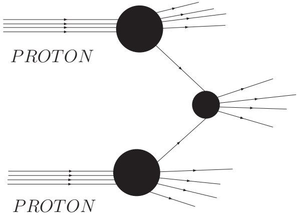
Application of unitarity-cut methods: the six-photon amplitudes

BERNICOT Christophe

based on JHEP **01** (2008) 059

PSI - Villigen

INTRODUCTION - MOTIVATIONS



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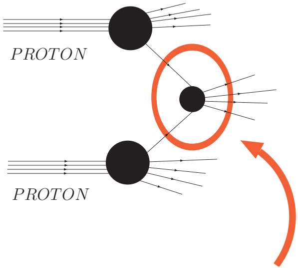

$$\textbf{PP} \longrightarrow 4 \text{ jets} = \begin{cases} 2q + 4g \longrightarrow 0 \\ 4q + 2g \longrightarrow 0 \\ 6q \longrightarrow 0 \\ 6g \longrightarrow 0 \end{cases}$$

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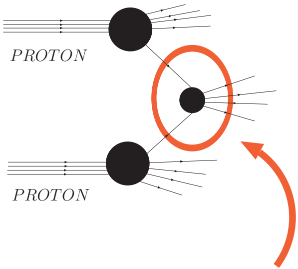
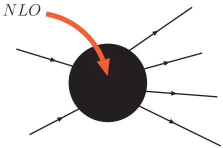


$$\mathbf{PP} \longrightarrow 4 \text{ jets} = \begin{cases} 2q + 4g \longrightarrow 0 \\ 4q + 2g \longrightarrow 0 \\ 6q \longrightarrow 0 \\ 6g \longrightarrow 0 \end{cases}$$

Computation of the sub-process:

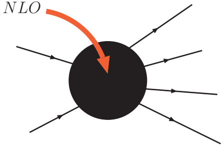
$$\gamma\gamma\gamma\gamma\gamma\gamma \longrightarrow 0$$

Difficulties of NLO calculation



to have a good prediction

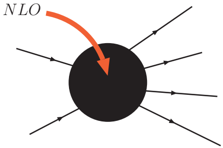
Difficulties of NLO calculation



to have a good prediction

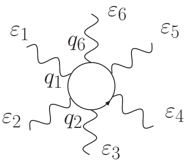
NLO calculation: 2 difficulties

Difficulties of NLO calculation



to have a good prediction

NLO calculation: 2 difficulties



$$tr\left(\varepsilon_1\gamma_{\mu_1}...\varepsilon_6\gamma_{\mu_6}\right)\int d^n q \frac{q_1^{\mu_1}...q_6^{\mu_6}}{(q_1^2+i\lambda)...(q_6^2+i\lambda)}$$

• no compact result.
⇒ How express polarisations vectors ε ???

• Classical reduction methods give $\sim$ 500000 scalars integrals.
⇒ FIND AN OTHER WAY !!!!

.... THE SOLUTIONS ....

COMPACT NOTATION

⋆ helicity amplitudes methods.

$$\text{Amplitude} = \sum_{i \in \text{helicity states of external photons}} \text{Amplitude}_i$$

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COMPACT NOTATION

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$$\text{Amplitude} = \sum_{i \in \text{helicity states of external photons}} \text{Amplitude}_i$$

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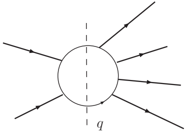
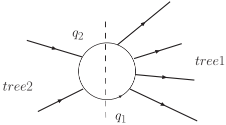
⋆ Polarisations vectors of photons:

$$\varepsilon_+^\mu(p) = \frac{\langle R - |\gamma^\mu|p-\rangle}{\langle R - |p+\rangle} \quad \varepsilon_-^\mu(p) = \frac{\langle R + |\gamma^\mu|p+\rangle}{\langle p + |R-\rangle}$$

$|R\rangle$ reference light-like vector.

UNITARITY OF A PROCESS

(Cutkosky,Bern [arXiv:hep-ph/9403226], Britto [arXiv:hep-th/0412103])

- $2\mathrm{Im}\left(\right.$  $\left. \right) = \int d^n q \, \delta\left(q_1^2\right) \delta\left(q_2^2\right)$ 

$$= \int d^n q \ \mathrm{tree1} \ \frac{i}{q_1^2 + i\lambda} \ \mathrm{tree2} \ \frac{i}{q_2^2 + i\lambda}$$

tree1,2 are on-shell.

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-

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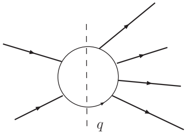

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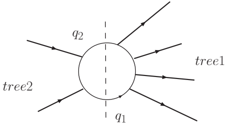

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- Calculation of the real part thanks to a dispersive relation

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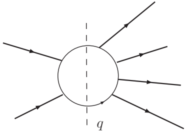
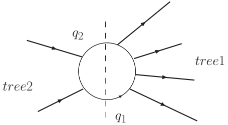
- $2\mathrm{Im}\Bigg($ 

$\Bigg) = \int d^n q \; \delta\left(q_1^2\right) \delta\left(q_2^2\right)$ 

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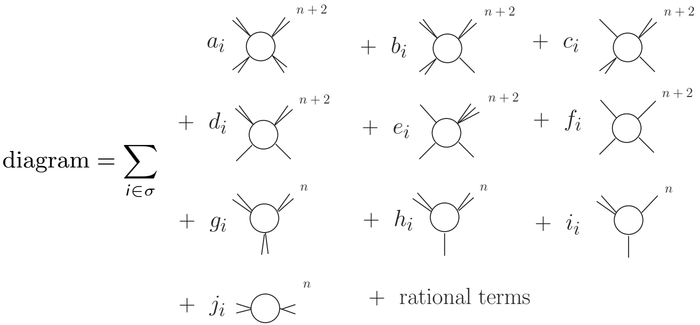
- Calculation of the real part thanks to a dispersive relation

- IMPROVEMENTS :
  - 3 or 4 cuts.
  - cuts in n dimensions.

NOT NEED RECONSTRUCTION!!!

Reduction of tensor integral

An amplitude: linear combination of scalar integrals.



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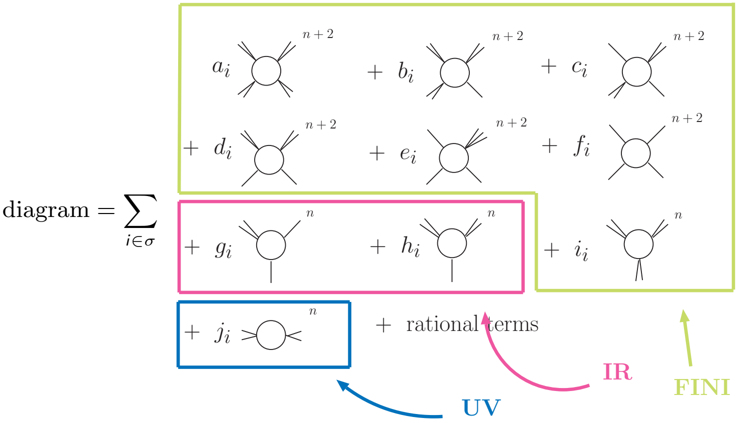

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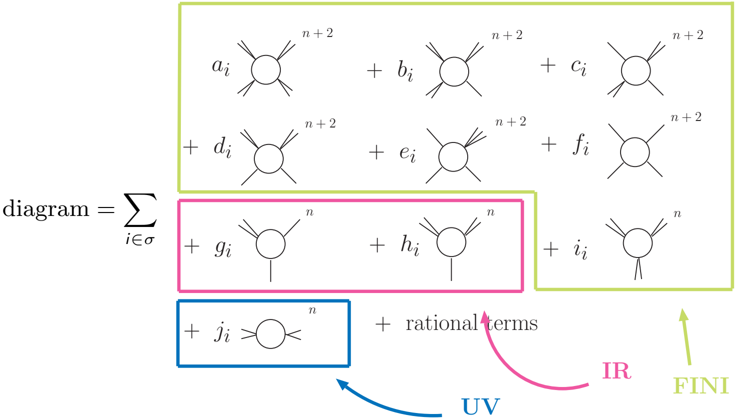
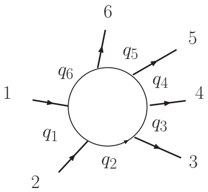


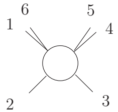
Only 10 coefficients to calculate. HOW ?????

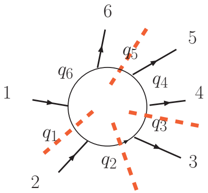
Computation of a box coefficient

exemple:



$$\int d^n q \frac{Num}{(q_1^2 + i\lambda)\,(q_2^2 + i\lambda)\ldots(q_6^2 + i\lambda)}$$

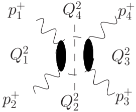
The coefficient α in front of: $\left(\right.$  $\left. \right)$ 4 cuts.



$$\alpha = \lim_{q_1^2, q_2^2, q_3^2 q_5^2 \to 0} \frac{Num}{(q_4^2 + i\lambda)\,(q_6^2 + i\lambda)}$$

Computation of the "++++" four-photon helicity amplitude in scalar *QED*

$$\text{Im } A^s(++++) = \sum_{\sigma(2,3,4)}$$



$$= \sum_{\sigma(2,3,4)} \int d^n Q \; \delta(Q_2^2)\delta(Q_4^2) A_{tree1} A_{tree2}$$

The two trees are on-shell

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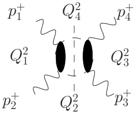

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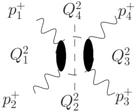

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$$Q = q + \mu \qquad A_{tree1} = -\mu^2 \frac{[12]}{\langle 12 \rangle} \sum_{1,2} \frac{1}{Q_1^2 + i\lambda}$$

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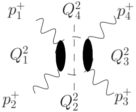

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$$\text{Im } A^s(++++) = \sum_{\sigma(2,3,4)} \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle} \int d^n Q \; \delta(Q_2^2)\delta(Q_4^2) \frac{\mu^2}{D_1^2} \frac{\mu^2}{D_3^3}$$

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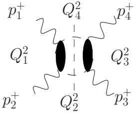

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$$\Rightarrow A^s(++++) = -\frac{1}{6} \sum_{\sigma(2,3,4)} \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle}$$

.... SIX-PHOTON AMPLITUDES ....

Six-photon amplitude in the past

Computation of the six-photon amplitude in *QED* theory:

- analytically/ G.Mahlon [arXiv:hep-ph/9311213]
  standard reduction

SIX-PHOTON AMPLITUDE IN THE PAST

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  Unitarity-cuts methods / Classical standard reduction
- numerically: G.Ossola, C.G.Papadopoulos,R.Pittau [arXiv:0704.1271[hep-ph]]
  Unitarity-cuts methods

Computation of the six-photon amplitudes

- $\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 \to 0$ C.Bernicot, J.Ph.Guillet [arXiv:hep-ph/0711.4713]

  Computation of the six-photon amplitude in three theories :

$$3 \text{ QED} : \begin{cases} \text{QED spinorielle} \\ \text{QED scalar} \\ \text{QED}^{\mathcal{N}=1} \end{cases}$$

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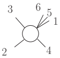
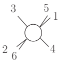
- 4 helicities amplitudes

$$A(++++++) = A(-+++++) = 0$$
$$A(--++++), \quad A(---+++)$$

Results for A_{--++++}

$$A_{--++++}^{scalar/spinor/\mathcal{N}=1} = \frac{i\ e^6}{2\pi^2} \sum_{\sigma(1,2)} \sum_{\sigma(3..6)} R^{scalar/spinor/\mathcal{N}=1} \left(\left[\text{diagram 1}\right]^{n+2} - \left[\text{diagram 2}\right]^{n+2} \right)$$





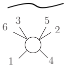
$$R^{scalar} = \frac{\langle 1341\rangle\langle 2342\rangle}{\langle 35\rangle\langle 45\rangle\langle 36\rangle\langle 46\rangle s_{34}}$$

$$R^{spinor} = -2\frac{\langle 1342\rangle^2}{\langle 35\rangle\langle 45\rangle\langle 36\rangle\langle 46\rangle s_{34}}$$

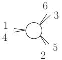
$$R^{\mathcal{N}=1} = -\frac{s_{34}\langle 12\rangle^2}{\langle 35\rangle\langle 45\rangle\langle 36\rangle\langle 46\rangle}$$

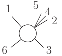
$$\langle ABCD\rangle = \langle p_A - |p_B+\rangle\langle p_B + |p_C-\rangle\langle p_C - |p_D+\rangle$$

Results for $A^{scalar/fermion}_{---+++}$

$$A^{scalar/spinor}_{---+++} = \frac{i\ e^6}{\pi^2} \sum_{\sigma(1,2,3)} \sum_{\sigma(4,5,6)} \Bigg(d^{scalar/spinor} \Bigg($$


$$\Bigg)^{n+2}$$

$$+ \frac{g^{scalar/spinor}}{12} \Bigg($$


$$\Bigg)^{n} + \frac{e^{scalar/spinor}}{4} \Bigg($$


$$\Bigg)^{n+2} + h.c. \Bigg)$$

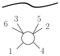
$$d^{scalar} = -\frac{\langle 24\rangle[16]}{\langle 45\rangle[31]} \frac{[1P_{425}2][6P_{425}4]}{[1P_{425}5][3P_{425}4]} \frac{\langle 1P_{425}4\rangle}{[1P_{425}4]}$$

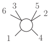
$$e^{scalar} = -\frac{\langle 2P_{425}1\rangle\langle 2P_{425}3\rangle[36][16]s_{425}}{\langle 4P_{425}1\rangle\langle 5P_{425}3\rangle\langle 5P_{425}1\rangle\langle 4P_{425}3\rangle} \frac{\langle 31\rangle}{[31]}$$

$$g^{scalar} = \frac{[4P_{25}1]}{[1P_{25}4]} \frac{[5P_{14}2]}{[2P_{14}5]} \frac{[6P_{25}3]}{[3P_{25}6]} \sum_{\gamma\pm} \frac{[1K_2^b1]}{[4K_2^b4]} \frac{[2K_2^b2]}{[5K_2^b5]} \frac{[3K_2^b3]}{[6K_2^b6]}$$

$$K_2^{b\,\mu} = \gamma_{\pm}\,(-P_{25})^{\mu} - s_{25}\,(P_{14})^{\mu}$$

$$\gamma_{\pm} = -P_{25}.P_{14} \pm \sqrt{\Delta} \qquad \Delta = (P_{25}.P_{14})^2 - P_{14}^2 P_{25}^2$$

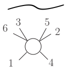


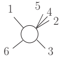
$$^{n+2} = \text{“Finite part”} \Bigg($$


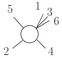
$$\Bigg)^{n}$$

Results for $A^{\mathcal{N}=1}_{---+++}$

$$A^{\mathcal{N}=1}_{---+++} = \frac{i\ e^6}{\pi^2} \sum_{\sigma(1,2,3)} \sum_{\sigma(4,5,6)} d^{\mathcal{N}=1}$$

$$\Bigg(\langle 1 P_{425} 4 P_{425} 1 \rangle \Big($$


$$\Big)^{n+2} + \frac{s_{13} \Big(\text{[diagram 2]} \Big)^{n+2} + s_{45} \Big(\text{[diagram 3]} \Big)^{n+2}}{2} \Bigg)$$


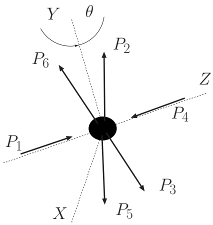


$$d^{\mathcal{N}=1} = \frac{[6 P_{425} 2]^2}{[31]\langle 45 \rangle [1 P_{425} 5][3 P_{425} 4]}$$

NO TRIANGLE !!!!!! probably just an accident

… a Plot of those amplitudes …

- Nagy-Soper kinematical configuration



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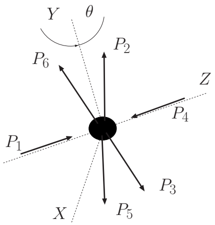

-

$$\begin{cases} \overrightarrow{p_2} = (-33.5, -15.9, -25.0) \\ \overrightarrow{p_3} = (11.0, 13.2, 22.0) \\ \overrightarrow{p_5} = (12.5, -15.3, -0.3) \\ \overrightarrow{p_6} = (10.0, 18.0, 3.3) \end{cases}$$

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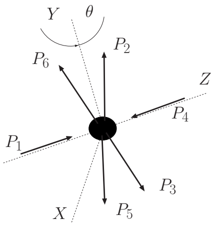

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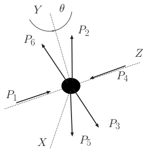
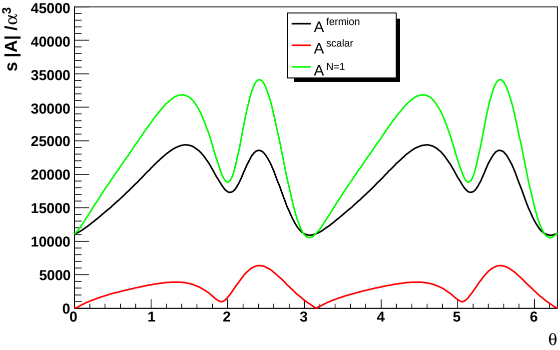
-

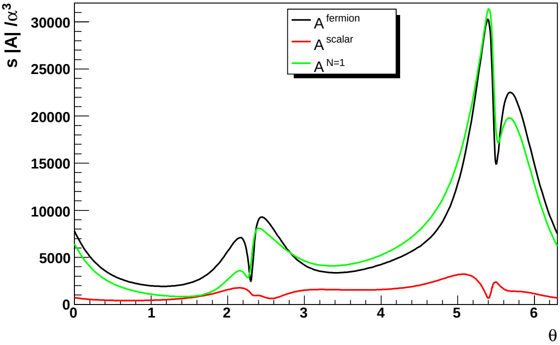
rotation of the final state around the Y-axis

The MHV amplitude

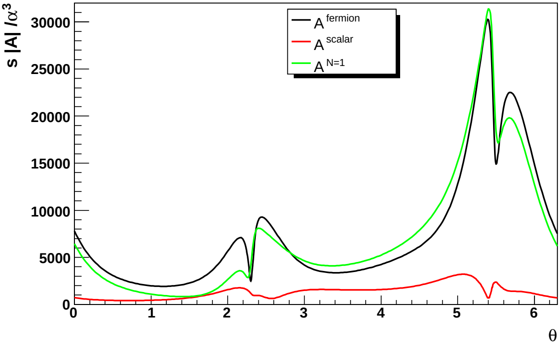




The NMHV amplitude

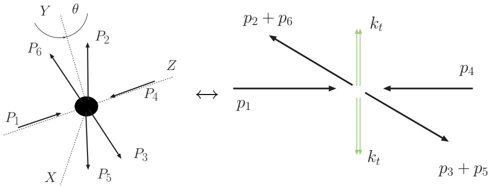


The NMHV amplitude



What are those dips ????????

... What is it ???



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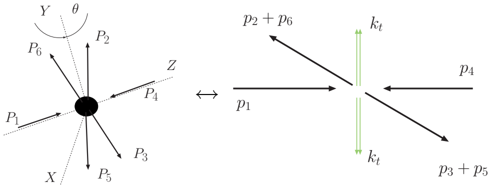

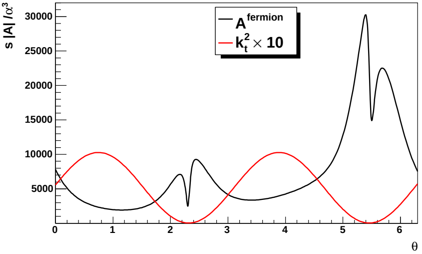

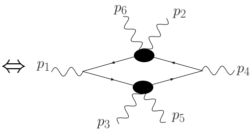

DOUBLE PARTONS SCATTERING (reach for $k_t = 0$)

Approach of this singularity

Change Nagy-Soper configuration to reach the singularity :

$$\begin{cases} \overrightarrow{p_2} = (-33.5, -15.9 - \Delta^y, -25.0) \\ \overrightarrow{p_3} = (11.0, 13.2 + \Delta^y, 22.0) \\ \overrightarrow{p_5} = (12.5, -15.3 + \Delta^y, -0.3) \\ \overrightarrow{p_6} = (10.0, 18.0 - \Delta^y, 3.3) \end{cases}$$

Singularity reach for $\Delta^y = 1,05$

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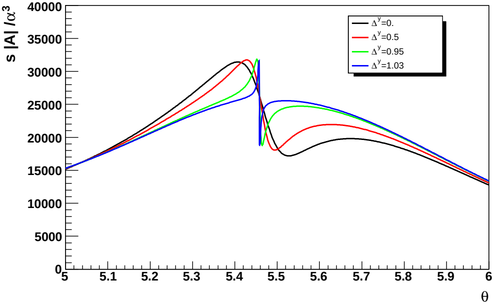
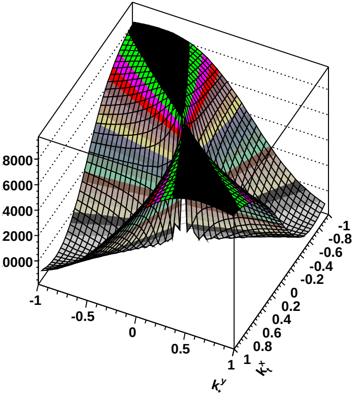
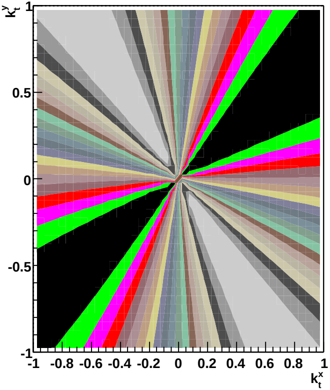


IS IT A DIVERGENCE OR NOT???

... Is it a divergence ???

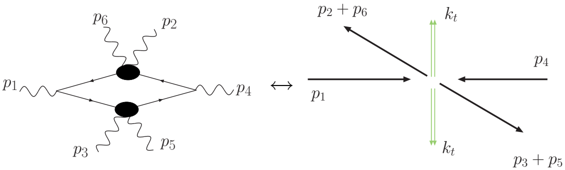
"Double parton scattering" reach for $k_t = 0$.
Amplitude around this singularity





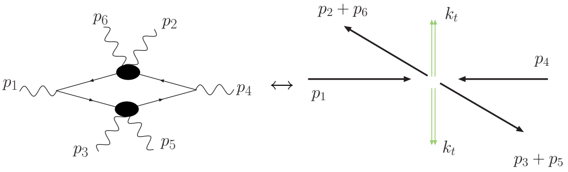
NO DIVERGENCE

Where is the divergences



$$\Leftrightarrow \begin{cases} \langle 1 P_{435} 4 \rangle \propto |k_t| \\ \langle 4 K_2^b 4 \rangle \propto |k_t|^2 \end{cases}$$

Where is the divergences



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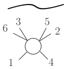
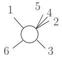
$$A^{\mathcal{N}=1}_{---+++} = \frac{i\ e^6}{\pi^2} \sum_{\sigma(1,2,3)} \sum_{\sigma(4,5,6)} d^{\mathcal{N}=1}$$

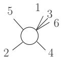
$$\Bigg(\langle 1 P_{425} 4 P_{425} 1 \rangle \Big($$



$$\Big)^{n+2} + \frac{s_{13} \Big($$



$$\Big)^{n+2} + s_{45} \Big($$



$$\Big)^{n+2}}{2} \Bigg)$$

$$d^{\mathcal{N}=1} = \frac{[6 P_{425} 2]^2}{[31]\langle 54 \rangle [1 P_{425} 4][3 P_{425} 5]}$$

.... WITH A MASSIVE LOOP ....

The four-photon amplitudes

- Extend to QCD. "t" has a significant mass.

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- 4 photons amplitudes in three theories

$$\text{QED}, \text{scalar QED}, \text{QED}^{\mathcal{N}=1}$$

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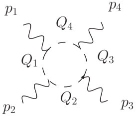
- Three helicities amplitudes :

$$A(++++), A(-+++), A(--++)$$

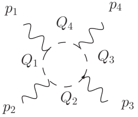
The origin of the "rational terms"



$$\Rightarrow \int d^n Q \frac{\varepsilon_1.Q_1 \ \varepsilon_2.Q_2 \ \varepsilon_3.Q_3 \ \varepsilon_4.Q_4}{(Q_1^2 - m^2 + i\lambda)\ldots(Q_4^2 - m^2 + i\lambda)}$$

Q n-dimensional momentum : $\quad Q = q + \mu$

The origin of the "rational terms"



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four-dimensional helicity scheme

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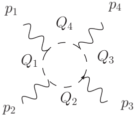

$$\Rightarrow \int d^n Q \frac{\varepsilon_1.Q_1\ \varepsilon_2.Q_2\ \varepsilon_3.Q_3\ \varepsilon_4.Q_4}{(Q_1^2 - m^2 + i\lambda)\ldots(Q_4^2 - m^2 + i\lambda)}$$

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reduce denominator

$$q_1^2 \to Q_1^2 - m^2 + (m^2 + \mu^2)$$

- Introduce "extra-integrals"

$$K_4^n = \int d^n Q \frac{\left(m^2+\mu^2\right)^2}{(Q_1^2+i\lambda)\ldots(Q_4^2+i\lambda)} = -\frac{1}{6} + O(m^2)$$

$$J_4^n = \int d^n Q \frac{m^2+\mu^2}{(Q_1^2+i\lambda)\ldots(Q_4^2+i\lambda)} = 0 + O(m^2)$$

$$J_3^n = \int d^n Q \frac{m^2+\mu^2}{(Q_1^2+i\lambda)\ldots(Q_3^2+i\lambda)} = -\frac{1}{2} + O(m^2)$$

Massless theory, those extra-integrals create rational terms.

In all order of ϵ.

$$A_4^{scalar}{}_{(++++)} = i\,\frac{4e^4}{(4\pi)^{n/2}} \sum_{\sigma(2,3,4)} \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} K_4^n$$

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In all order of ϵ.

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$$A_4^{scalar}{}_{(--++)} = i\frac{4e^4}{(4\pi)^{n/2}} \frac{\langle 12\rangle}{[12]} \frac{[34]}{\langle 34\rangle} \left(-\frac{2tu}{s} I_4^{n+2} + \sum_{\sigma(1,2)} \left(\frac{t-u}{s} I_2^n + 4\frac{u}{s} J_3^n \right) + \sum_{\sigma(2,3,4)} K_4^n \right)$$

Conclusion

$\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4 + \gamma_5 + \gamma_6 \rightarrow 0$

$A^{scalar}, A^{fermion}, A^{\mathcal{N}=1}$

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Efficient methods to calculate one massless loop:

- helicity amplitude
- unitarity

Conclusion

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Efficient methods to calculate one massless loop:

- helicity amplitude
- unitarity

Future prospect ....

-Improve the methods to a massive loop

-6 photons massive in a loop

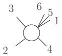
-6 massive bosons

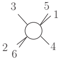
- Process with an infrared divergence.

- $q\overline{q} + 4g \to 0$

Gram Determinant

$$A^{scalar/spinor/\mathcal{N}=1}_{--++++} = \frac{i\ e^6}{2\pi^2} \sum_{\sigma(1,2)} \sum_{\sigma(3..6)} R^{scalar/spinor/\mathcal{N}=1} \Bigg(\left[\text{diagram 1}\right]^{n+2} - \left[\text{diagram 2}\right]^{n+2} \Bigg)$$





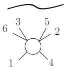
$$R^{scalar} = \frac{\langle 1341\rangle \langle 2342\rangle}{\langle 35\rangle \langle 45\rangle \langle 36\rangle \langle 46\rangle s_{34}}$$

$$R^{spinor} = -2 \frac{\langle 1342\rangle^2}{\langle 35\rangle \langle 45\rangle \langle 36\rangle \langle 46\rangle s_{34}}$$

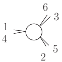
$$R^{\mathcal{N}=1} = - \frac{s_{34} \langle 12\rangle^2}{\langle 35\rangle \langle 45\rangle \langle 36\rangle \langle 46\rangle}$$

Gram determinant

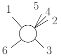
$$A^{scalar/spinor}_{---+++} = \frac{i\ e^6}{\pi^2} \sum_{\sigma(1,2,3)} \sum_{\sigma(4,5,6)} \Bigg(d^{scalar/spinor} \Bigg($$



$$\Bigg) + \frac{g^{scalar/spinor}}{12} \Bigg($$



$$\Bigg) + \frac{e^{scalar/spinor}}{4} \Bigg($$



$$\Bigg) + h.c. \Bigg)$$

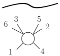
$$d^{scalar} = -\frac{\langle 24\rangle[16]}{\langle 45\rangle[31]} \frac{[1P_{425}2][6P_{425}4]}{[1P_{425}5][3P_{425}4]} \frac{\langle 1P_{425}4\rangle}{[1P_{425}4]}$$

$$e^{scalar} = -\frac{\langle 2P_{425}1\rangle\langle 2P_{425}3\rangle[36][16]s_{425}}{\langle 4P_{425}1\rangle\langle 5P_{425}3\rangle\langle 5P_{425}1\rangle\langle 4P_{425}3\rangle} \frac{\langle 31\rangle}{[31]}$$

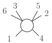
$$g^{scalar} = \frac{[4P_{25}1]}{[1P_{25}4]} \frac{[5P_{14}2]}{[2P_{14}5]} \frac{[6P_{25}3]}{[3P_{25}6]} \sum_{\gamma_\pm} \frac{[1K_2^b1]}{[4K_2^b4]} \frac{[2K_2^b2]}{[5K_2^b5]} \frac{[3K_2^b3]}{[6K_2^b6]}$$

$$K_2^{b\,\mu} = \gamma_\pm \left(-P_{25}\right)^\mu - s_{25}\left(P_{14}\right)^\mu$$

$$\gamma_\pm = -P_{25}.P_{14} \pm \sqrt{\Delta} \qquad \Delta = (P_{25}.P_{14})^2 - P_{14}^2 P_{25}^2$$



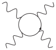
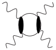
$$= \text{“Finite part”} \Bigg($$



$$\Bigg)$$

Use unitarity

- Gathering diagrams according to the branch cut, to make on-shell trees :

 $+ \ldots. \rightarrow$ 

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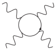
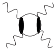
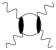
 $+ \ldots. \rightarrow$ 

- Set the decomposition:

 $= \sum_i a_i \vartheta_i$

Use unitarity

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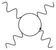
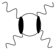
 $+ \ldots. \rightarrow$ 

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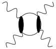
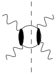
 $= \sum_i a_i \vartheta_i$

- Cut and integrate:

 $\Big|_s = \sum_i a_i \vartheta_i|_s$

With identification we obtain the coefficients a_i.