Predictions for Higgs signal and background processes with many-particle final states at the LHC

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Contents

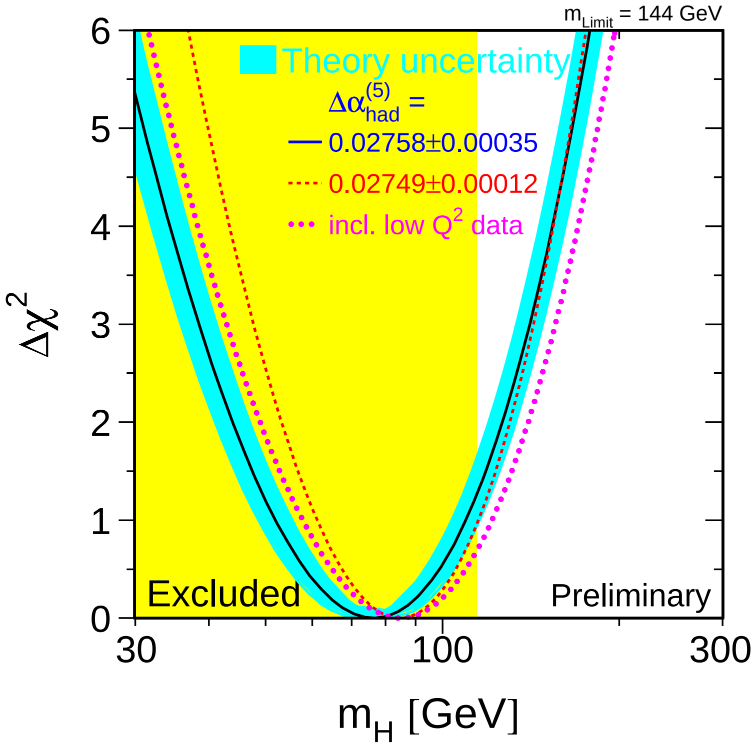
1 Introduction

Experiments at LEP/SLC/Tevatron

- confirmation of Standard Model as quantum field theory
  (quantum corrections significant)
- top mass m_t indirectly constrained by quantum corrections
  $\leftrightarrow$ in agreement with m_t measurement of Tevatron
- Higgs mass M_H indirectly constrained by quantum corrections
  $\hookrightarrow$ impact on Higgs searches

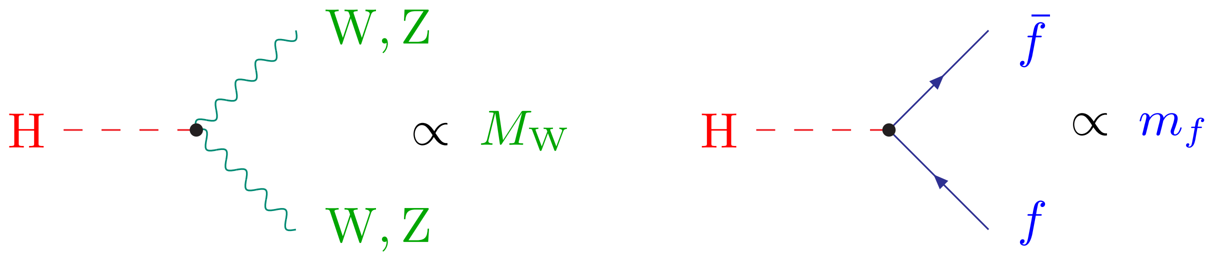
Great success of precision physics

- $M_\mathrm{H} > 114.4\,\mathrm{GeV}$ (LEPHIGGS '02)
  $\mathrm{e^+e^-} \not\to \mathrm{ZH}$ at LEP2
- $M_\mathrm{H} < 144\,\mathrm{GeV}$ (LEPEWWG '07)
  fit to precision data
  i.e. via quantum corrections



Higgs search at present and future colliders

Higgs bosons couple proportional to particle masses:



$\Rightarrow$ Higgs production mainly via coupling to W/Z bosons or top quarks

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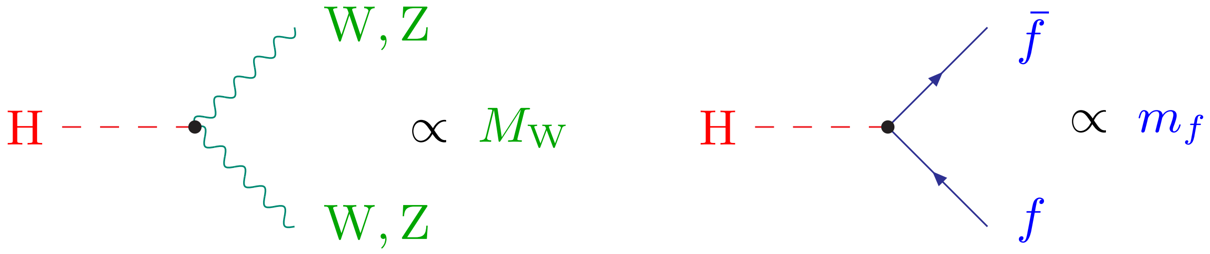

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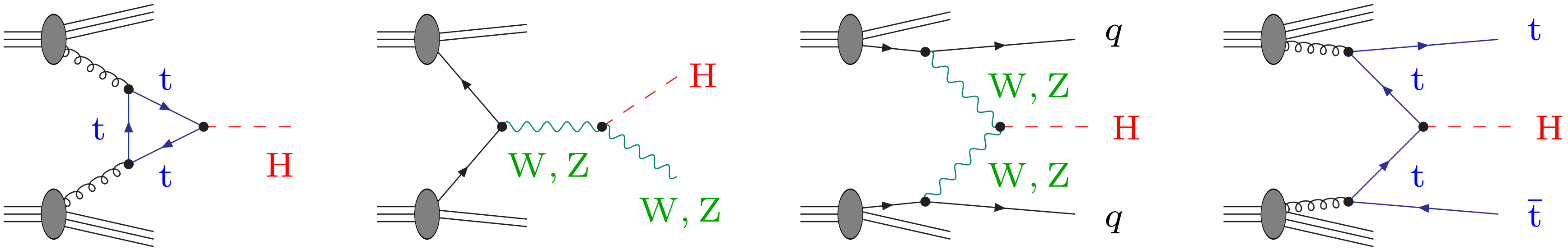
Processes at hadron colliders ($p\bar{p}/pp$):



Higgs search at present and future colliders

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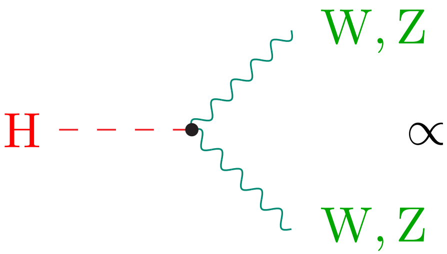
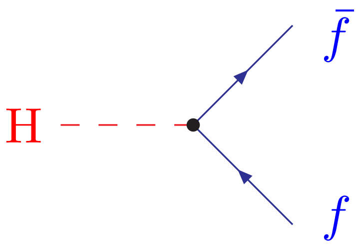


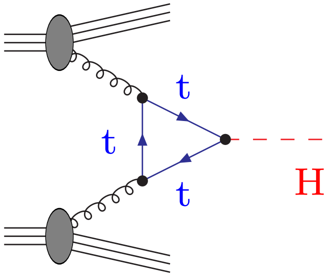
$\propto M_\mathrm{W}$

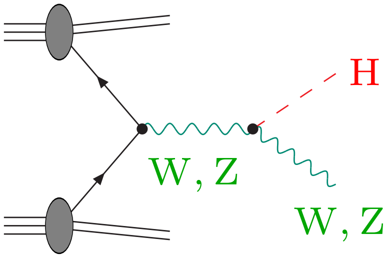


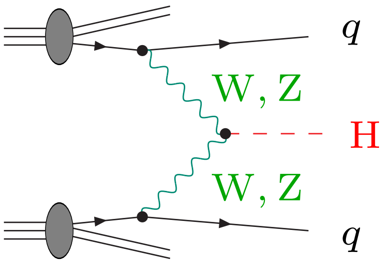
$\propto m_f$

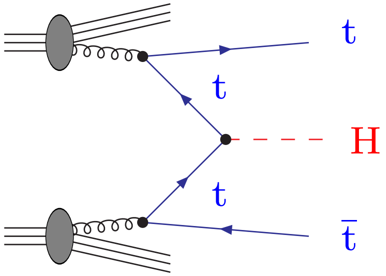
⇒ Higgs production mainly via coupling to W/Z bosons or top quarks

Processes at hadron colliders ($\mathrm{p\bar{p}/pp}$):

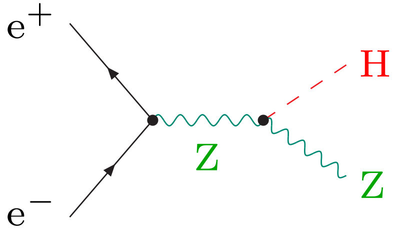


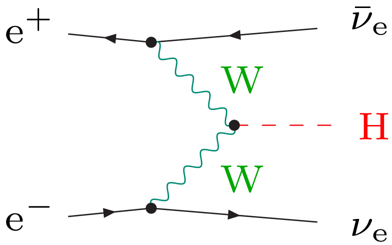


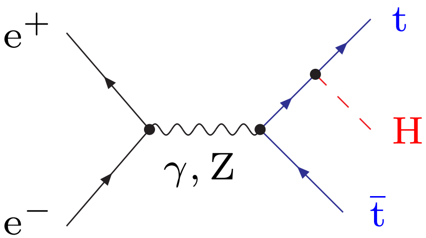




Processes at $\mathrm{e^+e^-}$ colliders:

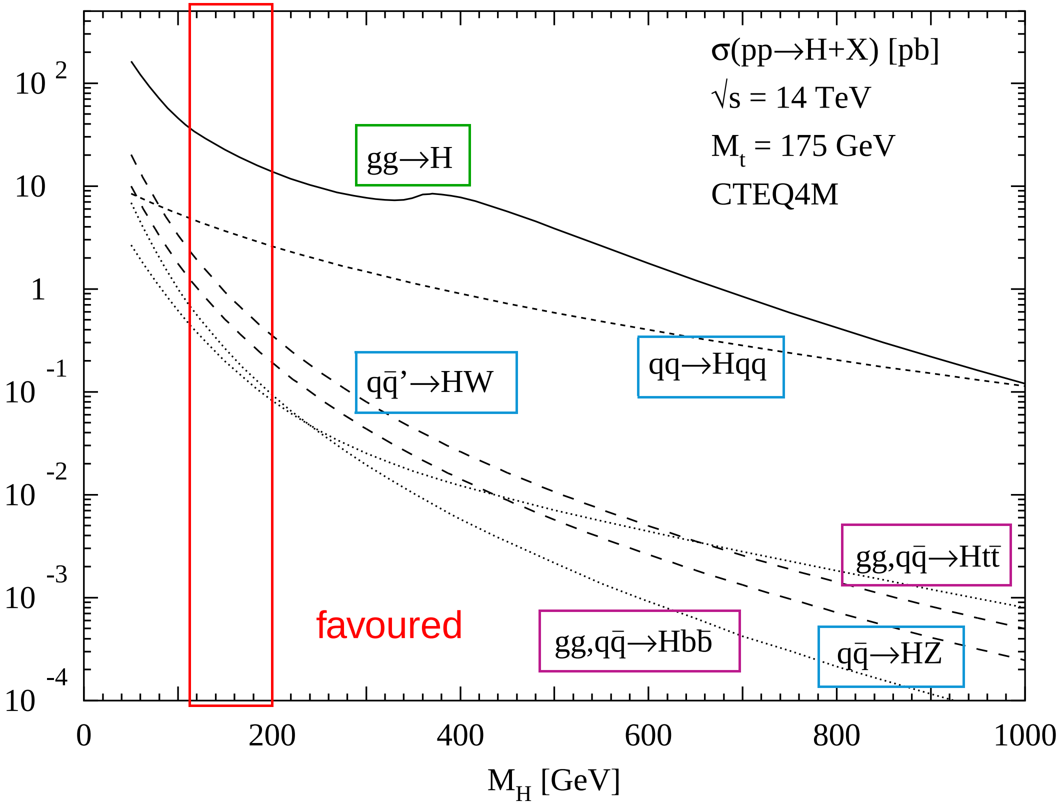




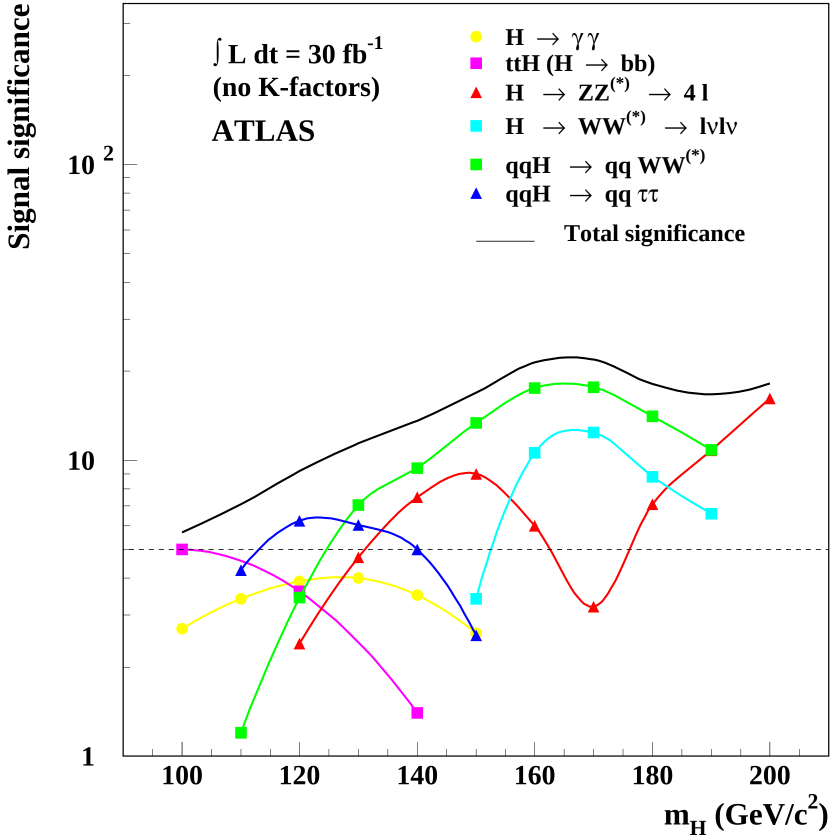


Cross sections and significance of the Higgs signal at the LHC

Spira et al. '98



ATLAS '03



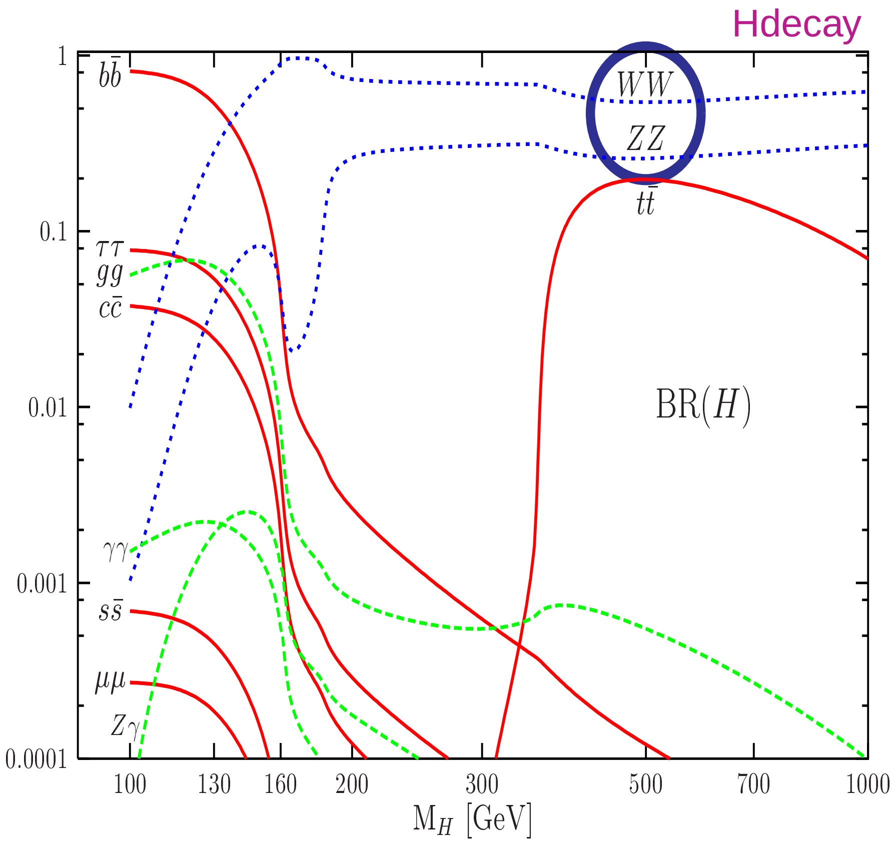
Typical size perturbative corrections at next-to-leading order (NLO):

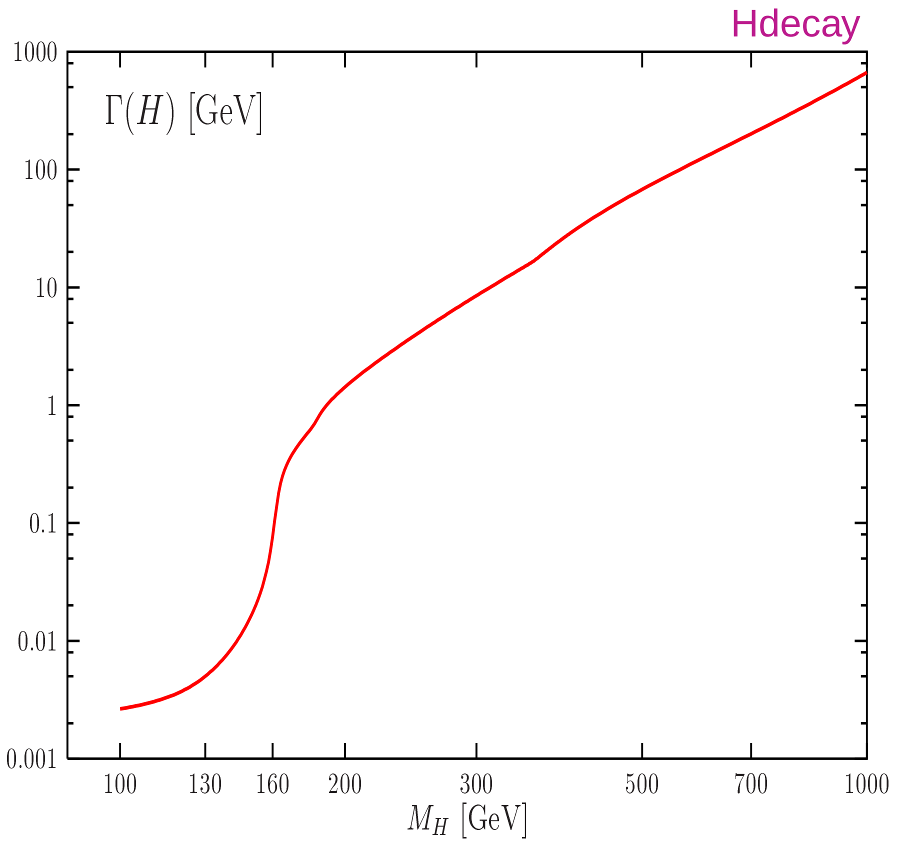
QCD: $\mathcal{O}(\alpha_s) \sim 10{-}100\%$ Electroweak: $\mathcal{O}(\alpha) \sim 10\%$

$\hookrightarrow$ calculate / control higher orders to reduce theoretical uncertainty down to the level of PDF ($q\bar{q} \sim 5\%$, $gg \sim 10\%$) and experimental uncertainties

Complication: many channels involve multi-particle final states.

2 The decays Higgs ⟶ WW/ZZ ⟶ 4 fermions





Importance of decays $\mathrm{H} \to \mathrm{WW}^{(*)}/\mathrm{ZZ}^{(*)}$ at the LHC:

– most important Higgs decay channels for $M_\mathrm{H} \gtrsim 125\,\mathrm{GeV}$

– most precise determination of M_H via $\mathrm{H}{\to}\mathrm{ZZ}{\to}4l$ for $M_\mathrm{H} \gtrsim 130\,\mathrm{GeV}$

Theoretical description of $\mathrm{H}\to\mathrm{WW}^{(*)}/\mathrm{ZZ}^{(*)}$:

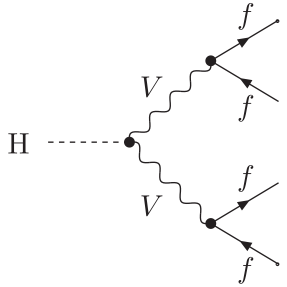
- previous work on partial decay widths not sufficient:
  - $\mathcal{O}(\alpha)$ corrections to $\mathrm{H}\to\mathrm{WW}/\mathrm{ZZ}$ with stable W's/Z's
    Fleischer, Jegerlehner '81; Kniehl '91; Bardin, Vilenskii, Khristova '91
  - lowest-order predictions for $\mathrm{H}\to\mathrm{WW}^{(*)}/\mathrm{ZZ}^{(*)}$
    e.g. by Hdecay (Djouadi, Kalinowski, Spira '98)
- however: proper description of distributions required
  - for the kinematical reconstruction of Z's, W's, and H
    $\hookrightarrow$ invariant-mass distributions
  - for the verification of spin 0 and CP parity of the Higgs boson
    $\hookrightarrow$ angular and invariant-mass distributions
    Nelson '88; Soni, Xu '93; Chang et al.'93;
    Skjold, Osland '93; Barger et al.'93;
    Arens, Sehgal '94; Buszello et al.'02; Choi et al.'0

Recent progress:

- PROPHECY4F: Monte Carlo generator for $\mathrm{H}\to\mathrm{WW}/\mathrm{ZZ}\to 4f$
  with EW and QCD corrections
  Bredenstein, Denner, S.D., Weber '06
- combination of production and decay:
  $(gg\to\mathrm{H}$ in NNLO QCD$)\otimes(\mathrm{H}\to\mathrm{WW}/\mathrm{ZZ}\to 4l$ in LO$)$
  Anastasiou et al. '07,'08;
  Frederix, Grazzini '08; Grazzini '08

Survey of Feynman diagrams for NLO EW and QCD corrections to $\mathrm{H} \to 4f$

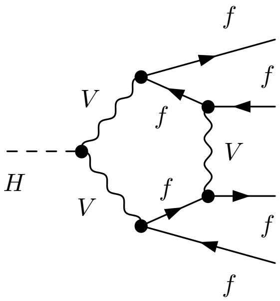
Lowest order:



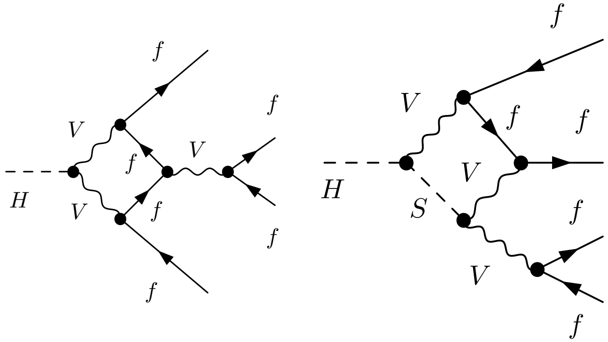
Typical one-loop diagrams:

diagrams = $\mathcal{O}(200-400)$

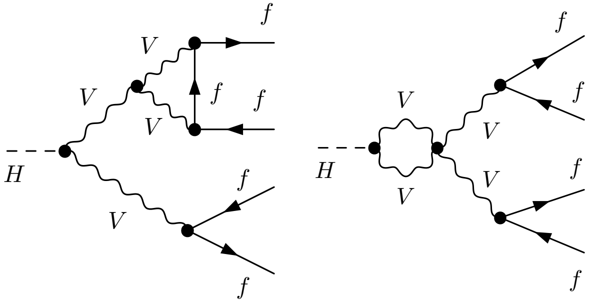
pentagons



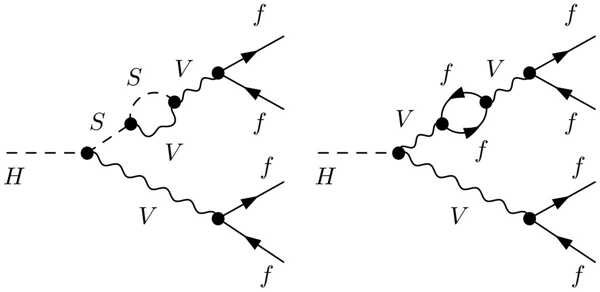
boxes



vertices



self-energies



$+$ photon / gluon bremsstrahlung

Features of PROPHECY4F: Bredenstein, Denner, S.D., Weber '06

- $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha_\mathrm{s})$ corrections to all channels $\mathrm{H} \to \mathrm{WW/ZZ} \to 4f$
- final-state radiation off leptons beyond $\mathcal{O}(\alpha)$ via structure functions
- leading 2-loop heavy-Higgs effects $\propto G_\mu^2 M_\mathrm{H}^4$
  Ghinculov '95; Frink, Kniehl, Kreimer, Riesselmann '96
- multi-channel Monte Carlo integration (checked by VEGAS)
  Berends, Kleiss, Pittau '94; Kleiss, Pittau '94
- improved Born approximation for simplified evaluation

Main complications in the loop calculation:

- numerical instabilities in Passarino–Veltman reduction of tensor integrals
  $\hookrightarrow$ new reduction methods developed Denner, S.D. '02,'05
- gauge-invariant treatment of W and Z resonances
  $\hookrightarrow$ "complex-mass scheme" Denner, S.D., Roth, Wieders '05

New concepts already used in $\mathcal{O}(\alpha)$ correction to $\mathrm{e}^+\mathrm{e}^- \to 4f$ Denner, S.D., Roth, Wieders '05

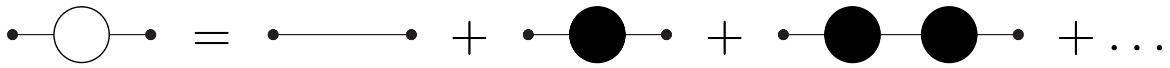
The complex-mass scheme for unstable particles

Problem of unstable particles:

description of resonances requires resummation of propagator corrections

$\hookrightarrow$ mixing of perturbative orders potentially violates gauge invariance

Dyson series and propagator poles (scalar example)



$$G^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2-m^2} + \frac{\mathrm{i}}{p^2-m^2}\,\mathrm{i}\Sigma(p^2)\,\frac{\mathrm{i}}{p^2-m^2} + \ldots = \frac{\mathrm{i}}{p^2-m^2+\Sigma(p^2)}$$

$\Sigma(p^2) =$ renormalized self-energy, $m =$ ren. mass

stable particle: $\mathrm{Im}\{\Sigma(p^2)\} = 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole for real value of p^2,
renormalization condition for physical mass m: $\Sigma(m^2) = 0$

unstable particle: $\mathrm{Im}\{\Sigma(p^2)\} \neq 0$ at $p^2 \sim m^2$

$\hookrightarrow$ location μ^2 of propagator pole is complex,
possible definition of mass M and width Γ: $\mu^2 = M^2 - \mathrm{i}M\Gamma$

The complex-mass scheme at NLO

Basic idea: mass^2 = location of propagator pole in complex p^2 plane

$\hookrightarrow$ consistent use of complex masses everywhere !

Application to gauge-boson resonances:

- replace $M_\mathrm{W}^2 \;\rightarrow\; \mu_\mathrm{W}^2 = M_\mathrm{W}^2 - \mathrm{i}M_\mathrm{W}\Gamma_\mathrm{W}, \qquad M_\mathrm{Z}^2 \;\rightarrow\; \mu_\mathrm{Z}^2 = M_\mathrm{Z}^2 - \mathrm{i}M_\mathrm{Z}\Gamma_\mathrm{Z}$

  and define (complex) weak mixing angle via $c_\mathrm{W}^2 \;=\; 1 - s_\mathrm{W}^2 \;=\; \frac{\mu_\mathrm{W}^2}{\mu_\mathrm{Z}^2}$

- virtues:
  - ◇ gauge-invariant result (Slavnov–Taylor identities, gauge-parameter independence)
    $\hookrightarrow$ unitarity cancellations respected !
  - ◇ perturbative calculations as usual (loops and counterterms)
  - ◇ no double counting of contributions (bare Lagrangian unchanged !)

- drawbacks:
  - ◇ unitarity-violating spurious terms of $\mathcal{O}(\alpha^2)$ $\rightarrow$ but beyond NLO accuracy !
    (from t-channel/off-shell propagators and complex mixing angle)
  - ◇ complex gauge-boson masses also in loop integrals

Comparison to other proposals:

- naive fixed-width schemes:

  $\displaystyle\frac{1}{p^2-M^2} \to \frac{1}{p^2-M^2+\mathrm{i}M\Gamma}$ in all or at least in resonant propagators

  ↪ breaks gauge invariance only mildly (?),
  but partial inclusion of widths in loops screws up singularity structure

- pole expansions Stuart ’91; Aeppli et al. ’93, ’94; etc.

  ↪ consistent, gauge invariant,
  but not reliable at threshold or in off-shell tails of resonances

- effective field theory approach Beneke et al. ’04; Hoang, Reisser ’04

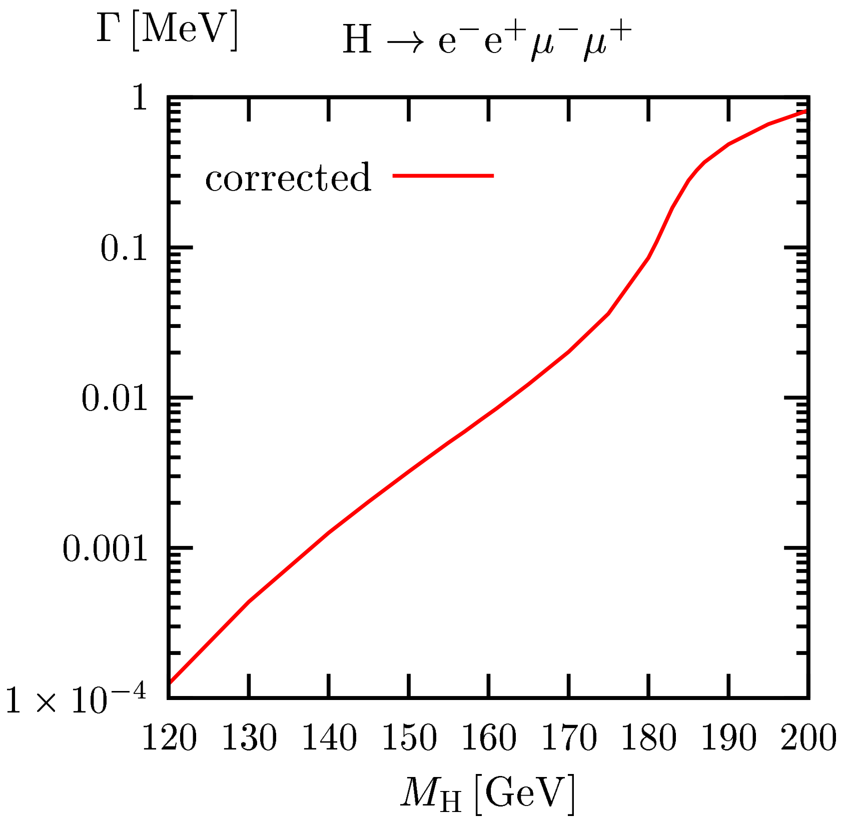
  ↪ gauge invariant, involves pole expansions,
  but can be combined with threshold expansions

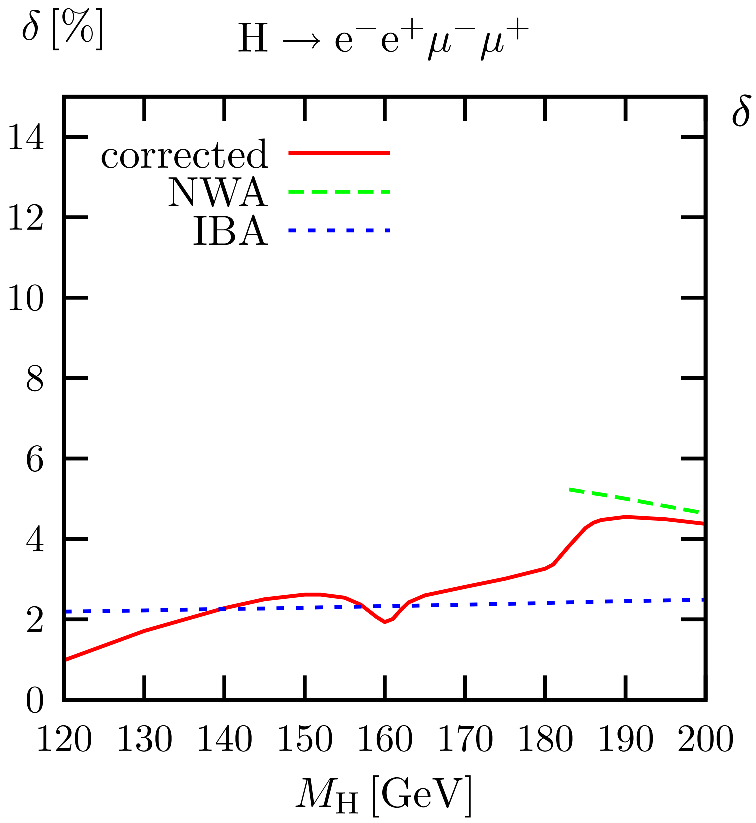
- complex-mass scheme Denner, S.D., Roth, Wackeroth ’99; Denner, S.D., Roth, Wieders ’05

  ↪ gauge invariant, valid everywhere in phase space

Some results for $\mathbf{H} \to \mathbf{ZZ} \to \boldsymbol{4l}$

Partial decay width for $\mathrm{H} \to \mathrm{ZZ} \to \mathrm{e}^-\mathrm{e}^+\mu^-\mu^+$ G_μ-scheme





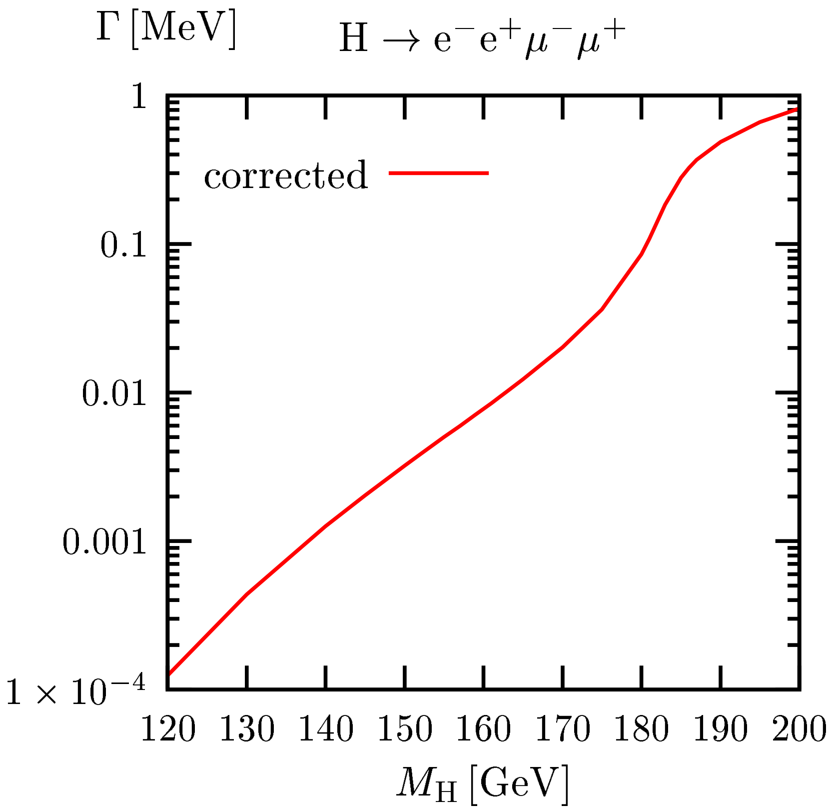
$$\delta = \frac{\Gamma}{\Gamma_\mathrm{Born}} - 1$$

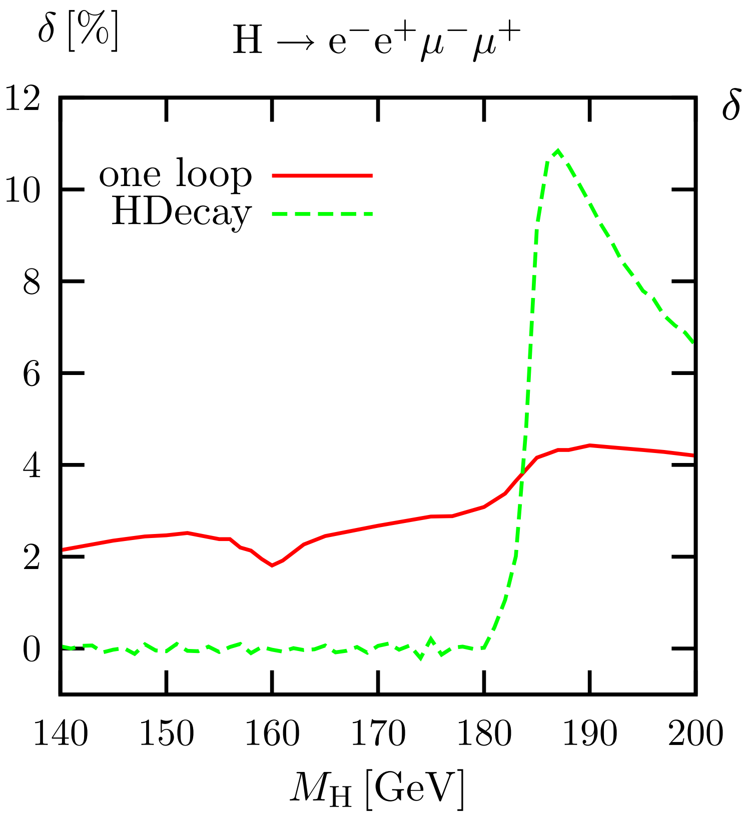
Bredenstein, Denner, S.D., Weber ’06

↑ threshold effect in loops for $M_\mathrm{H} \sim 2M_\mathrm{W}$

↑ kinematic threshold at $M_\mathrm{H} \sim 2M_\mathrm{Z}$

Comparison with HDECAY





$$\delta = \frac{\Gamma}{\Gamma_{\mathrm{Born}}} - 1$$

↑ threshold effect in loops for $M_H \sim 2M_W$

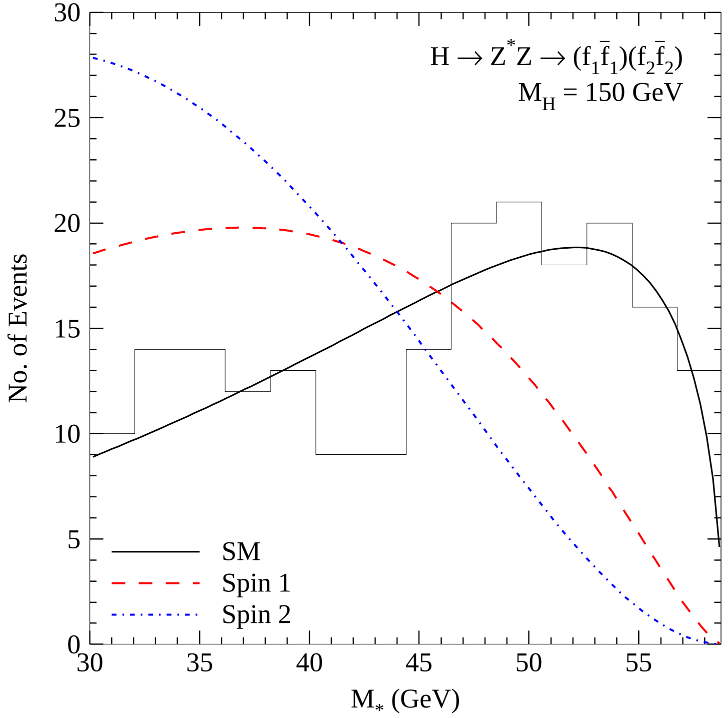
↑ kinematic threshold at $M_H \sim 2M_Z$

Note: peak structure in HDECAY is an artefact of the on-shell approximation above threshold.

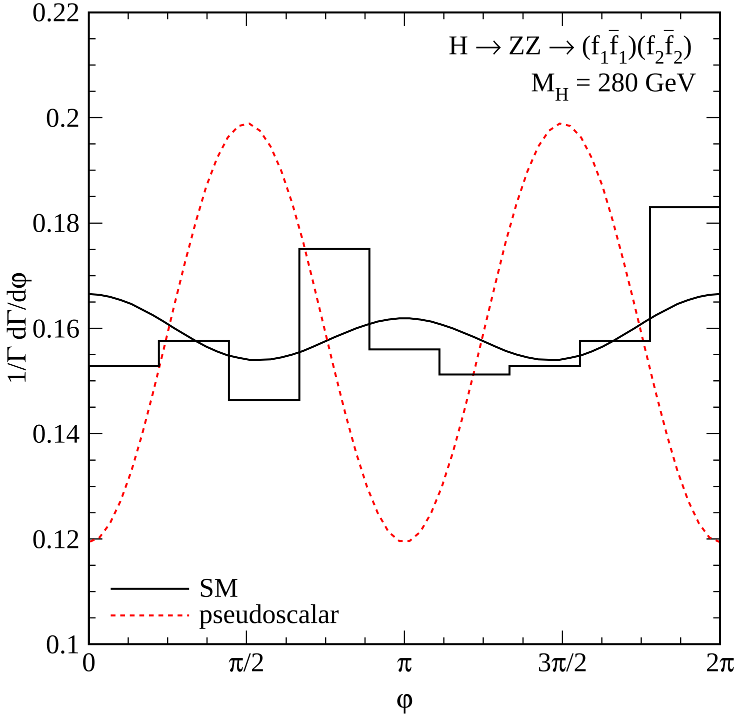
Sensitivity of distributions to non-standard effects in $\mathrm{H} \to \mathrm{ZZ} \to f_1 \bar{f}_1 f_2 \bar{f}_2$

Choi, Miller, Mühlleitner, Zerwas '02

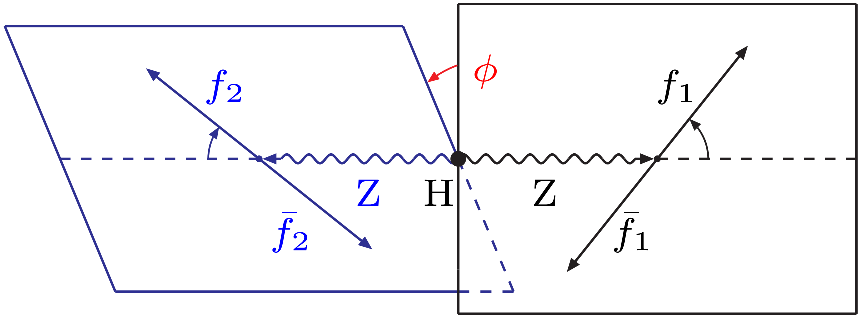
invariant Z mass:



angle between Z decay planes:



$$M_* = M_{f_1 \bar{f}_1}$$



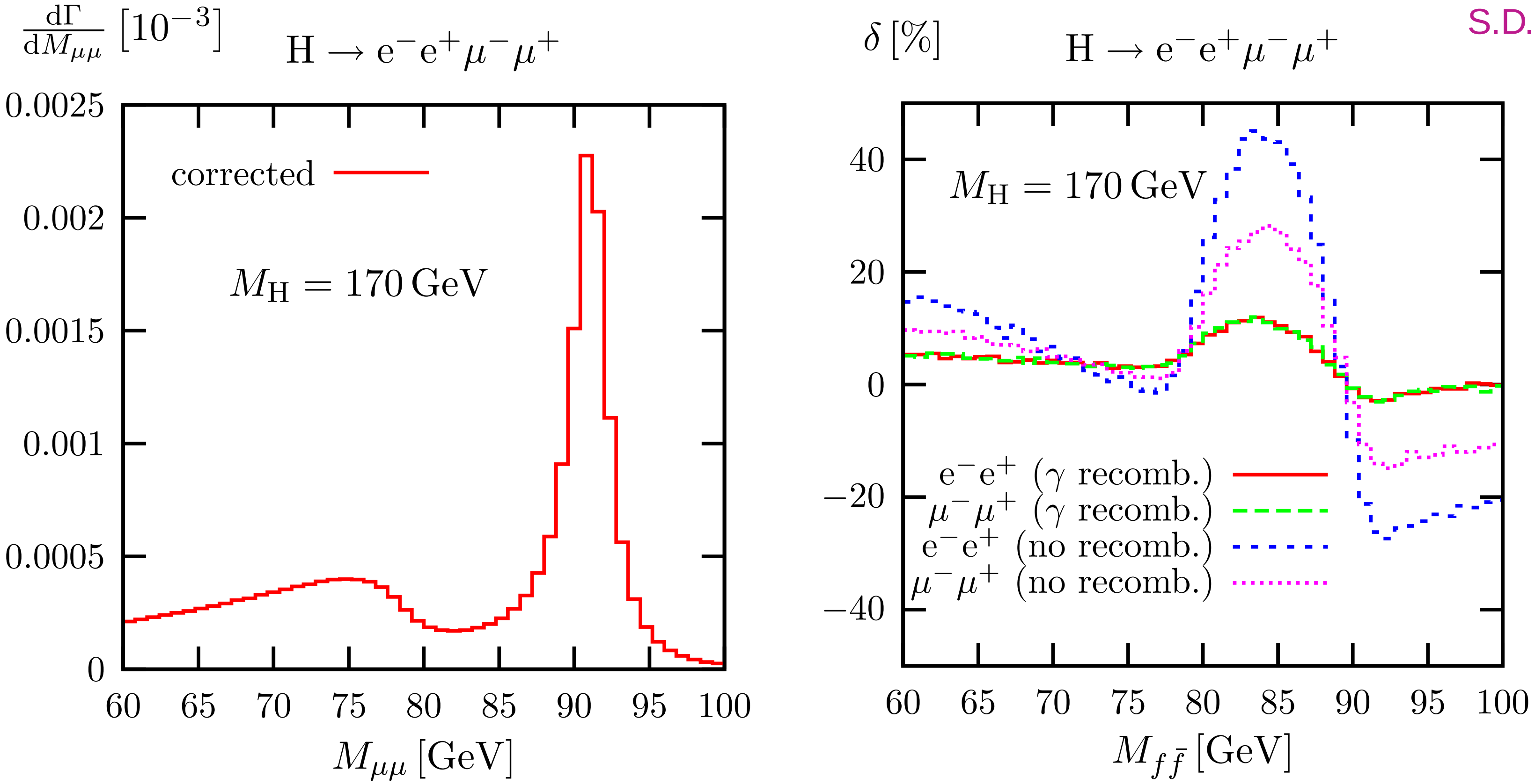
histograms = SM simulation for $L = 300\,\mathrm{fb}^{-1}$

$\hookrightarrow$ distributions sensitive to spin and parity

Electroweak corrections to the invariant Z mass

G_μ-scheme

Bredenstein, Denner, S.D., Weber ’06



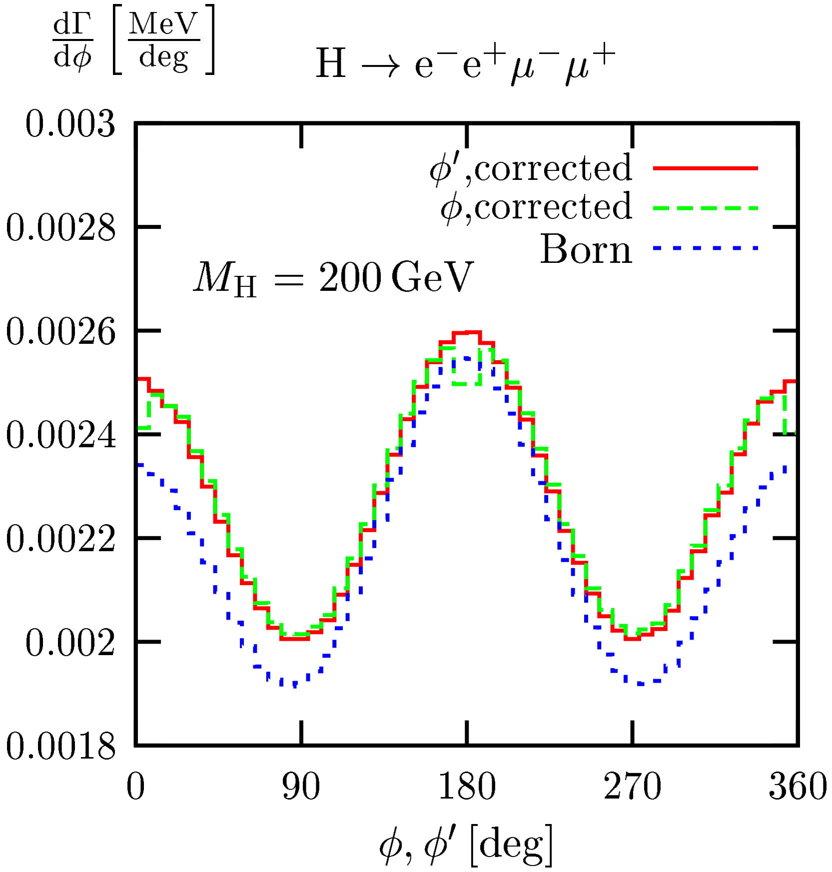
γ recombination if $M_{e\gamma/\mu\gamma} < 5\,\text{GeV}$

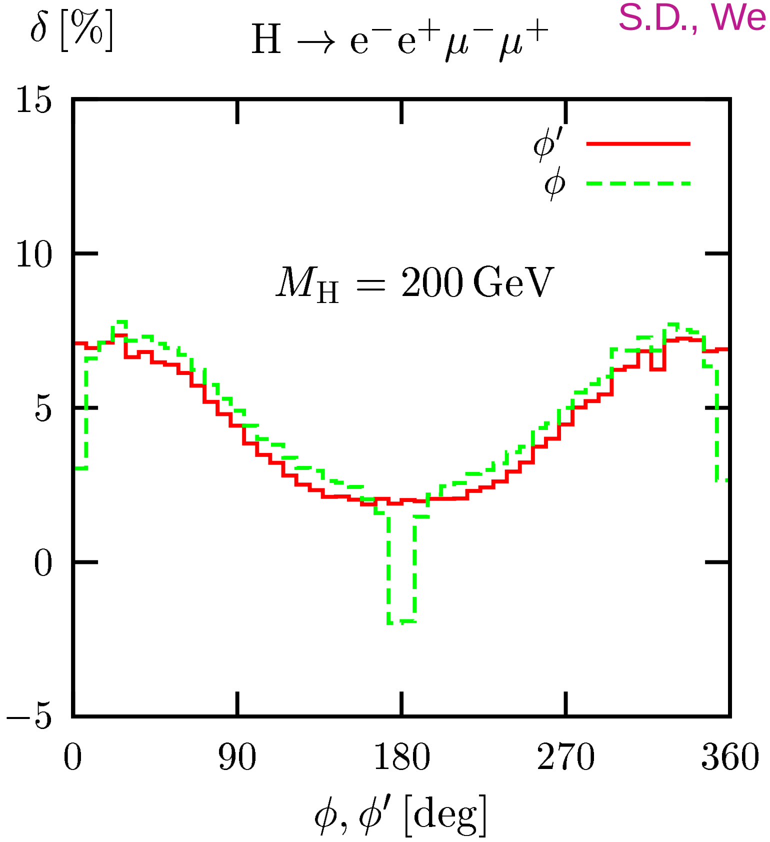
Large corrections from photon radiation in Z reconstruction

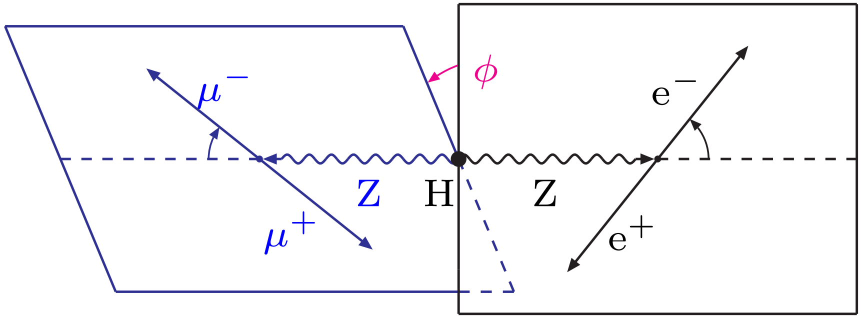
Electroweak corrections to the angle between Z decay planes

G_μ-scheme

Bredenstein, Denner, S.D., Weber ’06







$$\cos\phi = \frac{\left(\mathbf{p}_{e^-e^+}\times\mathbf{p}_{e^-}\right)\left(-\mathbf{p}_{\mu^-\mu^+}\times\mathbf{p}_{\mu^-}\right)}{\left|\mathbf{p}_{e^-e^+})\times\mathbf{p}_{e^-}\right|\left|-\mathbf{p}_{\mu^-\mu^+})\times\mathbf{p}_{\mu^-}\right|}$$

$$\cos\phi' = \frac{\left(\mathbf{p}_{e^-e^+}\times\mathbf{p}_{e^-}\right)\left(\mathbf{p}_{e^-e^+}\times\mathbf{p}_{\mu^-}\right)}{\left|\mathbf{p}_{e^-e^+}\times\mathbf{p}_{e^-}\right|\left|\mathbf{p}_{e^-e^+}\times\mathbf{p}_{\mu^-}\right|}$$

Combination of $gg \to \mathrm{H}$ production with $\mathrm{H} \to \mathrm{WW/ZZ} \to 4l$ decay

QCD corrections to $gg \to \mathrm{H}$:

- complete NLO correction known
  Graudenz, Spira, Zerwas '93
  Djouadi, Graudenz, Spira, Zerwas '95

- NNLO correction known for $m_\mathrm{t} \to \infty$

$$K = \frac{\sigma_\mathrm{NNLO}}{\sigma_\mathrm{LO}} \sim 2.0$$

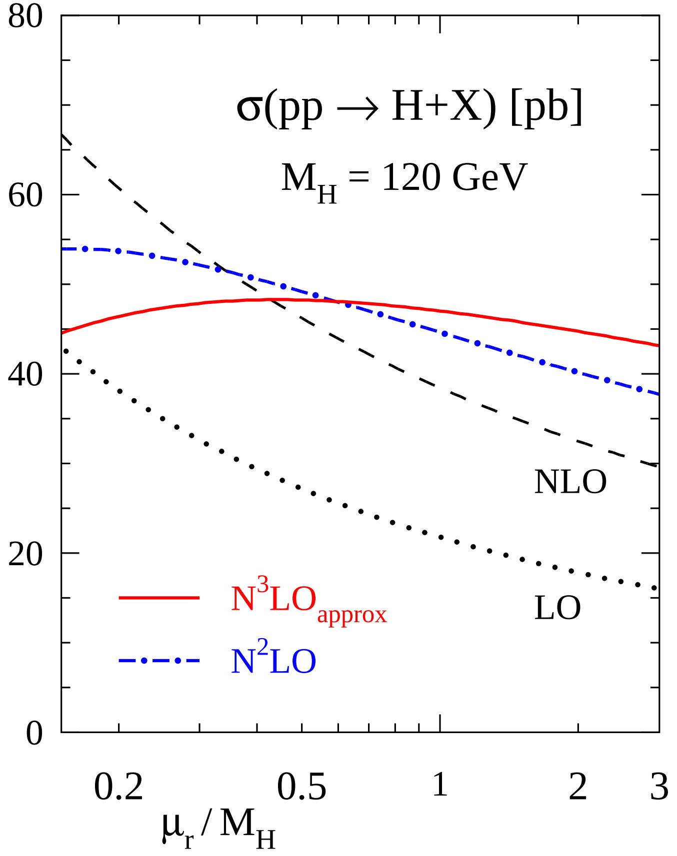
Harlander, Kilgore '01,'02
Catani, de Florian, Grazzini '01
Anastasiou, Melnikov '02
Ravindran, Smith, van Neerven '03,'04
Anastasiou, Melnikov, Petriello '05

- soft-gluon resummation
  up to NNNLO for $m_\mathrm{t} \to \infty$

Catani et al. '03
Moch, Vogt '05

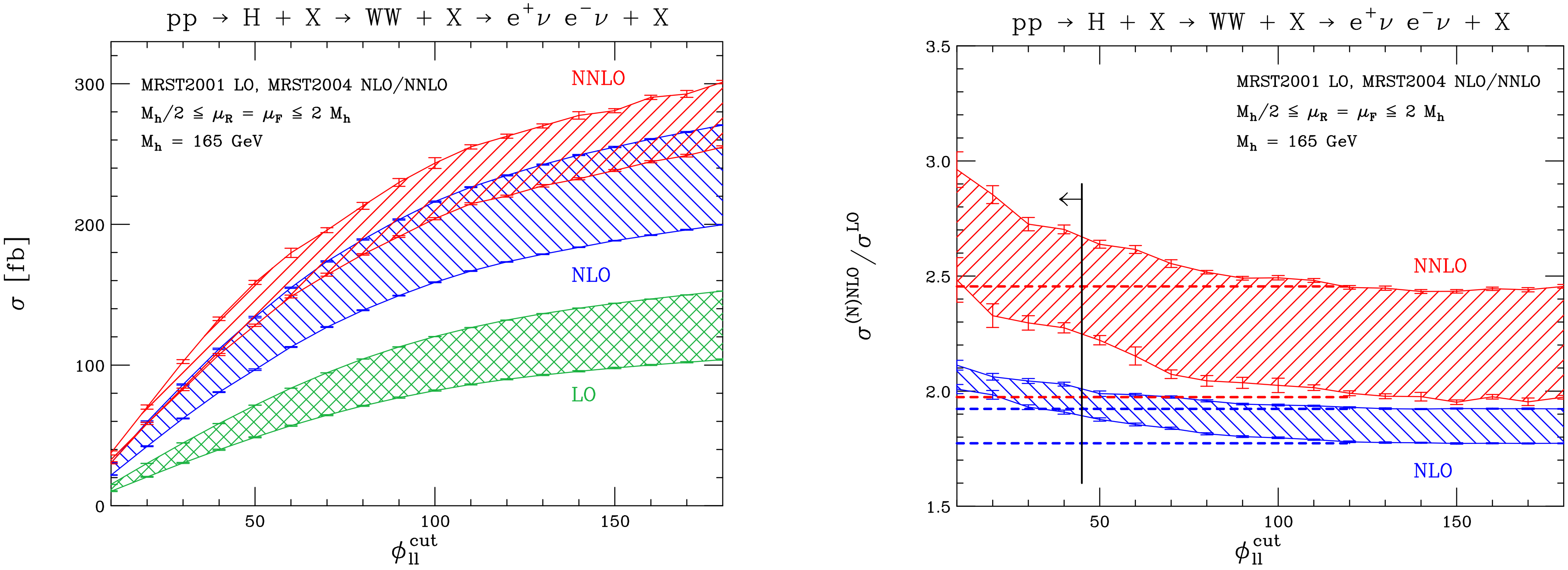
- residual scale uncertainty $\sim\ 5{-}10\%$

Moch, Vogt '05



NNLO QCD corrections to $\mathrm{gg} \to \mathrm{H} \to \mathrm{WW} \to ll\nu\nu$

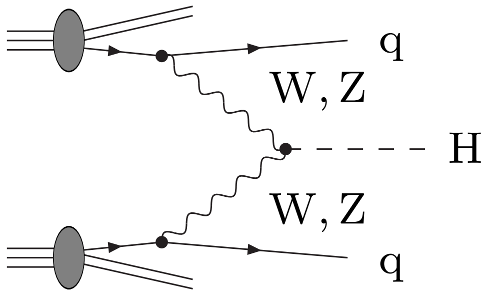
Anastasiou, Dissertori, Stöckli ’07



ϕ_{ll} = angle between charged decay leptons in the transverse plane

K factors in general depend on decay phase space.

3 Higgs production via weak vector-boson fusion (VBF)



colour exchange between quark lines suppressed

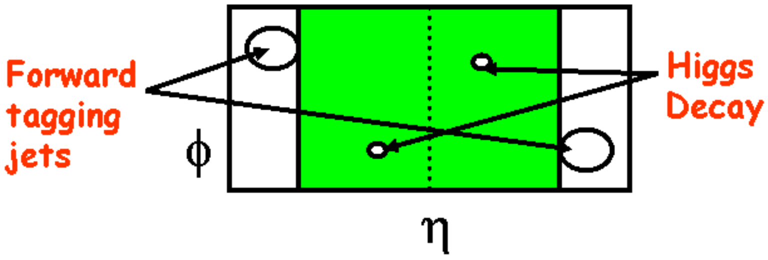
$\Rightarrow$ small QCD corrections

Han, Valencia, Willenbrock '92; Spira '98;
Djouadi, Spira '00; Figy, Oleari, Zeppenfeld '03

$\hookrightarrow$ t-channel approximation (vertex corrections)

VBF cuts and background suppression:

signature $=$ Higgs + 2jets



- 2 hard "tagging" jets demanded:

  $p_{\mathrm{Tj}} > 20\,\mathrm{GeV}, \quad |y_{\mathrm{j}}| < 4.5$

- tagging jets forward–backward directed:

  $\Delta y_{\mathrm{jj}} > 4, \quad y_{\mathrm{j1}} \cdot y_{\mathrm{j2}} < 0.$

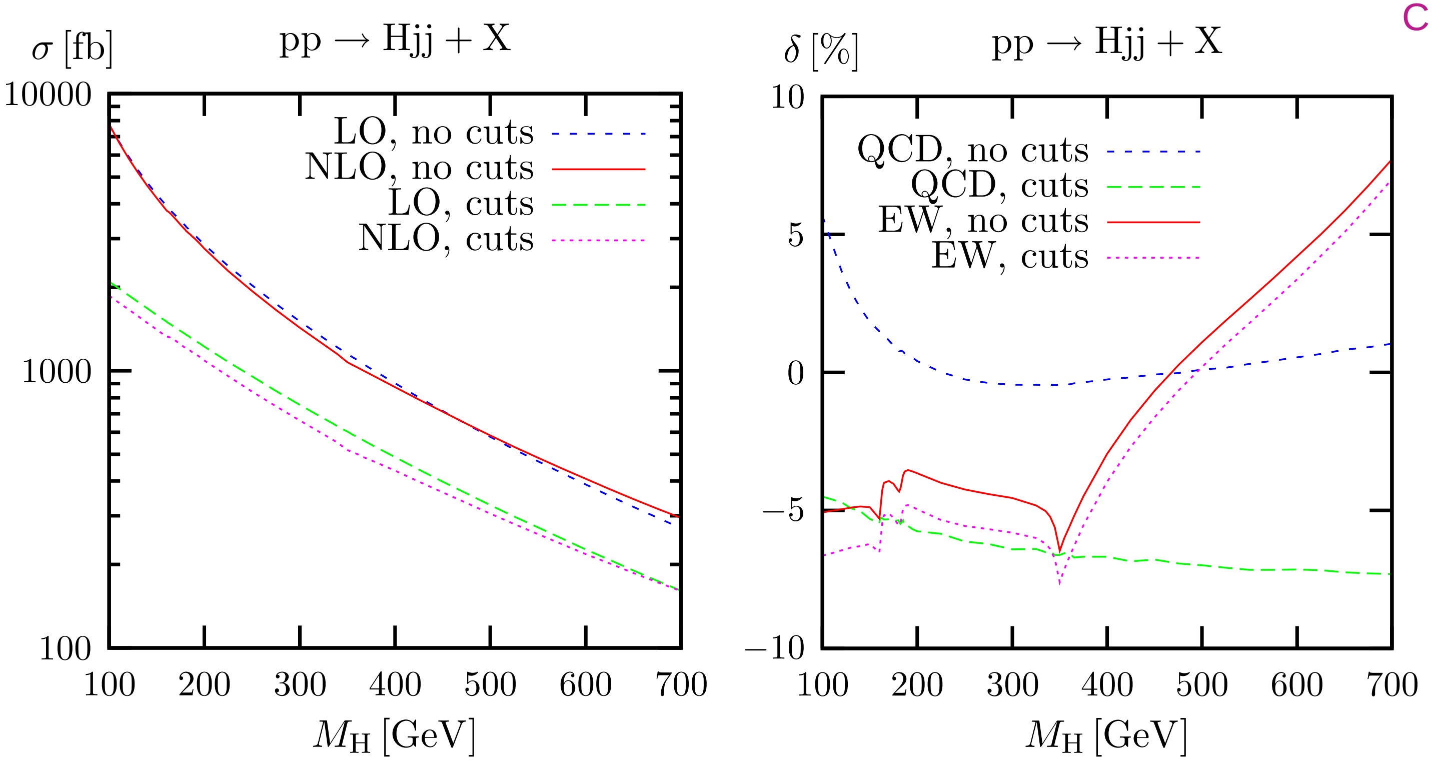
$\hookrightarrow$ Suppression of background

- from other (non-Higgs) processes,
  such as $\mathrm{t\bar{t}}$ or WW production Zeppenfeld et al. '94-'99

- induced by Higgs production via gluon fusion,
  such as $gg \to gg\mathrm{H}$ Del Duca et al. '06; Campbell et al. '06

Recent progress: complete NLO QCD and EW corrections

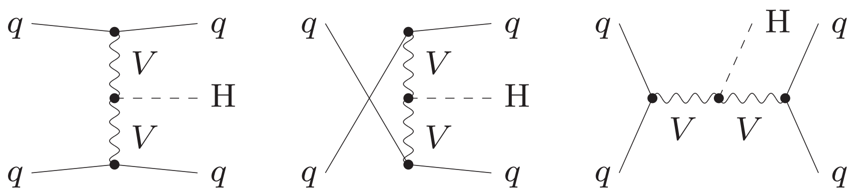
Ciccolini, Denner, S.D. ’07



QCD and EW corrections are of same size !

$(\mu_{\rm ren} = \mu_{\rm fact} = M_{\rm W})$

Features of the calculation:

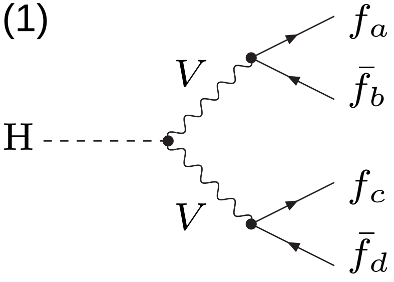


- NLO corrections to all LO diagrams and interferences included:
- leading 2-loop heavy-Higgs effects $\propto G_\mu^2 M_{\rm H}^4$
  Ghinculov ’95; Frink, Kniehl, Kreimer, Riesselmann ’96
- fully flexible Monte Carlo generator

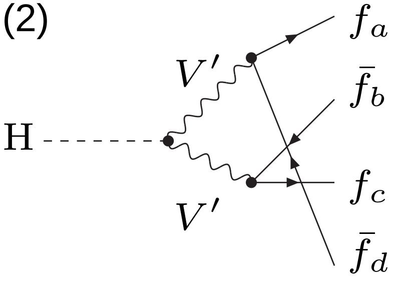
Classification of QCD corrections

Possible Born diagrams:

(1)



(2)



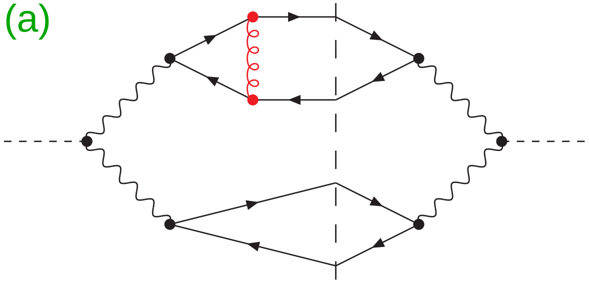
diagrams (2) only for

$q\bar{q}q\bar{q}$ and $q\bar{q}q'\bar{q}'$ channels

(q' = weak-isospin partner of q)

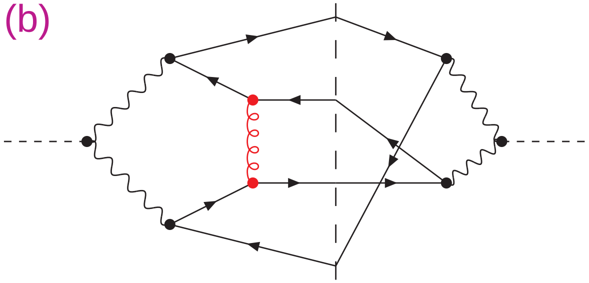
Classification of QCD corrections into four categories: (typical diagrams shown)

(a)

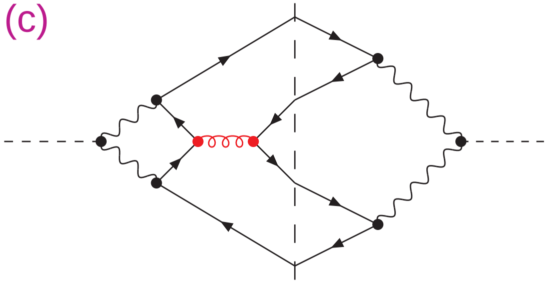


(a) contains previously known “t-channel approximation”

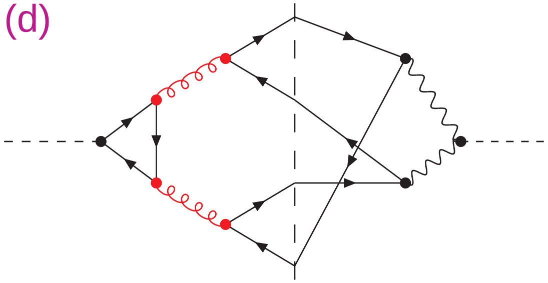
(b)



(c)



(d)



(b,c,d) = corrections to interferences (only for $q\bar{q}q\bar{q}$ and $q\bar{q}q'\bar{q}'$ channels)

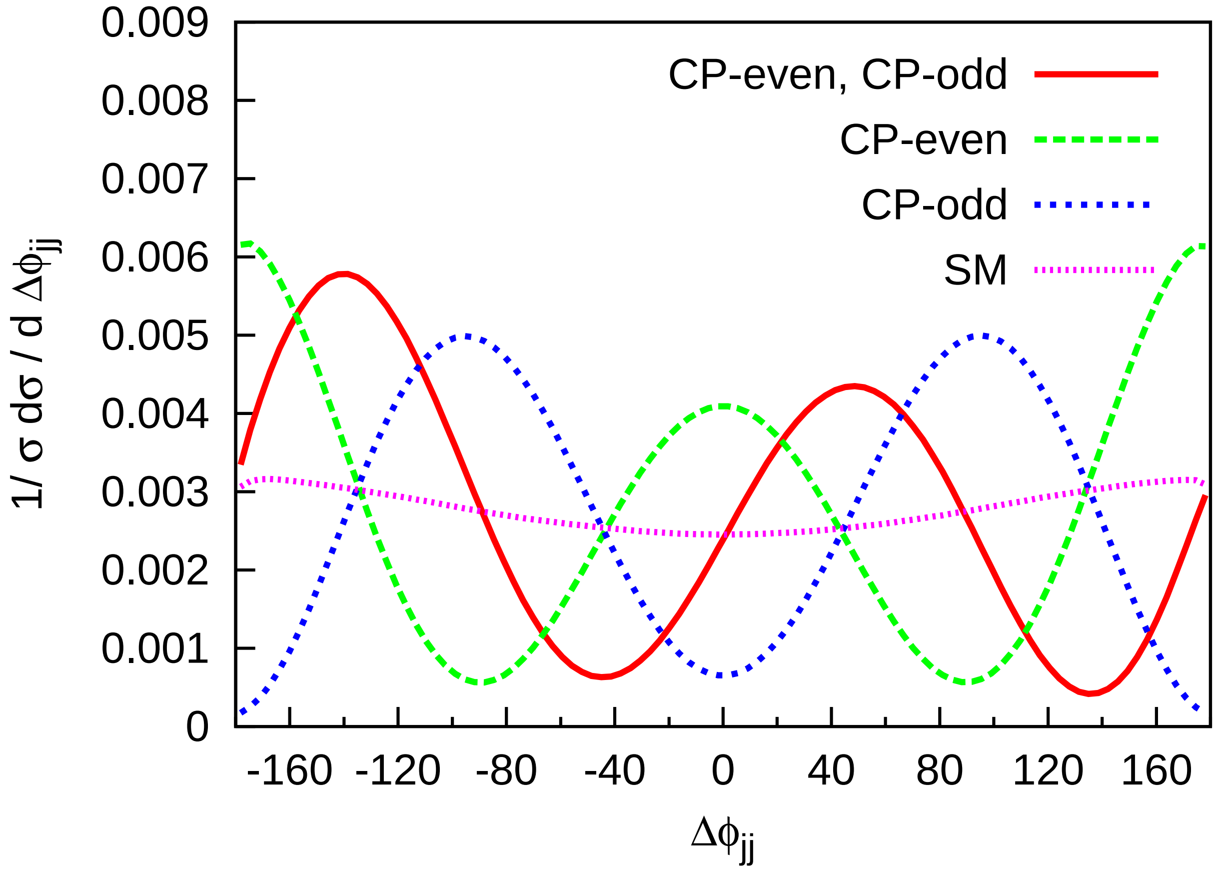
Size of specific corrections and subcontributions to cross sections:

	no cuts		VBF cuts		
$M_{\rm H}\,[\,{\rm GeV}]$	$120{-}200$	700	$120{-}200$	700	
various corrections:					
$\delta_{\rm QCD(a)}\,[\%]$	$4{-}0.5$	$+1$	≈ -5	-7	$\mathcal{O}(5{-}10\%)$
$\delta_{\rm QCD(b+c+d)}\,[\%]$	$\lesssim 0.2$	-0.1	< 0.1	< 0.1	negligible
$\delta_{{\rm EW},qq}\,[\%]$	≈ -6	$+6$	≈ -7	$+5$	$\mathcal{O}(5{-}10\%)$
$\delta_{{\rm EW},q\gamma}\,[\%]$	$\approx +1$	$+2$	$\approx +1$	$+2$	
$\delta_{G_\mu^2 M_{\rm H}^4}\,[\%]$	< 0.1	$+4$	< 0.1	$+4$	negligible for $M_{\rm H} < 400\,{\rm GeV}$
specific contributions:					
$\Delta_{s-{\rm channel}}\,[\%]$	$30{-}10$	1	< 0.6	< 0.1	negligible with VBF cuts
$\Delta_{t/u-{\rm interference}}\,[\%]$	< 0.5	< 0.1	< 0.1	< 0.1	negligible
$\Delta_{\rm b-quarks}\,[\%]$	≈ 4	1	≈ 2	1	

Distribution in the azimuthal angle difference $\Delta\phi_{\mathrm{jj}}$ of the tagging jets

Sensitivity to non-standard effects:

Hankele, Klämke, Zeppenfeld, Figy ’06



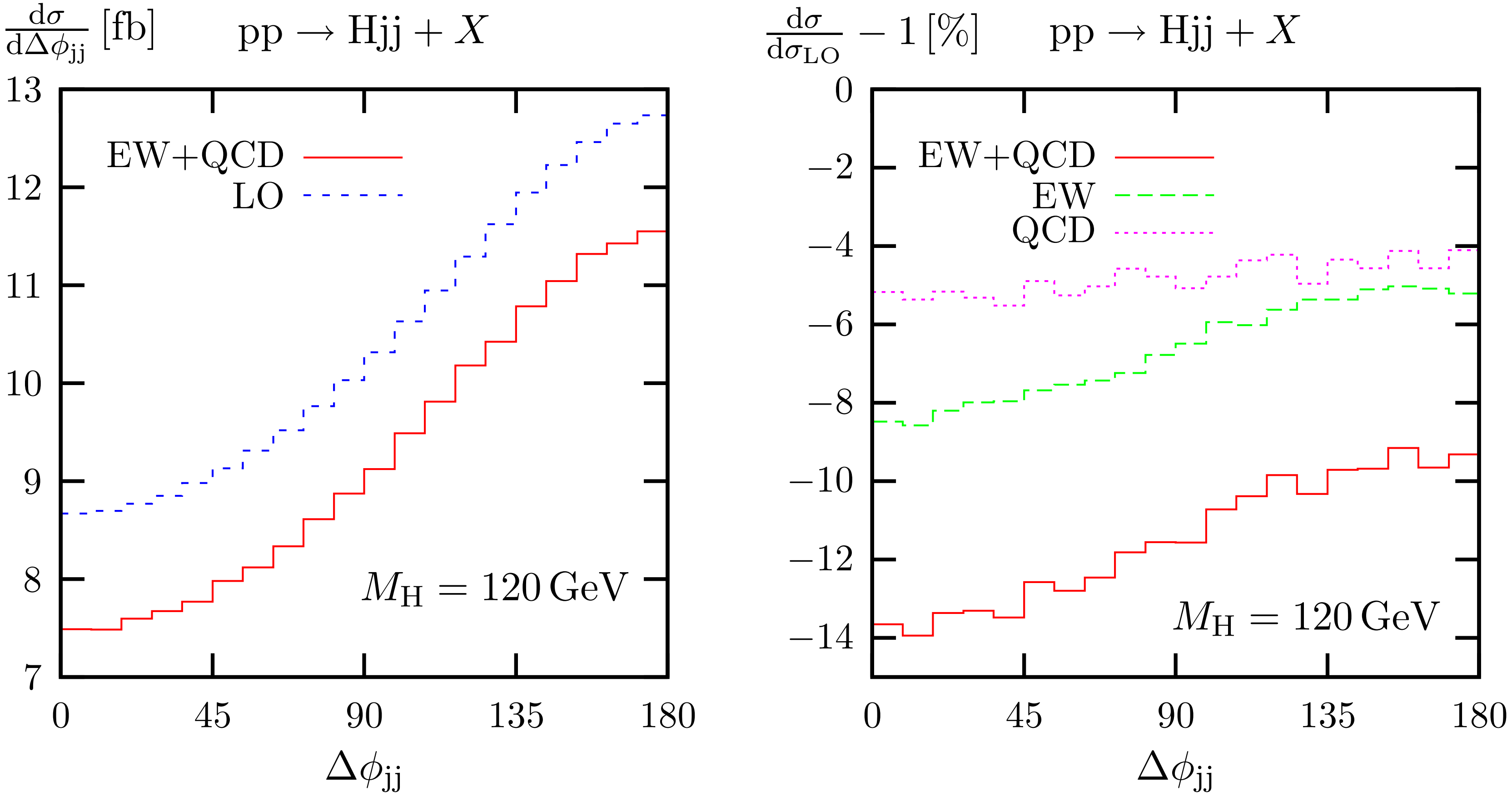
(Individual contributions without SM)

CP-even: $\mathcal{L} \propto HW^{+}_{\mu\nu}W^{-,\mu\nu}, \qquad \Gamma^{HW^+W^-}_{\mu\nu} \propto g_{\mu\nu}(k_+k_-) - k_{+,\nu}k_{-,\mu}$

CP-odd: $\mathcal{L} \propto H\tilde{W}^{+}_{\mu\nu}W^{-,\mu\nu}, \qquad \Gamma^{HW^+W^-}_{\mu\nu} \propto \epsilon_{\mu\nu\rho\sigma}k^{\rho}_{+}k^{\sigma}_{-}$

Corrections to the $\Delta\phi_{jj}$ distribution:

Ciccolini, Denner, S.D. ’07



Corrections induce small distortions (which are larger for p_T and y distributions).

4 Background processes with multi-particle final states

At the LHC the background to some signals cannot be measured !

↪ precise predictions for many background processes required

Examples for important missing NLO predictions for background:

"Les Houches wishlist '05"

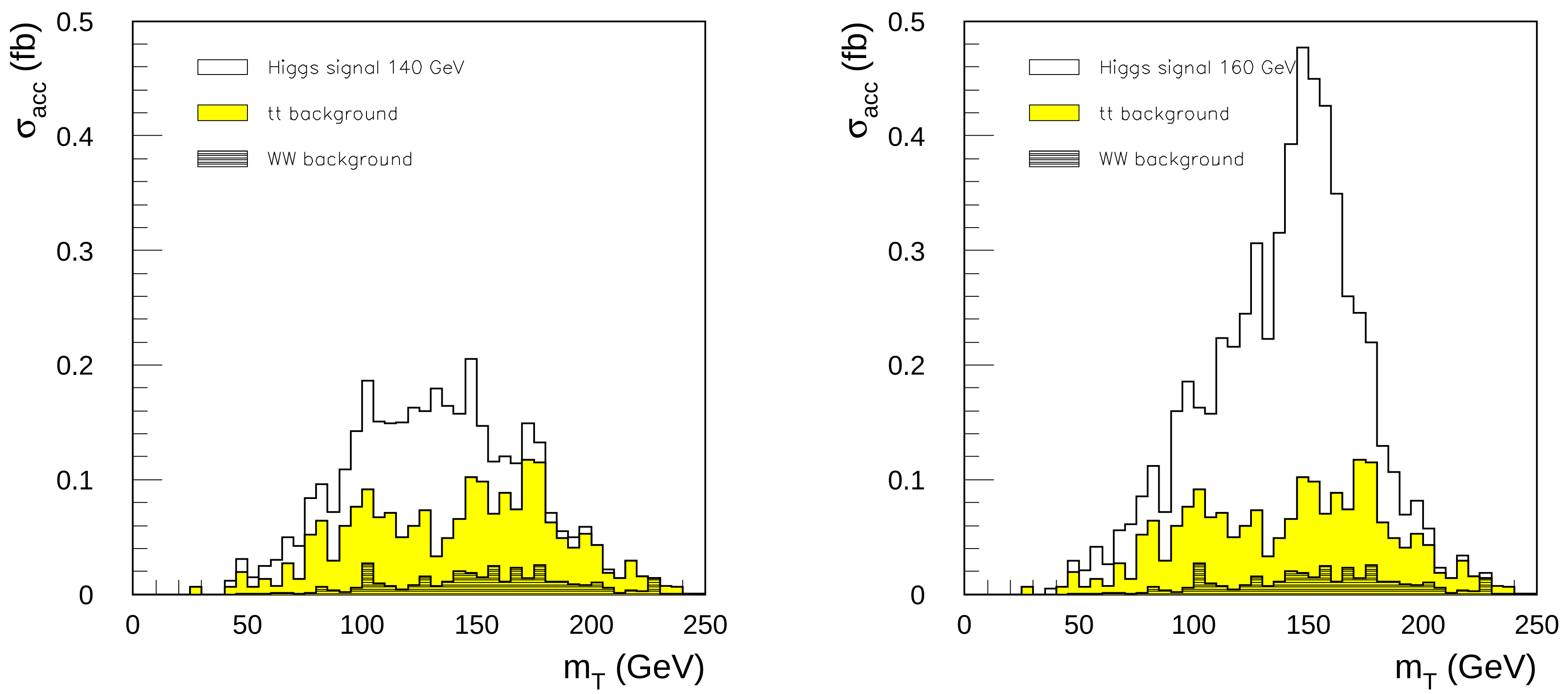
	background for
$pp \to WW + \mathrm{jet}$	$t\bar{t}H$, new physics
S.D., Kallweit, Uwer '07; Campbell, R.K.Ellis, Zanderighi '07	
$pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
$pp \to t\bar{t} + 2\mathrm{jets}$	$t\bar{t}H$
$pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
$pp \to VV + 2\mathrm{jets}$	VBF $\to H \to VV$
VBF: Jäger et al. '06; Bozzi et al. '07	
$pp \to V + 3\mathrm{jets}$	$t\bar{t}$, new physics
$pp \to VVV$	SUSY tri-lepton
ZZZ: Lazopoulos et al. '07	

↪ Many long-termed NLO calculations for theorists !

(several 10^4 diagrams, many "(wo)men-decades")

Note: calculations only possible with technical progress of recent years

An example: simulation of $\mathrm{H} \to \mathrm{WW}$ via VBF at ATLAS



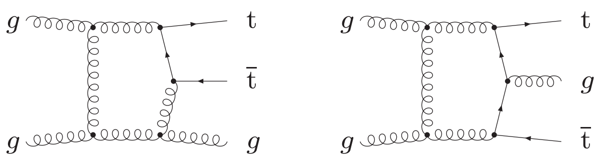
Higgs signal appears as “Jacobian peak” in transverse mass of the W-boson pair.

($\mathrm{t\bar{t}j}$ is major background component.)

NLO QCD corrections to $\mathrm{pp} \to \mathrm{t\bar{t}}$+jet $+ X$ and $\mathrm{pp} \to \mathrm{W^+W^-}$+jet $+ X$

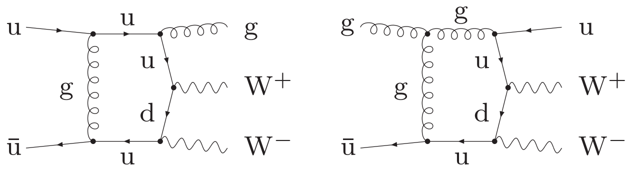
S.D., Uwer, Weinzierl ’07 S.D., Kallweit, Uwer ’07

- $\mathrm{t\bar{t}}$+jet:
  - ◇ understand top-quark dynamics
  - ◇ background to $\mathrm{t\bar{t}H}$ and Higgs via VBF



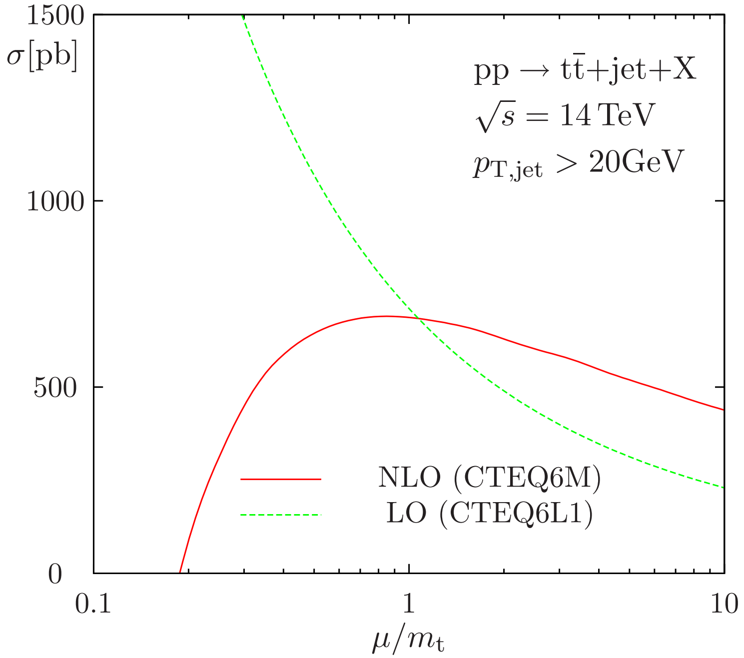
etc.

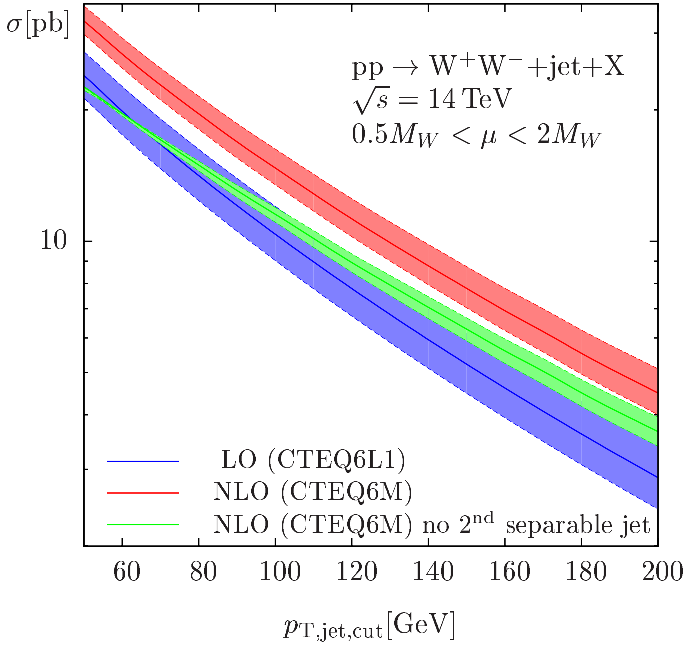
- WW+jet:
  - ◇ background to $\mathrm{H \to WW}$
  - ◇ background to SUSY searches



etc.

Cross sections at the LHC: NLO corrections significantly stabilize predictions





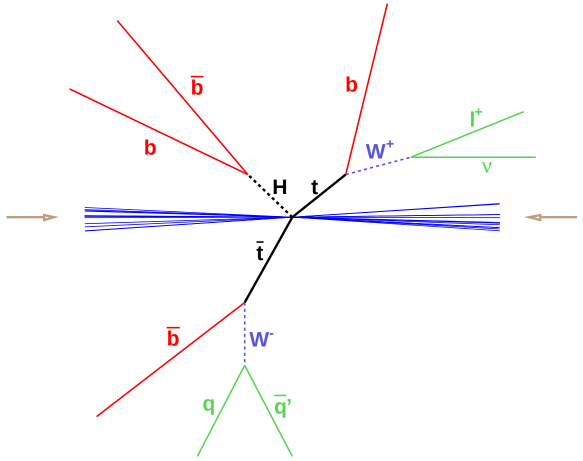
$ttH, H \to b\bar{b}$ (30 fb^{-1})

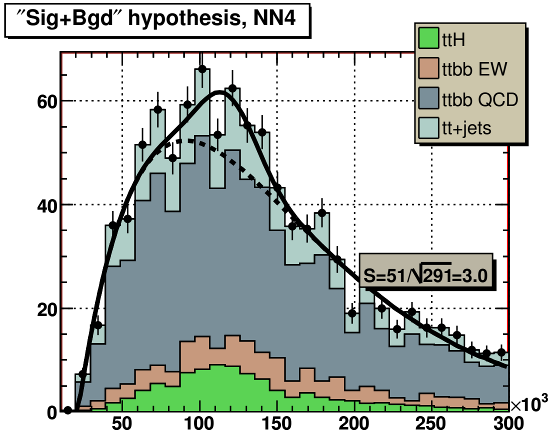
Slide borrowed from S.Horvat (MPI/ATLAS) '07

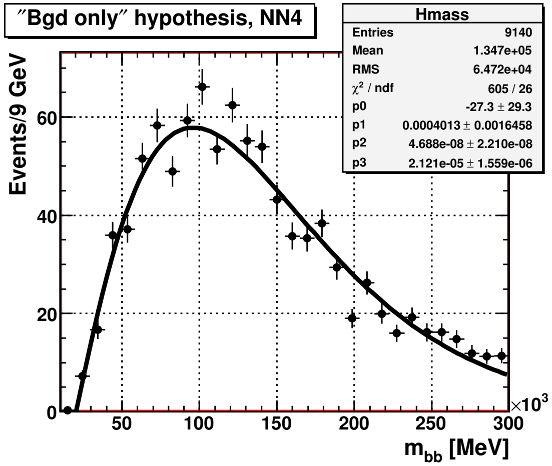
Backgrounds: $t\bar{t}jj$, $t\bar{t}b\bar{b}$ (irreducible).

Many discriminating variables:

jj (W): mass, momentum, $\Delta R(j,j)$ $(\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2})$
bjj (top-quark): mass, $\Delta R(b, jj)$
$b\ell\nu$ (top-quark): mass, $\Delta R(b, \ell\nu)$
$bjj, b\ell\nu$ (tt-pair): mass, $\Delta R(bjj, b\ell\nu)$







Important issues:

- NLO cross-sections for the signal, *ttbb*, *ttjj*.
- Signal and background shape are very similar.

$\Rightarrow$ Essential to reduce the theoretical uncertainties on x-sections.

5 Technical issues in “NLO multi-leg calculations”

Complications in NLO corrections to many-particle processes:

- huge amount of algebra, long final expressions
  - ↪ computer algebra / automation
- multi-dimensional phase-space integration
  - ↪ Monte Carlo techniques
- complicated structure of singularities and matching of virtual and real corrections
  - ↪ subtraction R.K.Ellis et al. ’81; S.D.Ellis et al. ’89; Mangano et al. ’92; Kunszt/Soper ’92; Frixione et al. ’96; Nagy/Z. Trócsányi ’96; Campbell et al. ’98; Catani/Seymour ’96; S.D. ’99; Phaf/Weinzierl ’01; Catani et al. ’02

    and slicing techniques

    Giele/Glover ’92; Giele et al. ’93; Keller/Laenen ’98; Harris/Owens ’01, etc.
- numerically stable evaluation of one-loop integrals with up to 5,6,. . . external legs
  - ↪ techniques to solve problems with inverse kinematical (e.g. Gram) det’s

    Stuart et al. ’88/’90/’97; v.Oldenborgh/Vermaseren ’90; Campbell et al. 96; Ferroglia et al. ’02; del Aguila/Pittau ’04; Binoth et al. ’02/’05; Denner/S.D. ’02/’05; v.Hameren et al. ’05; R.K.Ellis et al. ’05; Anastasiou/Daleo ’05; Ossola et al. ’06/’07; Lazopoulos et al. ’07; Forde ’07; R.K.Ellis et al. ’07; Kilgore ’07; Giele et al. ’08

    [But: many proposed methods not (yet?) used in complicated applications]
- treatment of unstable particles, issue of complex masses

Dipole subtraction formalism

→ process-independent treatment of singularities in real NLO corrections

worked out for

- QCD with massless partons (Catani, Seymour ’96)
- γ radiation off massive fermions (S.D. ’99)

} QCD with massive partons
Phaf, Weinzierl ’01
Catani, S.D., Seymour, Trócsányi ’02

basic idea: NLO correction to process with m partons

$$\sigma^{\mathrm{NLO}} = \underbrace{\int_{m+1}\left[\mathrm{d}\sigma^{\mathrm{real}} - \mathrm{d}\sigma^{\mathrm{sub}}\right]}_{\text{finite}} + \underbrace{\int_{m}\left[\mathrm{d}\sigma^{\mathrm{virtual}} + \mathrm{d}\bar{\sigma}_1^{\mathrm{sub}}\right]}_{\text{finite}} + \int_0^1 \mathrm{d}x \, \underbrace{\int_{m}\left[\mathrm{d}\sigma^{\mathrm{fact}}(x) + \left(\mathrm{d}\bar{\sigma}^{\mathrm{sub}}(x)\right)_+\right]}_{\text{finite}}$$

conditions on $\mathrm{d}\sigma^{\mathrm{sub}}$:

- sum rule: $-\int_{m+1}\mathrm{d}\sigma^{\mathrm{sub}} + \int_{m}\mathrm{d}\bar{\sigma}_1^{\mathrm{sub}} + \int_0^1 \mathrm{d}x \int_{m}\left(\mathrm{d}\bar{\sigma}^{\mathrm{sub}}(x)\right)_+ = 0$
- asymptotics: $\sigma^{\mathrm{sub}} \sim \sigma^{\mathrm{real}}$ in all collinear/IR regions

Strategy for extracting or translating IR (soft / collinear) singularities in loops:

Idea: convert integrals $I^{(D)}$ in $D{=}4{-}2\epsilon$ dim.

$\to$ 4-dim. integrals $I^{(\lambda)}$ with mass regulator λ

Procedure: consider finite and regularization-scheme-independent difference

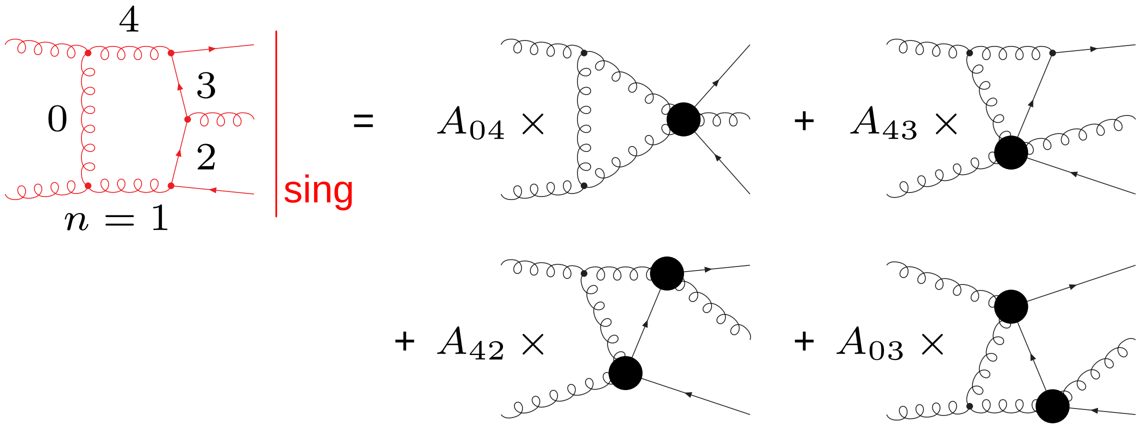
$$\left[I^{(D)} - I^{(D)}_{\rm sing}\right]\Big|_{D\to4} = \left[I^{(\lambda)} - I^{(\lambda)}_{\rm sing}\right]\Big|_{\lambda\to0}$$

$$\Rightarrow\ I^{(D)} = I^{(D)}_{\rm sing} + \left[I^{(\lambda)} - I^{(\lambda)}_{\rm sing}\right]\Big|_{\lambda\to0} + \mathcal{O}(\epsilon)$$

Note: mass-singular part can be universally constructed from 3-point integrals

$\hookrightarrow$ general result known explicitly S.D. ’03 Beenakker et al. ’01

An example from $gg \to t\bar{t}g$:



$\Big\}\ \frac{1}{\epsilon^2},\ \frac{1}{\epsilon}$ $\ \ln^2\lambda,\ \ln\lambda$

$\Big\}\ \frac{1}{\epsilon}$ $\ \ln\lambda$

Numerical evaluation of one-loop integrals

Passarino–Veltman reduction of tensor to scalar integrals

$\hookrightarrow$ inverse Gram determinants of external momenta

$\hookrightarrow$ serious numerical instabilities where $\det(\mathrm{Gram}) \to 0$
(at phase-space boundary but not only !)

Our solutions:

Denner, S.D., Nucl.Phys. B734 (2006) 62 [hep-ph/0509141]

- 1- and 2-point integrals → stable direct calculation
- 3- and 4-point integrals → two hybrid methods
  - (i) Passarino–Veltman $\oplus$ seminumerical method $\oplus$ analytical special cases
  - (ii) Passarino–Veltman $\oplus$ expansions in small Gram and other kin. determinants
- 5- and 6-point integrals
  - $\hookrightarrow$ stable reduction to lower-point integrals without Gram determinants

$\Rightarrow$ Techniques ready for further applications

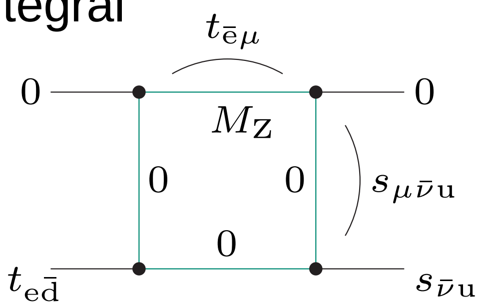
(dim. regularization for IR singularities possible; complex masses supported)

Practical experience

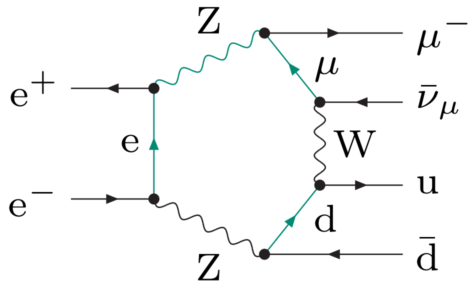
$\hookrightarrow$ Power + reliability of techniques can only be assessed via non-trivial applications !

A typical example with small Gram determinant:

Box integral



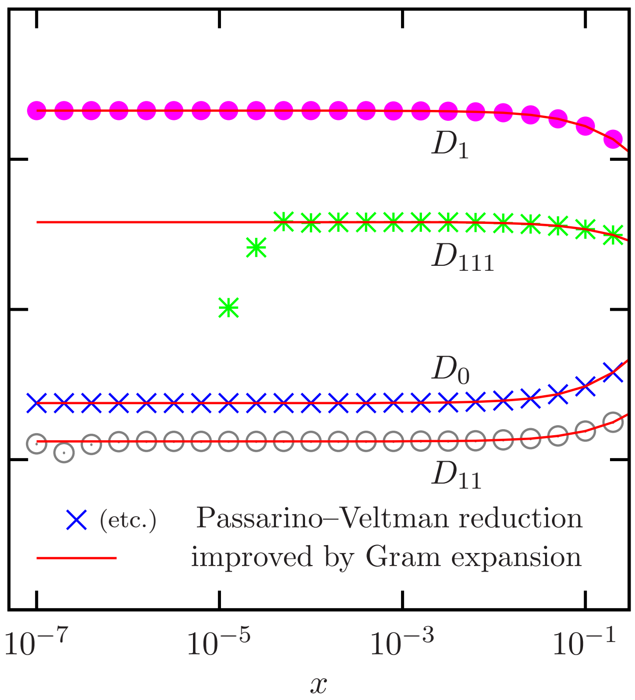
appears, e.g., in subgraph of diagram



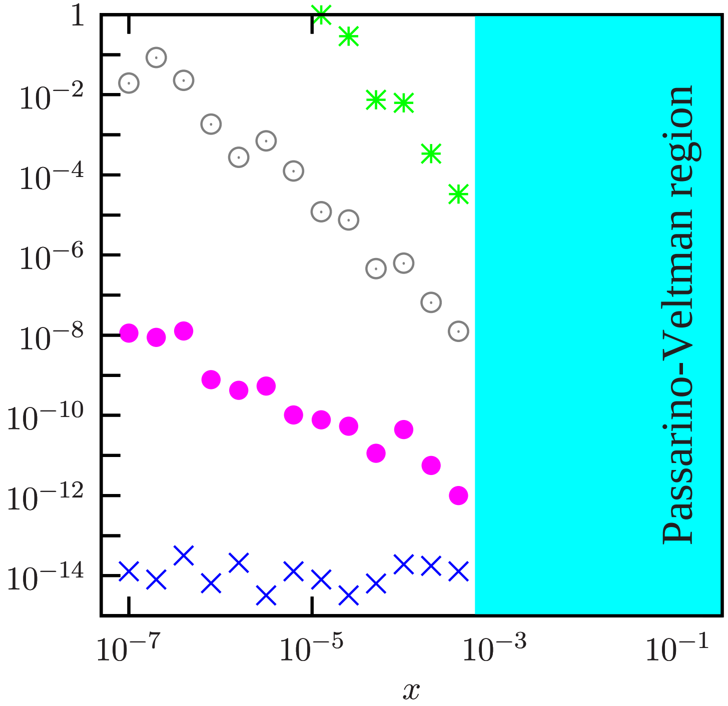
Gram det.: $\det(\mathrm{Gram}) \to 0$ if $t_{e\bar d} \to t_{\mathrm{crit}} \equiv \dfrac{s_{\mu\bar\nu u}(s_{\mu\bar\nu u} - s_{\bar\nu u} + t_{\bar e\mu})}{s_{\mu\bar\nu u} - s_{\bar\nu u}}$

Numerical comparison: maximal tensor rank $= 6$ (similar to $\mathrm{ee} \to 4f$ application)

Absolute predictions



Relative deviations from "best"



$$x \equiv \frac{t_{e\bar d}}{t_{\mathrm{crit}}} - 1$$

$s_{\mu\bar\nu u} = +2\times10^4\ \mathrm{GeV}^2$

$s_{\bar\nu u} = +1\times10^4\ \mathrm{GeV}^2$

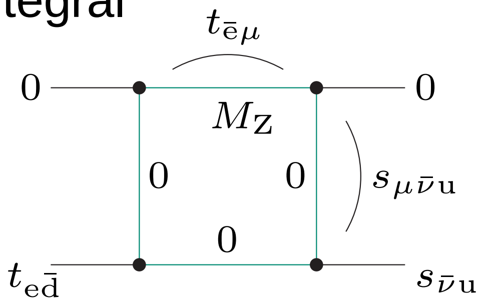
$t_{\bar e\mu} = -4\times10^4\ \mathrm{GeV}^2$

$t_{\mathrm{crit}} = -6\times10^4\ \mathrm{GeV}^2$

PV reduction breaks down, but Gram exp. stable for $\det(\mathrm{Gram}) \to 0$!

A typical example with small Gram determinant:

Box integral



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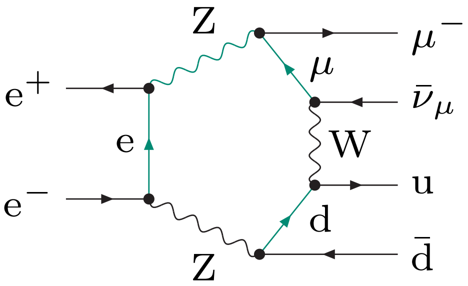

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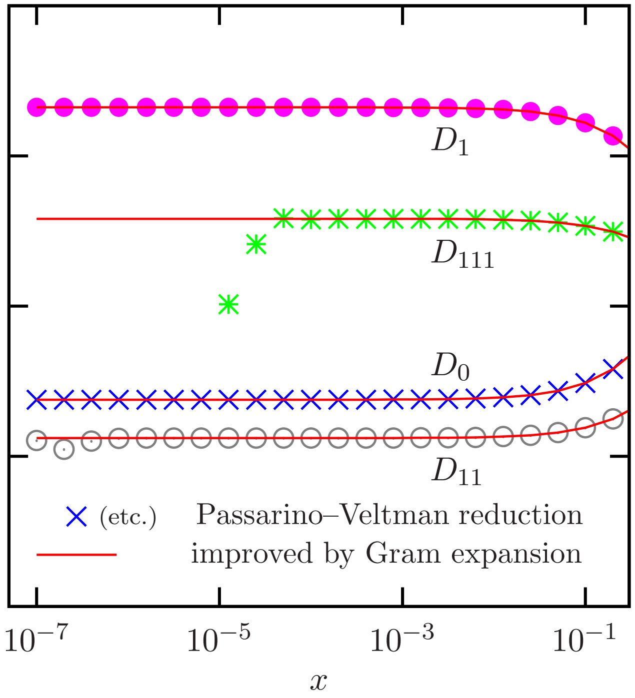
Numerical comparison: maximal tensor rank $= 12$

Absolute predictions



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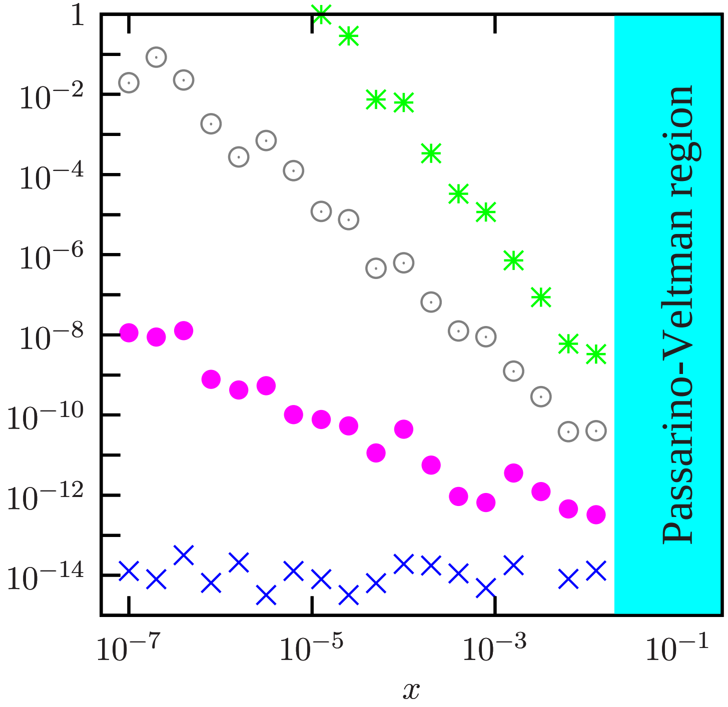

$$x \equiv \frac{t_{e\bar{d}}}{t_{\text{crit}}} - 1$$

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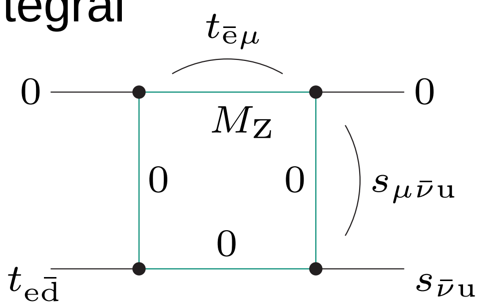

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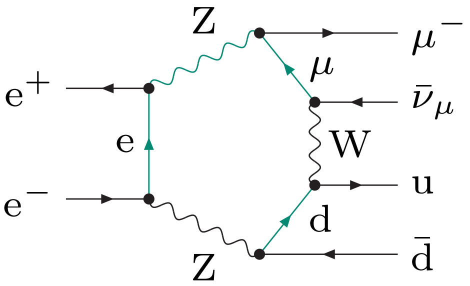

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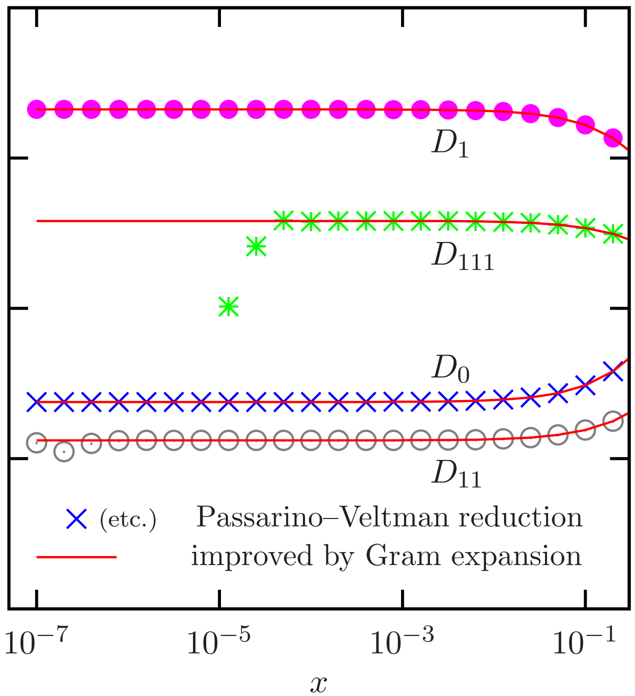
Numerical comparison: maximal tensor rank $= 25$

Absolute predictions



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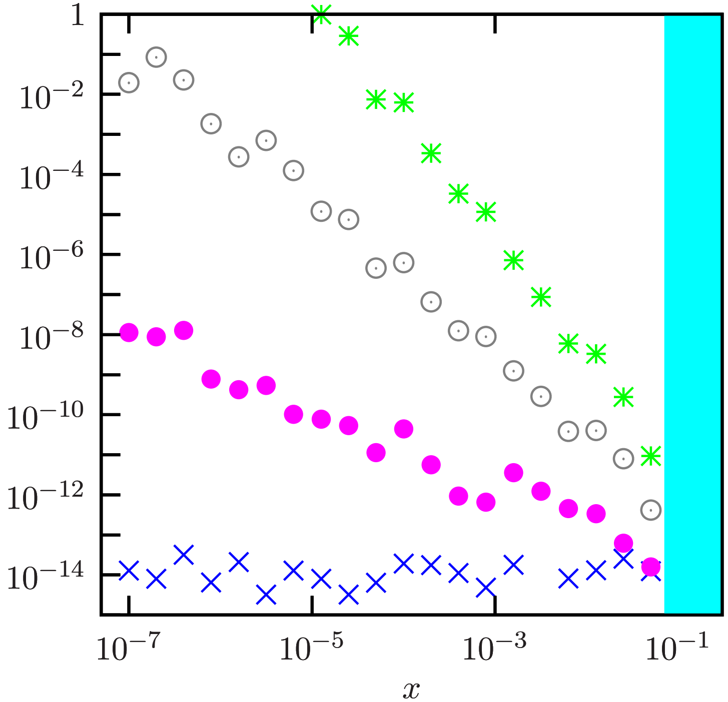

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PV reduction breaks down,
but Gram exp. stable
for $\det(\mathrm{Gram}) \to 0$!

6 Conclusions

Radiative corrections and the search for the Higgs boson

- Bounds on the Higgs mass from LEP2 search and precision physics:
  $114\,\mathrm{GeV} < M_\mathrm{H} \lesssim 200\,\mathrm{GeV}$

- LHC has sensitivity to SM-like Higgs up to $M_\mathrm{H} \lesssim 1\,\mathrm{TeV}$
  QCD corrections = substantial part of predictions
  - ◇ signal processes up to $\mathcal{O}(5–20\%)$ known in SM
    $\hookrightarrow$ continuous refinements (e.g. QCD resummations, EW corrections)
  - ◇ extended Higgs sectors (THDM, MSSM, etc.)
    $\hookrightarrow$ many improvements necessary (e.g. $\mathrm{pp} \to \mathrm{b\bar{b}h/H/A}$)
  - ◇ background processes
    $\hookrightarrow$ hard work at theoretical frontier (e.g. $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$)

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$\Rightarrow$ Theory is on track, but there is still a long way !

Please support young people
who take the challenge !

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Otherwise . . .