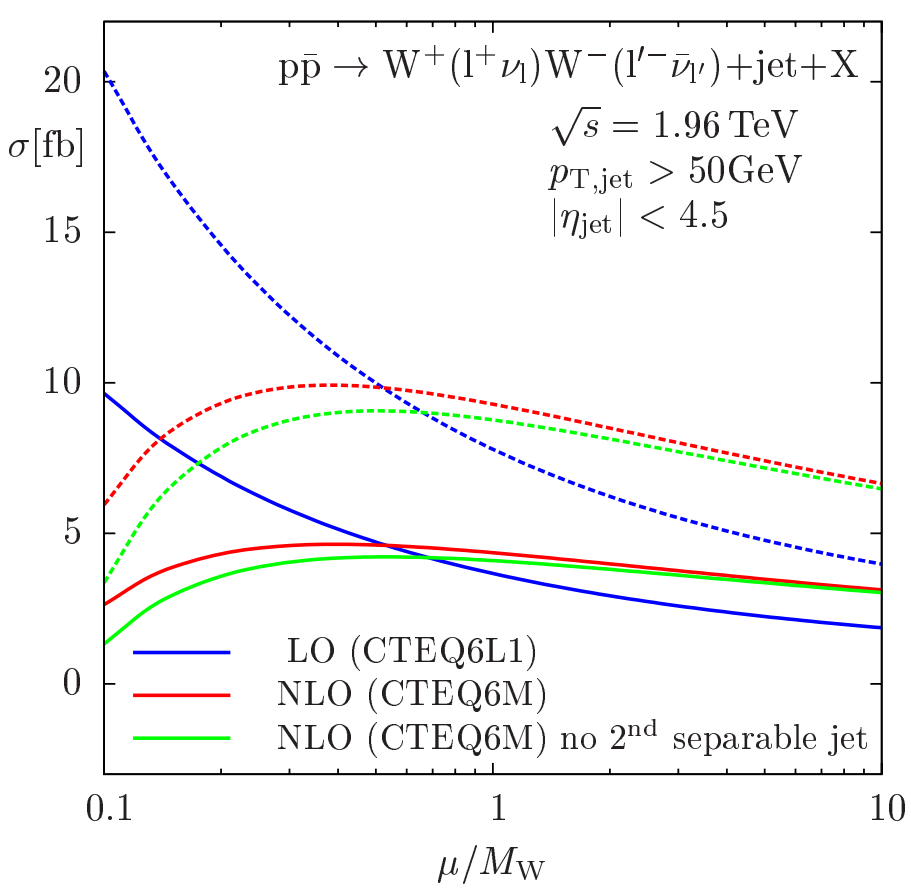


- Dashed lines: no leptonic cuts applied
  $\hookrightarrow\ \sigma_{\mathrm{W}^+(\mathrm{l}^+\nu_\mathrm{l})\mathrm{W}^-(\mathrm{l}'^-\bar{\nu}_{\mathrm{l}'})+\mathrm{jet}} = \sigma_{\mathrm{W}^+\mathrm{W}^-+\mathrm{jet}} \times (BR_{\mathrm{W}->\mathrm{l}\nu_\mathrm{l}})^2$
- Solid lines: Leptonic cuts applied: $p_{\mathrm{T,lep}} \geq 25\,\mathrm{GeV}$, $|\eta_{\mathrm{lep}}| \leq 2.5$, $p_{\mathrm{T,miss}} \geq 25\,\mathrm{GeV}$

Preliminary numerical results

LO versus NLO cross section at the Tevatron ($\mu = \mu_{\rm fact} = \mu_{\rm ren}$): PRELIMINARY!



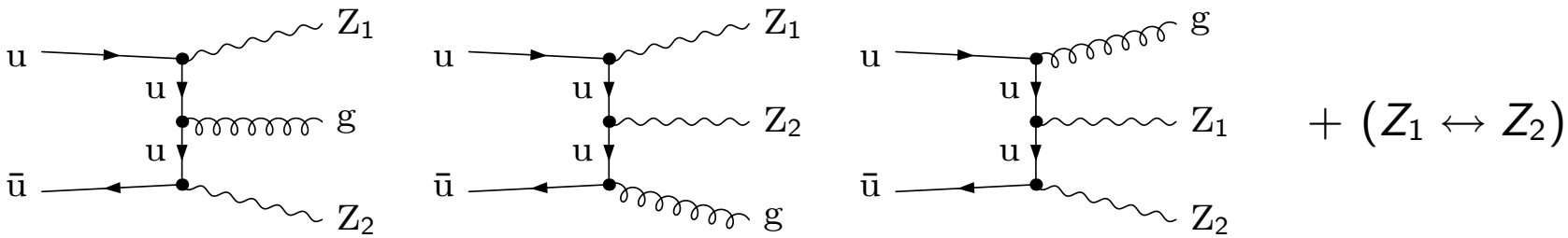


- Dashed lines: no leptonic cuts applied
  $\hookrightarrow\ \sigma_{W^+(l^+\nu_l)W^-(l'^-\bar{\nu}_{l'})+\text{jet}} = \sigma_{W^+W^-+\text{jet}} \times (BR_{W->l\nu_l})^2$
- Solid lines: Leptonic cuts applied: $p_{T,\text{lep}} \geq 25\,\text{GeV}$, $|\eta_{\text{lep}}| \leq 2.5$, $p_{T,\text{miss}} \geq 25\,\text{GeV}$

LO subprocesses for ZZ/WZ+jet production

ZZ+jet: 6 partonic channels in LO: $u\bar{u} \to ZZg$, $ug \to ZZu$, $g\bar{u} \to ZZ\bar{u}$, $d\bar{d} \to ZZg$, $dg \to ZZd$, $g\bar{d} \to ZZ\bar{d}$

Diagrams for $u\bar{u}$ initial state:

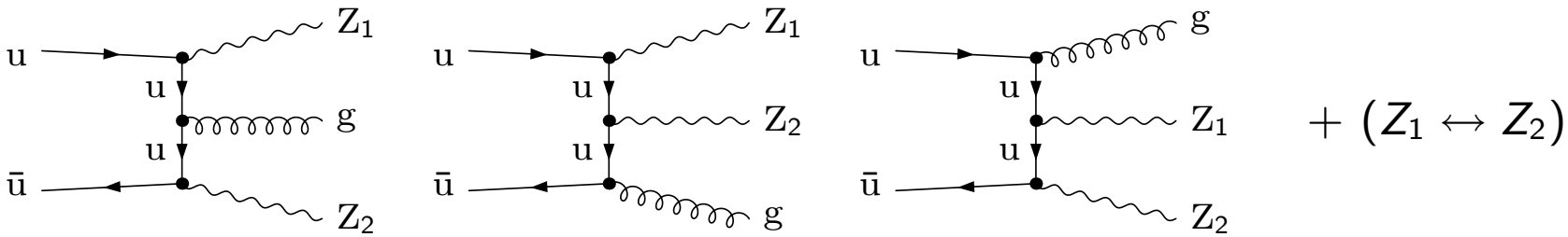


$+ (Z_1 \leftrightarrow Z_2)$

LO subprocesses for ZZ/WZ+jet production

ZZ+jet: 6 partonic channels in LO: $u\bar{u} \to ZZg$, $ug \to ZZu$, $g\bar{u} \to ZZ\bar{u}$, $d\bar{d} \to ZZg$, $dg \to ZZd$, $g\bar{d} \to ZZ\bar{d}$

Diagrams for $u\bar{u}$ initial state:

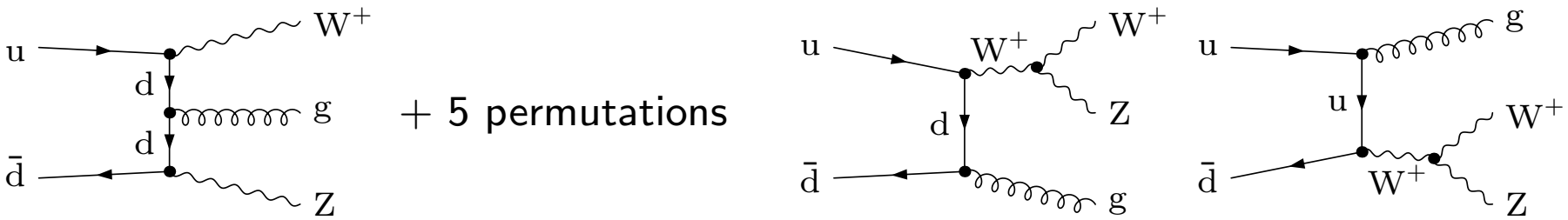


$+ (Z_1 \leftrightarrow Z_2)$

W^+Z+jet: 3 partonic channels in LO: $u\bar{d} \to W^+Zg$, $ug \to W^+Zd$, $g\bar{d} \to W^+Z\bar{u}$,

W^-Z+jet: 3 partonic channels in LO: $d\bar{u} \to W^-Zg$, $dg \to W^-Zu$, $g\bar{u} \to W^-Z\bar{d}$

Diagrams for $u\bar{d}$ initial state:

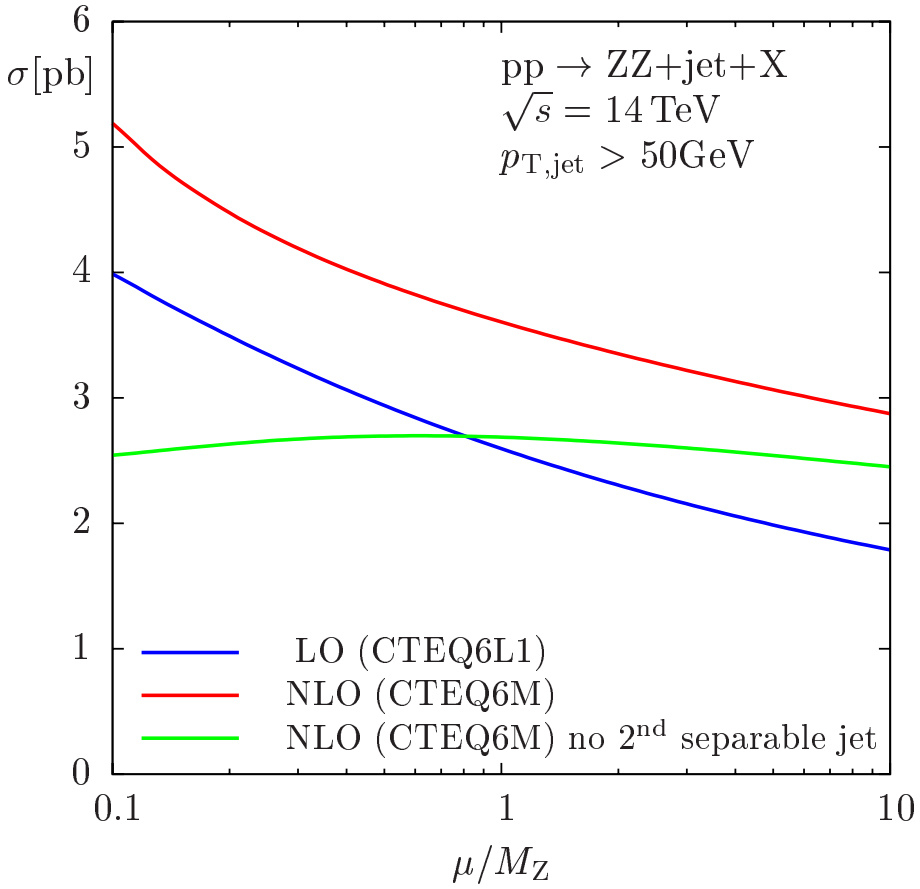


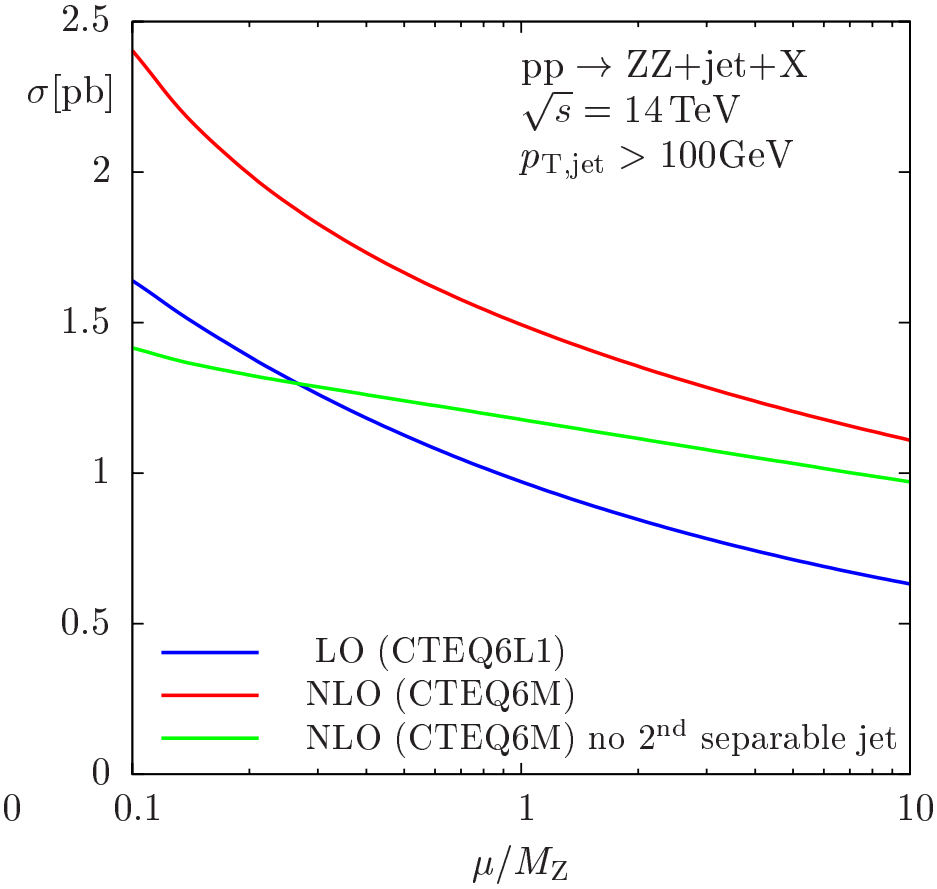
↪ explicit CKM dependence at LO for WZ+jet production

Preliminary numerical results

Similar calculation performed for $\mathrm{pp} \to \mathrm{ZZ{+}jet{+}X}$: PRELIMINARY!

LO versus NLO cross section at the LHC ($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$):



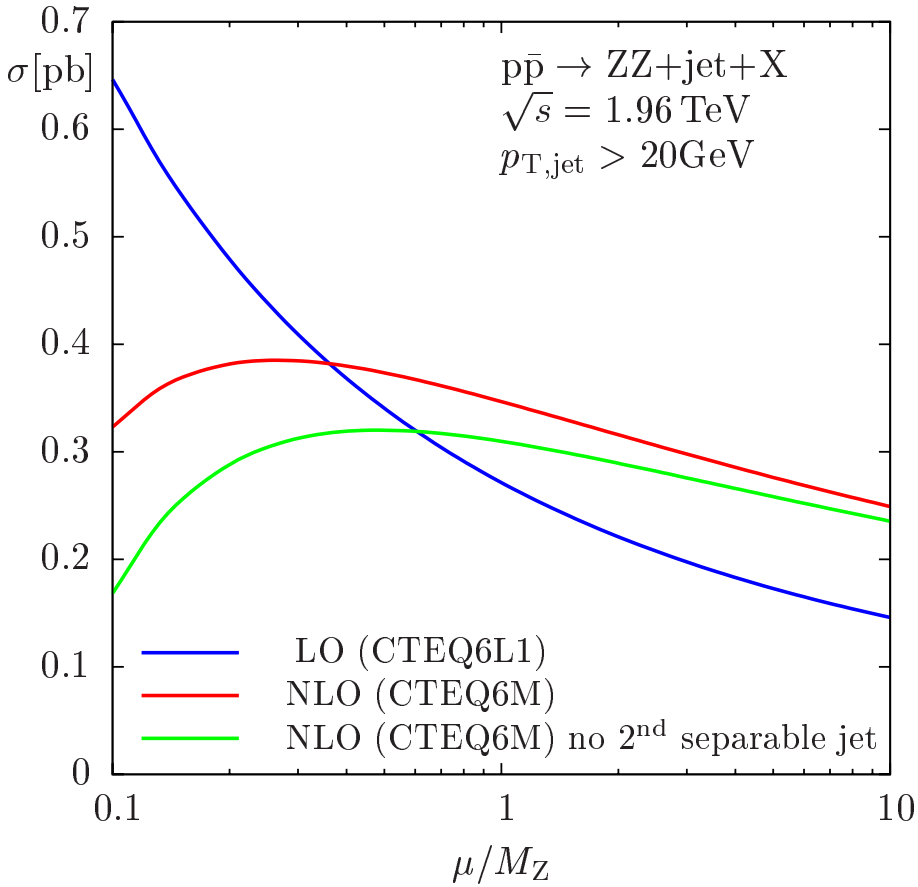


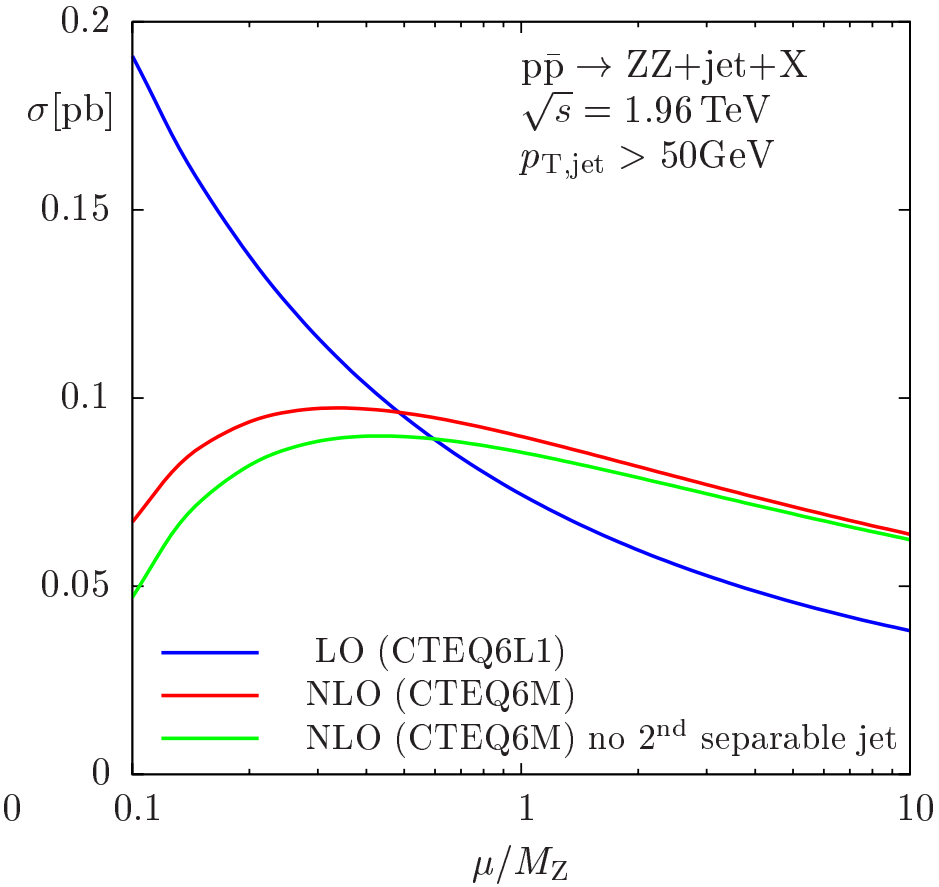
- Scale dependence stabilizes at NLO for genuine ZZ+jet production.
- But: Significant scale dependence is introduced by ZZ+2jets events.

Preliminary numerical results

Similar calculation performed for $\mathrm{p\bar{p}} \to \mathrm{ZZ{+}jet{+}X}$: PRELIMINARY!

LO versus NLO cross section at the Tevatron ($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$):



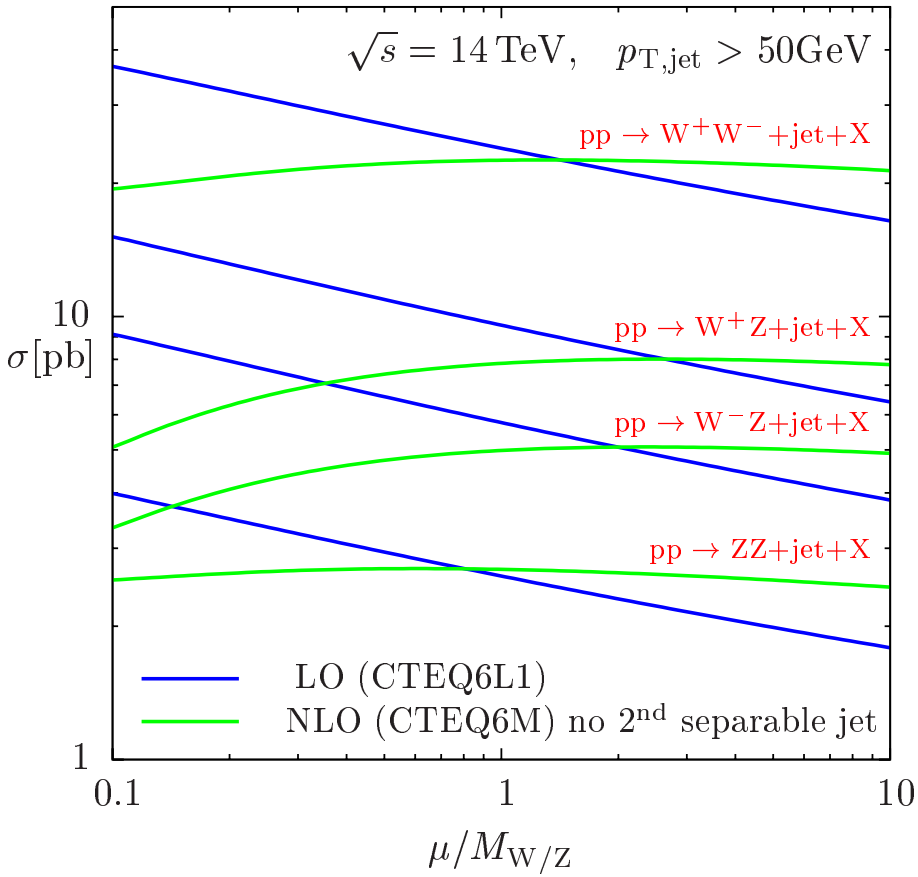


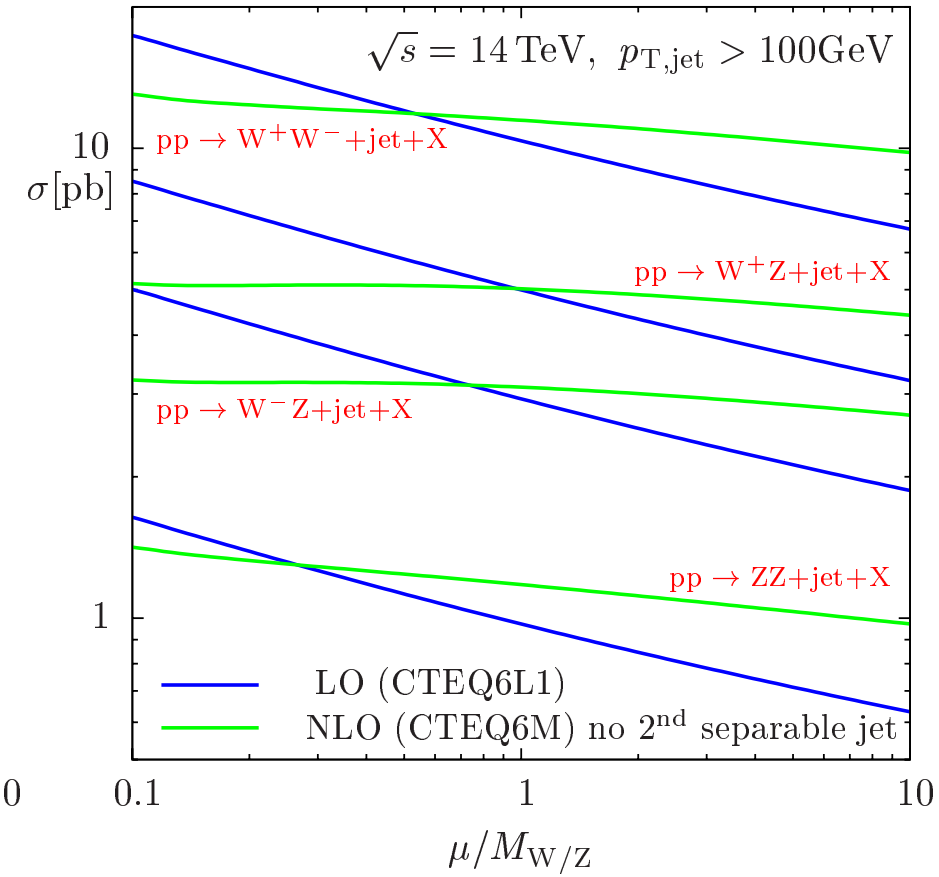
↪ Scale dependence stabilizes at NLO.
(only small influence from ZZ+2jets production)

Preliminary numerical results

Summary of results for $\mathrm{pp} \to \mathrm{VV{+}jet{+}X}$: PRELIMINARY!

LO versus NLO cross section at the LHC ($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$):



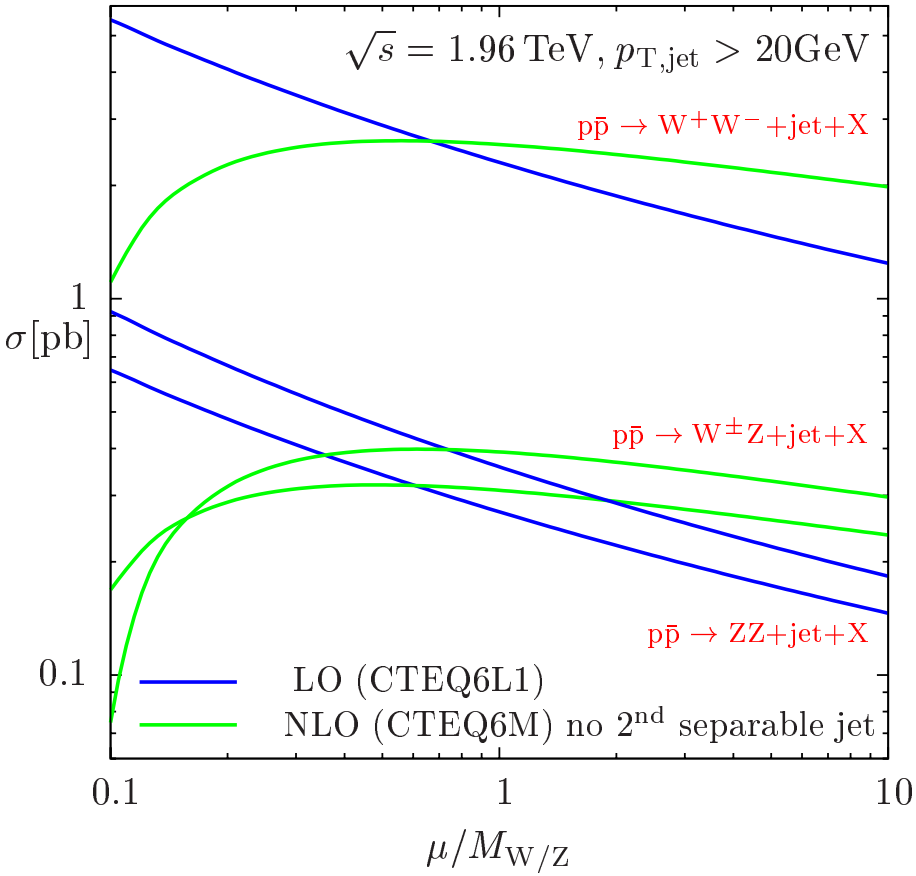


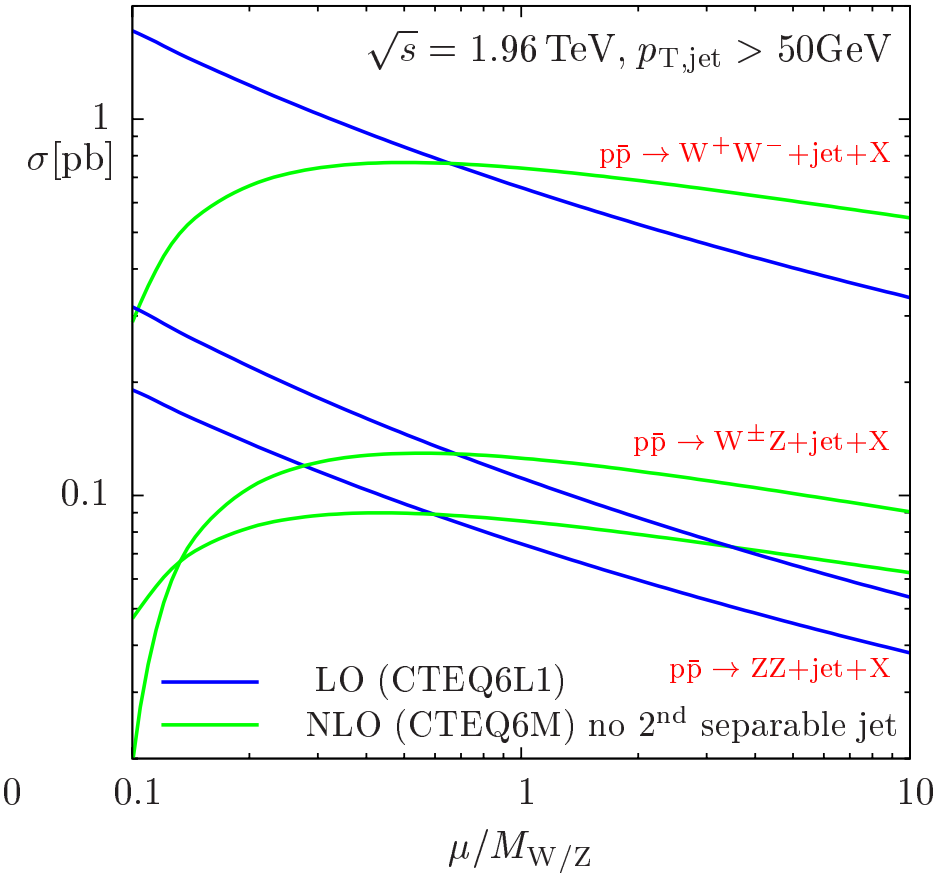
↪ Scale dependence stabilizes at NLO for genuine VV+jet production.

Preliminary numerical results

Summary of results for $\mathrm{p\bar{p}} \to \mathrm{VV{+}jet{+}X}$: PRELIMINARY!

LO versus NLO cross section at the Tevatron ($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$):





↪ Scale dependence stabilizes at NLO for genuine $\mathrm{VV{+}jet}$ production.

Conclusions & Outlook

Conclusions

$\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{WW} + \mathrm{jet} + X$

- Important background process for Higgs and other searches at Tevatron/LHC
- WW(+jets) production used for EW gauge-boson coupling analysis at the LHC

NEW: $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{WW} + \mathrm{jet} + X$ at NLO QCD

- Tevatron: NLO correction stabilizes LO cross sections
- LHC: Significant reduction of scale uncertainty for genuine WW+jet production
  But: significant scale dependence via WW+2jets events
- Comparison with [Ellis et al.], [Binoth et al.] in progress

Outlook & work in progress

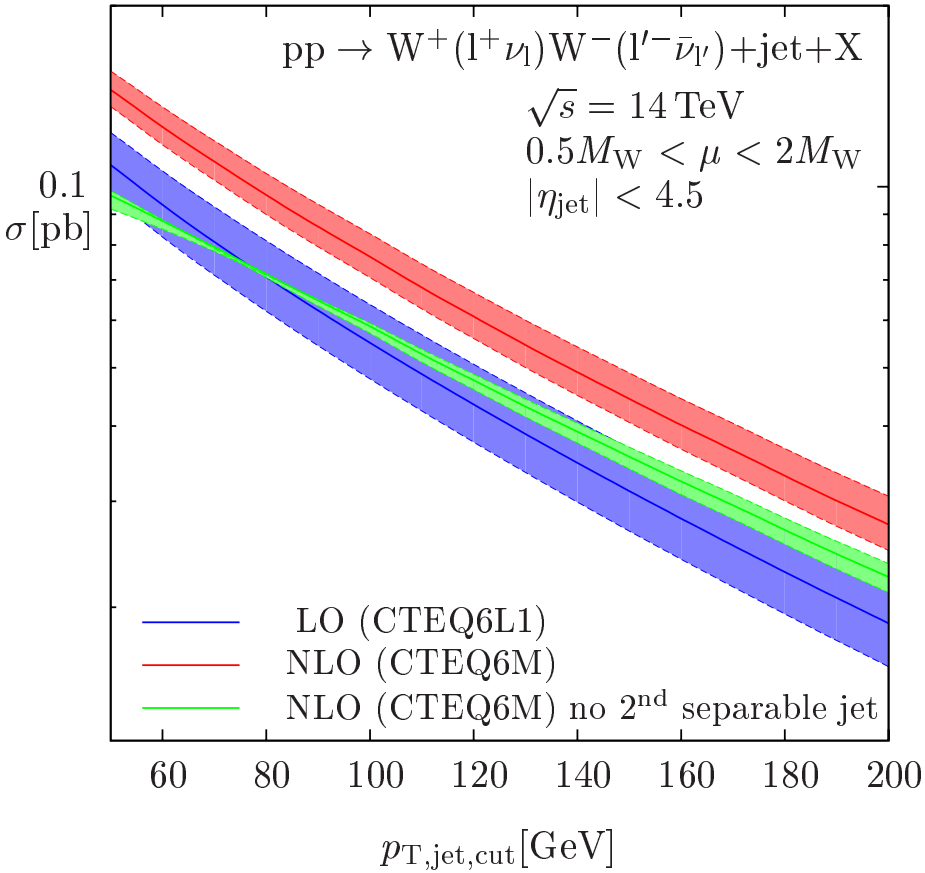
- Implementation of leptonic W decays and distributions
- Calculation of similar processes of the class $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{WW}/\mathrm{WZ}/\mathrm{ZZ} + \mathrm{jet} + X$
- Methods not yet exhausted $\longrightarrow$ more complicated applications ($2 \to 4$) feasible!

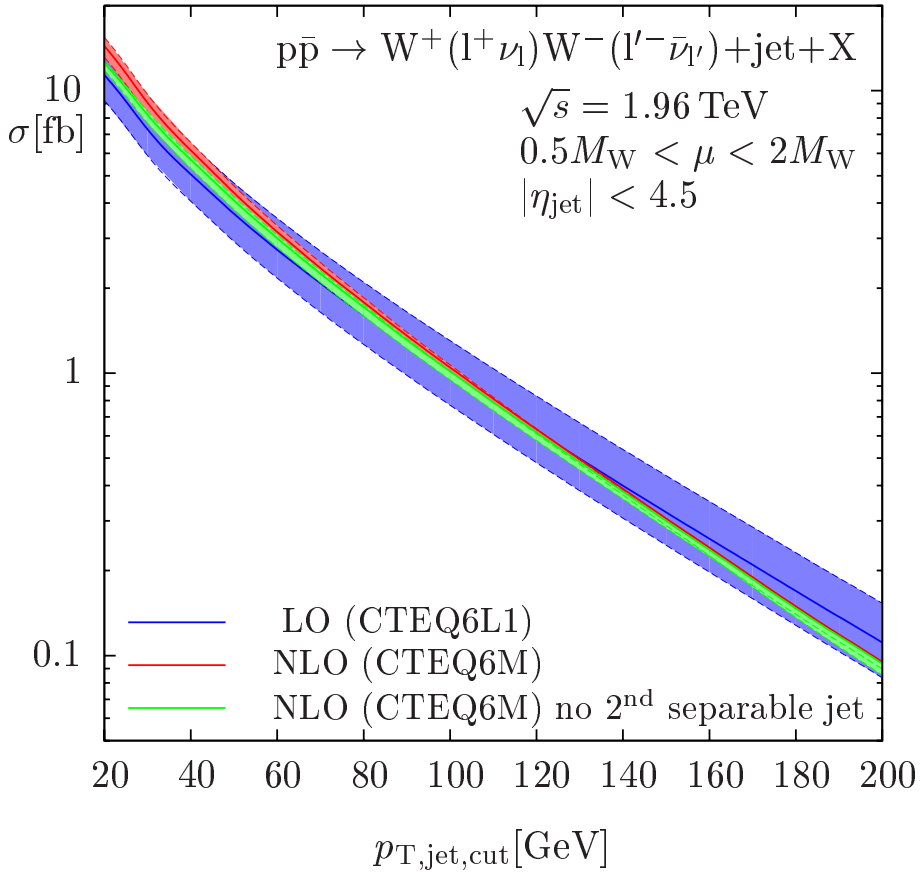
Backup slides

Preliminary numerical results

LO versus NLO cross section for $p_{\mathrm{T,cut,jet}}$-variation at LHC and Tevatron:
($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$)

PRELIMINARY!





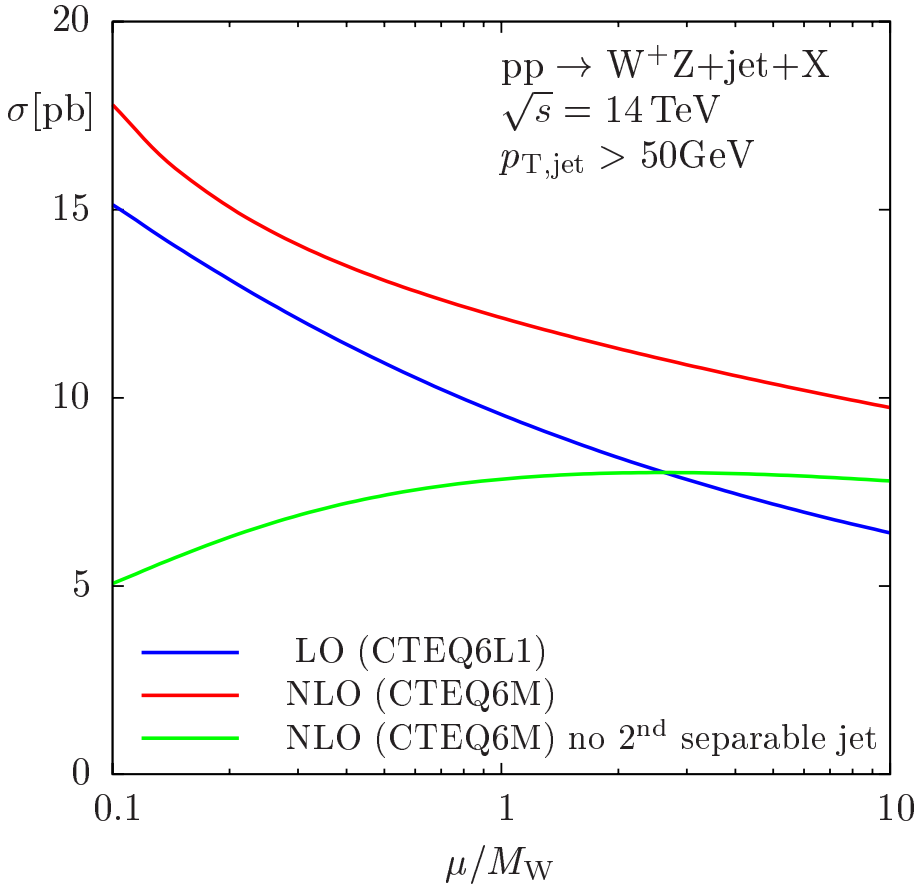
Leptonic cuts applied: $p_{\mathrm{T,lepton}} \geq 25\,\mathrm{GeV}$, $|\eta_{\mathrm{lepton}}| \leq 2.5$ $p_{\mathrm{T,miss}} \geq 25\,\mathrm{GeV}$

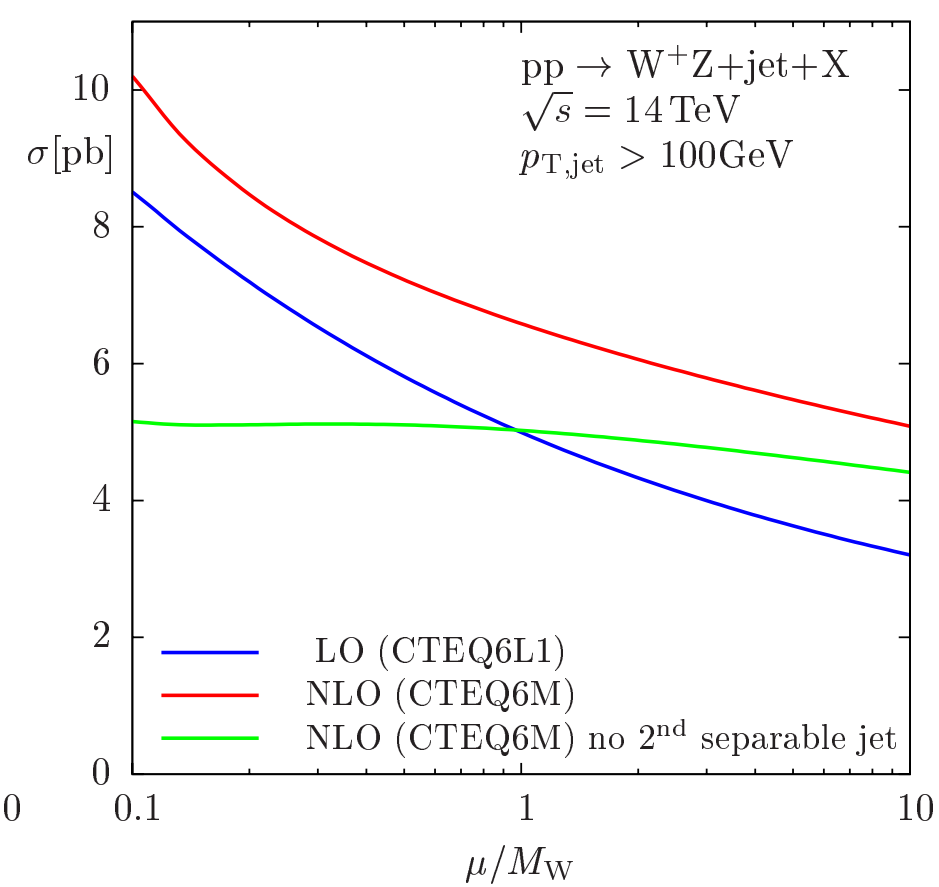
↪ Significant stabilisation of scale dependence is achieved at NLO
(at LHC especially for genuine $W^+(l^+\nu_l)W^-(l'^-\bar{\nu}_{l'})$+jet production)

Preliminary numerical results

Similar calculation performed for $pp \to W^+Z+jet+X$: PRELIMINARY!

LO versus NLO cross section at the LHC ($\mu = \mu_{fact} = \mu_{ren}$):



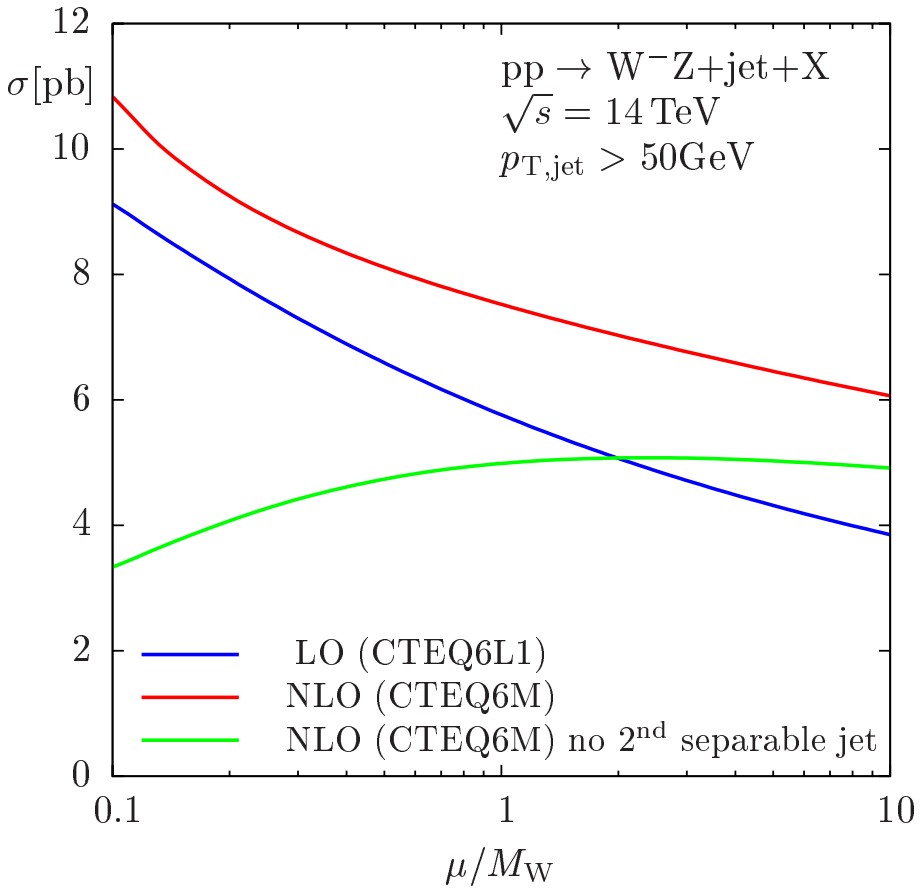


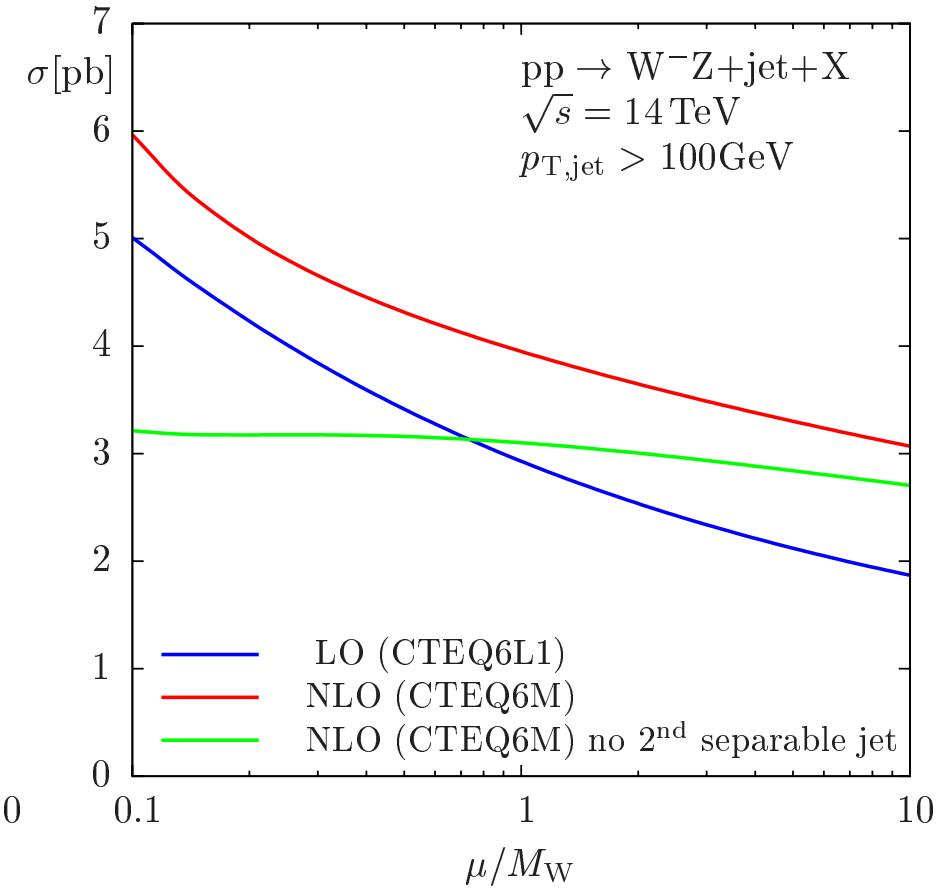
- Scale dependence stabilizes at NLO for genuine W^+Z+jet production.
- But: Significant scale dependence is introduced by W^+Z+2jets events.

Preliminary numerical results

Similar calculation performed for $\mathrm{pp} \to \mathrm{W^-Z{+}jet{+}X}$: PRELIMINARY!

LO versus NLO cross section at the LHC ($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$):



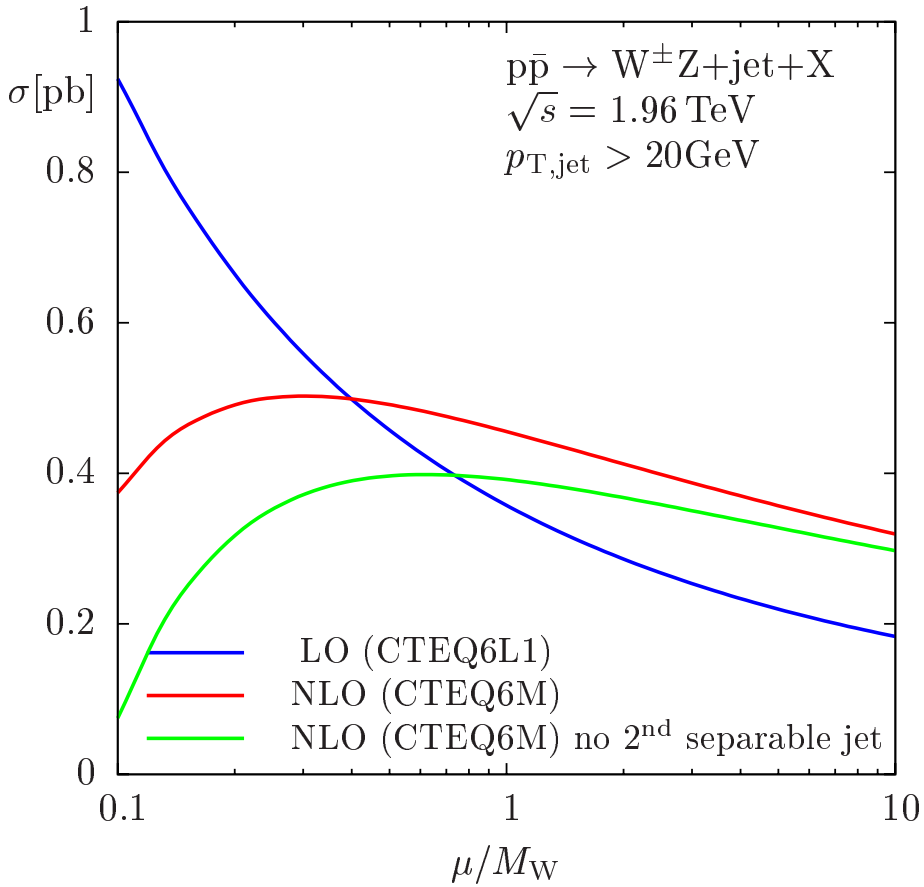


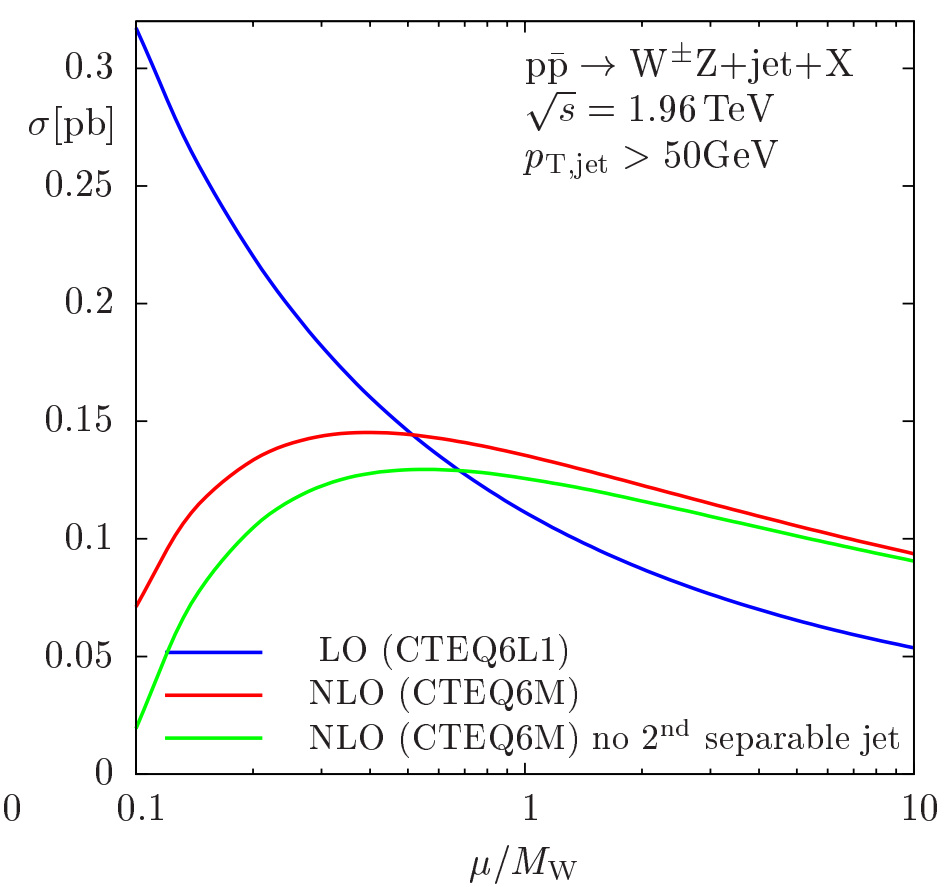
- Scale dependence stabilizes at NLO for genuine $\mathrm{W^-Z{+}jet}$ production.
- But: Significant scale dependence is introduced by $\mathrm{W^-Z{+}2jets}$ events.

Preliminary numerical results

Similar calculation performed for $\mathrm{p\bar{p}} \to \mathrm{W^{\pm}Z{+}jet{+}X}$: PRELIMINARY!

LO versus NLO cross section at the Tevatron ($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$):



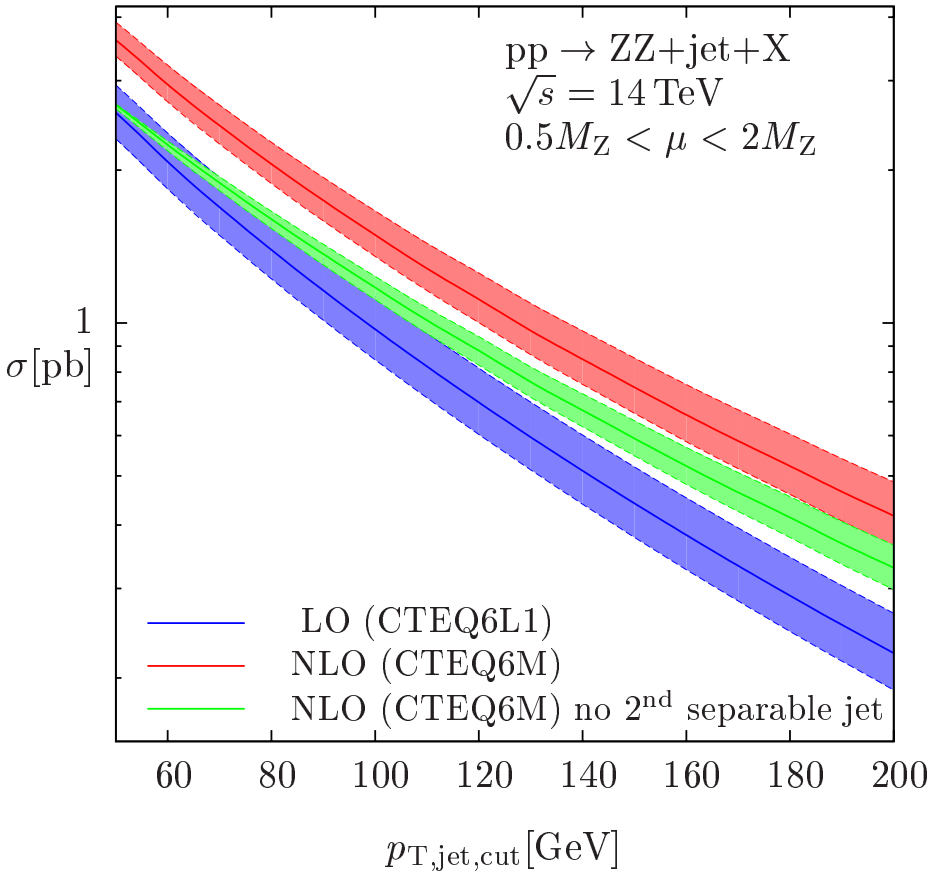


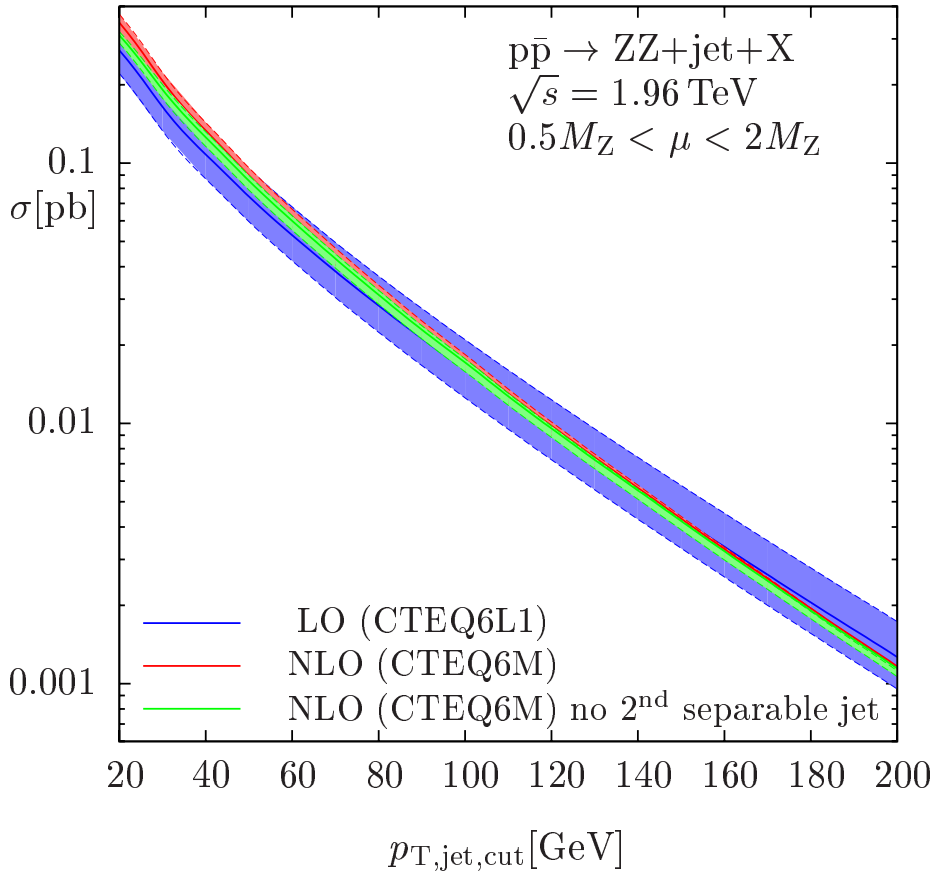
↪ Scale dependence stabilizes at NLO.
(only small influence from $\mathrm{WZ{+}2jets}$ production)

Preliminary numerical results

Similar calculation performed for $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{ZZ{+}jet{+}X}$: PRELIMINARY!

LO versus NLO cross section for $p_{\mathrm{T,cut,jet}}$-variation at LHC and Tevatron:
($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$)



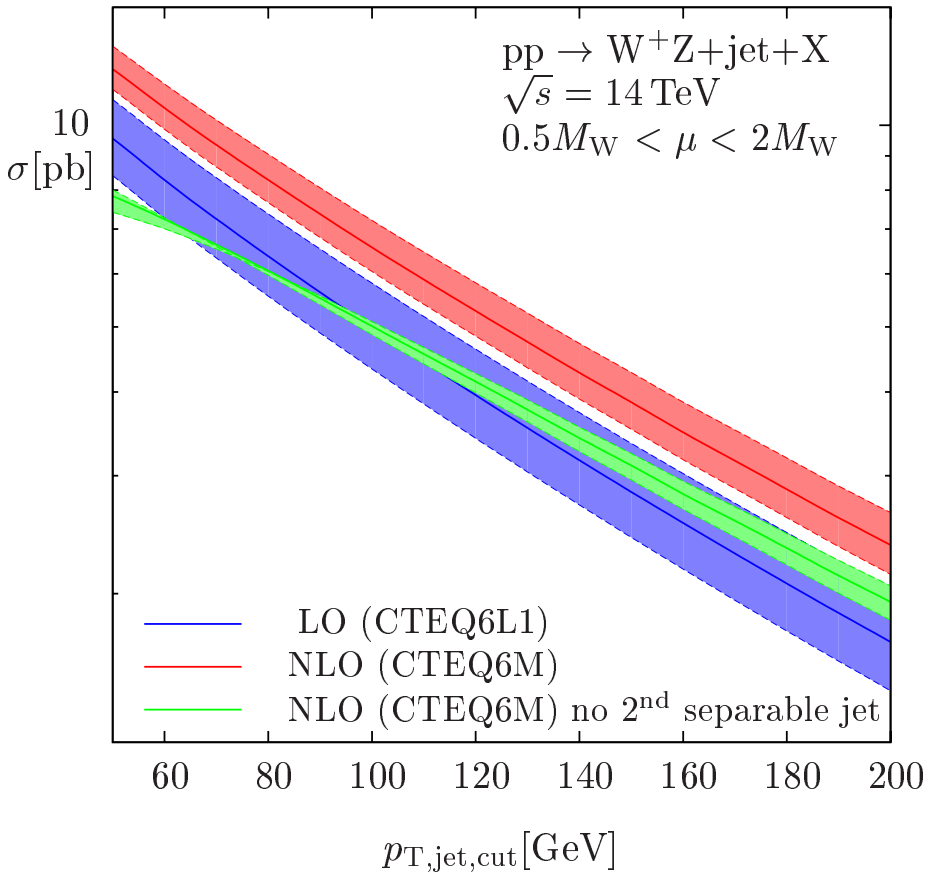


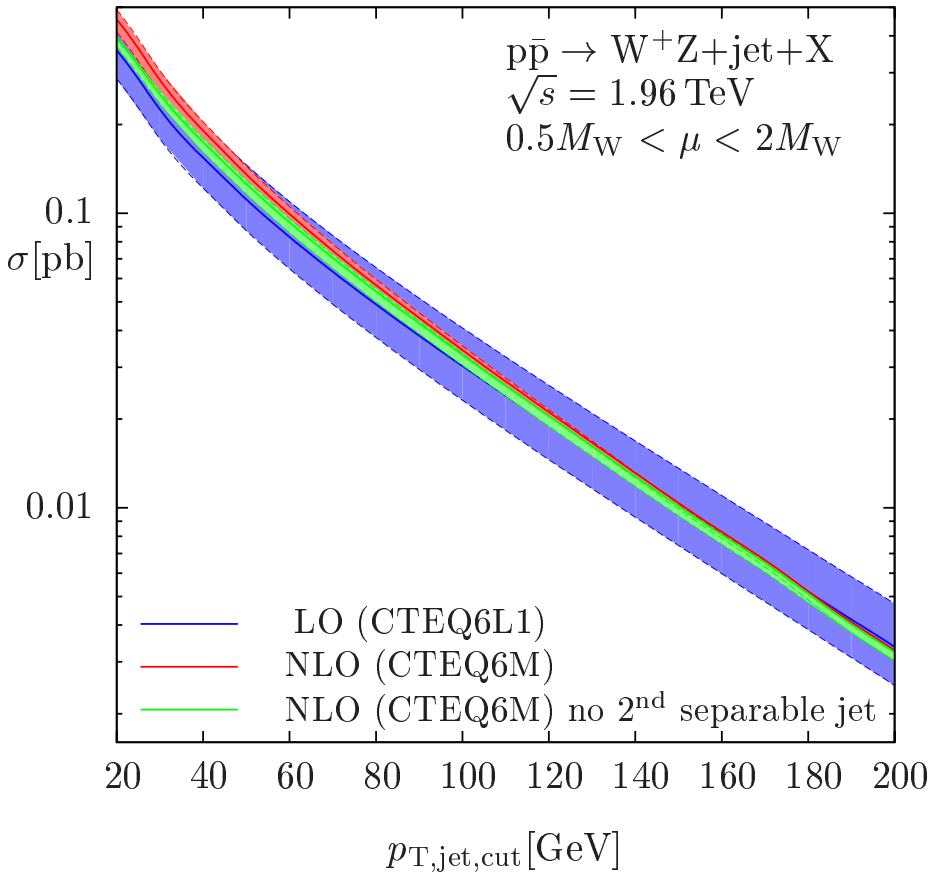
↪ Significant stabilisation of scale dependence is achieved at NLO
(at LHC especially for genuine ZZ+jet production)

Preliminary numerical results

Similar calculation performed for $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{W^+Z{+}jet{+}X}$: PRELIMINARY!

LO versus NLO cross section for $p_{\mathrm{T,cut,jet}}$-variation at LHC and Tevatron:
($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$)



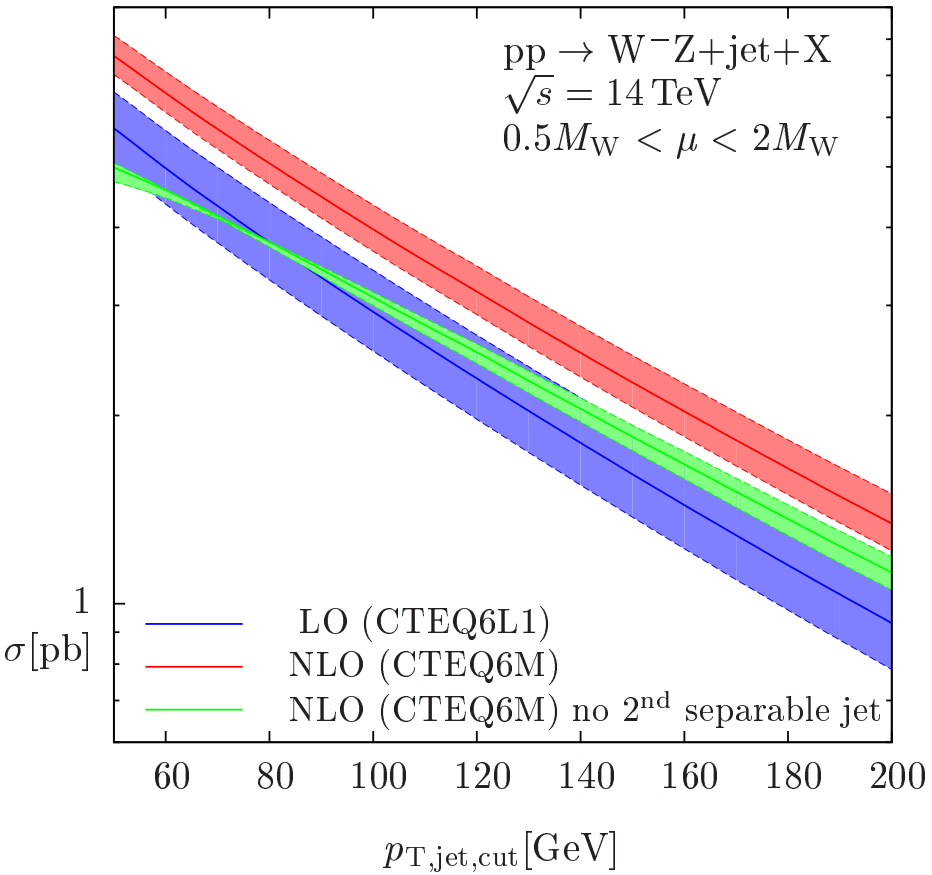


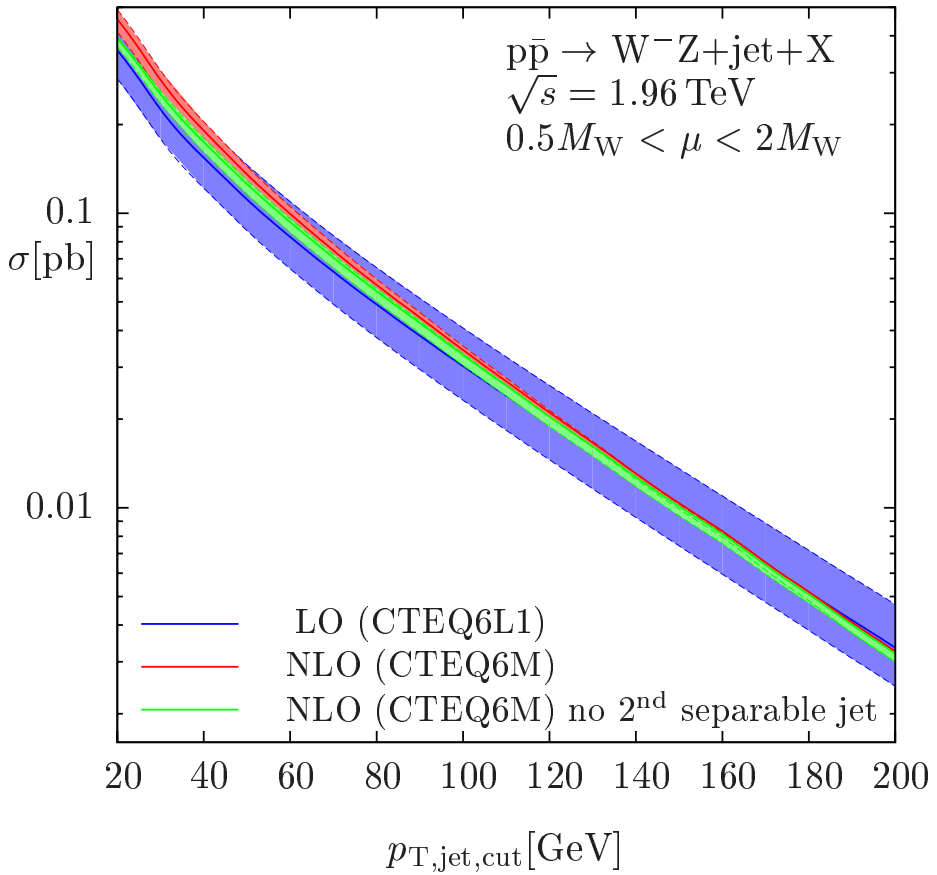
↪ Significant stabilisation of scale dependence is achieved at NLO
(at LHC especially for genuine $\mathrm{W^+Z{+}jet}$ production)

Preliminary numerical results

Similar calculation performed for $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{W^-Z{+}jet{+}X}$: PRELIMINARY!

LO versus NLO cross section for $p_{\mathrm{T,cut,jet}}$-variation at LHC and Tevatron:
($\mu = \mu_{\mathrm{fact}} = \mu_{\mathrm{ren}}$)





↪ Significant stabilisation of scale dependence is achieved at NLO
(at LHC especially for genuine $\mathrm{W^-Z{+}jet}$ production)