Towards reliable predictions for multiparticle processes at the LHC

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- Motivation
- Higgs-boson production in vector-boson fusion
- Production of a top-anti-top and a bottom-anti-bottom pair
- Conclusion

The Large Hadron Collider

In 2008 the Large Hadron Collider (LHC) will go into operation

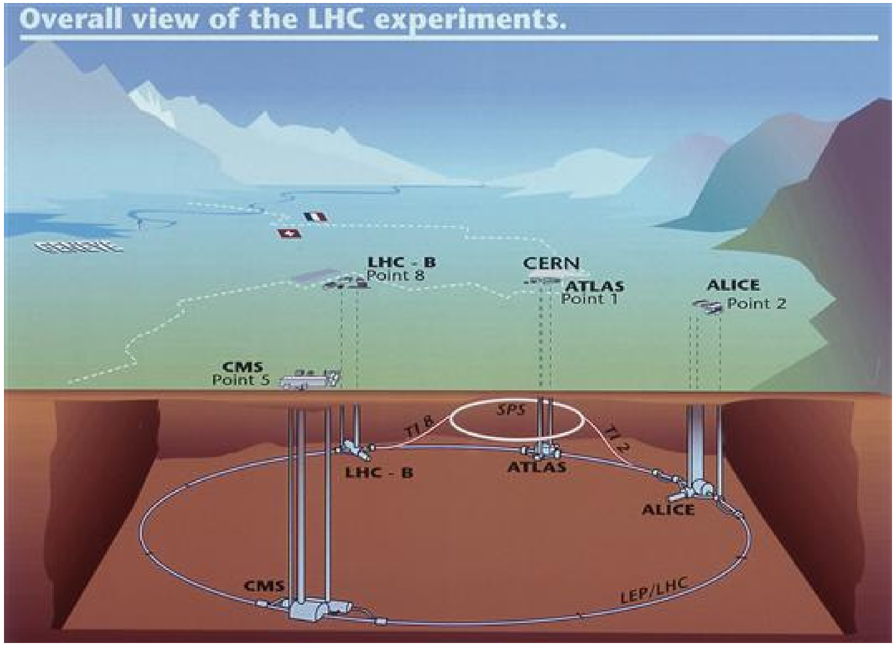
proton–proton collider

energy: $E_{\mathrm{CMS}} = 14\,\mathrm{TeV}$

luminosity: $10^{33} – 10^{34}\,\mathrm{s}^{-1}\mathrm{cm}^{-2}$

integrated lum.: $10 – 100\,\mathrm{fb}^{-1}/\mathrm{y}$

experiments:
ATLAS, CMS, LHC-B, ALICE



tasks for the LHC

- search for the Higgs boson (last missing piece of Standard Model) and study of its properties: mass, width, couplings, spin, parity
- search for physics beyond the Standard Model: supersymmetry, compositeness, extra dimensions, strings, . . .

Importance of NLO calculations for LHC

Importance of multiparticle processes at the LHC

- most interesting signals: heavy particles decaying into jets, leptons, photons $\Rightarrow$ multiparticle final states
- irreducible backgrounds to these signals (often not fully accessible to measurements)

importance of NLO QCD corrections at the LHC

- at LHC systematics better than at Tevatron; very high statistics
- typical size of NLO corrections $\mathcal{O}(10-100\%)$
- need NLO to reduce scale and other higher-order uncertainties (high powers of α_S!),

challenges for NLO programs

- reliable predictions: numerical stability
- sufficient speed

Technical problems and solutions in multiparticle NLO

- number and complexity of loop diagrams grow very fast
  - $\hookrightarrow$ computer algebra / automation

    use alternative ways to calculate amplitude Ossola, Pittau, Papadopoulos '07

    Bern, Dixon, Kosower, Dunbar, Britto, Cachazo, Feng, Forde, R.K.Ellis, Giele, Kunszt, Melnikov, Anastasiou, Mastrolia, . . .
- gauge-invariant treatment of unstable particles, issue of complex masses
  - $\hookrightarrow$ "complex-mass scheme" Denner, Dittmaier, Roth, Wieders, '05
- numerically stable evaluation of one-loop integrals with up to 5,6,. . . external legs (Gram determinants lead to instabilities)
  - $\hookrightarrow$ ◇ new reduction schemes

    ◇ higher numerical precision in critical regions
- complicated structure of singularities and matching of virtual and real corrections
  - $\hookrightarrow$ subtraction and slicing techniques

  Catani, Seymour, Dittmaier, Troscanyi, Phaf, Weinzierl; Giele, Glover, Keller, Laenen, . . .
- multi-dimensional phase-space integration
  - $\hookrightarrow$ Monte Carlo techniques Berends, Kleiss, Pittau, . . .

State of NLO calculations for the LHC

Many active groups/people

- MCFM: Campbell, R.K.Ellis, Zanderighi, Giele
- GOLEM: Binoth, Guffanti, Guillet, Heinrich, Karg, Kauer, Reiter, Sanguinetti
- Blackhat: Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maître
- Lazopoulos, Melnikov, Petriello
- Ossola, Papadopoulos, Pittau
- Bredenstein, Denner, Dittmaier, Kallweit, Pozzorini
- Uwer, Weinzierl, . . .

State of the art: several $2 \to 3$, but very few $2 \to 4$ calculations

- $\mathrm{e^+e^-} \to$ 4f (EW) Denner, Dittmaier, Roth, Wieders '05
- $\mathrm{e^+e^-} \to \mathrm{HH}\nu\bar{\nu}$ (EW)
  GRACE: Boudjema, Fujimoto, Ishikawa, Kaneko, Kurihara, Shimizu, Kato, Yasui '05
- $\gamma\gamma \to \mathrm{t\bar{t}b\bar{b}}$ (QCD) Lei, Wen-Gan, Liang, Ren-You, Yi '07

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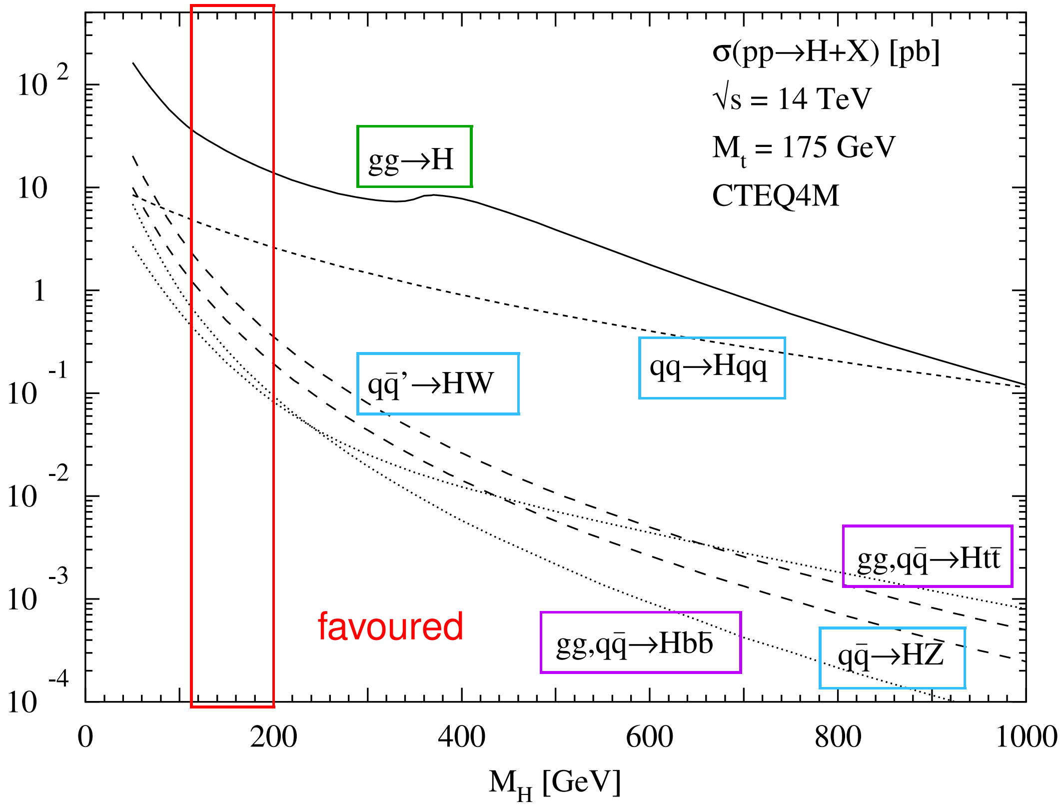
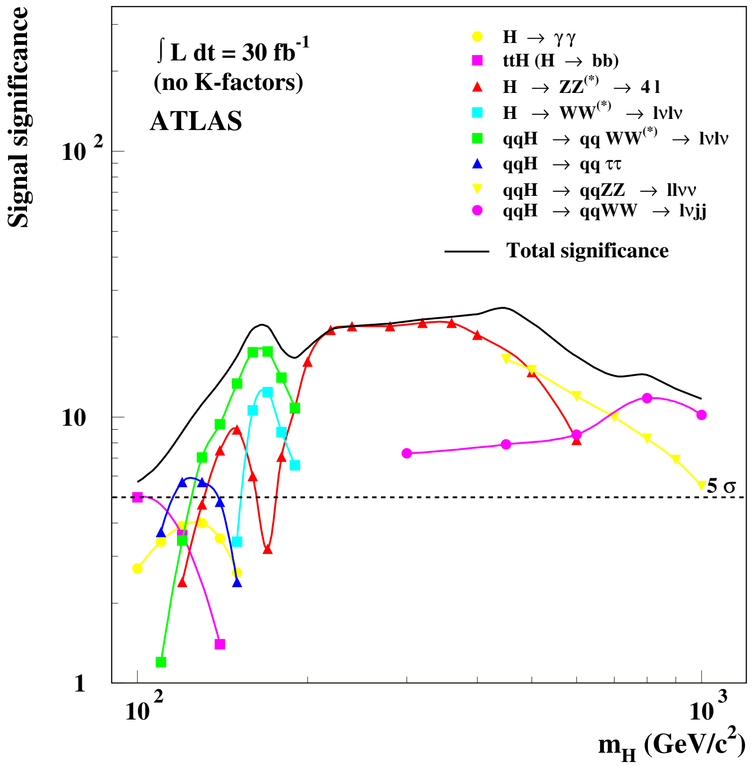
$$\boxed{\mathrm{pp} \to \mathrm{H} + 2\mathrm{jets} + X}$$

Significance of Higgs signal at LHC

Spira et al. '98



ATLAS



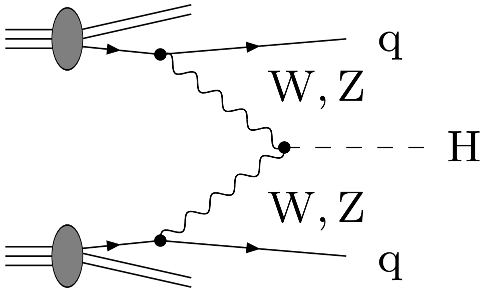
Importance of vector-boson fusion (VBF) $\mathrm{pp} \to \mathrm{H} + 2\mathrm{jets} + X$:

– important Higgs-production process for $100\,\mathrm{GeV} \lesssim M_{\mathrm{H}} \lesssim 200\,\mathrm{GeV}$ and large Higgs boson masses

– measurement of HVV couplings

expected statistical uncertainty for $\sigma \times B$: 5–10% Zeppenfeld et al. '00

Process topology of Higgs production via VBF



process dominated by t- and u-channel diagrams

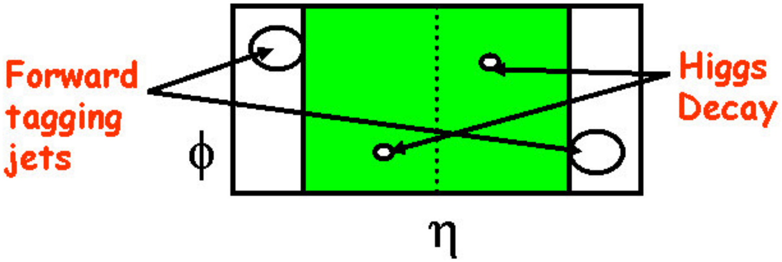
$\Rightarrow$ t-channel approximation

dominant contribution has two forward jets

VBF cuts and background suppression:

- 2 hard “tagging” jets demanded:
  $p_{\mathrm{Tj}} > 20\,\mathrm{GeV}, \quad |y_{\mathrm{j}}| < 4.5$
- tagging jets forward–backward directed:
  $\Delta y_{\mathrm{jj}} > 4, \quad y_{\mathrm{j1}} \cdot y_{\mathrm{j2}} < 0$

signature $=$ Higgs + 2jets

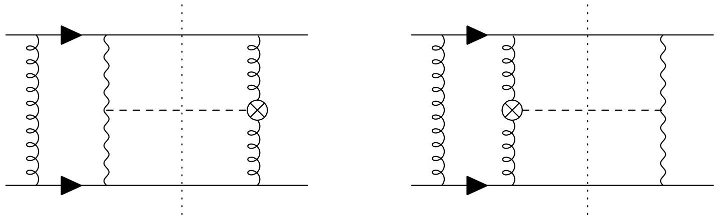


$\hookrightarrow$ suppression of background

- from other (non-Higgs) processes,
  such as $\mathrm{t\bar{t}}$ or WW production   Zeppenfeld et al. ’94–’99
- induced by Higgs production via gluon fusion,
  such as $gg \to gg\mathrm{H}$ Del Duca et al. ’06; Campbell et al. ’06

Results on Higgs+2jets production in NLO

- NLO QCD corrections to VBF in "t-channel approximation" (vertex corrections)
  colour exchange between quark lines suppressed $\Rightarrow$ small QCD corrections
  - total cross section — Han, Valencia, Willenbrock '92; Spira '98; Djouadi, Spira '00
    $\hookrightarrow$ corrections $\sim 5{-}10\%$, residual scale dependence: few per cent
  - realistic cuts distributions — Figy, Oleari, Zeppenfeld '03; Berger, Campbell '04
    $\hookrightarrow$ corrections $\sim 10{-}20\%$, strongly phase-space dependent
- NLO QCD corrections to gluon-initiated channels — Campbell, R.K.Ellis, Zanderighi '06
  (effective $\mathrm{H}gg$ coupling) $\Rightarrow$ contribution to VBF $\sim 5\%$ — Nikitenko, Vazquez '07
- complete NLO QCD+EW corrections to VBF — Ciccolini, Denner, Dittmaier '07
  $\hookrightarrow$ NLO QCD $\sim$ NLO EW $\sim 5{-}10\%$ → discussed in this talk !
- QCD loop-induced interferences between VBF and gluon-initiated channels



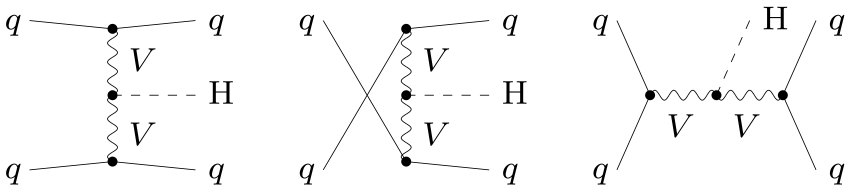
Andersen, Binoth, Heinrich, Smillie '07
Bredenstein, Hagiwara, Jäger '08
→ impact $\lesssim 10^{-3}\,\%$ (negligible!)

- SUSY QCD+EW corrections — Hollik, Plehn, Rauch, Rzehak '08
  $\hookrightarrow$ $|\mathrm{MSSM}-\mathrm{SM}| \lesssim 1\%$ for SPS points ($2{-}4\%$ for low SUSY scales)

Details of the NLO calculation for H + 2jets

EW production of Higgs+2jets in LO

- many subcontributions from qq, $q\bar{q}$, and $\bar{q}\bar{q}$ channels (64 different initial states)
- each channel receives contributions from one or two topologies (“t, u, s”):



all contributions and interferences taken into account in LO and NLO

EW production of Higgs+2jets in NLO

- partonic channels for
  - one-loop diagrams: qq, $q\bar{q}$, $\bar{q}\bar{q}$
  - real QCD corrections qq, $q\bar{q}$, $\bar{q}\bar{q}$ (gluon emission), qg, $\bar{q}g$ (gluon induced)
  - real QED corrections qq, $q\bar{q}$, $\bar{q}\bar{q}$ (photon emission), $q\gamma$, $\bar{q}\gamma$ (photon induced)
- collinear initial-state singularities from QCD and QED splittings

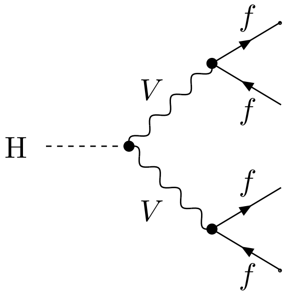
$\hookrightarrow$ factorization and PDF redefinition for QCD and QED singularities

Recycling strategy: obtain all LO and NLO amplitudes via crossing from NLO EW and QCD corrections to $\mathrm{H} \to \mathrm{WW}/\mathrm{ZZ} \to 4f$

Bredenstein, Denner, Dittmaier, Weber ’06

Survey of Feynman diagrams for $\mathrm{H} + 2\mathrm{jets}$

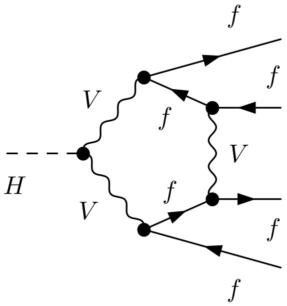
Lowest order:



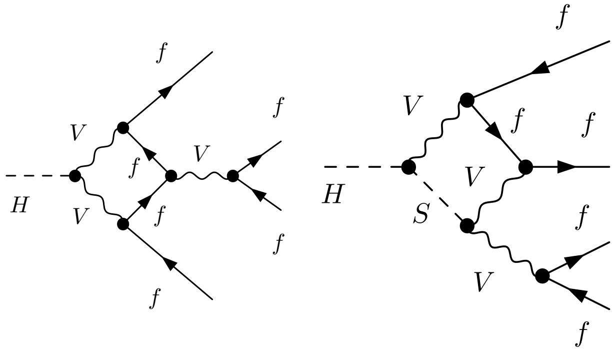
$V = W, Z$

typical one-loop diagrams: # diagrams = $\mathcal{O}(200)$ per tree diagram

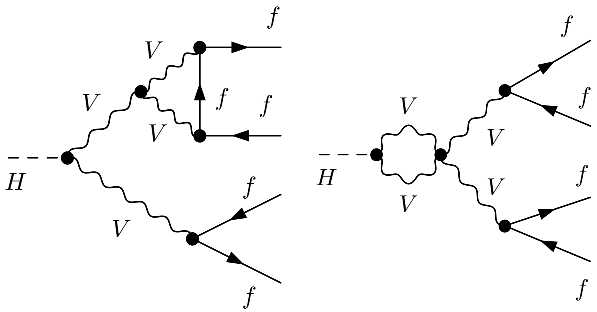
6/8 pentagons



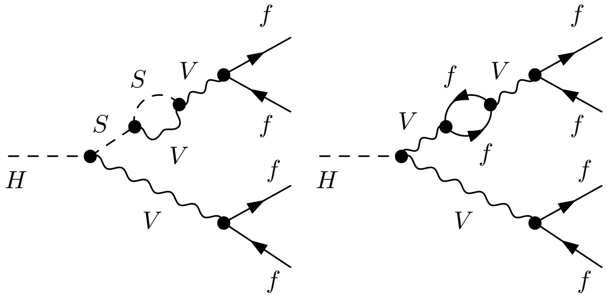
14/24 boxes



vertices



self-energies



$+$ tree graphs with real photons and gluons

Calculation of virtual corrections for $\mathrm{H} + 2\mathrm{jets}$

Tools: crucial methods already developped for $\mathrm{e}^+\mathrm{e}^- \to 4f$
Denner, Dittmaier, Roth, Wieders ’05

- generation of Feynman diagrams with FeynArts version 1 and 3
  Küblbeck, Böhm, Denner, Eck ’90,’92; Hahn ’01
- algebraic simplifications using two independent in-house programs implemented in *Mathematica*, one building upon FORMCALC
  Hahn, Perez-Victoria ’99, Hahn ’00
- reduction of tensor integrals according to
  Denner, Dittmaier, NPB658 (2003)175 [hep-ph/0212259], NPB734 (2006) 62 [hep-ph/0509141]
  $\hookrightarrow$ numerically stable results
- scalar integrals: evaluated with standard techniques and analytic continuation for complex masses

contributions

- complete NLO QCD and electroweak corrections
- leading two-loop corrections $\propto G_\mu^2 M_\mathrm{H}^4$ to VVH vertex in large M_H limit
  Ghinculov ’95; Frink et al. ’96

Algebraic reduction of tensor integrals

For details see Denner, Dittmaier NPB734 (2006) 62 [hep-ph/0509141]

- 1- and 2-point integrals: numerically stable direct calculation
- 3-point and 4-point integrals: Passarino–Veltman reduction

  $\hookrightarrow$ inverse Gram determinants of up to three momenta

  $\hookrightarrow$ serious numerical instabilities where $\det G \to 0$

  (at phase-space boundary, but also within phase space!)

  two hybrid methods

  (i) Passarino–Veltman $\oplus$ expansions in small Gram and other kinematical determinants (see also R.K.Ellis et al. '05)

  (ii) Passarino–Veltman $\oplus$ analytical special cases $\oplus$ semi-numerical method (in this calculation for checks only) (numerical calculation of logarithmic Feynman-parameter integral and algebraic reduction to this basis integral) (see also Binoth et al. '05; Ferroglia et al. '02)
- 5-point integrals $\to$ five 4-point integrals Melrose '65; Denner, Dittmaier '02, '05

  6-point integrals $\to$ six 5-point integrals (see also Binoth et al. '05)

  without inverse Gram determinants and simultaneous reduction of rank by one

Real corrections for $\mathrm{pp} \to \mathrm{H} + 2\mathrm{jets} + X$

Matrix elements

- Weyl-van der Waerden spinor technique Dittmaier '98 ⇒ compact expressions

soft and collinear singularities:

- regularized with infinitesimal photon/gluon and quark masses
- dipole subtraction formalism Catani, Seymour '96; Dittmaier '99; Diener, Dittmaier, Hollik '05

$$\int \mathrm{d}\sigma_{2\to4} = \int \left[\mathrm{d}\sigma_{2\to4} - \sum_{\substack{i,j=1\\ i\neq j}}^{4} \mathrm{d}\sigma_{2\to4}^{\mathrm{dipole},ij} \right] + \sum_{\substack{i,j=1\\ i\neq j}}^{4} \mathcal{F}_{ij} \otimes \mathrm{d}\sigma_{2\to3}$$

numerically stable/efficient, but 12 subtraction terms

- Phase-space slicing Giele,Glover '92; Giele et al. '93; Keller,Laenen '98; Harris,Owens '01

$$\int \mathrm{d}\sigma_{2\to4} = \int_{\substack{E>\delta_{\mathrm{s}}\sqrt{\hat{s}}/2\\ \cos\theta<1-\delta_{\mathrm{c}}}} \mathrm{d}\sigma_{2\to4} + F(\delta_{\mathrm{s}},\delta_{\mathrm{c}}) \otimes \mathrm{d}\sigma_{2\to3}$$

numerical cancellations/CPU-consuming, but “simple” and important check

phase-space integration

- multi-channel Monte Carlo integration with adaptive optimization
  Berends, Kleiss, Pittau '94; Kleiss, Pittau '94

  ~ 250 channels to map peaks from all propagators and dipoles in all partonic channels

Checks on the calculation for pp $\longrightarrow$ H + 2jets + X

- UV structure of virtual corrections
  $\hookrightarrow$ independence of reference mass μ of dimensional regularization
- IR structure of virtual + soft-gluon/photon corrections
  $\hookrightarrow$ independence of $\ln m_\gamma$ (m_γ = infinitesimal photon mass)
- mass singularities of virtual + collinear gluon/photon corrections
  $\hookrightarrow$ independence of $\ln m_{f_i}$ (m_{f_i} = small masses of external fermions)
- gauge invariance of amplitudes with $\Gamma_{\mathrm{W}}, \Gamma_{\mathrm{Z}} \neq 0$
  $\hookrightarrow$ identical results in 't Hooft–Feynman and background-field gauge
  Denner, Dittmaier, Weiglein '94
- real corrections
  $\hookrightarrow$ squared amplitudes compared with MADGRAPH Stelzer, Long '94
- combination of virtual and real corrections
  $\hookrightarrow$ identical results with two-cutoff slicing and dipole subtraction
- two completely independent calculations of all ingredients!

Set-up for numerical calculation of $\mathrm{H}+2\mathrm{jets}$

Definition of observables

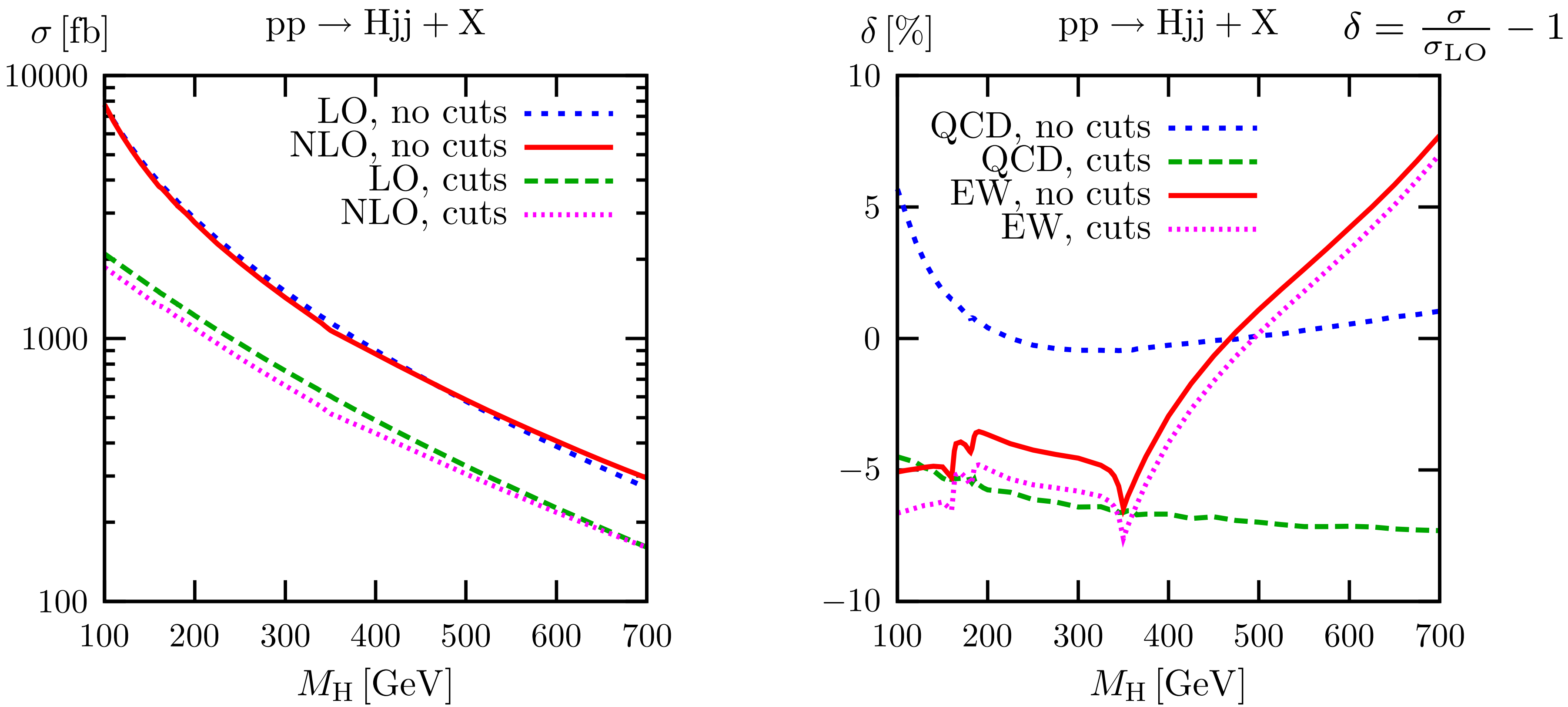
- Jet definition: k_T algorithm as used at Tevatron run II Blazey et al. '00
  $\hookrightarrow$ clusters partons with $|\eta| < 5$ into jets with jet resolution $D = 0.8$
  photons included in clustering
- VBF cuts: following Figy, Zeppenfeld '04
  - 2 hard “tagging” jets demanded: $p_{\mathrm{Tj_1}} > p_{\mathrm{Tj_2}} > 20\,\mathrm{GeV}, \quad |y_{\mathrm{j}1,2}| < 4.5$
  - tagging jets forward–backward directed: $|y_{\mathrm{j_1}} - y_{\mathrm{j_2}}| > 4, \quad y_{\mathrm{j_1}} \cdot y_{\mathrm{j_2}} < 0$
  - no cuts on Higgs momentum (should be adjusted to specific decays)

NLO settings:

- central scales: $\mu_\mathrm{R} = \mu_\mathrm{F} = M_\mathrm{W}$
- PDFs: MRST2004QED which includes QED corrections and γ PDF
  with $\alpha_\mathrm{S}(M_\mathrm{Z}) = 0.1187$, b-quark contributions neglected
- $\alpha_\mathrm{s}(\mu_\mathrm{R})$ with 5 active flavours (top-quark decoupled) and two-loop running
- α defined in G_μ scheme: $\alpha_{G_\mu} = \sqrt{2}G_\mu M_\mathrm{W}^2(1 - M_\mathrm{W}^2/M_\mathrm{Z}^2)/\pi$
  $\hookrightarrow$ absorbs running of α from $Q = 0$ to EW scale and $\Delta\rho$ in $Wq\bar{q}'$ coupling

Total cross section for $\mathrm{pp} \to \mathrm{H} + 2\mathrm{jets} + X$

Ciccolini, Denner, Dittmaier ’07



- QCD and EW corrections are of same generic size ($\sim 5\%$)
- sensitivity to cuts: large for QCD, small for EW corrections
- heavy-Higgs corrections at $M_\mathrm{H} \sim 700\,\mathrm{GeV}$: $\underbrace{G_\mu M_\mathrm{H}^2}_{\text{1-loop}} \sim \underbrace{(G_\mu M_\mathrm{H}^2)^2}_{\text{2-loop}} \sim 4\%$

  $\hookrightarrow$ breakdown of perturbation theory
- scale uncertainty $\sim$ 2–3% within $M_\mathrm{W}/2 < \mu_{\mathrm{R/F}} < 2M_\mathrm{W}$ in NLO ($\sim 10\%$ in LO)

Corrections and subcontributions to cross sections

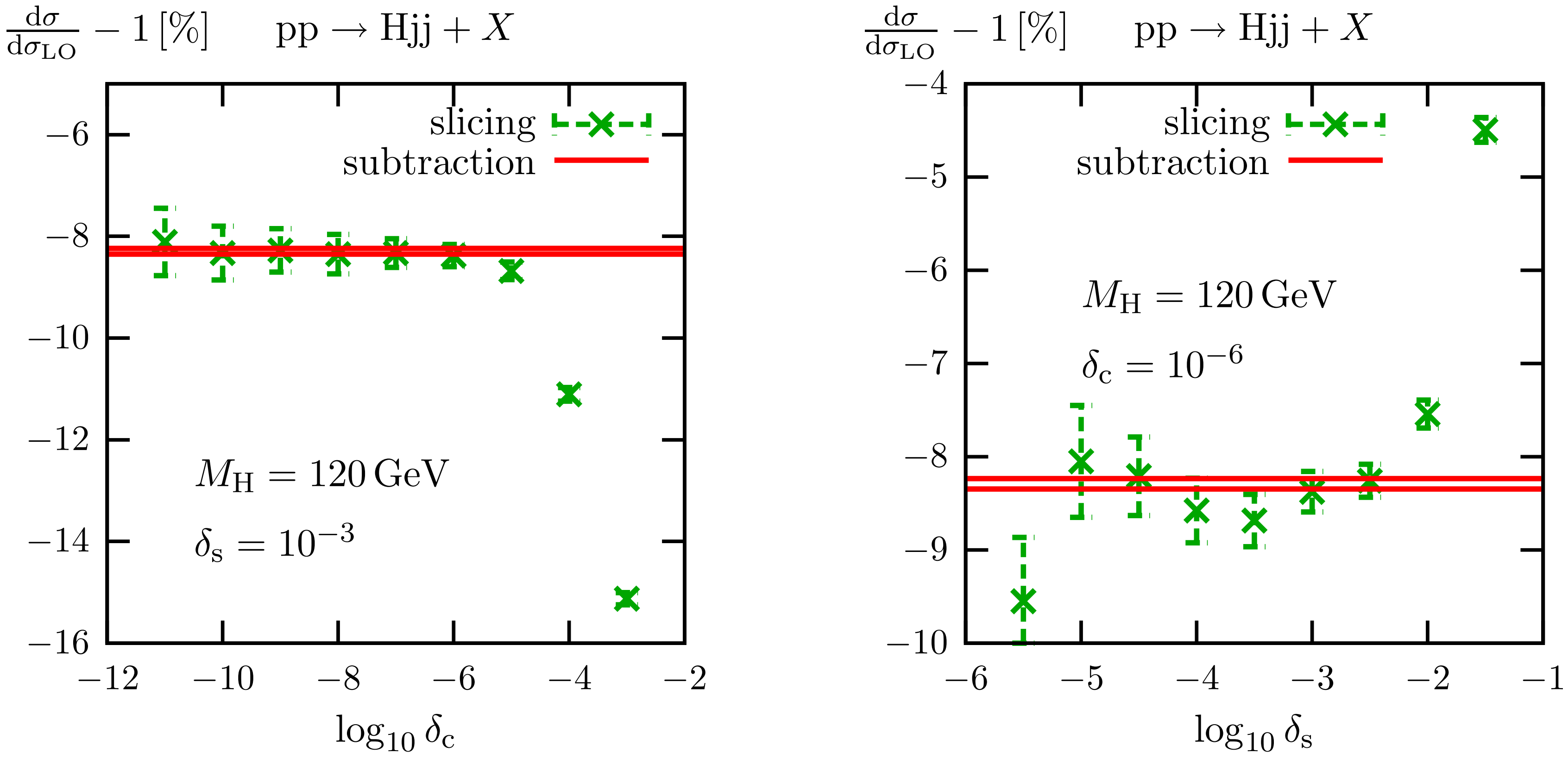
Ciccolini, Denner, Dittmaier ’07

	no cuts		VBF cuts		
$M_\mathrm{H}\,[\mathrm{GeV}]$	$120{-}200$	700	$120{-}200$	700	
various corrections:					
$\delta_{\mathrm{QCD(diag)}}\,[\%]$	$4{-}0.5$	$+1$	≈ -5	-7	$\mathcal{O}(5{-}10\%)$
$\delta_{\mathrm{QCD(int)}}\,[\%]$	$\lesssim 0.2$	-0.1	< 0.1	< 0.1	negligible
$\delta_{\mathrm{EW},qq}\,[\%]$	≈ -5	$+6$	≈ -7	$+5$	$\mathcal{O}(5{-}10\%)$
$\delta_{\mathrm{EW},q\gamma}\,[\%]$	$\approx +1$	$+2$	$\approx +1$	$+2$	$\mathcal{O}(1\%)$
$\delta_{G_\mu^2 M_\mathrm{H}^4}\,[\%]$	< 0.1	$+4$	< 0.1	$+4$	negligible for $M_\mathrm{H} < 400\,\mathrm{GeV}$
specific contributions:					
$\Delta_{s-\mathrm{channel}}\,[\%]$	$30{-}10$	1	< 0.6	< 0.1	negligible with VBF cuts
$\Delta_{t/u-\mathrm{interference}}\,[\%]$	< 0.5	< 0.1	< 0.1	< 0.1	negligible
$\Delta_{\mathrm{b-quarks}}\,[\%]$	≈ 4	1	≈ 2	1	

Comparison between subtraction and slicing methods

VBF cuts $\mu_F = \mu_F = M_H$

Ciccolini, Denner, Dittmaier ’07

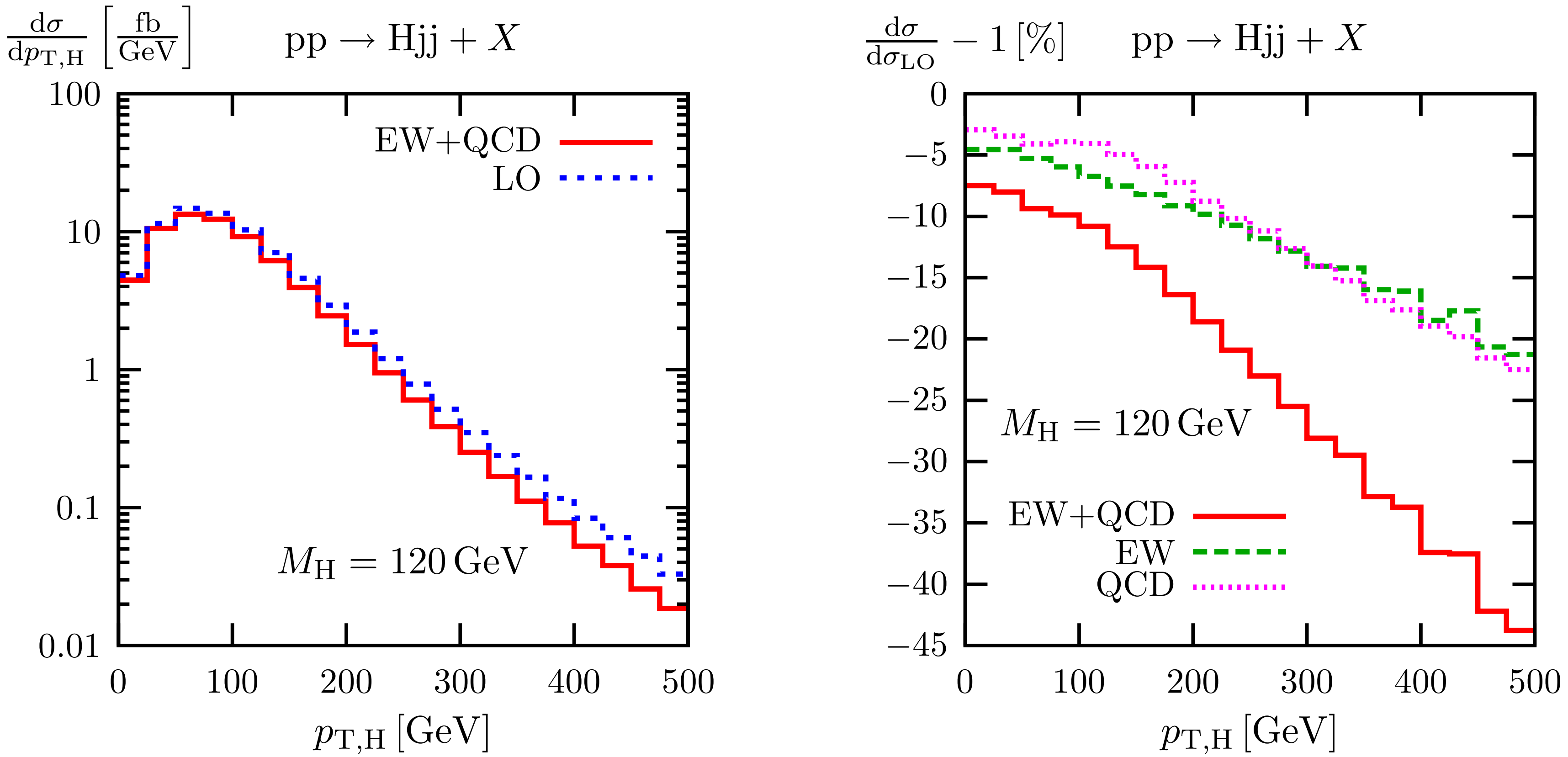


- slicing cuts in partonic CM frame:
  soft region: $E_{\gamma,g} < \delta_s \frac{\sqrt{\hat{s}}}{2}$, collinear cone: $1 - \cos(\theta_{\{\gamma,g\}q}) < \delta_c$
- slicing: 10^9 events, 144 CPU h subtraction: 10^8 events, 64 CPU h
  $\Delta\sigma/\sigma_{LO} = 0.2\%$ $\Delta\sigma/\sigma_{LO} = 0.06\%$

Distribution in transverse momentum of Higgs boson

VBF cuts

Ciccolini, Denner, Dittmaier ’07



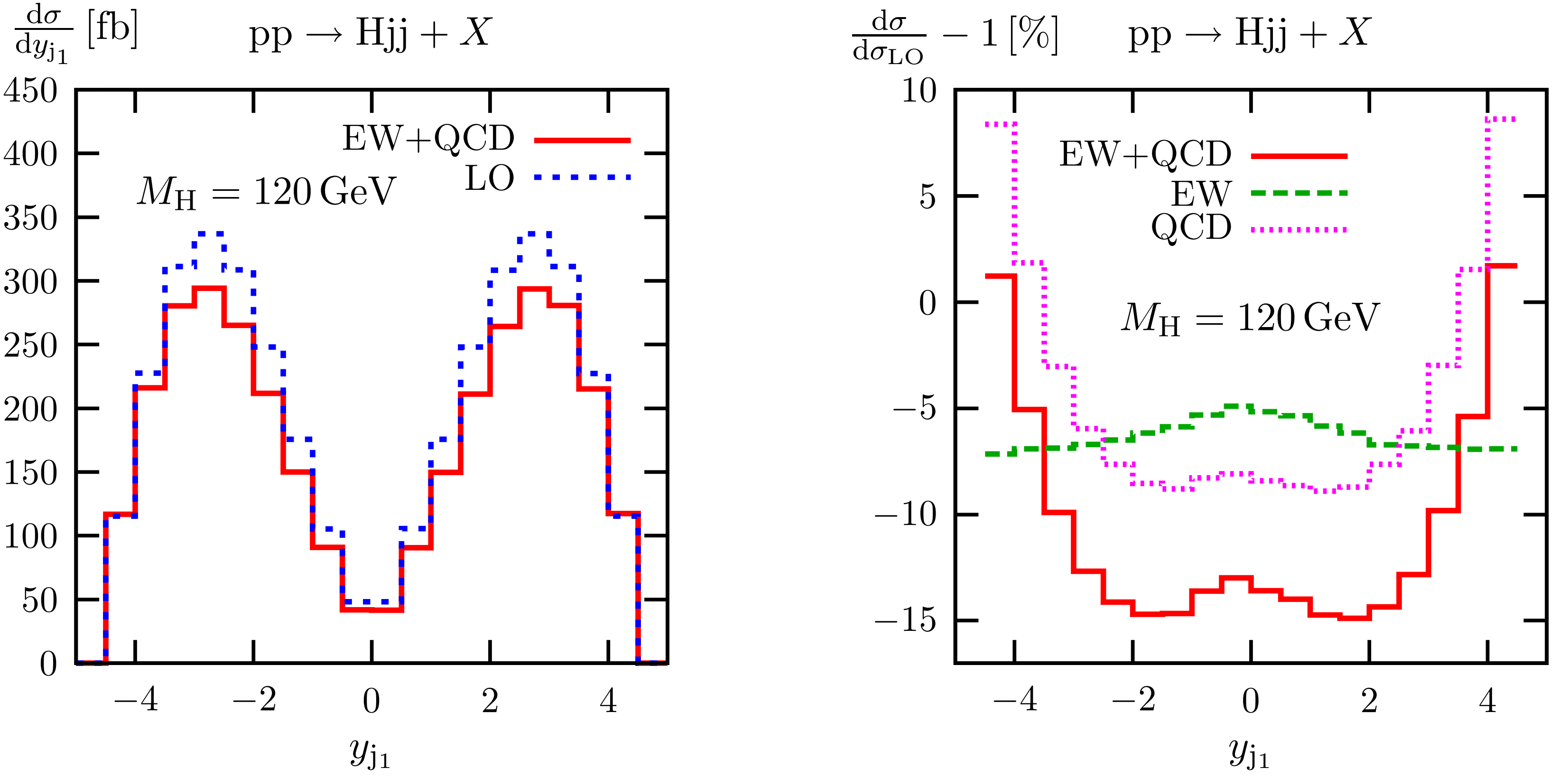
EW and QCD corrections similar

EW corrections -20% at $p_{T,H} = 500\,\text{GeV}$

Distribution in the rapidity of the harder tagging jet

VBF cuts

Ciccolini, Denner, Dittmaier ’07

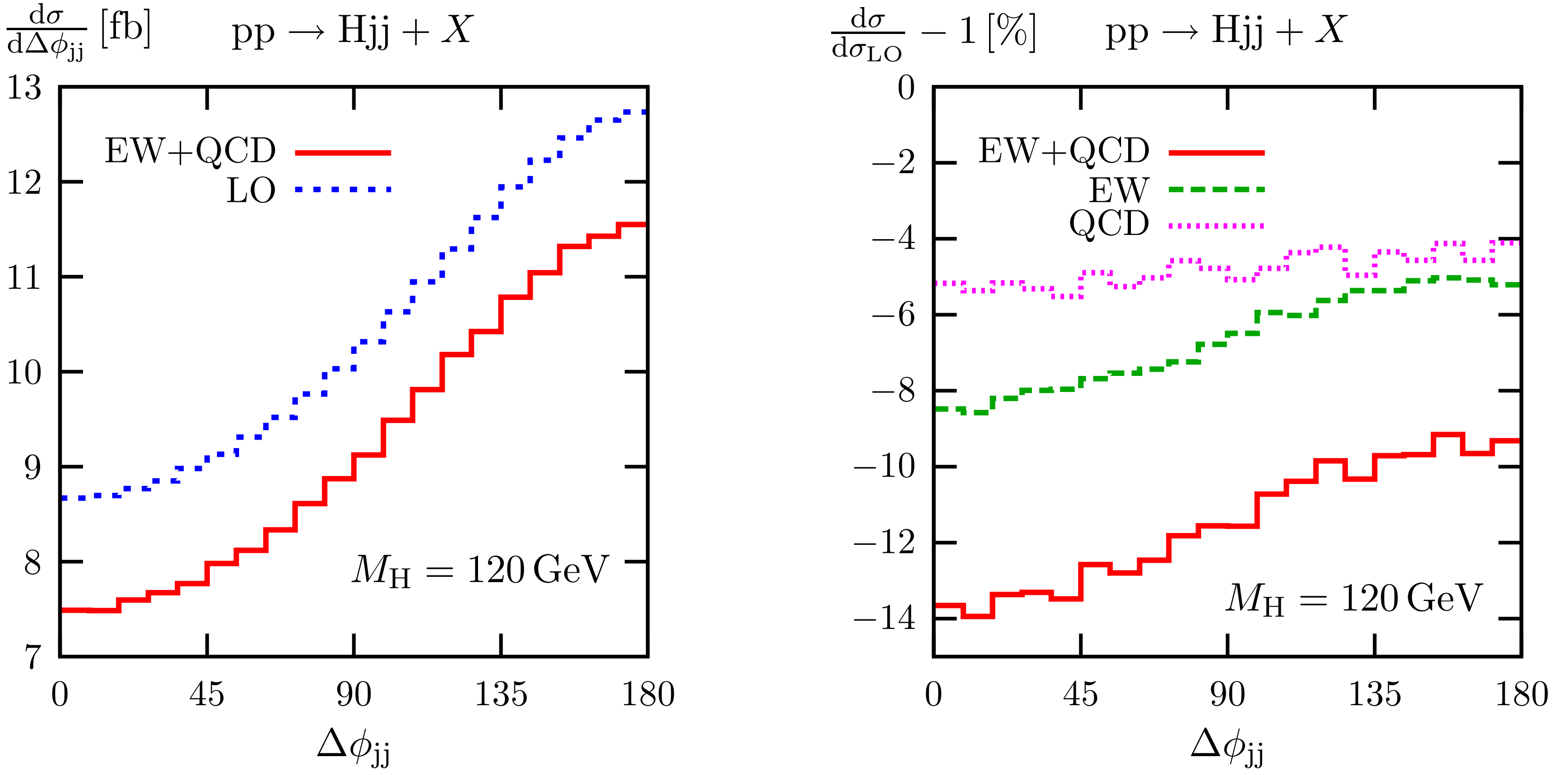


- tagging jets forward–backward
- QCD corrections distort shape significantly
- EW corrections depend only weakly on rapidity y_{j_1} $(-4\% - -7\%)$

Distribution in azimuthal angle separation of tagging jets

VBF cuts

Ciccolini, Denner, Dittmaier ’07



distribution in $\Delta\phi_{jj}$ sensitive to non-standard HVV couplings Figy, Zeppenfeld ’04

EW corrections yield distortion of distribution by 4%

$$\mathrm{pp} \longrightarrow \mathrm{t\bar{t}b\bar{b}} + X$$

Background processes: Les Houches '05 wishlist

	Reaction	background for	existing calculations
1.	VVj	$\mathrm{t\bar{t}H}$, new physics	$\mathrm{WW}j$: Dittmaier, Kallweit, Uwer '07 $\mathrm{WW}j$: Campbell, R.K.Ellis, Zanderighi '07 $\mathrm{WW}j$: Binoth, Guillet, Karg, Kauer, Sanguinetti (in progress)
2.	$\mathrm{t\bar{t}b\bar{b}}$	$\mathrm{t\bar{t}H}$	this talk
3.	$\mathrm{t\bar{t}}jj$	$\mathrm{t\bar{t}H}$	–
4.	$VV\mathrm{b\bar{b}}$	VBF$\to \mathrm{H} \to VV$, $\mathrm{t\bar{t}}$, NP	–
5.	$VVjj$	VBF$\to \mathrm{H} \to VV$	VBF: Jäger, Oleari, Zeppenfeld '06 + Bozzi '07
6.	$Vjjj$	new physics	–
7.	VVV	SUSY trilepton signal	ZZZ: Lazopoulos, Melnikov, Petriello '07 WWZ: Hankele, Zeppenfeld '07 VVV: Binoth, Ossola, Papadopoulos, Pittau '07

Motivation for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}} + X$

Technical

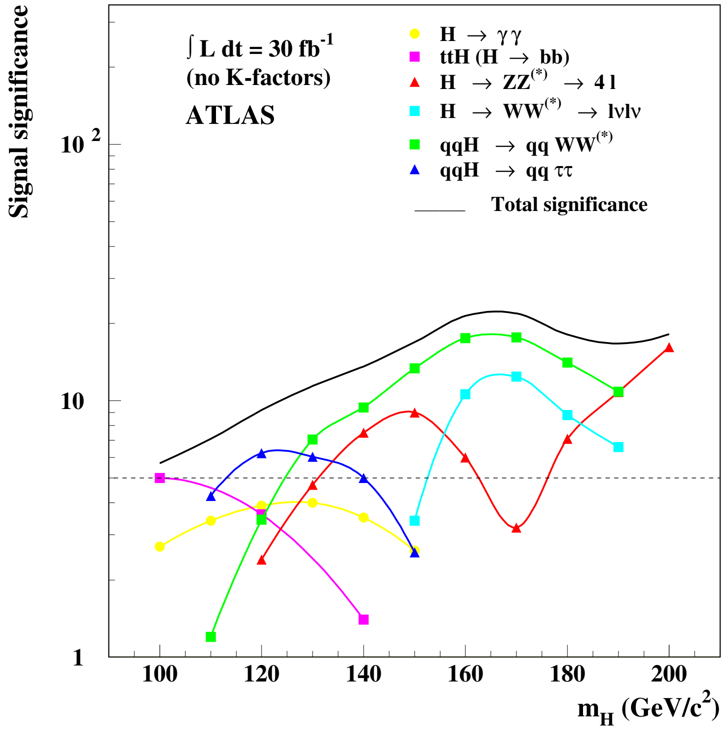
validate NLO algorithms by calculating a non-trivial LHC process

- $2 \to 4$ process involving hexagons
- massless and massive particles, 6 coloured legs

phenomenological

associated $\mathrm{t\bar{t}H(H \to b\bar{b})}$ production

- can be observed in $\mathrm{H} \to \mathrm{b\bar{b}}$ channel
- exploits large BR($H \to \mathrm{b\bar{b}}$) for light H
- measurement of top Yukawa coupling

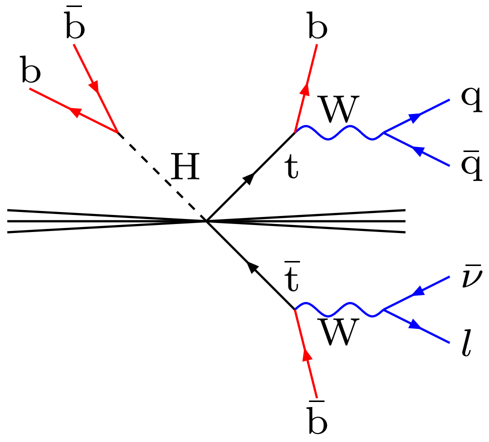


Relevance of $\mathrm{t\bar{t}b\bar{b}}$ for analysis of $\mathrm{t\bar{t}H}$ production

Proposed analysis (ATLAS TDR)

- select final state $\mathrm{b\bar{b}b\bar{b}}jjl\nu$ (4 b-quarks!)
- reconstruct $\mathrm{t\bar{t}b\bar{b}}$ (b-tagging crucial)
- select region $|m_{\mathrm{b\bar{b}}} - M_{\mathrm{H}}| < 30\,\mathrm{GeV}$

Richter-Was and Sapinski, ATL-PHYS-98-132



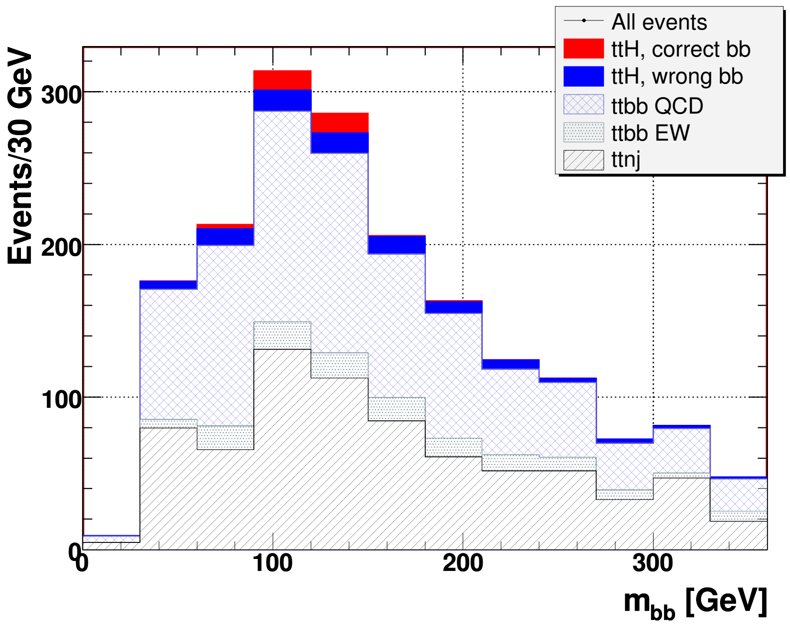
backgrounds

- small S/B
- dominant B: $\mathrm{t\bar{t}b\bar{b}}$ (QCD+EW), $\mathrm{t\bar{t}}jj$
- 20% uncertainty on B kills measurement!
- data do not provide enough precision on normalization and shape of B
- scale dependence at LO > 100%

⇒ NLO crucial for reliable B prediction and ttH measurement

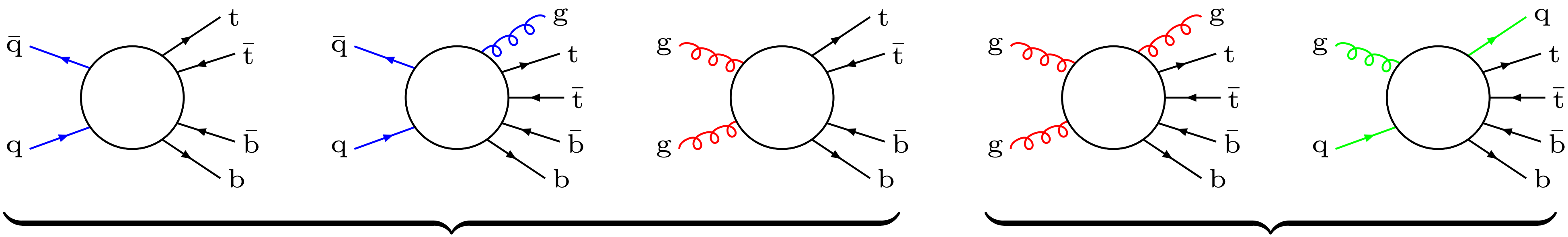
Benedetti at al. '07

private communication S. Kotov, ATLAS '08



$2 \to 4$ and $2 \to 5$ Feynman diagrams to $\mathrm{t\bar{t}b\bar{b}}$

Quark–antiquark and gluon induced processes



this talk

work in progress

quark–antiquark channel

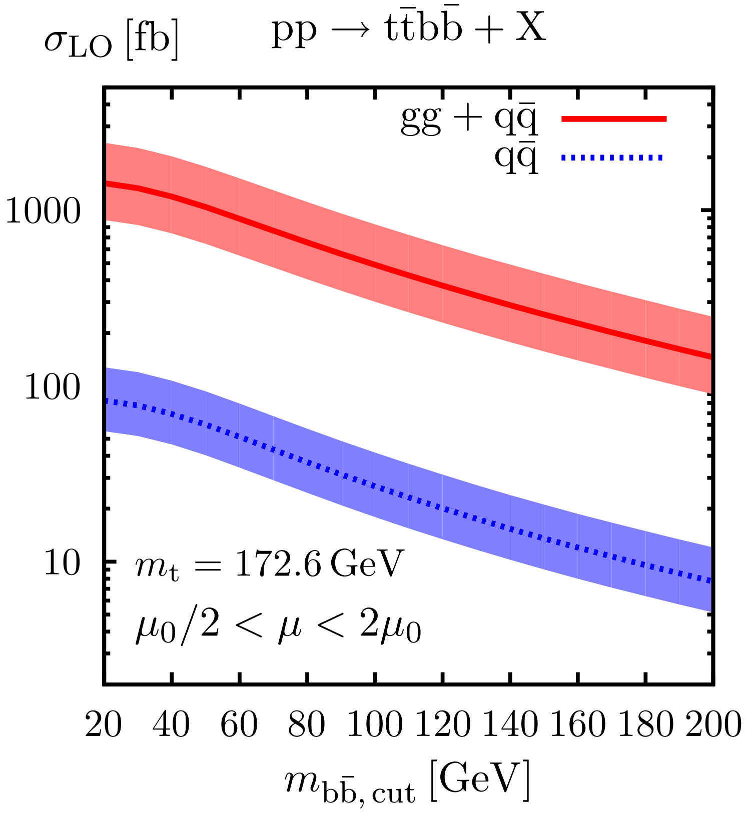
- 5 times less diagrams than gg channel
- not sufficient for LHC (small fraction of σ)
- demonstrate feasibility of calculation

diagrams and impact on σ_{LO}

	$q\bar{q}$	gg	qg
LO	7	36	
virtual	188	1003	
real	64	341	64
$\sigma/\sigma_{\mathrm{tot}}$	5%	95%	

Dependence of LO cross section on $\mathrm{b\bar{b}}$ invariant-mass cut

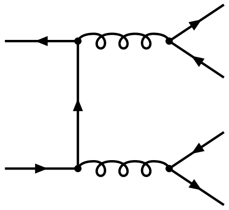
Bredenstein, Denner, Dittmaier, Pozzorini



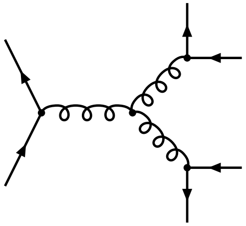
- relative weight: $\sigma_{\mathrm{gg}} \simeq 20\sigma_{\mathrm{q\bar{q}}}$
- $\mathrm{q\bar{q}}$ and gg have similar shape
- scale dependence $\sim$ factor 2
  $\mu_0 = m_{\mathrm{t}} + m_{\mathrm{b\bar{b}},\mathrm{cut}}/2$

Tree and one-loop contributions to $\bar{q}q \to t\bar{t}b\bar{b}$

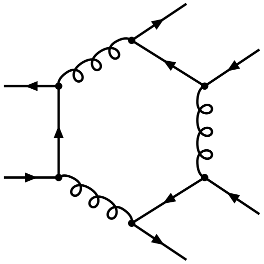
Tree (7) and one-loop (188) diagrams



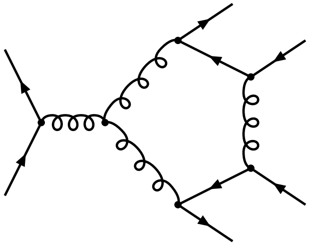
6



1



8 hexagons



24 pentagons

etc.

two independent calculations

- generation of diagrams with FeynArts 1.0 and 3.2 Küblbeck et al. '90,'92; Hahn '01
- algebraic simplifications using two independent in-house programs implemented in *Mathematica*, one using FormCalc 5.2 Hahn '06 for preliminary algebraic manipulations (Dirac algebra, covariant decomposition)
- reduction of tensor integrals according to

  Denner, Dittmaier, NPB658 (2003)175 [hep-ph/0212259], NPB734 (2006) 62 [hep-ph/0509141]

  $\hookrightarrow$ numerically stable results

top quarks massive and bottom quarks massless

Structure of one-loop contributions to $\bar{\mathrm{q}}\mathrm{q} \longrightarrow \mathrm{t\bar{t}b\bar{b}}$

Standard matrix elements and colour structures for individual diagram Γ

$$\mathcal{M}^{(\Gamma)} = \underbrace{\mathcal{C}^{(\Gamma)}}_{\text{factorized colour structure}} \sum_m \mathcal{F}_m^{(\Gamma)}(\{p_a \cdot p_b\}) \underbrace{\hat{\mathcal{M}}_m(\{p_a\},\{\lambda\})}_{\text{standard matrix elements}}$$

form factors $\mathcal{F}_m^{(\Gamma)}$

$$\mathcal{F}_m^{(\Gamma)}(\{p_a \cdot p_b\}) = \sum_{j_1 \cdots j_R} \mathcal{K}^{(\Gamma)}_{m,j_1\ldots j_R}(\{p_a \cdot p_b\}) \underbrace{T_{j_1\ldots j_R}(\{p_a \cdot p_b\})}_{\text{tensor loop coefficients}}$$

computed numerically diagram by diagram (no analytic reduction to scalar integrals)

main goals

- reduction to small set of standard matrix elements $\hat{\mathcal{M}}_m$
- fast and stable numerical evaluation of tensor integrals $T_{j_1\ldots j_R}$

$\bar{\mathrm{q}}\mathrm{q} \to \mathrm{t}\bar{\mathrm{t}}\mathrm{b}\bar{\mathrm{b}}$: colour structure, rational terms

- six colour structures for $\bar{\mathrm{q}}\mathrm{q} \to \mathrm{t}\bar{\mathrm{t}}\mathrm{b}\bar{\mathrm{b}}$

$$\begin{array}{ccc} 1 \otimes T^a \otimes T^a, & T^a \otimes 1 \otimes T^a, & T^a \otimes T^a \otimes 1, \\ 1 \otimes 1 \otimes 1, & f^{abc} T^a \otimes T^b \otimes T^c, & d^{abc} T^a \otimes T^b \otimes T^c \end{array}$$

- exploit colour factorization for individual diagrams
  3^{n_4} colour-factorized amplitudes for each loop diagram with n_4 4-gluon vertices ⇒ only one colour-factorized amplitude for most diagrams

rational terms originate from $1/(D-4)$ poles of tensor loop integrals

$$\mathcal{K}^{(\Gamma)}_{m,j_1\ldots j_R}(D) \underbrace{T_{j_1\ldots j_R}}_{\frac{R_{j_1\ldots j_R}}{(D-4)} + T^{\mathrm{finite}}_{j_1\cdots j_R}} = \mathcal{K}^{(\Gamma)}_{m,j_1\ldots j_R}(4)\, T_{j_1\ldots j_R} + \mathcal{K}'^{(\Gamma)}_{m,j_1\ldots j_R}(4)\, R_{j_1\ldots j_R} + \mathcal{O}(D-4)$$

- residues $R_{j_1\ldots j_R}$ of tensor integrals explicitly available
- after $(D-4)$-expansion continue calculation in $D=4$

Reduction of standard matrix elements for $\mathrm{\bar{q}q} \longrightarrow \mathrm{t\bar{t}b\bar{b}}$

After Dirac algebra + Dirac equation: $\mathcal{O}(10^3)$ Dirac structures of type

$$\underbrace{\bar{v}(p_1)\ldots\gamma^\mu\gamma^\nu\not{p}_3\ldots u(p_2)}_{\mathrm{q\bar{q}\ chain}}\ \underbrace{\bar{v}(p_3)\ldots\gamma^\mu\gamma^\nu\gamma^\rho\not{p}_6\ldots u(p_4)}_{\mathrm{t\bar{t}\ chain}}\ \underbrace{\bar{v}(p_5)\ldots\gamma^\rho\not{p}_2\not{p}_3\ldots u(p_6)}_{\mathrm{b\bar{b}\ chain}}$$

after cancellation of $1/(D-4)$ poles work in 4 dimensions
use Chisholm identity

$$\mathrm{i}\varepsilon^{\alpha\beta\gamma\delta}\gamma_\delta\gamma^5 = \gamma^\alpha\gamma^\beta\gamma^\gamma - g^{\alpha\beta}\gamma^\gamma + g^{\alpha\gamma}\gamma^\beta + g^{\beta\gamma}\gamma^\alpha$$

and identities that can be derived therefrom, like

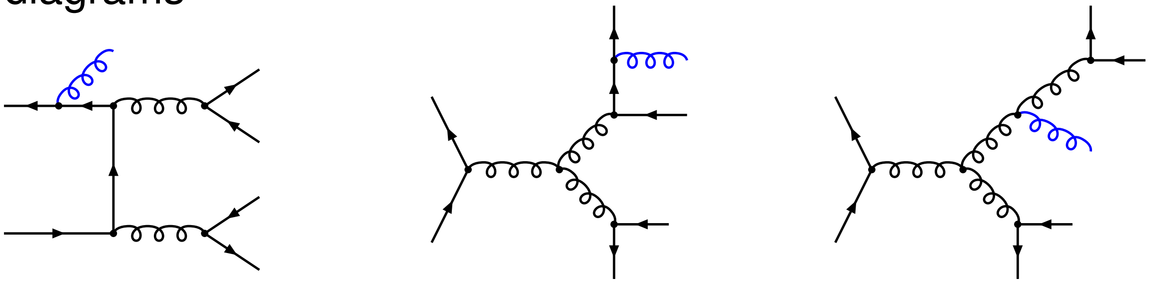
$$\begin{aligned}\gamma^\mu\gamma^\alpha\gamma^\beta\omega_\pm\otimes\gamma_\mu\omega_\mp &= \gamma^\mu\omega_\pm\otimes\gamma^\alpha\gamma^\beta\gamma_\mu\omega_\mp\\ \gamma^\mu\gamma^\alpha\gamma^\nu\omega_\pm\otimes\gamma_\mu\gamma^\beta\gamma_\nu\omega_\mp &= 4\gamma^\beta\omega_\pm\otimes\gamma^\alpha\omega_\mp\end{aligned}$$

construct a sophisticated algorithm to reduce # of γ-contractions and $\not{p}$-terms
⇒ reduction of all Dirac structures to $\lesssim 200$ standard matrix elements
without introducing new denominators that might spoil numerical stability

Real corrections for $\bar{q}q \to t\bar{t}b\bar{b}$

Diagrams and matrix elements for $\bar{q}q \to t\bar{t}b\bar{b}g$

- 64 Feynman diagrams



etc.

- analytically with Weyl–van der Waerden spinors Dittmaier '98
  and with `Madgraph 4.1.33` Maltoni, Stelzer

soft and collinear singularities

- regularized dimensionally or with infinitesimal gluon and quark masses
- dipole subtraction Catani, Seymour '96; Catani, Dittmaier, Seymour, Trócsányi '02
  30 subtraction terms
- phase-space slicing Giele, Glover '92; Giele et al. '93; Keller, Laenen '98; Harris, Owens '01
- initial-state collinear singularities cancelled by $\overline{\mathrm{MS}}$ redefinition of PDFs

phase-space integration

- adaptive multi-channel Monte Carlo Berends, Kleiss, Pittau '94; Kleiss, Pittau '94
  $\mathcal{O}(300)$ channels to map all peaks from propagators and dipoles

Checks on the calculation for $\bar{q}q \longrightarrow t\bar{t}b\bar{b}$

- leading order checked against SHERPA Gleisberg et al. ’03
- virtual corrections
  - cancellation of UV, soft and collinear singularities
  - independent calculations agree pointwise and after phase space integration
- real corrections
  - cancellation of soft and collinear singularities
  - squared amplitudes agree with MADGRAPH Stelzer, Long ’94
  - independent calculations agree after phase space integration
- combination of virtual and real corrections
  - subtraction terms in independent calculations agree pointwise
  - independent calculations agree after phase space integration
  - identical results with two-cutoff slicing and dipole subtraction
- two completely independent calculations of all ingredients!

Set-up for numerical calculation of $\mathrm{pp} \longrightarrow \mathrm{t\bar{t}b\bar{b}} + X$

Definition of observables

- jet definition: k_{T} algorithm as used at Tevatron run II Blazey et al. ’00
  - select massless partons (g, q and b) with $|\eta| < 5$
  - reconstruct jets with $\sqrt{\Delta\phi^2 + \Delta y^2} > D = 0.8$
- cuts for b jets:
  - require two b-jets with $p_{\mathrm{T},j} > 20\,\mathrm{GeV}$ and $y_j < 2.5$ (b tagging!)
  - $\mathrm{b\bar{b}}$ invariant mass: $m_{\mathrm{b\bar{b}}} > m_{\mathrm{b\bar{b}},\mathrm{cut}}$
  - no cuts on top momentum

NLO settings: [LO obtained with LO α_{S}, LO PDFs and 1-loop running]

- central scale: $\mu_0 = m_{\mathrm{t}} + m_{\mathrm{b\bar{b}},\mathrm{cut}}/2$ $(m_{\mathrm{t}} + M_{\mathrm{H}}/2$ for $\mathrm{t\bar{t}H})$
- PDFs: CTEQ6M with $\alpha_{\mathrm{S}}(M_{\mathrm{Z}}) = 0.118$, b-quark contributions neglected
- $\alpha_{\mathrm{s}}(\mu_{\mathrm{R}})$ with 5 active flavours (top-quark decoupled) and two-loop running
- top mass: $m_{\mathrm{t}} = 172.6\,\mathrm{GeV}$

NLO results for $\mathrm{pp} \longrightarrow \mathrm{t\bar{t}b\bar{b}} + X$ at the LHC

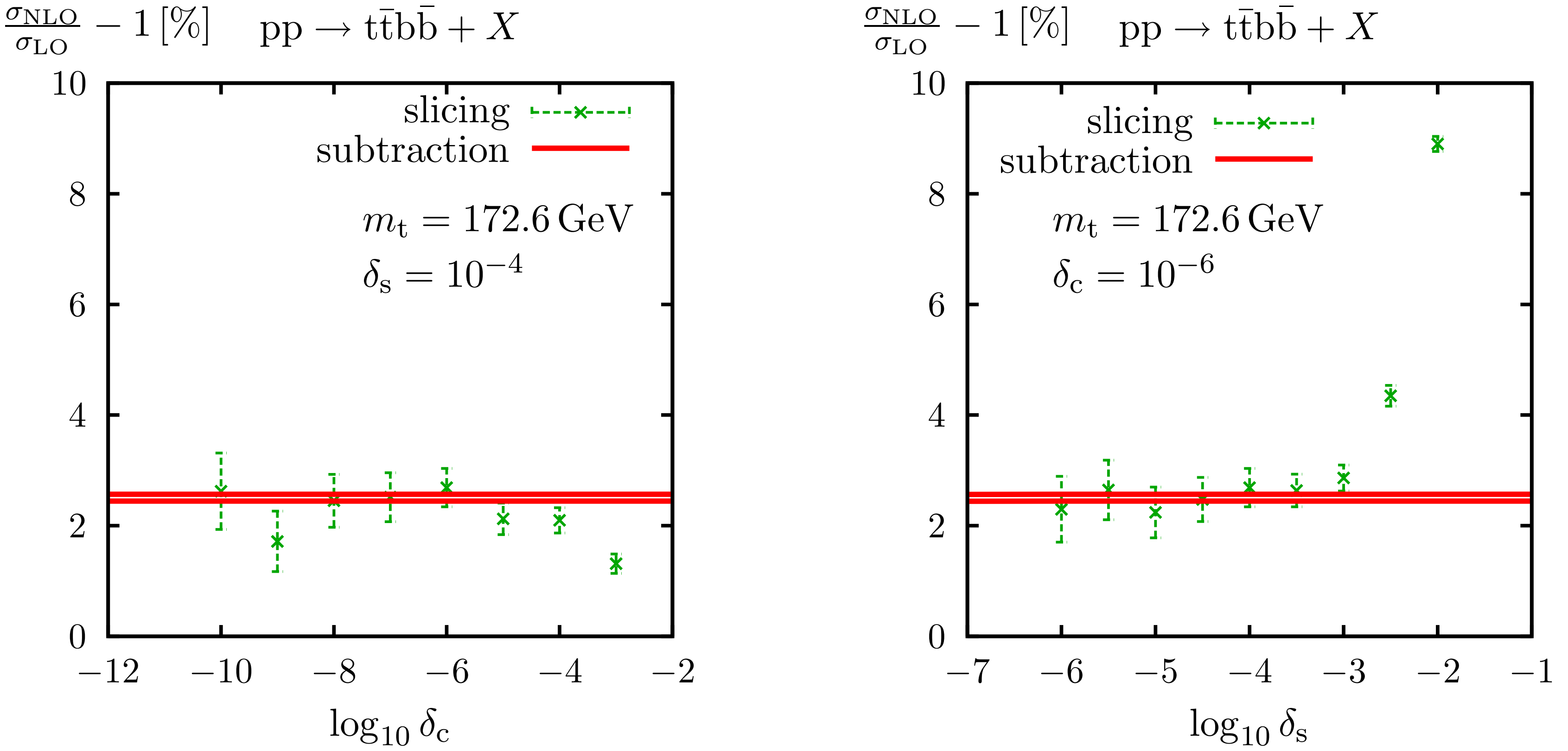
The following results are PRELIMINARY

- $\mathrm{q\bar{q}}$ channel only
- not all cross checks completed (but almost!)

Subtraction versus slicing for $\bar{q}q \longrightarrow t\bar{t}b\bar{b}$

PRELIMINARY

Bredenstein, Denner, Dittmaier, Pozzorini

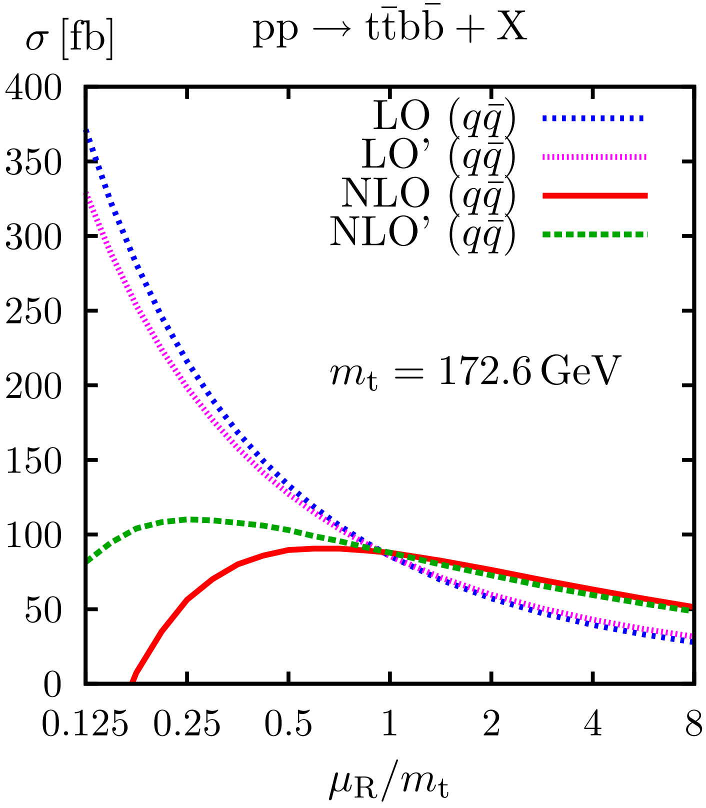


- slicing cuts in partonic CM frame:
  soft region: $E_g < \delta_s \frac{\sqrt{\hat{s}}}{2}$, collinear cone: $1 - \cos(\theta_{gq}) < \delta_c$
- slicing: 10^9 events, subtraction: 2×10^8 events
- relative NLO correction: $\sim 2.5\%$, $\sigma_{LO} = 85.520(26)\,fb$, $\sigma_{NLO} = 87.698(56)\,fb$

LO and NLO scale dependence for $\bar{q}q \rightarrow t\bar{t}b\bar{b}$

PRELIMINARY

Bredenstein, Denner, Dittmaier, Pozzorini



LO, NLO: $\mu_F = \mu_R$

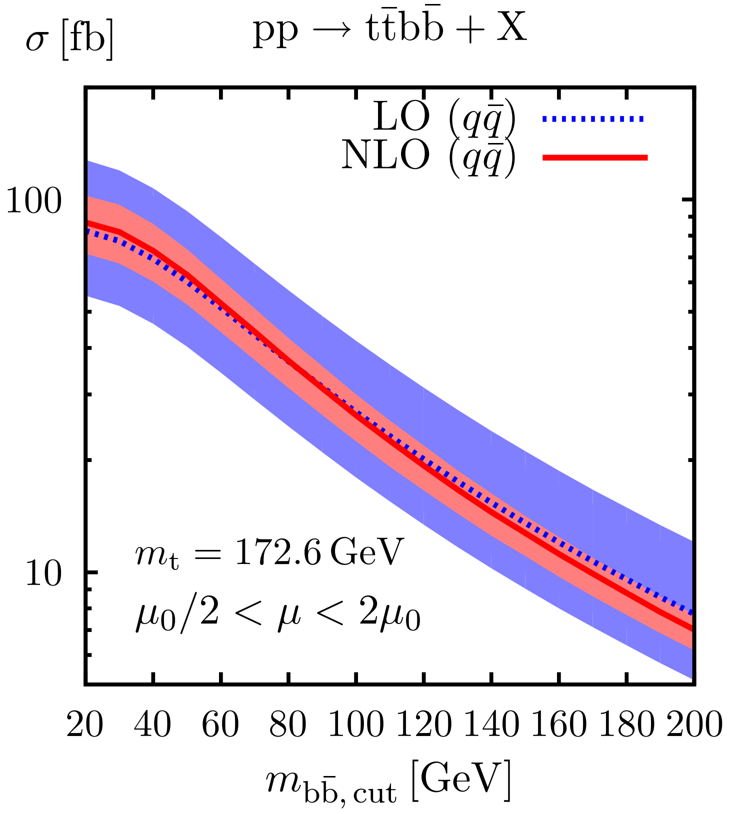
LO', NLO': $\mu_F = m_t^2/\mu_R$

- central scale $\mu_0 = m_t$
- dominant dependence from $\alpha_S(\mu_R)^4$
- μ_F-dependence much smaller
- huge LO dependence: up to factor 4
  $m_t/2 < \mu < 2m_t$: $+55\%, -50\%$
- stabilization at NLO (close to maximum)
  $m_t/2 < \mu < 2m_t$: $\pm 17\%$

Dependence on $\mathrm{b\bar{b}}$ invariant mass cut for $\mathrm{\bar{q}q \to t\bar{t}b\bar{b}}$

PRELIMINARY

Bredenstein, Denner, Dittmaier, Pozzorini



- strong reduction of scale dependence
- NLO consistent with LO uncertainty band
- shape of $m_{\mathrm{b\bar{b}}}$ distorted by corrections

Speed and precision of calculation

Runtime and statistical precision $(\Delta\sigma/\sigma_{\mathrm{LO}})$ with 3 GHz Intel Xeon processor

	$\sigma/\sigma_{\mathrm{LO}}$	# events (after cuts)	$\Delta\sigma/\sigma_{\mathrm{LO}}$	runtime	time/event
tree	74.8%	6×10^{7}	3×10^{-4}	64min	60μs
virtual	-4.3%	0.34×10^{7}	2.5×10^{-4}	12h	13ms
real + dipoles	32.0%	11×10^{7}	5×10^{-4}	31h	1ms
all	2.5%		6×10^{-4}	70h	

- speed of virtual corrections (13 ms/event) very encouraging
- for same precision $(\Delta\sigma/\sigma)$ virtual corrections require less CPU-time than real corrections (scale-dependent statement!)

Conclusions

Conclusions

NLO calculations for multiparticle processes at the LHC

- necessary for adequate exploitation of experimental data
- lots of progress in recent years, many active groups

Higgs production in vector-boson fusion $\mathrm{pp} \to \mathrm{H} + 2\mathrm{jets} + X$

- important channel for Higgs-boson search and study
- complete electroweak and QCD NLO corrections known
  electroweak corrections $\sim -5\%$, comparable to QCD corrections
  $\hookrightarrow$ theoretical accuracy below uncertainties from PDFs and experiment

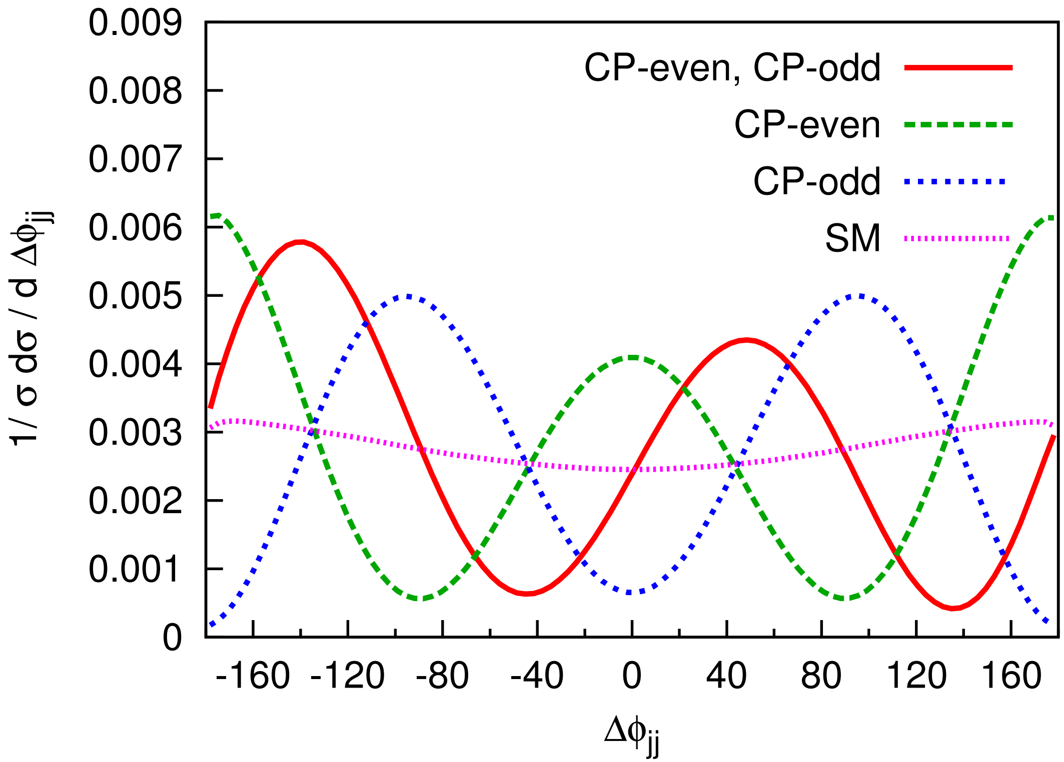
NLO calculation for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}} + X$

- very important for $\mathrm{t\bar{t}H}$ measurement
- $2 \to 4$ LHC process
- first results for $\mathrm{q\bar{q}}$ channel available (scale dependence reduced by factor 3)
- calculation of gg channel in progress

Backup slides

BSM effects in azimuthal angle difference

Azimuthal angle difference $\Delta\phi_{\mathrm{jj}}$ of tagging jets is sensitive to BSM effects:



Hankele, Klämke,
Zeppenfeld, Figy ’06

(Individual contributions
without SM)

CP-even: $\mathcal{L} \propto HW^{+}_{\mu\nu}W^{-,\mu\nu}, \qquad \Gamma^{HW^+W^-}_{\mu\nu} \propto g_{\mu\nu}(k_+k_-) - k_{+,\nu}k_{-,\mu}$

CP-odd: $\mathcal{L} \propto H\tilde{W}^{+}_{\mu\nu}W^{-,\mu\nu}, \qquad \Gamma^{HW^+W^-}_{\mu\nu} \propto \epsilon_{\mu\nu\rho\sigma}k^{\rho}_{+}k^{\sigma}_{-}$

Complex-mass scheme at NLO

Denner, Dittmaier, Roth, Wieders ’05

Basic idea: (renormalized) mass^2 = location of propagator pole in complex p^2 plane

$\hookrightarrow$ consistent use of complex masses everywhere!

application to gauge-boson resonances:

- replace $M_{\mathrm{W}}^2 \to \mu_{\mathrm{W}}^2 = M_{\mathrm{W}}^2 - \mathrm{i} M_{\mathrm{W}} \Gamma_{\mathrm{W}}, \qquad M_{\mathrm{Z}}^2 \to \mu_{\mathrm{Z}}^2 = M_{\mathrm{Z}}^2 - \mathrm{i} M_{\mathrm{Z}} \Gamma_{\mathrm{Z}}$

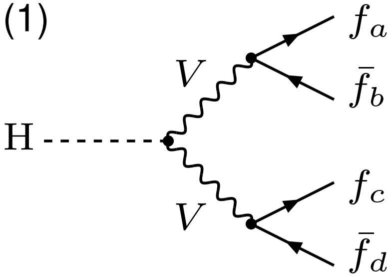
  and define (complex) weak mixing angle via $\cos^2\theta_{\mathrm{w}} \equiv c_{\mathrm{w}}^2 = 1 - s_{\mathrm{w}}^2 = \frac{\mu_{\mathrm{W}}^2}{\mu_{\mathrm{Z}}^2}$
- virtues:
  - gauge-invariant result (Ward identities, Slavnov–Taylor identities)

    $\hookrightarrow$ gauge-parameter independence, unitarity cancellations!
  - perturbative calculations as usual (complex counterterms!)
  - no double counting (bare Lagrangian unchanged!)
- drawbacks: spurious terms of $\mathcal{O}(\alpha\Gamma/M) = \mathcal{O}(\alpha^2) \Rightarrow$ beyond NLO accuracy (from Γ in t-channel/off-shell propagators and complex mixing angle)
- loop integrals with complex masses

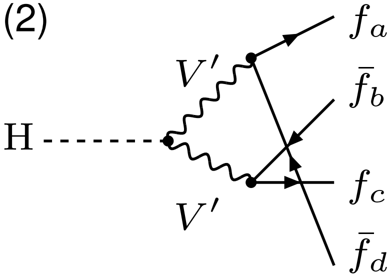
Classification of QCD corrections

Possible Born diagrams:

(1)



(2)



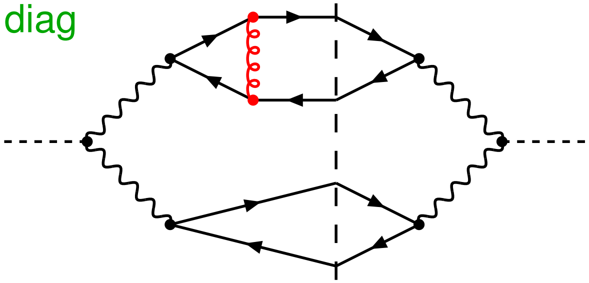
diagrams (2) only for

$q\bar{q}q\bar{q}$ and $q\bar{q}q'\bar{q}'$ channels

(q' = weak-isospin partner of q)

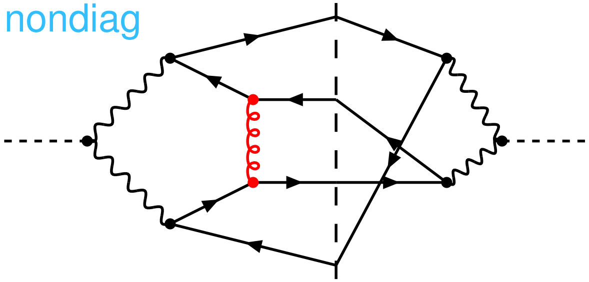
classification of QCD corrections into four categories: (typical diagrams shown)

diag

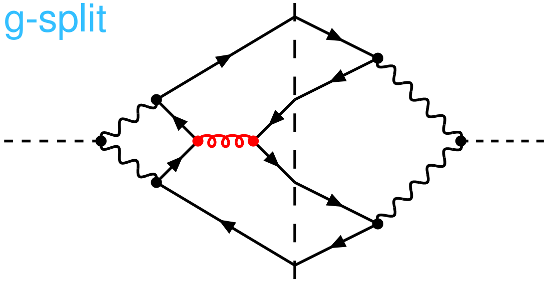


diag = corrections squared tree diagrams

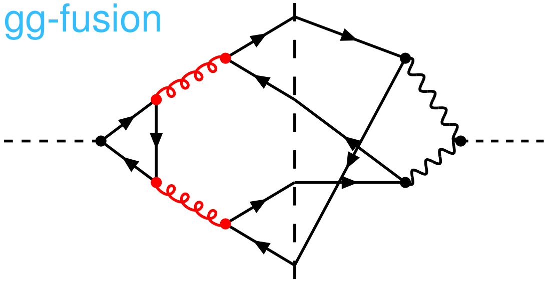
nondiag



g-split



gg-fusion



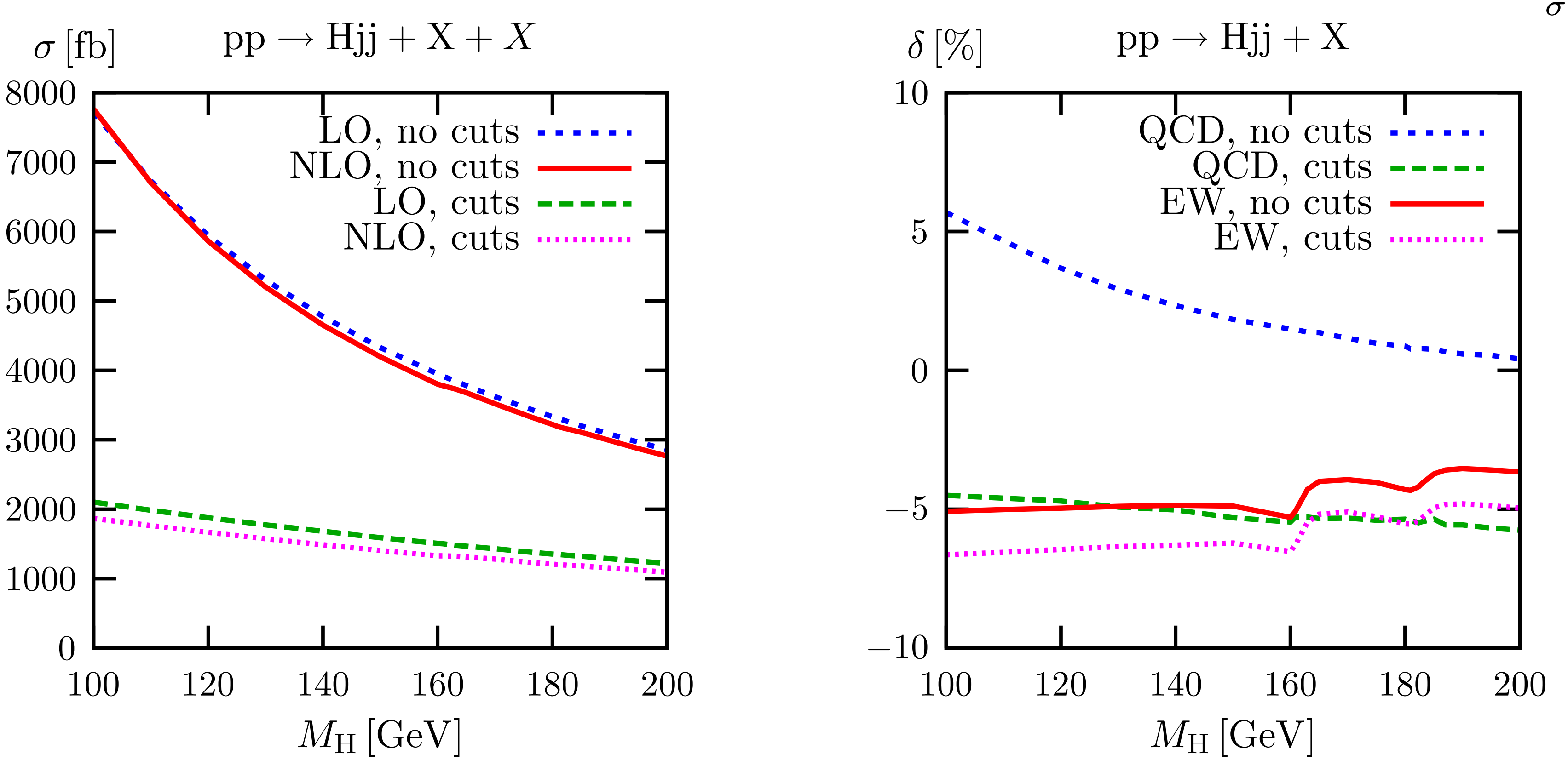
nondiag, g-split, gg-fusion = corrections to interferences (only for $q\bar{q}q\bar{q}$ and $q\bar{q}q'\bar{q}'$ channels)

gg-fusion int.: see also Anderson, Smillie ’06 and in $\mathcal{O}(\alpha^2\alpha_\mathrm{s}^3)$ Anderson et al. ’07, Bredenstein et al. ’08

Total cross section for $\mathrm{pp} \to \mathrm{H} + 2\mathrm{jets} + X$

Ciccolini, Denner, Dittmaier '07

$$\delta = \frac{\sigma}{\sigma_{\mathrm{LO}}} - 1$$



↑ WW threshold for $M_{\mathrm{H}} \sim 2M_{\mathrm{W}}$

↑ ZZ threshold for $M_{\mathrm{H}} \sim 2M_{\mathrm{Z}}$

electroweak (EW) corrections of similar size as QCD corrections

EW corrections -4% – -7%

Total cross section for pp $\longrightarrow$ H + 2jets + X

VBF cuts

Ciccolini, Denner, Dittmaier '07

$M_{\rm H}\ [\rm GeV]$	120	150	200	400	700
$\sigma_{\rm LO}\ [\rm fb]$	1876.3(5)	1589.8(4)	1221.1(3)	487.31(9)	160.67(2)
$\sigma_{\rm NLO}\ [\rm fb]$	1665(1)	1407.5(8)	1091.3(5)	435.4(2)	160.36(5)
$\delta_{\rm EW}\ [\%]$	$-6.47(2)$	$-6.27(2)$	$-4.98(1)$	$-3.99(1)$	$6.99(2)$
$\delta_{{\rm EW},qq}\ [\%]$	$-7.57(2)$	$-7.42(2)$	$-6.19(1)$	$-5.37(1)$	$5.44(2)$
$\delta_{\gamma\text{-induced}}\ [\%]$	1.10	1.15	1.22	1.38	1.55
$\delta_{\rm QCD}\ [\%]$	$-4.77(4)$	$-5.20(4)$	$-5.65(3)$	$-6.67(3)$	$-7.18(2)$
$\delta_{\rm QCD,diag}\ [\%]$	$-4.75(4)$	$-5.17(4)$	$-5.66(4)$	$-6.63(3)$	$-7.18(2)$
$\delta_{\rm QCD,nondiag}\ [\%]$	-0.011	$-0.0052(1)$	$0.0032(1)$	0.0030	0.0022
$\delta_{\text{g-split}}\ [\%]$	$-0.0085(1)$	$0.0084(1)$	0.027	0.014	0.0074
$\delta_{\text{gg-fusion}}\ [\%]$	-0.030	-0.030	$-0.028(1)$	-0.020	-0.014
$\delta_{G_\mu^2 M_{\rm H}^4}\ [\%]$	0.0035	$0.0086(1)$	0.027	0.43	$4.06(1)$

electroweak corrections -6.5% – $+7\%$

photon-induced corrections $\sim 1\%$

interference corrections $\sim 0.02\%$

10^8 weighted events

~ 100 CPU h on Xeon 3 GHz PC per cross section

Total cross section for $\mathrm{pp} \to \mathrm{H} + 2\mathrm{jets} + X$

No cuts

Ciccolini, Denner, Dittmaier '07

M_H [GeV]	120	150	200	400	700
σ_LO [fb]	5943(1)	4331(1)	2855.4(6)	900.7(1)	270.51(4)
σ_NLO [fb]	5872(2)	4202(2)	2765(1)	871.8(3)	294.33(9)
δ_EW [%]	$-4.94(2)$	$-4.91(2)$	$-3.67(1)$	$-2.97(1)$	$7.74(2)$
$\delta_{\mathrm{EW},qq}$ [%]	$-5.79(2)$	$-5.92(2)$	$-4.85(1)$	$-4.50(1)$	$5.99(2)$
$\delta_{\gamma\text{-induced}}$ [%]	0.85	1.00	1.18	1.53	1.75
δ_QCD [%]	$3.75(5)$	$1.94(3)$	$0.49(3)$	$-0.24(3)$	$1.06(3)$
$\delta_{\mathrm{QCD,diag}}$ [%]	$3.97(3)$	$2.04(3)$	$0.55(3)$	$-0.06(3)$	$1.14(3)$
$\delta_{\mathrm{QCD,nondiag}}$ [%]	$0.010(2)$	$0.027(2)$	$0.050(1)$	0.026	0.013
$\delta_{\text{g-split}}$ [%]	$-0.015(1)$	$0.059(1)$	$0.110(1)$	$0.040(1)$	$0.017(1)$
$\delta_{\text{gg-fusion}}$ [%]	$-0.19(1)$	-0.20	-0.22	-0.24	$-0.11(1)$
$\delta_{G_\mu^2 M_\mathrm{H}^4}$ [%]	0.0027	0.0073	0.025	0.42	$4.03(1)$

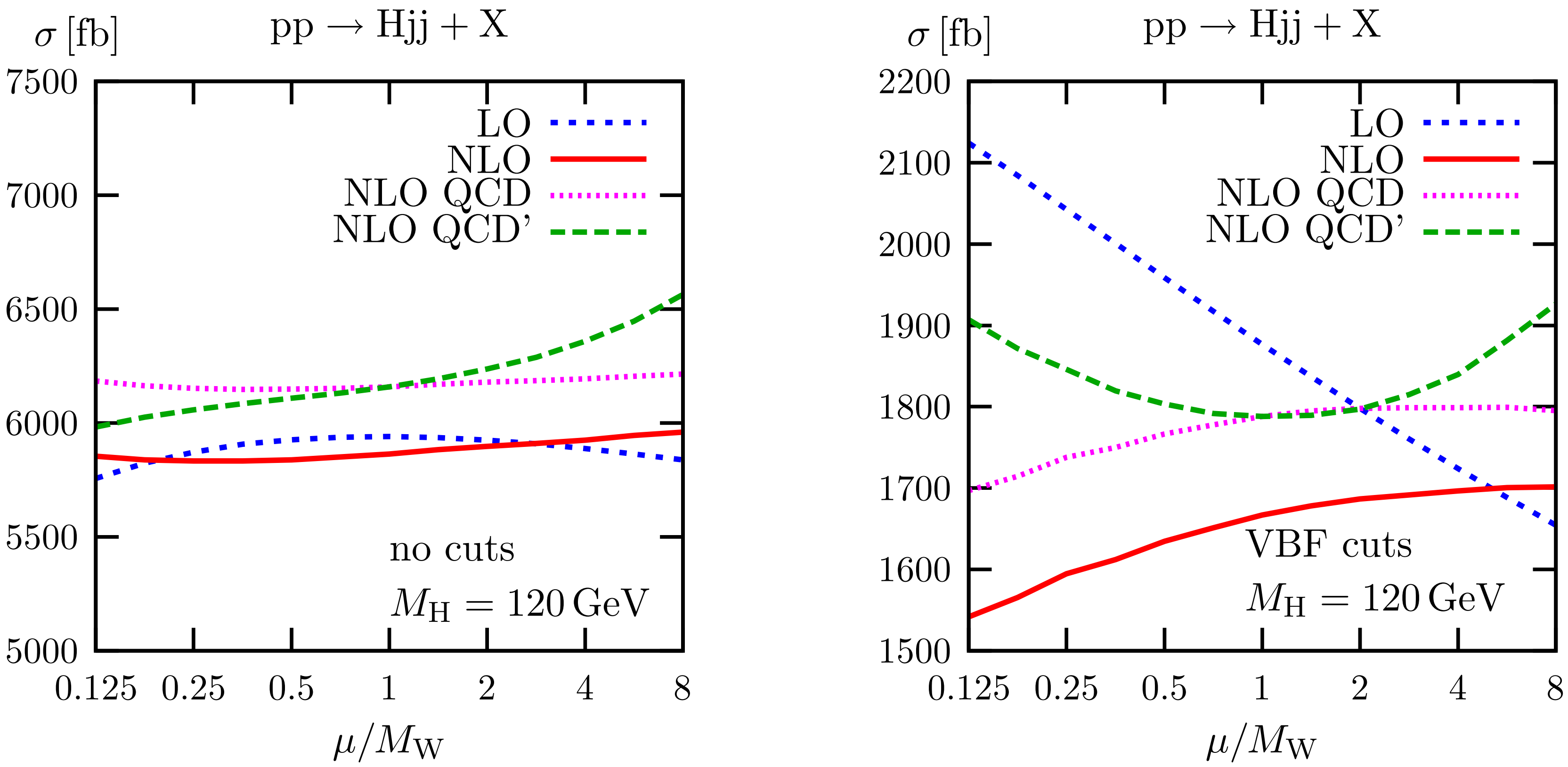
electroweak corrections -5% – $+8\%$
photon-induced corrections $\sim 1\%$
interference corrections $\sim 0.1\%$

5×10^7 weighted events
~ 100 CPU h on Xeon 3 GHz PC
per cross section

Scale dependence for $\mathrm{pp} \to \mathrm{H} + \mathrm{2jets} + X$

$M_{\mathrm{H}} = 120\,\mathrm{GeV}$

Ciccolini, Denner, Dittmaier ’07



$\mu_{\mathrm{R}} = \mu_{\mathrm{F}} \equiv \mu$ for LO, NLO and NLO QCD

$\mu_{\mathrm{R}} = M_{\mathrm{W}}^2/\mu$ for NLO QCD’

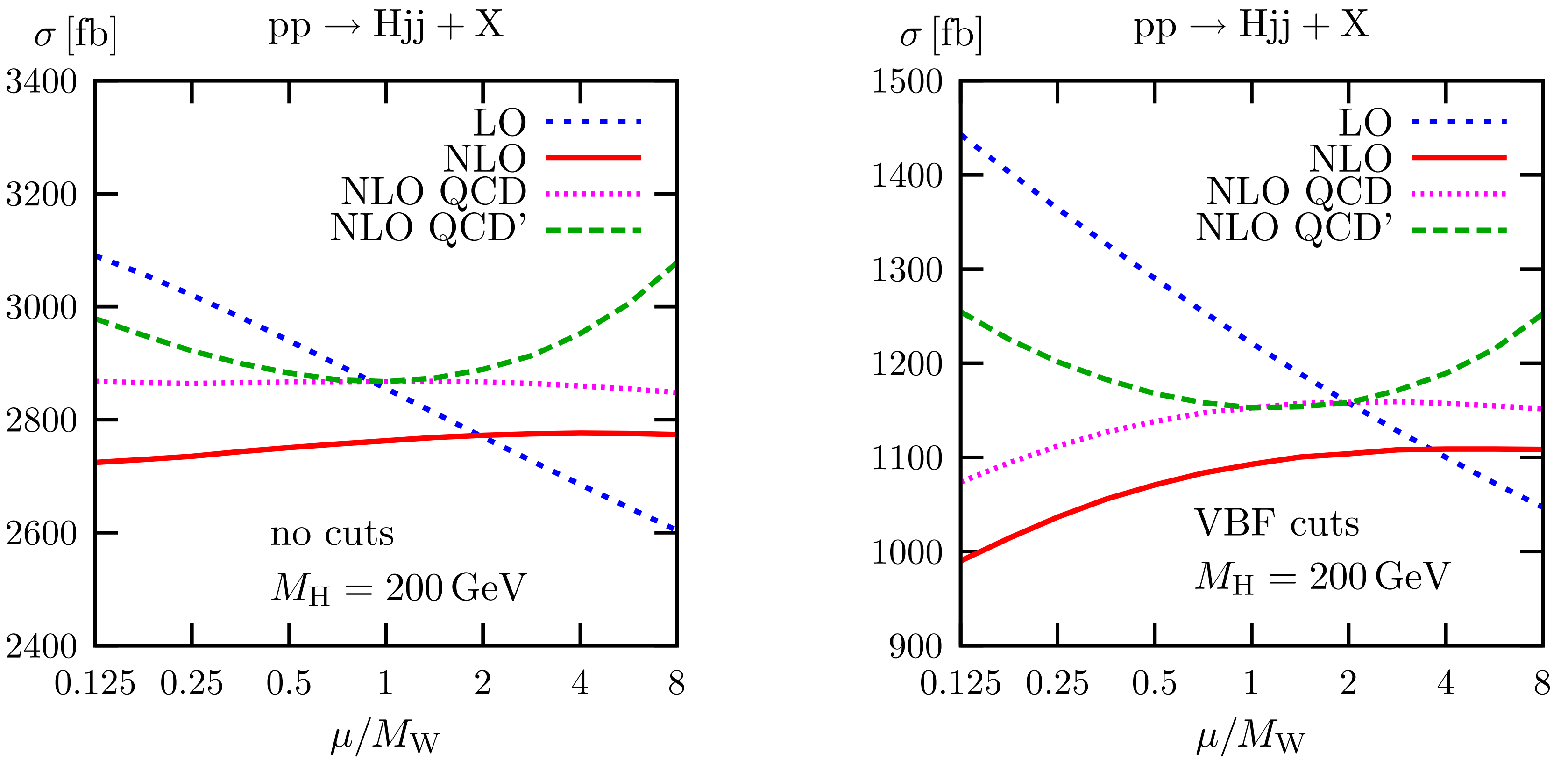
scale dep.: $M_{\mathrm{W}}/8 < \mu < 8M_{\mathrm{W}}$: NLO $\pm 8\%$, $\pm 9\%$ (LO $\pm 14\%$)

$M_{\mathrm{W}}/2 < \mu < 2M_{\mathrm{W}}$: NLO $\pm 2\%$, $\pm 2\%$ (LO $\pm 5\%$)

Scale dependence for $\mathrm{pp} \to \mathrm{H} + 2\mathrm{jets} + X$

$M_\mathrm{H} = 200\,\mathrm{GeV}$

Ciccolini, Denner, Dittmaier ’07



$\mu_\mathrm{R} = \mu_\mathrm{F} \equiv \mu$ for LO, NLO and NLO QCD

$\mu_\mathrm{R} = M_\mathrm{W}^2/\mu$ for NLO QCD’

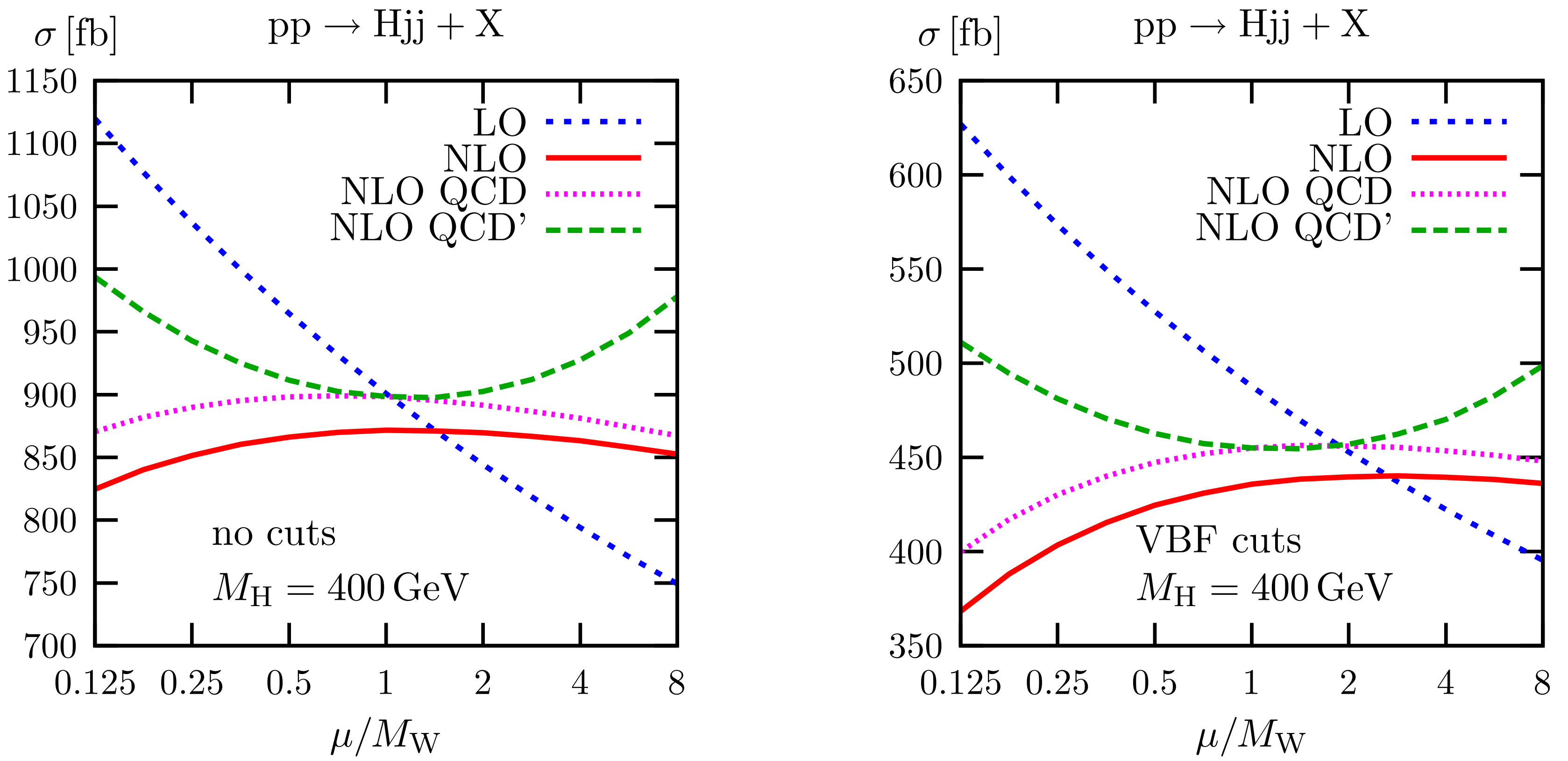
scale dep.: $M_\mathrm{W}/8 < \mu < 8M_\mathrm{W}$: NLO ${}^{-2\%}_{+9\%}$, $\pm 11\%$ (LO $\pm 9\%$, $\pm 18\%$)

$M_\mathrm{W}/2 < \mu < 2M_\mathrm{W}$: NLO $\pm 1\%$, $\pm 2\%$ (LO $\pm 3\%$, $\pm 6\%$)

Scale dependence for $\mathrm{pp} \to \mathrm{H} + 2\mathrm{jets} + X$

$M_{\mathrm{H}} = 400\,\mathrm{GeV}$

Ciccolini, Denner, Dittmaier ’07



$\mu_{\mathrm{R}} = \mu_{\mathrm{F}} \equiv \mu$ for LO, NLO and NLO QCD

$\mu_{\mathrm{R}} = M_{\mathrm{W}}^2/\mu$ for NLO QCD’

scale dep.: $M_{\mathrm{W}}/8 < \mu < 8M_{\mathrm{W}}$: NLO $\pm 11\%$, $\pm 15\%$ (LO $\pm 24\%$, $\pm 29\%$)

$M_{\mathrm{W}}/2 < \mu < 2M_{\mathrm{W}}$: NLO $\pm 1\%$, $\pm 3\%$ (LO $\pm 7\%$, $\pm 8\%$)

Comparison with existing NLO QCD calculations

Tuned comparison: only squared t- and u-channel diagrams, no interferences

VVH2 by M. Spira based on Hahn, Valencia, Willenbrock ’92 no cuts

$M_{\mathrm{H}}\,[\mathrm{GeV}]$	120	150	170	200	400	700
$\sigma_{\mathrm{LO}}\,[\mathrm{fb}]$	4226.3(6)	3357.8(5)	2910.7(4)	2381.6(3)	817.6(1)	257.49(4)
$\sigma_{\mathrm{LO}}^{\mathrm{VV2H}}\,[\mathrm{fb}]$	4226.2(4)	3357.3(3)	2910.2(3)	2380.4(2)	817.33(8)	257.40(3)
$\sigma_{\mathrm{NLO}}\,[\mathrm{fb}]$	4424(4)	3520(3)	3052(3)	2505(2)	858.4(7)	268.2(2)
$\sigma_{\mathrm{NLO}}^{\mathrm{VV2H}}\,[\mathrm{fb}]$	4415(1)	3519.7(8)	3055.8(7)	2503.4(6)	858.8(2)	268.03(6)

agreement within 0.2% $\sim$ statistical error

VBF cuts: VBFNLO Zeppenfeld et al. VBF cuts

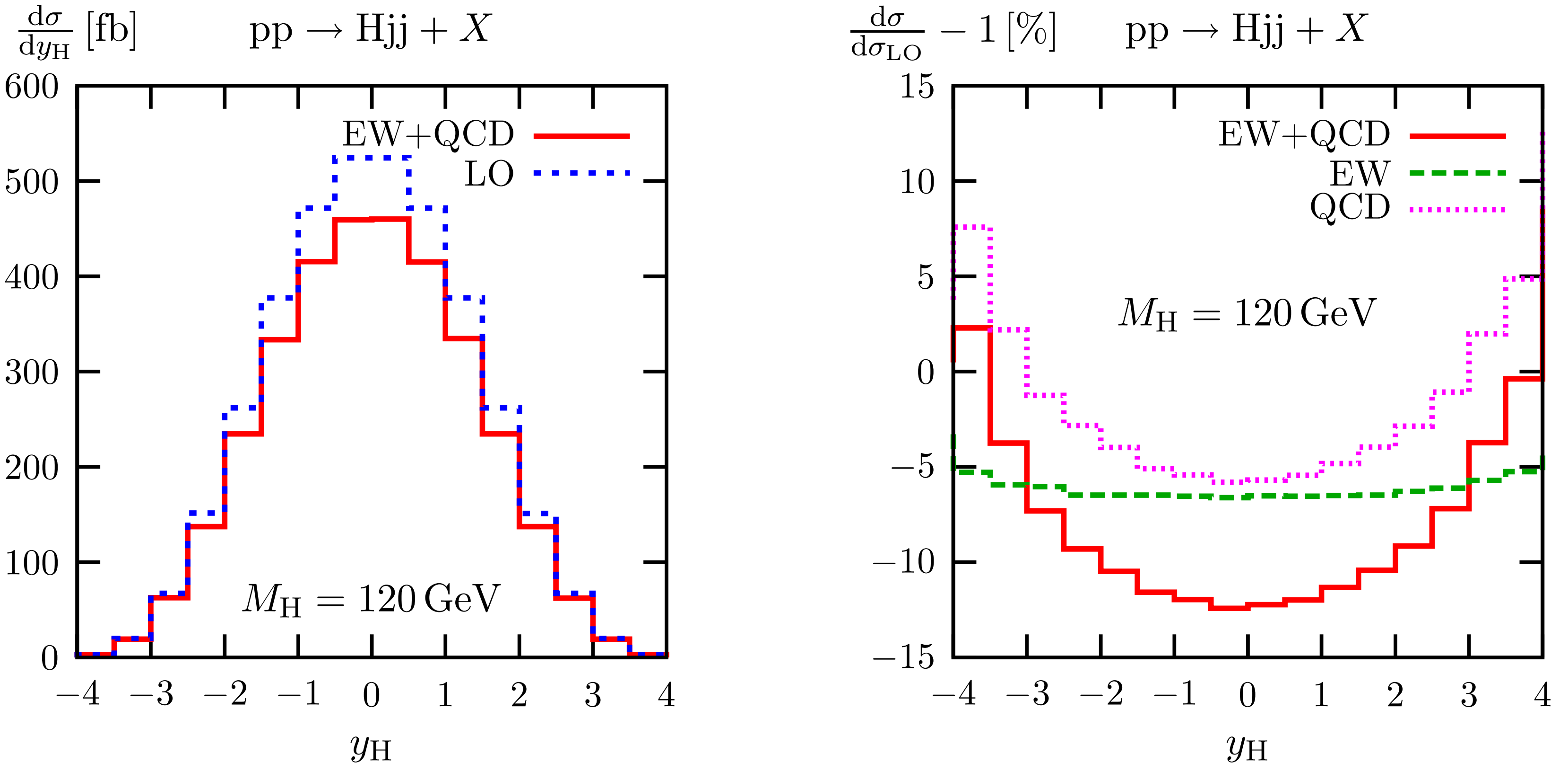
$M_{\mathrm{H}}\,[\mathrm{GeV}]$	120	150	170	200	400	700
$\sigma_{\mathrm{LO}}\,[\mathrm{fb}]$	1686.2(3)	1433.4(2)	1290.3(2)	1106.8(1)	451.27(5)	153.68(2)
$\sigma_{\mathrm{LO}}^{\mathrm{VBFNLO}}\,[\mathrm{fb}]$	1686.90(5)	1433.79(4)	1290.42(4)	1106.97(3)	451.31(1)	153.689(4)
$\sigma_{\mathrm{NLO}}\,[\mathrm{fb}]$	1728(2)	1463(1)	1313(2)	1121(1)	444.8(3)	147.2(1)
$\sigma_{\mathrm{NLO}}^{\mathrm{VBFNLO}}\,[\mathrm{fb}]$	1728.8(2)	1461.7(2)	1311.7(1)	1119.8(1)	444.71(3)	147.14(1)

agreement within 0.1% $\sim$ statistical error

Distribution in the rapidity of the Higgs boson

VBF cuts

Ciccolini, Denner, Dittmaier ’07



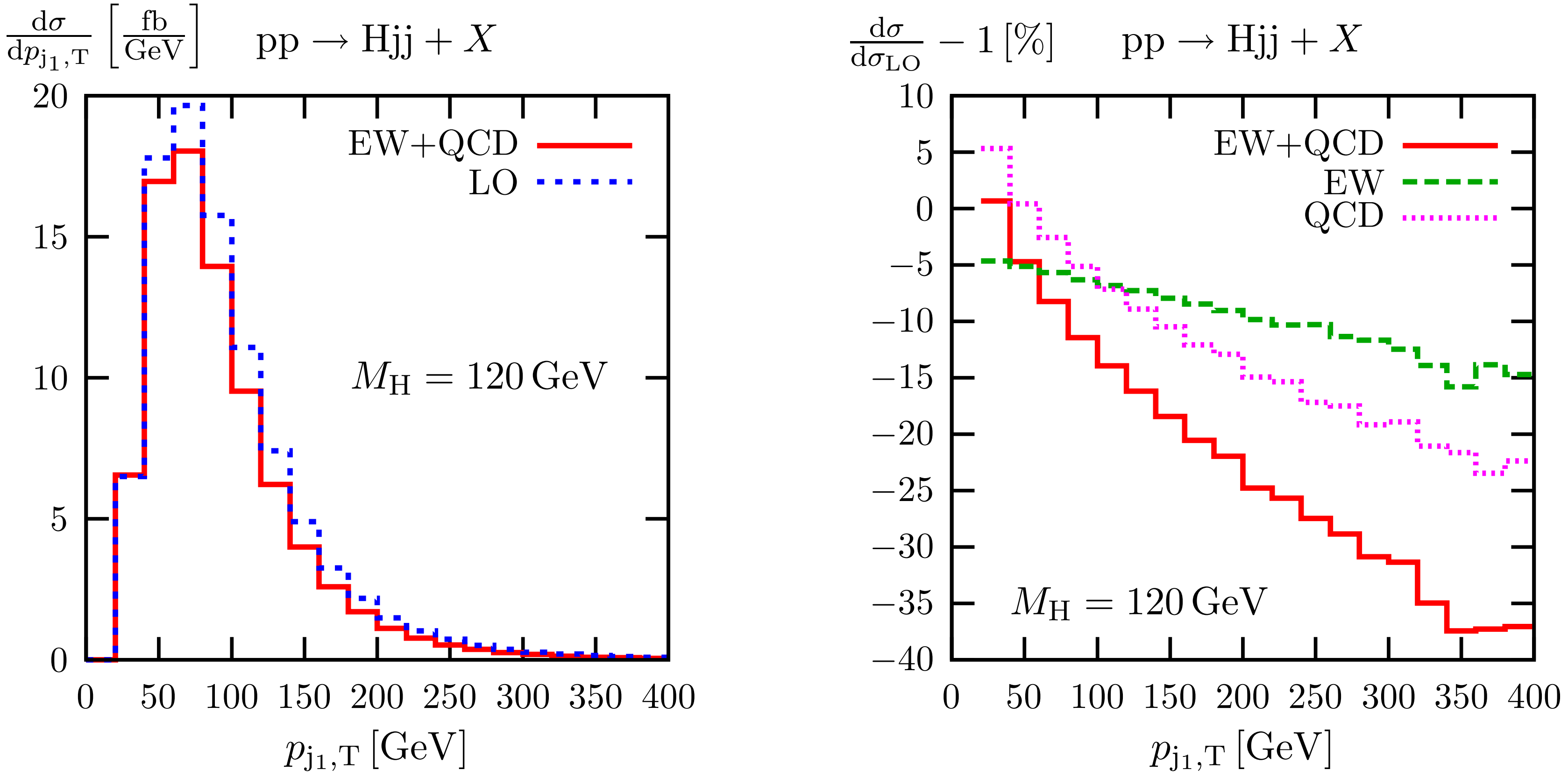
QCD corrections peak in forward and backward direction

EW corrections depend weakly on y_H

Distribution in transverse momentum of harder tagging jet

VBF cuts

Ciccolini, Denner, Dittmaier ’07



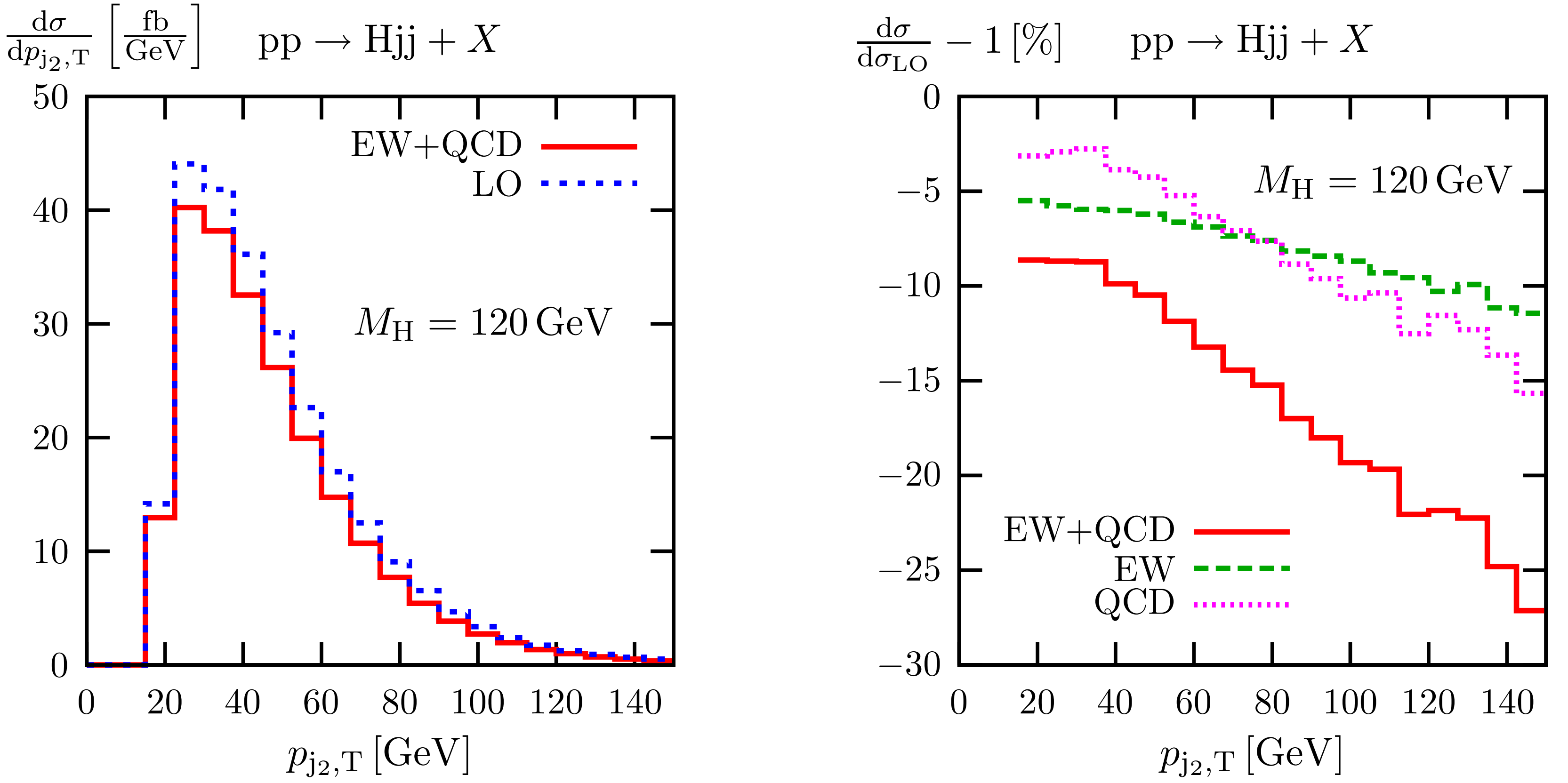
QCD and EW corrections become more and more negative for large p_{T,j_1}

QCD and EW corrections add up for large p_{T,j_1}

Distribution in transverse momentum of softer tagging jet

VBF cuts

Ciccolini, Denner, Dittmaier '07



QCD and EW corrections become more and more negative for large p_{T,j_2}

QCD and EW corrections add up for large p_{T,j_1}

Standard matrix elements for $\bar{\mathrm{q}}\mathrm{q} \longrightarrow \mathrm{t}\bar{\mathrm{t}}\mathrm{b}\bar{\mathrm{b}}$

25 types of standard matrix elements (polarization dependent)

- 10 of "massless" type: one Dirac matrix per chain

$$\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\ \bar{v}(p_3)\gamma^\mu\omega_\beta u(p_4)\ \bar{v}(p_5)\gamma^\mu\omega_\rho u(p_6)$$

$$\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\ \bar{v}(p_3)\not{p}_j\omega_\beta u(p_4)\ \bar{v}(p_5)\not{p}_k\omega_\rho u(p_6)$$

- 15 of "massive" type: 2/0 Dirac matrices inside the $\mathrm{t}\bar{\mathrm{t}}$ chain*

$$\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\ \bar{v}(p_3)\not{p}_j\gamma^\mu\omega_\beta u(p_4)\ \bar{v}(p_5)\not{p}_k\omega_\rho u(p_6)$$

$$\bar{v}(p_1)\gamma^\mu\omega_\alpha u(p_2)\ \bar{v}(p_3)\not{p}_j\not{p}_j\omega_\beta u(p_4)\ \bar{v}(p_5)\gamma^\mu\omega_\rho u(p_6)$$

$$\bar{v}(p_1)\gamma^\mu\omega_\alpha u(p_2)\ \bar{v}(p_3)\gamma^\mu\gamma^\nu\omega_\beta u(p_4)\ \bar{v}(p_5)\gamma^\nu\omega_\rho u(p_6)$$

$$\bar{v}(p_1)\gamma^\mu\omega_\alpha u(p_2)\ \bar{v}(p_3)\omega_\beta u(p_4)\ \bar{v}(p_5)\gamma^\mu\omega_\rho u(p_6)$$

$$\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\ \bar{v}(p_3)\omega_\beta u(p_4)\ \bar{v}(p_5)\not{p}_k\omega_\rho u(p_6)$$

*price to pay for the presence of massive top quarks