Towards a complete NNLO Prediction of the $\bar{B} \to X_s\gamma$ decay rate

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Collaborators:
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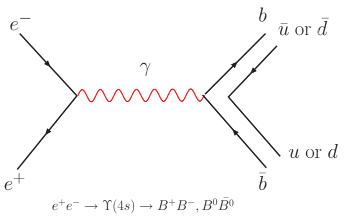
Outline

- Motivations $\&$ theoretical framework
- Current state-of-the-art for NNLO corrections
- $\mathcal{O}\left(\alpha_s^2\right)$ corrections to $\langle s\gamma|O_{1,2}|b\rangle$
  - first step: $\mathcal{O}\left(\alpha_s^2\, n_f\right)$ contribution
  - second step: bosonic contribution
    $\rightarrow$ applied techniques for the calculation of masters
- Summary and conclusions

Motivations

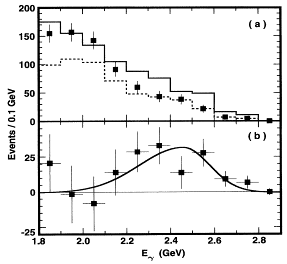
- $\bar{B} \to X_s\gamma$ most precise short-distance information currently available for $\Delta B = 1$ FCNC




PHYSICAL REVIEW LETTERS

First Measurement of the Rate for the Inclusive Radiative Penguin Decay $b \to s\gamma$

(CLEO Collaboration)



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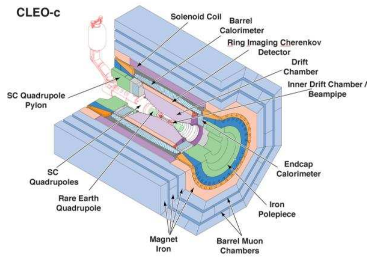
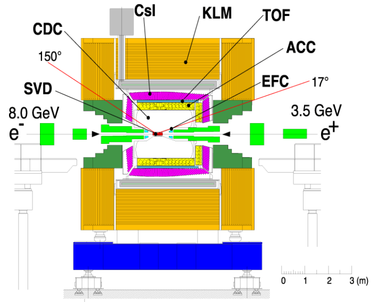


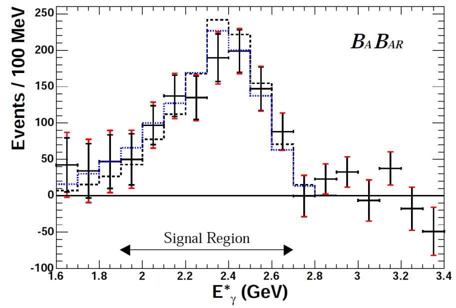


$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm exp}_{E_\gamma > 1.6\,\rm GeV} = (3.55 \pm 0.26) \times 10^{-4}$$

[HFAG2006]

- less sensitive to non-perturbative effects
  dominant ones: $\mathcal{O}(\Lambda^2/m_b^2)$, $\mathcal{O}(\Lambda^2/m_c^2)$, $\mathcal{O}(\alpha_s\Lambda/m_b)$

$$\Longrightarrow \Gamma(\bar{B} \to X_s\gamma) \approx \Gamma(b \to X_s^{parton}\gamma)$$
$$= \Gamma(b \to s\gamma) + \Gamma(b \to s\gamma g) + \ldots$$



Motivations

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First Measurement of the Rate for the Inclusive Radiative Penguin Decay $b \to s\gamma$

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We have measured the inclusive $b \to s\gamma$ branching ratio to be $(2.32 \pm 0.57 \pm 0.35) \times 10^{-4}$, where the first error is statistical and the second is systematic. Upper and lower limits on the branching ratio,

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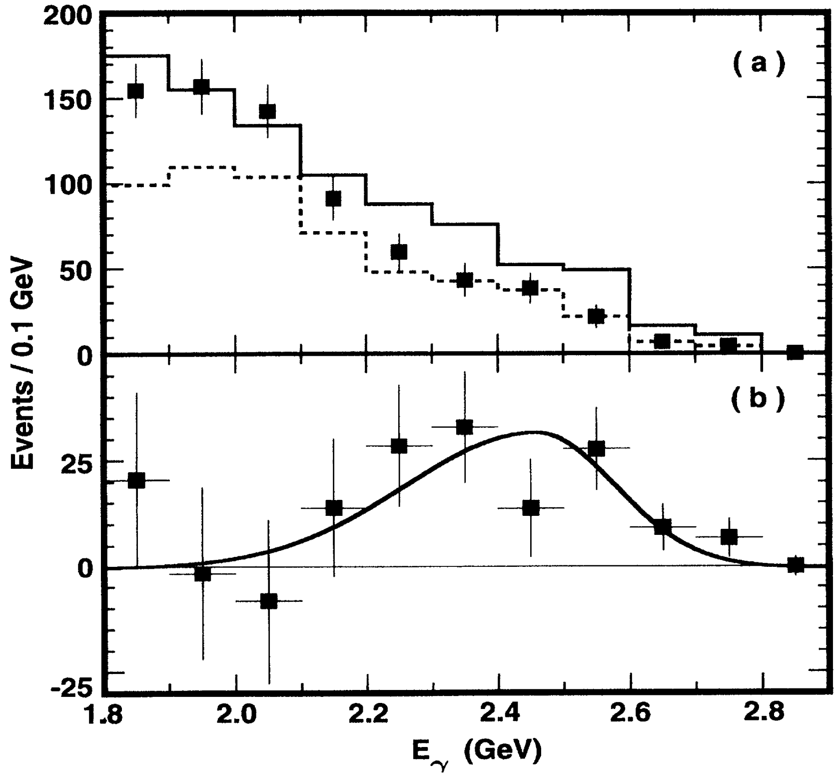

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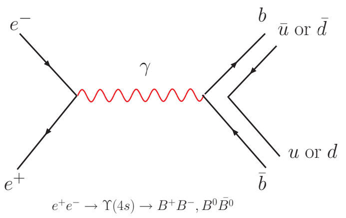

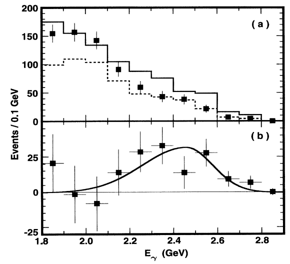

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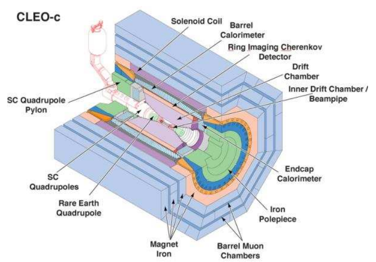

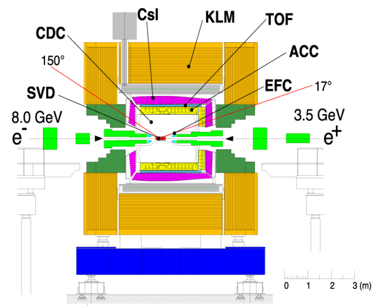

$$\boxed{\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm exp}_{E_\gamma > 1.6\,{\rm GeV}} = (3.55 \pm 0.26) \times 10^{-4}}$$ [HFAG2006]

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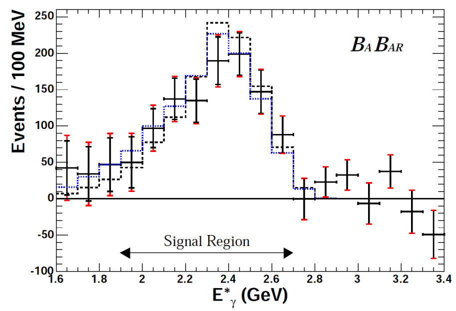
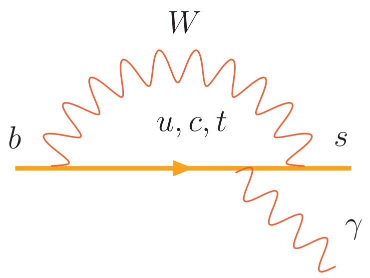
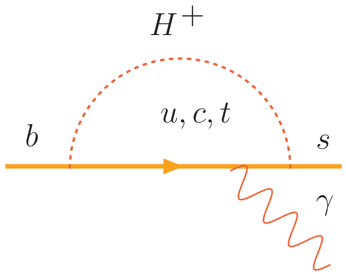


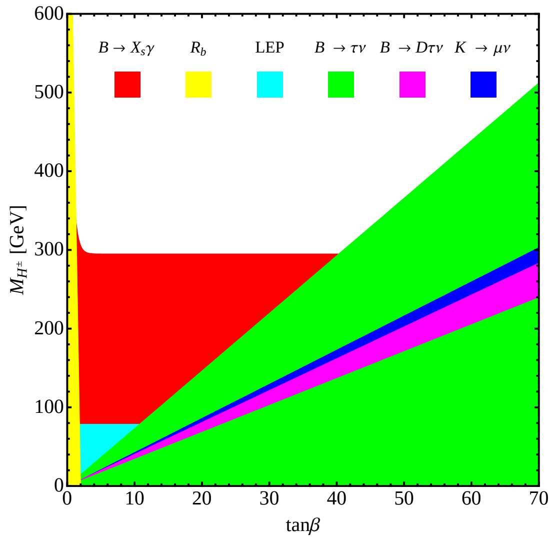
$$\Longrightarrow \quad \Gamma(\bar{B} \to X_s\gamma) \quad \approx \quad \Gamma(b \to X_s^{parton}\gamma)$$
$$= \quad \Gamma(b \to s\gamma) + \Gamma(b \to s\gamma g) + \ldots$$

Motivations

- loop induced in SM and highly sensitive to new physics which is not suppressed by factors of α as compared to SM contributions







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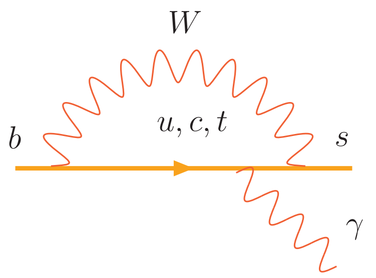
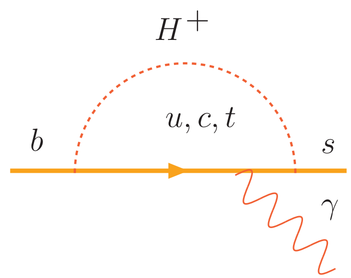
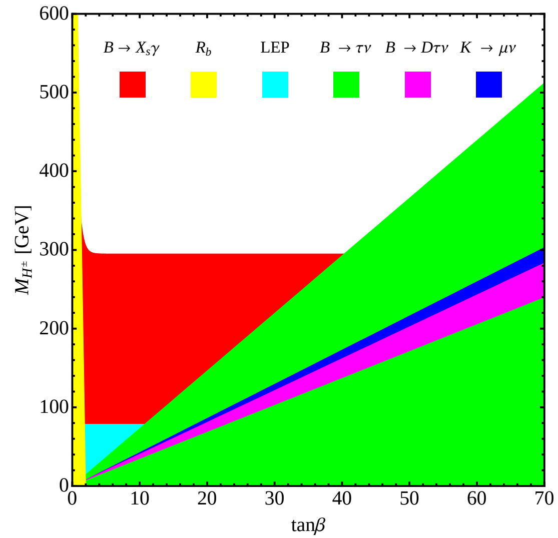




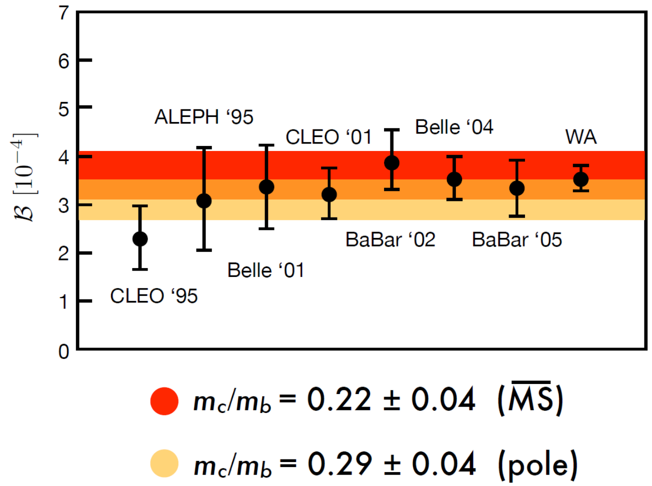


- Experimental precision already better than theoretical NLO prediction
  - $\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{th,NLO}}_{E_\gamma > 1.6\,\mathrm{GeV}} = (3.57 \pm 0.30) \times 10^{-4}$

    [Misiak et al 2001,Buras et al 2002]
  - $\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{exp}} = (3.55 \pm 0.26) \times 10^{-4}$

    [HFAG 2006]

  Super-B factory: 5% uncertainty possible (more statistics, lower E_γ)



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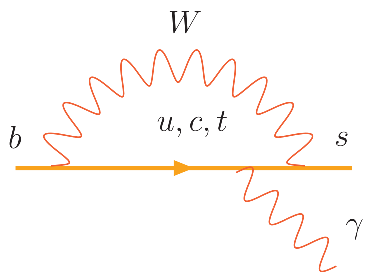

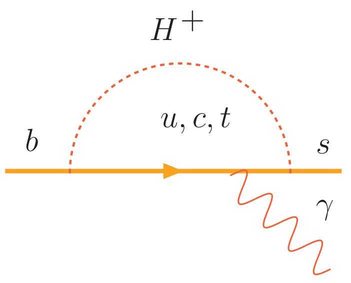

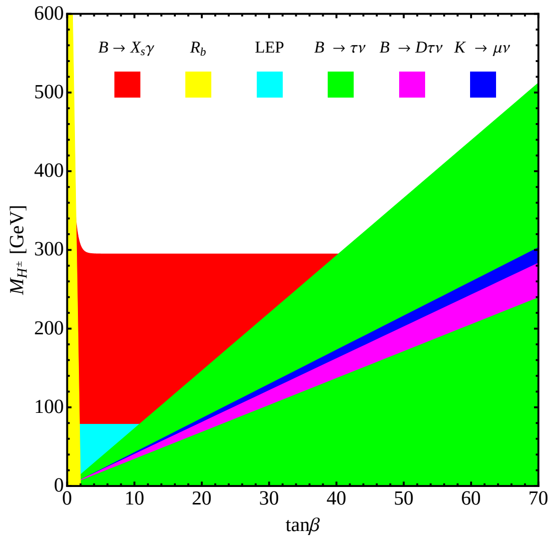

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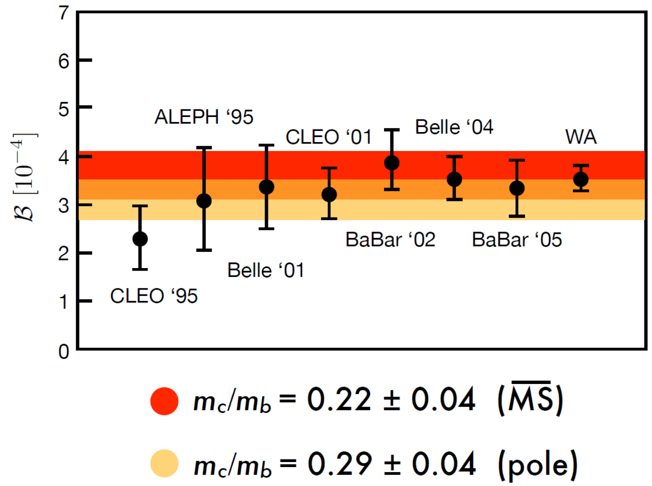

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  Super-B factory: 5% uncertainty possible
  (more statistics, lower E_γ)

$\Rightarrow$ strong constraints on new physics require better theoretical precision

Motivations

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[HFAG 2006]

- Contributions to the theory prediction

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma>1.6\,\rm GeV} = \mathcal{B}(\bar{B} \to X_c e\bar{\nu})_{\rm exp} \left[\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to ce\bar{\nu})}\right]_{\rm LO\ EW} f\left(\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right)\times$$

$$\times\Bigg\{\underbrace{1 + \underset{\substack{\textbf{NLO}\\ \sim 25\%}}{\mathcal{O}(\alpha_s)} + \underset{\substack{\textbf{NNLO}\\ \sim 7\%}}{\mathcal{O}(\alpha_s^2)} + \underset{\sim 4\%}{\mathcal{O}(\alpha_{\rm em})}}_{\textbf{perturbative corrections}} + \underbrace{\underset{\sim 1\%}{\mathcal{O}\left(\frac{\Lambda^2}{m_b^2}\right)} + \underset{\sim 3\%}{\mathcal{O}\left(\frac{\Lambda^2}{m_c^2}\right)} + \underset{<\sim 5\%}{\mathcal{O}\left(\frac{\Lambda}{m_b}\alpha_s\right)}}_{\textbf{non-perturbative corrections}}\Bigg\}$$

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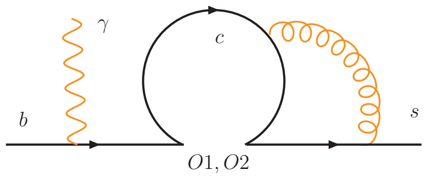
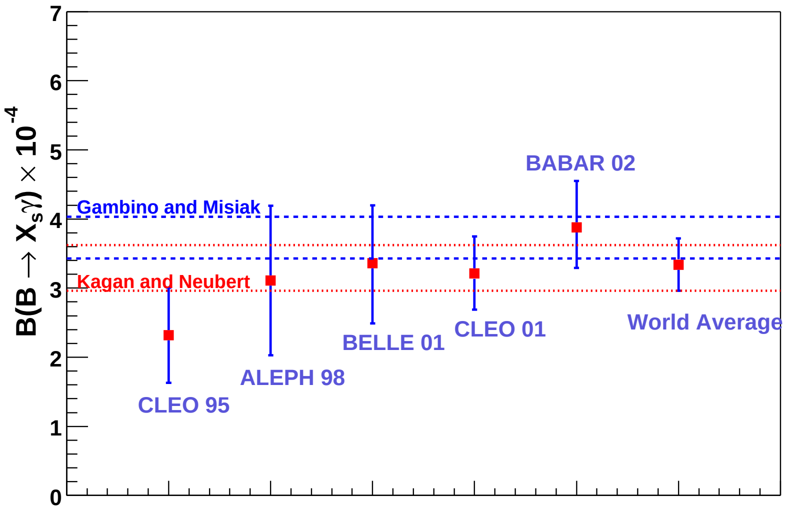
expected NNLO corrections to $\mathcal{B}$ ($\sim 7\%$) are of the same size as the experimental error

Motivations

- Charm quark mass definition ambiguity
  - dependence of $\mathcal{B}(\bar{B} \to X_s\gamma)^{theo}$ on m_c enters through the $\langle s\gamma|O_{1,2}|b\rangle$ which start contributing at $\mathcal{O}(\alpha_s)$
  - $m_c^{pole}/m_b^{pole} = 0.29 \pm 0.02$
    $\mathcal{B}(\bar{B} \to X_s\gamma)^{theo} = (3.32 \pm 0.30) \times 10^{-4}$
  - $\overline{m}_c(m_b/2)/m_b^{pole} = 0.22 \pm 0.04$
    $\mathcal{B}(\bar{B} \to X_s\gamma)^{theo} = (3.70 \pm 0.30) \times 10^{-4}$



[Aubertetal 02]



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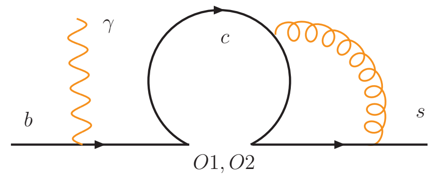

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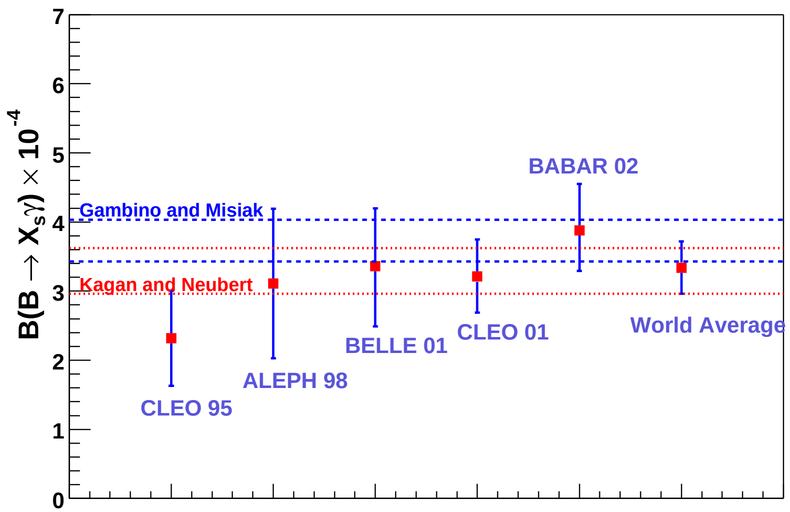


- difference between using $\overline{m}_c(\mu)$ and m_c^{pole} is a NNLO effect in the branching ratio
  $\Longrightarrow$ resolving the ambiguity requires going to the NNLO level

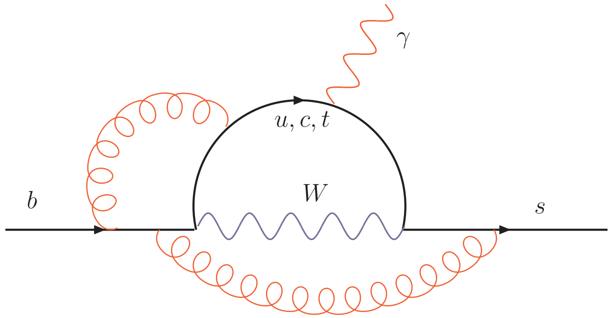
Theoretical framework

- diagrams involve scales with large hierarchy

  $M_W\,, M_t \gg m_b \gg m_s \implies$ large $\log\left(\frac{M_W^2}{m_b^2}\right)$

  $\longrightarrow$ resummation of $\alpha_s \log\left(\frac{M_W^2}{m_b^2}\right)$ is necessary

  using RG techniques



- start by introducing an effective theory without the heavy fields

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD \times QED}(u,d,s,c,b) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i(\mu)\, O_i(\mu)$$

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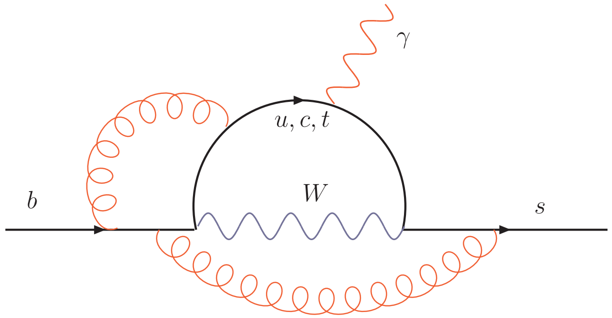

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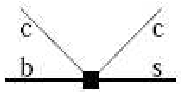
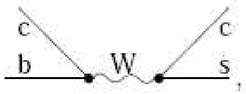
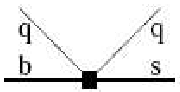
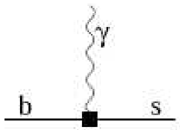
$O_{1,2}$ =


$= (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b)$, **from**


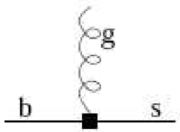
, $|C_i(m_b)| \sim 1$

$O_{3,4,5,6}$ =


$= (\bar{s}\Gamma_i b)\sum_q(\bar{q}\Gamma_i' q)$, $|C_i(m_b)| < 0.07$

O_7 =


$= \frac{e m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}$, $C_7(m_b) \simeq -0.3$

O_8 =


$= \frac{g m_b}{16\pi^2}\, \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}$, $C_8(m_b) \simeq -0.15$

Theoretical framework

Calculation done in three steps:

- Matching find the Wilson coefficients $C_i(\mu)$ by comparing the full and the effective theory at the mass scale $\mu \approx M_W$
  $\Rightarrow$ no large logarithms and only vacuum diagrams

- Mixing compute the anomalous dimensions of the operators and solve the renormalization group equations to go down with the Wilson coefficients to $\mu \approx m_b$

$$\frac{d}{d\mu} C_j(\mu) = C_i(\mu)\gamma_{ij}(\mu)$$

- Matrix elements calculate the matrix elements of all the operators at $\mu \approx m_b \Rightarrow$ no large logarithms as no heavy masses are present

Current state-of-the-art for NNLO corrections

1. Matching
   - 2-loop matching for ($O_1, \ldots, O_6$) [Bobeth,Misiak,Urban 00]
   - 3-loop matching for O_7 and O_8 [Misiak,Steinhauser 04]
2. Mixing
   - 3-loop: $(O_1, \ldots, O_6)$ and $(O_7,\, O_8)$ sectors [Gorbahn,Haisch 05] [Gorbahn,Haisch,Misiak 05]
   - 4-loop $(O_1, \ldots, O_6) \longrightarrow (O_7,\, O_8)$ [Czakon,Haisch,Misiak 06]

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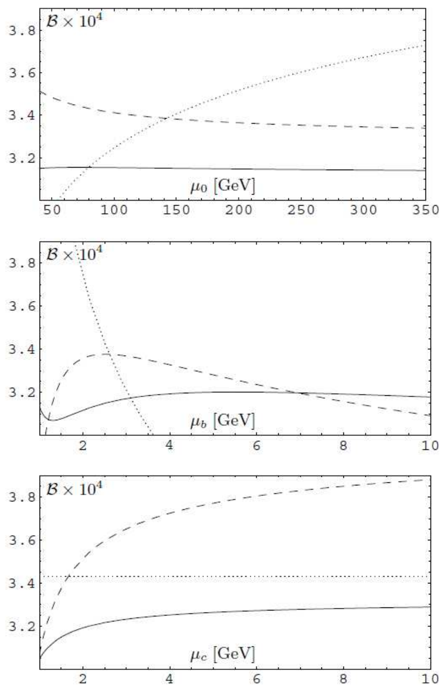
3. Matrix elements
   - $O_1,\ O_2,\ O_7,\ O_8$ large β_0 [Bieri,Greub,Steinhauser 03]
   - O_7 [Blokland,Czarnecki,Misiak,Slusarczyk,Tkachov 05] [Asatrian,Hovhannisyan,Poghosyan,Ewerth,Greub,Hurth 06]
   - O_7, photon spectrum [Melnikov,Mitov 05] [Asatrian,Ewerth,Ferroglia,Gambino,Greub 06]
   - O_1, O_2 leading term for $m_c \gg m_b$ [Misiak,Steinhauser 06]

The NNLO estimated Branching Ratio

$$\boxed{\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm theo}_{E_\gamma > 1.6\,{\rm GeV}} = (3.15 \pm 0.23) \times 10^{-4}}$$

[Misiak et al 06] [Misiak,Steinhauser 06]

- Decomposition of Uncertainty
  - non-perturbative $\quad 5\% \quad \mathcal{O}(\alpha_s \Lambda/m_b)$
  - parametric $\quad 3\% \quad \alpha_s(M_Z)$, $\mathcal{B}^{exp}_{SL}$, $m_c \ldots$
  - m_c interpolation $\quad 3\% \quad$ ($O_{1,2}$ matrix elements)
  - higher order $\quad 3\% \quad$ (μ_b, μ_c, μ_0 dependence)



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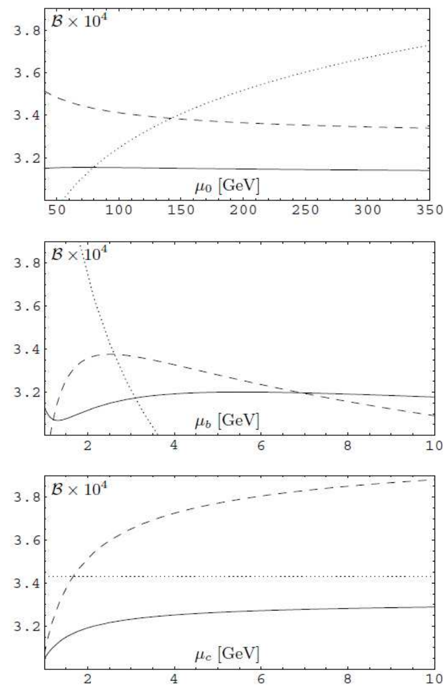

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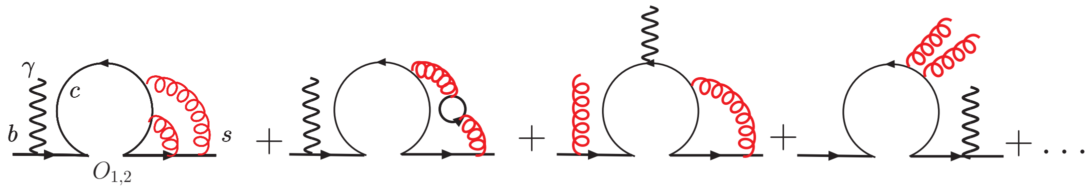
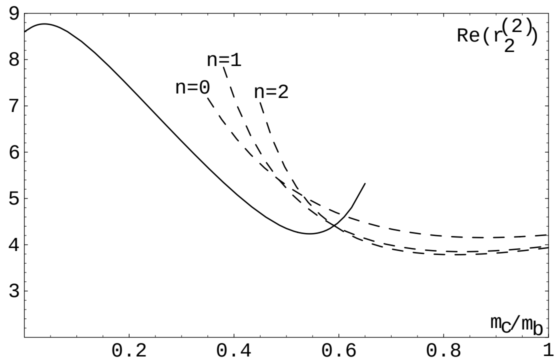
- source of the interpolation uncertainty is the missing $\boldsymbol{\mathcal{O}}\ (\boldsymbol{\alpha_s^2})$ correction to $\langle s\gamma | O_{1,2} | b \rangle$

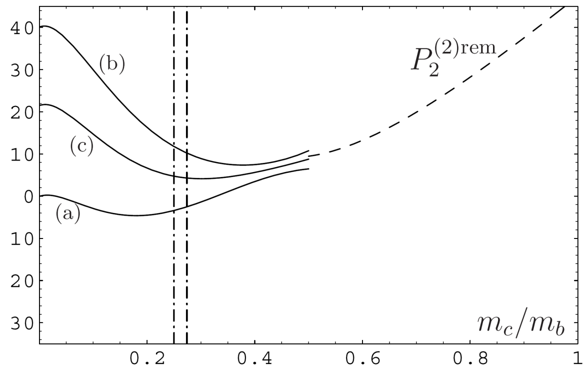


More about the interpolation uncertainty

- $\mathcal{O}\left(\alpha_s^2\right)$ perturbative contribution to $\mathcal{B}(\bar{B} \to X_s\gamma)$: $$P_2^{(2)} = \sum_{i,j=1}^{8} C_i^{(0)} C_j^{(0)} \left(n_f\, A_{ij} + B_{ij}\right)$$
- using large β_0 approx. $$P_2^{(2)} = \sum_{i,j=1}^{8} C_i^{(0)} C_j^{(0)} \left(\frac{-3}{2}\beta_0\, A_{ij} + B'_{ij}\right) = P_2^{(2),\beta_0} + P_2^{(2),rem}$$
  - $P_2^{(2),\beta_0}$ known for $\langle s\gamma|O_{1,2,7,8}|b\rangle$



- expansions in limits $m_c/m_b \to 0$ and $m_c \gg m_b$ match nicely for Re$\langle s\gamma|O_2|b\rangle^{\beta_0}$
- good approximation already for $n = 0$
- no large $c\bar{c}$ threshold effects at $m_c = m_b/2$

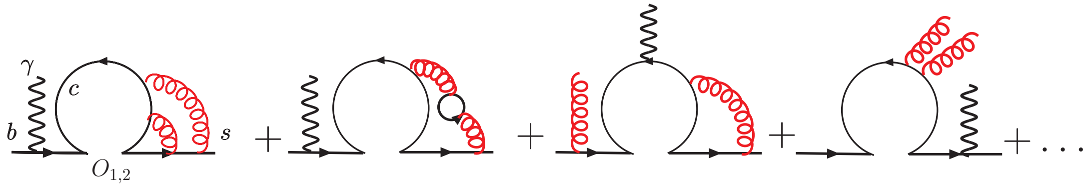


- calculate the leading term of large m_c expansion for $P_2^{(2),rem}$ and interpolate to physical m_c
- making assumptions for $P_2^{(2),rem}$ at $m_c = 0$ is the source of the interpolation uncertainty

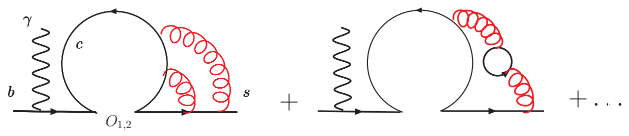
Reducing the overall uncertainty of $\mathcal{B}(\bar{B} \to X_s\gamma)^{\text{theo,NNLO}}_{E_\gamma > 1.6\,\text{GeV}}$

- removing the interpolation uncertainty

$\Longrightarrow$ need a complete calculation of $\langle \boldsymbol{s\gamma} | \boldsymbol{O_{1,2}} | \boldsymbol{b} \rangle$ at $m_c \neq 0$



$\longrightarrow$ working on the virtual part [R. B, Czakon, Schutzmeier]



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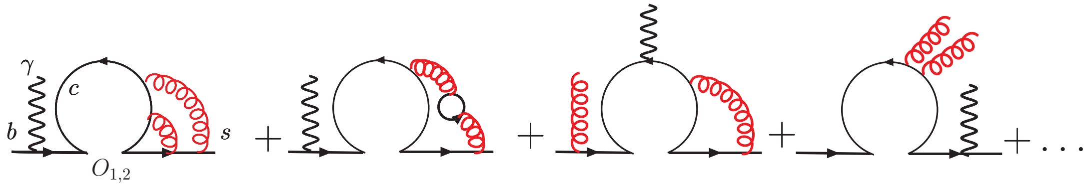

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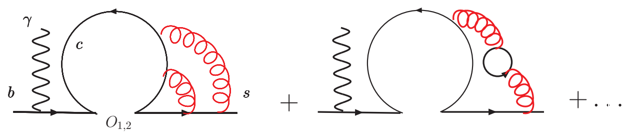

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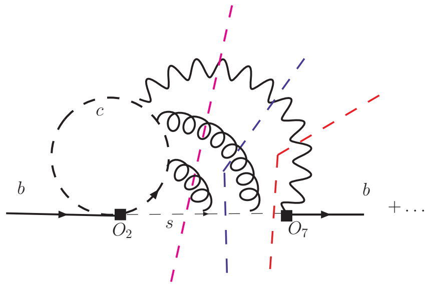

in progress [R. B, Czakon, Schutzmeier]

Removing the interpolation uncertainty: virtual part

- approx. 400 3-loop on-shell vertex diagrams with two scales m_b & m_c
- around 500 masters are involved in the bare amplitude
- symbolic reduction down to masters is not yet complete for the full 3-loop vertex
- $\mathcal{O}\left(\alpha_s^2 n_f\right)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$:

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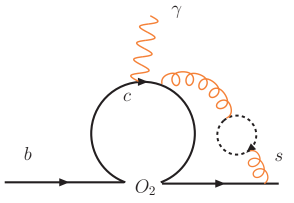

- masters were calculated with Mellin Barnes
  - first way: a numerical integration of the MB representations is performed for specific values of z using the `MB` package
    [MB : Czakon 05] ,
    [MBrepresentation : Chachamis, Czakon 06]
  - second way:
    - perform an expansion in $z = m_c^2/m_b^2$ by closing contours
    - coefficients of the expansion are given by at most a 1-dimensional MB integral expressed as a sum over residues
    - sum these infinite series using **XSummer** [Moch & Uwer 05]

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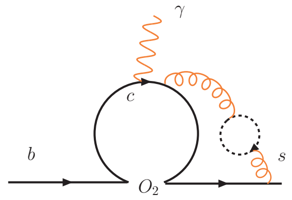

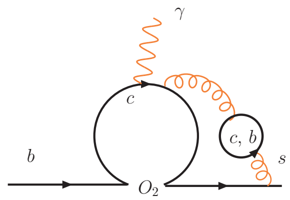

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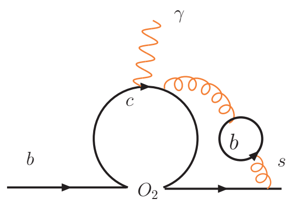
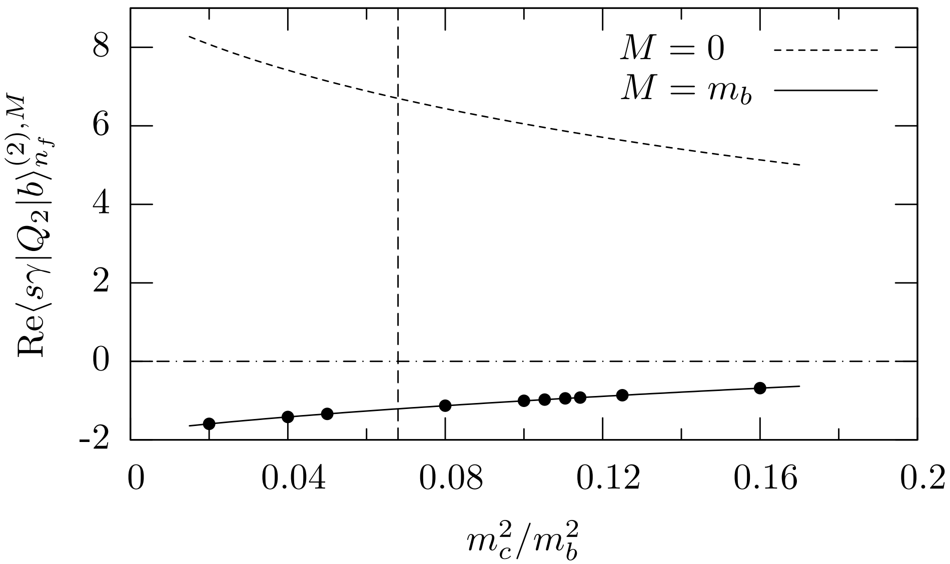
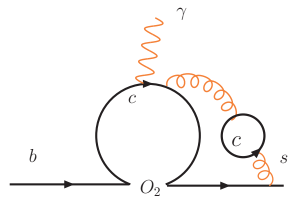
- MB alone was not enough to calculate all the masters due to poor convergence
- use differential equations solved numerically
  - boundaries were obtained using diagrammatic large mass expansion for $m_c \gg m_b$

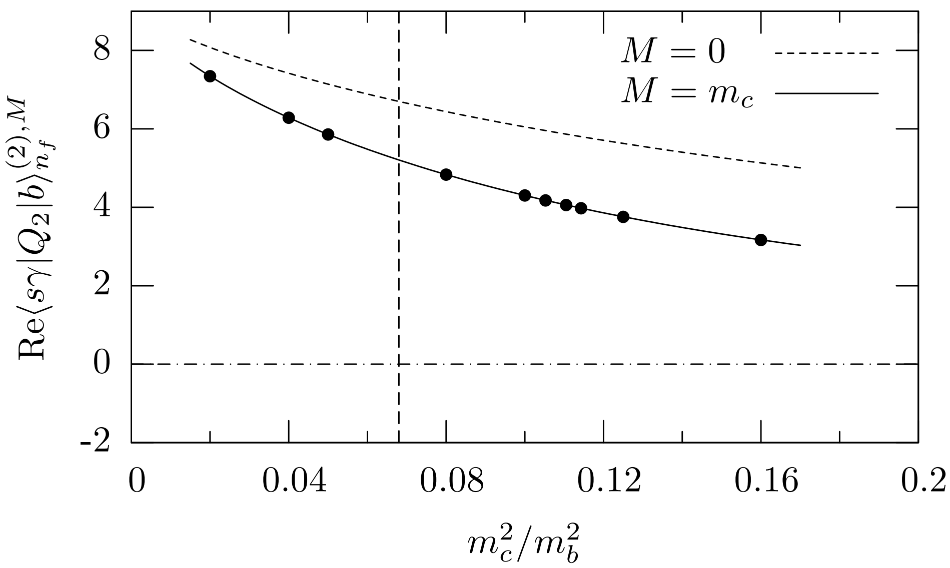
$\langle s\gamma|O_2|b\rangle_{\mathcal{O}(\alpha_s^2 n_f)}$

- Results for the massive fermionic contributions:









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- less pronounced differences for the c-quark $\longrightarrow$ moderate negative corrections wrt. massless approximation

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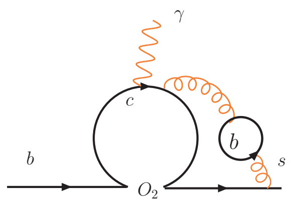

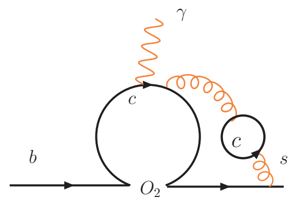

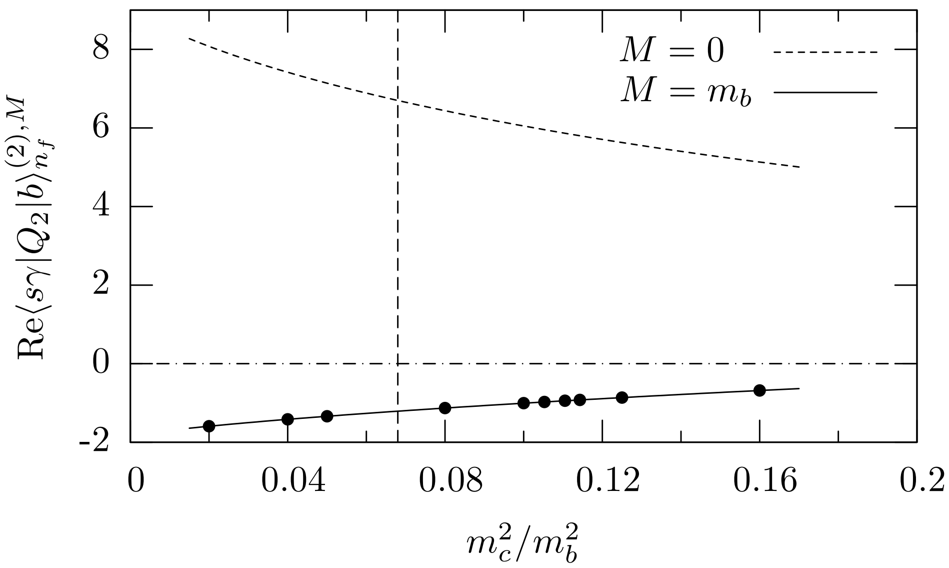

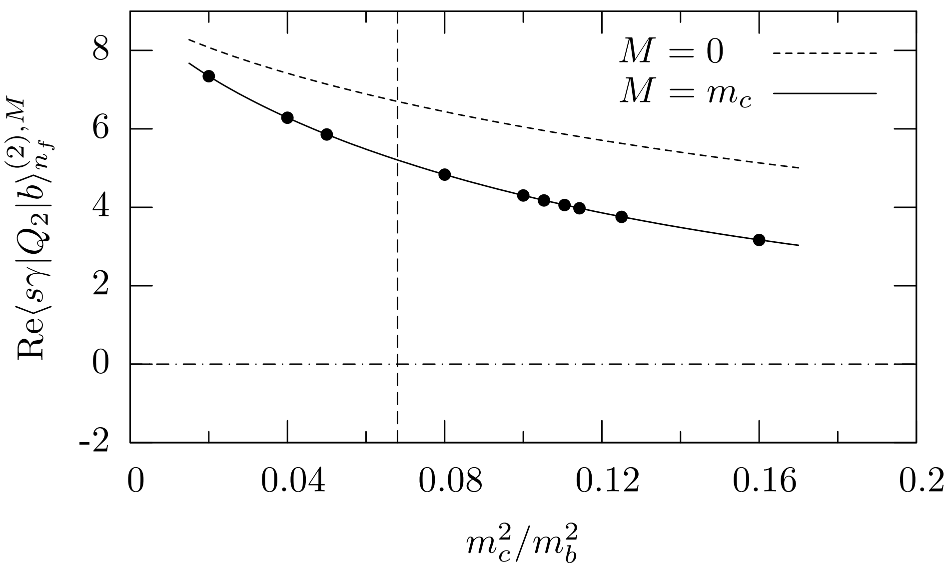

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numerical impact of the mass corrections on $\mathcal{B}(\bar{B} \to X_s\gamma) = +$ 1.1% for $\mu_b = 2.5$ GeV

Reducing the interpolation uncertainty

- calculating $\mathcal{O}\left(\alpha_s^2\right)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$ at $m_c = 0$ helps significantly in reducing the interpolation uncertainty

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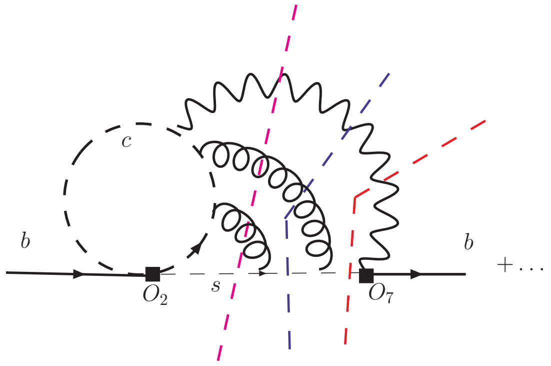

up to 4-particle cuts: $\gamma s, \gamma s g, \gamma s g g, \gamma s q \bar{q}$

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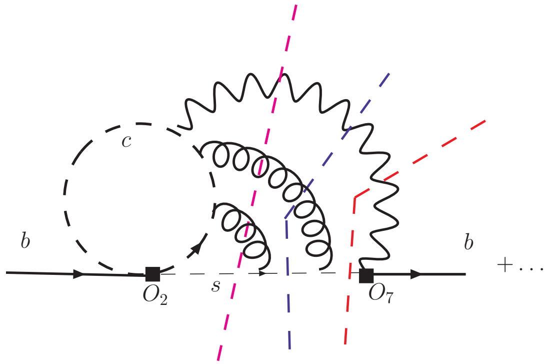
$\Longrightarrow$ 

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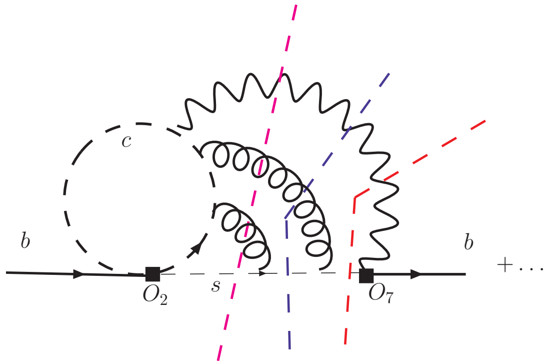
$\Longrightarrow$ 

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    - Mellin Barnes [Smirnov '99, Tausk '99]
    - Differential equations [Gehrmann, Remiddi '00]
    - Sector decomposition [Binoth, Heinrich '00]
    - Nested Sums [Moch, Uwer, Weinzierl '01]
    - Difference equations [Laporta '01]

How to get the masters ?

the 4-loop on-shell cut masters:

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  - do we know how to use it for integrals with on-shell unitarity cuts ?
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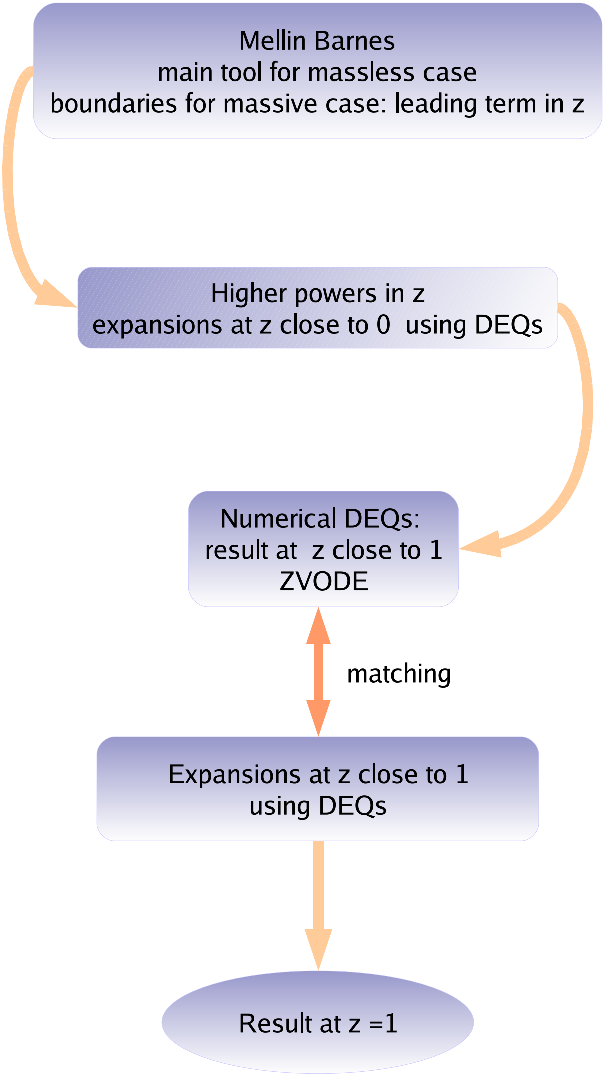
so what is the way out ?

Winning strategy: combining methods

Merging methods is the way to go, but a long chain of steps:



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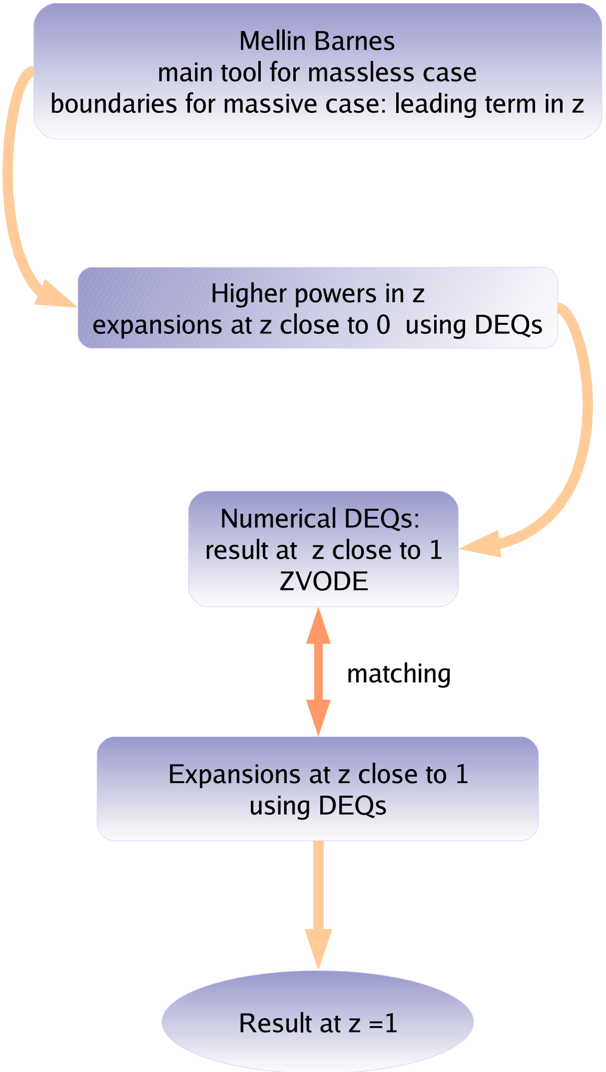

- evaluation of off-shell master integrals $V_i(z,\epsilon)$ with help of numerical differential equations (deqns) [Caffo, Czyz, Remiddi 98]

$$\frac{d}{dz} V_i(z,\epsilon) = A_{ij}(z,\epsilon) V_j(z,\epsilon), \quad z = p_b^2/m_b^2$$

- Idea:
  - calculate integrals at some "simple" point (e.g. $p_b^2 \ll m_b^2$)
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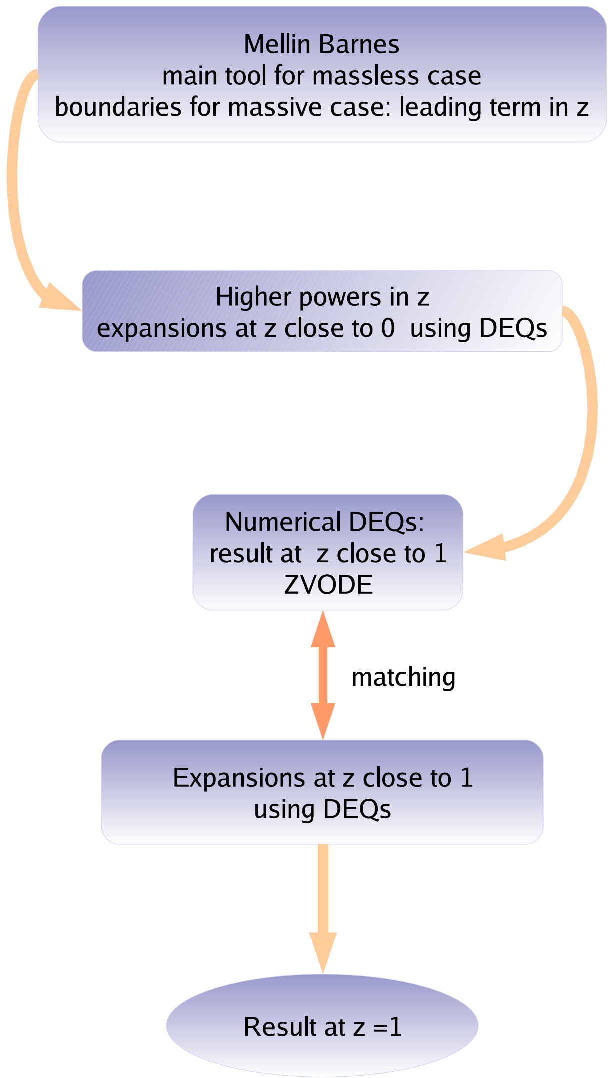

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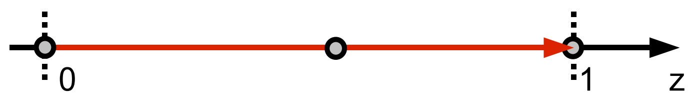

$\rightarrow$ but:deqns singular in both endpoints! (and on naive contour $z \in \mathbb{R}$)
$\Rightarrow$ solution:combine expansions with numerical integration in complex plane

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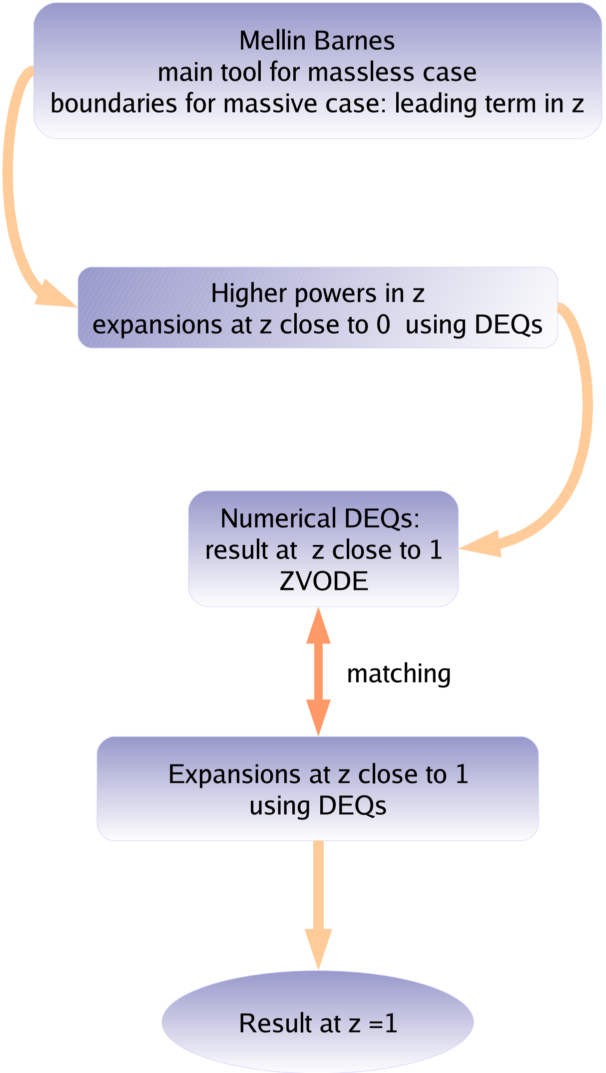


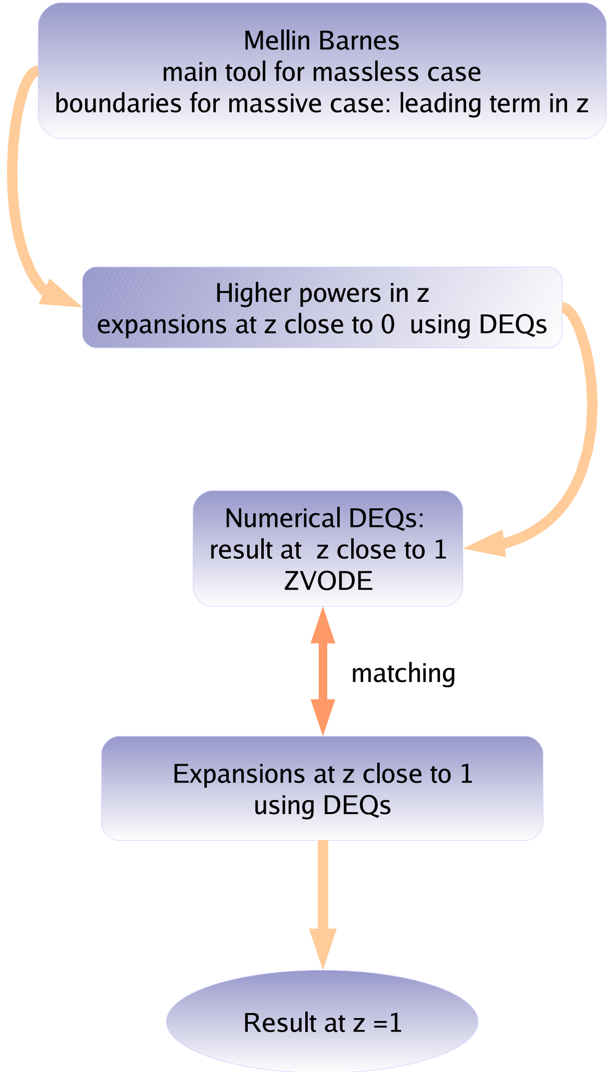
- expand in ϵ and z in the limit $z \to 0$ with ansatz:

$$V_i(z,\epsilon) = \sum_{nmk} c^0_{inmk} \epsilon^n z^m \log^k z$$

- solve recursively for c^0_{inmk} up to high powers in z
- boundary conditions:
  - Mellin Barnes $\&$ diagrammatic large-mass expansions for $p_b^2 \ll m_b^2$
    $\Rightarrow$ high precision values for $z \approx 0$

Winning strategy: combining methods

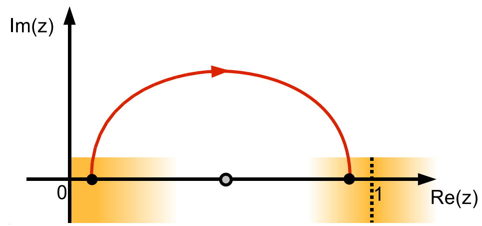
Merging methods is the way to go, but a long chain of steps:



- expand in ϵ and z in the limit $z \longrightarrow 0$ with ansatz:

$$V_i(z,\epsilon) = \sum_{nmk} c^0_{inmk} \epsilon^n z^m \log^k z$$

- solve recursively for c^0_{inmk} up to high powers in z
- boundary conditions:
  - Mellin Barnes & diagrammatic large-mass expansions for $p_b^2 \ll m_b^2$
    $\Rightarrow$ high precision values for $z \approx 0$
- use these values as starting point for numerical integration (in **complex** plane) up to $z \approx 1$ (**ZVODE**, `Hindmarsh et al`)



- perform another power logarithmic expansion around $z \longrightarrow 1$ and solve coefficients c^1_{inmk} recursively
- use numerical integration to fix the remaining c^1_{inmk}

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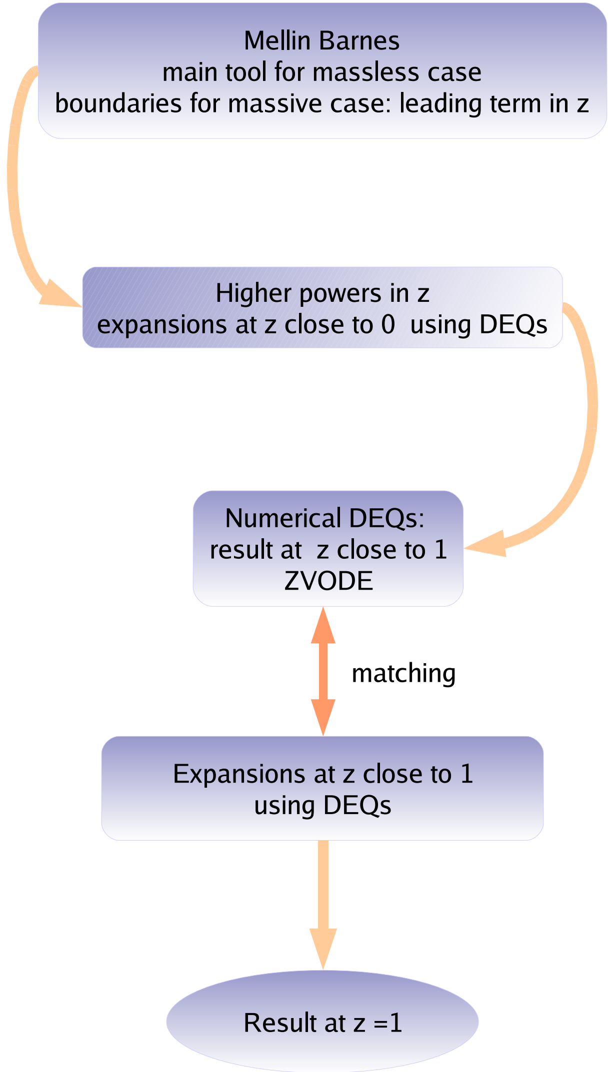

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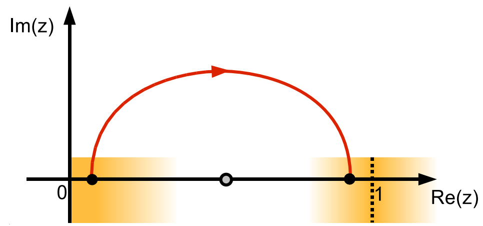

- perform another power logarithmic expansion around $z \to 1$ and solve coefficients c^1_{inmk} recursively
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result for $z = 1$ is the leading term

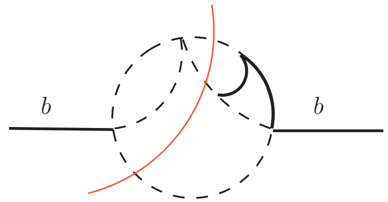
Boundaries for DEQs: 2- and 3-particle cuts

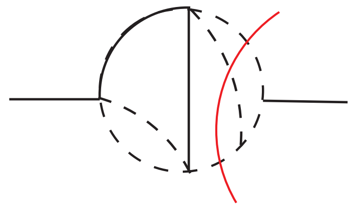
- derive a MB representation for loops on the left and the right of the cut
- integrate over the phase space analytically
- perform an analytic continuation in ε for $\varepsilon \longrightarrow 0$ [MB.m, M.Czakon]
- expand in $z = p_b^2/m_b^2$ where $p_b^2 \ll m_b^2$ by closing contours in the multi-fold MB integrals
- use Barnes Lemmas to remove some integrations if possible
- for multi-fold MB integrals (up to 3) integrate numerically $\longrightarrow$ we use a C++ implementation

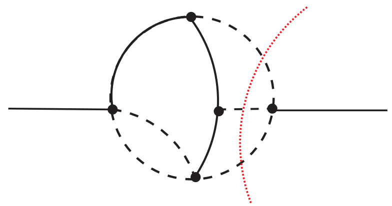
  of the double-exponential integration method [H. Takahasi & M. Mori] in quad-double precision

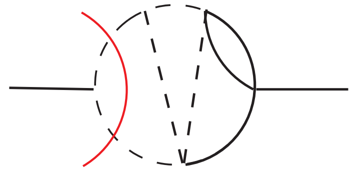
  ($\approx$ 64 digits) based on the qd library [Bailey, Hida, Li]

  $\Rightarrow$ all boundaries obtained with at least 16 digits



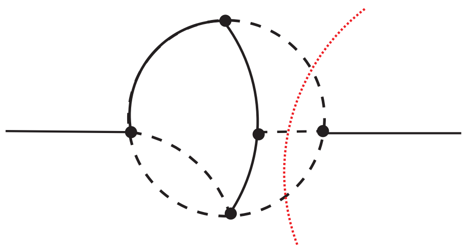






a boundary example

`PR214(1,1,1,1,1,1,1,1,0,0,0,0,0,0) =`



- a three-parameter MB representation:

$$\frac{1}{(2\pi i)^3}\int_{-i\infty}^{i\infty} dy_1 dy_2 dy_3\, (z)^{y_3}\, s_{12}^{y_1}\, s_{13}^{y_2}\, \Gamma(\ldots) \qquad \text{with } \; z = p_b^2/m_b^2$$

- after integrating over the three-particle phase space:

$$\frac{1}{(2\pi i)^3}\int_{-i\infty}^{i\infty} dy_1 dy_2 dy_3 (z)^{1-2ep+y_1+y_2+y_3}\, \Gamma(\ldots)$$

- After analytic continuation $\varepsilon \to 0$ and Laurent expansion in ε, the leading power of z for $z \to 0$ is extracted from the remaining MB integrals by taking residues
  $\Rightarrow$ only one MB-parameter is left for terms of order z up to $\mathcal{O}(\varepsilon^2)$

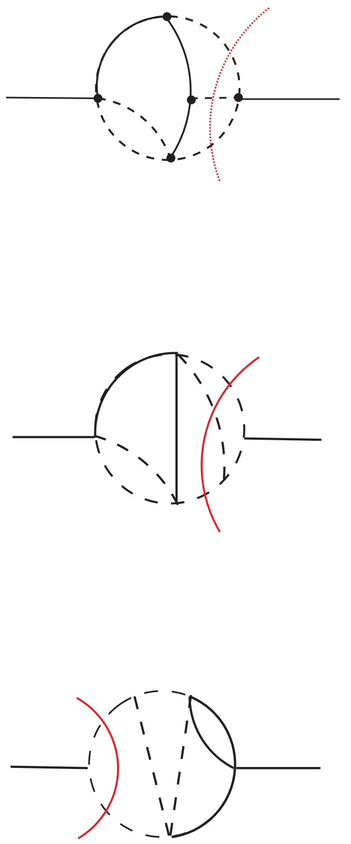
$$\texttt{PR214(1,1,1,1,1,1,1,1,0,0,0,0,0,0)} = I * \pi * ($$
$$\mathbf{z} * (3.7500000000000000 + 0.5/\varepsilon + 16.4800659331517735 * \varepsilon$$
$$+(-1. - 7.5000000000000000 * \varepsilon) * Log[z] + (1. * \varepsilon) * Log[z]^2 \;\;)$$

DEQs: deep expansions

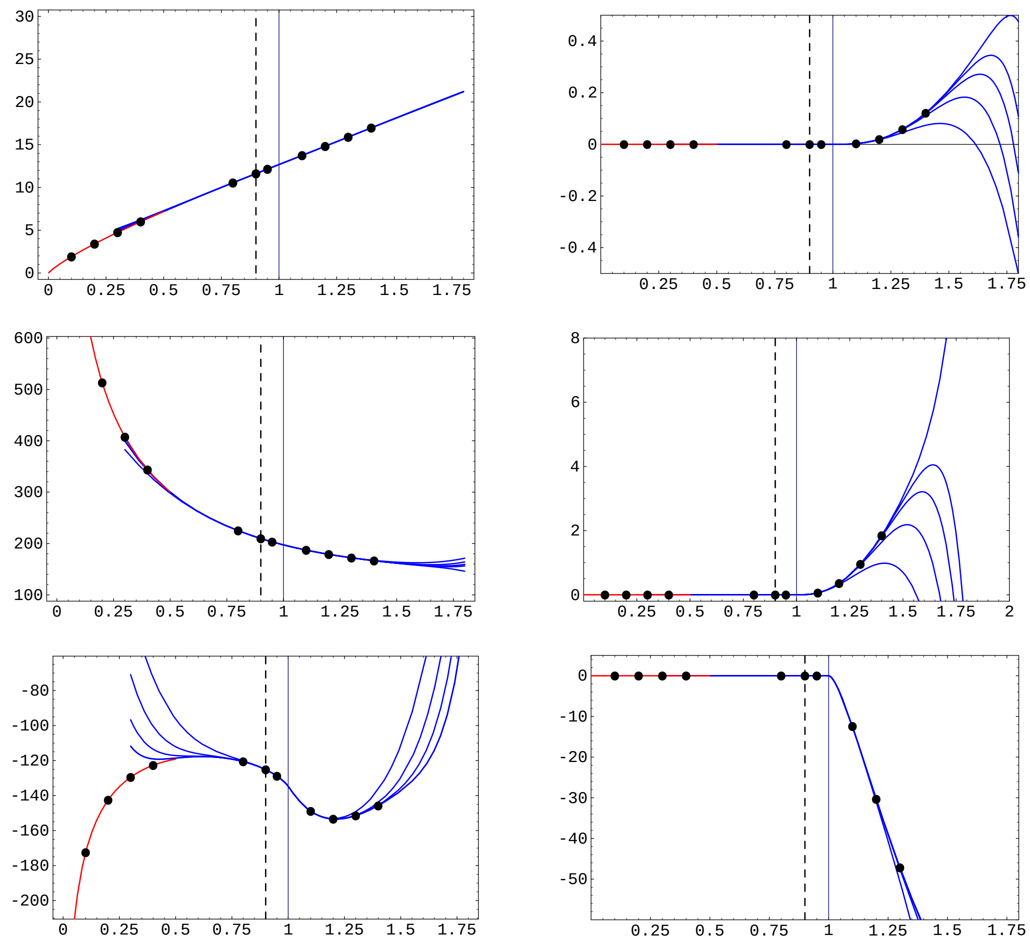
$$
\begin{aligned}
&\texttt{PR214(1,1,1,1,1,1,1,1,0,0,0,0,0,0)} = I * \pi * (\\
&\mathbf{z} * (3.750000000000000 + 0.5/\varepsilon + 16.48006593315177 35 * \varepsilon \\
&+(-1. - 7.5000000000000000 * \varepsilon) * Log[z] + (1. * \varepsilon) * Log[z]^2) + \\
&\mathbf{z^2} * (0.2361111111111111 + 0.01388888888888888/\varepsilon + 1.707779609254215 9 * \varepsilon \\
&+(-0.0277777777777777 - 0.4722222222222222 * \varepsilon) * Log[z] \\
&+(0.0277777777777777 * \varepsilon) * Log[z]^2) + \\
&\mathbf{z^3} * (0.03418209876543209 + 0.0009259259259259 26/\varepsilon + 0.347796161193079416 * \varepsilon \\
&+(-0.001851851851851852 - 0.06836419753086419 7 * \varepsilon) * Log[z] \\
&+(0.001851851851851852 * \varepsilon) * Log[z]^2) + \\
&\mathbf{z^4} * (0.008763888888888888 + 0.0000892857142857142 8/\varepsilon + 0.1108357508610868 9 * \varepsilon \\
&+(-0.0001785714285714285 7 - 0.0175277777777777 77 * \varepsilon) * Log[z] \\
&+(0.0001785714285714285 7 * \varepsilon) * Log[z]^2) + \\
&\mathbf{z^5} * (0.003259523809523809 5 + 0.0000105820105820582/\varepsilon + 0.0467270178563159 7 * \varepsilon \\
&+(-0.0000211640211640211 - 0.0065190476190476190 * \varepsilon) * Log[z] \\
&+(0.0000211640211640211 * \varepsilon) * Log[z]^2) + \cdots + \mathcal{O}(z^{19}))
\end{aligned}
$$

Some Results for 2- and 3-particle cuts

Preliminary results: sample masters with 2- and 3-particle cuts



Im | Re

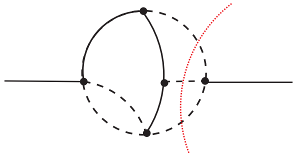


$x = p_b^2/m_b^2$ | $x = p_b^2/m_b^2$

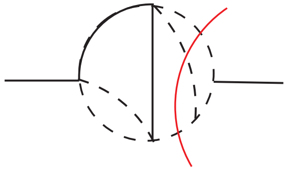
- Expansions:
  - $x \rightarrow 0$: up to x^{18}
  - $x \rightarrow 1$: up to $(1-x)^{12}$
- Numerical integration: starts at $x_0 = 0.02$
- Matching: done at $x_1 = 0.9$

Some Results for 2- and 3-particle cuts

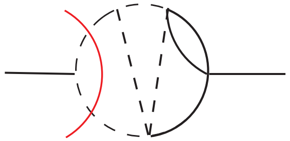
- Preliminary results at $x = 1$: sample masters with 2- and 3-particle cuts



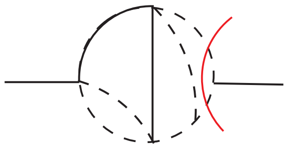
$$= \frac{1.4514\, i}{\varepsilon} + 11.6173\, i + \mathcal{O}\left(\varepsilon\right)$$



$$= \frac{3.14159\, i}{\varepsilon^3} + \frac{20.0142\, i}{\varepsilon^2} + \frac{77.1378\, i}{\varepsilon} + 209.713\, i + \mathcal{O}\left(\varepsilon\right)$$



$$= \frac{-2.0944\, i}{\varepsilon^3} - \frac{12.5778\, i}{\varepsilon^2} - \frac{35.6402\, i}{\varepsilon} - 125.153\, i + \mathcal{O}\left(\varepsilon\right)$$



$$= \frac{-2.0944\, i}{\varepsilon^3} - \frac{4.91208\, i}{\varepsilon^2} - \frac{30.5699\, i}{\varepsilon} - 40.7068\, i + \mathcal{O}\left(\varepsilon\right)$$

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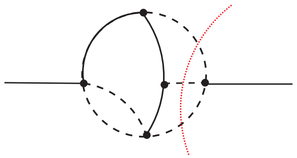
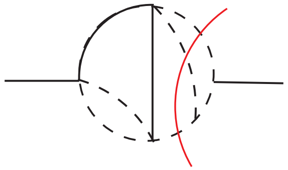
- Preliminary results at $x = 1$: sample masters with 2- and 3-particle cuts



$$= \frac{1.4514\, i}{\varepsilon} + 11.6173\, i + \mathcal{O}\left(\varepsilon\right)$$



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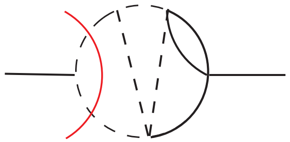
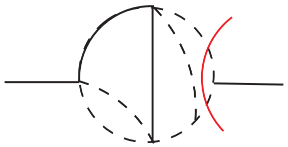


$$= \frac{-2.0944\, i}{\varepsilon^3} - \frac{12.5778\, i}{\varepsilon^2} - \frac{35.6402\, i}{\varepsilon} - 125.153\, i + \mathcal{O}\left(\varepsilon\right)$$



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masters with 2-particle cuts are obtained with two independent calculations
$\longrightarrow$ cross checks will be done soon

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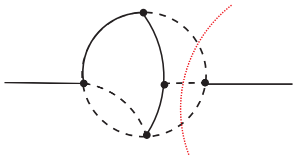

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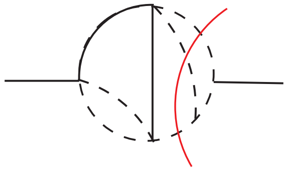

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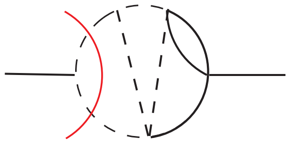

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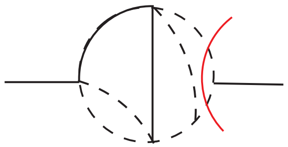

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masters with 2-particle cuts are obtained with two independent calculations
$\longrightarrow$ cross checks will be done soon

- what we have:
  - masters with massless internal lines:
    all 2- and 3-particle cuts
    all 4-particle cuts but one
  - masters with b-quark internal lines:
    2- and 3-particle cuts are almost there

- still to be calculated: masters with 4-particle cuts and internal b-lines

Summary

- Matching current and future experimental precision for $\bar{\boldsymbol{B}} \longrightarrow \boldsymbol{X_s\gamma}$ decay necessitates NNLO corrections on the theory side
  crucial missing piece: $O(\alpha_s^2)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$

- Reducing the interpolation uncertainty: needs $O(\alpha_s^2)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$ at $m_c = 0$
  $\rightarrow 70\%$ of the project is completed

- Removing the interpolation uncertainty: needs $O(\alpha_s^2)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$ at physical m_c
  $\longrightarrow$ completed the fermionic contribution

  $\rightarrow$ massless case: calculated in two ways and confirmed the findings of [Bieri, Greub, Steinhauser 03]

  $\rightarrow$ massive case: impact on the branching ratio $+1.1\%$ for $\mu_b = 2.5$GeV

  $\longrightarrow$ bosonic contribution: work in progress

Summary

- From Misiak's talk at the Flavour Dynamics workshop in Albufeira Portugal, 6th November 2007

Currently known contributions $\mathcal{B}(\bar{B} \to X_s\gamma)$ that have not been included in the estimate $\underset{(\pm 7.3\%)}{(3.15 \pm 0.23)} \times 10^{-4}$ in hep-ph/0609232:

• New/old large-β_0 bremsstrahlung effects [Ligeti, Luke, Manohar, Wise, 1999] [Ferroglia, Haish, 2007, to be published]	$\Rightarrow$	$+2.0\%$ in the BR
• Four-loop mixing into the $b \longrightarrow sg$ operator Q_8 [Czakon, Haisch, MM, hep-ph/0612329]	$\Rightarrow$	-0.3% in the BR
• Charm mass effects in loops on gluon lines in K_{77} [Asatrian, Ewerth, Gabrielyan, Greub, hep-ph/0611123] [Czarnecki, Pak, to be published]	$\Rightarrow$	$+0.3\%$ in the BR
• Charm and bottom mass effects in loops on gluon lines in the three-loop $b \longrightarrow s\gamma$ matrix elements of Q_1 and Q_2 [Boughezal, Czakon, Schutzmeier, arXiv:0707.3090]	$\Rightarrow$	$+1.1\%$ in the BR
• Non-perturbative $\mathcal{O}\left(\alpha_s \frac{\Lambda}{m_b}\right)$ effects in the term $\sim C_7C_8$ [Lee, Neubert, Paz, hep-ph/0609224]	$\Rightarrow$	-1.5% in the BR
	Total:	$+1.6\%$ in the BR

- cancellation between the shifts from the different contributions
- next update of the prediction of $\mathcal{B}(\bar{B} \to X_s\gamma)$ will be provided once the complete $\boldsymbol{\mathcal{O}}\left(\boldsymbol{\alpha_s^2}\right)$ correction to $\langle s\gamma|O_{1,2}|b\rangle$ is finished